Optimization of Satellite Constellation
Reconfiguration

by

Uriel Scialom

Submitted to the Department of Aeronautics and Astronautics
in partial fulfillment of the requirements for the degree of

Master of Science in Aeronautics and Astronautics

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

August 2003

© Uriel Scialom, MMIII. All rights reserved.

The author hereby grants to MIT permission to reproduce and
distribute publicly paper and electronic copies of this thesis document
in whole or in part.

Author .................................................................
Department of Aeronautics and Astronautics
August 22, 2003

Certified by ..............................................................
Olivier L. de Weck
Robert N. Noyce Assistant Professor of Aeronautics and Astronautics
and Engineering Systems
Thesis Supervisor

Accepted by .............................................................
Edward M. Greitzer
H.N. Slater Professor of Aeronautics and Astronautics
Chair, Committee on Graduate Students
Optimization of Satellite Constellation Reconfiguration

by

Uriel Scialom

Submitted to the Department of Aeronautics and Astronautics
on August 22, 2003, in partial fulfillment of the
requirements for the degree of
Master of Science in Aeronautics and Astronautics

Abstract

Traditional satellite constellation design has focused on optimizing global or zonal coverage with a minimum number of satellites. The number and size of individual satellites determines the overall constellation capacity. In some instances, it is desirable to deploy a constellation in stages to gradually expand capacity. This requires launching additional satellites and reconfiguring on-orbit satellites. A methodology for optimizing orbital reconfiguration of satellite constellations is thus presented in this thesis. The purpose of the project was to find a technical way for transforming an initial constellation with low capacity into a new constellation with higher capacity. The general framework was applied to LEO constellations of communication satellites. The modules implemented allow prediction of time and cost to reconfigure a fixed initial constellation $A$ into a final constellation $B$. Due to the complexity of the problem and consequently to the inevitable approximations, the simulation provides estimates. The main steps are to assign on-orbit satellites from $A$ to occupy slots of $B$. The other issue is to plan launches and transfers. From the planning, the simulation returns reconfiguration time and cost, and coverage capacity during transfers. Although some interesting tendencies seem clearly to come out from the results, this thesis is primarily focused on concepts and ideas. As a consequence, the methodology and simulation tool will be useful as a foundation for future works on that field.

Thesis Supervisor: Olivier L. de Weck
Title: Robert N. Noyce Assistant Professor of Aeronautics and Astronautics and Engineering Systems
Acknowledgments

Firstly, I would like to express my gratitude to my research advisor, Professor Olivier de Weck for his guidance. I particularly appreciated his help and kindness, when I was in search of a topic for my S.M. thesis. Professor Manuel Martinez-Sanchez need also to be thanked for his availability. The work done by Mathieu Chaize, graduate student in the Space Systems Laboratory, was a foundation for this thesis. I would like thus to thank him warmly.

Finally, my family, especially my parents, Edith and Daniel, need to be acknowledged for their support during all these years of study.

I was supported during this academic year 2002-2003 by a MIT Department of Aeronautics and Astronautics Fellowship. The funds were provided by the Goldsmith Scholarship. This project was also supported during the summer 2003 by the Defense and Advanced Research Project Agency: Systems Architecting for Space Tugs-Phase B contract ‡ F29601-97-K-0010/CLIN11.
Contents

1 Introduction 21
  1.1 Motivation .............................................. 21
    1.1.1 Globalstar and Iridium History .................... 21
    1.1.2 Staged Deployment and Economic Opportunity ........ 22
  1.2 Literature Review ...................................... 24
    1.2.1 Review of previous Work on Satellite Constellation Reconfiguration .......... 24
    1.2.2 Applications of Constellation Reconfiguration .......... 26
  1.3 Orbital Reconfiguration Definition ...................... 27
    1.3.1 Generic Definition .................................. 27
    1.3.2 Particular Case .................................... 27
  1.4 Thesis Purpose and Organization ........................ 28

2 Problem Description and Background 31
  2.1 Issues Raised by the Reconfiguration ..................... 32
  2.2 A Two-Phased-Reconfiguration Scenario ................... 35
  2.3 Formal Mathematical Problem Definition .................. 35

3 Project Framework: Satellite Constellation Reconfiguration 39
  3.1 Framework Development .................................. 39
    3.1.1 Objectives ........................................ 39
    3.1.2 Parameters ........................................ 41
    3.1.3 Design Variables .................................. 41

7
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1.4</td>
<td>Constraints</td>
<td>43</td>
</tr>
<tr>
<td>3.1.5</td>
<td>Block and $N^2$ Diagrams</td>
<td>44</td>
</tr>
<tr>
<td>3.2</td>
<td>Four Optimizations for One Problem</td>
<td>48</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Assignment Problem and Auction Algorithm</td>
<td>48</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Loop for assigning the Ground Satellites</td>
<td>51</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Launch Vehicle Selection Process</td>
<td>52</td>
</tr>
<tr>
<td>3.2.4</td>
<td>The Optimal Transfer Schedule</td>
<td>52</td>
</tr>
<tr>
<td>4</td>
<td>Detailed Description of the Reconfiguration Framework Modules</td>
<td>55</td>
</tr>
<tr>
<td>4.1</td>
<td>Constellation Module</td>
<td>55</td>
</tr>
<tr>
<td>4.1.1</td>
<td>Presentation of Polar and Walker constellations</td>
<td>55</td>
</tr>
<tr>
<td>4.1.2</td>
<td>Module Utilization</td>
<td>57</td>
</tr>
<tr>
<td>4.2</td>
<td>Astrodynamics Module</td>
<td>58</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Background</td>
<td>58</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Astrodynamics Module Inputs and Outputs</td>
<td>60</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Chemical Propulsion Scenario</td>
<td>61</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Electric Propulsion Scenario</td>
<td>66</td>
</tr>
<tr>
<td>4.3</td>
<td>Assignment Modules</td>
<td>67</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Assignment Methodology</td>
<td>67</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Efficiency of the Auction Algorithm</td>
<td>68</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Sensitivity of the Assignment to the relative Phasing of the two Constellations</td>
<td>71</td>
</tr>
<tr>
<td>4.4</td>
<td>Launch Module and Launch Schedule</td>
<td>73</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Iridium Constellation Deployment</td>
<td>74</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Description of the two Simulations available</td>
<td>75</td>
</tr>
<tr>
<td>4.4.3</td>
<td>Choice of one Simulation</td>
<td>76</td>
</tr>
<tr>
<td>4.4.4</td>
<td>Problems posed by the Launch Planning</td>
<td>77</td>
</tr>
<tr>
<td>4.5</td>
<td>Transfer Schedule</td>
<td>79</td>
</tr>
<tr>
<td>4.6</td>
<td>Coverage Module</td>
<td>80</td>
</tr>
<tr>
<td>4.6.1</td>
<td>Determination of the Partial Coverage during the Reconfiguration</td>
<td>80</td>
</tr>
</tbody>
</table>
4.6.2 Inputs/Outputs of Coverage Module ............................................. 82
4.6.3 Start of Operations for Constellation B ...................................... 83
4.7 Cost Module ................................................................................. 84
  4.7.1 Launch Cost ........................................................................... 84
  4.7.2 Transfer Cost ......................................................................... 84
  4.7.3 Cost of Service Outage ............................................................... 85
  4.7.4 Cost to Produce new Satellites ................................................. 89

5 Results .......................................................................................... 93
  5.1 Total Reconfiguration Energy Requirements .................................. 94
    5.1.1 Section Purpose ................................................................. 94
    5.1.2 Diagram of Altitude versus Inclination "Roadmap" .................. 95
    5.1.3 Comments .......................................................................... 96
  5.2 Case Study ................................................................................ 97
    5.2.1 Assignment Modules Results ................................................. 97
    5.2.2 Comparison of two Strategies for Chemical Propulsion ........ 102
    5.2.3 Comparison for Electric Propulsion ......................................... 105
    5.2.4 Conclusion ........................................................................... 107
  5.3 Impact of the Scenario and of the Propulsion System ....................... 107
    5.3.1 Scenario Impact .................................................................. 108
    5.3.2 Impact of the Propulsion Systems .......................................... 109
    5.3.3 Recommendation ................................................................. 111
  5.4 Tradespace Exploration ................................................................ 113
  5.5 Economic Opportunity of Multiple Reconfigurations Path ............... 116
  5.6 High Fidelity Reconfiguration Simulation ...................................... 118
    5.6.1 Structure of a High Fidelity Constellation Reconfiguration Sim-
    ulation .......................................................................................... 119
    5.6.2 Removal of Simplifying Assumptions ....................................... 120
    5.6.3 Tools for a High Fidelity Simulation ......................................... 121
    5.6.4 Visualization, Preliminary Study ............................................ 124
6 Recommendations and Conclusions

6.1 Summary ......................................................... 129
6.2 Conclusions ...................................................... 130
6.3 Recommendations for Future Work ............................ 131
## List of Figures

1-1 IRIDIUM constellation. Picture from [LUT00] .......................... 22

1-2 Schema of a Constellation Reconfiguration.

   \( N(A) \) = number of satellites in configuration \( A \).
   \( N(B) \) = number of satellites in configuration \( B \).
   \( N(B) \geq N(A) \) .......................... 28

2-1 The first phase: the launch phase of new satellites. ................. 36

2-2 The second phase: the transfer phase of the on-orbit satellites. .... 36

3-1 Constellation Design Variables. The design variables that can be modified to increase the capacity after the deployment are the altitude \( (h) \) and the elevation angle \( (\epsilon) \). This drives the number of satellites, since global coverage has to be maintained. .......................... 42

3-2 Orbital Constellation Reconfiguration Block Diagram ............. 43

3-3 Project matrix \( N^2 \). See the Master Table (Table 3.2) for the explanation of the indexes. .......................... 45

3-4 Expected variation of the service outage cost and of the fuel cost with respect of the \( I_{sp} \) .......................... 47

3-5 Assignment problem for a reconfiguration between constellation \( A \) with \( N_{sats}(A) \) satellites and constellation \( B \) with \( N_{sats}(B) \) satellites. .... 49

4-1 Iridium coverage simulated by the module. .......................... 57

4-2 Globalstar coverage simulated by the module. .......................... 57
4-3 Definition of the orbital elements. Extracted from [LUT00] and slightly modified. ........................................... 59

4-4 Transition matrix $\Delta V_{1,j}$ ........................................... 60

4-5 Trajectories in electric and chemical propulsion. Figure modified from [SMAD99], Fig 7.9 page 185 ................................. 62

4-6 Change of orbital plane. Both inclination and right ascension of ascending node are changed. ................................. 63

4-7 Representation of an orbital plane. ........................................... 64

4-8 $\Delta V_{\text{tot}}$ versus angle of relative phasing. No reassignment loop in this case. 72

4-9 $\Delta V_{\text{tot}}$ versus angle of relative phasing. Launch Vehicle Capacity of 3 satellites................................. 73

4-10 $\Delta V_{\text{tot}}$ versus angle of relative phasing. Launch Vehicle Capacity of 5 satellites................................. 74

4-11 Intersection between the coverage surfaces of two satellites. Area of double coverage. ........................................... 81

4-12 Coverage distribution in the case of Polar constellations. 28 satellites, $h = 1600$ km, $i=90$ deg ........................................... 82

4-13 Coverage distribution in the case of Walker constellations. Walker 28/7/2, $h = 1600$ km, $i=59$ deg ........................................... 82

4-14 Coverage surfaces of $A$ and $B$. SIMPLE CASE ................................. 83

4-15 $C_s$ versus time. The capacity of $A$ reduces gradually after $t_A$ and the capacity of $B$ increases gradually. Once the capacity of $B$ exceeds that of $A$ (crossover), service is switched over. ........................................... 87
"Constellation Recon®guration Map". Diagram of inclination versus altitude. "eps" represents the minimum elevation of the constellation. The values T/P/F are also indicated close to the asterisk representing each constellation.

$0 \leq \Delta V \leq 2 \text{ km/s}$: cheap reconfiguration.

$2 \leq \Delta V \leq 3 \text{ km/s}$: medium expense reconfiguration.

$\Delta V > 3 \text{ km/s}$: expensive reconfiguration.

5-1 Representation of the initial assignment. .................................................. 99

5-2 Assignment of ground satellites process. The first four launches go to plane 3 and fill that plane entirely. .................................................. 100

5-3 Reassignment of ground satellites. Penalization of one on-orbit satellite. Fifth launch. .................................................. 102

5-4 Sixth launch. .................................................................................. 103

5-5 $\Delta C$ versus $I_{sp}/(\eta P)$ drawn for the five electric propulsion systems. ..... 110

5-6 Fuel Cost and Service Outage Cost function of $I_{sp}$. .......................... 112

5-7 Fuel cost versus service outage cost. .................................................. 114

5-8 Legend .......................................................................................... 114

5-9 Economic opportunity of the Path 1. .................................................. 117

5-10 Economic opportunity of the Path 2. .................................................. 118

5-11 High Fidelity Simulation Framework .................................................. 120

5-12 Hi Fi Simulation Process .................................................................. 122

5-13 Static Visualization of the transfer of a single S/C. .............................. 124

5-14 Animated visualization of the reconfiguration. From "AnimationGUI".  

The dots represent the satellites in motion. .................................................. 125

5-15 Position of the six satellites before the transfers. Projection on $z = 0$. 126

5-16 Position of the six satellites after the transfer phase. There are 3 satellites per orbit. However, repositioning is necessary. .................. 127
List of Tables

3.1 Simulator Design Vector \( \mathbf{x} \) .............................................. 42
3.2 Master Table ................................................................. 46
3.3 Costs table for the simple case ........................................... 50
4.1 Comparison of the two strategies ......................................... 64
4.2 Electric propulsion systems .................................................. 67
4.3 Random assignment results .................................................. 69
4.4 SA results ............................................................................ 71
4.5 Deployment of the Iridium constellation. ................................. 75
4.6 Time between two successive launches. ................................... 78
5.1 Assignment for chemical propulsion. Reconfiguration of the constella-
    tion \( A \) to \( B \) with \( h_b = 1200km \) and \( \epsilon_b = 5deg \) ...................... 98
5.2 Number of ground satellites assigned in each plane after the first run.
5.3 Final Assignment for chemical propulsion. Reconfiguration of the con-
    stallation \( A \) to \( B \) with \( h_b = 1200km \) and \( \epsilon_b = 5deg \). In bold are
    represented the changes due to the reassignment of the ground satellites.101
5.4 Cost breakdown obtained with the first strategy. ........................ 104
5.5 Cost breakdown obtained with the second strategy. \( \epsilon \) means that the
    cost is negligible. ................................................................. 105
5.6 Cost breakdown obtained with the first strategy with resistojet. ....... 106
5.7 Cost breakdown obtained with the second strategy with resistojet. .... 106
5.8 Impact of the transfer scenarios using chemical propulsion .......... 109
5.9 Sensitivity of the total cost with respect to the propulsion system. ... 110
Nomenclature

Abbreviations

EOL  End of Life
GEO  Geosynchronous Equatorial Orbit
LCC  Life Cycle Cost
LEO  Low Earth Orbit
MSDO Multidisciplinary System Design Optimization
MEO  Medium Earth Orbit
NPV  Net Present Value
OTV  Orbital Transfer Vehicle
RAAN Right Ascension of the Ascending Node
SC   Service Charge
TFU  Theoretical First Unit

Symbols

$A$  Initial constellation
$A_u$ Average user activity (in Erl)
$B$  Final constellation
$C$  constellation type
$C_{fuel}$ Cost to launch the additional mass of fuel
$C_{launch}$ Cost to launch new satellites
$C_{outage}$ Cost of service interruption
$C_{prod}$ Cost to produce new satellites
$C_{tot}$ Total Cost
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cov</td>
<td>Coverage matrix</td>
</tr>
<tr>
<td>Cov_mean</td>
<td>Mean value of the constellation coverage during reconfiguration</td>
</tr>
<tr>
<td>D_a</td>
<td>Average user activity (in minutes per month)</td>
</tr>
<tr>
<td>D_A</td>
<td>Antenna diameter</td>
</tr>
<tr>
<td>ΔV_{i,j}</td>
<td>Transition matrix</td>
</tr>
<tr>
<td>ΔV_tot</td>
<td>Total DeltaV for reconfiguration</td>
</tr>
<tr>
<td>Δf_c</td>
<td>Per-channel bandwidth</td>
</tr>
<tr>
<td>e</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>h</td>
<td>Altitude</td>
</tr>
<tr>
<td>i</td>
<td>Inclination</td>
</tr>
<tr>
<td>I</td>
<td>Net investment</td>
</tr>
<tr>
<td>ISL</td>
<td>Inter satellite links</td>
</tr>
<tr>
<td>I_sp</td>
<td>Specific impulse</td>
</tr>
<tr>
<td>J</td>
<td>Objective function</td>
</tr>
<tr>
<td>LCC</td>
<td>Life cycle cost</td>
</tr>
<tr>
<td>m_d</td>
<td>Satellite dry mass</td>
</tr>
<tr>
<td>m_w</td>
<td>Satellite wet mass</td>
</tr>
<tr>
<td>n</td>
<td>Multiplicity of coverage</td>
</tr>
<tr>
<td>N_max</td>
<td>Maximum number of subscribers the system can support</td>
</tr>
<tr>
<td>N_sats</td>
<td>Number of satellites in a constellation</td>
</tr>
<tr>
<td>N_user</td>
<td>Maximum number of simultaneous users</td>
</tr>
<tr>
<td>Ω</td>
<td>Longitude of the ascending node</td>
</tr>
<tr>
<td>p</td>
<td>Number of planes</td>
</tr>
<tr>
<td>P_r</td>
<td>Propulsion System</td>
</tr>
<tr>
<td>P_t</td>
<td>Satellite transmit power</td>
</tr>
<tr>
<td>q</td>
<td>constraints vector</td>
</tr>
<tr>
<td>r</td>
<td>Discount Rate</td>
</tr>
<tr>
<td>R</td>
<td>Orbit Radius</td>
</tr>
<tr>
<td>s</td>
<td>Scenario</td>
</tr>
<tr>
<td>S_l</td>
<td>Launch Schedule</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Transfer Schedule</td>
</tr>
<tr>
<td>$T_{\text{launch}}$</td>
<td>Duration of the Launch Phase</td>
</tr>
<tr>
<td>$T_{\text{total}}$</td>
<td>Total time of reconfiguration</td>
</tr>
<tr>
<td>$T_{\text{transfer}}$</td>
<td>Transfer time</td>
</tr>
<tr>
<td>$T_{\text{sys}}$</td>
<td>Lifetime of the system</td>
</tr>
<tr>
<td>$U_S$</td>
<td>Global system utilization</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle between ascending nodes</td>
</tr>
<tr>
<td>$x$</td>
<td>Design vector</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Minimum elevation angle</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Propulsion efficiency</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Orbit rotation speed</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Satellite true anomaly</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Motivation

1.1.1 Globalstar and Iridium History

Technical successes but economic failures, the two first “big” Low Earth Orbit constellations of satellites, IRIDIUM and GLOBALSTAR, have revealed the problems resulting from demand uncertainty of large capacity systems. Operational since November 1998, the IRIDIUM constellation constituted of 66 satellites placed in polar orbits (Figure 1-1) had to declare bankruptcy only thirteen months after its launch. During the design process at the beginning of the 1990’s, the company whose target was the global business traveler, imagined it could attract 3 million customers. However, by the time IRIDIUM was deployed, the marketplace had been transformed by the development of terrestrial cellular networks. That is why one year after its launch, IRIDIUM had attracted only 50,000 customers, ultimately leading to bankruptcy. In 2000, although with different technical characteristics, the GLOBALSTAR constellation had to file for Chapter 11 protection for the same reasons. Those two examples show the high economic risks encountered by developers of commercial space systems. Ten years can indeed separate conceptual design from deployment. An alternative approach to deal with uncertainty in future demand in the case of satellite constellations is represented by a “staged deployment” strategy [CHAI03].
1.1.2 Staged Deployment and Economic Opportunity

It is possible to reduce the economic risks, by initially deploying a smaller constellation with low capacity that can be increased when the market conditions are good. The work done by Mathieu Chaize [CHAI03] has revealed the economic opportunity “staged deployment” represents for LEO constellations of communication satellites.

The “staged deployment” strategy would imply to design a flexible system that can adapt to highly uncertain market conditions. This approach is opposed to the traditional way of designing satellite constellation with fixed capacity from a specific set of requirements. The flexibility has to be specified prior to the deployment of the system. “Real options” to achieve the changes in capacity have to be found. In the present study, the “real option” considered is the extra-fuel carried by each satellite to achieve the orbital reconfiguration. Real options would not be necessarily utilized after the launch of the constellation. This new approach could trouble many designers and decision makers. They will need to understand the value of investing in such flexibility. For this, Chaize [CHAI03] has evaluated the economic opportunity for “staged deployment” when the size of the market targeted is close to the one Iridium originally expected.

The results are quite encouraging. Defining the value of flexibility as the money potentially saved by staged deployment compared to a fixed architecture, Chaize
shows that a decrease in the life cycle costs between 20 % and 45 % could be reached by this strategy when overaging over a range of future, uncertain demand scenarios. In addition to reducing the economic risk associated with the deployment of a large capacity system, this strategy could allow to reduce the costs as well. This decrease in the costs is, however, sensitive to the value of the discount rate. The higher the discount rate is, the more the costs are reduced. As a consequence, flexibility represents a real challenge for designers and a new way of thinking. Indeed, the best architectures for staged deployment are not necessarily the ones on the Pareto Front of “Life Cycle Cost” versus “Constellation Capacity” but the ones that give maximum flexibility such that future capacities can be changed “on-demand”. However, an estimation of the extra costs associated with reconfiguration is needed to determine the price to pay for flexibility. If the cost to reconfigure the constellation is smaller than the price we are willing to pay for it, it would be economically advantageous to undertake the reconfiguration. In other words, the reconfiguration is relevant if and only if its price appears to be lower than the a priori value of flexibility. The purpose of this thesis is notably to estimate the additional cost required by an orbital reconfiguration (see Section 1.4) and is a complement of the work done by Chaize. Due to the inevitable assumptions required by such a complex system, the reconfiguration costs will be approximated to a good order of magnitude. If the economic opportunity revealed is 20-45 % of $LCC$, taking the example of Iridium with a $LCC$ of approximately 5.5 $B\$, the economic opportunity will be in the range of 1.1-2.5 $B\$. So, to be sufficiently precise the cost estimate must be within 2-5% of the $LCC$. In other words, the reconfiguration cost must be known to within $\pm 100 - 250 M\$ in the case of an Iridium-like configuration.
1.2 Literature Review

1.2.1 Review of previous Work on Satellite Constellation Reconfiguration

First, we will mention as a reference some work on “static” optimization of satellite constellations. After this, a short review of previous work on constellation reconfigurations will be presented.

