TEN SIMPLE RULES FOR MATHEMATICAL WRITING

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ON WRITING

- "Easy reading is damn hard writing" (Hawthorne)
- "Word-smithing is a much greater percentage of what I am supposed to be doing in life than I would ever have thought" (Knuth)
- "I think I can tell someone how to write but I can't think who would want to listen" (Halmos)

WHAT IS MATH WRITING?

- Writing where mathematics is used as a primary means for expression, deduction, or problem solving.
- Examples that are:
  - Math papers and textbooks
  - Analysis of mathematical models in engineering, physics, economics, finance, etc
- Examples that are not:
  - Novels, essays, letters, etc
  - Experimental/nonmathematical scientific papers and reports

WHAT IS DIFFERENT ABOUT MATH WRITING?

- Math writing blends two languages (natural and math)
  - Natural language is rich and allows for ambiguity
  - Math language is concise and must be unambiguous
- Math writing requires slow reading
  - Often expresses complex ideas
  - Often must be read and pondered several times
  - Often is used as reference
  - Usually must be read selectively and in pieces

WHY THIS TALK?

- Experience is something you get only after you need it …
- One current model: The conversational style
  - “Mathematics should be written so that it reads like a conversation between two mathematicians on a walk in the woods” (Halmos)
  - “Talk to your readers as you write” (Strang)
  - Very hard to teach to others (“Effective exposition is not a teachable art. There is no useful recipe …” Halmos)
  - Controversial (where do proofs start and end? … I am not sure what the assumptions are … I can’t find what I need … etc)
- Instead we will advocate a structured style
  - Offers specific verifiable rules that students can follow and thesis advisors can check
  - Allows room to develop and improve over time

SOURCES

- General style books
  - Strunk and White, “The Elements of Style” (www)
  - Fowler and Aaron, “The Little Brown Handbook”
  - Venolia, “Write Right!”
- Halmos, “How to Write Mathematics”
- Knuth, et al, “Mathematical Writing” (www)
- Kleiman, “Writing a Math Phase Two Paper,” MIT (www)
- Krantz, “A Primer of Mathematical Writing”
- Higham, “Handbook of Writing for the Mathematical Sciences”
- Alley, “The Craft of Scientific Writing”
- Thomson, “A Guide for the Young Economist”

RULES OF THE GAME

- **Small rules:**
  - Apply to a single sentence (e.g., sentence structure rules, mathspeak rules, comma rules, etc)
- **Broad rules:**
  - Apply to the entire document
  - General style and writing strategy rules
  - Are non-verifiable (e.g., organize, be clear and concise, etc)
- **Composition rules (our focus in this talk):**
  - Relate to how parts of the document connect
  - Apply to multiple sentences
  - Are verifiable

SOME EXAMPLES OF SMALL RULES I

- **Break up long blocks of text into simpler ones:**
  - Few lines and verbs per sentence; few sentences per paragraph.
  - 2-3-4 rule: Consider splitting every sentence of more than 2 lines, every sentence with more than 3 verbs, and every paragraph with more than 4 "long" sentences.
- **Mathspeak should be "readable"**
  - BAD: Let k>0 be an integer.
  - GOOD: Let k be a positive integer or Consider an integer k>0.
  - BAD: Let $x \in R^n$ be a vector.
  - GOOD: Let x be a vector in R^n or Consider a vector $x \in R^n$.
- **Don't start a sentence with mathspeak**
  - BAD: Proposition: f is continuous.
  - GOOD: Proposition: The function f is continuous.

