Graph-theoretical Considerations in Design of Large Telescope Arrays for Robustness and Scalability

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We study the staged deployment of large telescope array configurations as an optimization problem subject to cost, performance and network robustness. The LOFAR (LOw Frequency ARray), the worlds largest telescope array, is planned to be built in stages, with current funding allowing for O(100) stations over 100km. This new generation of telescope arrays requires new system design principles and modeling techniques. We develop a staged optimization framework, representing large telescope arrays as generalized networks, with nodes as the telescope stations and edges as the cable links between them. We model network design and growth with both graph-theoretical and physical metrics. Additionally, we model the probability of failure of each topology from random failure of a node or a link along the baseline. We make recommendations about the best cost-performance and robustness trade-off configurations, while comparing two different staging principles. The modeling framework introduced is applicable to various engineering problems susceptible to network representation, in telecommunication networks, transportation routes, and space exploration systems.

I. Introduction

Multiple antenna radio astronomy, also known as interferometry, uses many spread-out linked antennas to create an aperture equivalent to the aperture of a telescope with the diameter of the antenna array. Various size large telescopes exist, like the Very Large Array (VLA) in New Mexico and the Very Long Baseline Interferometry (VLBI) projects, in which stations are too far to be linked with conventional cable, and the data is recorded, transported remotely and then correlated. The applications of such grand-scale astronomy go beyond deep space observation. Some scientific results due to the VLBI include the motion of the Earth's tectonic plates, definition of the celestial reference frame, understanding the deep structure of the Earth, creating better atmospheric models, imaging high-energy particles being ejected from black holes and imaging the surfaces of nearby stars at radio wavelengths.¹

The design of large systems with such wide scientific applications is costly and difficult to plan. This study models the staged deployment of telescope array configurations as an optimization problem subject to performance, cost, and network robustness.

A. Motivation

The LOw Frequency ARray (LOFAR) is an example of an innovative effort to create a breakthrough in sensitivity for astronomical observations at radio-frequencies below 250 MHz.⁴ Numerous clusters of antennas collect signals to be sent to a receiver and analyzer. Half of the cost in designing such systems lies in the steel and moving structure. This is why single apertures much larger than the equipment are unaffordable. With large telescope arrays one can collect signals separately and interpret the image as a whole, while reducing the infrastructure cost significantly. In the case of LOFAR, numerous simple antennas, on the order of 25000 in the full design, organized in clusters, are planned to be built in stages. Phase 1 is currently funded with O(10000) antennas on maximum baseline of 100 km.

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Figure 1. Very Large Array, Socorro, New Mexico, (image courtesy NRAO / AUI / NSF) 2,3

As envisioned such a complex system has not only to be designed to reach scientific objectives at affordable cost, but also designed to be extensible. Extensibility indicates the capability of the system to increase performance under an increased load when more resources are added. In addition, an array with a diameter on the order of hundreds of kilometers with thousands of stations has a large probability of failure, either due to equipment resisting environmental conditions or random station or link failure along the baseline. To ensure network resilience and sustained functionality after random failure, the array needs to be designed both for extensibility and robustness.

To meet the above objectives, we modeled large telescope arrays as generalized networks, with nodes as telescope stations and edges as the cable links between them. Their design and growth are modeled with both graph-theoretical and physical metrics like resolution. Recommendations are made about the best cost-performance and robustness trade-off configurations.

B. Problem Formulation

Our goal is to find an optimum configuration of telescope stations subject to performance and cost metrics such that this configuration is expandable (optimally) and relatively robust (resilient to random failure).

Problem setup: The first stage has m stations with coordinates (\vec{x}^m, \vec{y}^m) , and the second stage has n stations with coordinates (\vec{x}^n, \vec{y}^n) , all enclosed in a circular space of constant diameter d, scaled with the size of the array. First-stage points are a subset of the second-stage points due to the legacy constraint. The arc set of the first-stage array is also a subset of the arc set of the second-stage array, so that the original infrastructure like laid cable, roads, trenches are kept and expenses for recabling and rebuilding the network are avoided.