“Static” optimization of satellite constellations has been extensively studied over the past fifteen years, since constellations of satellites are the only way to achieve global coverage. These studies were principally demanded by the space communications systems manufacturers. However, they represent interesting applications for military purposes as well. The goal was actually to achieve global coverage while minimizing the necessary number of satellites. Two methods were proposed to solve this issue. The first one organizes the satellites into inclined circular orbital planes. This type of constellations was named “Walker constellation” with reference to the author of this method [Wal77]. The other method is based on the utilization of polar orbits and is due to Adams and Rider [AR87]. Adams and Lang [AL98] compared those two types of constellations. Depending of the fold of coverage (multiplicity \( n \)), the coverage requirement, the launch vehicle capability or the sparing strategy, they explain what type of constellations is more efficient. A table of the optimal constellations achieving global coverage is provided. In the case of zonal coverage, other types of constellations have been proposed including elliptical orbits. Crossley [CROSS03] used Genetic Algorithms (GA) to optimize constellations for zonal coverage.

Contrary to “static” optimization of satellite constellations, very few studies exist on optimization of satellite constellation reconfiguration. The past studies on “constellation reconfiguration” have principally focused on the constellation maintenance problem. No studies have considered reconfigurations to increase the constellation capacity. However, the literature on reconfiguration for maintenance has described some interesting concepts, applicable in our case. Seroi et al. [Ser02] pointed out the
complexity of space systems such as satellite constellations. The authors discussed the difficulty to optimize maintenance. In order to replace failed satellites or satellites at EOL ("End of Life"), they advise to launch new satellites by the means of launch vehicles with variable capacity. One contribution of this article is the utilization of an optimization technique called Dynamic Programming with Reinforcement Learning. The Dynamic Programming whose purpose is to optimize a decision sequence is a mathematical model of an agent that adapts his decision with respect to time. The Reinforcement Learning allows to push back the limits of this method which would otherwise require very high computation capacity. With Reinforcement Learning, the data are not stored during the process. The solution obtained is thus not the optimal one, however it is very close to the optimum.

In the same field, Ahn and Spencer [AS02] studied the optimal reconfiguration for formations of satellites after a failure of one of the satellites. The constellation considered was a formation flying satellite constellation. The goal was to find the maneuver cost that minimizes the total fuel usage among the individual satellites that remain operational. Their strategy was to prevent any unbalanced propellant usage. Depleting the propellant of one constellation member while not using any propellant from the other constellation members can cause early failure of another formation member and would necessitate to add other replacement satellites to the constellation, which is costly.

Techsat21, an Air Force Research Laboratory program is a good example of orbital reconfiguration. Saleh, Hastings and Newman [JHSN01] have briefly described this program. Focused on lightweight and low-cost clusters of micro-satellites, this program intends to reconfigure the geometry of the different clusters of a space based radar system. The purpose is to change the system’s capability by geometry modification, from a radar mode whose resolution is 500 meters to a geo-location mode with a resolution of 5 km.
1.2.2 Applications of Constellation Reconfiguration

Section 1.1.2 pointed out the economic opportunity represented by “staged deployment” and “orbital reconfiguration” for commercial satellite constellations. However, there are also non-commercial applications. Saleh et al. [JHSN01] explained that defense oriented space systems could be a field of application for flexibility in design. In defense systems, the development times are of the order of 5 to 10 years, and changes in system usage are very probable. Below are two examples for military applications. The military applications consist primarily of tracking targets or monitoring particular regions of the globe.

The next generation of radars will allow the Pentagon to track moving targets in uncrowded areas (see [Sin03]). A program for Space Based Radar satellites will be deployed in a staged manner. The Pentagon plans to begin launching a first constellation into Low Earth orbit in 2012. This constellation will have small gaps in its coverage early in the deployment. The initial constellation will indeed consist of between 9 and 12 satellites, which will lead to coverage gaps as long as 5 minutes. The capacity will be improved in 2015 with the launch of a second set of satellites. This program is thus a good example of “staged deployment”. However the on-orbit satellites will not be reconfigured to form a constellation with the launched satellites. The launched satellites will rather occupy the remaining free spots of the initial constellation not fully populated. We can notice that there is no orbital reconfiguration in this example. There is also no continuous global coverage requirement. This example points out the differences in requirements between military and commercial applications.

An example of orbital reconfiguration for military purposes is given by Henry and Sedwick [HS01]. Henry and Sedwick explain that orbital planes can drift using small variations of the satellites altitudes. This drifting would allow the constellation to be reconfigured in order to focus the capacity on a particular region. Repeating ground tracks (resonant orbits) in LEO are crucial in achieving higher coverage in some areas at the expense of coverage in other areas. This strategy would be very interesting
with radar constellations or optical reconnaissance satellites when a conflict arises in a particular region of the globe.

1.3 Orbital Reconfiguration Definition

As explained in the last section, the term “reconfiguration” for constellations of satellites has been principally used to designate the set of necessary maneuvers to recover service after the failure of a satellite. In this thesis, the term reconfiguration will be employed in a more ambitious way since it requires the motion of an entire constellation. Subsection 1.3.1 will present a generic definition for Satellite Constellation Reconfiguration, whereas the Subsection 1.3.2 will describe a particular type of Reconfiguration. Only this type of Reconfiguration will be studied in the thesis.

1.3.1 Generic Definition

A possible definition for Satellite Constellation Reconfiguration is: deliberate change of the relative arrangements of satellites in a constellation by addition or substraction of satellites and orbital maneuvering in order to achieve desired changes in coverage, performance or capacity.

1.3.2 Particular Case

This subsection describes the particular case of Satellite Constellation Reconfiguration, studied in this thesis. Firstly, this thesis will focus on constellations with a single global coverage requirement (Iridium and Globalstar type). This requirement adds difficulties compared to the examples described in the literature review. Moreover, the goal of the reconfiguration will be to increase the capacity of a constellation after its initial deployment. Satellites need to be added in stages. The on-orbit satellites are then “reconfigured” to form a new constellation with the additional satellites incorporated. In other words, they are transferred from their initial orbit into a new trajectory, lower in altitude.
Chapter 4: Thesis Purpose and Organization

The purpose of this thesis is to study in detail the technical way to achieve a constellation reconfiguration in the sense explained in the previous section. The staged deployment strategy itself is not the topic of this thesis. The thesis is only focused on the study of one reconfiguration and proposes an optimal set of maneuvers to reconfigure an initial constellation into a bigger one with more satellites and thus higher capacity. The purpose of this thesis is to study in detail the technical way to achieve a reconfiguration.

1.4 Thesis Purpose and Organization

Chapter 2 consists of the Solution Framework. The Block Diagram of the entire baseline scenario for the reconfiguration is proposed. The different questions raised by the reconfiguration maneuvers are presented, and a solution framework is developed to describe the formal problem. Chapter 3 will describe the formal problem of one reconfiguration and propose an optimal set of maneuvers to reconfigure an initial constellation into a bigger one with more satellites and thus higher capacity.

The reconfiguration consists thus of repositioning on-orbit satellites into another constellation. Figure 1-2 summarizes the reconfiguration process.

\[ N(A) = \text{number of satellites in configuration } A \]
\[ N(B) = \text{number of satellites in configuration } B \]

Figure 1-2: Schema of a Constellation Reconfiguration.
optimization procedure ("framework") is presented, and the objective vector, design vector and fixed parameters are described. This chapter shows that four separate optimizations could be required in the proposed method to solve the various assignment, maneuver and timing problems of reconfiguration.

Chapter 4 describes in detail the different modules implemented to run the framework. The assumptions and limitations of each module are discussed carefully.

In Chapter 5, the procedure is executed and interesting results are presented. This chapter will first discuss the interest in terms of fuel consumption to reconfigure a constellation depending on its type (Walker or Polar), inclination, altitude and of the type, inclination and altitude of the final configuration. For this purpose, reconfigurations in three regions (LEO, MEO and GEO) will be studied. Reconfigurations between these three regions will also be discussed. After this, the entire simulator will be run for a sample of reconfigurations of Polar constellations in Low Earth Orbit. This limitation to LEO Polar constellations does not alter the general scope of the methodology. Different scenarios and strategies will be commented in this case. At the end of this chapter, the economic opportunity of different paths of reconfigurations proposed by Chaize [CHAI03] will be discussed and a discussion will be made on High Fidelity Simulation. The simple framework will be compared with a High Fidelity Simulation Framework and several questions will be answered. How can such a simulation be built? What assumptions should be removed from the simple framework? What tools could be used and how do the high fidelity results compare with the simple model?

Finally, Chapter 6 summarizes the findings and sets recommendations for future studies in this field. In particular the issue of deploying a satellite constellation in several layers of different altitudes should be studied as an alternative or complement to orbital reconfiguration. Chapter 6 will also point out the limitations of the framework and all the improvements that could be made in a more detailed study.
Chapter 2

Problem Description and Background

The purpose of this thesis is to describe concepts and ideas that could be applied in an orbital reconfiguration of satellite constellations. Due to the complexity of the problem, some assumptions have to be made. As described briefly in Section 1.3, new satellites have to be launched in order to increase the capacity of the constellation. It will be assumed that all satellites, the on-orbit satellites and the satellites on the ground, are built and equipped the same, except for their fuel load. The dry mass $m_d$ would be identical, but not the wet mass, $m_w$. This assumption is not obvious since if the altitude of the satellites is changed, the hardware requires some modifications. Indeed, to produce a particular beam pattern on the ground, the characteristics of the antenna depends on the altitude of the satellites. Reconfiguration within the satellites themselves therefore also needs to be achieved. However, this problem will not be considered in the present thesis. The satellites considered in this thesis will have characteristics close to Iridium satellites. Particularly the dry mass will be the same: 700 $kg$. The extra-mass of fuel necessary to achieve eventual transfers is not taken into account in these 700 $kg$. The choice of small satellites is judicious. Indeed the extra fuel mass needed on a satellite is driven by its dry mass. The higher the dry mass is, the higher the fuel mass necessary to achieve the maneuver.

In the following section, the different issues raised by an orbital reconfiguration
are outlined. Then Section 2.2 will introduce a scenario in two distinct phases: the launch of new satellites followed by the transfer of the on-orbit satellites.

2.1 Issues Raised by the Reconfiguration

Below is a list of questions that this thesis intends to answer. First of all, some notations must be specified. As explained in Chapter 1, a reconfiguration is a set of orbital maneuvers to evolve from a constellation $A$ to a constellation $B$. The number of satellites in the initial and final constellation will be called, $N_{sats}(A)$ and $N_{sats}(B)$ respectively. As reconfigurations we consider only an increase capacity, we have $N_{sats}(B) > N_{sats}(A)$. The number of satellites to be launched is thus $N_{sats}(B) - N_{sats}(A)$. In a first approach, spare satellites are not considered.

- The first question to be answered is to determine the optimal maneuvers for transferring the $N_{sats}(A)$ on-orbit satellites into slots of the constellation $B$. Those maneuvers will have to minimize the total Delta V noted $\Delta V_{total}$ for the entire reconfiguration summed over all on-orbit satellites.

$$\Delta V_{total} = \sum_{k=1}^{N_{sats}(A)} \Delta V_{k^{th} satellite}$$  \hspace{1cm} (2.1)

Each satellite of the initial constellation will be assigned to a slot of the new constellation in order to minimize this $\Delta V_{total}$. Knowing the propulsion system utilized for the transfer (particularly the $Isp$), the value $\Delta V_{k^{th} satellite}$ will allow to compute the additional mass of fuel necessary to achieve the transfer of the $k^{th}$ satellite of the constellation $A$. Alternative objective functions for optimizing the reconfiguration are discussed in Chapter 6. Instead of minimizing the total $\Delta V$, we could have chosen to minimize the difference in $\Delta V$ between satellites.

- Another problem is set by the assignment of the satellites on the ground. All the satellites of each launch have to be assigned to the same orbital plane. This strategy allows to prevent a costly non-coplanar repositioning in terms of fuel.
The launcher delivers its on-board satellites to the plane they are assigned to. The capacity of the chosen launcher is a parameter that has to be taken into account during the assignment process. This constraint makes the assignment process more difficult. There are actually two overlapping assignments: the assignment of the launched satellites and the assignment of the on-orbit satellites. The second assignment is dependent on the first one.

- To maneuver the on-orbit satellites, several possibilities are available. The satellites can utilize their own propulsion system or be transported by a Space Tug. Currently, no universal space tug exists. Only Orbital Transfer Vehicle (OTV) that add a preprogrammed amount of $\Delta V$ beyond LEO exist [SMAD99]. This solution is currently being studied in the Space Systems Laboratory. Nevertheless, due to the impossibility to use a database of prices to estimate the cost of this solution, this option was not considered. The study is limited to extra fuel carried by the satellites at the time of the initial launch. The question is to determine what kind of propulsion systems should be utilized for doing the transfer. Two different types of systems are considered: electric propulsion and chemical propulsion. In electric propulsion, some different thrusters will be studied: Arcjet, Resistojet, Plasma Thruster, Hall Thruster and Ion Engines. The advantage of chemical propulsion is a priori the speed for achieving the transfers. But due to the low $I_{sp}$, the extra mass of fuel will be high. In electric propulsion, the high $I_{sp}$ will entail a lower mass of fuel but the time for transferring the on-orbit satellites will be higher leading to outage costs. An achievement of this thesis will be to determine to which effect the reconfiguration cost is more sensitive. A tradeoff is thus expected.

- A launch strategy has to be found for the satellites on the ground. Two types of difficulties have to be solved. First of all, a launcher (or several launchers) should be selected. The choice of the launcher(s) depends obviously on the satellite characteristics such as mass and size and on the orbital destination. For instance to launch into a Polar orbit (inclination $\approx 90$ deg.), some launch-
ers, depending on the launch site, are better suited. The other problem is to determine a launch planning. The required time for launching all the satellites on the ground, is a limiting factor for the reconfiguration. To estimate this time is a difficult task. The launch market is not easily modeled: the interactions between the customers, the satellites manufacturers and the launch companies are very complex in practice.

- During the transfers of the on-orbit satellites, the constellation will not be 100% operational. There will obviously obviously some holes in the coverage entailing service outages. The lost revenue due to these outages has to be quantified. It is a part of the total reconfiguration cost. The order of magnitude of this cost has to be compared notably with the other chunks of the total cost.

- The on-orbit satellites can not be transferred randomly to their new positions in the constellation $B$. The service outages depend directly on these transfers. The idea is to find an optimal transfer schedule allowing to maximize the coverage capability of the constellation during the transfer.

The project is not a classic Multidisciplinary System Design Optimization ($MSDO$) problem. The purpose is not to find a best design, but more to find an optimal way to move a constellation $A$ into a constellation $B$. Constellation $A$ is given a-priori. It is “Design” in the sense that some possibilities exist to do this maneuver and that the methodology will allow to determine the optimal one, in terms of time, energy ($\Delta V$), cost or partial coverage. However, the project deals clearly with a system (i.e. a constellation of satellites) and will require some optimizations that will be presented further in this thesis. The multidisciplinary aspect of the project is lastly obvious, since it requires notions of astrodynamics, propulsion, cost modelling, launch modelling and operations research.
2.2 A Two-Phased-Reconfiguration Scenario

As explained previously, a reconfiguration consists of two types of maneuvers: the orbital transfer of $N_{sats}(A)$ on-orbit satellites to slots of constellation $B$ and the launch of $N_{sats}(B) - N_{sats}(A)$ satellites to occupy the remaining slots of $B$. It was decided that the transfer phase occurs only after the ground satellite launch phase. This strategy allows to minimize the risk of service interruption. Indeed if the two phases are realized at the same time (or if the launch phase occurs later) and if unfortunately one launch fails, the delay for replacing the lost satellites in the failed launch will entail a period of longer service outage. Indeed the constellation $A$ is no longer at full capacity and the final one is not complete in such a scenario. With this two-phased-scenario (launch first, then transfer), a launch failure will just entail to postpone the beginning of the transfer phase, the constellation $A$ remaining 100% operational during this delay. Figures 2-1 and 2-2 summarize this scenario in two distinct phases. Figure 2-1 shows the launch of the new satellites. During this phase, the on-orbit satellites remain in their initial orbits (in other words, in the initial configuration $A$). The new satellites are directly sent to their final orbits and slots in $B$. Figure 2-2 indicates the transfer of the on-orbit satellites of $A$ to the remaining slots of the configuration $B$. Those slots are represented with dashed lines.

2.3 Formal Mathematical Problem Definition

The formal mathematical problem definition could be written as:

$$\begin{align*}
\min \quad & \text{Cost}(A \rightarrow B) \\
\text{s.t.} \quad & Isp_{\text{min}} \leq Isp \leq Isp_{\text{max}} \\
\quad & L.V. \text{max payload volume} \\
\quad & 200 \leq h_a \leq h_b \leq h_{\text{GEO}} \\
\text{given} \quad & \text{constellation A,B} \\
\quad & M_{\text{sat}}(\text{dry})
\end{align*}$$
Figure 2-1: The first phase: the launch phase of new satellites.

First phase:
launch of new satellites

Figure 2-2: The second phase: the transfer phase of the on-orbit satellites.

Second phase:
transfer of the on-orbit satellites

Final slot
CHAPTER 2 SUMMARY

This chapter dealt with the descriptions of the problem and background. The different questions raised by the Reconfiguration were described in detail which allow the reader to have a global understanding of the problem. Those questions will be answered at least partially in the next chapters. Lastly, the end of this chapter explained why a two-phased scenario was chosen.
Chapter 3

Project Framework: Satellite Constellation Reconfiguration

This Chapter will develop a framework to study the orbital reconfiguration problem. In the first section, design variables, parameters, constraints and objectives will be identified. A version of the block diagram (Figure 3-2) utilized for this project will be commented. The second section will point out the four optimization methods needed in this model.

3.1 Framework Development

3.1.1 Objectives

The overall objectives for the solution to the orbital reconfiguration problem are:

- the minimization of the Energy necessary to execute the transfers or, in other words the minimization of the total Delta V for transferring all the on-orbit satellites: \( \Delta V_{\text{total}} \).

- the minimization of the total time, \( T_{\text{tot}} \), required for reconfiguration. This time is, as the two phases are separated the sum of the launch phase time and of the transfer phase time: \( T_{\text{total}} = T_{\text{launch}} + T_{\text{transfers}} \).
• the minimization of the total cost, $C_{tot}$ of reconfiguration including the production cost of new satellites, the cost for launching these new satellites, the cost of the fuel necessary for the on-orbit satellite transfers and the cost due to service outages: $C_{total} = C_{prod} + C_{launchs} + C_{fuel} + C_{outage}$. The cost of the ground segment is not included (impossibility to obtain even rough estimations). Moreover it is not totally relevant in this case.

• the mean value in % of the constellation coverage during the transfer operations $Cov_{mean}$ should be maximized. This mean coverage could be computed as an integrated metric over time:

$$Cov_{mean} = \frac{1}{T_{transfers}} \int_0^{T_{transfers}} cov(t) dt. \quad (3.1)$$

where $cov(t)$ represents the coverage value with respect to time. It will be explained in Chapter 4 how Equation 3.1 is computed in detail.

The objector vector $J$ is summarized in Equation 3.2.

$$J = \begin{bmatrix} \Delta V_{total} \\ T_{total} \\ C_{total} \\ Cov_{mean} \end{bmatrix} = \begin{bmatrix} \text{Total Delta V} \\ \text{Total Time} \\ \text{Total Cost} \\ \text{Mean Coverage} \end{bmatrix} \begin{bmatrix} \text{[m/sec]} \\ \text{[s]} \\ \text{[B$\$]} \\ \text{[%]} \end{bmatrix} \quad (3.2)$$

The relationships between these objectives are too complex to determine a-priori. However, some trends are expected. The minimization of the Energy necessary to execute the transfers will also minimize the cost of extra-fuel. Nevertheless, the $\Delta V$ has no impact on the other chunks of the cost and on the two other objectives: time and coverage. The Time is minimized in the sense that the best strategy allowing to compress the launch schedule of new satellites could be chosen. The time to launch the new satellites, $T_{launch}$, is however independent of the other objectives. The time to execute the transfers, $T_{transfers}$ will have an influence on the cost of service outage $C_{outage}$ and on the mean coverage $Cov_{mean}$. But it is difficult to determine the effect on both objectives. This is explained in more detail in Section 3.1.5.
3.1.2 Parameters

The parameters represent fixed quantities that are not changeable during the constellation reconfiguration process. Satellite characteristics such as dry mass ($\approx 700 Kg$) are among these parameters. The constellation studied will only insure single coverage (i.e. $n = 1$) on the Earth’s surface. Both the initial and final constellation will satisfy this. The initial constellation will be considered as a constant, except for the study in Chapter 5 of different possible reconfigurations depending on the altitude (LEO, MEO, GEO) and on the type of constellation (Polar or Walker). The parameters characterizing a satellite constellation are: the type ($C_a$ for the constellation $A$) the altitude ($h_a$ for $A$) and the minimum elevation angle ($\epsilon_a$ for $A$). Figure 3-1 shows these parameters. However it is assumed that $A$ was designed and optimized separately. The design phase of $A$ is not the purpose of this study.

In this chapter, the constellation reconfiguration framework is set up in general. However for applications, particular values for $A$ will be picked in a later chapter. For the constellation $A$, the baseline parameters will be fixed respectively to:

- $C_a =$’polar’
- $h_a = 2000 km$
- $\epsilon_a = 5 deg.$

This initial constellation will be studied because its low capacity could be reasonably improved by a reconfiguration: only 21 satellites are needed in this particular constellation, to be compared with the 66 satellites of the IRIDIUM constellation ($h = 780$ km, $\epsilon = 8$ deg). This constellation $A$ represents the initial deployment.

3.1.3 Design Variables

The design variables are the propulsion system chosen for the on-orbit satellite transfer, $P_r$, and the final constellation characteristics: the altitude $h_b$ and the elevation $\epsilon_b$. There are two methods to increase the capacity of a constellation: either the altitude is decreased or the min elevation angle is increased. This condition implies natural
Figure 3-1: Constellation Design Variables. The design variables that can be modified to increase the capacity after the deployment are the altitude ($h$) and the elevation angle ($\epsilon$). This drives the number of satellites, since global coverage has to be maintained.

bounds for $h_b$ and $\epsilon_b$: $h_b \leq h_a$ and $\epsilon_b \geq \epsilon_a$. The design vector, $x$, embodies the architectural design decisions and is subject to the bounds or discrete choices shown in Table 3.1. The constellation $B$ was included in the Design Vector, because $B$ is not fixed like the constellation $A$ which is given initially and a-priori. Chaize [CHAI03] has shown that several paths of reconfiguration exist depending on the value of the discount rate and also on the market opportunities. The constellation $B$ is thus not known initially. Some constellation candidates exist potentially, hence this choice to put the parameters of $B$ in the Design Vector.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
<th>$x_{LB}$</th>
<th>$x_{UB}$</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_b$</td>
<td>const. type</td>
<td>Polar</td>
<td>Walker</td>
<td>[-]</td>
</tr>
<tr>
<td>$h_b$</td>
<td>altitude</td>
<td>200</td>
<td>36000</td>
<td>[km]</td>
</tr>
<tr>
<td>$\epsilon_b$</td>
<td>min elevation</td>
<td>5</td>
<td>35</td>
<td>[deg]</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Propulsion System</td>
<td>$Is_p = 200$</td>
<td>$Is_p = 10000$</td>
<td>[s]</td>
</tr>
</tbody>
</table>
3.1.4 Constraints

Due to the particularity of the project, no constraints have been imposed. A constraint on the Total Time could have been chosen, for instance limiting the operations duration to 400 days. But the uncertainty about the launch schedule and the consequent approximations concerning the planning render the imposition of such a constraint impractical.
3.1.5 Block and $N^2$ Diagrams

As shown in Figure 3-2, The Block Diagram of the framework is complex. The different modules are inter-connected. The $N^2$ matrix (see Figure 3-3) allows a better understanding of the connections between the different modules. Seven modules are implemented in the framework:

- The constellation module (on the left of the diagram) determines from the altitudes and elevations, the characteristics of the constellation $A$ and $B$: number of satellites, number of planes, angle between ascending line nodes called, $\alpha$, and inclination $i$ [Wal77], [AL98], [AR87].