SOME EXAMPLES OF SMALL RULES II

- Use active voice ("we" is better than "one")
- Minimize "strange" symbols within text
- Make proper use of "very," "trivial," "easy," "nice," "fundamental," etc
- Use abbreviations correctly (e.g., cf., i.e., etc.)
- Comma rules
- "Which" and "that" rules
- … ETC

BROAD RULES
SOME EXAMPLES

- Language rules/goals to strive for: precision, clarity, familiarity, forthrightness, conciseness, fluidity, rhythm
- Organizational rules (how to structure your work, how to edit, rewrite, proofread, etc)
- "Down with the irrelevant and the trivial" (Halmos)
- "Honesty is the best policy" (Halmos)
- "Defend your style" (against pushy copyeditors -Halmos)
- … ETC

MATH WRITING WITHOUT MATH PROOFS

- **Is it OK to skip proofs?**
  - Rigorous proofs are the essence of mathematical writing
  - A mathematician relies on proofs to gain intuition
  - … but many readers prefer no proofs (at least to start …)
- **Intuitive math writing: An alternative to a proof based development**
  - Explain mostly in words (some) math results, and give refs
  - State precisely a few (if any) theorems … place (some) proofs in appendixes
  - Use suggestive natural language to describe the intuition behind theorems/algorithms
  - A challenge: Intuitive math writing is trickier/more demanding than rigorous proof-based writing
- **Example: Bertsekas/Tsitsiklis probability book**

PRINCIPLES OF INTUITIVE MATH WRITING

- Do not cut corners
  - Better to skip a proof than to give a sloppy proof
- Maintain math-like rigor in natural language
  - Without math, precise language becomes more important
- Use natural language and logic to replicate math arguments (without excessive formality)
- Explain the range and limitations of your development, and use well-chosen (simple and intuitive) examples to demonstrate
- Aim to provide enough explanation/intuition so mathematicians can believe your argument and even construct a rigorous proof

PRACTICAL TIPS FOR INTUITIVE MATH WRITING

- Loose language is the easiest way to confuse the reader
- Define terms rigorously, use them consistently
  - Avoid ambiguous or undefined terms
  - Don't use multiple terms with essentially the same meaning, e.g., don't use "likelihood" or "chance" (in place of "probability"), "random values" (in place of "random variables"), etc
- Use well-designed and suggestive figures
- Overexplain when addressing a cross-disciplinary audience (but don't overdo it)
- Consider taking a rigorous math course as preparation for intuitive math writing!

THE TEN COMPOSITION RULES

- Structure rules (break it into digestible pieces)
  - Organize in segments
  - Write segments linearly
  - Consider a hierarchical development
- Consistency rules (be boring creatively)
  - Use consistent notation and nomenclature
  - State results consistently
  - Don't underexplain - don't overexplain
- Readability rules (make it easy for the reader)
  - Tell them what you'll tell them
  - Use suggestive references
  - Consider examples and counterexamples
  - Use visualization when possible

