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Optimal Reconfiguration of Satellite Constellations with the Auction Algorithm

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Traditionally, satellite constellation design has focused on optimizing global, zonal or regional coverage with a minimum number of satellites. In some instances, however, it is desirable to deploy a constellation in stages to gradually expand capacity. This requires launching additional satellites and reconfiguring the existing on-orbit satellites. Also, a constellation might be retasked and reconfigured after it is initially fielded for operational reasons. This paper presents a methodology for optimizing orbital reconfigurations of satellite constellations. The work focuses on technical aspects for transforming an initial constellation A into a new constellation, B, typically with a larger number of satellites. A general framework was developed to study the orbital reconfiguration problem. The framework was applied to low Earth orbit constellations of communication satellites. This paper specifically addresses the problem of determining the optimal assignment for transferring on-orbit satellites in constellation A to constellation B such that the total ΔV for the reconfiguration is minimized. It is shown that the auction algorithm, used for solving general network flow problems, can efficiently and reliably determine the optimum assignment of satellites of A to slots of B. Based on this methodology, reconfiguration maps can be created, which show the energy required for transforming one constellation into another as a function of type (Street-of-Coverage, Walker, Draim), altitude, ground elevation angle and fold of coverage. Suggested extensions of this work include quantification of the tradeoff between reconfiguration time and ΔV, multiple successive reconfigurations, balancing propellant consumption within the constellation during reconfiguration as well as using reconfigurability as an objective during initial constellation design.

Nomenclature

$\mathcal{A}_{AB}$	Assignment Matrix
$\mathbf{A}$	Initial constellation
$\mathbf{B}$	Final constellation
F	Rel. phasing between orbits, $360/T$, deg
I_{sp}	Specific Impulse, sec
P	Number of orbital planes
S	Number of satellites per plane
T	Total number of satellites
a	Semi-major axis, km
c_{ij}	Cost per unit flow along arc (i,j)
f_{ij}	Amount of flow along arc (i,j)
h	Circular orbital altitude, km
i	common inclination, deg
$\Delta\mathbf{V}_{ij}$	Transition matrix, km/s
ΔV_{total}	Total ΔV for reconfiguration, km/s
Ω	Longitude of the ascending node, deg
α	Angle between ascending nodes, deg
ϵ	Minimum ground elevation angle, deg
θ	Satellite true anomaly, deg

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Introduction

LOW Earth Orbit (LEO) constellations of satellites, such as IRIDIUM[1] and GLOBALSTAR,[2] have revealed the problems resulting from demand uncertainty of large capacity systems. Although both systems were technically successful, they failed economically due to market changes that had taken place between conceptual design and the time they became operational. An alternative approach to deal with uncertainty in future demand, in the case of satellite constellations, is a "staged deployment" strategy.[3,4] It is possible to reduce the economic risks, by initially deploying a smaller constellation with low capacity that can be increased when the market conditions are good. The "staged deployment" strategy requires a flexible system that can adapt to uncertain market conditions. One aspect of this flexibility, in the LEO satellite constellation case, would be the ability to reconfigure the satellites' orbits. This paper focuses on the optimization of such an orbital reconfiguration, so that the total ΔV for the reconfiguration is minimized. Figure 1 shows a hypothetical example of a reconfiguration of an optimally phased polar constellation $\mathbf{A}$ at $h_A = 2000$ km into a polar constellation $\mathbf{B}$ at $h_B = 1200$ km. Both configurations achieve single fold, continuous global coverage.

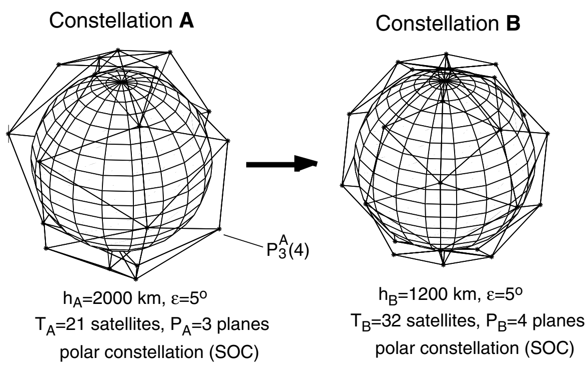

Fig. 1 Hypothetical reconfiguration of constellation A into constellation B. Lines between satellites represent inter-satellite links (ISL).

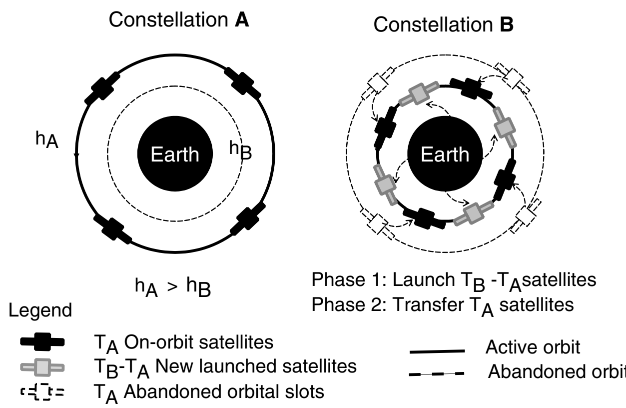

Fig. 2 Two phase orbital reconfiguration concept from constellation A into constellation B

Definitions

The term "reconfiguration" for constellations of satellites has been traditionally used to designate the set of necessary maneuvers to recover service after the failure of a satellite. An example would be the replacement of a failed satellite by phasing an existing on-orbit spare into the appropriate orbital slot.[5,6] In this paper, the term *reconfiguration* will be employed in a more ambitious way, since it will refer to the motion of an entire constellation. Satellite constellation reconfiguration may be defined in general as *a deliberate change of the relative arrangements of satellites in a constellation by addition or subtraction of satellites and orbital maneuvering in order to achieve desired changes in coverage or capacity.* The reconfiguration thus consists of repositioning on-orbit satellites into another configuration and in some instances of launching new satellites for completing the spots of the new constellation.

Each spot in a constellation is designated as $P_j^i(k)$, where i is the constellation designation, j is the orbital plane number and k is the slot number in the j-th plane. The designation of a satellite as $P_3^A(4)$, for example, indicates that it belongs to the 4-th slot of the 3rd plane of Constellation **A** (Fig.1). Figure 2 conceptually visualizes the reconfiguration process. We will consider primarily reconfiguration from higher to lower altitudes, whereby the new orbital planes of **B** can either be populated entirely by existing on-orbit satellites from **A**, entirely by newly launched satellites or by a mix of both. Alternatively, a constellation can be reconfigured by inserting orbital planes, or increasing the number of satellites per plane, $S = T/P$, while keeping the altitude, h, constant.

Literature Review

Static optimization of satellite constellations has been extensively studied over the past thirty years at increasing levels of sophistication. *Static*, here refers to a constellation, whose altitude and relative arrangement of orbital slots is time invariant. The goal of these studies was usually the same: To achieve global, zonal or regional coverage while minimizing the necessary number of satellites.[5] Three methods were proposed to solve this problem. The first one organizes the satellites into inclined circular orbital planes, whose nodal crossings are evenly spaced. Such *Walker* constellations are named after the original author of this method.[7–9] Follow-on research into common-altitude, inclined circular orbit constellations is credited to Mozhaev,[10,11] Ballard[12] and more recently, Lang.[13,14] The second method, *Street of Coverage* or SOC for short, is based on the utilization of continuous bands of coverage, usually resulting in polar orbits as described by Adams and Rider.[15] Adams and Lang[6] compared those two types of constellations. Depending on the fold of coverage (multiplicity or diversity, n), the coverage requirement, the launch vehicle capability, or the sparing strategy, they explain which type of constellation is more efficient. For single fold global coverage and $T < 20$ *Walker* constellations are more efficient, while for $T > 20$, i.e. at lower altitudes, polar SOC constellations are more efficient, see Figure 5. The third method developed by *Draim*[16] uses elliptical, highly eccentric orbit (HEO) satellites to achieve global continuous, or regional coverage with, in some cases, even fewer satellites. Tables of optimal constellations achieving global coverage have been developed by various authors and are the typical source of departure (**A**) and arrival (**B**) configurations in this paper. In the case of global polar constellations, analytical expressions exist to find the optimal constellation, given h, ϵ and n-fold of coverage. For *Walker* constellations designated as $T/P/F/i$, one typically resorts to numerical optimization. Crossley[17] and co-authors used Genetic Algorithms to optimize constellations for zonal coverage.

Contrary to "static" optimization of satellite constellations, very few studies exist on optimization of satellite constellation reconfiguration. The past studies on constellation reconfiguration have principally focused on the constellation maintenance problem. However, the literature on reconfiguration for maintenance has described some interesting concepts, applicable in our case.