- From the characteristics and altitudes of both constellations, the astrodynamics module computes two transition matrices that will be explained further in detail in this thesis. These transition matrices return time and $\Delta V$ of each possible transfer for each satellite.

- On the top are represented the assignment modules. This pair of modules allows the optimal assignment of both the on-orbit satellites and the satellites on the ground. The inputs are the transition matrix and the characteristics of the selected launcher. These modules return one of the objectives: the optimal Delta $V$ for transferring all the on-orbit satellites $\Delta V_{total}$.

- The Launch module in the diagram center selects the best launcher for this problem, for the $N(B) - N(A)$ satellites only.

- At the bottom, are represented the schedule modules (for the launches and transfers). They allow to determine another objective: the total time for reconfiguration $T_{total}$.

- From the schedule, the partial coverage module calculates the coverage capacity during the process for both initial and final constellation. It allows to find the fourth objective: the average coverage value in % maintained by the constellation coverage during the operations, $Cov_{mean}$.
Lastly on the right is represented the cost module. The second objective: the total cost $C_{\text{total}}$ is estimated by this module.

We will now describe briefly how the tradeoffs between these four objectives will be resolved. Two factors will influence the results: the type of propulsion system (actually the value of the $I_{sp}$ of this system) and the way of transferring the on-orbit satellites. Cost and Time will vary with the $I_{sp}$. Obviously the Time required for transfers increase with the $I_{sp}$, since thrust levels and accelerations decrease. The impact on the cost is more difficult to determine. Two parts of the total cost are dependent on the $I_{sp}$: the cost of service outage and the fuel cost. The cost of service outage will probably increase with increasing transfer time and also with the $I_{sp}$. The higher the $I_{sp}$ will be, the longer the transfers will be and thus the higher the cost of service interruption will be. On the other hand, the fuel cost depending on the extra-fuel mass, the fuel cost will surely decrease with increasing $I_{sp}$. Indeed the higher the $I_{sp}$ is, the lower the mass necessary to achieve the transfers is. Figure 3-4
<table>
<thead>
<tr>
<th>Index</th>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$C_a$</td>
<td>constellation type of $A$</td>
<td>[-]</td>
</tr>
<tr>
<td>2</td>
<td>$C_b$</td>
<td>constellation type of $B$</td>
<td>[-]</td>
</tr>
<tr>
<td>3</td>
<td>$h_a$</td>
<td>altitude of $A$</td>
<td>km</td>
</tr>
<tr>
<td>4</td>
<td>$h_b$</td>
<td>altitude of $B$</td>
<td>km</td>
</tr>
<tr>
<td>5</td>
<td>$\epsilon_a$</td>
<td>min elevation angle of $A$</td>
<td>deg</td>
</tr>
<tr>
<td>6</td>
<td>$\epsilon_b$</td>
<td>min elevation angle of $B$</td>
<td>deg</td>
</tr>
<tr>
<td>7</td>
<td>$P_r$</td>
<td>propulsion system</td>
<td>[-]</td>
</tr>
<tr>
<td>8</td>
<td>$N_a$</td>
<td>number of satellites in $A$</td>
<td>[-]</td>
</tr>
<tr>
<td>9</td>
<td>$N_b$</td>
<td>number of satellites in $B$</td>
<td>[-]</td>
</tr>
<tr>
<td>10</td>
<td>$p_a$</td>
<td>number of planes in $A$</td>
<td>[-]</td>
</tr>
<tr>
<td>11</td>
<td>$p_b$</td>
<td>number of planes in $B$</td>
<td>[-]</td>
</tr>
<tr>
<td>12</td>
<td>$i_a$</td>
<td>inclination of $A$</td>
<td>deg</td>
</tr>
<tr>
<td>13</td>
<td>$i_b$</td>
<td>inclination of $B$</td>
<td>deg</td>
</tr>
<tr>
<td>14</td>
<td>$\alpha_a$</td>
<td>angle between ascending nodes $A$</td>
<td>deg</td>
</tr>
<tr>
<td>15</td>
<td>$\alpha_b$</td>
<td>angle between ascending nodes $B$</td>
<td>deg</td>
</tr>
<tr>
<td>16</td>
<td>$\Delta V_{i,j}$</td>
<td>transition matrix for energy</td>
<td>$[N_b \times N_b]$ km/s</td>
</tr>
<tr>
<td>17</td>
<td>$i_{i,j}$</td>
<td>transition matrix for time</td>
<td>$[N_b \times N_b]$ s</td>
</tr>
<tr>
<td>18</td>
<td>$Ass$</td>
<td>Assignment of the satellites into slots of $B$</td>
<td>$[N_b \times 1]$</td>
</tr>
<tr>
<td>19</td>
<td>$Cap_{\text{launcher}}$</td>
<td>Capacity of the launcher selected</td>
<td>sats</td>
</tr>
<tr>
<td>20</td>
<td>$\Delta V^\text{modif}_{i,j}$</td>
<td>transition matrix modified by the reassignment</td>
<td>$[N_b \times N_b]$ km/s</td>
</tr>
<tr>
<td>21</td>
<td>$Ass^\text{modif}$</td>
<td>New Assignment</td>
<td>$[N_b \times 1]$</td>
</tr>
<tr>
<td>22</td>
<td>$Name_{\text{launcher}}$</td>
<td>Name of the launcher selected</td>
<td>[-]</td>
</tr>
<tr>
<td>23</td>
<td>$Cost_{\text{launcher}}$</td>
<td>Cost of the launcher selected</td>
<td>[-]</td>
</tr>
<tr>
<td>24</td>
<td>$Time_{\text{transfer}}$</td>
<td>Transfer time for all the satellites</td>
<td>$[N_b \times 1]$ s</td>
</tr>
<tr>
<td>25</td>
<td>$S_t$</td>
<td>Transfer schedule</td>
<td>$[N_b \times 3]$ s</td>
</tr>
<tr>
<td>26</td>
<td>$S_l$</td>
<td>Launch schedule</td>
<td>s</td>
</tr>
<tr>
<td>27</td>
<td>$Cov$</td>
<td>Coverage matrix</td>
<td>%</td>
</tr>
<tr>
<td>28</td>
<td>$\Delta V_{\text{total}}$</td>
<td>Total $\Delta V$ for transferring all the satellites</td>
<td>km/s</td>
</tr>
<tr>
<td>29</td>
<td>$T_{\text{total}}$</td>
<td>Total Time to reconfigure $A$</td>
<td>s</td>
</tr>
<tr>
<td>30</td>
<td>$Cov_{\text{mean}}$</td>
<td>Mean coverage during the operations</td>
<td>%</td>
</tr>
<tr>
<td>31</td>
<td>$C_{\text{total}}$</td>
<td>Total Cost to reconfigure $A$</td>
<td>$B$</td>
</tr>
</tbody>
</table>

Table 3.2: Master Table
summarizes this issue. If the cost of outage and the fuel cost are of the same order of magnitude which is assumed on this figure, an optimal \( Isp \) minimizing the total cost will exist. Nevertheless if the costs have not the same order of magnitude, this tradeoff does not exist anymore. In commercial applications, we can guess that the cost of the service outage will be much higher than in military applications. The military systems have no requirement to insure the service, except during conflicts, contrary to commercial systems which have customers on a continuous basis. Slow reconfiguration might be more acceptable to the military during regeneration phases between major conflicts. One of the contributions of this thesis will be to determine in which situation the commercial systems are: in the situation depicted by Figure 3-4 or in the situation where the cost of service outage is much higher than the fuel cost.

The way of transferring the satellites will influence the cost, but also the coverage. If the transfer schedule is compressed, the cost of service outage will surely be different than if the schedule is staggered. It is difficult to say a priori which case will entail a lower cost. Moreover a compressed schedule will entail a very low cover-
age during the operations, whereas a staggered schedule will entail a higher coverage
during the transfers. If a requirement concerning coverage and time is required by
the manufacturer, a tradeoff has to be found concerning the schedule. Indeed the
schedule must be sufficiently compressed to respect the timing requirement but not
too much in order to insure the coverage requirement.

3.2 Four Optimizations for One Problem

Four optimizations would be required ideally to solve the overall problem. The com-
plete assignment of the satellites in the constellation $B$ utilizes two methods in series
(see the Subsection 3.2.1 and 3.2.2). The Launch Selection Process would require an
optimization technique to determine the best launcher to launch the ground satellites.
An optimization loop (see the loop at the bottom of the Block Diagram-Figure 3-2)
is lastly required to determine the optimal transfer schedule.

3.2.1 Assignment Problem and Auction Algorithm

The problem of putting $N(B)$ satellites ($N_{sats}(A)$ from the initial constellation and
the rest from the ground) in $N(B)$ slots can be represented as an assignment problem
where the necessary total Delta V to achieve the transfers has to be minimized. The
flow Network in Figure 3-5 distinguishes the arcs coming from the on-orbit satellites
and the arcs coming from the launched satellites. This distinction is due to the
approximation that the Delta V necessary to reposition the ground satellites in their
respective orbits is negligible. So, $\Delta V_{\text{launched satellite}} \approx 0$. Obviously the $\Delta V$
from the ground ($\approx 7.5 \text{ km/s}$) is not included. This $\Delta V$ is part of the Launch Process,
since this impulse is given by the Launcher upper stages and has not to be taken
into account for the satellites themselves (it is taken into account in the launch cost,
however).

The flow network can be interpreted if we consider that there are $N(B)$ persons
and $N(B)$ projects. We wish to assign a different person to each project while min-
imizing a linear cost function of the form $\sum_{i=1}^{N(B)} \sum_{j=1}^{N(B)} c_{i,j} f_{i,j}$ where $f_{i,j} = 1$ if the
Figure 3-5: Assignment problem for a reconfiguration between constellation A with $N_{sats}(A)$ satellites and constellation B with $N_{sats}(B)$ satellites.

The $i^{th}$ person is assigned to the $j^{th}$ project, and $f_{i,j} = 0$ otherwise. In the example studied, the coefficient $c_{i,j}$ represents the delta V for transferring the $i^{th}$ satellite of the constellation A to the $j^{th}$ slot of B: this value will be called $\Delta V_{i,j}$ in the rest of the thesis.

Bertsimas and Tsitsiklis [BERT97] explain that a very efficient method for solving this flow network problem exists: ”the auction algorithm”. The idea is to represent the situation as a bidding mechanism whereby persons bid for the most profitable projects. It can be visualized by thinking about a set of contractors who compete for the same projects and therefore keep lowering the price they are willing to accept for any given project. The auction algorithm is described below:

- A typical iteration starts with a set of prices $p_1, \ldots, p_n$ for the different projects, a set $S$ of assigned persons, and a project $j_i$ assigned to each person $i$ of $S$. At the beginning of the algorithm, the set $S$ is empty.

- Each unassigned person finds a best project $k_i$ by maximizing the profit $p_k - c_{i,k}$.
over all $k$. Let $k'_i$ be a second best project, that is,

$$p_{k'_i} - c_{1,k'_i} \geq p_k - c_{1,k} \quad \text{for all } k \neq k_i$$  \hspace{1cm} (3.3)

Let

$$\Delta_{k_i} = (p_k - c_{1,k}) - (p_{k'_i} - c_{1,k'_i})$$  \hspace{1cm} (3.4)

Person $i$ "bids" $p_{k_i} - \Delta_{k_i} - \epsilon$ for project $k_i$.

- Every project for which there is at least one bid is assigned to a lowest bidder; the old holder of the project (if any) becomes unassigned. The new price $p_i$ of each project that has received at least one bid is set to the value of the lowest bid.

The auction algorithm terminates after a finite number of stages with a feasible assignment. Moreover if the cost coefficients $c_{i,j}$ are integer and if $0 < \epsilon < \frac{1}{n}$, the auction algorithm terminates with an optimal solution [BERT97]. $\epsilon$ is an intermediate of calculation. Without $\epsilon$, the algorithm would often be deadlocked. In terms of time of calculation, the auction algorithm is very efficient since it runs in time $O(n^4 \max c_{i,j})$.

To illustrate this method, the algorithm will be applied on a simple case that could be solved by hand. Two persons 1 and 2, and two projects $A$ and $B$ are considered. The purpose is to assign each person to a project, knowing the cost that each project will imply for 1 and 2. Table 3.3 indicates these costs. The obvious assignment minimizing the total cost would be to assign project $A$ to 1 and project $B$ to 2. We will check that the algorithm returns the same result. First of all, a set of prices for the different projects must be chosen. Arbitrarily 10,000 and 20,000 $ were chosen for $p_A$ and $p_B$, respectively. $\epsilon$ was chosen equal to 0 in this example, since $\epsilon$ has no influence on the solution. In the first iteration, each person finds a best project

\begin{table}[h]
\centering
\caption{Costs table for the simple case}
\begin{tabular}{|c|c|c|}
\hline
 & Project $A$ & Project $B$ \\
\hline
Person 1 & 5,000 $ & 10,000 $ \\
Person 2 & 5,000 $ & 1,000 $ \\
\hline
\end{tabular}
\end{table}
maximizing the profit. For person 1, the profit of project $A$ is $p_A - c_{1,A} = 5,000$ and the profit of project $B$ is $p_B - c_{1,B} = 10,000$. So, 1 will bid for project $B$. The value of the bid is $p_A - \Delta = 20,000 - 5,000 = 15,000$ where $\Delta$ represents the difference between the profit of the two projects. For the second person the situation is as follows: $p_A - c_{2,A} = 5,000$ and $p_B - c_{2,B} = 19,000$. 2 will bid for $B$ also. The value of the bid is $20,000 - 14,000 = 6,000$. There are two bids for $B$ and zero for $A$. $B$ is assigned to the lowest bid, so to 2. The new price of $B$ is the value of the bid of 2: $p_{B}^{modif} = 6,000$. For the second iteration, only 1 is considered. The profit of project $A$ is still 5,000$ for 1, whereas the profit of $B$ is now equal to $6,000 - 10,000 = -4,000$. Logically 1 chooses the project $A$ and is assigned to this project. The auction algorithm works well on this simple example.

3.2.2 Loop for assigning the Ground Satellites

By imposing $\Delta V_{\text{launched satellite}} = 0$ in the flow network depicted in Figure 3-5, the auction algorithm proceeds by assigning first the on-orbit satellites into slots of the constellation $B$. The launched satellites go then to the remaining slots. Although the Delta V is minimized with this method, this approach is not satisfactory. Recall that there is a strong constraint concerning the ground satellites. In other words the satellites of the same launch have to be assigned to the same plane in order to prevent a costly repositioning. The assignment returned by the auction algorithm will not necessarily satisfy this constraint. That is why the implementation of a loop for assigning the ground satellites (on top of the Block Diagram-Figure 3-2) was necessary. The goal of this loop is to refine the assignment returned by the auction algorithm so that the new assignment satisfies the launch constraint. The loop will be described in detail in Chapter 4 and an assignment process will be discussed in Chapter 5. However, the assignment refinement can penalize some on-orbit satellites in terms of $\Delta V$ by changing their final slots, the transfer to the new reassigned slot being too costly. Indeed after the reassignment, some satellites may need such a high quantity of extra-fuel that the transfer could become technically infeasible. A tolerable limit concerning the extra-mass of fuel has to be chosen. If the satellite needs
a mass of extra-fuel above this limit, the manufacturer could decide to abandon this satellite (this satellite becoming effectively a spare satellite) and to launch a new one instead. All these conceptual problems will be discussed further.

### 3.2.3 Launch Vehicle Selection Process

The Launch Selection Process should in theory find the optimal subset of launch vehicles that can deploy all the satellites on the ground at minimum cost, time or even risk depending of the objective defined by the manufacturer. However in the simulator, parallel launches will not be considered, as they are not supported by the existing launch infrastructure at the present time. This simplification leads to a significant increase of the time to deploy the ground satellites. But it gives a realistic order of magnitude for this deployment time.

### 3.2.4 The Optimal Transfer Schedule

The optimal transfer schedule is the schedule minimizing the cost of service interruption. The idea would be to find a feedback link between the transfer schedule module and the module computing the cost of service outage. As explained previously, it is difficult to predict which strategy between a compressed schedule and a staggered schedule will imply low cost of service outage. Intuitively, the cost of service outage will depend on the partial coverage maintained during the transfer phase and on the duration of this transfer phase. We can suppose that the higher the transfer time will be, the higher this cost will be. Also, the lower the coverage will be during this phase, the higher the cost will be. Nevertheless those two effects are opposed. Indeed if the schedule is compressed, the time needed to achieve all the transfers will be short, but the coverage during this period will be low since many satellites are moving at once (the assumption is that the satellites are not operational during the transfers). In the case of a staggered schedule, the time needed to achieve the transfers will be higher since few satellites are moving at once. Logically the coverage will be higher in this case. The purpose of the loop would be to determine to which effect the cost
of service outage is more sensitive: coverage or time. If the two effects have roughly
the same influence on the cost a tradeoff between staggered and compressed schedule
would have to be found. In this case, the loop would return an intermediate schedule
between staggered and compressed. However, due to the difficulty to find a feedback
between the transfer schedule module and the module computing the cost of service
outage, the loop was not implemented in the present framework. This concept should
be considered during future studies. Nevertheless, in order to show the influence of the
way of transferring the satellites, some different schedules (staggered or compressed)
were implemented. In Chapter 4, those scenarios will be briefly described. The type
of scenario was thus added to the Design Vector. An additional complication would
be added if the geographic non-uniformity of the demand were taken into account.
This thesis assumes that demand is uniformly distributed around the globe.

Another issue should be considered concerning the transfer schedule: the capacity
of the mission control center to be able to monitor/command several transfers simulta-
neously. A user-specified capacity should be theoretically found. Can we monitor
and command all the satellites simultaneously, only three, only two? It is difficult to
say in a generic solution of this problem.
CHAPTER 3 SUMMARY

The purpose of this chapter was to develop a framework to study the orbital reconfiguration problem. The Block Diagram is the key figure of this chapter since it points out the complexity of this problem. Let’s describe briefly this Diagram. First, the type, altitude and min elevation angle are used to obtain the constellation parameters: number of satellites, planes, inclination. Next the assignment problem is solved. This assignment puts each satellite into a slot of the final configuration in order to minimize the total $\Delta V$. The characteristics of the launcher selected allow to deduce the launch schedule. Then the best transfer schedule is deduced from a loop allowing to minimize the cost of service outage. From the two schedules, the total time of reconfiguration is deduced, the coverage during the operations as well. Finally, the cost is determined from the number of new satellites to be built and launched, from the amount of fuel necessary to achieve the transfer and from the value of the outage cost.

This framework was kept as general as possible. This framework could be thus applied to every type of reconfiguration (LEO, MEO or GEO for altitude, polar, Walker or even geosynchronous for the type) and for every type of applications (military or commercial). In the following chapters, the framework will be applied to particular cases.
Chapter 4

Detailed Description of the Reconfiguration Framework Modules

In this chapter, the different modules implemented for the framework are described in detail. Inputs, outputs and functions are presented for each module. Since the focus of this thesis is to develop a framework applicable to a deeper study of constellation reconfiguration, the theory on which the modules are based is carefully described. The assumptions and limitations are also pointed out. The Block Diagram 3-2, presented in Chapter 3 depicts how these modules are connected.

4.1 Constellation Module

INPUTS $C, h, \epsilon$ \Rightarrow OUTPUTS $T, P, \alpha, i$

4.1.1 Presentation of Polar and Walker constellations

Two methods have been developed to generate optimal constellations of large number of satellites for continuous coverage: the Walker method and the Streets of Coverage method. Adams and Lang [AL98] explain the differences between these two methods.
The constellations generated by these two methods are respectively called Walker constellations [Wal77] and polar constellations (Rider is often credited for the Iridium design [Rid85]). The Walker method proposes to organize the satellites using inclined circular orbital planes, whereas the Streets of Coverage method utilizes polar orbits. The Walker constellations are characterized by an uniform distribution of the ascending nodes (RAAN) for the different planes. This is not the case for polar constellations that are optimally phased between co-rotating interfaces. RAAN’s are uniformly spaced for polar constellations with arbitrary inter-plane phasing. Moreover the polar constellations need many satellites per plane and the best coverage is obtained at poles, while Walker constellations have few satellites per plane and a best coverage at mid-latitude close to the inclination of orbits. However in both case, the number of satellites per plane depends on altitude. It decreases when the altitude increases in order to maintain global coverage.

The parameters necessary to describe a Walker constellation are the total number of satellites in constellation $T$ ($N$ alternate nomenclature), the number of commonly inclined orbital planes $P$, the relative phasing parameter $F$ and the common inclination for all satellites $i$. The parameters necessary to describe a Polar constellation are $T$ and $P$ as well. But $\alpha$, the angular separation between ascending nodes needs to be known, since in this case $\alpha$ is different from $180/P$ deg in the case of optimal phasing. However, the inclination, $i$, for polar constellations is close to 90 deg.

Adams and Rider [AR87] have given a condition for global coverage for polar constellations:

$$\pi = (P - 1)(\alpha) + \phi$$  \hspace{1cm} (4.1)

where $P$ is the number of planes, $\alpha$ the angle between co-rotating orbits (also called angular separation between ascending nodes) and $\phi$ the angle between counter-rotating orbits. For Walker optimization, no closed form is known and we resort to heuristic optimization.
4.1.2 Module Utilization

The "constellation module" was implemented by de Weck and Chang. From the characteristics of the constellation, type (Walker or Polar), altitude \((h)\), elevation \((\epsilon)\) and multiplicity of coverage \((n, n \text{ was limited to 1 for the project})\), the module returns the parameters defined in the previous section: \(T, P, i\) and \(\alpha\).

To benchmark this module the characteristics of Iridium and Globalstar were utilized. Iridium is a polar constellation with an altitude of 780 km, and an elevation \(\epsilon\) of 8.2 deg. With these inputs, the module returns values for \(T, P, i\) and \(\alpha\) of 66 satellites, 6 planes, 90 deg and 30 deg. These values correspond to the real characteristics of the Iridium constellation. Figure 4-1 depicts the coverage of this constellation. An asterisk indicates subsatellite point (Nadir point) for each satellite in the constellation. Lines indicate the coverage area.

Globalstar is a Walker constellation, with an altitude of 1400 km and an elevation angle of 10 deg. The module returned in this case \(T=50 \text{ sats, } P=5 \text{ planes, } i=90\) deg and \(\alpha=89.4\) deg (see Figure 4-2), whereas the real Globalstar constellation has \(T = 48\).
The results returned by the module being trustworthy, the module was applied in
the framework. It is a key module linking the design vector and the parameters to
the other modules.