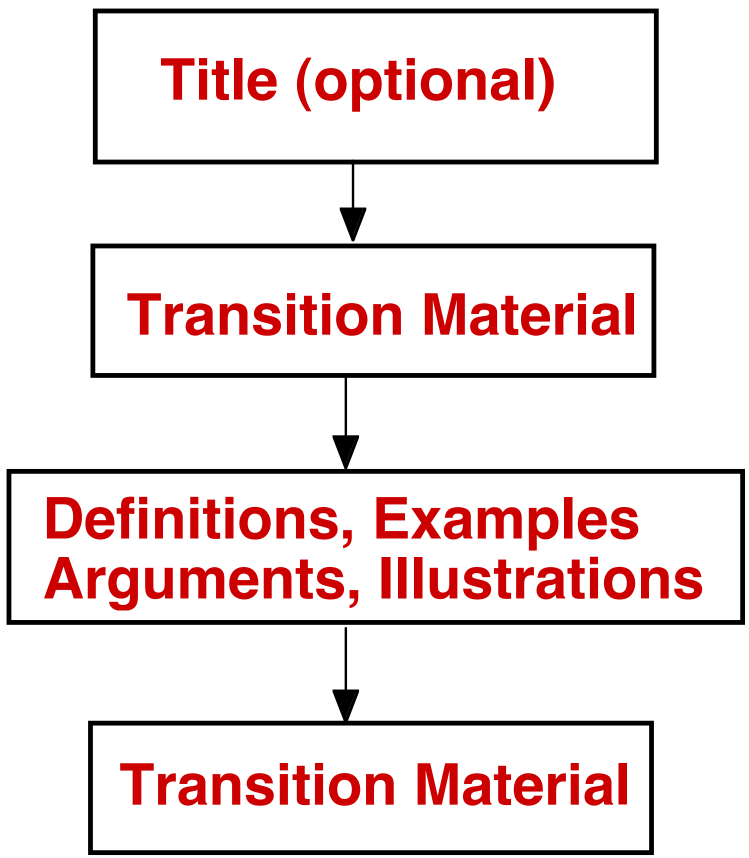
1. ORGANIZE IN SEGMENTS

- "Composition is the strongest way of seeing" (Weston)
- Extended forms of composition have a fundamental unit:
  - Novel — Paragraph
  - Film — Scene
  - Slide presentation — Slide
  - Evening news program — News report
- Key Question: What is the fundamental unit of composition in math documents?
- Answer: A segment, i.e., an entity intended to be read comfortably from beginning to end
- Must be not too long to be tiring, not too short to lack content and unity

SEGMENTATION PROCESS

- Examples of segments:
  - A mathematical result and its proof
  - An example
  - Several related results/examples with discussion
  - An appendix
  - A long abstract
  - A conclusions section
- A segment should "stand alone" (identifiable start and end, transition material)
- Length: 1/2 page to 2-3 pages

SEGMENT STRUCTURE



EXAMPLE OF SEGMENTATION: A SECTION ON PROB. MODELS

• Sample space - Events	(1 page)
• Choosing a sample space	(0.5 page)
• Sequential models	(0.75 page)
• Probability laws - Axioms	(1.25 page)
• Discrete models	(2 pages)
• Continuous models	(1 page)
• Properties of probability laws	(2 pages)
• Models and reality	(1.25 page)
• History of probability	(1 page)

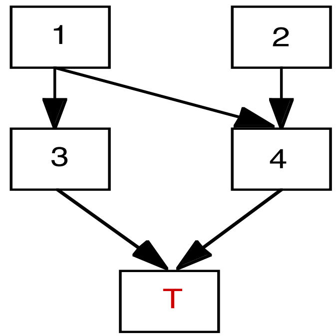
See Sec. 1.2 of Bertsekas and Tsitsiklis probability book

2. WRITE SEGMENTS LINEARLY

- Question: What is a good way to order the flow of deduction and dependency?
- General rule: Arguments should be placed close to where they are used (minimize thinking strain)
- Similarly, definitions, lemmas, etc, should be placed close to where they are used
- View ordering as an optimization problem
- A linear/optimal order is one that positions arguments (definitions, lemmas) so as to minimize the total number of “crossings” over other arguments (definitions, lemmas), subject to the dependency constraints. Depth-first order is usually better.

