Partitioned Conduction Modes in Surface Integral Equation-Based Impedance Extraction

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Abstract

In this paper, the conduction mode basis functions previously developed for a volume integral formulation are partitioned and employed as surface basis functions in an surface integral equation formulation. Implementation examples on package wires, rings and transmission lines show that the partitioned conduction modes improve the surface integral solver accuracy, efficiency and consistency over a wide range of frequency.

I. Introduction

As integrated circuits operate at increasingly higher speed, methods are needed for quasi-static and full wave electromagnetic analysis of packages and on-chip interconnect over a very wide range of frequencies, up to tens of gigahertz (GHz). Such methods need to account for the effects of distributed resistive, capacitive and inductive impedance throughout the entire chip and package layout in an accurate and efficient manner. The development of accelerated integral equation solvers such as FastCap[1], FastHenry[2], and PFFT [3], together with several techniques for handling skin and proximity effects [4], [5], [6], [7], has made the realization of the aforementioned features feasible in electromagnetic analysis tools. In particular, surface integral formulations have gained wide acceptance in recent years because they avoid frequency-dependent discretization of the interior of conductors or substrates typical of volume methods. The formulation described in this paper addresses many issues currently plaguing the surface integral solver FastImp [8]. One of the shortcomings of FastImp is that it switches between two different techniques for computing current through each contact surface depending on the operating frequencies applied to the conductor, thus breaking the continuity of the solution. Secondly, FastImp's computation of impedance lacks accuracy at low frequencies due to its use of a simple centroid collocation scheme. Thirdly, at high frequencies, the formulation cannot adequately capture the current as it crowds near the sides and corners of a contact surface without using a much refined discretization that is computationally expensive. To make clear of this difficulty, consider that in order to capture skin and proximity effects accurately in FastImp, not only the dimension of contact panels must be narrower than the skin depth, but the dimension of non-contact panels near the edges and corners of the contacts must be narrower as well. This implies that when discretizing long wires, such as in transmission line configurations, many tightly interacting long and thin panels are produced. The problem of these high aspect-ratioed panels is that they worsen the efficiency of clustering-based fast solvers [1], [2], [3].

II. Background: FastImp

The surface integral equation formulation used in FastImp as proposed in [8] is as follows:

$$\frac{1}{2}\overline{E}(\overline{r}) = \int_{S_i} dS' \left[G_1(\overline{r},\overline{r'}) \frac{\partial \overline{E}(\overline{r'})}{\partial n} - \frac{\partial G_1(\overline{r},\overline{r'})}{\partial n} \overline{E}(\overline{r'}) \right] \tag{1}$$

$$-\frac{1}{2}\overline{E}(\overline{r}) = \int_{S} ds' \left[G_0(\overline{r},\overline{r'}) \frac{\partial \overline{E}(\overline{r'})}{\partial n} - \frac{\partial G_0(\overline{r},\overline{r'})}{\partial n} \overline{E}(\overline{r'}) \right] + \bigtriangledown \phi(\overline{r}) \tag{2}$$

$$\phi(\overline{r}) = \int_{S_i} ds' G_0(\overline{r},\overline{r'}) \frac{\rho_s(\overline{r'})}{\varepsilon} \tag{3}$$

$$\nabla \cdot \overline{E} = 0, \tag{4}$$

where S_i is the surface of the ith conductor and S is the union of all conductor surfaces. The unknowns associated with the above formulation are the electric field $\overline{E}$, the surface normal derivative of the electric field $\frac{\partial \overline{E}}{\partial n}$, the surface charge density ρ_s, and the electric potential ϕ. Green's functions G_0 and G_1 are defined as $G_0(\overline{r},\overline{r'}) = \frac{e^{jk_0|\overline{r}-\overline{r'}|}}{4\pi|\overline{r}-\overline{r'}|}$ and $G_1(\overline{r},\overline{r'}) = \frac{e^{jk_1|\overline{r}-\overline{r'}|}}{4\pi|\overline{r}-\overline{r'}|}$ with $k_0 = \omega\sqrt{\varepsilon\mu}$ and $k_1 = \sqrt{\omega^2\mu\varepsilon - j\omega\mu\sigma}$, where ω is the excitation frequency, ε is the dielectric permittivity, and μ is the magnetic permeability. Associated with this formulation are the boundary conditions:

$$\hat{n}\cdot E(\overline{r}) = \frac{j\omega\rho_s(\overline{r})}{\sigma} \;\; \overline{r} \in S_{i_{nc}}, \qquad E(\overline{r}) = \hat{n}E_n(\overline{r}) \;\; \overline{r} \in S_{i_c}, \qquad \text{and} \qquad \frac{\partial E(\overline{r})}{\partial n} = 0 \quad \overline{r} \in S_{i_c}, \tag{5}$$

where $S_{i_{nc}}$ and S_{i_c} denote the non-contact and contact surfaces of the ith conductor, respectively. $\hat{n}$ is the outward normal unit vector of the conductor surface. The discretization used in FastImp is based on first partitioning the conductor surface into regular rectangular

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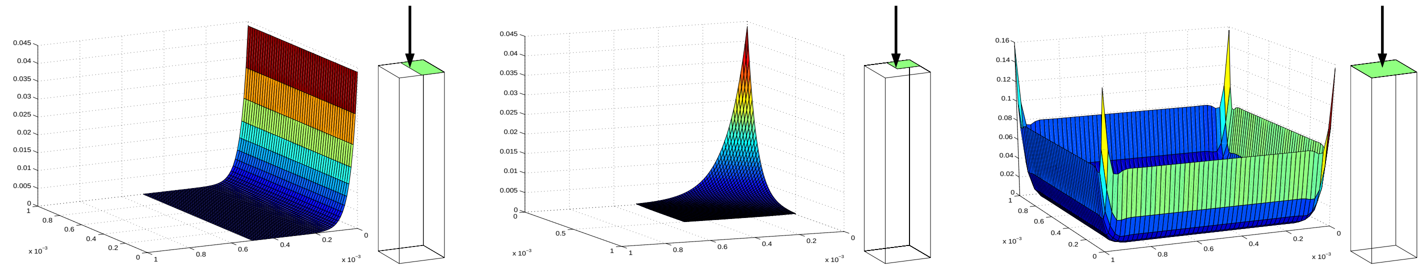

Fig. 1. Contact electric field representation using a side mode (left), a corner mode (center) and a combination of all eight modes (right) with their respective area of optimal coverage. The arrows indicate the direction of current flow.

panels and then solving the system of equations in (1)-(5)using a piecewise constant collocation scheme. In evaluating contact current, at low frequencies, FastImp uses a surface integral approximation that takes into account the sum of all the current flowing through the discretized panels on each contact. At high frequencies where skin effects generate fields that vary exponentially along a wire cross section, this piecewise-constant representation often requires too many panels to be an efficient method for impedance extraction. Therefore, FastImp uses line integrations involving the magnetic field around the cross sections near the contacts to compute contact current at high frequencies.

III. Background: Volume Conduction Mode Basis Functions

A volume integral formulation that models efficiently the skin effect of a conductor at high frequencies was first introduced in [7], and then extended in [9] to capture proximity effects and transmission line resonances. In that approach, it was recognized that the normal electric field on a cross-section of a conductor can be represented by the infinite series: $E_n(x,y) = \sum_k C_k e^{-a_k x} e^{-b_k y}$, where n denotes the normal to a cross-section, and coordinate system *(x,y)* specifies any point contained in the plane of the cross section. C_k's are scalar coefficients. Exponents a_k and b_k satisfy the constraint $a_k^2 + b_k^2 = \left(\frac{1+j}{\delta}\right)^2$. Each term in the infinite series represents a "volume conduction mode."

IV. Partitioned Conduction Modes in FastImp

The main observation of this paper is that the volume-based conduction mode basis functions can be modified to be used in a surface formulation so as to accurately and efficiently model contact surface current flow. For the volume-based method, a set of conduction mode basis functions is applied to each subdivided segment volume of the conductor. On the other hand, for the surface-based method, the conduction mode basis functions are only applied on each conductor contact surface, thereby reducing the number of unknowns in comparison to the piece-wise constant surface method, especially at high operating frequencies.