4.2 Astrodynamics Module

INPUTS \( Pr, T, P, i, \alpha, h \) \( \Rightarrow \) OUTPUTS \( \Delta V_{i,j}, t_{i,j} \)

4.2.1 Background

An orbital slot in a constellation is defined by 6 orbital elements (\( a, e, i, \Omega, \omega, \nu \)).
The inclination \( i \) with respect to the equator and the longitude of the ascending node
\( \Omega \) define the orbital plane of the satellite, \( \Omega \) representing the angle from the vernal
equinox to the ascending node. The ascending node is the point where the satellite
passes through the equatorial plane moving from south to north. The semi-major axis
\( a \) describes the size of the elliptic orbit, whereas the eccentricity \( e \) describes the shape.
The argument of perigee \( \omega \) is the angle from the ascending node to the eccentricity
vector. It allows to find the position of the perigee of the ellipse in the orbital plane
and thus gives the orientation of the ellipse. Finally, the true anomaly \( \nu \) gives the
position of the satellite on the ellipse with respect to the perigee. The true anomaly is
the only orbital element depending on time. Thus, in order to determine the position
of a satellite unambiguously, a time reference (=Epoch) needs to be defined. The
other 5 elements are constant. Figure 4-3 summarizes this.

However, the orbits considered in this thesis are circular. This assumption simplifies
the orbital elements since \( e = 0 \) in a circular orbit and \( a = R_{earth} + h \), where \( R_{earth} \)
is the mean radius of the Earth and \( h \) the altitude of the orbit. In addition, it is no
longer possible to define a perigee because all the points of the orbit have the same
altitude. \( \omega \) and \( \nu \) are replaced by \( \theta \) which represents the angle between the satellite
and the line of nodes. As a consequence, only 4 orbital elements are necessary to
determine the exact position of a slot in the case of circular orbits: the altitude \( h \),
the inclination \( i \), the longitude of the ascending node \( \Omega \) and the true anomaly \( \theta \).
A transfer for reconfiguration will imply changes to those 4 parameters. The change of $\Omega$ and $i$ will allow to put the satellite in the right orbital plane and the change of altitude $h$ will place the satellite in the right orbit. However, once these first maneuvers are achieved, the satellite and the final slot may have different true anomaly $\theta$. The satellite may thus need to be repositioned into the final orbit (phasing). Different strategies exists for this rendezvous maneuver. One strategy will be described briefly in the case of chemical propulsion in Section 4.2.3. The module implemented to compute the transfer time and $\Delta V$ will not consider explicitly the problem of true anomaly phasing. Typically the $\Delta V$ for phasing is significantly smaller than for changing $i$, $\Omega$ and $h$. The true anomaly, $\theta$, should be taken into account during an eventual high fidelity simulation (explained in Section 5.6). The $\Delta V$ returned by the module for each satellite to each slot will thus depend only on the initial and final altitudes $h_a$ and $h_b$, on the initial and final inclinations $i_a$ and $i_b$ and on the initial and final longitudes of ascending node $\Omega_a$ and $\Omega_b$. In other words, the $\Delta V$ depends
of the characteristics of the initial and final planes: \( \Delta V = f(i_a, i_b, h_a, h_b, \Omega_a, \Omega_b) \). An allowance is made for “worst-case” phasing \( \Delta V \) for each satellite. This assumption neglects the effect of true anomaly and simplifies the assignment problem. Indeed in this case, we do not need to maintain a one-to-one assignment from \( N(A) \) to \( N(B) \). A many-to-one assignment to the appropriate orbital plane of \( B \) is sufficient.

4.2.2 Astrodynamics Module Inputs and Outputs

Recall that this module computes \( \Delta V \) and transfer time from each position on the initial constellation \( A \) to each slot in constellation \( B \). It returns two transition matrices: \( \Delta V_{i,j} \) and \( t_{i,j} \). The inputs are the propulsion system (represented by its \( I_{sp} \)) and the characteristics of the constellations \( A \) and \( B \): altitude \( h \), number of satellites \( N \) and planes \( P \), inclination, \( i \) and angle \( \alpha \) between ascending nodes of neighboring planes.

In Figure 4-4, the typical form of a transition matrix is depicted. We obtain block matrices, since all the satellites of a same plane need the same \( \Delta V \) to be transferred into a plane of the final constellation.

Now we will add a specific example of a transition matrix \( \Delta V_{i,j} \) for the reconfiguration from the polar constellation \( A \) with altitude \( h_a = 36,000 \) km and min elevation
angle $\epsilon_a = 2$ deg. to the Walker constellation $B$ with altitude $h_b = 30,000$ km and min elevation angle $\epsilon_b = 2$ deg. $A$ has 4 satellites (2 planes of 2) and $B$ has 6 satellites (2 planes of 3). The inclination $i_b$ of $B$ is 52.2 deg. The values are in km/s.

\[
\Delta V_{ij} = \begin{pmatrix}
2.6 & 2.6 & 2.6 & 4.9 & 4.9 & 4.9 \\
2.6 & 2.6 & 2.6 & 4.9 & 4.9 & 4.9 \\
4.9 & 4.9 & 4.9 & 2.6 & 2.6 & 2.6 \\
4.9 & 4.9 & 4.9 & 2.6 & 2.6 & 2.6 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 
\end{pmatrix}
\]

We obtain a block matrix. All the satellites of plane 1 of $A$ need a $\Delta V$ of 2.6 km/s to go to plane 1 of $B$ and a $\Delta V$ of 4.9 km/s to go to plane 2. As well, all the satellites of plane 2 of $A$ need a $\Delta V$ of 2.6 km/s to go to plane 2 of $B$ and a $\Delta V$ of 4.9 km/s to go to plane 1. The two ground satellites are assigned a transfer $\Delta V$ of zero.

Two types of propulsion were chosen to do the transfers: chemical propulsion or electric propulsion. Each type of transfer will now be briefly described. Figure 4-5 summarizes the differences between these transfers.

In chemical propulsion, the transfer trajectory is a semi-ellipse. In electric propulsion, the satellite spirals between its initial and final position.

### 4.2.3 Chemical Propulsion Scenario.

The chemical thruster selected for the transfers is an $OF_2$ and $B_2H_6$ Bipropellant with an $I_{sp}$ of 430 sec.

Two strategies were considered for the transfers in chemical propulsion. The first one consists of three different phases:

- a Hohmann transfer for altitude combined with either the inclination change or the node line change
- a simple plane change (before or after the Hohmann transfer depending on
which strategy minimizes the $\Delta V$ required) to change the parameter that was held constant during the Hohmann transfer (longitude of ascending node $\Omega$ or inclination $i$)

- the repositioning in the new orbit consisting of a true anomaly phasing

This strategy was chosen primarily because simple expressions exist for the angular changes when only $\Omega$ or $i$ are changed. When only $i$ varies, the angle between the initial and final planes is: $\Delta i = i_2 - i_1$. When $\Omega$ varies, the angle is: $\Delta \Omega \cdot \sin(i)$ with $\Delta \Omega = \Omega_2 - \Omega_1$.

In the case of a simple plane change, the expression utilized to compute the $\Delta V$ is

$$\Delta V = 2V_i \sin(\lambda/2)$$

(4.2)

where $V_i$ is the initial velocity and $\lambda$ is the angle increment required.

In the case of the plane change combined with Hohmann transfer, the expression is

$$\Delta V = (V_i^2 + V_f^2 - 2V_iV_f \cos(\lambda))^{1/2}$$

(4.3)

where $V_i$ is the initial velocity, $V_f$ is the final velocity and $\lambda$ is the angle change.
required.

The second strategy consists of changing the inclination and the longitude of the ascending node at the same time. At the nodal crossing point of the two orbital planes (initial and final), a maneuver is performed that changes both the inclination and right ascension of the ascending node. This maneuver is illustrated in Figure 4-6. The plane change is also combined with a Hohmann transfer, allowing the change of altitude. Actually this second strategy combines the two first phases of the first strategy. As for the first strategy, the second one ends with a repositioning phase for true anomaly, \( \theta \) in the new orbit.

![Figure 4-6: Change of orbital plane. Both inclination and right ascension of ascending node are changed.](image)

The difficult part of this strategy is to estimate the angle between the two orbital planes. In this case, there is no simple analytical expression for this angle. In the coordinates defined in Figure 4-7, the normal vector of the orbital plane \( \vec{n} \) is equal to

\[
\vec{n} = \begin{cases} 
\sin(i)\sin(\Omega) \\
-\sin(i)\cos(\Omega) \\
\cos(i)
\end{cases}
\]  

(4.4)

If we define \( \vec{n}_a \) and \( \vec{n}_b \) the normal vector of respectively the initial and final orbit
planes, the angle $\alpha$ between the two planes can be obtained from the expression

$$
\cos(\alpha) = \vec{n}_a \cdot \vec{n}_b = \sin(i_a)\sin(\Omega_a)\sin(i_b)\sin(\Omega_b) + \sin(i_a)\cos(\Omega_a)\sin(i_b)\cos(\Omega_b) + \cos(i_a)\cos(i_b)
$$

(4.5)

Now a comparison between the two strategies will be made. Table 4.1 summarizes the results obtained with both strategies for four different transfers. $\Delta V_1$ is the $\Delta V$ computed with the first strategy, $\Delta V_2$ the $\Delta V$ computed with the second one. The angles are in degrees, $\Delta V$ is in km/s.

<table>
<thead>
<tr>
<th>$\Omega_a$</th>
<th>$i_a$</th>
<th>$h_a$</th>
<th>$\Omega_b$</th>
<th>$i_b$</th>
<th>$h_b$</th>
<th>$\Delta V_1$</th>
<th>$\Delta V_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>1000</td>
<td>20</td>
<td>45</td>
<td>1000</td>
<td>5.6</td>
<td>5.6</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>1000</td>
<td>45</td>
<td>20</td>
<td>1000</td>
<td>2.3</td>
<td>1.9</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>2000</td>
<td>20</td>
<td>45</td>
<td>1000</td>
<td>5.4</td>
<td>5.4</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>2000</td>
<td>45</td>
<td>20</td>
<td>1000</td>
<td>2.4</td>
<td>2</td>
</tr>
</tbody>
</table>

The second strategy seems to be more cost efficient in terms of $\Delta V$. Moreover the transfer is shorter with the second option, since the time spent between the Hohmann transfer and the simple plane change is suppressed. In the first strategy, the satellite should wait on its trajectory until its current orbit intersects the desired plane. It is not the case anymore with the second strategy, since the simple plane change is
However, even in the second strategy, the time to do the entire transfer is roughly approximated because of the difficulty to know the exact satellite position during the maneuver, since these calculations are carried out independent of the true anomaly $\theta$. The discussion of high fidelity simulation will consider true anomaly, thus anchoring the reconfiguration to a particular time (Epoch) reference. We have $T_{\text{transfer}} = T_{\text{Hohmann}} + T_{\text{phasing}}$. The duration of the repositioning phase is very complex to determine. Indeed this stage depends obviously on the satellites already arrived in the orbit in constellation $B$ and on their positions on this orbit (i.e. the value of their respective true anomaly). An extreme value corresponding to the worst case was chosen to estimate the phasing duration. It is assumed that the $\Delta V$ required to reposition the satellite is reasonably small with respect to the $\Delta V$ needed for the first phase. The goal is to find a value for the $\Delta V_{\text{phasing}}$ allowing a transfer in a reasonable time, without consuming an important part of the extra mass. From the work done by Chaize on time vs $\Delta V$ for phasing maneuvers in the Appendix of [CHAI03], the following value was chosen arbitrarily and applied in the framework:

$$\Delta V_{\text{phasing}} = 0.5 \text{km/s} \quad (4.6)$$

This value corresponds to a consumption of approximately 90 kg of fuel for the $I_{sp}$ of 430 s and a $S/C$ mass of 700 kg. A repositioning with high thrust will actually consist of doing a Hohmann transfer to a slightly lower orbit, then drifting at higher speed and finally doing a reverse Hohmann transfer in order to rendezvous with the final slot. The $\Delta V$ required for this repositioning is a function of the time $\Delta t$ to do the transfer. Assume the satellite has to be moved forward by an angle $\Delta \Theta$, the expression between time and $\Delta V_{\text{phasing}}$ is given by$^3$:

$$\Delta V \approx \frac{2}{3} \frac{R}{\Delta t - \frac{\omega}{2}} \Delta \Theta \quad (4.7)$$

where $R$ is the orbit radius and $\omega$ the orbit rotation speed.

$^3$Rocket Propulsion course 16.512 -Professor Martinez-Sanchez MIT
The worst case corresponds to a value of $\Delta \Theta = \pi$. This value was applied in the module in order to estimate the time of phasing and also the total time of transfer.

To conclude this section, the $\Delta V$ computed for chemical propulsion is reasonably precise and the transfer time a good approximation. In this model, both $\Delta V$ and transfer time do not depend on the chemical thruster utilized. Time and $\Delta V$ are thus constants with respect of the type of chemical thrusters. This can be explained, since both represent impulsive transfers, i.e. the burn time is very small compared to the transfer time: $T_{\text{burn}} \ll T_{\text{transfer}}$.

### 4.2.4 Electric Propulsion Scenario.

Contrary to chemical propulsion, the transfer with electric propulsion consists of only one stage with continuous thrust. The $\Delta V$ does not depend on the propulsion system. Indeed, $\Delta V$ is given by the expression\(^4\):

$$\Delta V = (V_i^2 + V_f^2 - 2V_iV_f \cos(\frac{\pi}{2}\lambda))^{1/2} \quad (4.8)$$

where $V_i$ is the initial velocity, $V_f$ is the final velocity and $\lambda$ is the angle change required. This expression is very close to the expression obtained in an Hohmann transfer in chemical propulsion. The main difference is the $\pi/2$ inside the cosine. For the angles $\lambda$ considered (between 0 and $\pi/2$), the $\Delta V$ in electric propulsion will be higher than the $\Delta V$ required for impulsive transfers. Indeed $\cos(\frac{\pi}{2}\lambda) \leq \cos(\lambda)$. So, $\Delta V_{\text{electric}} \geq \Delta V_{\text{impulsive}}$.

Table 4.2 summarizes the characteristics of the electric propulsion systems studied. The order of magnitude of the efficiencies, $\eta$ and of the input power come from [SMAD99]-Chapter 17. The Powers were also chosen, knowing that the satellites considered have a transmitter power of 1 kW.

Now the transfer time will be estimated. The acceleration during the electric transfer is assumed to be constant. The transfer Delta $V$, $\Delta V_{\text{transfer}}$ is obtained from the expression: $\Delta V = a.T_{\text{transfer}}$ where $a$ is the acceleration of the spacecraft and

\(^4\)Space Propulsion course 16.522-Professor Martinez-Sanchez MIT
Table 4.2: Electric propulsion systems

<table>
<thead>
<tr>
<th>Name</th>
<th>$Isp$ (sec)</th>
<th>Power (kW)</th>
<th>$\eta$</th>
<th>Manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistojet 500</td>
<td>500</td>
<td>2</td>
<td>1</td>
<td>N$_2$H$_4$-Primex</td>
</tr>
<tr>
<td>Arcjet</td>
<td>580</td>
<td>2</td>
<td>0.34</td>
<td>Teflon-Primex</td>
</tr>
<tr>
<td>Plasma Thruster (PPT)</td>
<td>1200</td>
<td>1.5</td>
<td>0.094</td>
<td>Xenon-Loral</td>
</tr>
<tr>
<td>Hall Thruster</td>
<td>1600</td>
<td>1.5</td>
<td>0.43</td>
<td>Xenon-Loral</td>
</tr>
<tr>
<td>Ion Engine</td>
<td>3280</td>
<td>2.5</td>
<td>0.66</td>
<td>Xenon-NASA</td>
</tr>
</tbody>
</table>

\[ T_{\text{transfer}} \] the time to achieve the transfer. Moreover $a = \frac{F}{M}$ where $M$ is in a first approximation the satellite dry mass and $F$ the thrust. From [SMAD99], the thrust is given by the following expression:

\[
F = \frac{2\eta P}{I_{sp} g} \tag{4.9}
\]

where $P$ is the power and $\eta$ the propulsive efficiency. The transfer time is thus given by:

\[
T_{\text{transfer}} = \frac{M g I_{sp} \Delta V}{2\eta P} \tag{4.10}
\]

Contrary to the $\Delta V$, the transfer time varies with the type of electric propulsion system. Moreover it was assumed that the satellite arrives directly in its final slot. That is why a phasing step is not considered for electric propulsion.

4.3 Assignment Modules

INPUTS $\Delta V_{i,j} \Rightarrow$ OUTPUTS $Ass$

4.3.1 Assignment Methodology

The matrix $\Delta V_{i,j}$ is utilized by the auction algorithm to determine the best assignment in terms of fuel consumption. However as explained previously, the assignment returned by the algorithm does not match necessarily with the launch vehicle(s) capacity. This problem explained the necessity to implement a loop to assign the launched satellites after the auction algorithm module. The idea is to refine the assignment,
so that it could match with the vehicle capacity. First of all, from the assignment returned, the number of slots occupied by the ground satellites is obtained for each plane of $B$. If the repartition of the ground satellites does not correspond to the launcher capacity, one or several position(s) occupied by the satellites of $A$ are set free in the plane considered in order to permit a launch to this plane. The method corresponds to a reassignment for the satellites on the ground. The initial matrix $\Delta V_{i,j}$ is modified to take into account this reassignment. When a satellite (say the $p^{th}$) is assigned to a slot of constellation $B$ (say the $j^{th}$), we prevent the other satellites to go to this position by setting $\Delta V_{i,j} = 2000$ for all $i \neq p$. Once all the satellites on the ground are reassigned, the auction algorithm is run another time with the modified transition matrix $\Delta V_{m}^{i,j}$. The method is very efficient in terms of time of calculation since the auction algorithm is run only twice.

This reassignment process is explained in detail in Chapter 5 with a case study.

4.3.2 Efficiency of the Auction Algorithm

In this subsection, the efficiency of the auction algorithm is discussed. During the implementation of the framework, this study was led to prove the interest and reliability of the auction algorithm compared to other methods. The values given in this subsection are not universal, since the framework was not completely implemented but the purpose of this study was only to test different methods of assignment. The auction algorithm was first compared to assignments randomly generated. After this, an interesting comparison with the heuristic technique called simulated annealing ($SA$) was performed. The reconfiguration considered for this part was the reconfiguration from the polar constellation $A$ with $h_{a} = 2000 \ km$ and $\epsilon_{a} = 5 \ deg.$ to the polar constellation $B$ with altitude $h_{b} = 1000 \ km$ and elevation $\epsilon_{b} = 5 \ deg.$ The loop for assigning the ground satellites was not be taken into account: only the initial assignment returned by the auction algorithm was considered. The auction algorithm returned, according to Equation (2.1), an optimal value of $\Delta V_{\text{total}} = 26.5 \ km/s$ which corresponds to an average value per satellite of $1.25 \ km/s$. To ensure that the algorithm works well, seven random assignments were generated. From the matrix
Table 4.3: Random assignment results

<table>
<thead>
<tr>
<th>Experiments</th>
<th>$\Delta V_{total}$ (km/s)</th>
<th>$\Delta V_{avg}$ per satellite (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>144.8</td>
<td>6.9</td>
</tr>
<tr>
<td>2</td>
<td>156.7</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>140.2</td>
<td>6.7</td>
</tr>
<tr>
<td>4</td>
<td>111.8</td>
<td>5.3</td>
</tr>
<tr>
<td>5</td>
<td>142.6</td>
<td>6.8</td>
</tr>
<tr>
<td>6</td>
<td>122.7</td>
<td>5.8</td>
</tr>
<tr>
<td>7</td>
<td>147.5</td>
<td>7.0</td>
</tr>
</tbody>
</table>

$\Delta V_{i,j}$, the $\Delta V_{total}$ in each case is computed. The results are compiled in Table 4.3.

The results given by the auction algorithm seem to be very good compared to a non-optimized assignment. The auction algorithm returns only one of the optimal assignment. Indeed, the best assignment is non unique, since only assigning to the correct plane of constellation $B$ really matters. Knowing that 21 satellites had to be assigned into 40 slots, the size of the full-factorial solution space is $C_{40}^{21} \approx 1.31 \times 10^{11}$. Of these $C_{40}^{21}$ possibilities, only $(C_{8}^{7})^3 = 512$ assignments would return the optimal value of 26.5 km/s. The constellation $A$ is constituted of 3 planes of 7 and the constellation $B$ of 5 planes of 8. Changing the slot of one satellite in the same plane would not change the $\Delta V_{tot}$. These considerations explain why $(C_{8}^{7})^3$ optimal assignments exist. If timing (true anomaly) and exact phasing between $A$ and $B$ were taken into account, this would change, since then the $\Delta V$ for true anomaly rephasing in one orbital plane of $B$ would distinguish between satellites within one orbital plane, rather than assigning them all the same $\Delta V_{phase}$. This is explored further in Section 5.6. In that case, a single optimal assignment can exist. It would be very sensitive to the exact transfer schedule.

The comparison with SA will also prove the efficiency of the auction algorithm. Kirkpatrick et al.[KIRK83] described the different steps in order to utilize a Simulated Annealing Algorithm. The steps are explained below.

- Step 1: Choice of the Initial Design Vector $X_0$. First of all, the assignment was chosen in the form of a vector whose size is the size of the final constellation:
The \( N_{sats}(B) \). The \( i^{th} \) coordinate indicates in which slot the \( i^{th} \) satellite of \( A \), will go. The initial design vector (or assignment) was chosen arbitrarily: the \( i^{th} \) satellite of \( A \) goes to the \( i^{th} \) slot of \( B \). (in other words, \( X_0(i) = i \)) Why this choice? Because this initial assignment would imply a \( \Delta V_{total} \) of 128.7 km/s to do the maneuver. It is far from the optimal value of \( \Delta V_{total} = 26.5 \) km/s. The initial assignment could thus be easily-improved.

- Step 2: Perturb \( X_i \) to obtain a neighboring assignment \( X_{i+1} \)

\( X_{i+1} \) is a neighbor of \( X_i \) if \( X_{i+1} \) is obtained from a modification of \( X_i \). The perturbation chosen was to determine randomly two positions of \( X_i \) and to invert these two positions for obtaining \( X_{i+1} \).

- Step 3: Choice of the initial system temperature.

It was chosen to first take an initial temperature on the order of magnitude of the expected range of the objective function. The range of the \( \Delta V_{tot} \) is around 100 km/s. After several attempts inconclusive with an initial temperature of 100 km/s for different cooling schedules, an initial temperature of 1 km/s was finally chosen.

- Step 4: Choice of the cooling schedule.

There are two most common cooling schedules. The first one is to explore several perturbations at a given temperature, then reduce the temperature to a predetermined value, and repeat. The second is to reduce the temperature between each perturbation, but by a smaller amount. The second approach was selected. At each iteration the temperature were reduced by 0.001 km/s.

- Step 5: Terminating the SA algorithm.

The algorithm is terminated when \( T \) reaches the value 0, i.e. after 1000 iterations in this case.