Seroi *et al.*[18] pointed out the complexity of space systems such as satellite constellations. The authors discussed the difficulty to optimize maintenance. In order to replace failed satellites or satellites at the end of their design life, they suggest launching new satellites by means of launch vehicles with variable capacity. They utilize an optimization technique called *Dynamic Programming with Reinforcement Learning.* Dynamic Programming is implemented via a mathematical model of an agent that adapts his decision with respect to time. The Reinforcement Learning allows to push back the limits of this method which would otherwise require very high computational capacity.

Ahn and Spencer[19] studied the optimal reconfiguration for formations of satellites after a failure of one of the satellites. The constellation considered was a cluster of formation flying satellites. The goal was to find the maneuver cost that minimizes the total fuel usage among the individual satellites that remain operational. Their strategy was to prevent any unbalanced propellant usage. Depleting the propellant of one constellation member, while not using any propellant from the other constellation members can cause early failure of another formation member and would necessitate the premature addition of replacement satellites to the formation.

Techsat21, an Air Force Research Laboratory program is a good example of orbital reconfiguration. Saleh, Hastings, and Newman[20] briefly describe this program. Focused on lightweight and low-cost clusters of micro-satellites, this program intended to reconfigure the geometry of different clusters of a space based radar system. The purpose was to change the system's capability by geometry modification, from a radar mode with 0.5 km resolution to a geo-location mode with 5 km resolution.

A number of researchers have thus worked on various types of "reconfiguration" issues. This study, however, deals with determining how a low capacity constellation can be *reconfigured* into a higher capacity constellation in an optimal way. The optimality criterion (objective function) is the minimization of the total ΔV required for the reconfiguration.

Orbital Reconfiguration

Simplifying Assumptions

Due to the complexity of the orbital reconfiguration problem, some simplifying assumptions were made in this study. For instance, it was assumed that new satellites have to be launched in order to increase the capacity of the constellation, and that all satellites, the on-orbit satellites and the satellites to be launched from the ground, are identical except for their propellant load. This assumption is not obvious, since if the altitude of the satellites is changed, the hardware has to operate reliably over a range of altitudes. For instance, in order to produce a particular beam pattern on the ground, the characteristics of the antenna depend on the altitude of the satellites. Radiation shielding requirements differ by altitude. Therefore, realistically, reconfiguration within the satellites themselves also needs to be achieved. However, this problem was not considered in the present analysis. The article focuses on inter-satellite reconfiguration rather than on intra-satellite reconfiguration.

The satellites considered in this study had characteristics similar to those of Iridium satellites. Particularly the dry mass was the same: 700 kg. The extra-mass of propellant necessary to achieve eventual transfers was not included in the 700 kg, but represents an additional mass.

As explained earlier, a "reconfiguration" in the context of this article is considered to be a set of orbital maneuvers in order to evolve from an initial circular constellation **A** to a new circular constellation **B**. The number of satellites in the initial and final constellation are denoted as T_A and T_B, respectively. It was also assumed that $T_B \geq T_A$ since only those reconfigurations that involved an increase in capacity were considered. The number of satellites to be launched is thus: $T_B - T_A$. In a first approach, spare satellites were not considered. Note, however, that the main contribution of this article, the solution procedure for the satellite assignment to the new orbital slots via the auction algorithm, is independent of the exact configuration of satellite constellations **A** and **B**.

The Orbital Reconfiguration Problem

The orbital reconfiguration problem essentially has two parts. The first issue is to determine the optimal maneuvers for transferring the T_A on-orbit satellites into slots of the new constellation **B**. Those maneuvers will have to minimize the total ΔV, denoted ΔV_{total}, for the entire reconfiguration summed over all on-orbit satellites.

$$\Delta V_{total} = \sum_{k=1}^{T_A} \Delta V_{k^{th} satellite} \tag{1}$$

Each satellite of the initial constellation **A** needs to be assigned to a slot of the new constellation **B**, such that ΔV_{total} is minimized. The additional mass of fuel necessary to achieve the transfer of the k_{th} satellite of constellation **A** is computed from the specific impulse, I_{sp}, of the propulsion system utilized for the transfer and the value of $\Delta V_{k_{th} satellite}$:

$$\frac{m_f}{m_{i,k}} = \exp\left[\frac{-\Delta V_{k^{th}satellite}}{g \cdot I_{sp}}\right] \tag{2}$$

and the extra fuel mass to carry on the k-th satellite is

$$\Delta m_{k,A\rightarrow B} = m_{i,k} - m_f \tag{3}$$

where m_f is the operating mass, once the k-th satellite arrives in orbit **B**. The difference in orbital velocity between the circular orbits of constellation **A** and **B** serves as a lower ceiling for $\Delta V_{k_{th}satellite}$:

$$\Delta V_{A\rightarrow B} = \sqrt{\frac{\mu_E}{r_B}} - \sqrt{\frac{\mu_E}{r_A}} \tag{4}$$

with the gravitational constant $\mu_E = 3.986{\cdot}10^5$ km^3/s^2 and orbital radius $r_A = r_E + h_A$. A lower bound for $\Delta V_{k^{th}satellite}$ for the case shown in Figure 1 is thus $\Delta V_{A\rightarrow B} = 0.355$ km/s. The actual energy requirement, however, will be higher. This is because for the k-th satellite to reach its assigned slot in **B**, a combination of plane change and mean anomaly phasing is often required in addition to the necessary altitude change.[5]

Note, that instead of minimizing the total ΔV_{total}, the difference in ΔV between the satellites could be minimized. This alternative objective function is discussed in the last section.

The second part of the orbital reconfiguration problem is the assignment of the $T_B - T_A$ satellites that have to be launched from Earth. Ideally, all satellites in a launch vehicle should be assigned to the same orbital plane. This would prevent costly non-coplanar repositioning of the new satellites. The capacity, in terms of number of satellites, of the chosen launcher is a parameter that has to be taken into account during the assignment process. This constraint makes the assignment process more complicated.

In summary, two overlapping transfer assignments are required: the assignment of the $T_B - T_A$ launched satellites and the assignment of the T_A on-orbit satellites to slots in constellation **B**, expressed by the assignment matrix $\mathcal{A}_{AB}$.

A Two-Phased-Reconfiguration Scenario

It makes sense that the transfer phase of the T_A satellites occurs only after the ground satellite launch phase has been successfully completed. This strategy minimizes the risk of service interruptions. If the two phases are realized at the same time (or if the launch phase occurs later) and if one launch fails, the delay for replacing the lost satellites would entail a period of service outage. Constellation **A** would no longer be fully operational, while constellation **B** would not yet be completed. A very undesirable situation. With this two-phased-scenario (launch first, then transfer), a launch failure will only postpone the beginning of the transfer phase, while constellation **A** still remains 100% operational during the delay.

Figure 3 summarize this scenario in two distinct phases. Figure 3(a) shows the launch of the new satellites with a launch vehicle of capacity T_{LV}. During this phase, the on-orbit satellites remain in their initial orbits at altitude h_A. The new satellites are directly sent to their final orbits in slots of constellation **B**. Only one orbital plane is shown for clarity.

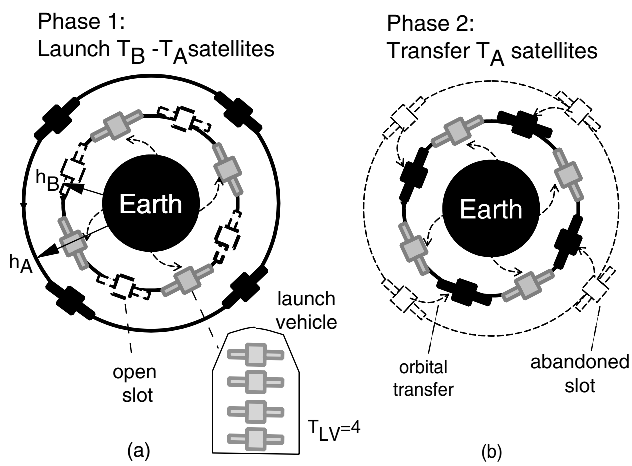

Fig. 3 (a)The first phase: launching new satellites, (b) The second phase: transferring on-orbit satellites

Figure 3(b) indicates the transfer of the on-orbit satellites of **A** to the remaining slots of configuration **B**. These open slots are represented with dashed lines.

Framework for Orbital Reconfiguration Analysis

A framework for systematically analyzing the orbital reconfiguration of satellite constellations has been developed. This framework is based on several steps (modules), and allows the study of various factors (such as ΔV requirements, cost, reconfiguration time, partial coverage *etc.*) associated with an orbital reconfiguration scenario.[21] This paper however, only focuses on a subset of the framework, namely the Constellation, Astrodynamics, and the Assignment Modules, and only the ΔV requirements are studied. Figure 4 shows a schematic representation of the section of the framework that is employed in this particular study.