In our method, the normal electric field distributions on a contact, E_n, can be accurately represented by just a few of the infinitely many conduction modes. Specifically, we use eight such modes. Four modes are "side modes" and they capture the field exponentially decaying from each side of a conductor contact cross section. A side mode is specified by letting $a_k = \frac{1+j}{\delta}$ and $b_k = 0$. The combination of all four side modes is able to account for most of the high frequency conductor field distribution on a contact. At extremely high frequencies, four additional "corner" modes are necessary to account for the extra distributions decaying from the four corners of the contact. A corner mode is specified by letting $a_k = \frac{1}{\sqrt{2}}(\frac{1+j}{\delta})$ and $b_k = \frac{1}{\sqrt{2}}(\frac{1+j}{\delta})$.

However, if each one of the eight conduction modes were implemented over the entire area of the contact surface, numerical difficulties would arise at low frequencies due to the fact that the relative flatness of the contact electric field at low frequencies makes all the conduction modes resemble each other. This lack of distinctions between modes introduces linear dependency between the system of discretized integral equations at low frequencies. The system thus becomes ill-conditioned and solving it using an iterative method would require many iterations. This problem can be rectified by confining conduction modes to areas on the contact that reduce the amount of overlaps between modes while maintaining their effectiveness. For instance, the coverage of each mode can be reduced to a half or a quarter of the contact area. Therefore the four side modes are confined to the left half, right half, top half and bottom half of the contact. The four corner modes are confined to the upper left quarter, lower left quarter, upper right quarter and lower right quarter of the contact. Figure 1 (on the left) and Figure 1 in the center show the electric field distributions of one partitioned side mode and one partitioned corner mode, respectively.

In general, the electric field on a contact can be written as a weighted sum of field contributions from all eight partitioned conduction modes. That is, $\overline{E}_{contact}(\overline{r}) = \hat{n}_c \sum_{j=1}^{N} C_j W_j(\overline{r})$, where W_j denotes the jth conduction mode basis function, $\hat{n}_c$ is the surface normal of the contact, and N is the total number of conduction modes used. Figure 1 (on the right) shows an example of contact electric field representation using all eight conduction mode basis functions, that is, four partitioned side modes and four partitioned corner modes.

The integral equations (1)-(5) are discretized using a combination of partitioned conduction mode basis functions representing the contact normal electric field, and piecewise-constant surface panel basis functions representing the non-contact fields. A standard Galerkin technique is applied when field evaluations are made on the contact surfaces while a centroid collocation scheme is used when field evaluations are made on the non-contact surfaces. This hybrid scheme is utilized to produce a system of discretized

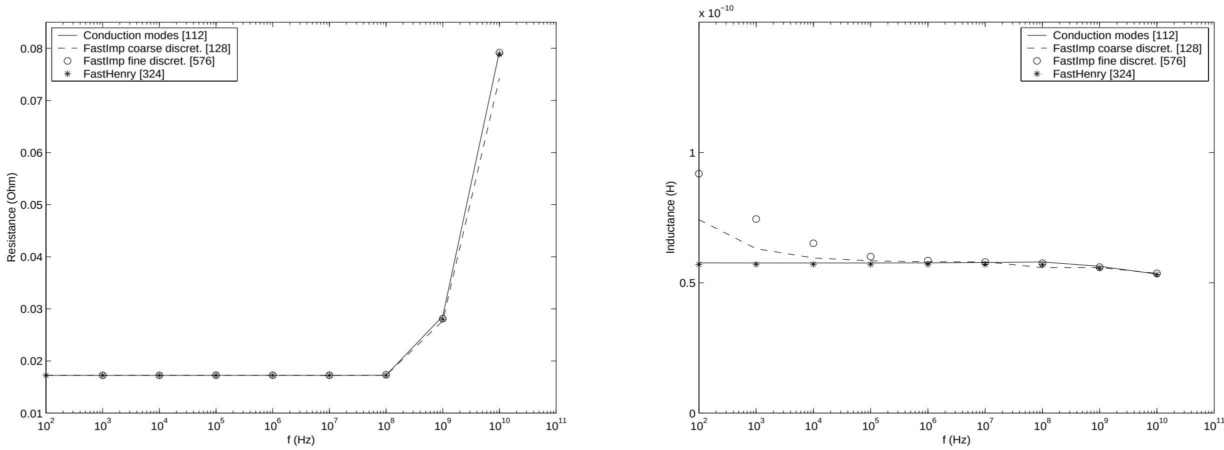

Fig. 2. Straight wire results. Left: resistance ($Re(Z)$). Right: inductance ($Im(Z)/j\omega$).

equations that can be solved using an iterative method.