The auction algorithm returns a \( \Delta V \) of 26.5 km/s in a CPU time of around 1.2 sec. The \( SA \) algorithm was run several times with the cooling schedule and initial
Table 4.4: SA results

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\Delta V_{\text{total}}$ (km/s)</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.9</td>
<td>2.09</td>
</tr>
<tr>
<td>2</td>
<td>27.9</td>
<td>1.97</td>
</tr>
<tr>
<td>3</td>
<td>31.8</td>
<td>1.97</td>
</tr>
<tr>
<td>4</td>
<td>36.0</td>
<td>2.03</td>
</tr>
<tr>
<td>5</td>
<td>31.8</td>
<td>2.09</td>
</tr>
<tr>
<td>6</td>
<td>29.3</td>
<td>2.03</td>
</tr>
<tr>
<td>7</td>
<td>26.5</td>
<td>2.19</td>
</tr>
<tr>
<td>8</td>
<td>30.5</td>
<td>2.09</td>
</tr>
<tr>
<td>9</td>
<td>27.9</td>
<td>2.31</td>
</tr>
<tr>
<td>10</td>
<td>30.4</td>
<td>2.42</td>
</tr>
</tbody>
</table>

temperature explained above. The results are summarized in Table 4.4. The CPU time is obtained for a particular computer\(^5\).

For each try, the initial assignment was well-improved (recall that the initial assignment corresponded to a $\Delta V_{\text{tot}}$ of 128.7 km/s). The SA algorithm returns a good assignment in terms of $\Delta V$ but it returns very rarely the best one (the same than the auction algorithm). In 10 attempts, the best one was obtained only once (Trial 7 in Table 4.4). Moreover, the time of CPU calculation is a little bit higher for SA. This study seems to support the reliability and speed of the auction algorithm compared to simulated annealing, at least empirically. SA is too dependent on the different parameters such as initial temperature and cooling schedule for being a competitive method in this context.

4.3.3 Sensitivity of the Assignment to the relative Phasing of the two Constellations

The relative phasing of the two constellations is the angle between the first plane of the initial constellation $A$ and the first plane of the final constellation $B$. In order to quantify the sensitivity of the assignment with respect of this angle, the $\Delta V$ total (after optimal assignment) was computed for different values of this angle.

\(^5\)Specifications of the computer utilized: Laptop Samsung VM7000 Series. Processor: Pentium II 400MHz. 64 MB RAM.
The reconfiguration considered is the reconfiguration from $A$ with $h_a = 2000 \text{ km}$ and $\epsilon_a = 5 \text{ deg}$ to $B$ with $h_b = 1200 \text{ km}$ and $\epsilon_b = 5 \text{ deg}$. Three graphs were drawn depending on the Launch Vehicle capacity. Figure 4-8 represents the variation in $\Delta V$ versus relative phasing angle when the reassignment loop is not considered. This case is applicable when the Launch Vehicle can only carry 1 satellite per launch or when a series of launch vehicles with variable capacity is available. The relative phasing that minimizes the $\Delta V_{\text{tot}}$ corresponds to an angle of $0 \text{ deg}$. In other words, the first plane of both constellations must be the same in order to minimize the fuel consumption. Figures 4-9 and 4-10 were drawn for a launch capacity of three and five satellites respectively. In both cases, the phasing minimizing the fuel consumption is no longer $0 \text{ deg}$. The best relative phasing is thus sensitive to the launch vehicle capacity. However, since the best $\Delta V$ obtained is very close to the $\Delta V$ obtained when the phasing angle is equal to 0, we decided to utilize this angle of 0 for the project.

![Graph 4-8: $\Delta V_{\text{tot}}$ versus angle of relative phasing. No reassignment loop in this case.](image-url)
Figure 4-9: $\Delta V_{tot}$ versus angle of relative phasing. Launch Vehicle Capacity of 3 satellites.

4.4 Launch Module and Launch Schedule

Launch Module: INPUTS $i_b, N_b - N_a, m_{dry}$ ⇒ OUTPUTS $Cap_{launcher}, Cost_{launcher}$

Launch Schedule Module: INPUTS $Cap_{launcher}, N_b - N_a$ ⇒ OUTPUTS $S_l$

This section describes the launch strategy chosen for the project implementation. Two modules of the Block Diagram (see Figure 3-2) are necessary to determine the launch strategy. The Launch Module returns the name and the cost of the Launch Vehicle considered, whereas the “Launch Schedule Module” returns a plan to launch the satellites from the ground. In Subsection 4.4.1, the deployment of the IRIDIUM constellation is carefully described. This example allows to determine an order of magnitude for the deployment time and launch vehicle capacity required in the case of LEO satellite constellations. In 4.4.2, two Launch Selection Processes\(^6\) will be briefly described. By comparison with the data of the IRIDIUM deployment, one simulation will be retained (Subsection 4.4.3). Finally, the problems posed by the launch planning will be discussed in Subsection 4.4.4.

\(^{6}\) available in the Space Systems Laboratory at MIT
4.4.1 Iridium Constellation Deployment

As shown in Table 4.5, the IRIDIUM constellation was deployed in just twelve months from three countries: Russia, USA and China. This deployment strategy proves that it is possible to launch 72 satellites into LEO in just one year with the current launch infrastructure.

The launch vehicles utilized and their respective capacity represent the other interesting data in this table. Seven satellites were launched with the Russian rocket Proton, five with the US Delta II and two with the Chinese Long March. Wertz and Larson [SMAD99] give the payload accommodation for the three Launch Vehicles. Proton with a diameter of 4.1 m and a length of 15.6 m has a payload accommodation of 206 m³, whereas Delta II has a payload accommodation of 56 m³ (diameter of 2.9 m and length of 8.5 m). It is remarkable to see that with a payload accommodations almost four times higher than Delta II, Proton only carried 7 satellites compared to the 5 satellites carried by Delta II. It is not due to a payload mass limitation, since Proton can carry 20 tons of payload to LEO [SMAD99]. The manufacturer must have considered that it was too risky to put more than 7 satellites per launch vehicle. This study gives orders of magnitude for the launcher capacity and the maximum number
May 5, 1997 5 satellites launched on Delta II
June 18 7 satellites launched on Proton
July 9 5 satellites launched on Delta II
August 20 5 satellites launched on Delta II
September 13 7 satellites launched on Proton
September 26 5 satellites launched on Delta II
November 8 5 satellites launched on Delta II
December 8 2 satellites launched on Long March
December 20 5 satellites launched on Delta II
February 18, 1998 5 satellites launched on Delta II
March 25 2 satellites launched on Long March
March 29 5 satellites launched on Delta II
April 6 7 satellites launched on Proton
May 2 2 satellites launched on Long March
May 17 5 satellites launched on Delta II

Table 4.5: Deployment of the Iridium constellation.

of satellites that can be put in one launch. This data will allow to benchmark the two simulations available and to keep the more reliable one. The order of magnitude for the deployment duration will allow to implement a realistic launch schedule module.

4.4.2 Description of the two Simulations available

One of the launch simulations available was coded in 2002 by Adam Ross\textsuperscript{7}. This simulation is based on an optimization approach to the launch vehicle selection process for satellite constellations proposed by Jilla and Munson [Jil00]. The selection problem is based on finding the optimal subset of launch vehicles that can deploy all of the satellites in a constellation at minimum cost and/or risk; while adhering to a set of satellite, political and availability constraint. The tool consists of a database containing information on all of the operational launch vehicles. The information of this database is then combined with the properties of the constellation to create a mathematical formulation of the launch vehicle selection problem as an integer program (IP). The optimization is solved via a branch-and-bound algorithm. This method is a priori attractive due to the impressive number of launch vehicles present in the database. Moreover the simulation is not limited to launch to Low Earth Orbit

\textsuperscript{7}graduate student-Space Systems Laboratory/MIT
The other simulation is part of the constellation simulator developed by de Weck and Chang [OLdW02]. This program takes mass and volume of an individual satellite, satellite altitude, and inclination as inputs, and returns a suggestion to the user on the types of launch vehicles to use, the launch sites and the cost. This program is limited to LEO constellations and only 6 launch vehicles are considered: Atlas IIIA, Delta II 7920, H-II A 202, Long March 2C, Pegasus XL and Ariane 5.

4.4.3 Choice of one Simulation

In order to determine the method that will be implemented in the simulator, the best way is to benchmark the results returned by both methods. The deployment of IRIDIUM was chosen as a reference: altitude of 780 km, mass of 700 kg, volume of 7 m$^3$ per satellite, inclination of 89 degrees and 72 satellites to be deployed. With these inputs, the method based on Jilla’s approach [Jil00] advices the launch vehicle Delta IV-M with a capacity of two satellites per launch. Knowing that 5 satellites were carried by the launch vehicle Delta II during the IRIDIUM deployment, the results returned by this simulation are surprising. Indeed Delta IV-M although bigger than Delta II has according to the module a lower capacity. These results seem to compromise the utilization of this module in the simulator to study orbital reconfiguration. In addition to this, the price to launch a Delta IV-M rocket is not realistic: 255 M$. Although less ambitious with a small database and limited to LEO study, the module of de Weck and Chang returns results that are more realistic in this context. With the characteristics of Iridium satellites, the module returns Ariane 5 with a capacity of 18 satellites per launch and a cost of 150 M$. The capacity of 18 satellites can be surprising, but knowing that Ariane 5 can carry 18 tons to LEO [SMAD99], it should be possible to put 18 Iridium satellites in one launch vehicle. Obviously a manufacturer would likely not consider to launch such an amount of satellites in the same launch, due to risk considerations. For the same reasons that Proton in the real deployment of Iridium (Section 4.4.1) was not used at full capacity, such a strategy would be too risky. The simulator implemented will take this into account by limiting
the number of satellites per Launch Vehicle:

\[ N_{\text{sats per vehicle}} = \min(Launcher\text{Capacity}, N_{\text{sats per plane}}, 6) \]

The limit of 6 satellites for one launch vehicle is inspired by Section 4.4.1. This limit aims to limit the risk of losing a high number of satellites in the case of a launcher failure. This number can be changed by the user depending on his risk tolerance level.

To summarize, the second method was incorporated into the simulator for constellation reconfiguration. This method is less ambitious, but the results are realistic.

4.4.4 Problems posed by the Launch Planning

This subsection describes briefly how the “Launch Schedule Module” works. The module takes as inputs the name of the Launcher and the number of launches to send in orbit all the \( N(B) - N(A) \) satellites on the ground. It returns in matrix form a plan for the different launches:

\[
S_l = \begin{pmatrix}
t_1 & N_{\text{sats}}(\text{launch 1}) \\
t_2 & N_{\text{sats}}(\text{launch 2}) \\
\vdots & \vdots \\
t_n & N_{\text{sats}}(\text{launch n})
\end{pmatrix}
\]

This planning allows to compute the duration of the launch phase.

During the initial implementation of the “Launch Schedule module”, a sophisticated model was applied to determine the schedule. The first step was to compute the availability of the launcher selected. From [SMAD99], this availability is given by the expression

\[
Av = 1 - [L(1 - Re)T_d/(1 - 1/S)]
\]  
(4.11)

where \( Av \) is the availability in %, \( Re \) the vehicle reliability, \( L \) the launch rate in units of flights per year, \( T_d \) the stand-down time following a failure in units of years and \( S \) the surge-rate capacity. Typical values for \( S \) are between 1.15 and 1.5 in the
Table 4.6: Time between two successive launches.

<table>
<thead>
<tr>
<th>Launcher</th>
<th>Country</th>
<th>Time between 2 launches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ariane 5</td>
<td>Europe</td>
<td>80 days</td>
</tr>
<tr>
<td>Atlas IIIA</td>
<td>USA</td>
<td>65 days</td>
</tr>
<tr>
<td>Delta II 7920</td>
<td>USA</td>
<td>42 days</td>
</tr>
<tr>
<td>H-II A202</td>
<td>Japan</td>
<td>100 days</td>
</tr>
<tr>
<td>Long March 2C</td>
<td>China</td>
<td>60 days</td>
</tr>
<tr>
<td>Pegasus XL</td>
<td>USA</td>
<td>60 days</td>
</tr>
</tbody>
</table>

United-States. From the availability and the number of flight per year, the length between two successive launches was calculated and hence also the launch schedule.

This method was not retained in the final version of the framework because the expression often returns negative availabilities. In addition to this, this model could be acceptable in a situation where the demand is much higher than the supply. However in the current launcher market, supply is generally higher than the demand. Ariane 5 has been launched only 14 times since 1996 and the inaugural launch. This is due not only to technical failures, but also to the absence of customers. Recall that Ariane 4 could be launched 14 times in just one year. There are not enough customers compared to the high number of Launch Systems. If a manufacturer decided to undertake a constellation reconfiguration, he would have no problem to find a Launch System available and to launch all the “new satellites” in a rapid manner. It is thus assumed that the availability of all the Launch System is equal to 1.

In order to determine a schedule, the time between two successive launches has to be estimated. As explained in the last section, the launch module incorporated in the simulator has a database limited to 6 launch vehicles. The time between two successive launches was extrapolated from the Iridium deployment for Delta II and Long March 2C. For Pegasus XL and Atlas III A, the records available on the Internet were utilized to deduce an estimation of this time. For Ariane 5 and H-II A202, the time between two successive launches was assumed. There are not sufficient records for these two launchers to give a statistically significant approximation of this time. Table 4.6 summarizes this.

---

8www.astronautix.com/lvs/pegsusxl.html
4.5 Transfer Schedule

INPUTS $t_{i,j}, s \Rightarrow$ OUTPUTS $S_t$

As explained in Chapter 3, the transfer schedule should be determined by an optimization loop whose objective would be to minimize the service outage cost. In view of the difficulty to find a feedback between the “Transfer Schedule Module” and the module computing the cost of service outage, it was finally decided to generate four possible scenarios for the satellite transfer phase. The goal is to show the influence of the timing of transferring the constellation $A$. The four scenarios are described below.

- **Scenario 1:**
  The first scenario consists of first transferring half of the satellites. Once this first stage is accomplished, the remaining half are transferred into their new positions. The first wave of transfers coincides with the last launch.

- **Scenario 2:**
  All the satellites begin their transfers at the same time. As for scenario 1, the transfer begins once the last launch is achieved.

- **Scenario 3:**
  The satellites are transferred sequentially. Once a satellite has reached its final slot, the transfer of the following can begin.

- **Scenario 4:**
  All the satellites finish their transfers at the same time. This scenario is redundant with Scenario 2 in chemical propulsion, since all the satellites have the same transfer time in our model, but not in electric propulsion.

In a High Fidelity Simulation, the satellites can not begin or finish their transfer exactly at the same time in chemical propulsion, since the transfer start must occur
when the S/C is on the crossing point between its original trajectory and final plane. Thus, all the satellites will not be obviously on their respective crossing point at the same time. However, due to the fact that true anomaly is not considered in our simple framework, the approximation consisting of considering transfers at the same time is understandable. Indeed, the purpose was to compare the effect of a staggered schedule (scenario 3) with a compressed schedule (scenario 2 and 4) on the outage cost (the scenario 1 is an intermediate situation between 2 and 3). That is why, these scenarios were implemented in the present framework.

Knowing the transfer time necessary for all the satellites, the module returns a transfer planning depending on the scenario selected. The output is a matrix called $S_t$.

$$
S_t = \begin{pmatrix}
\text{satellite n} & T_{\text{beginning-transfer}} & T_{\text{end-transfer}} \\
1 & t_1 & t_1 + T_{\text{transfer}(1)} \\
2 & t_2 & t_2 + T_{\text{transfer}(2)} \\
\vdots & \vdots & \vdots \\
N(A) & t_{N(A)} & t_{N(A)} + T_{\text{transfer}(N(A))}
\end{pmatrix}
$$

The total transfer time is: $t_{N(A)} + T_{\text{transfer}(N(A))} - t_1$.

### 4.6 Coverage Module

**INPUTS** $S_l, S_t$ ⇒ **OUTPUTS** $Cov$

#### 4.6.1 Determination of the Partial Coverage during the Re-configuration

A key issue of this module is to find a metric allowing to quantify the partial coverage, in other words the percentage of coverage available during operations. Due to the multiple intersection between the coverage surface of the different satellites of a constellation (see Figure 4-11), it is very difficult to quantify the impact of the absence
of one satellite on the total coverage, as it varies over time for both Walker and Polar constellations. Since the constellations considered in this thesis ensure only single coverage on all of the Earth’s surface (i.e. single global coverage), it was assumed that the partial coverage could be measured by the percentage of operational satellites, the satellites in transfer not being taken into account. This assumption is valid only in the case of single coverage over the Earth’s surface. Indeed, assume that the constellation has a double coverage on all the Earth surface. If one satellite is transferred, there will be no hole in the coverage since another satellite in the constellation covers the same areas (consequence of the “double global coverage”). In the case of single global coverage, the transfer of one satellite will entail obviously one hole in the coverage. However, the assumption estimating the partial coverage by the percentage of operational satellites represents a rough simplification, because even in the case of single global coverage some areas are covered with double or triple coverage. Figure 4-12 depicts the regions of single, double and triple coverage in the case of polar constellations. The polar constellation always concentrates the higher folds of coverage at the poles. Walker constellations usually have these higher folds near the latitude corresponding to the orbital inclination (see Figure 4-13 and [AL98]).
Figure 4-12: Coverage distribution in the case of Polar constellations. 28 satellites, $h = 1600$ km, $i = 90$ deg.

Figure 4-13: Coverage distribution in the case of Walker constellations. Walker 28/7/2, $h = 1600$ km, $i = 59$ deg.

As a consequence of this, a constellation with 80% of operational satellites will surely have a partial coverage higher than 80%. Nevertheless, this assumption represents a good order of magnitude of the partial coverage.

4.6.2 Inputs/Outputs of Coverage Module

The inputs of this module are the launch and transfer schedules $S_{l}$ and $S_{t}$. From the plan to launch the ground satellites and the plan to transfer the satellites of the constellation $A$, the module returns the coverage capacity of the initial and final constellation during the entire process. The output is a matrix called $Cov$.

$$
Cov = \begin{pmatrix}
\text{Date of operations (s)} & \text{coverage of } A \text{ in } \% & \text{coverage of } B \text{ in } \% \\
t_1 & 100\% & 0\% \\
t_2 & : & : \\
\vdots & \vdots & \vdots \\
t_{\text{final}} & 0\% & 100\%
\end{pmatrix}
$$

The dates of operations represent either the launch dates, or the dates of transfer start, or the dates of transfer end.
4.6.3 Start of Operations for Constellation B

It was decided that constellation $B$ starts operations once its coverage capacity exceeds the coverage capacity of $A$. Once $B$ has initiated service, $A$ ends operations. In this scenario, $A$ and $B$ are not in service at the same time. Another possible alternative would have been to consider that $A$ and $B$ could be operational in parallel. A simple case will illustrate this option. Consider a constellation $A$ with 4 satellites and a constellation $B$ with 8 slots, in other words $N_{sats}(A) = 4$ and $N_{sats}(B) = 8$. In order to simplify, it is considered that the coverage areas of all satellites are rectangular. Figure 4-14 represents the coverage of both constellations once two satellites of $A$ have been transferred to $B$. 4 ground satellites had been launched earlier. So, in this phase of the process, $A$ has two satellites in service and $B$ has six satellites in service. The partial coverage of $A$ (in light shading in the Figure) is 50 %, whereas the partial coverage of $B$ is 75 % (in dark shading in the Figure). In the legend of Figure 4-14, the coverage surface of one satellite of $A$ is the double of the coverage surface of one satellite of $B$, satellites of $A$ being higher in altitude than satellites of $B$.

![Figure 4-14: Coverage surfaces of $A$ and $B$. SIMPLE CASE](image)

If the model chosen for the simulator were applied to this simple case, the service would be insured by constellation $B$. However Figure 4-14 shows that if the coverages
of $A$ and $B$ were added (case where the constellations are in service in parallel), there would have been 100% coverage.

From this simple case, it can be concluded that if $A$ and $B$ were utilized at the same time, the partial coverage would be higher. However this alternative raises a lot of issues, concerning its feasibility (coordinated operation of two partial constellations with a subset of transitioning satellites). During the reconfiguration, the entire constellation would consist of two layers at different altitudes. The question is to know if it would be feasible technologically to connect two layers. This simple case of “hybrid constellations” needs to be studied in detail. For instance, if inter satellite links (ISL) are used, links between the layers need to be created which is a very challenging problem. For these reasons, the assumption consisting of utilizing only one constellation at the same time was upheld.

### 4.7 Cost Module

The total cost is the sum of four different costs: $C_{total} = C_{launch} + C_{transfer} + C_{outage} + C_{production}$. If the reconfiguration extends over more than one year, the costs need to be discounted to a particular fiscal year ($FY$).

#### 4.7.1 Launch Cost

The launch cost $C_{launch}$ is given by the Launch Module. It is obviously equal to the number of launches times the price of one launch. Launch integration cost, insurance costs or costs arising from launch failures are neglected, but could be included in the future.

#### 4.7.2 Transfer Cost

INPUTS $\Delta M, I_{sp} \Rightarrow$ OUTPUTS $C_{fuel}$

The transfers of the on-orbit satellites of constellation $A$ will imply costs for the
constellation manufacturer/operator. This cost should include the cost of the propulsion systems (it is assumed particularly for chemical transfer that the satellite own propulsion systems is not sufficient to achieve the transfers, hence the necessity to add a chemical thruster) and the cost to launch the extra-fuel necessary for the transfers. The “real option” considered in this thesis being the fuel necessary to achieve the transfers, a propulsion systems is necessary to utilize this fuel. If the “real option” considered had been the utilization of a Space Tug, the satellites to be transferred would not have needed a propulsion systems but perhaps would have needed to be outfitted with a compatible docking mechanism. The cost of the propulsion systems would have been replaced by the Space Tug utilization cost.

In the present case (fuel as “real option”), it was impossible to obtain prices of the propulsion systems or even a rough approximation. Indeed, costs are a very sensitive information in the aerospace industry. However assuming that this cost is negligible with respect to the cost to launch the extra-fuel, it was decided to focus only on this cost. So,

\[ C_{\text{transfers}} = C_{\text{fuel}} = C_{\text{kg of payload}} \cdot \Delta M \]  

(4.12)

where the cost coefficient \( C_{\text{kg of payload}} \) is the cost to launch a kg of payload to LEO. A mean value of \( C_{\text{kg of payload}} \) for different launchers (SMAD page 802 table 20-14 on “Launch Vehicles Costs”) was utilized: \( C_{\text{mean kg of payload}} = 13\text{K$}/\text{kg} \). \( \Delta M \) was obtained from \( \Delta V \) thanks to the Rocket equation:

\[ \Delta M = M_f [e^{\frac{\Delta V}{I_{\text{sp}}}} - 1] \]  

(4.13)

4.7.3 Cost of Service Outage

INPUTS \( Cov \) ⇒ OUTPUTS \( C_{\text{outage}} \)

The third cost is the cost due to service outage: \( C_{\text{outage}} \). It corresponds to the loss of revenue entailed by the on-orbit satellite transfers. During transfer operations, the satellites are indeed no longer operational, which implies a decrease in the
constellation capacity and therefore a temporarily reduced revenue.