Before discussing the satellite assignment problem in depth, the preceding steps used in carrying out the analysis are first described in some detail.

Constellation Module

This module computes the parameters that describe both the initial and final constellations. For **A** and **B** one specifies the type of constellation, C (SOC, Walker), as well as their circular altitudes, h_A, h_B, and

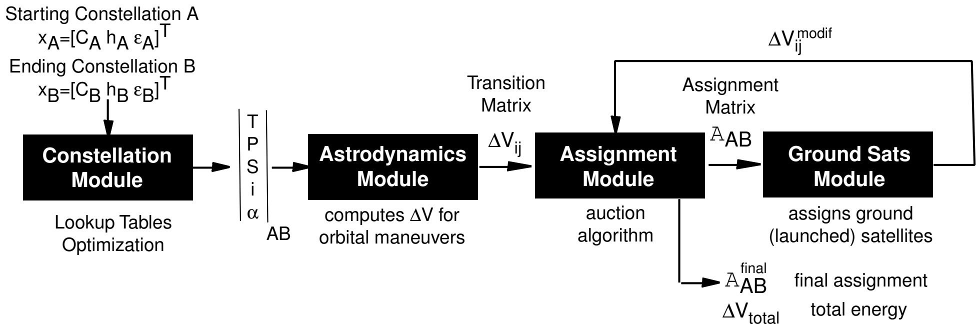

Fig. 4 Framework for studying orbital reconfiguration

ground elevation angles, ϵ_A, ϵ_B. The remainder of this paper assumes single fold coverage, $n = 1$, but this is not a limiting factor.

Adams and Lang[6] explain the differences between these two methods. The Walker constellations are characterized by a uniform distribution of the ascending nodes (RAAN) for the different planes. This is not the case for Polar constellations that are optimally phased between co-rotating interfaces. RAAN's are uniformly spaced for polar constellations with arbitrary inter-plane phasing. Moreover, the Polar constellations typically need many satellites per plane and the best coverage is obtained at the poles, while Walker constellations have fewer satellites per plane and a best coverage at mid-latitudes close to the inclination of the orbits. In both cases, the number of satellites per plane depends inversely on altitude. In order to maintain global coverage the number of satellites increases as the altitude decreases (for constant ϵ), see Figure 5.

Calculations for SOC constellations

For SOC constellations the module returns an optimal polar constellation based on the analytical expressions and optimization procedure outlined by Rider:[22]

The Earth nadir angle, η in radians, is defined as

$$\eta = \arcsin\left[\cos\left(\frac{\pi\epsilon}{180}\right)\frac{r_E}{r_E+h}\right] \tag{5}$$

Sometimes α is used for the nadir angle, but we will reserve α to represent the RAAN spacing of ascending nodes. The Earth central half-angle, θ (in radians), is

$$\theta = \frac{\pi}{2} - \frac{\pi\epsilon}{180} - \eta \tag{6}$$

For single street coverage we set $k = 1$ and $j = n/k$, where n is the desired multiplicity of coverage. For SOC constellations there is a closed form expression for P, but one must search for the smallest S, which will satisfy the global coverage condition given by Adams and Rider:[15]

$$\pi = \alpha(P-1) + \phi \tag{7}$$

where P is the number of planes, α is the angle between co-rotating orbits (also called angular separation between ascending nodes), and ϕ is the angle between counter-rotating orbits. The procedure is to search for the smallest value of S on the interval $S_i \in [S_{min}, S_{max}]$, where S_i is a trial number. Further details on this procedure are given by Rider.[22] The lower bound is defined by

$$S_{min} = \left\lceil \frac{j\cdot\pi}{\theta} \right\rceil \tag{8}$$

while S_{max} is usually set to a large number (> 50). Next, the half-street width's of coverage are calculated as:

$$c_1 = \arccos\left[\frac{\cos\theta}{\cos(1\cdot\pi/S_i)}\right] \tag{9}$$

and

$$c_j = \arccos\left[\frac{\cos\theta}{\cos(j\cdot\pi/S_i)}\right] \tag{10}$$

The number of orbital planes is obtained as

$$P_i = \left\lceil \pi\left[\arcsin\left[\frac{\sin c_1}{\cos(\phi_n\pi/180)}\right] + \arcsin\left[\frac{\sin c_j}{\cos(\phi_n\pi/180)}\right]\right]^{-1}\right\rceil \tag{11}$$

where ϕ_n is the latitude above which global n-fold coverage has to be achieved; $\phi_n = 0$ in this study. The resulting number of satellites T_i is then

$$T_i = P_i \cdot S_i \tag{12}$$

From these trials, the smallest number of total satellites is selected: $T = \min(T_i)$. For large SOC constellations (mainly at lower altitudes), the large constellation approximations provided by Adams and Rider[15] (Eq.26-29) are used:

$$T_{approx.} = \frac{4\sqrt{3}}{9}\cdot n\cdot\left(\frac{\pi}{\theta}\right)^2\cdot\cos\left(\frac{\phi\pi}{180}\right) \tag{13}$$

for optimally phased constellations and finally

$$\left[\frac{S}{P}\right]_{approx.} = \sqrt{3}\cdot\frac{j/k}{\cos(\phi\cdot(\pi/180))} \tag{14}$$

From this T, P, S and α are obtained for large SOC constellations. Additionally, the angular separation, α, between the ascending nodes needs to be known, since in this case α is different from $180/P$ deg in the case of optimal phasing. The inclination, i, for Polar constellations is close to 90 deg.

Table Lookup for Walker Constellations

The parameters necessary to describe optimal Walker constellations are the total number of satellites, T in the constellation, the number of commonly inclined orbital planes, P, the relative phasing parameter, F, and the common inclination for all satellites, i. The optimal Walker constellations are extracted by means of a lookup table, which was assembled by Lang[14] from numerical optimizations up to $n = 4$. Extrapolations are used for constellations with $T > 100$. Figure 5 shows an overview of the results obtained by the constellation module as a graph of the number of satellites (output), T, as a function of the inputs: constellation type, C, altitude h and diversity n, while the minimum ground elevation was held constant at $\epsilon = 0$.

Constellation Benchmarking

The characteristics of Iridium and Globalstar were utilized to benchmark this module, i.e. verify its validity. Iridium is a Polar constellation with an altitude of 780 km, and an elevation $\epsilon = 8.2$ deg. With these inputs, the module returns values for $T, P, i,$ and α of 66 satellites, 6 planes, 90 deg inclination, and 30 deg angular separation. These values are close to the actual characteristics of the Iridium constellation. Globalstar is a Walker constellation, with an altitude of 1400 km and an elevation angle of 10 deg. The module returned 50 satellites in 5 planes, whereas the deployed Globalstar constellation has 48 satellites in 8 planes inclined at 52 degrees. Figure 5 shows the results computed by the Constellation module, which match closely the results given by Chobotov and co-authors[5] in their Fig. 15.18. All the calculations were carried out with the constellation module discussed in this section. The main results for $n = 1, 2, 3, 4$ for SOC and Walker constellations between 250 km and 10,000 km altitude (assuming $\epsilon = 0$) are confirmed. The locations of Iridium($\epsilon = 8.2$) and Globalstar ($\epsilon = 10$) as well as constellations $\mathbf{A}(\epsilon = 5)$ and $\mathbf{B}(\epsilon = 5)$ from Figure 1 are also shown for convenience. The precision of the constellation module was therefore deemed sufficient in order to study the constellation reconfiguration problem in the subsequent sections.

Astrodynamics Module

This module calculates the ΔV and transfer time, T, from each position of the initial constellation $\mathbf{A}$ to each slot in $\mathbf{B}$. This article focuses on ΔV as the main objective to be minimized.