Two evaluation metrics are used. Total cable length, as a surrogate for cost, is computed as the sum of the lengths of all edges in the network, as shown in Equation 1. Stations are linked by a minimal spanning tree, so for any given set of points, there is a unique arc set A. The uv density, as a surrogate for performance, is computed as the number of unfilled uv points directly from the station coordinates as shown in Equation 2. These uv points are correlated with the points of a pre-computed uniform grid. A uv point is considered filled if a uv point of the array is within a certain radius. Then the uv density metric is computed as the percentage of unfilled points, as shown in Equation 3.

$$C(\vec{x}, \vec{y}, A) = \sum_{(i,j) \in A} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \ A - arc \ set \ of \ (\vec{x}, \vec{y})$$
(1)

$$u_{i,j} = \frac{x_i - x_j}{\lambda}, \ v_{i,j} = \frac{y_i - y_j}{\lambda}, \ \forall i, j, \ \lambda - observation \ wavelength$$
(2)

$$P(\vec{x}, \vec{y}, \lambda) = 1 - \frac{N_{uvactual}}{N_{uv}}$$
(3)

An example of a uniform grid and the filled *uv* points by a near log-spiral geometry is given in Figure 2.



Figure 2. A nominal grid (left) used to calculate the filled uv points for a log-spiral seeded geometry

Problem statement (2 stages): For given number of stations m and n and site diameter d_m , the optimization problem is formulated as:

find
$$(\vec{x}^m, \vec{y}^m), (\vec{x}^n, \vec{y}^n)$$
 (4)
ereby $J = J^m + J^n$ is minimized (5)

subject

$$J^{m} = w_{1}C(\vec{x}^{m}, \vec{y}^{m}, A) + w_{2}P(\vec{x}^{m}, \vec{y}^{m}, \lambda), \ J^{m} = w_{1}C(\vec{x}^{n}, \vec{y}^{n}, A) + w_{2}P(\vec{x}^{n}, \vec{y}^{n}, \lambda)$$
(6)

to
$$\{(x_i, y_i)\}_{i=1}^m \subset \{(x_i, y_i)\}_{i=1}^n$$
, and $A(\vec{x}^m, \vec{y}^m) \subset A(\vec{x}^n, \vec{y}^n)$, where $m < n$

$$0 \le \sqrt{x_i^2 + y_i^2} \le d_m, \ 0 \le \sqrt{x_i^2 + y_j^2} \le d_n, \ for \ i = 1 \dots m, \ j = 1 \dots n, \ d_n = \frac{n}{m} d_m \tag{8}$$

$$0 \le w_1, w_2 \le 1 \tag{9}$$

(5)

(7)

Here C and P are the cable length and uv density metrics as defined in Equations 1 and 3, $A(\vec{x}, \vec{y})$ contains the unique set of arcs between the points $\{\vec{x}, \vec{y}\}$, and w_1 and w_2 are arbitrary constant weights.

This problem formulation can be extended to any number of stages, defined as nested subsets in Equation 7. The metrics can also be modified to reflect robustness, flexibility, modularity or alternative cost and performance measures. Priorities (weights) can be assigned to different stages (in Equation 5) to account for the varying importance of present and future objectives. Such weighting can have impact not only on the resulting designs, but also on the optimization tools preferred to solve the problem.

С. **Previous Work**

1. Staged deployment and orbital reconfiguration of satellite constellations

Chaize studies the value of flexibility in design in an uncertain environment in the context of satellite constellations like Iridium.⁵ The Iridium company believed it could attract about 3 million customers, but was not designed to accommodate a variable marketplace, quickly transformed by the growth of terrestrial cellular networks. Thirteen months after beginning operations, Iridium filed for Chapter 11 protection. Chaize argues that this is a consequence of the traditional design of large capacity systems which optimizes for a specific objective (global capacity) and fails to account for demand uncertainty. He proposes a staged deployment strategy in which the system capacity is increased gradually and design decisions are made dynamically at each deployment stage. Using a multi-objective optimization framework, he compares the lifecycle costs obtained with flexibility with those obtained using traditional design techniques, and finds a 30% improvement in the lifecycle cost of flexible satellite constellations.

2.Studies on network topology, robustness and cost

An abundance of literature exists about network topology implications for cost, performance and robustness. Good references on network theory are the two extensive reviews in $Nature^6$ by Barabasi and in SIAM $Review^7$ by Newman. The three references presented below discuss topology optimization for performance and robustness.

Yang and Kornfeld⁸ study the optimality of the hub-and-spoke architecture for a FedEx delivery problem. They use mixed integer programming to model the network of next day air cargo delivery flights between a small number of city pairs. It turns out that hub-and-spoke is not always the desired architecture, but the preferred topology varies with the demand, aircraft type and city locations. Only aircraft flight-time operating costs are considered, without robustness to uncertain demand. The original intent of the study is to model the FedEx delivery network with 26 cities, which turn out to introduce more variables than the integer programming algorithms can handle. This study indicates that understanding network topology is crucial to airlines costs modeling and operations, and also that heuristic algorithms might be more suitable to deal with large nonlinear design spaces.