The utilization of the constellation yields for the manufacturer a revenue per user called service charge: \( S.C \) in US$/min. An empirical expression of the lower limit for this charge is given by Lutz and Werner [LUT00]:

\[
\text{service charge} = \frac{I(1 + k)^T}{(365.24.60)TC_sU_s}
\]  

(4.14)

where \( I \) is the net investment for the system including the costs of development, production, installation and initial system operation; \( k \) is the internal annual interest rate (Typical values are \( k = 5\ldots30\% \)); \( T \) is the period of time until amortization (Typical values are \( T = 3\ldots8 \) years); \( C_s \) is the global system capacity and \( U_s \) is the global system utilization (\( U_s = 5\ldots15\% \)).

The expression \( C_sU_s = NuDa365:120 \) allows to compute the quantity \( C_sU_s \). \( Nu \) is the number of users (subscribers) in the system and \( Da \) the average user activity in minutes per month (\( = 100\ldots150 \text{ min/month} \)).

For the project purpose, \( k \) was chosen equal to 15\%, \( T \) equal to 5 years and \( Da \) 120 min/month. Moreover, \( I \) was assumed to be equal to the Life Cycle Cost, since operations costs of the constellation are typically significantly smaller than \( I \). This assumption must be revisited in the future.

The revenue entailed by the constellation utilization is then approximated by the product of the service charge times the constellation’s utilized capacity. So, \( R = S.C \times C_sU_s \). It was assumed that during operations, the service charge of one configuration \( A \) or \( B \) is a constant, equal to the service charge when the configuration is fully operational (\( S.C.A \) and \( S.C.B \)) and that the global system capacity varies with respect to time. \( C_s(t) \) decreases when a satellite of \( A \) leaves its position in the initial configuration and increases when this satellite reaches its final slot in constellation \( B \). Figure 4-15 summarizes this.

From these assumptions, the cost of service outage can be computed as the dif-
Figure 4-15: $C_s$ versus time. The capacity of $A$ reduces gradually after $t_A$ and the capacity of $B$ increases gradually. Once the capacity of $B$ exceeds that of $A$ (crossover), service is switched over.

The difference of revenue entailed by the transfers:

$$C_{outage} = \int_{t_A}^{t_C} (R_A - R(t))dt + \int_{t_C}^{t_B} (R_B - R(t))dt \quad (4.15)$$

where $t_A$ represents the start of transfers, $t_B$ the time when $B$ is completed and $t_C$ the date when the configuration $B$ initiates service. The first part of the outage cost between $t_A$ and $t_C$ is a direct loss of revenue entailed by the transfer and the fact that $A$ is no longer fully operational. The second part is more a missed economic opportunity due to the fact that $B$ is not fully operational yet. Equation 4.15 can be also expressed as:

$$C_{outage} = \int_{t_A}^{t_C} S.C.A(C_{s,A} - C_s(t))U_s dt + \int_{t_C}^{t_B} S.C.A(C_{s,A} - C_s(t))U_s dt \quad (4.16)$$

In the case of partial coverage, the number of users and also the global system
capacity is obviously lower. In order to compute the global system capacity \( C_s \) during the transfers, it was assumed that \( C_s(C_{ov} = x\%) = C_{s,A/B} \times x\% \) where \( C_{s,A/B} \) represents the global capacity of either configuration \( A \) or \( B \) when the constellation is 100% operational and \( x \) is the partial coverage. The distribution of users is assumed thus to be homogeneous. This assumption is valid as well if it is considered that the holes in the coverage appear equally over very populated regions as well as over desert regions.

\( C_{outage} \) was computed in its discrete form for the project purpose:

\[
C_{outage} = \sum_{t_A}^{t_C} (S.C.A)C_{s,A}U_s.T_{ime}(x\%)(1 - x) + \sum_{t_C}^{t_B} (S.C.B)C_{s,B}U_s.T_{ime}(x\%)(1 - x)
\]

(4.17)

where \( x \) represents the partial coverage in \% and \( T_{ime}(x\%) \) the length of this period of coverage.

The investment \( I \) and the number of users \( N_u \) are obtained from the simulator implemented by de Weck and Chang [OLdW02]. This constellation simulator has been used to generate 1800 different architectures for the particular case of LEO personal communications systems. These 1800 architectures represent only polar constellations. The Design Vector of the simulator consists of the constellation type, \( C \), the altitude, \( h \), the minimum elevation angle, \( \epsilon \), the satellite transmitter power, \( P_t \), the satellite antenna size, \( D_A \), the per-channel bandwidth, \( \Delta f_c \) and the satellite lifetime, \( T_{sat} \). The last input is \( ISL \), the inter satellite links: \( ISL = 1 \) if intersatellite links are implemented and 0 otherwise. From these inputs, the simulator returns some outputs such as the Average Delay, the number of users \( (N_u) \), the Lifetime capacity (in min) and the Lifecycle Cost (in \( B\$ \)). The values utilized for running the baseline simulation are:

- \( C = \text{polar} \)
- \( P_t = 1000W \)
- \( D_A = 2.5m \)
- \( \Delta f_c = 40kHz \)
4.7.4 Cost to Produce new Satellites

The production of new satellites entails a cost. This cost is called $C_{production}$. A learning curve model was applied to determine this cost. The total production cost for $N$ units (ie $N$ satellites) is given by the expression [SMAD99]:

$$production\ cost = TFU.N^B$$  \hspace{1cm} (4.18)

where $B = 1 - \frac{\ln((100\%)/S)}{\ln 2}$

$TFU$ is the theoretical first unit cost and $S$ is the learning curve slope in percent. For less than 10 units produced, a 95% learning curve is generally applied. Between 10 and 50 units, a 90% learning curve and 85% for over 50 units is appropriate. The cost in terms of production induced by the reconfiguration can be estimated thus with:

$$C_{production} = TFU.(N_{sats}(B)^{B_s} - N_{sats}(A)^{B_s})$$  \hspace{1cm} (4.19)

It is the cost to produce the additional satellites to be launched. The cost of the first unit $TFU$ was obtained from the website of NASA Johnson Spaceflight Center\footnote{www.jsc.nasa.gov/bu2/SVLCM.html}. For a dry mass of 700 kg, the model returned a first unit cost of 29.89 millions US$ FY99. This value was applied to the simulator. In theory, the $N(B) - N(A)$ satellites will be produced when needed and thus several months or years after the first wave of production. This thesis assumes that it would be economically too risky to produce these satellites together with the $N(A)$ first ones and to put them in storage. If the market conditions are too bad to undertake a reconfiguration, these satellites would be lost.
Another expression to compute the production cost could be:

\[ C_{production} = TFU.(N_{sats}(B) - N_{sats}(A))^{B_a-b} \]  \hspace{1cm} (4.20)

This expression assumes that the production line was restart from the beginning and that all learning was lost since the initial production run. Even if the production line will be stopped several months, the technology is not lost. So, the production cost should be lower than the cost given by Equation (4.20). However, the production cost is surely higher than in Equation (4.19), because while technology and methodology are acquired, the personal must be trained again and this will entail additional cost, not captured in our model. In reality, the production cost is expected to be between these two situations.
CHAPTER 4 SUMMARY

In this chapter, the different modules implemented were presented in detail. For each module, concepts, assumptions and mathematical formulations were explained. This thesis intends to be a foundation for future work in the field of satellite constellation reconfiguration. For this, the assumptions and limits of each module were carefully pointed out. The framework could be thus easily improved. If the constellation module is trustworthy thanks to the benchmarking with the Iridium and Globalstar constellations, the other modules would need some refinements. The astrodynamics module seems to be acceptable for computing the $\Delta V$ using chemical propulsion. In electric propulsion, a lot of approximations were made. Thus, this module can not be considered completely trustworthy. The assignment modules work well. However, since we want all the on-orbit satellites to be exactly the same, it could be judicious to replace the criteria consisting of minimizing the total Delta V with another criteria allowing to minimize the difference in $\Delta V$ between satellites. Concerning the launch module, as explained previously, the database is too small. The coverage module is fine if the study is limited to single coverage. Lastly, the cost module is not sufficiently precise. All the submodules could be improved in a future study.
Chapter 5

Results

In this Chapter, the total energy necessary for different types of reconfigurations will first be discussed (Section 5.1). A “constellation reconfiguration map” of altitude versus inclination is depicted. After this, the study will be limited to the reconfiguration of LEO polar constellations. A case study will be presented and the results returned by the assignment modules will be studied in detail (Section 5.2). The impact of the propulsion system and of the transfer scenario is the subject of Section 5.3. Then, an exploration of the trade space of “outage cost versus fuel cost” is made in Section 5.4. Fixing the initial constellation $A$, outage cost and fuel cost will be computed for different final constellations ($B$), and different propulsion systems and scenarios. In Section 5.5, the predicted reconfiguration costs will be computed for the optimal paths of reconfigurations identified in Chaize’s thesis [CHAI03]. The purpose is to determine if the reconfiguration cost is smaller or larger than the $\approx 20-30\%$ “opportunity” in the $LCC$ revealed by Chaize. Finally, Section 5.6 will focus on High Fidelity Simulation. This section will explain how a High Fidelity Simulation can be built and what tools or software can be used for this.
5.1 Total Reconfiguration Energy Requirements

5.1.1 Section Purpose

The purpose of this section is to estimate the amount of energy needed to undertake a reconfiguration. Depending on the type of reconfiguration, the average quantity of $\Delta V$ (per satellite) necessary to achieve the maneuvers of the on-orbit satellites can be very different. Different types of reconfigurations are thus considered: reconfigurations in altitude, reconfigurations in inclination or reconfigurations coupling change in altitude and inclination. Reconfiguration in RAAN is not explored explicitly, but is an implicit function of the number of orbital planes in $A$ and $B$. Reconfiguration in altitude means that the type of constellation (Polar or Walker) is conserved during the reconfiguration, only the altitude varies. An example of reconfiguration in altitude is the reconfiguration from a GEO polar constellation into a MEO polar constellation. Inversely, a reconfiguration in inclination means that the altitude is conserved whereas the constellation type is changed. If the inclination change is large enough, the reconfiguration from a LEO Polar constellation into a LEO Walker constellation is a case of reconfiguration in inclination.

To capture the impact of the reconfiguration type on the energy consumption, the average $\Delta V$ per satellite is taken into account. Another possible criteria would have been to consider the maximum $\Delta V$ for reconfiguration in any satellite. The type of propulsion system considered is chemical propulsion. The results would be similar for electric propulsion. Moreover, only the top part of the Block Diagram (constellation module, astrodynamics module and auction algorithm), see Figure 3-2 is run in this section. The loop to assign the ground satellites is not taken into account. The purpose is indeed to show trends and orders of magnitude. What are the reconfigurations requiring high fuel consumption? The reconfigurations requiring reasonable fuel consumption? These are the questions answered by this section.
Figure 5-1: “Constellation Reconfiguration Map”. Diagram of inclination versus altitude. ”eps” represents the minimum elevation of the constellation. The values T/P/F are also indicated close to the asterisk representing each constellation.

- $0 \leq \Delta V \leq 2 \text{ km/s}$: cheap reconfiguration.
- $2 \leq \Delta V \leq 3 \text{ km/s}$: medium expense reconfiguration.
- $\Delta V > 3 \text{ km/s}$: expensive reconfiguration.

5.1.2 Diagram of Altitude versus Inclination “Roadmap”

A diagram of altitude versus inclination was drawn in order to point out the influence of the reconfiguration type on fuel consumption. Figure 5-1 depicts this diagram.

The asterisks represent the positions on the diagram of the constellations, the arrows representing the direction of reconfiguration. The middle of the arrow indicates the average $\Delta V$ per satellite required for this reconfiguration. We can think of this
Diagram as a “Reconfiguration Map”.

From the rocket equation applied for an $I_{sp}$ of 430 s, an extra-fuel mass of 700 kg corresponds to a $\Delta V$ of 2.9 km/s. Recall that the satellite dry mass is of 700 kg, such an extra-mass of fuel represents a high quantity of fuel for a single satellite. From this statement, it was considered that a reconfiguration needing an average $\Delta V$ above 3 km/s was an expensive one in terms of fuel consumption. Similarly, a reconfiguration requiring an average $\Delta V$ below 2 km/s could be considered as a “cheap” reconfiguration, since 2 km/s of $\Delta V$ represents a fuel mass of $\approx 400$ kg.

5.1.3 Comments

The diagram of altitude versus inclination indicates a very interesting tendency. First, the reconfigurations from polar-polar seem to be reasonable on average except for the reconfiguration from MEO to LEO. The $\Delta V$ required is around 1.8-1.9 km/s per satellite. The same tendency appears for the Walker-Walker reconfigurations. The consumption of energy is even lower. It can reach values below 1.5 km/s. The most expensive reconfigurations are the reconfigurations requiring a high angle inclination change, in other words the reconfigurations Walker-Polar or Polar-Walker. An exception is GEO where the $\Delta V$ required is relatively low, the other polar-walker reconfigurations considered are quite expensive. The reconfigurations in MEO or from GEO to MEO are, however, a little bit less expensive (between 2.7 km/s and 3.2 km/s) than the reconfigurations in LEO or from MEO to LEO which require $\Delta V$’s above 3.5 km/s. The most expensive reconfiguration appeared to be the reconfiguration from a MEO Walker constellation with the following characteristics T/P/F, 16/8/5 to a LEO polar constellation with the characteristics 36/4/0. This reconfiguration requires indeed an average $\Delta V$ per satellite of almost 4.5 km/s. This corresponds to an extra-fuel mass of 1.3 tons. Reconfigurations coupling change of altitude and change of inclination are, not surprisingly, very expensive.

To conduct reconfigurations in altitude seems to be reasonable. Inclination changes imply plane change, which are very expensive in LEO. These results can be explained because a polar constellation generally has fewer planes than a Walker one. To recon-
figure a polar into a Walker constellation or vice-versa will thus imply several plane changes with high angle increments.

5.2 Case Study

In this section, the framework developed to study reconfiguration of constellations is applied to a particular case. The reconfiguration of the LEO polar constellation \( A \) with altitude \( h_a = 2000 \text{ km} \) and min elevation angle \( \epsilon_a = 5 \text{ deg} \) to the LEO polar constellation \( B \) (altitude \( h_b \) of 1200 km and min elevation angle \( \epsilon_b \) of 5 deg). The study is limited to chemical propulsion and the transfer scenario 1 (i.e. two transfer waves) selected for the transfers. The purpose of this section is to demonstrate the assignment of the \( N(A) \) satellites into the slots of the constellation \( B \). This includes the loop to assign the ground satellites (Section 5.2.1). After this, two strategies to deal with the on-orbit satellites penalized by the reassignment will be presented and compared in Section 5.2.2 and 5.2.3. Finally, conclusions will be presented in Section 5.2.4.

5.2.1 Assignment Modules Results

Given the parameters defined above, the constellation module returns that \( A \) contains 21 satellites in 3 planes of seven satellites and \( B \) has 32 slots (4 planes of 8 satellites). Thus, 11 satellites need to be launched. A first run of the auction algorithm was achieved without taking into account the loop for assigning the satellites on the ground. Table 5.1 shows the assignment returned by the algorithm. Note that \( P_1^a(4) \) indicates the 4\(^{th} \) satellite in the 1\(^{st} \) plane of \( A \). This assignment represents a total Delta V of 40.5 km/s. A remarkable point is that all the satellites of a plane of \( A \) go to the same plane of \( B \). It sounds logical, since the \( \Delta V \) depends only of the initial and final plane characteristics.

As explained in Chapter 3, the auction algorithm proceeds by assigning first the on-orbit satellites, the launched satellites going then to the remaining spots. In this case study, the launch vehicle selected can carry two satellites per launch. Six launches
Table 5.1: Assignment for chemical propulsion. Reconfiguration of the constellation $A$ to $B$ with $h_b = 1200 km$ and $\epsilon_b = 5 deg$

<table>
<thead>
<tr>
<th>Initial Position in $A$</th>
<th>Final Slot in $B$</th>
<th>$\Delta V$ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_a^1(1)$</td>
<td>$P_b^1(8)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P_a^1(2)$</td>
<td>$P_b^1(7)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P_a^1(3)$</td>
<td>$P_b^1(6)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P_a^1(4)$</td>
<td>$P_b^1(5)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P_a^1(5)$</td>
<td>$P_b^1(4)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P_a^1(6)$</td>
<td>$P_b^1(3)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P_a^1(7)$</td>
<td>$P_b^1(2)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P_a^2(1)$</td>
<td>$P_b^2(8)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^2(2)$</td>
<td>$P_b^2(7)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^2(3)$</td>
<td>$P_b^2(6)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^2(4)$</td>
<td>$P_b^2(5)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^2(5)$</td>
<td>$P_b^2(4)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^2(6)$</td>
<td>$P_b^2(3)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^2(7)$</td>
<td>$P_b^2(2)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^3(1)$</td>
<td>$P_b^3(8)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^3(2)$</td>
<td>$P_b^3(7)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^3(3)$</td>
<td>$P_b^3(6)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^3(4)$</td>
<td>$P_b^3(5)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^3(5)$</td>
<td>$P_b^3(4)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^3(6)$</td>
<td>$P_b^3(3)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P_a^3(7)$</td>
<td>$P_b^3(2)$</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Figure 5-2: Representation of the initial assignment.

would be necessary: 5 launches of 2 satellites and 1 launch of 1 satellite. Considering the distribution of the ground satellites in the first assignment (see Table 5.2), it can be deduced that this repartition does not match with the launcher capacity. A reassignment is thus necessary. Figure 5-2 depicts this first assignment and the problem raised by the capacity of the launch vehicles. The loop for assigning the

Table 5.2: Number of ground satellites assigned in each plane after the first run.

<table>
<thead>
<tr>
<th>Plane</th>
<th>Number of ground satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

satellites on the ground will be thus considered and the process of ground satellites assignment detailed. Plane 3 contains no satellites of A, the four first launches will therefore go to plane 3 of B. Figure 5-3 summarizes this. However, the fifth launch requires to free one slot in plane 1, 2 or 4 in order to allow a launch of two satellites. There is indeed in these planes only one slot each reserved for ground satellites. The
Figure 5-3: Assignment of ground satellites process. The first four launches go to plane 3 and fill that plane entirely.

loop finally chooses to free a slot in plane 4. The satellites of the fifth launch are thus put in the plane 4 (see Figure 5-4). The module had the choice to put the satellites of the fifth launch also to plane 1 and plane 2. But since it is better to penalize a satellite with high $\Delta V$ required to achieve the transfer, it was chosen to free a slot in plane 4. Planes 2 and 4 have a $\Delta V$ of 2.5 km/s per satellite. Plane 1 has a $\Delta V$ of 0.85 km/s per satellite. A fifth launch into this plane was prevented. Why choose the plane where the average $\Delta V$ is higher? Because it allows to reduce the gain in $\Delta V$ after the reassignment of the satellites of $A$.

Once the reassignment is done, the auction algorithm is run a second time with the matrix $\Delta V_{i,j}^{\text{modif}}$. The new assignment is summarized in Table 5.3. The Delta V of 50.5 km/s obtained with this second run points out the influence of the loop for assigning the launched satellites. This influence is higher if the launch vehicle has a high capacity, which is not the case in this example. Indeed, only one on-orbit satellite is penalized by this reassignment. But this satellite is severely penalized with a $\Delta V$ of almost 12.5 km/s which represents a mass of extra fuel of 12.8 tons.
Table 5.3: Final Assignment for chemical propulsion. Reconfiguration of the constellation $A$ to $B$ with $h_b = 1200km$ and $e_b = 5deg$. In bold are represented the changes due to the reassignment of the ground satellites.

<table>
<thead>
<tr>
<th>Initial Position in $A$</th>
<th>Final Slot in $B$</th>
<th>$\Delta V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^a_1(1)$</td>
<td>$P^b_1(8)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P^a_1(2)$</td>
<td>$P^b_1(7)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P^a_1(3)$</td>
<td>$P^b_1(6)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P^a_1(4)$</td>
<td>$P^b_1(5)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P^a_1(5)$</td>
<td>$P^b_1(4)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P^a_1(6)$</td>
<td>$P^b_1(3)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P^a_1(7)$</td>
<td>$P^b_1(2)$</td>
<td>0.85</td>
</tr>
<tr>
<td>$P^a_2(1)$</td>
<td>$P^b_2(2)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_2(2)$</td>
<td>$P^b_2(6)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_2(3)$</td>
<td>$P^b_2(5)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_2(4)$</td>
<td>$P^b_2(7)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_2(5)$</td>
<td>$P^b_2(8)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_2(6)$</td>
<td>$P^b_2(4)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_2(7)$</td>
<td>$P^b_2(3)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_3(1)$</td>
<td>$P^b_4(7)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_3(2)$</td>
<td>$P^b_4(6)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_3(3)$</td>
<td>$P^b_4(8)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_3(4)$</td>
<td>$P^b_4(1)$</td>
<td>12.5</td>
</tr>
<tr>
<td>$P^a_3(5)$</td>
<td>$P^b_4(4)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_3(6)$</td>
<td>$P^b_4(3)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P^a_3(7)$</td>
<td>$P^b_4(5)$</td>
<td>2.5</td>
</tr>
</tbody>
</table>
5.2.2 Comparison of two Strategies for Chemical Propulsion

In view of such an extra-mass of fuel, two options are conceivable depending on the feasibility to add to a satellite with a 700 kg dry mass an extra-mass of almost 13 tons. From the data available for the different launch systems, we know that it is possible to launch such a mass in LEO. Ariane 5 has indeed a capacity of 18 tons into LEO. Nevertheless we can doubt the technical and economical feasibility for a single satellite to transport such an extra mass. Wertz and Larson [SMAD99] provide data on Orbital Transfer Vehicles. From this source, we can estimate an order of magnitude for the ratio $\frac{\text{Mass}_{\text{total}}}{\text{Burnout-Mass}}$. For the Orbital Transfer Vehicles called IUS manufactured by Boeing, we obtain a ratio around 13. In the case of the satellite penalized, the ratio obtained is 18. It is a little bit higher than the previous case. No study indicates the infeasibility of such a ratio. Nevertheless, for such high ratios, the rocket equation dictates that most of the fuel would be used to push fuel around, rather than useful mass.

In this section, two strategies will be thus described with respect to this penalized
satellite. The first strategy will consist of assuming that this satellite can transport the necessary fuel to achieve its transfer. The simulator will be run and results returned. The second strategy consists of assuming that it is impossible technically. The option is thus to abandon this satellite and to replace it with a new launched satellite. The abandoned satellite becomes in this strategy a spare satellite in the new constellation $B$ but might not be able to be used in all planes of $B$. The impact of this strategy on cost, time, $\Delta V$ and coverage will be described.

If we run the simulator without taking into account the problem raised by this satellite, the following results are obtained:

- $\Delta V_{total} = 50.5382 \ km/s$ (which represents a $\Delta M$ of 21.2 tons)
- $T_{total} = 300 \ days$ (only 21 hours of this for the transfer phase)
- $C_{total} = 0.5659 \ B\$
- $Cov_{mean} = 59\%$

Table 5.4 provides the cost breakdown. It is interesting to notice that the chunks have
the same order of magnitude except for the cost of outage. It can be explained, if we consider that with all satellites from A transferring simultaneously, most of the 300 days are consumed by launches which do not result in service outage of constellation A.