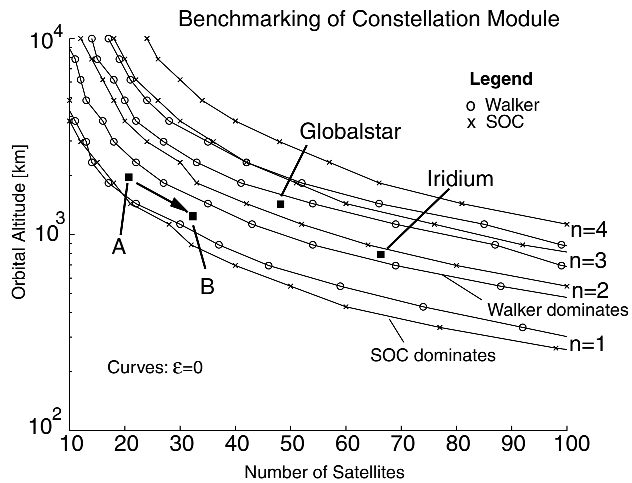

Fig. 5 Benchmarking of the Constellation Module

Review of Orbital Elements

An orbital slot in a constellation is defined by six orbital elements $(a, e, i, \Omega, \omega, \theta)$. The inclination, i, with respect to the equator and the longitude of the ascending node, Ω, define the orbital plane of the satellite. The element Ω represents the angle from the vernal equinox to the ascending node (RAAN). The ascending node is the point where the satellite passes through the equatorial plane moving from south to north. The semi-major axis, a, describes the size of the elliptic orbit, whereas the eccentricity, e, describes the shape. The argument of perigee, ω, is the angle from the ascending node to the eccentricity vector. It allows finding the position of the perigee of the ellipse in the orbital plane and thus gives the orientation of the ellipse. Finally, the true anomaly, θ, gives the position of the satellite on the ellipse with respect to the perigee. The true anomaly is the only orbital element dependent on time. Thus, in order to determine the position of a satellite unambiguously, a time reference (or Epoch) needs to be defined. The other five elements are constant. Figure 6 illustrates the orbital elements described above.

The orbits considered in this study were circular. With that assumption, $e = 0$ and $a = r_E + h$, where r_E is the mean radius of the Earth and h is the altitude of the orbit. Consequently, only four orbital elements (h, i, Ω, and θ) are needed to determine the exact position of a slot in the case of circular orbits.

Astrodynamics Module Assumptions

A transfer for reconfiguration purposes therefore implies changes to these four parameters. The change of Ω and i will allow to put the satellite in the right orbital plane and the change of altitude h will place the satellite in the right orbit. However, once these first maneuvers are achieved, the satellite and the final slot may have different true anomaly (phase), θ. The satel-

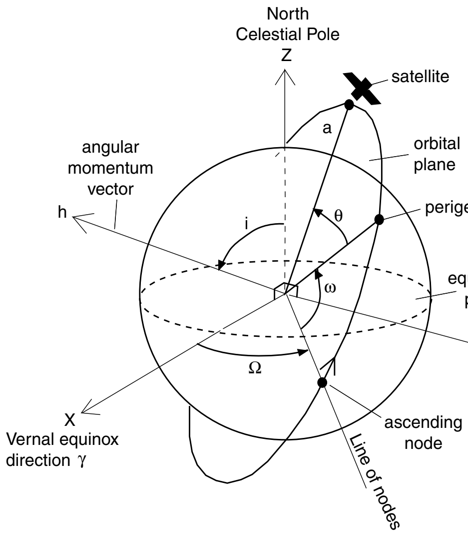

Fig. 6 Classical orbital elements in the Geocentric Inertial (GCI) frame

lite may thus need to be repositioned (phased) into the final orbital slot. Different strategies exist for this rendezvous maneuver. Typically, the ΔV for phasing is significantly smaller than that needed for changing i, Ω, and h.

The exact ΔV required for phasing is not exactly predictable, due to the strong sensitivity to the exact timing of burns of the transfer maneuvers and the variety of strategies to achieve phasing (see below). One minute of time difference in LEO corresponds to roughly 4 degrees of phase. It was therefore decided to assume a "worst case" ΔV phasing allowance for each satellite assigned to the same orbital plane of **B**. This allowed decoupling the orbital reconfiguration problem between **A** and **B** from a specific Epoch and timing of individual maneuvers. Another benefit of this important assumption is that it simplifies the subsequent assignment problem. A cumbersome one-to-one assignment from T_A to T_B is replaced by a simpler assignment of a particular satellite of constellation **A** to the appropriate orbital plane of **B**.

Therefore, the ΔV requirement returned by the module for each satellite to transfer from **A** to each slot in **B** depends only on the initial and final altitudes, h_A and h_B, on the initial and final inclinations, i_A and i_B, and on the initial and final longitudes of ascending node, Ω_A and Ω_B. In other words, ΔV depends mainly on the characteristics of the initial and final planes: $\Delta V = f(i_A, i_B, h_A, h_B, \Omega_A, \Omega_B) + \Delta V_{phasing}$.

Orbital Transfer Calculations

This subsection briefly summarizes the orbital transfer calculations. First, the plane and altitude changes are discussed, followed by the phasing maneuvers. The transfers are assumed to be based on chemical (impulsive) propulsion.

Plane and Altitude Changes

This subsection discusses the strategy for altitude and plane changes ($h_{A\to B}$, $i_{A\to B}$ and $\Omega_{A\to B}$) to transfer satellites from constellation **A** to assigned slots, $P_j^B(k)$, in **B**. The first option consists of three phases:

1. Hohmann transfer for altitude change, combined with either the inclination change, $i_{A\to B}$, or the node line change,$\Omega_{A\to B}$.
2. Simple plane change before or after the Hohmann transfer, depending on which sequence minimizes ΔV, to change the parameter that was held constant during the Hohmann transfer (Ω or i).
3. True anomaly phasing to correct $\Delta\theta$, as described below.

When only i varies, the angle between the initial and final planes is: $\Delta i = i_B - i_A$. When Ω varies, the angle is: $\Delta\Omega \cdot sin(i)$ with $\Delta\Omega = \Omega_B - \Omega_A$. In the case of a simple plane change, the expression utilized to compute the ΔV is

$$\Delta V = 2V_A sin(\lambda/2) \tag{15}$$

where V_A is the initial (departure) velocity and λ is the angle increment.

In the case of the plane change combined with Hohmann transfer, the expression is

$$\Delta V_{transfer} = (V_A^2 + V_B^2 - 2V_A V_B cos(\lambda))^{1/2} \tag{16}$$

where V_A is the initial velocity, V_B is the final velocity and λ is the angle change required.

The second strategy consists of changing i and Ω at the same time. This maneuver is performed at the nodal crossing point of the two orbital planes (initial and final). The plane change is also combined with a Hohmann transfer, allowing the change of altitude $\Delta h = h_B - h_A$. This second strategy combines the two first phases of the first strategy, followed by $\Delta\theta$ phasing. In this case, there is no simple analytical expression for the transfer angle, λ. The normal vector of an orbital plane $\vec{n}$ is equal to

$$\vec{n} = \left\{ \begin{array}{c} sin(i)sin(\Omega) \\ -sin(i)cos(\Omega) \\ cos(i) \end{array} \right\} \tag{17}$$

If we define $\vec{n}_A$ and $\vec{n}_B$ as the normal vectors of the initial and final orbit planes, the angle λ between the two planes can be obtained from the expression

$$cos(\lambda) = \vec{n}_A \cdot \vec{n}_B = sin(i_A)sin(\Omega_A)sin(i_B)sin(\Omega_B) + sin(i_A)cos(\Omega_A)sin(i_B)cos(\Omega_B) + cos(i_A)cos(i_B) \tag{18}$$

Now a comparison between the two strategies will be made. Table 1 summarizes the results obtained with both strategies for four different transfers. ΔV_1 is the ΔV computed with the first strategy, ΔV_2 the ΔV computed with the second one. The angles are in degrees, ΔV is in km/s.

Table 1 Comparison of two orbital transfer strategies

Ω_A	i_A	h_A	Ω_B	i_B	h_B	ΔV_1	ΔV_2
10	0	1000	20	45	1000	5.6	5.6
0	10	1000	45	20	1000	2.3	1.9
10	0	2000	20	45	1000	5.4	5.4
0	10	2000	45	20	1000	2.4	2

The second strategy appears to be more cost efficient in terms of ΔV. Moreover the transfers are shorter with the second option, since the time spent between the Hohmann transfer and the simple plane change is suppressed. In the first strategy, the satellite should wait on its trajectory until its current orbit intersects the desired plane. This is not the case with the second strategy, since the simple plane change is suppressed. The second strategy is incorporated in the Astrodynamics module. The ΔV for transferring a satellite from its slot $P_m^A(n)$ to the j-th plane of $\mathbf{B}$ must be augmented by the energy required to phase the satellite into its target slot, $P_j^B(k)$.

Phasing Maneuver

The phasing maneuver can be executed as either a sub- or a super-synchronous transfer with respect to the circular reference orbit of constellation $\mathbf{B}$. The transfer time depends on $\Delta\theta$, i.e. the difference in true anomaly of the satellite and its target orbital slot, $P_j^B(k)$. If $0^o < \Delta\theta < 180^o$, the slot is said to be ahead of the spacecraft in the direction of the orbital velocity vector. If $180^o < \Delta\theta < 360^o$, the slot is behind the satellite, see Figure 7.