Another example is a study done jointly with the American Airlines Operations Research Department and the Georgia Institute of Technology.⁹ The authors assess flight network robustness due to canceling flights because of disruptions. Canceling a single flight causes cascading cancellations of a sequence of flights that starts and ends at the same airport. It is claimed that fleet assignment and aircraft rotation with short cycles will be less sensitive to cancelations. The lower bound for the number of cycles is estimated using the hub connectivity of the fleet assignment. It is shown that solution models perform better than traditional assignment models.

Mathias and Gopal¹⁰ study whether the small-world topology arises as a consequence of a trade-off between maximal connectivity and minimal wiring. They perform a single-stage simulated annealing optimization with two opposing metrics: connectivity, modeled as the physical (Euclidean) distance between any two nodes and wiring, modeled as the average distance between all pairs of vertices. The optimization goal is to minimize the weighted sum of the two metrics. The authors claim that small-world behavior arises when the two metrics are given equal priority and show results of hub emergence and evolution for varying weights. This claim is proven by evaluating the generated networks with graph statistical measures. The interesting conclusion from this study is that optimization in the tension of two opposing metrics gives rise to scale-free and small-world networks. This has implications in our interest in telescope arrays robustness.

II. Algorithms - Heuristics for Two-stage Optimization

A. Static Multiobjective Optimization

The challenge of a multiobjective problem rests in the natural tension between vital objectives. For example, optimizing a fighter jet for speed means sacrificing fuel efficiency. Understanding trade-offs between cost and performance and schedule and risk to meet requirements¹¹ is *static multiobjective optimization* - the problem of finding a fixed-point design with multiple objectives.

The two design objectives, relevant to telescope array topology, are described by Equations 1 and 3. The optimization goal is to position both the nodes and the links in the network. The placement of the antennas (nodes) determines the fidelity of the obtained image, but it also affects the cost of building the infrastructure, like the power distribution, site preparation and laying connecting cable. In network terminology, the configuration of the nodes affects the coverage and the connectivity of the network. Various metrics represent coverage and linkage, all naturally opposing. Intuitively, a spread-out net has good coverage, but large diameter, long shortest paths and overall large total linkage. Cohanim uses a genetic algorithm framework to rapidly explore the objective space of the two cited metrics.¹² Figure 3 shows results from telescope configurations optimized for minimum cable length (left-most), maximum coverage (middle) and both, equally weighted (right-most). The top plots show the geometry, the bottom show the corresponding uv density points. The trade-off between cost and performance is clear: low-cost designs have the worst relative coverage, while the best performance designs have the greatest cable length.



Figure 3. Telescope array configurations with 27 stations optimized for : (a) minimum cable length, a VLAlike configuration, (b) maximum coverage or minimum uv density, a circular configuration, (c) both metrics, a randomized log-spiral geometry

B. Dynamic (Multistage) Optimization

Complex systems can scale with time (extend or shrink) or evolve or both. Extensibility (scaling) means preserving the nature of the elements of the system and their function, while increasing their number or size. Evolution means changing fundamental form and/or function. Here we consider only scaling, that is, adding stations to a telescope array and connecting them to the existing infrastructure without changing it. Single-step staging is defined as designing a m-station configuration and a n-station configuration such that m < n and the n-array contains the m-array. The second stage is strictly larger than the first $(n - m \gg 1)$ since we are interested in extensibility, not accretion (slow small-scale growth).

Two diametrically opposite approaches are used to design arrays for extensibility: a forward-looking technique, which optimizes the initial design; and a backward-looking technique, which optimizes the final goal. In the forward approach, the GA-optimized first-stage array is augmented to second stage optimally using a simulated annealing algorithm. In the backward approach, the GA-optimized second-stage array is reduced optimally to first stage using the same simulated annealing algorithm. The simulated annealing algorithm procedure is shown in Table 1. The overall algorithm procedure is presented in Figure 4. Notice