Table 5.4: Cost breakdown obtained with the first strategy.

<table>
<thead>
<tr>
<th>Cost</th>
<th>Cost in $B$</th>
<th>percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outage</td>
<td>0.0009</td>
<td>0.1%</td>
</tr>
<tr>
<td>Fuel</td>
<td>0.2753</td>
<td>48.6%</td>
</tr>
<tr>
<td>Production</td>
<td>0.1696</td>
<td>29.9%</td>
</tr>
<tr>
<td>Launch</td>
<td>0.1201</td>
<td>21.2%</td>
</tr>
<tr>
<td>Total</td>
<td>0.5659</td>
<td>100%</td>
</tr>
</tbody>
</table>

The second strategy consisting of abandoning the penalized satellite and to launch another one, we can a priori guess some tendencies. The $\Delta V_{total}$ will obviously decrease, since the 13 tons of fuel to transfer the satellite are no longer necessary. The time to do the operations will increase due to the delay to launch another satellite. Concerning the cost, the Launch and production costs will obviously increase, the fuel cost should decrease. It is more difficult to imagine the impact of this abandonment on the cost of service outage. However, this impact will surely be slight. Practically, to estimate these objectives in this second strategy, the launch and transfer schedules $S_l$ and $S_t$ have been modified. The matrix line corresponding to the penalized satellites was suppressed in $S_t$ and a line corresponding to the launch of the replacing satellite was added in $S_l$. The following results were obtained:

- $\Delta V_{total} = 38.0572 \text{ km/s} \ (\Delta M = 21.2 - 12.8 = 8.4 \text{ tons})$
- $T_{total} = 360 \text{ days}$
- $C_{total} = 0.4336 \text{ B$}$
- $ Cov_{mean} = 60.575\%$

As for the first strategy, a cost breakdown was done. Table 5.5 summarizes this. As forecast, the fuel cost decreases a lot in this second strategy, whereas the launch and
production cost increase slightly. The outage cost is almost the same and remain negligible. The total cost obtained is of the same order of magnitude but, lower than for the first strategy. The difference for the case considered is 130M$ in favor of abandonment. And the abandonment strategy has the advantage of having an on-orbit spare in constellation $B$, albeit not for all slots of $B$.

5.2.3 Comparison for Electric Propulsion

Now the same comparison will be done with an electric propulsion system. Resistojet ($I_{sp} = 500$ sec.) was selected for this purpose.

If we run the simulator without considering the problem posed by the penalized satellite, we obtain:

- $\Delta V_{total} = 54.45 \text{ km/s}$ (which represents a $\Delta M$ of 19.5 tons. The penalized satellite must carry 11.7 tons of fuel.)
- $T_{total} = 580.5$ days (300 days for the launch phase and $\approx 280$ days for the transfer phase)
- $C_{total} = 0.6217 \text{ B$}$
- $Cov_{mean} = 59\%$

The cost breakdown is summarized in Table 5.6.

Contrary to chemical propulsion, the transfer time is a function of the $\Delta V$ required to achieve this transfer. Abandoning the penalized satellite will thus decrease
Table 5.6: Cost breakdown obtained with the first strategy with resistojet.

<table>
<thead>
<tr>
<th>Cost</th>
<th>Value in B$</th>
<th>percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outage</td>
<td>0.078</td>
<td>12.5%</td>
</tr>
<tr>
<td>Fuel</td>
<td>0.254</td>
<td>40.8%</td>
</tr>
<tr>
<td>Production</td>
<td>0.1696</td>
<td>27%</td>
</tr>
<tr>
<td>Launch</td>
<td>0.12</td>
<td>19.3%</td>
</tr>
<tr>
<td>Total</td>
<td>0.6217</td>
<td>100%</td>
</tr>
</tbody>
</table>

relatively the total time of reconfiguration. The results given by the second strategy are:

- \( \Delta V_{\text{total}} = 40.35 \text{ km/s} \)
- \( T_{\text{total}} = 300 \text{ days (launch phase)} + 60 \text{ days (launch of a replacement satellite)} + 168.5 \text{ days (transfer phase)} = 529 \text{ days} \)
- \( C_{\text{total}} = 0.487 \text{ B$} \)
- \( Cov_{\text{mean}} = 63.63\% \)

The second strategy seems thus to be less expensive and quicker in electric propulsion. Concerning the different chunks of the total cost, the comments are the same as for chemical propulsion. The difference is also of 130M$ in favor of abandonment. However, in this case there is a gain of around 50 days in favor of abandonment. The abandonment strategy is thus very interesting in electric propulsion in the case where a spacecraft is severely penalized by the reassignment.

Table 5.7: Cost breakdown obtained with the second strategy with resistojet.

<table>
<thead>
<tr>
<th>Cost</th>
<th>Value in B$</th>
<th>percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outage</td>
<td>0.0624</td>
<td>12.8%</td>
</tr>
<tr>
<td>Fuel</td>
<td>0.10</td>
<td>20%</td>
</tr>
<tr>
<td>Production</td>
<td>0.1846</td>
<td>38%</td>
</tr>
<tr>
<td>Launch</td>
<td>0.14</td>
<td>28.7%</td>
</tr>
<tr>
<td>Total</td>
<td>0.487</td>
<td>100%</td>
</tr>
</tbody>
</table>
5.2.4 Conclusion

The second strategy is a conceivable and interesting alternative. If further study shows that a limit exists for the extra-mass of fuel, this method, consisting of abandoning those satellites heavily penalized by the reassignment and of launching new ones in their place, is applicable. The cost is of the same order of magnitude and even lower. Obviously, there is a delay due to the extension of the launch phase. In electric propulsion, this delay is compensated by the abandon of the “slowest” satellite. In chemical, this delay could entail loss of economical opportunity, since the final configuration with higher revenue is postponed. Thus in chemical, the manufacturer could have the choice between a quicker but more expensive strategy (penalization strategy) or a longer but less expensive strategy (abandonment strategy).

However, for the end of this chapter, it will be assumed that the first strategy is technologically feasible. Only this strategy will be applied further.

5.3 Impact of the Scenario and of the Propulsion System

In this section, the framework is run for the same reconfiguration than considered in the last section: reconfiguration to the constellation $B$ with altitude $h_b = 1200 \ km$ and elevation $\epsilon_b = 5 \ deg$. However, the purpose of this section is to determine the influence on the overall results, notably cost and time of the 6 different propulsion systems (chemical, arcjet, resistojet, plasma thruster, hall thruster and ion engine) and of the four scenarios for the transfers of the on-orbit satellites. The propulsion systems and the scenarios were presented in Chapter 4. Subsection 5.3.1 will focus on the impact of the scenarios, whereas the Subsection 5.3.2 will deal with the different propulsion systems. Subsection 5.3.3, based on the observations done in the two previous subsections, will recommend a scenario and a propulsion system to solve this particular reconfiguration problem.
5.3.1 Scenario Impact

In order to determine the influence of the scenarios selected for the transfers, the propulsion system was fixed. The propulsion system utilized for this purpose was chemical. The framework was run for the 4 scenarios presented in the previous chapter. Table 5.8 summarizes the results returned by the simulator for the four scenarios. Some comments can be made about this. First of all, scenarios 2 and 4 return exactly the same results. It is not surprising since those scenarios consist of transferring all satellites at the same time. The satellites begin their transfers together in the second scenario, whereas they finish their transfers at once in the fourth scenario. Apparently, this distinction has little influence given our assumptions. For now, only the second scenario will be considered. Scenario 2 seems to be the most interesting one in terms of cost and time compared to the first and third ones: approximately 11 hours to achieve the transfers and a cost of service outage of 0.88 M$. Scenario 1 leads to similar results: 21 hours for the transfer phase and a cost of service outage of 0.95 M$. Scenario 3, consisting of transferring all the satellites sequentially is less advantageous with a transfer phase of around 9 days and a cost of service outage slightly higher with 5.6 M$ (however in this example the service outage cost is quite negligible compared to the other costs, assuming that no customer will permanently be lost due to the 9 day service interruption). Moving many satellites at once (all or half) seems to be the right strategy. It allows to compress the schedule and to reduce the service interruption. Nevertheless, moving many satellites at once is also high risk, since all satellites need to be tracked and controlled simultaneously from the ground. Knowing that in chemical the burns must occur at precise instants, it is unlikely to happen that way in real life. The size of the qualified personnel necessary to track and control all satellites would be too high. For this, only the third scenario (moving the satellites sequentially) is realistic in chemical and only this scenario will be considered later. Maybe an intermediate scenario between the second and the third would be technically feasible: moving two, three or four satellites at once. In electrical propulsion, the transfer phase does not require such attention since this phase
Table 5.8: Impact of the transfer scenarios using chemical propulsion

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Propulsion</th>
<th>$T_{transfer}$ (hours)</th>
<th>Cov$_{mean}$ (%)</th>
<th>$C_{tot}$ (B$)</th>
<th>$C_{outage}$ (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-two waves</td>
<td>Chemical</td>
<td>21.4</td>
<td>59</td>
<td>0.5659</td>
<td>0.95</td>
</tr>
<tr>
<td>2-begin together</td>
<td>Chemical</td>
<td>10.8</td>
<td>34.375</td>
<td>0.5658</td>
<td>0.88</td>
</tr>
<tr>
<td>3-staggered</td>
<td>Chemical</td>
<td>224.7 $\approx$ 9.3 days</td>
<td>78.29</td>
<td>0.5705</td>
<td>5.6</td>
</tr>
<tr>
<td>4-end together</td>
<td>Chemical</td>
<td>10.8</td>
<td>34.375</td>
<td>0.5658</td>
<td>0.88</td>
</tr>
</tbody>
</table>

is much longer and there are no single impulsive burns. We will thus assume that Scenario 2 is feasible in electric propulsion. To conclude this subsection, scenario 2 appears least costly in terms of service outage. It seems that the cost of service outage is much more sensitive to the length of the transfer time than to the coverage capability. Although the mean coverage maintained during the operations is $34.375\%$ for scenario 2 to be compared with the $78.29\%$ of scenario 3, the compressed scenario entails the least loss of revenue.

5.3.2 Impact of the Propulsion Systems

The purpose of this section is to show the influence of the propulsion systems on the cost and time.

Firstly, we will show the sensitivity of the cost with respect of the scenario for the six propulsion systems considered in this thesis. The difference of the cost obtained with scenario 3 with the cost obtained with scenario 2 is computed. This difference is written as $\Delta C = C_3 - C_2$. Table 5.9 shows the results. A graph of $\Delta C$ versus $I_{sp}/(\eta P)$ is drawn in Figure 5-6. The points are on a straight line. This can be interpreted by assuming that in electric propulsion, the other costs are negligible with respect of the cost of service outage. Knowing that the cost of service outage depends linearly of the transfer time and that the time is linear with $I_{sp}/(\eta P)$, we can explain this.

Thus, we can deduce that, the higher the transfer time is, the larger the gap in terms of total cost between the compressed and staggered scenarios is. In chemical propulsion, this gap is slight. In electric propulsion, the gap increases almost proportionally to $I_{sp}$.
Table 5.9: Sensitivity of the total cost with respect to the propulsion system.

<table>
<thead>
<tr>
<th>Propulsion</th>
<th>$C_3$ (in $B$)</th>
<th>$C_2$ (in $B$)</th>
<th>$\Delta C$ (in $B$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical</td>
<td>0.5705</td>
<td>0.5658</td>
<td>0.0047</td>
</tr>
<tr>
<td>Resistojet</td>
<td>0.8812</td>
<td>0.595</td>
<td>0.2862</td>
</tr>
<tr>
<td>Arcjet</td>
<td>1.6246</td>
<td>0.6473</td>
<td>0.9773</td>
</tr>
<tr>
<td>PPT</td>
<td>11.8377</td>
<td>2.086</td>
<td>9.7517</td>
</tr>
<tr>
<td>Hall Thruster</td>
<td>3.6781</td>
<td>0.835</td>
<td>2.8431</td>
</tr>
<tr>
<td>Ion Engine</td>
<td>2.99</td>
<td>0.713</td>
<td>2.277</td>
</tr>
</tbody>
</table>

Figure 5-6: $\Delta C$ versus $Isp/(\eta P)$ drawn for the five electric propulsion systems.

Now, only the 5 electric propulsion systems will be considered. The scenario considered is scenario 2 which is assumed to be technically realizable in electric propulsion as explained in the last section. Table 5.10 summarizes the results obtained. We notice the following:

- 1) Cost and Time are very sensitive to the propulsion systems chosen. The transfer times are very high for electric propulsion. We should notice that it is due in part to the penalized satellite whose transfer time is very high.

- 2) The times for completing the transfers $T_{transfers}$ are too high with Ion Engine, Hall Thruster and Plasma thruster: from 1115 days (around three years) with Ion Engine for the second scenario to 4775 days (13 years) with Plasma thrusters. These values indicate that these propulsion systems are not viable in
Table 5.10: Impact of the propulsion systems

<table>
<thead>
<tr>
<th>Propulsion</th>
<th>$C_{total}$ (in B$)</th>
<th>$C_{fuel}$ (in B$)</th>
<th>$C_{outage}$ (in B$)</th>
<th>$T_{transfers}$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistojet</td>
<td>0.595</td>
<td>0.2540</td>
<td>0.0511</td>
<td>140</td>
</tr>
<tr>
<td>Arcjet</td>
<td>0.6473</td>
<td>0.1832</td>
<td>0.1744</td>
<td>478</td>
</tr>
<tr>
<td>PPT</td>
<td>2.086</td>
<td>0.0563</td>
<td>1.74</td>
<td>4775</td>
</tr>
<tr>
<td>Hall Thruster</td>
<td>0.835</td>
<td>0.0389</td>
<td>0.5072</td>
<td>1391</td>
</tr>
<tr>
<td>Ion Engine</td>
<td>0.713</td>
<td>0.0169</td>
<td>0.4064</td>
<td>1115</td>
</tr>
</tbody>
</table>

3) Resistojet and Arcjet utilization, even if a little bit more expensive than chemical (0.595 and 0.6473 billions dollars respectively) and much longer (around 140 days for transfers with resistojet and one and a half years with arcjet) remain possible options.

4) The fuel cost decreases as the $I_{sp}$ increases. This fuel cost is an important portion of the total cost in chemical propulsion (around 50%), whereas in electric propulsion the fuel cost is negligible except for resistojets and arcjets. Figure 5-7 depicts this. From this figure, we conclude that in this example of reconfiguration the costs of fuel and outage are not of the same order of magnitude from an $I_{sp}$ of 700 s. In Chapter 3, we were expecting a tradeoff between those two costs, with an optimal $I_{sp}$. The figure depicts an optimum for an $I_{sp}$ around 430 sec. This optimum is obtained for a chemical thruster. The points obtained in the chemical propulsion field ($I_{sp}$ below 430 sec) were generated with scenario 3. The points obtained in electric propulsion ($I_{sp}$ above 430 sec.) were generated with scenario 2. From this picture, the best option in terms of cost seems to be the chemical thruster with the highest $I_{sp}$.

5.3.3 Recommendation

In this subsection, the results obtained with chemical propulsion and staggered schedule (scenario 3) will be compared with the results obtained with the most advantageous electric propulsion systems in a compressed schedule (scenario 2). From the
results obtained in the last subsection, the best electric systems both in terms of Time and Cost is Resistojet: 140 days of transfer and a total cost $C_{total}$ of 0.595 $B$. Recall that the transfers last around 9 days and that the total cost is 0.57 $B$ for chemical propulsion for the scenario 3. From this, we would advise to a manufacturer to utilize chemical propulsion even with a staggered schedule. The costs are of the same order of magnitude, but the reconfiguration time is less, by far, in chemical propulsion. Scenario 3 corresponding to a sequential schedule where the satellites move one by one, can be improved by compressing a little bit the schedule: moving two or three satellites at once will surely decrease the cost of outage (but very slightly) and the transfer time. Chemical propulsion is thus the best option at the present time. Indeed, this recommendation might change in the future if high-$I_{sp}$, high-thrust propulsion devices are developed. Moreover, this recommendation is made for a particular reconfiguration $A$ to $B$. For another reconfiguration, the costs
Table 5.11: Reconfiguration Trade Space. $s$ is the transfer scenario.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>values</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_b$</td>
<td>‘Polar’</td>
<td></td>
</tr>
<tr>
<td>$h_b$</td>
<td>800,1200,1600</td>
<td>km</td>
</tr>
<tr>
<td>$\epsilon_b$</td>
<td>5,20</td>
<td>deg</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Ch, Res, Arc, PPT, Hall, Ion</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>1,2,3,4</td>
<td></td>
</tr>
</tbody>
</table>

could be lower in electric than in chemical. However, the time of reconfiguration will be always lower in chemical. The next section will show that the recommendation given in this subsection should be interpreted carefully.

5.4 Tradespace Exploration

In this subsection, a tradespace of service outage vs fuel cost is explored. The purpose of this subsection is to confirm that chemical transfers are characterized by high fuel cost and low cost of service outage and that in electrical transfer, the fuel cost decreases and the service outage cost increases with increasing $I_{sp}$. Recall that the design vector is made up of the altitude and minimum elevation angle of the constellation B ($h_b$ and $\epsilon_b$) and of the propulsion systems. For this trade space exploration, we add to the design vector the transfer scenario (1, 2, 3 or 4). The reassignment loop was not considered for this study. First the continuous elements are discretized (altitude and minimum elevation). 3 altitudes are chosen (800, 1200 and 1600) and two min elevation angles (5 and 20 degrees). Table 5.11 summarizes the trade space. This results in a total of 144 possibilities. This trade space is represented in Figure 5-8 with corresponding legend in Figure 5-9: service outage vs fuel cost. A Pareto Front appears clearly in this tradespace. The chemical transfers are at the bottom on the right of the tradespace, characterizing high fuel cost and low outage cost. Each propulsion system is characterized by a zone on this diagram. For the electric transfers, the higher the $I_{sp}$ is, the more to the top-left of the tradespace the points are. The fuel cost dominates all the reconfigurations in chemical and most of the reconfigurations with Resistojet (except with scenario 3). The outage cost
Figure 5-8: Fuel cost versus service outage cost.

![Graph showing fuel cost versus service outage cost with Pareto front, fuel cost dominates, and outage cost dominates areas marked.]

LEGEND:

**Propulsion:**
- Chemical
- Resistojet
- Arcjet
- Plasma Thruster
- Hall Thruster
- Ion Engine

**Scenario:**
- x scenario 1
- o scenario 2
- + scenario 3
- * scenario 4

Figure 5-9: Legend

dominate all the reconfigurations in Ion Engine, Plasma Thruster and Hall Thruster. The reconfigurations with Arcjet are on the limit characterized by the straight line on the figure.

There are on the figure, 5 Pareto-optimal reconfigurations. Table 5.12 provides
Table 5.12: Pareto Optimal Reconfigurations

<table>
<thead>
<tr>
<th>numero</th>
<th>$h_b$ [km]</th>
<th>$\epsilon_b$ [deg]</th>
<th>scenario</th>
<th>$Pr$ [-]</th>
<th>$C_{fuel}$ [M$]$</th>
<th>$C_{outage}$ [M$]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>800</td>
<td>20</td>
<td>2</td>
<td>chemical</td>
<td>54</td>
<td>0.26</td>
</tr>
<tr>
<td>2</td>
<td>1600</td>
<td>20</td>
<td>2</td>
<td>chemical</td>
<td>32</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>1600</td>
<td>20</td>
<td>2</td>
<td>Resistojet</td>
<td>6.8</td>
<td>1.46</td>
</tr>
<tr>
<td>4</td>
<td>1600</td>
<td>20</td>
<td>2</td>
<td>Arcjet</td>
<td>5.8</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>1600</td>
<td>20</td>
<td>2</td>
<td>Ion Engine</td>
<td>1</td>
<td>11.6</td>
</tr>
</tbody>
</table>

a listing of these points. All the pareto-optimal reconfigurations are done with the second scenario (compressed schedule) proving that this scenario minimizes the cost of service outage. Moreover, except for the first one, all the points concern the reconfiguration to the constellation $B$ with an altitude of 1600 km and an elevation angle of 20 deg. This particular configuration is very close in altitude to the initial one, which can explain why the fuel consumption, and so the fuel cost is lower than for the other reconfigurations. This configuration $B$ has also a large number of slots, because of the elevation angle of 20 deg ($N_{sats}(B) = 60$). A large number of slots in the final constellation provides more possibilities for the assignment of the on-orbit satellites. This can explain also why the fuel cost is low. Which is surprising in these points, compared to the last section is the fact that electric propulsion seems less expensive than chemical propulsion. If we add outage and fuel costs, we obtain 32M$ for chemical propulsion, 8M$ for resistojet, 11M$ for arcjet and 13M$ for Ion Engine. We can explain this while considering that for this particular reconfiguration, the transfer times are not very high even in electric propulsion. Consequently, the outage cost are not very high, explaining the costs obtained for the different propulsion system. In addition to this, knowing that the reassignment loop was not implemented for this tradespace, no satellite were penalized, preventing from higher outage cost and fuel cost (see the case study Section 5.2).
5.5 Economic Opportunity of Multiple Reconﬁgurations Path

Chaize has shown that optimal paths of reconﬁgurations exist for the staged deployment depending of the value of the discount rate $r$ [CHAI03]. He describes 5 paths. Table 5.13 summarizes these 5 paths.

Table 5.13: The 5 paths of reconﬁguration

<table>
<thead>
<tr>
<th>Path</th>
<th>$r$</th>
<th>Arch. 1</th>
<th>Arch. 2</th>
<th>Arch. 3</th>
<th>Arch. 4</th>
<th>Arch. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0%</td>
<td>800km, 5deg</td>
<td>400km, 5deg</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5%</td>
<td>2000km, 5deg</td>
<td>800km, 5deg</td>
<td>400km, 5deg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10-30%</td>
<td>2000km, 5deg</td>
<td>1600km, 5deg</td>
<td>1200km, 5deg</td>
<td>800km, 5deg</td>
<td>400km, 5deg</td>
</tr>
<tr>
<td>4</td>
<td>35%</td>
<td>2000km, 5deg</td>
<td>1600km, 5deg</td>
<td>1200km, 5deg</td>
<td>800km, 5deg</td>
<td>400km, 5deg</td>
</tr>
<tr>
<td>5</td>
<td>40-100%</td>
<td>1600km, 5deg</td>
<td>1200km, 5deg</td>
<td>800km, 5deg</td>
<td>400km, 5deg</td>
<td>400km, 20deg</td>
</tr>
</tbody>
</table>

Each path entails a gain on the Life Cycle Cost $LCC$. However, Chaize when computing the Life Cycle Cost of the staged deployment $LCC^{staged}$ has not taken into account the cost of reconﬁgurations, in other words the cost of the fuel necessary to achieve the transfers and the cost of the service outage. For path 1, 2 and 3, we will thus compare the gain in the $LCC$ with the cost of the different reconﬁgurations added and determine if the staged deployment strategy still represents an economic interest.