The sub-synchronous transfer is initiated with a burn in the direction opposite the velocity vector, placing the satellite in an orbit with lower perigee (less energy) but with the same apogee, see Figure 7(b). This "accelerates" the spacecraft with respect to the slot. If the perigee is selected correctly, the spacecraft can then rendezvous with the assigned slot, $P_j^B(k)$, at the apogee of the circular reference orbit after an integer number of periods (k), see Figure 7(c). A final impulse is then given to place the satellite back in the reference orbit. The super-synchronous transfer is identical, except for a higher perigee, which slows the satellite down with respect to the target slot. The phasing transfer time, $T_{phasing}$, was computed by Chaize[3] as:

$$T_{phasing} = \left(\frac{\pi - \Delta\theta}{\pi} + k\right)\Pi_B \tag{19}$$

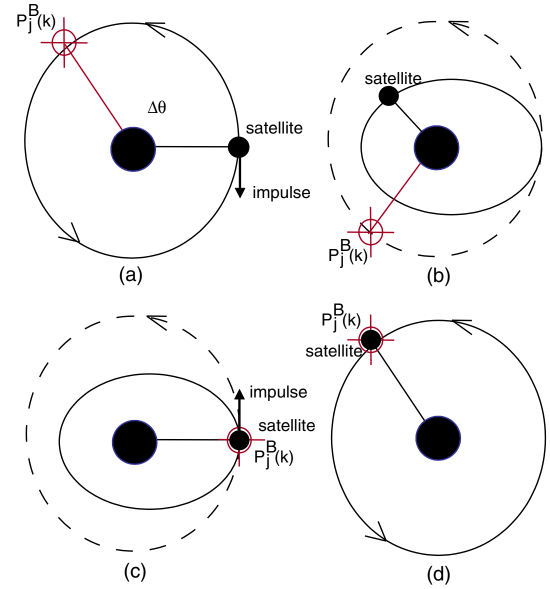

Fig. 7 Decomposition of a sub-synchronous transfer and rendezvous between the satellite and assigned orbital slot, $P_j^B(k)$.

where k is the integer number of orbital revolutions of the slot between the time the phasing maneuver is initiated and when rendezvous occurs and Π_B is the orbital period of the reference circular orbit in constellation $\mathbf{B}$:

$$\Pi_B = 2\pi\sqrt{\frac{r_B^3}{\mu_E}} \tag{20}$$

with $\mu_E = GM_E$. The ΔV budget for the phasing maneuver consists of two impulses as described above and can be calculated[3] as:

$$\Delta V_{phs}^{sub} = 2\sqrt{\mu_E}\left(\sqrt{\frac{1}{r_B}} - \sqrt{\frac{2}{r_B} - \left(\frac{\pi - \Delta\theta}{\pi} + k\right)^{-\frac{2}{3}}\frac{(k+1)^{\frac{2}{3}}}{r_B}}\right) \tag{21}$$

for a sub-synchronous transfer and as

$$\Delta V_{phs}^{super} = 2\sqrt{\mu_E}\left(\sqrt{\frac{2}{r_B} - \left(\frac{\pi - \Delta\theta}{\pi} + k\right)^{-\frac{2}{3}}\frac{k^{\frac{2}{3}}}{r_B}} - \sqrt{\frac{1}{r_B}}\right) \tag{22}$$

for a super-synchronous phasing maneuver. The essential design variable of this maneuver is k.

A separate approach for the phasing maneuver is to use two subsequent Hohmann transfers, which requires a total of four burns. The equations necessary to calculate this phasing maneuver can be found in Wertz and Larson.[23] A comparison of the three phasing strategies in terms of the tradeoff between $\Delta V_{phasing}$ and $T_{phasing}$ is shown in Figure 8. The double Hohmann transfers are represented with a line because these transfers depend on the altitude of the lower orbit considered, which is continuous. Sub-synchronous and

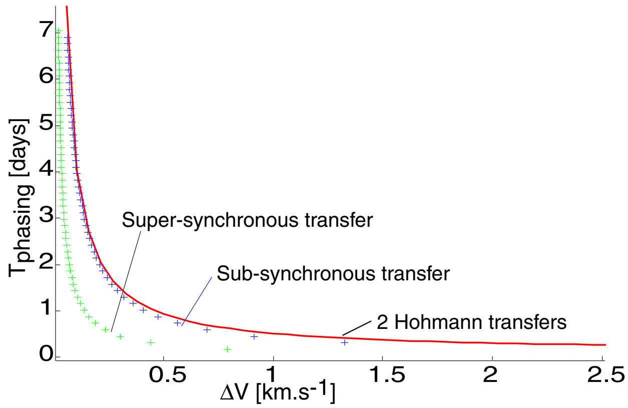

Fig. 8 ΔV **and** $T_{phasing}$ **for different transfers (**$e_B = 0$, $r_B = 11400$ **km and** $\Delta\theta$**=270 deg).**

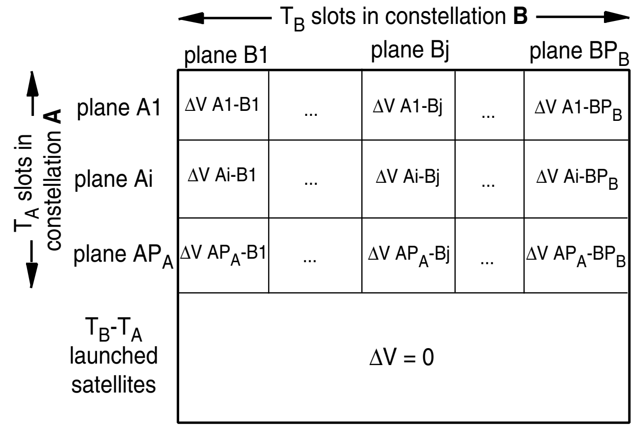

Fig. 9 General Form of $\Delta \mathbf{V}_{ij}$ **Transition Matrix**

super-synchronous transfers depend on k which is a discrete integer. We see from this example that the two-burn phasing maneuver is preferable with respect to both ΔV and $T_{phasing}$ compared to any of the double Hohmann transfers. In this example, the super-synchronous phasing seems to be preferable because the orbital slot is initially "behind" the satellite. The worst-case for phasing corresponds to $\Delta\theta = 180$ degrees. This property can be used to determine upper limits on $T_{phasing}$ and $\Delta V_{phasing}$, when the transfer phasing angle $\Delta\theta$ is not exactly known, as was argued earlier. A conservative phasing ΔV allowance of 0.5 km/s will be used in the subsequent analysis, as this number is close to the "knee" in the curve of Figure 8. Theoretically one can drive $\Delta V_{phasing}$ arbitrarily small if one is willing to wait a long time to complete the orbital reconfiguration. The phasing ΔV and time can be adjusted for different constellation reconfiguration scenarios based on the relationships shown in this section.

Astrodynamics Module Outputs

The module returns two transition matrices: $\Delta \mathbf{V}_{ij}$ and $\mathbf{T}_{ij}$. The inputs are the propulsion system (represented by its I_{sp}) and the characteristics of the constellations **A** and **B**: altitude h, number of satellites T and planes P, inclination i, and angle α between ascending nodes of neighboring planes. Figure 9 shows the typical form of the $\Delta \mathbf{V}_{ij}$ transition matrix, which has block sub-matrices, since all the satellites of a same plane need the same ΔV to be transferred into a plane of the final constellation.

Equation (23) shows the transition matrix $\Delta \mathbf{V}_{ij}$ obtained for the reconfiguration from an optimal polar (SOC) constellation **A** with altitude $h_A = 36,000$ km and minimum elevation angle $\epsilon_A = 2$ deg to an optimal Walker constellation **B** with altitude $h_B = 30,000$ km and minimum elevation angle $\epsilon_B = 2$ deg. **A** has 4 satellites (2 planes of 2) and **B** has 6 satellites (2 planes of 3). The inclination i_B of **B** is 52.2 deg. The entry in the i^{th} row and j^{th} column of $\Delta \mathbf{V}_{ij}$ is the ΔV in km/s that is needed to place the i^{th} satellite of **A** in the j^{th} slot of **B**.

$$\Delta \mathbf{V}_{ij} = \begin{pmatrix} 2.6 & 2.6 & 2.6 & 4.9 & 4.9 & 4.9 \\ 2.6 & 2.6 & 2.6 & 4.9 & 4.9 & 4.9 \\ 4.9 & 4.9 & 4.9 & 2.6 & 2.6 & 2.6 \\ 4.9 & 4.9 & 4.9 & 2.6 & 2.6 & 2.6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{23}$$

As mentioned above, $\Delta \mathbf{V}_{ij}$ is a block matrix. The two satellites (one and two) of plane 1 of **A** need a ΔV of 2.6 km/s to go to any of the three slots in plane 1 of **B** and a ΔV of 4.9 km/s to go to any of the three slots of plane 2 of **B**. Similarly, all the satellites of plane 2 (*i.e.* satellites three and four) of **A** need a ΔV of 2.6 km/s to go to plane 2 of **B** and a ΔV of 4.9 km/s to go to plane 1. The two ground-launched satellites are assigned a transfer ΔV of zero, as it is assumed that this energy is provided by the upper stage(s) of the launch vehicle.

