Numerical Techniques for Extracting Geometrically Parameterized Reduced Order Interconnect Models from Full-wave Electromagnetic Analysis

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Abstract

In this paper we give a fairly abstract description of a projection approach for generating geometrically parameterized reduced-order models directly from the numerical solution of Maxwell's equations. The method is without much justification, so a spiral inductor example is given to motivate further study.

I. Introduction

As analog and RF circuit designers make more extensive use of computational optimization to size transistors, they are increasingly interested in extending such approaches to simultaneously optimizing the interconnect. The difficulty is that it is much too computationally expensive to use general numerical electromagnetic analysis to reassess interconnect performance at every step of an optimization procedure. A faster method is needed to characterize the impact of geometric changes on interconnect performance. One approach, particularly successful for functional interconnect structures such as spiral inductors, is to develop an approximate parameterized model [1]. As with any manual macromodeling approach, there is a disadvantage in that an expert is required to redevelop the model when there is any significant change in technology or nominal geometry.

A much more flexible alternative is to find an approach for extracting geometrically parameterized low-order interconnect models directly from numerical electromagnetic analysis. The idea of generating parameterized reduced-order models is not new, recent approaches have been developed for interconnect that focus on statistical performance evaluation [2], [3] and clock skew minimization [4]. One recently developed technique for generating geometrically parameterized models of physical systems assumed a linear dependence on the parameter, and was applied to reducing a discretized linear partial differential equation using a projection approach [5]. Resistor-capacitor (RC) models of interconnect are linearly dependent on width and spacing parameters, at least for simplified models, and are typically reduced with respect to the Laplace transform parameter using projection approaches [6], [7], [8], [9], [10], [11], [12], [13], [14]. This fortuitous combination of attributes made generating geometrically-parameterized reduced-order models of RC interconnect a natural candidate to drive the development of projection-based multiparameter model reduction techniques [15], [16].

In [17], a method was presented for extracting geometrically parameterized models of the impedance of spiral inductors directly from a discretized integral formulation of Maxwell's equations. The method was based on the idea that the matrix entries in the discretized equations could be approximated as a polynomial function of both frequency and geometric parameters. Then, the matrix coefficients of the polynomial was used to determine a set of projection vectors which would ensure moment matching. As was pointed out in [18], frequency dependence is often better handled with Laguerre style interpolating functions. In addition, in [19] it was noted that near optimal projection vectors can be derived by sampling in the frequency domain. In this paper we very briefly sketch out a generalization of the approach in [17], one which combines the above observations. That is, in the next section we give a projection procedure based on using sampling that allows for arbitrary

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interpolating functions. Afterwards we show results from a spiral inductor example and then conclude this brief paper.

II. A General Projection Procedure

A number of approaches to discretizing Maxwell's equations will, when applied to interconnect analysis, generate a system of equations of the form

$$E(P)x = Bu \quad y = Cx \tag{1}$$

where u is the input, y is the output, x is an n-length vector of internal "states" related to the input and output through matrices B and C respectively, P is a vector of parameters, and and $E(P)$ is a parameter-dependent $n \times n$ matrix. Note that in general, the vector of parameters will include both frequency and geometric parameters.

Consider representing the parametric variation of E with a set of p interpolating functions. For example, these functions could be polynomial with respect to the geometric elements of the vector P, but Leguerre with respect to frequency. Then the parameterized system of equations would be

$$\left(\theta_1(P)E_1 + \ldots + \theta_p(P)E_p\right)x = Bu \quad y = Cx \tag{2}$$

where the matrices $E_1, \ldots, E_p$ are $n \times n$ interpolating function coefficient matrices.

Following [16], [17], one can produce a reduced order system using congruence transformations on the individual matrices $E_1, \ldots, E_p$

$$\left(\theta_1(P)V^T E_1 V + \ldots + \theta_p(P)V^T E_p V\right)\hat{x} = V^T Bu \quad y = CV\hat{x} \tag{3}$$

where $\hat{x}$ is the reduced state vector of length $q << n$, and the projection matrix V has size $n \times q$. The $q \times q$ reduced matrices are given by $\hat{E}_i = V^T E_i V$.