Table 5.14: Gain on the $LCC$

<table>
<thead>
<tr>
<th>Path</th>
<th>$LCC^{staged}$ in B$</th>
<th>$LCC^{trad}$ in B$</th>
<th>gain in B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
<td>1.95</td>
<td>0.65</td>
</tr>
<tr>
<td>4</td>
<td>1.25</td>
<td>1.95</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>1.05</td>
<td>1.9</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Path 1 only requires one reconﬁguration. Utilizing the simulator developed during the project, we obtain that the fuel cost for this reconﬁguration $C_{fuel}$ is equal to $\approx 0.25B$ whereas the cost of outage $C_{outage}$ is around 8 $M$. There is thus the possibility to save $0.4 - 0.25 = 0.15B$ with this path compared to a traditional
Figure 5-10: Economic opportunity of the Path 1.

fixed deployment. Figure 5-10 summarizes this. This represents a savings of $150M (≈ 7.5% of LCC). Whether this potential (average) savings is worthwhile, given the technical risks involved and other uncertainties is a managerial decision. The framework was able to quantify the problem and provide a decision basis.

Path 2 requires two reconconfigurations. The reconconfiguration from the architecture 1 to the architecture 2 entails a fuel cost $C_{fuel}$ of 0.2 $B\$ and a service outage cost of 3 $M\$. The reconconfiguration from architecture 2 to architecture 3 represents a fuel cost of 0.25 $B\$. We can thus save with this path $0.5 - 0.2 - 0.25 = 0.05 \ B\$. Figure 5-11 summarizes this.

Path 3 requires 4 reconconfigurations. The gain entailed by this path is around 0.65 $B\$ (see Table 5.14). The total reconconfiguration cost for these 4 reconconfigurations is estimated to 0.825 $B\$. With our model, there is no economic opportunity in this case. Nevertheless, a more detailed study of the reconconfiguration from the Architecture 2 to the Architecture 3 shows that a satellite is penalized by the reassignment (analogous case than in Section 5.2). The case study Section 5.2 has shown that abandoning the penalized satellite and launching a new one can reduce the cost of more than 100 $M\$, allowing perhaps an economic opportunity. However due to the approximations, it is difficult to have a precise conclusion for this path. The gain in LCC is of the same
Figure 5-11: Economic opportunity of the Path 2.

To conclude this section, the higher the number of reconfigurations required for a path is, less interesting the economic opportunity seems to be.

5.6 High Fidelity Reconfiguration Simulation

The purpose of the Reconfiguration Framework developed in this thesis was to return on order of magnitude for the cost and time of reconfiguration. Nevertheless, the modules implemented do not allow for virtual execution of the reconfiguration, for optimization and rough planning of the reconfiguration maneuvers. This framework allows in order of magnitude for the cost and time of reconfiguration. This thesis was to return the economic opportunity of the Path 2.
5.6.1 Structure of a High Fidelity Constellation Reconfiguration Simulation

As explained in Chapter 4, the assignment process does not distinguish between satellites in the same plane. The satellites are assigned to the right plane, in order to minimize the total Delta V, $\Delta V_{tot}$, but the assignment to the right slot within a plane is not considered explicitly in our model. The High Fidelity Constellation Reconfiguration Simulation should be thus capable of distinguishing the different slots in a same plane. As a consequence of our simplifications, the transfer time was roughly and conservatively approximated for each satellite. A High Fidelity Simulation would allow to determine the exact position of each spacecraft and the exact time of transfer. Particularly, in chemical propulsion, once the satellite has executed the semi-ellipse of the Hohmann transfer, this satellite is on the right orbit, but not necessarily in the right position (or slot) on this orbit. Hence, the necessity to rephase this satellite. A High Fidelity Simulation would allow to know the exact position of the satellite on this orbit and the exact position of the other satellites already-present on this orbit. The knowledge of the relative position of the satellite with respect of the other satellites already-present in the orbit, would allow to deduce by which angle $\Delta \Theta$, the satellite has to be moved in order to reach its final slot. From Equation (4.7) explained in Chapter 4, we would be capable to deduce the exact time of transfer for this satellite. Recall that in the actual simulation, we approximate $\Delta \Theta$ arbitrarily by $\pi$ (worst-case situation). The High Fidelity Simulation would therefore yield a lower transfer time.

The explanations given in this subsection are valid only for chemical propulsion. In electric propulsion, the trajectories are different. We have for now not enough knowledge of the transfer in electric propulsion, except that the trajectory is a spiral. Thus, it is difficult for instance to say, where the S/C arrives on the right orbit after the transfer. Since a LEO orbit is on the order of 100 min, one degree of true anomaly corresponds to 16.7 sec, which is small compared to the total ”spiral” transfer time of 40-300 days. We have assumed previously that the spacecraft arrives directly on the
right slot, neglecting a repositioning phase. This assumption is not obvious and would need further studies. For all these reasons, this chapter on High Fidelity Simulation will only focus on chemical transfers.

5.6.2 Removal of Simplifying Assumptions

In order to know the exact trajectory of each satellite, we need a time reference or Epoch. At this time reference, the true anomaly of the different satellites on their respective orbits need to be known for configuration $A$. So, the actual simulation has to be modified in order to take into account the true anomaly $\theta$. Figure 5-12 depicts a possible Framework for the High Fidelity Simulation. Let’s consider the satellite $i$ of $A$. Its true anomaly at $t = 0$ is $\theta = \theta_i$ (very important information).

The actual framework is very useful since it allows to know to which plane of $B$, 

![Figure 5-12: High Fidelity Simulation Framework](image-url)

![Figure 5-12: High Fidelity Simulation Framework](image-url)
the satellite $i$ will be transferred (top part of Figure 5-12). But it is not sufficient to determine the exact slot in this final plane. The knowledge of the final plane will allow also to determine the true anomaly of the crossing point of the initial orbit and final plane. Let’s call $\theta_{\text{crossing}}$ this true anomaly. From this true anomaly and from the transfer scenario selected, the Hi Fi Simulation will be capable to determine when the satellite can begin its transfer. $t_{\text{start}}$ corresponds to the beginning of the Hohmann transfer. $t_{\text{start}}$ has to correspond to an instant when the satellite is on the crossing point, since $t_{\text{start}}$ is the time of the first burn. After this, the simulation determines the time and true anomaly of arrival in the final orbit, $t_{\text{arrival}}$ and $\theta_{\text{arrival}}$. It should be easy to compute, since we used the Hohmann strategy. If the S/C transferred is the first satellite arrived in the final orbit, this S/C will not need to be rephased in the orbit and will be a reference for the other satellites in that plane. However, if the S/C is not the first arrived, it will need to be repositioned. The phasing module will allow this. The Inputs to this module would be the true anomaly of the satellite considered for the repositioning, here the $i^{th}$ and the true anomalies of the satellites already present. From this, the module will deduce which slots are free and to which of these slots the satellite $i$ must be assigned. The module will choose the slot the closest to the satellite, in order to minimize the repositioning phase. The Outputs of this module are the time when the transfer is achieved, $t_{\text{end}}$ and the final slot for the satellite $i$. Figure 5-13 summarizes this. Various uncertainties could be included in the future.

5.6.3 Tools for a High Fidelity Simulation

Matlab and the Spacecraft Control Toolbox\(^1\) should be sufficient to simulate the reconfiguration process and allow a visualization. However, a convenient way to code the High Fidelity Simulation has to be found. The process explained in the last section, has to be applied to a large number of satellites, the $N_{\text{sat}}(A)$ satellites of constellation $A$, sometimes in parallel depending on the transfer scenario selected. A very high number of parameters are thus necessary. Once the different parameters

\(^1\)Product of Princeton Satellite Systems
Figure 5-13: Hi Fi Simulation Process
(initial true anomaly $\theta_i$, start transfer time $t_{\text{time}}$, position of the crossing point $\theta_{\text{crossing}}$, arrival on the new orbit time $t_{\text{arrival}}$, end of transfer time $t_{\text{end}}$ and final slot in $B$) for the different satellites are determined, a visualization would be possible, thanks to the Spacecraft Control Toolbox. Three functions from this toolbox would be very useful:

- The function “RV2El” allows to compute the six orbital elements, defined in Chapter 4 of a new trajectory. For instance, at the crossing point where the burn occurs, knowing the cartesian coordinates of this point and the final velocity obtained after the increment of velocity ($\vec{V}_{\text{after}} = \vec{V}_{\text{before}} + \Delta \vec{V}$), the function “RV2El” returns the orbital elements of the new trajectory (in our case, the orbital elements of the transfer ellipse). It is very useful, since these elements are difficult to compute manually.

- The function “RVFromKepler” allows to deduce the position and speed of a S/C with respect of the time. The inputs are the 6 orbital elements of the S/C and the instants when we want to know its position and speed. This function assumes that the orbital elements are given at $t = 0$. With this function, the trajectory with respect to time is easily generated.

- The function “AnimationGUI” shows in 3D the satellite motions. The inputs are the coordinates of the different S/C with respect to time.

These three functions could be used as follows. For instance, we know the 6 orbital elements of the initial orbit of the S/C. Knowing this and the time we want to represent its initial orbit, we first use “RVFromKepler”. At the crossing point where the transfer begins, we use “RV2El” to determine the orbital elements of the transfer ellipse. After this, from these new orbital elements, we can determine the position of the satellite on this ellipse with respect to time by using again “RVFromKepler”. The process is the same for the other phases. In order to run the animation, we concatenate the different trajectories (initial orbit, ellipse of transfer, final orbit and repositioning phase) and utilize “AnimationGUI”. The next section will show different
results obtained with this method. The repositioning phase was not considered in our preliminary study.

The utilization of a software like STK could be also considered for the visualization. Graphically STK is superior to the Spacecraft Toolbox. However, it is not compatible with the Matlab environment. A careful study of STK should be made in a future study.

5.6.4 Visualization, Preliminary Study

With the tools described in the last subsection, a first attempt of static and animated visualization was done for a single spacecraft. Figure 5-14 shows the results obtained for the transfer in chemical propulsion of a S/C, initially in an equatorial orbit with a radius of 30,000 km to an orbit at 25,000 km, inclined at 45 deg. We see clearly on this figure that the transfer is coupled with a plane change. Using “AnimationGUI”,

Figure 5-14: Static Visualization of the transfer of a single S/C.

we can obtain an animation of this transfer.

After this first conclusive attempt, a more ambitious visualization was undertaken:
the reconfiguration of the polar constellation $A$ constituted of two planes of 2 satellites (orbit radius of 42,000 km) to the Walker constellation $B$ (2 planes of 3 satellites, orbit radius of 36,000 km and inclination of 52.2 deg). The first planes of $A$ and $B$ have the same longitude of the ascending node: $\Omega_1 = 0$ deg. Also, the planes 2 of $A$ and $B$ have the same longitude of the ascending node: $\Omega_2 = 90$ deg.

The first step was to run the astrodynamics module and the auction algorithm to determine, which assignment minimizes the $\Delta V_{total}$. The transition matrix of this particular case was computed in Chapter 4 (see Section 4.2.2). The best assignment is obviously to transfer the plane 1 of $A$ into the plane 1 of $B$ and the plane 2 of $A$ into the plane 2 of $B$. Once this first step done, the different start time of transfers and arrival, $t_{\text{start}}$ and $t_{\text{arrival}}$ were computed manually for the four satellites to be transferred. It was assumed that the two launched satellites were already in the final orbits (one on each final plane). After this, the functions defined in the last section were applied to generate the different trajectory. Figure 5-15 represents the

Figure 5-15: Animated visualization of the reconfiguration. From “AnimationGUI”. The dots represent the satellites in motion.

environment in 3D generated by AnimationGUI.
Figures 5-16 and 5-17 show respectively the positions of the different satellite before and after the transfers. These figures represent projections on the axis $z = 0$. Before the transfers, we see clearly that the launched satellite are lower in altitude.

On Figure 5-16, the planes of $A$ and the orbits of $B$ are represented.

Figure 5-17 shows the position of the satellites on their final orbit. The satellites are on both orbits not uniformly distributed. This visualization underlines the necessity of a repositioning phase. For instance, on orbit 1, one satellite is chasing the other.

5.6.5 Results comparison with simple (Epoch-independent) Model

The advantage of the high fidelity simulation compared to the actual simulation is to return precise times for the transfer time and exact $\Delta V$ for rephasing. The simple framework is currently conservative in terms of transfer time. The objectives sensitive in our model to the transfer times are the mean coverage and cost of service outage (plus obviously the total time). However, the cost of service outage in chemical propulsion is quite negligible. So, the high fidelity simulation will not have an
important impact on the cost, at least in chemical propulsion. The impact on the mean coverage is difficult to predict. The transfer time would be relatively reduced in an High Fidelity Simulation and this would surely have an impact on the coverage.

To conclude, in a High Fidelity Simulation, we will obtain costs very close to those obtained in the simple framework. There is no influence on the $\Delta V_{tot}$, except for phasing. The transfer time would surely be relatively reduced, and thus, we can predict a reduction of the total time.
CHAPTER 5 SUMMARY:

This chapter yielded some very interesting results. First, the “reconfiguration constellation map” could be a very useful tool for a manufacturer. Indeed, with this map, the manufacturer can determine which types of reconfiguration are reasonable, in terms of fuel consumption. This chapter describes also, for a particular case, the problems posed by the satellite assignment to the slots of constellation $B$. The launcher capacity constraint can entail penalization of on-orbit satellites resulting in a very costly transfer. The abandonment strategy of penalized satellite could be very judicious, particularly in electric propulsion. However, electric propulsion does not seem to be the right choice. Indeed even in a staggered schedule, the transfers seem quicker and less expensive in chemical propulsion. One achievement of this chapter is the comparison of the $LCC$ gain to the cost of reconfiguration for some optimal paths. This study indicates that savings are possible when the path requires only 1 or 2 reconfigurations. However gain in $LCC$ and reconfiguration costs have the same order of magnitude. Finally, the last section explained the concept of High Fidelity Simulation. Some visualizations of simple cases are presented.
Chapter 6

Recommendations and Conclusions

6.1 Summary

The goal of this thesis was to develop a framework to study the orbital satellite constellation reconfiguration problem and to apply it to a particular case of LEO reconfiguration of polar constellations. This study allows to find an order of magnitude for the time and cost to achieve the set of maneuvers, as well as computing the amount of extra fuel to place on the initial satellites. In Chapter 1, it was explained that the failure of the traditional approach for designing LEO constellations entailed the necessity to develop a new strategy called “staged deployment”. To deploy a constellation in a staged manner will require several orbital reconfigurations. The goal of this thesis is to estimate the price induced by a reconfiguration from an initial configuration $A$ to a configuration $B$.

In Chapter 2, the different issues raised by the reconfiguration are carefully described. This chapter presents the questions that this thesis answers, at least partially and points out the difficulty implied by a reconfiguration both technically and economically. Section 2.2 describes a scenario in two phases and explains why it was decided to initially launch all the additional satellites before beginning the transfer phase (risk reduction).

Chapter 3, where the Project Framework is developed and described is the key chapter of this thesis. Applicable to every type of reconfiguration, this Framework is
as general as possible. In Section 3.1, the Orbital Constellation Reconfiguration Block Diagram is presented and discussed. Section 3.2 discusses the four optimizations necessary for this study. The Auction Algorithm described in detail in this section, is one of those optimizations utilized and a very effective method.

Chapter 4 consists of a detailed description of the different modules. Inputs, Outputs and functions are described. Moreover, the assumptions and limitations of each module are shown. In view of the model complexity, some simplifications were inevitable. The results obtained should be therefore be carefully commented, since they represent only approximations.

Chapter 5 presents some interesting results. In Section 5.1, a “constellation reconfiguration map” altitude versus inclination exposes the types of reconfiguration that are most interesting in terms of fuel consumption. Section 5.2 consisting of a case study explains, how the satellite assignment on the final slots of $B$ works. Two strategies dealing with the problem of penalized satellites are presented in this case study. Section 5.3 indicates the impact of the different propulsion systems and scenarios on the cost and time of reconfiguration. Section 5.4 consists of a trade space exploration of cost outage versus fuel cost. Section 5.5 shows the economic opportunity of several reconfiguration paths. Lastly, Section 5.6 explains in detail what would be needed to construct an High Fidelity Simulation. A first attempt at visualization is shown at the end of this section.

6.2 Conclusions

The results obtained seem to preconize the utilization of chemical thruster to transfer the on-orbit satellites of the constellation $A$, particularly because of the low transfer time. However for the total costs, some reconfigurations (depending on the altitude or elevation angle of the final constellation) and/or strategy (penalized vs abandonment) will be economically advantageous in electric (Hall Thruster and Plasma Thruster are not retained). It will be the choice of the manufacturer to decide which propulsion systems and strategy is better suited for its requirement. For instance, if the manu-
facturer has a strong requirement concerning the reconfiguration time, he will choose to utilize chemical transfer and to penalize some satellites if necessary. Nevertheless, in chemical (and even in Resistojet) the mass of extra fuel necessary to do the transfer could be relatively high compared to the satellite dry mass. The question is to know if it is realistic to launch and to carry such an extra-mass of fuel.

Concerning the transfer scenario, the need for tracking and monitoring the satellites precisely in a chemical transfer has implied the choice of a staggered schedule for the transfers, the satellites moving sequentially. For electric transfer, a compressed schedule could be utilized.

From the simulation, we could quantify the price to pay to reconfigure a constellation and to determine if a “staged deployment” strategy remains economically advantageous. Chaize [CHAI03] provides an estimation of the gain resulting from a staged deployment strategy on the life cycle cost ($LCC$). The decrease in the life cycle costs in his study is estimated between 20 and 45%. The study in Chapter 5 of 3 different paths of reconfiguration has shown that a staged deployment strategy could be economically advantageous, particularly for paths with a low number of reconfiguration. There is a range of opportunity. However as explained in the last section, the approximations retained for the simulation implementation entail to interpret carefully the results. We are only sure that the traditional life cycle cost $LCC^{\text{trad}}$ is of the same order of magnitude that the sum $LCC^{\text{staged}}$ plus cost of reconfiguration. Moreover, the economic opportunity of 7.5% shown in Section 5.5 must be carefully balanced with the technical risks of achieving the reconfiguration. Satellite manufacturers are traditionally very reluctant to relocate or reconfigure assets that are operational on-orbit. Further detailed analysis has to be performed to overcome this barrier.

### 6.3 Recommendations for Future Work

Some refinements can improve the framework.

- The main limitation in the thesis is that the transfer time is roughly approxi-
mated. For a precise study, one should consider true anomaly or Epoch for each satellite. However such a method could be very costly in time of calculation and difficult to implement. Section 5.6 has explained this issue.

Also, the actual computation of $\Delta V$ in electric propulsion is very approximate. Indeed the S/C mass was assumed constant and equal to the dry mass $m_d$ during the transfer. It is obviously false, since the S/C mass varies from $m_w$ to $m_d$ during the transfer. The computation of the exact $\Delta V$ in electric propulsion would require numerical analysis which is very costly.

- The metric adopted for the satellite assignment was to minimize the $\Delta V_{total} = \sum_{k=1}^{N_{sats}(A)} \Delta V_k$. This metric is convenient, but since we want all satellites to be the same for commonality, manufacturing and launch reasons, it would be maybe more interesting to minimize the variance of the required $\Delta V$ in the constellation. So, to minimize $\sum_{k=1}^{N_{sats}(A)} (\Delta V_k - \frac{\Delta V_{tot}}{N_{sats}(A)})^2$. The gap between the satellites consumption will be lower, although the $\Delta V_{total}$ could be higher.

- The Launch Module utilized for the thesis purpose should be improved in order to consider more launch vehicles in its database. Moreover parallel launches should be considered, inspired by the Iridium deployment. Another strategy allowing to suppress the complex loop for assigning the ground satellites would have to consider a set of launch vehicles with variable capacity. It would be thus possible to respect the first assignment returned by the auction algorithm without penalizing on-orbit satellites by a reassignment. This method could unfortunately increase significantly the cost to launch the satellites from the ground.

- The metric chosen to quantify the coverage capacity of the constellation during the transfers, in other words the percentage of operational satellites is satisfying only for single coverage. If a constellation with double or triple coverage is studied, another metric would have to be found. Indeed in this case, we could have several satellites in transfer and still have a global coverage of 100% over the Earth’s surface. The relative position of the satellites with their respective
neighbors would have to be taken into account. Indeed if one satellite is in
transfer, its neighbors flying over the same region can insure coverage instead
of it. In the case of multiple coverage, the transfers should be organized so that
a region is not overly penalized by the transfers. For instance, two satellites
flying over the same region should not be transferred at the same time. So, the
transfer schedule has to be modified in order to distinguish the satellites and to
take into account their relative positions. Double or Triple coverage is a more
complex issue than single coverage.

- The cost module could be relatively improved in a future study. For instance,
the fuel cost was quantified thanks to an average for several launchers of the cost
to launch a kg of payload to LEO. It is not completely satisfying. A launch
planning for the on-orbit satellites should be determined, knowing the exact
amount of fuel carried by each satellite. From the total mass of the satellites,
we could determine exactly which launcher or series of launch vehicles could
be utilized for this first deployment and so, the exact price to launch these
satellites with their extra-fuel. After this, we could compare with the price to
launch these satellites empty. The difference of price would give the exact fuel
cost.

The production model could be also improved. Two models were proposed for
this cost (Equations 4.19 and 4.20). A future study could determine which
model is nearest the reality or propose another one. The price of a single S/C
production could be also checked and refined.

Lastly, the method for estimating the cost of service outage is rough. Maybe an-
other model would improve this estimation. A model where the non-uniformity
of the users distribution would be taken into account and potential, permanent
loss of paying customers due to service outage.

- This thesis deals mainly with a single reconfiguration from $A$ to $B$. Multiple-
Reconfigurations $A \rightarrow B \rightarrow C$ would likely be very expensive or impossible by
carrying all the fuel onboard for 2-3 or more future reconfigurations. Already
for a single reconfiguration, the extra fuel mass per satellite of constellation $A$ is large, see Figure 5-1. It is unclear whether enough fuel could be carried for multiple reconfigurations. This needs to be investigated further. Moreover due to the rocket equation, most of the fuel would be used to push fuel around. An alternative to this problem would be the exploration of fuel depots at strategic locations on orbit that could refuel satellites adaptively as needed. It could be also judicious to consider the utilization of a Space Tug as a “real option”, instead of extra-fuel. The Space Tug would transport the satellite to be transferred.

- Also, an alternative to Re-Con might be staged deployment with multiple circular layers. Similar to an onion, the satellites of $A$ would remain in their orbits. The additional satellites would form a new orbital shell at a lower altitude. Constellation $B$ would thus contain two layers (or more) of satellites. This would permit not to have to maneuver the satellites in $A$. Fuel is saved with this strategy, but the difficulty is shifted over to the electronics, since one must now provide Inter satellite links between satellites at different altitudes and the precise phasing between satellites in the orbital shells must be tightly controlled to avoid any holes in coverage. This would be more “staged deployment” than reconfiguration.
Bibliography


[DWMH01] Raymond J. Sedwick David W. Miller and Kate Hartman. Evolutionary growth of mission capability using distributed satellite sparse aper-
tures: application to NASA’s soil moisture mission (ex-4). 2001 (UNPUBLISHED)


