HMMT November 2020 Integration Bee Finals

Sponsored by Five Rings Capital

November 14, 2020

A Message from our Sponsor, Five Rings Capital

Our Contestants (in alphabetical order)

- Adithya Balachandran

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- Harry Chen

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- Jack Moffatt

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- Harry Chen
- Jack Moffatt
- Jordan Hochman

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  - If 0 or 4 people get the integral - no points for anyone
  - If 1 person gets the integral, then +3 for them, -1 for others
  - If 2 people get the integral, then +2 for them, and -2 for others
  - If 3 people get the integral, then +1 for them, and -3 for the one person that did not.

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- If there is a tie at the end, we will have a tie-breaking integral estimation question.

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- For the spectators - feel free to follow along and try these integrals as well!

Any questions before we start?

Problem 1

Evaluate the following Integral (in terms of a):

$$\int_0^1 \lfloor \log_a(x) \rfloor \, dx$$

Solution 1

$$\frac{a}{1-a}$$

Problem 2

Evaluate the following Integral:

$$\int_0^1 \sqrt{x+\sqrt{x}}\, dx$$

Solution 2

$$\frac{7}{12}\sqrt{2} + \sinh^{-1}(1)/4 = \frac{7}{12}\sqrt{2} + \frac{\log(1+\sqrt{2})}{4}$$

Problem 3

Evaluate the following Integral:

$$\int_0^{\pi/2} \frac{\log(2\sin^2 x)}{\log(\cot(x))} dx$$

Solution 3

$$-\frac{\pi}{2}$$

Problem 4

Evaluate the following Integral:

$$\lim_{n\to\infty} \int_{[0,1]^n} x_1 + x_1 x_2^2 + x_1 x_2^2 x_3^3 + \cdots + x_1 x_2^2 \cdots x_n^n \, dx^n$$

This notation means that n integrals are taken over the variables $x_1, x_2, \ldots, x_n \in [0, 1]$.
For example, the integral for $n = 2$ is $\int_0^1 \int_0^1 x_1 + x_1 x_2^2 \, dx_1 \, dx_2$.

Solution 4

$$e - 2$$

Problem 5

Evaluate the following Integral:

$$\int \frac{e^{2x}}{(x^2-1)^2} \cdot \frac{2x^2-3x-1}{x+1}\, dx$$

Solution 5

$$\frac{e^{2x}}{(x-1)(x+1)^2} + C$$

Problem 6

Evaluate the following Integral:

$$\int_0^{\pi/2} \sqrt[2020]{\tan(x)}\, dx$$

Solution 6

$$\frac{\pi}{2}\sec\left(\frac{\pi}{4040}\right)$$

Problem 7

Define $f_n(x) = \frac{\pi}{2}\sin(f_{n-1}(x))$ and $f_0(x) = x$. Evaluate

$$\int_0^{\pi} \lim_{n\to\infty} f_n(x)\,dx$$

Solution 7

$$\frac{\pi^2}{2}$$

Problem 8

Evaluate the following Integral:

$$\int_0^{\pi/2} \frac{dx}{\sin x + \cos x + 1}$$

Solution 8

$$\log 2$$

Problem 9

Evaluate the following Integral:

$$\int_0^1 \left(\frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor\right) x \, dx$$

Solution 9

$$1 - \frac{\pi^2}{12}$$

Problem 10

Evaluate the following Integral:

$$\int_0^{\pi} \left(\frac{\pi}{2} - x\right) \tan x \, dx$$

Solution 10

$$\pi \log 2$$

Problem 11

Estimate the following Integral:

$$\int_0^1 \pi^{x-x^\pi} \, dx$$

Solution 11

$$\approx 1.35717608899$$