Consider finding $p \times q$ sufficiently independent sets of parameters to use for P, and then derive $q \times p$ associated x's by solving(3). If the projection matrix V is computed from those x's using the prescription in [19], then one is led to a perhaps surprising conclusion. The reduced E_i matrices, the $\hat{E}_i$'s, can be determined without explicitly finding or projecting the E_i's. The reduced coefficient matrices can be found by fitting.

III. A Simple Spiral Inductor Example

As an simple example of these parameterized reduction techniques, we reprise a 600 um x 600 um two-turn spiral inductor example originally presented in [17]. We considered three parameters, the excitation frequency (ranging from 0 to 20 gigaherz), the wire width (ranging from one to five microns), and the wire seperation (also ranging from one to five microns). To compute the impedance of the spiral inductor, we used a piecewise-constant discretization of a volume integral formulation [20], [21]. In particular, the spiral inductor's volume currents and surface charges were discretized into a total of 422 piecewise constant filaments and panels. Then a Galerkin method was applied to the volume integral formulation, resulting in a 422×422 $E(P)$ matrix which depends on frequency, wire width and wire spacing. Note that for this simple example, there is no ground plane or substrate.

To generate a parameterized reduced order model, we assumed that the matrix E was an affine function of frequency, which is equivalent to assuming the quasi-static case, and that the dependence on width and spacing was second order. Nine points in the geometric parameter space were used to determine the coefficients of the $E(p)$ matrix. Finally, twelve vectors were used to compute the reduced order model, but the ones used were based on moment-matching [17].

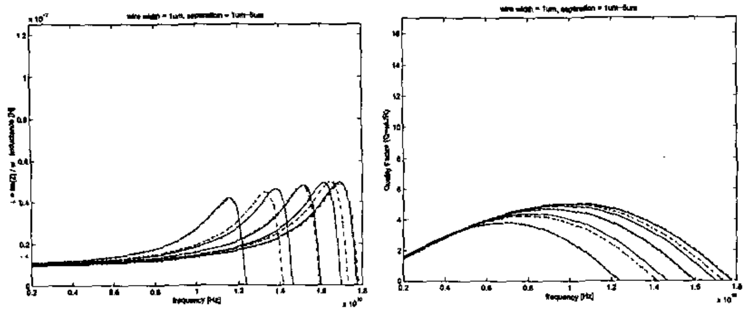

Fig. 1. Above: inductance. Below: quality factor. The five continuous lines in each plot correspond to the response of the original model for the two-turn inductor. The dash-dotted lines are the response of the parameterized reduced order model. The five pairs of curves correspond to wire width W=1um, and wire separation d = 1um, 2um, 3um, 4um, 5um.

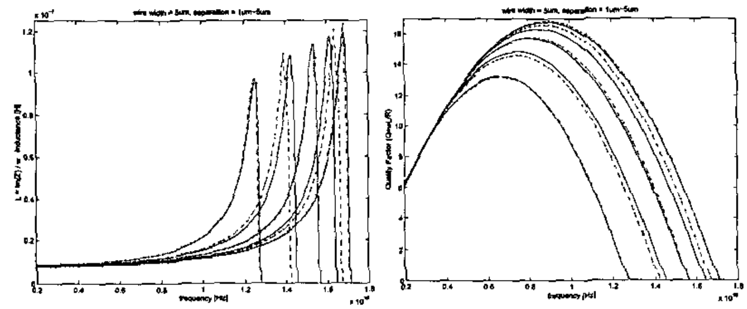

Fig. 2. Same curves as in Fig. 1, obtained here for a different wire width W = 5um. The largest errors (4% in amplitude near the peak for the quality factor, and 3% in position of the resonance peak for the inductance) are observed at d=2um and d=4um, which are farthest from the interpolation data points.

In Fig. 1 and 2 we show inductance and the quality factor of the inductor. The original model (continuous lines) has order 422 while the reduced model after interpolation and congruence transformation has order 12 (dash-dotted lines). Changing wire width and wire separation has a significant effect on inductor quality factor and on the position of the first resonance, but this dependence is successfully, but not perfectly, captured by the parameterized reduced order model.

IV. Conclusions and Acknowledgements

In this brief paper we outlined an approach for generating geometrically parameterized reduced-order models. We gave little insight into how to pick interpolation functions or parameter sets. Clearly, the method requires further study.The authors

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