Backward SA	Forward SA
1. Start with a GA-optimized <i>n</i> -array and a <i>m</i> -subset; set initial temperature and cooling step	1. Start with a GA-optimized m -array and a n -superset; set initial temperature and cooling step
2. Evaluate the energy $E(\bar{x}_i^m, \bar{y}_i^m)$ of the <i>m</i> -array	2. Evaluate the energy $E_i(\bar{x}_i^n, \bar{y}_i^n)$ of the <i>n</i> -array
3. Perturb the <i>m</i> -array by removing and adding a station from the <i>n</i> -array, while keeping the m – <i>array</i> connected	3. Perturb the <i>n</i> -array by choosing another set of $n - m$ stations complimentary to the base <i>m</i> -array from a randomly-generated larger sample
4. Evaluate the energy $E(\bar{x}_{i+1}^m, \bar{y}_{i+1}^m)$ of the new <i>m</i> -array	4. Evaluate the energy $E_i(\bar{x}_{i+1}^n, \bar{y}_{i+1}^n)$ of the new <i>n</i> -array
5. If $E(\bar{x}_{i+1}^m, \bar{y}_{i+1}^m) < E(\bar{x}_i^m, \bar{y}_i^m)$ then keep $\{\bar{x}_{i+1}^m, \bar{y}_{i+1}^m\}$ as the new solution	5. If $E(\bar{x}_{i+1}^n, \bar{y}_{i+1}^n) < E(\bar{x}_i^n, \bar{y}_i^n)$ then keep $\{\bar{x}_{i+1}^n, \bar{y}_{i+1}^n\}$ as the new solution
6. If $E(\bar{x}_{i+1}^m, \bar{y}_{i+1}^m) > E(\bar{x}_i^m, \bar{y}_i^m)$ then keep $\{\bar{x}_{i+1}^m, \bar{y}_{i+1}^m\}$ with probability $e^{-\Delta/T}$, where $\Delta = E(\bar{x}_{i+1}^m, \bar{y}_{i+1}^m) - E(\bar{x}_i^m, \bar{y}_i^m)$	6. If $E(\bar{x}_{i+1}^n, \bar{y}_{i+1}^n) > E(\bar{x}_i^n, \bar{y}_i^n)$ then keep $\{\bar{x}_{i+1}^n, \bar{y}_{i+1}^n\}$ with probability $e^{-\Delta/T}$, where $\Delta = E(\bar{x}_{i+1}^n, \bar{y}_{i+1}^n) - E(\bar{x}_i^n, \bar{y}_i^n)$
7. Update temperature according to cooling schedule (cool_step = 1.01): $T_{new} = T/1.01, T \equiv T_{new}$	7. Update temperature according to cooling sched- ule (cool_step = 1.01): $T_{new} = T/1.01, T \equiv T_{new}$
8. If $E(\bar{x}_{current}^m, \bar{y}_{current}^m) \leq 0.1 E(\bar{x}_1^m, \bar{y}_1^m)$ or $T < 10^{-6}$ then terminate algorithm and save current solution. Otherwise, iterate.	8. If $E(\bar{x}_{current}^n, \bar{y}_{current}^n) \leq 0.1 E(\bar{x}_1^n, \bar{y}_1^n)$ or $T < 10^{-6}$ then terminate algorithm and save current solution. Otherwise, iterate.

Table 1. SA algorithm steps for backward and forward staging techniques

that in both staging principles, the legacy of the initial condition is preserved. In particular, for backward optimization, a valid subset of the initial array is a locally connected subset of stations. An example is given in Figure 5. The best subset of the second-stage circle is the semi-circle of filled points. We expect that the backward staging principle is superior due to its embedded knowledge of the future.

III. Extensibility Results for Radio Telescope Arrays

A. Design Variables and Metrics

The design vector consists of station coordinates (\vec{x}, \vec{y}) placed strategically in a circular space with diameter proportionate to the array size. For 60 stations, the benchmark is 400 km, similar to the LOFAR plan. This means that for 30 stations the site diameter will be 200 km. Each experiment is performed twice, with forward and backward staging techniques. The optimization framework is tested for three stages, with 27, 60 and 99 stations per stage. To initialize the algorithm, regular array geometries (circular, Y-shaped, triangular, Reuleaux triangular, log-spiral) and random configurations were seeded, shown in Figure 6. The design objectives, cost and performance, are implemented as total array cable length and imagining quality (uv density metric), as described in Section I.