Orbital Assignment

This module is shown in Figure 4 and computes the optimal assignment of satellites from **A** to slots of constellation **B**. The task of doing optimal orbital reconfiguration can be considered to be a network flow problem. In general a network is a directed graph, which consists of a number of nodes and a set, A_c, of arcs that represent the connections between pairs of nodes. A network is typically visualized by thinking of some material that flows on each arc, where f_{ij} denotes the amount of flow through the arc that connects nodes i and j. It is also assumed that there is a cost per unit flow, c_{ij} along the arc (i, j). The general minimum cost network flow problem deals with the minimization of a linear cost function of the form $\sum_{(i,j)\in A_c} c_{ij}f_{ij}$, over all feasible flows.

The Transportation Problem

There are several special cases of the network flow problem. One such case is the assignment problem which is really a specific case of the more general transportation problem.[24]

In the transportation problem there are m suppliers and n consumers. The issue is to transport goods from the suppliers to the consumers at minimum cost. It is assumed that the i^{th} supplier can provide s_i amount of goods, and the j^{th} consumer has demand d_j for the goods. It also assumed that the total supply is equal to the total demand of all the consumers. In this case, the problem is formulated as:

$$
\begin{array}{lll}
\text{minimize} & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}f_{ij} & \\
\text{subject to} & \sum_{i=1}^{m} f_{ij} = d_j, & j = 1, ..., n \\
 & \sum_{j=1}^{n} f_{ij} = s_i, & i = 1, ..., m \\
 & f_{ij} \geq 0 & \forall i, j
\end{array}
\tag{24}
$$

Note, that the first equality constraint specifies that the demand of each consumer must be fulfilled, and the second constraint implies that the entire supply of each supplier must be shipped.

The Assignment Problem

As mentioned earlier, a specific case of the transportation problem is the assignment problem. In this case the number of suppliers is equal to the number of consumers. Furthermore, each supplier has unit supply and each consumer has unit demand. It has been proven[24] that one can always find an optimal solution in which every f_{ij} is either 0 or 1. This means that every supplier i, is assigned to a unique and distinct consumer j. Thus for each i there is a unique j for which $f_{ij} = 1$. The problem is therefore:

$$
\begin{array}{lll}
\text{minimize} & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}f_{ij} & \\
\text{subject to} & \sum_{i=1}^{m} f_{ij} = 1, & j = 1, ..., n \\
 & \sum_{j=1}^{n} f_{ij} = 1, & i = 1, ..., m \\
 & f_{ij} \geq 0 & \forall i, j
\end{array}
\tag{25}
$$

This problem was applied to assigning the T_B slots of the new constellation **B** to the T_A on-orbit satellites and $T_B - T_A$ launched satellites. The "goods" were satellites that needed to be supplied to the "consumers", *i.e.* the slots of constellation **B**. The "cost per unit flow" of transportation was the ΔV requirement. The problem was thus to make assignments such that the necessary total ΔV_{total} to achieve all transfers was minimized.

Figure 10 shows a flow network that represents the satellite assignment problem. Obviously, the ΔV needed to place the satellites in orbit from the ground ($\approx$ min 7.5 km/s) is not included. This ΔV is part of the launch process, since this impulse is given by the launcher's upper stages.

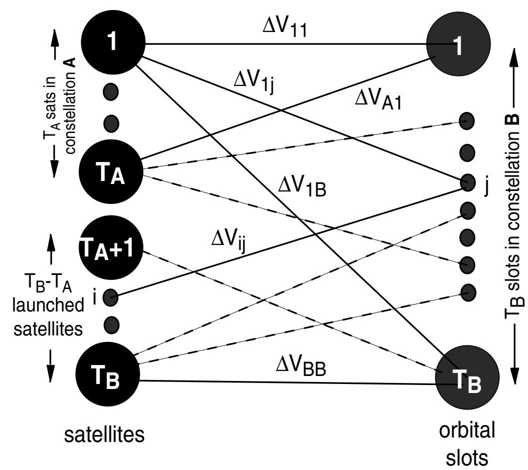

Fig. 10 Assignment problem for a reconfiguration between constellation A with T_A satellites and constellation B with T_B orbital slots

The Auction Algorithm

Bertsimas and Tsitsiklis[24] explain that an efficient method for solving the assignment problem is "the auction algorithm". One interpretation of this algorithm is that there are T_B persons and T_B projects. It is desired to assign a different person to each project while minimizing a linear cost function of the form $\sum_{i=1}^{T_B}\sum_{j=1}^{T_B} c_{ij}f_{ij}$ where $f_{ij} = 1$ if the i^{th} person is assigned to the j^{th} project, and $f_{ij} = 0$ otherwise. In summary, the idea is to represent the situation as a bidding mechanism whereby persons bid for the most profitable projects. It can be visualized by thinking about a set of contractors who compete for the same projects and therefore keep lowering the price they are willing to accept for any given project. In the constellation reconfiguration case, the "persons" are the satellites, the "projects" are the slots in the constellation, and the coefficient c_{ij} represented the ΔV_{ij} for transferring the i^{th} satellite of the constellation **A** to the j^{th} slot of **B**.

The auction algorithm essentially consists of two main parts: the bidding phase, and the assignment phase. In the bidding phase there is a set of prices $p_1, ..., p_n$ for the n projects. Each unassigned person finds a best project, j, by maximizing the profit $p_j - c_{ij}$, and "bids" for it by accepting a lower price. The price is lowered by:

$$\text{(best profit)-(second best profit)-}\epsilon_{auc}$$

The parameter ϵ_{auc}, is a small positive number. It is used to prevent a deadlock in the algorithm, since if there are two equally profitable projects, a bidder will not be able to lower the price of either one of them. Note, that this is the maximum amount by which the price could be lowered before the best project ceases to be the best one.

In the assignment phase the projects are assigned to the lowest bidders. The new price of each project is set to the value of the lowest bid, and any old holder of the project is unassigned.

The specific steps performed in the algorithm are as follows:

- A typical iteration starts with a set of prices $p_1, ..., p_n$ for the different projects, a set S of assigned persons, and a project j_i assigned to each person i of S. At the beginning of the algorithm, the set S is empty.

- Each unassigned person finds a best project k_i by maximizing the profit $p_k - c_{ik}$ over all k. Let $k_i^{'}$ be a second best project, that is,

  $p_{k_i'} - c_{ik_i'} \geq p_k - c_{ik} \forall k \neq k_i$

  Let $\Delta_{k_i} = (p_k - c_{ik}) - \left(p_{k_i'} - c_{ik_i'}\right)$

  Person i "bids" $p_{k_i} - \Delta_{k_i} - \epsilon_{auc}$ for project k_i.

- Every project for which there is at least one bid is assigned to a lowest bidder; the old holder of the project (if any) becomes unassigned. The new price, p_i, of each project that has received at least one bid is set to the value of the lowest bid.

The auction algorithm terminates after a finite number of stages with a feasible assignment. Moreover if the cost coefficients c_{ij} are integers and if $0 < \epsilon_{auc} < 1/n$, the auction algorithm terminates with an optimal solution.[24] In terms of time of calculation, the auction algorithm is very efficient since it runs in time $O(n^4 \max c_{ij})$. The combination of this algorithm with the orbital reconfiguration problem is the main contribution of this article.

Example

To illustrate this method, the algorithm can be applied to a simple case. Two persons 1 and 2, and two projects A and B are considered. The purpose is to assign each person to a project, knowing the cost that each project will imply for 1 and 2. Table 2 indicates these costs.

Table 2 Cost table for simple assignment case

	Project A	Project B
Person 1	\$5,000	\$10,000
Person 2	\$5,000	\$1,000

The obvious assignment minimizing the total cost would be to assign project A to 1 and project B to 2. The algorithm can be tested to see if it returns the same result.

First, a set of prices for the different projects are chosen. The values for p_A and p_B are therefore, arbitrarily chosen to be \$10,000 and \$20,000, respectively. The value for ϵ_{auc} is set equal to 0 in this example, since ϵ_{auc} has no influence on the solution.