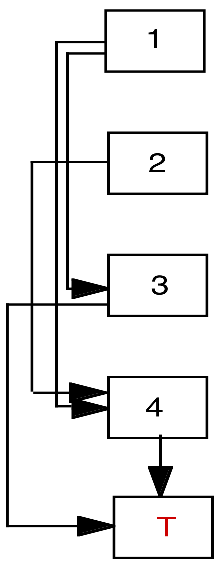
EXAMPLES OF ORDERING

Dependency Graph of Arguments



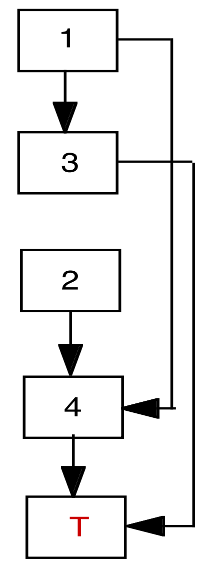
Level 1 Arguments

Level 2 Arguments



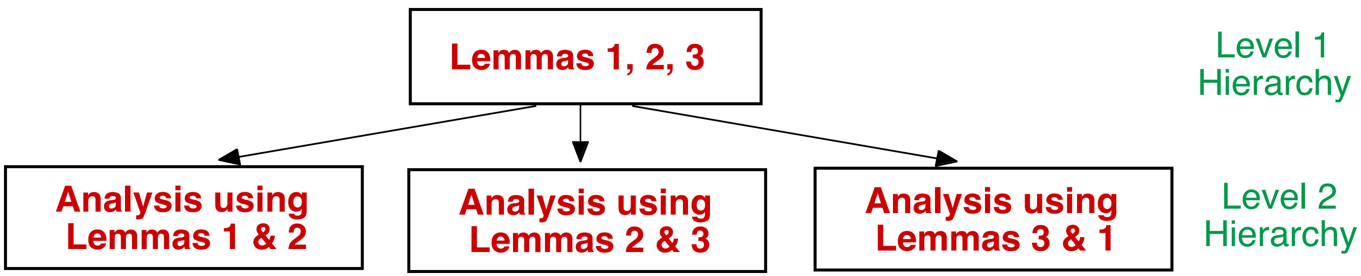
Nonlinear

Linear



3. CONSIDER A HIERARCHICAL DEVELOPMENT

- **Arguments/results used repeatedly may be placed in special segments for efficiency**



- **Possibly create special segments for special material (e.g., math background, notation, etc)**
- **Analogy to subroutines in computer programs**

4. USE CONSISTENT NOTATION

- Choose a notational style and stick with it
- Examples:
  - Use capitals for random variables, lower case for values
  - Use subscripts for sequences, superscripts for components
- Use suggestive/mnemonic notation. Examples: S for set, f for function, B for ball, etc
- Use simple notation. Example: Try to avoid parenthesized indexes: $x(m,n)$ vs x_{mn}
- Avoid unnecessary notation:
  - BAD: Let X be a compact subset of a space Y. If f is a continuous real-valued function over X, it attains a minimum over X.
  - GOOD: A continuous real-valued function attains a minimum over a compact set.

5. STATE RESULTS CONSISTENTLY

- Keep your language/format simple and consistent (even boring)
- Keep distractions to a minimum; make the interesting content stand out
- Use similar format in similar situations
- Bad example:
  – Proposition 1: If A and B hold, then C and D hold.
  – Proposition 2: C' and D' hold, assuming that A' and B' are true.
- Good example:
  – Proposition 1: If A and B hold, then C and D hold.
  – Proposition 2: If A' and B' hold, then C' and D' hold.

6. DON'T OVEREXPLAIN - DON'T UNDEREXPLAIN

- Choose a target audience level of expertise/background (e.g., undergraduate, 1st year graduate, research specialist, etc)
- Aim your math to that level; don't go much over or under
- Explain potentially unfamiliar material in separate segment(s)
- Consider the use of appendixes for background or difficult/specialized material

7. TELL THEM WHAT YOU'LL TELL THEM

- Keep the reader informed about where you are and where you are going
- Start each segment with a short introduction and perhaps a road map
- Don't string together seemingly aimless statements and surprise the reader with "we have thus proved so and so"
- Announce your intentions/results, e.g., "It turns out that so-and-so is true. To see this, note …"
- Tell them what you told them

8. USE SUGGESTIVE REFERENCES

- Frequent numbered equation/proposition referencing is a cardinal sin
- It causes page flipping, wastes the reader’s time, and breaks concentration
- Refer to equations/results/assumptions by content/name (in addition to number), e.g., Bellman’s equation, weak duality theorem, etc
- Repeat simple math expressions
- Remind the reader of unusual notation, and earlier analysis
- Dare to be repetitive (but don’t overdo it)

9. CONSIDER EXAMPLES AND COUNTEREXAMPLES

- "Even a simple example will get three-quarters of an idea across" (Ullman)
- Examples should have some spark, i.e., aim at something the reader may have missed
- Illustrate definitions/results with examples that clarify the boundaries of applicability
- Use counterexamples to clarify the limitations of the analysis, and the need for the assumptions

10. USE VISUALIZATION WHEN POSSIBLE

- “A picture is worth a thousand words”
- Keep figures simple and uncluttered
- Use substantial captions
- Captions should reinforce and augment the text, not repeat it
- Use a figure to illustrate the main idea of a proof/argument with no constraint of math formality
- Prefer graphs over tables

THE END

"Bad thinking never produces good writing"
(Lamport)

Good writing promotes good thinking …