V. Implementation Results: A straight package wire example

For our first simple implementation example we considered a straight wire with square ($10\mu m$ by $10\mu m$) cross-section, length $100\mu m$, and conductivity $5.8\times 10^7\frac{S}{m}$. Result comparisons are made among FastImp's coarse uniform piecewise-constant 4X4X6 surface discretization (4x4 panels on the contact faces, 4x6 on non-contact, for a total of 128 basis functions); our surface conduction mode method (same 4x6 non-contact surface discretization but only 8 partitioned conduction modes per contact for a total of 112 basis functions); and FastImp's fine non-uniform piecewise-constant 12X12X6 discretization (total of 576 basis functions). We used as a reference for accuracy calculations the results generated by a fine 324 thin-filament volume discretization in FastHenry so that the cross-section of the smallest filament was within $\frac{1}{8}$th of the skin depth at the highest frequency used in this example.

The analytic 0.0172Ω DC resistance value is met by all methods in the resistance comparison plot in Figure 2 (on the left). When compared to the FastHenry's reference solution at high frequencies, our partitioned conduction mode method yields similar resistance accuracy (e.g. 1% at 10GHz) as the much more expensive FastImp's fine piecewise-constant discretization. FastImp's coarse piecewise-constant discretization, on the other hand, shows a degraded resistance accuracy of (e.g. 5% at 10GHz). Inductance calculations in Figure 2 (on the right), show poor accuracy at low frequencies when using FastImp's coarse (e.g. 61% at 100Hz) or fine (e.g. 30% at 100Hz) piecewise-constant discretizations. Our contact partitioned conduction mode method has a maximum error of 1% from FastHenry's inductance at 100Hz.

On average, in this example our method is as accurate as FastImp's fine piecewise-constant discretization which uses 5 times more basis functions, and it is 5% more accurate than FastImp's coarse piecewise-constant discretization which uses a similar number of basis functions.

VI. Implementation Results: a Ring example

Consider a ring with an inner radius of $20\mu m$ and a square cross-section of $10\mu m$ by $10\mu m$. Result comparisons are made among FastImp's coarse uniform piecewise-constant 4X4X8 discretization (544 basis functions); our method using the same non-contact surface discretization plus 8 conduction modes per contact (528 basis functions); and FastImp's fine non-uniform 12X12X8 discretization (1824 basis functions). The reference results are generated by FastHenry using a volume discretization of 1568 filaments, where the cross-section of the smallest filament was within $\frac{1}{4}$th of the skin depth at the highest frequency used.

In Figure 3 (on the left) it is noticeable a non-physical dip in resistance at 100MHz in both sets (fine and coarse) of FastImp's piecewise-constant discretizations. Such dip is due to FastImp's switching between two different methods for computing contact current. Our partitioned conduction mode method has evidently corrected this discontinuity in solution by implementing one consistent method for computing contact current across all frequencies.

At low frequencies, Figure 3 (in the center) demonstrates that our method provides the best inductance evaluations with a maximum error of 9% at 100Hz while FastImp's coarse discretization yields an error of 32% and its fine discretization yields an error of 30% at the same frequency. At high frequencies, we have observed that our method produces the same small error (e.g. 2% at 30GHz) as the FastImp's fine piecewise-constant discretization scheme, which uses 3.5 times more basis functions. Alternatively, in this example our method is 6.5 times more accurate that FastImp's coarse piecewise-constant discretization that uses the same number of basis functions.