In the first iteration, each person finds a best project maximizing the profit. For person 1, the profit of project A is $p_A - c_{1A} = \$5,000$ and the profit of project B is $p_B - c_{1B} = \$10,000$. The values of c_{1A} and c_{1B} are \$5,000 and \$10,000 respectively as shown in Table 2. Person 1 will bid for project B since it yields a greater profit. The value of the bid is $p_B - \Delta = \$20,000 - \$5,000 = \$15,000$, where Δ represents the difference between the profit of the two projects. For person 2, the profits for the two projects are $p_A - c_{2A} = \$5,000$ and $p_B - c_{2B} = \$19,000$. Person 2 will also bid for project B. The value of the bid is $\$20,000 - \$14,000 = \$6,000$. There are two bids for B and zero for A. Project B is assigned to the lowest bidder, *i.e.* to person 2. The new price of B is the value of the bid of 2: $p_B^{\text{modif}} = \$6,000$.

For the second iteration, only person 1 is considered. The profit of project A is still $\$5,000$ for 1, whereas the profit of project B is now equal to $\$6,000 - \$10,000 = -\$4,000$. Person 1, therefore, bids for project A and is assigned to that project and the algorithm terminates.

Application to Orbital Reconfiguration

For the orbital reconfiguration case, the auction algorithm was applied in a similar fashion, where the satellites bid for various slots in the new constellation **B**.

Due to the condition $\Delta V_{\text{launched satellite}} = 0$ in the flow network (depicted in Figure 10), the auction algorithm first assigns the T_A on-orbit satellites into slots of the new constellation **B**. The launched satellites are then assigned to the remaining slots. Although the ΔV is minimized with this method, this approach is not entirely satisfactory. This is because satellites of the same launch should be assigned to the same plane. The assignment returned by the auction algorithm would not necessarily satisfy that constraint, given a certain capacity of the launch vehicle. The assignment was therefore refined with an additional loop (as shown in Figure 4) to reassign ground-launched satellites if necessary.

Assignment Module

The Assignment Module uses the transition matrix $\Delta\mathbf{V}_{i,j}$ to determine the most energy efficient assignment of satellites to orbital slots. However, as explained previously, the assignment is performed in a loop in order to refine the assignment, so that it matches with the launch vehicle capacity.

First of all, from the initial assignment $\mathcal{A}_{AB}$, the number of slots occupied by the ground satellites is obtained for each plane of **B**. If the repartition of the ground satellites does not correspond to the launcher capacity, one or several position(s) occupied by the satellites of **A** are set free in the plane considered in order to permit additional launched satellites to be placed in that plane. The method corresponds to a reassignment of some of the ground-launched satellites.

The initial matrix $\Delta\mathbf{V_{ij}}$ is modified to take that reassignment into account. When a satellite (say the p^{th}) is assigned to a slot of constellation **B** (say the j^{th}), other satellites are prevented from going to that position by setting $\Delta V_{i,j} = 2000$ for all $i \neq p$. Once all the satellites on the ground are reassigned, the auction algorithm is run a second time with the modified transition matrix $\Delta\mathbf{V_{i,j}^{modif}}$, resulting in a modified assignment, $\mathcal{A}_{AB}^{modif}$. This method turns out to be very efficient in terms of calculation time, since the auction algorithm is run only twice. This reassignment process is explained in detail again in the following case study.

Auction Algorithm Benchmarking

During implementation of the framework, the auction algorithm was compared to two other methods of assignment. The first method consisted of using randomly generated assignments. The second method used was Simulated Annealing (SA),[25] which is a well known combinatorial optimization technique.

The reconfiguration scenario used in making the comparisons was the reconfiguration of a polar SOC constellation **A** with $h_A = 2000$ km and $\epsilon_A = 5$ deg to a polar SOC constellation **B** with altitude $h_B = 1000$ km and elevation $\epsilon_B = 5$ deg (similar to Figure 1). The constellation **A** consisted of 3 planes of 7 satellites each, and constellation **B** was comprised of 5 planes of 8 satellites each. The loop for assigning the ground satellites was not be taken into account, and only the initial assignment returned by the auction algorithm was considered. The auction algorithm, according to Equation (1), returned an optimal value of $\Delta V_{total} = 26.5$km/s which represents an average ΔV per satellite of 1.25 km/s (for the 21 on-orbit satellites).

In the first method, seven random assignments were generated. From the matrix $\Delta\mathbf{V}_{ij}$, the ΔV_{total} in each case was computed. Table 3 shows the results.

Table 3 Random assignment ΔV results (benchmarking)

Experiment	$\Delta V_{total}(km/s)$	$\Delta V_{avg}(km/s)$ per sat.
1	144.8	6.9
2	156.7	7.5
3	140.2	6.7
4	111.8	5.3
5	142.6	6.8
6	122.7	5.8
7	147.5	7.0

The results given by the auction algorithm were much better than those returned by a non-optimized assignment. Note, that the auction algorithm returns only one of the optimal assignments. In fact, the best assignment is non-unique, since only assigning to the correct plane of constellation **B** really matters. Knowing that 21 satellites had to be assigned into 40 slots, the size of the full-factorial solution space is $C_{40}^{21} \approx 1.31 \times 10^{11}$. Of these C_{40}^{21} possibilities, only $(C_8^7)^3 = 512$ assignments would return the optimal value of 26.5 km/s. Changing the slot of one satellite in the same plane would not change the ΔV_{total}. These considerations explain why $(C_8^7)^3$ optimal assignments exist.

As mentioned earlier, Simulated Annealing was also used to compare the efficiency of the auction algorithm. The different steps of SA are described by Kirkpatrick et al.[25] The initial assignment vector was chosen arbitrarily resulting in a ΔV_{total} of 128.7 km/s. Perturbations to the assignment were generated by randomly inverting the assigned slots of pairs of satellites. If the perturbation was beneficial, the modified assignment was always accepted. If the new assignment resulted in a higher ΔV_{total}, the probability of accepting that new assignment (') was equal to $\exp[-(\Delta V'_{total} - \Delta V_{total})/T]$, where the SA system temperature, T, was gradually lowered until the assignment appeared to be *frozen*.

The auction algorithm returned a ΔV of 26.5 km/s in a CPU time of around 1.2 sec. The SA algorithm was run several times and the results are summarized in Table 4. The CPU times (in the table) were obtained from a Samsung VM700 Series laptop with a 400 MHz, Pentium II processor and 64 MB of RAM.

For each try, the initial assignment was well-improved (recall that the initial assignment corresponded to a ΔV_{total} of 128.7 km/s). The SA algorithm returned a good assignment in terms of ΔV, but it very rarely returned the best one (which was the 26.5 km/s as returned by the auction algorithm). In ten attempts, the best one was obtained only once (Trial 7 in Table 4). Moreover, the computation time was slightly higher for SA.

This study supports the reliability and speed of the auction algorithm compared to Simulated Annealing for this assignment problem, at least empirically. SA

Table 4 Simulated Annealing ΔV results (benchmarking)

Experiment	$\Delta V_{total}(km/s)$	CPU Time (s)
1	27.9	2.09
2	27.9	1.97
3	31.8	1.97
4	36.0	2.03
5	31.8	2.09
6	29.3	2.03
7	26.5	2.19
8	30.5	2.09
9	27.9	2.31
10	30.4	2.42

is too dependent on the different parameters such as initial temperature and cooling schedule for being a competitive method in this context.

Case Study

In order to demonstrate practical use of the auction algorithm to orbital reconfiguration, the process of assigning T_A satellites to **B**, the framework was applied to a particular orbital reconfiguration case. The reconfiguration of the LEO polar (SOC) constellation **A** with altitude $h_A = 2000$ km and minimum ground elevation angle $\epsilon_A = 5$ degrees to a LEO polar (SOC) constellation **B** (altitude h_B of 1200 km and minimum elevation angle ϵ_B of 5 degrees). This scenario is depicted in Figure 1. The study was limited to chemical (impulsive) propulsion and the two-phase transfer scenario. The study also included the loop to assign the ground satellites.

Given the parameters defined above, the constellation module computed that for continuous, global, single-fold coverage, **A** contained 21 satellites in 3 planes of 7 satellites and **B** had 32 slots (4 planes of 8 satellites). Thus, 11 satellites needed to be launched.

Assignment Module Results

A first run of the auction algorithm was achieved without taking into account the loop for assigning the satellites on the ground. Table 5 shows the assignment returned by the algorithm. Note that $P_1^A(4)$ indicates the 4^{th} satellite in the 1^{st} plane of **A**. This assignment represents a total ΔV of 40.5 km/s. All the satellites of a plane of **A** go to the same plane of **B**. This seems intuitive, since the ΔV depends only on the initial and final plane characteristics, plus the phasing allowance.

