

A Quasi-Convex Optimization based Model Order Reduction Code

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Model Reduction Problem

Given

- Order pairs (i.e. frequency samples/frequency response)

$$(t_i, G(e^{jt_i})) \in [0, 2\pi] \times \mathbf{C}, \quad \forall i = 1, 2, \dots, N \quad (1)$$

- Integer m (i.e. reduced order)
- (Optional) Weights on frequency response match error

$$W_i \in \mathbf{R}, W_i \geq 0, \quad i = 1, 2, \dots, N$$

Find a *discrete time* transfer function of order m

$$G_r(z) := \frac{p(z)}{q(z)} := \frac{\sum_{k=1}^m p_k z^k}{\sum_{k=1}^m q_k z^k} \quad (2)$$

such that

$$\max_i |W_i (G(e^{jt_i}) - G_r(e^{jt_i}))| \rightarrow \min \quad (3)$$

and

$$q(z) \neq 0, \forall z \in \mathbf{C}, |z| \geq 1$$

Relaxation Setup

Finding G_r is a difficult problem. Therefore a relaxation is solved instead [1].

Find trigonometric polynomials

$$a(t) = 1 + a_1 \cos(t) + a_2 \cos(2t) + \dots + a_m \cos(mt) \quad (4)$$

$$b(t) = b_0 + b_1 \cos(t) + b_2 \cos(2t) + \dots + b_m \cos(mt) \quad (5)$$

$$c(t) = c_1 \sin(t) + c_2 \sin(2t) + \dots + c_m \sin(mt) \quad (6)$$

such that

$$\begin{aligned} \hat{G}_r(e^{jt}) &:= \frac{b(t) + jc(t)}{a(t)} \\ \max_i |W_i (G(e^{jt_i}) - \hat{G}_r(e^{jt_i}))| &\rightarrow \min \end{aligned} \quad (7)$$

and

$$a(t) > 0, \forall t \in [0, 2\pi]$$

Reconstructing Reduced Model

See [1] for detail. Once trigonometric polynomials $a(t), b(t), c(t)$ are found, a spectral factorization can be used to reconstruct an approximation of reduced transfer function denominator $q(z)$. In addition, the numerator $p(z)$ can be found by solving a convex optimization problem.

Itemized Explanation of Input/Output arguments

```
% function [Gr,xa,xb,xc,pp,qq,y,output] = run_mor(G,t,m,W,r)
%
% Use relaxation scheme to solve
% min max_i |W(i)*(G(i) - Gr(t(i)))| subject to stability of Gr
%
% master file for running ellip_rpn_mor.m
%
% Inputs:
% G --- a vector of frequency response samples of the original system
% t --- a vector of frequency samples corresponding to G
% m --- order of the reduced system
% W --- a vector of frequency weight (of length t) (default = ones(length(t),1)
% r --- desired value of relative tolerance (default = 0.9)
%
% Outputs:
% Gr --- discrete-time transfer function of the reduced model Gr(z) = pp(z)/qq(z)
% xa --- denominator of optimal relaxation a(t) = 1+xa(1)*cos(t)+xa(2)*cos(2*t)+...
% xb --- real part of numerator of optimal relaxation b(t) = xb(1)+xb(2)*cos(t)+xb(3)*cos(2*t)+...
% xc --- imaginary part of numerator of optimal relaxation c(t) = xc(1)*sin(t)+xc(2)*sin(2*t)+...
% pp --- numerator of Gr(z)
% qq --- denominator of Gr(z)
% y --- optimal objective value of the relaxation
% output --- miscellaneous algorithmic output
```

Input arguments

- G : a vector of length N storing frequency response of full model as in (1).
- t : a vector of length N storing frequency samples at which G is sampled. See (1).
- m : an nonnegative integer specifying the order of reduced model.
- W : (optional) a vector of length N . Weights on objective functions in (3) and (7).
- r : (optional) $r \in (0, 1)$. An algorithmic parameter specifying the accuracy of the optimization. The result is more accurate when r is small.

Output arguments

- Gr : discrete-time transfer function object of the reduced model.
- xa : a length m vector of the coefficients of trigonometric polynomial $a(t)$ in (4). $xa[1] = a_1, xa[2] = a_2, \dots$
- xb : a length $m+1$ vector of the coefficients of trigonometric polynomial $b(t)$ in (5). $xb[1] = b_0, xb[2] = b_1, \dots$
- xc : a length m vector of the coefficients of trigonometric polynomial $c(t)$ in (4). $xc[1] = c_1, xc[2] = c_2, \dots$
- pp : a length $m+1$ vector of coefficients of numerator of $G_r(z)$ in (2). $pp[1] = p^m, pp[2] = p^{m-1}, \dots$

- qq : a length $m + 1$ vector of coefficients of denominator of $G_r(z)$ in (2). qq[1] = q^m , qq[2] = q^{m-1}, \dots
- y : optimal objective value of (7).
- output: internal algorithmic log.

References

- [1] KC. Sou, L. Daniel, and A. Megretski. A Quasi-Convex Optimization Approach to Parameterized Model Order Reduction. In *IEEE/ACM Design Automation Conference*, June 2004.