Figure 4. Algorithms flowchart, forward and backward staging principles in parallel



Figure 5. Example of backward staging. The filled circles segment in the left plot is the selected optimal subset of the wider GA-optimized array; the right plot shows the first-stage array and the second-stage array with their uv density plots

B. Static Optimization Insights

Cohanim performed optimizations for arrays of 27, 60, 100, and 160 stations with similar geometric seeds.¹² It turns out that geometry types are clustered in the trade-off plot between decreasing cable length (cost) and improving array performance. Our results confirm that circular geometries are best optimized for the uv metric and VLA-like for the cable metric, so they assume the two corners of the Pareto front. Nadir-utopia solutions can be regular geometries, like Reuleaux triangles, or they can be derived from regular geometries, like slightly randomized VLA-like for minimum cable and circular shapes with inward protruding arms for minimum uv density. Random and log-spiral geometries often remain sub-optimal. Figure 7 shows second-stage geometries on the Pareto front for backward and forward runs.



Figure 6. Geometric seeds: VLA (Y-shaped), circles, triangles, Reuleaux triangle, log-spirals and random



Figure 7. Second-stage backward (left) and forward (right), 60 stations, Pareto fronts for different geometric seeds: VLA-like (pluses), triangular (triangles), Reuleaux triangular (diamonds), circular (circles), random (crosses) and log spiral (stars)

C. Dynamic Optimization Results

There are two sets of interesting questions related to extensible arrays. The first is about the benefit of extensible design: is there a lifecycle cost saving compared to a traditional fixed-point design approach? The second question is about the philosophy of staging. What approach, a forward or a backward-looking, will be more favorable and in what cases?

1. Lifecycle Benefits of Staging Optimization

Redesigning large engineering systems during their lifetime is a complicated process which involves partial or substantial replacement, or extension. Making scalability part of the design process, results in easier operation, better performance and reduces overall cost. To compare the traditionally optimal designs with the optimum for many stages, we compare single-stage nadir designs (closest to utopia in the population) and the best design sequences for three stages with 27, 60 and 99 stations.

Figure 11 shows the three populations for both backward and forward strategies. Clearly, nadir-utopia points in each stage are not the same historically (the evolution on the same array). The best design path points are very close to the nadir-points of the population, and could even coincide with them, as in the case of backward, third stage, but they can never overlap completely. The other interesting observation is that the best design points are usually close to the Pareto front for their own stage, even though they do not coincide with the nadir-point. For the designer this means that even for configurations where the best metrics are not selected, the best trade-off is.

2. Backward versus Forward Staging Principles

The forward and backward staging principles have diametrically opposite optimality priorities. A goal of this study is to assess their relative performance under different design objectives. Given their extremal nature, it is unlikely that either is an ideal approach. As mentioned earlier, we expect backward to be superior in most cases, because it assumes an optimal future.

• As designed, backward is more deterministic, while forward is more flexible with its greater degree of randomness. Backward works with a static predetermined geometry, and hence produces reduced versions of Pareto-optimal designs in the first stage. This makes the backward strategy almost predictable. Forward chooses new designs from a randomly generated set in the design space, thus allowing for the emergence of new geometries. The forward strategy is much less predictable than the backward approach, though certain behavior can be deduced from the goals of the optimizer.

• Per stage comparison confirms that backward and forward are suitable for different stages, but it also reveals consistent patterns in their relative performance. The first-stage results for 27 stations are compared in Figure 8 (left). The first-stage forward stations are a GA result, hence a balanced set of designs with satisfactory performance and best possible cable lengths for that performance. The backward results for 27 stations are a second-time SA optimization of optimum 99-station designs. They are naturally low-cost (lowest cable length), but because of the reductionist philosophy of backward, they have poor performance.



Figure 8. Backward versus forward comparison per stage; left: first stage, 27 stations with random seeds, middle: second stage, 60 stations with VLA-like seeds, right: third stage, 99 stations with circular seeds

The second stage comparison, given for VLA-like seeds in Figure 8 (middle), invokes similar conclusions, though the differences are not as striking as in the first-stage case. This is the passing (intermediate) stage between the domination stages of each strategy. Another viewpoint of the per stage comparison is an assessment of the best each strategy can do at every stage. Results are shown in Figure 9. At its best, the forward strategy is always more costly (Figure 9 right), but it outperforms the backward strategy in the first-stage (Figure 9 middle).

In summary, the forward is always the more expensive approach and the better performer, except in the case of third stage, where the backward strategy is the winner in both objectives.

• Our results show that the forward and backward principles are fundamentally different approaches. The hypothesis that the backward strategy will be better overall has clearly been disproved. The superiority of backward in the third stage cannot outweigh its poor performance in the early stages. Backward is the strategy to follow only if the goal is to construct intermediate steps or building blocks to reach a vital final goal. In an uncertain environment, with a lot of intermediate goals, and desired sustained system performance at all times, forward is the strategy to take. It not only allows adaptation to changing requirements, like budget or policy constraints, but it also allows attaching priorities to different stages.



