A Quasi-Convex Optimization based Model Order Reduction Code

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Itemized Explanation of Input/Output arguments

```
% function [Gr,y,xa,xb,xc,pp,qq,output] = run_mor(G,t,m,W,r)
%
% Use relaxation scheme to solve
% min max_i |W(i)*(G(i) - Gr(t(i))| subject to stability of Gr
%
% master file for running ellip_rpn_mor.m
%
% Inputs:
% G --- a vector of frequency response samples of the original system
% t --- a vector of frequency samples corresponding to G
% m --- order of the reduced system
% W --- a vector of frequency weight (of length t) (default = ones(length(t),1)
% r --- desired value of relative tolerance (default = 0.9)
%
% Outputs:
% Gr --- discrete-time transfer function of the reduced model Gr(z) = pp(z)/qq(z)
% y --- optimal lower bound of H-inf norm error of |Gr(z)-G(z)|
% xa --- denominator of optimal relaxation a(t) = 1+xa(1)*cos(t)+xa(2)*cos(2*t)+...
% xb --- real part of numerator of optimal relaxation b(t) = xb(1)+xb(2)*cos(t)+xb(3)*cos(2*t)+...
% xc --- imaginary part of numerator of optimal relaxation c(t) = xc(1)*sin(t)+xc(2)*sin(2*t)+...
% pp --- numerator of Gr(z)
% qq --- denominator of Gr(z)
% output --- miscellaneous algorithmic output
```

Input arguments

- G : a vector of length N storing frequency response of full model as in (1).
- t : a vector of length N storing frequency samples at which G is sampled. See (1).
- m : an nonnegative integer specifying the order of reduced model.
- W : (optional) a vector of length N. Weights on objective functions in (3) and (7).
- r : (optional) $r \in (0, 1)$. An algorithmic parameter specifying the accuracy of the optimization. The result is more accurate when r is small.

Output arguments

- Gr : discrete-time transfer function object of the reduced model.
- y : optimal lower bound of $||G(z) - G_r(z)||_\infty$ That is, optimal objective value of (7) multiplied by r.
- xa : a length m vector of the coefficients of trigonometric polynomial $a(t)$ in (4). xa[1] $= a_1$, xa[2] $= a_2, \ldots$.

- xb : a length $m+1$ vector of the coefficients of trigonometric polynomial $b(t)$ in (5). xb[1] $= b_0$, xb[2] $= b_1, \ldots$.
- xc : a length m vector of the coefficients of trigonometric polynomial $c(t)$ in (4). xc[1] $= c_1$, xc[2] $= c_2, \ldots$.
- pp : a length $m+1$ vector of coefficients of numerator of $G_r(z)$ in (2). pp[1] $= p^m$, pp[2] $= p^{m-1}, \ldots$.
- qq : a length $m+1$ vector of coefficients of denominator of $G_r(z)$ in (2). qq[1] $= q^m$, qq[2] $= q^{m-1}, \ldots$.
- output: internal algorithmic log.

Model Reduction Problem

Given

- Order pairs (i.e. frequency samples/frequency response)

$$\left(t_i, G(e^{jt_i})\right) \in [0, 2\pi] \times \mathbf{C}, \quad \forall\, i = 1, 2, \ldots, N \tag{1}$$

- Integer m (i.e. reduced order)
- (Optional) Weights on frequency response match error

$$W_i \in \mathbf{R},\, W_i \geq 0, \quad i = 1, 2, \ldots, N$$

Find a *discrete time* transfer function of order m

$$G_r(z) := \frac{p(z)}{q(z)} := \frac{\sum\limits_{k=1}^{m} p_k z^k}{\sum\limits_{k=1}^{m} q_k z^k} \tag{2}$$

such that

$$\max_i |\, W_i \left(G(e^{jt_i}) - G_r(e^{jt_i})\right) | \to \min \tag{3}$$

and

$$q(z) \neq 0, \forall\, z \in \mathbf{C}, |z| \geq 1$$

Relaxation Setup

Finding G_r is a difficult problem. Therefore a relaxation is solved instead [1].

Find trigonometric polynomials

$$\begin{aligned} a(t) &= 1 + a_1\cos(t) + a_2\cos(2t) + \ldots + a_m\cos(mt) && (4)\\ b(t) &= b_0 + b_1\cos(t) + b_2\cos(2t) + \ldots + b_m\cos(mt) && (5)\\ c(t) &= c_1\sin(t) + c_2\sin(2t) + \ldots + c_m\sin(mt) && (6) \end{aligned}$$

such that

$$\begin{aligned} &\hat{G}_r(e^{jt}) := \tfrac{b(t)+jc(t)}{a(t)} \\ &\max_i |\, W_i \left(G(e^{jt_i}) - \hat{G}_r(e^{jt_i})\right) | \to \min \end{aligned} \tag{7}$$

and

$$a(t) > 0, \forall\, t \in [0, 2\pi]$$

Reconstructing Reduced Model

See [1] for detail. Once trigonometric polynomials $a(t), b(t), c(t)$ are found, a spectral factorization can be used to reconstruct an approximation of reduced transfer function denominator $q(z)$. In addition, the numerator $p(z)$ can be found by solving a convex optimization problem.

References

[1] KC. Sou, L. Daniel, and A. Megretski. A Quasi-Convex Optimization Approach to Parameterized Model Order Reduction. In *IEEE/ACM Design Automation Conference*, June 2004.