VII. Implementation Results: a two wires Transmission line

Consider two typical IC package wires, each 5mm long with $10\mu m$ by $40\mu m$ on the cross section, and situated $10\mu m$ apart. The two wires are shorted at one end and are excited with an ideal voltage source. Result comparisons are made between FastImp's coarse

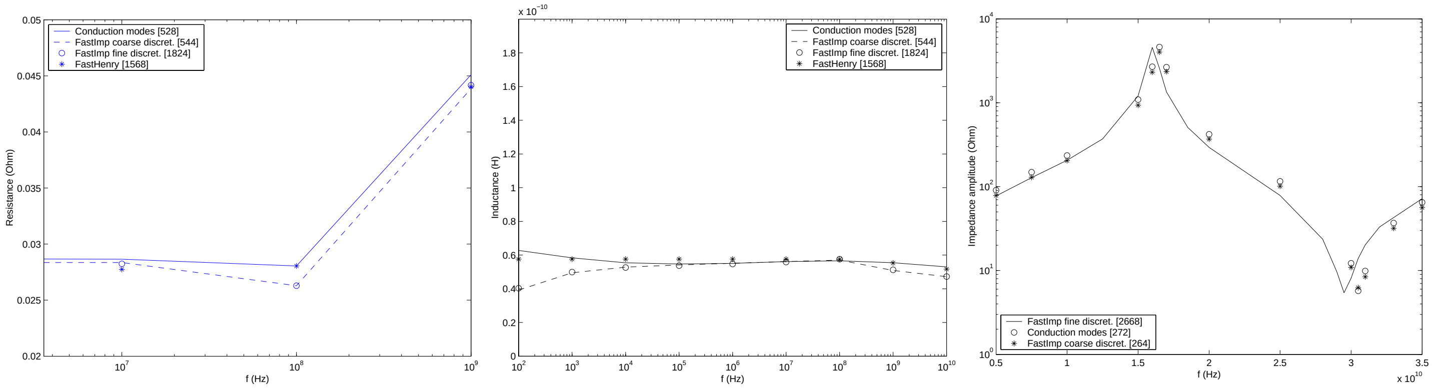

Fig. 3. On the left: resistance of the ring with a closer view of the “dip” in FastImp’s resistance results occurring at 100MHz due to the switching of contact current evaluation method. In the center: inductance of the ring. On the right: impedance amplitude vs. frequency for a shorted T-line plotted on a log amplitude scale.

piecewise-constant 2X3X10 discretization (264 basis functions), and our method using the same non-contact surface discretization, but with 8 conduction modes per contact (272 basis functions). In order to produce an accurate reference solution for the comparison we used the FastImp piecewise-constant approach with a very fine and non-uniform 8x9 discretization on the contact faces, so that panels near the edges were about one skin depth. In the original FastImp approach such discretization implies a similar fine discretization on the non-contact panels near the contacts and therefore along the entire wire length. In order to avoid long and skinny panels that cannot be efficiently handled by PFFT, a minimum of 35 sections along the length must then be used, thereby producing a very large number (2668) of basis functions even for this very simple problem.

The results shown in Figure 3 (on the right) from both the conduction mode method and FastImp’s coarse piecewise-constant discretization have a worst-case error of 3% in the position of the first quarter-wavelength impedance resonance when compared to the fine discretization solution. The results produced by FastImp’s coarse discretization has a worst-case error of 11% measured on the impedance amplitude at the same resonance. Our conduction mode yields a very small error of only 1% at that resonance. As a final observation, for accuracies of 1% to 3%, our method uses 10 times fewer basis functions than the FastImp fine piecewise-constant discretization. Since those 10 times extra basis functions in the piecewise-constant discretization are mostly tightly interacting panels, they cannot efficiently exploit the O(N log N) fast-solver acceleration techniques [1], [2], [3], hence our final save in memory and time is a square factor of $10^2 = 100$.

VIII. Conclusion

In this paper, it is shown that the conduction modes, originally developed in [7] for a volume integral method, can be applied to a surface integral method such as FastImp [8]. Furthermore, these basis functions can be modified to promote their linear independence while still maintaining their effectiveness. It has also been observed that in order to capture skin and proximity effects, the original FastImp needs to use a very fine piecewise-constant discretization on contact faces, which implies a similarly fine discretization on non-contact faces in order to avoid high aspect ratios on tightly interacting panels that are unfavorable for fast-solver [3]. For accuracies within 1% to 3% in a transmission line example, by using only up to eight conduction modes as basis functions on the contacts, we could use a 10 times coarser discretization of the non-contact surfaces. Therefore a saving factor of 100 in memory and simulation time can be achieved with the new method. Overall, using these partitioned conduction modes as basis functions on the contacts offers improved accuracy, consistency across frequencies, and increased efficiency in impedance evaluations when compared with the existing piecewise-constant approach in FastImp.

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