Figure 9. Lowest achievable energy per state for backward and forward strategies (left); minimum uv density metric (middle) and minimum cable length (right); evidently forward staging always costs more than backward staging, but in terms of coverage the two methods are superior in different stages

IV. Network Analysis; Robustness

Robustness is defined as the system's ability to respond to changes in the external conditions or internal organization while maintaining relatively normal behavior. *Topological robustness* is the impact of a disturbance on structural integrity. For example, if an airline hub airport is suddenly closed, most of the airline's flights will not be able to follow their normal itinerary, will not be able to fly at all, or will have to be rerouted. *Functional and dynamical robustness* reflects changes in the network dynamics and functionality due to environmental or internal disturbances. In the airline example, this corresponds to the inability of the airline to transport passengers, or serve certain destinations. Loss in function can result in loss of market share and thus revenue (or profit) loss. This is why understanding network class types and their relative robustness is essential to designing networks and preventing failure.

We calculate topological and functional robustness as the percentage of surviving nodes and percentage loss of function respectively, after a random node or link failure. Single node/edge failures are averaged over all nodes/edges to provide a network-wide measure. Here, we briefly outline some of our results.

As expected, we find that robustness is a function of the network topology - similar geometric patterns have similar robustness metric. Figure 10 shows color-coded robustness on a cable length versus uv density plot. The dark and light patterns correspond to different geometries. These robustness patterns can be mapped to Figure 7 (left). The dark low-robust patch in the center of Figure 10 corresponds to circles on Figure 7. Using this color mapping, we establish that random configurations are relatively the most robust of all, with some really good designs (white squares) away from the Pareto front. VLA-like geometries (top left corner of Figure 10) also perform well, as expected, followed by Reuleaux triangles, triangles and down to the least robust circles.



Figure 10. Topological robustness metric indicated by color scale for each design point in objectives space (lighter is more robust), second-stage backward strategy, all geometries, 60 stations

A. Tension between Pareto optimality and Robustness

One of the primary reasons for studying the robustness of telescope arrays is the belief that optimal designs are not necessarily robust to external influence. This conjecture is shown to be a strong statement. The intuition behind comes from the vulnerability of highly-optimized systems argument.¹³ In the context of graphs, this means that the network may be designed to be highly resilient to random deletion of nodes, but lose structure and functionality if targeted at specific high-degree nodes.

As seen in Figure 10, the most robust designs (color-coded in white) are usually far from the Pareto front, while the robustness of the Pareto designs varies considerably. In fact, robustness has little to do with optimality, but more with topology. Scale-free arrays like random and Y-shaped are more robust compared to uniform-degree arrays like circular, triangular and Reuleaux triangular.

The tension between Pareto optimality and robustness is evident when comparing the best design paths based on cost and performance and the best robustness paths. Figure 11 shows three subsequent stages for both backward and forward techniques. For both strategies, the most robust path is far behind the Pareto front and the best cost-performance trade-off path. The good news is that for different strategies, the robust design choice requires a sacrifice in only one metric. In the case of backward staging, robustness is achieved at greater cost but relatively the same performance. For forward staging, the cost to design robustly is the same as the optimal path's, but the performance is worse with a higher *uv* density. With this insight, we can clearly term the backward strategy a *cost-optimizer*, and the forward a *performance-optimizer*.



Figure 11. Three stages, backward (left) and forward (right), with best design paths in terms of traditional metrics (black diamonds close to the Pareto front) and robustness (red diamonds behind the Pareto front); first stage (27 stations) is shown in grey, second stage (60 stations) in blue and third stage (99 stations) in green

V. Conclusion

This work extended previous studies done on network optimization of telescope arrays to multiple stages. It was confirmed that the best extensible configurations are not the single-stage winners, due to penalties for extensibility paid in the first or last stage of the design depending on the strategy. Another hypothesis confirmed is that robust arrays do not reside at the trade-off front of cost and performance, but are instead suboptimal.

Perhaps the most surprising conclusion is that the backward staging strategy is not superior for the staging of telescope arrays. It was found suitable only when the end state of the system is the primary goal, for which intermediate building blocks are needed. For sustained performance throughout all stages, embedded flexibility in the design to future budget or demand uncertainties, forward is the recommended strategy.

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