As explained earlier, the auction algorithm first assigns the on-orbit satellites. The launched satellites then go to the remaining spots. In this case study, the selected launch vehicle could carry two satellites per launch, i.e. $T_{LV} = 2$. Six launches would be necessary for eleven ground satellites: five launches of two satellites and one launch of one satellite (spares not considered). Examining the results in Table 5 it becomes clear that there is an uneven distribution of the "empty slots" in the planes of **B**. Table 6 shows the number of ground satellites that are needed to fill each plane of **B**. This distribution is not suitable for the selected launch vehicle, since plane changes would be required to fill the gaps in planes 1, 2 and 4 of constellation **B**.

Table 5 Initial assignment matrix, $\mathcal{A}_{AB}$, for reconfiguration of constellation A to B with $h_A = 2000$ km, $h_B = 1200$ km and $\epsilon_A = \epsilon_B = 5$ degrees

Position in A	Final Slot in B	ΔV (km/s)
$P_1^A(1)$	$P_1^B(8)$	0.85
$P_1^A(2)$	$P_1^B(7)$	0.85
$P_1^A(3)$	$P_1^B(6)$	0.85
$P_1^A(4)$	$P_1^B(5)$	0.85
$P_1^A(5)$	$P_1^B(4)$	0.85
$P_1^A(6)$	$P_1^B(3)$	0.85
$P_1^A(7)$	$P_1^B(2)$	0.85
$P_2^A(1)$	$P_2^B(8)$	2.5
$P_2^A(2)$	$P_2^B(7)$	2.5
$P_2^A(3)$	$P_2^B(6)$	2.5
$P_2^A(4)$	$P_2^b(5)$	2.5
$P_2^A(5)$	$P_2^B(4)$	2.5
$P_2^A(6)$	$P_2^B(3)$	2.5
$P_2^A(7)$	$P_2^B(2)$	2.5
$P_3^A(1)$	$P_4^B(8)$	2.5
$P_3^A(2)$	$P_4^B(7)$	2.5
$P_3^A(3)$	$P_4^B(6)$	2.5
$P_3^A(4)$	$P_4^B(5)$	2.5
$P_3^A(5)$	$P_4^B(4)$	2.5
$P_3^A(6)$	$P_4^B(3)$	2.5
$P_3^A(7)$	$P_4^B(2)$	2.5

Table 6 Number of ground satellites assigned to each plane of B after the first run of the auction algorithm

Plane	# of ground satellites
1	1
2	1
3	8
4	1

Figure 11(a) depicts this first assignment and shows the problem that results due to the capacity of the launch vehicles (LVs). A reassignment was necessary using the refinement loop in the algorithm of the Assignment Module (Figure 4). Since plane 3 contains no satellites of **A**, the first four launches (LV1-4) will therefore go to plane 3 of **B**.

The fifth and sixth launches require freeing one slot in plane 1, 2 or 4 in order to allow a launch of two satellites to the same plane, since each of these planes have only one slot reserved for ground satellites. The assignment loop chooses to free a slot in plane 4 by

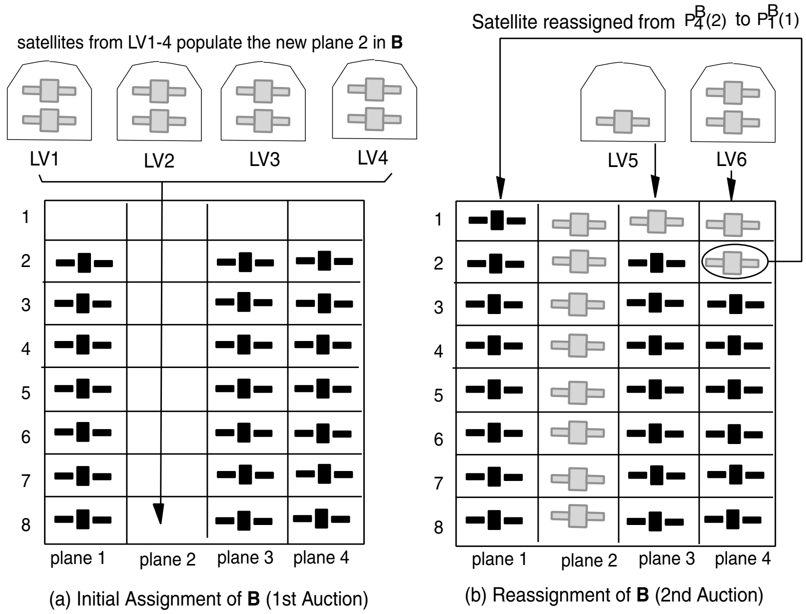

Fig. 11 Representation of (a) initial and (b) final satellite assignment to slots of B for case study.

reassigning it to the empty slot in plane 1: $P_4^B(2) \rightarrow P_1^B(1)$. In the fifth launch a single satellite is used to fill plane 3, while the sixth and final launch uses a pair of new satellites to fill plane 4, see Figure 11(b). Once the reassignment is done, the auction algorithm is run a second time with the matrix $\Delta \mathbf{V}_{\mathbf{i,j}}^{\mathbf{modif}}$. The final assignment is summarized in Table 7.

The ΔV of 50.5 km/s obtained after the second auction points out the influence of the loop for assigning the launched satellites. This influence is higher if the launch vehicle has a higher capacity, which would then require moving more satellites from their original assignments in order to allow for same plane launches. In this example only one on-orbit satellite is penalized by the reassignment.

Table 7 Final assignment matrix, $\mathcal{A}_{AB}^{final}$, Reconfiguration of constellation A to B with $h_A = 2000$ km, $h_B = 1200$ km and $\epsilon_A = \epsilon_B = 5$ degrees

Position in A	Final Slot in B	ΔV (km/s)
$P_1^A(1)$	$P_1^B(8)$	0.85
$P_1^A(2)$	$P_1^B(7)$	0.85
$P_1^A(3)$	$P_1^B(6)$	0.85
$P_1^A(4)$	$P_1^B(5)$	0.85
$P_1^A(5)$	$P_1^B(4)$	0.85
$P_1^A(6)$	$P_1^B(3)$	0.85
$P_1^A(7)$	$P_1^B(2)$	0.85
$P_2^A(1)$	$P_2^B(2)$	2.5
$P_2^A(2)$	$P_2^B(6)$	2.5
$P_2^A(3)$	$P_2^B(5)$	2.5
$P_2^A(4)$	$P_2^B(7)$	2.5
$P_2^A(5)$	$P_2^B(8)$	2.5
$P_2^A(6)$	$P_2^B(4)$	2.5
$P_2^A(7)$	$P_2^B(3)$	2.5
$P_3^A(1)$	$P_4^B(7)$	2.5
$P_3^A(2)$	$P_4^B(6)$	2.5
$P_3^A(3)$	$P_4^B(8)$	2.5
$P_3^A(4)$	$P_1^B(1)$	12.5
$P_3^A(5)$	$P_4^B(4)$	2.5
$P_3^A(6)$	$P_4^B(3)$	2.5
$P_3^A(7)$	$P_4^B(5)$	2.5

Constellation Reconfiguration Map

The framework presented here was used to create a *Constellation Reconfiguration Map.* Depending on the type of reconfiguration, the average quantity of ΔV (per satellite) necessary to achieve the maneuvers of the on-orbit satellites can be very different. Different types of reconfigurations were considered: reconfigurations in altitude, reconfigurations in inclination, and reconfigurations in both altitude and inclination. Reconfiguration in RAAN was not explored explicitly, but it is an implicit function of the number of orbital planes in **A** and **B**. A reconfiguration in altitude only conserves the type of constellation (polar SOC or Walker). An example of reconfiguration in altitude is the reconfiguration from a GEO polar constellation into a MEO polar constellation. Inversely, a reconfiguration in inclination conserving the altitude can change the constellation type. If the inclination change is large enough, the reconfiguration from a LEO Polar constellation into a LEO Walker constellation can - at least theoretically - be considered.

The loop to assign the ground satellites was not taken into account. The main purpose was to see trends and orders of magnitude. A diagram of altitude versus inclination was drawn in order to point out the influence of the reconfiguration type on fuel consumption. Figure 12 shows this diagram. The black dots represent the positions of the constellations as well as their parameters, T, P, F, ϵ. The arrows represent the direction of reconfiguration. The number at the middle of the arrow indicates the average ΔV per satellite required for that reconfiguration.

For an I_{sp} of 430 s, an extra-fuel mass of 700 kg corresponds to a ΔV of 2.9 km/s, as computed from the Rocket Equation (2). Since the satellite dry mass in the analysis was taken to be 700 kg, such an extra-mass of fuel represents a high quantity of fuel for a single satellite. Therefore, a reconfiguration needing an average ΔV above 3 km/s is an expensive one in terms of fuel consumption. Similarly, a reconfiguration requiring an average ΔV below 2 km/s could be considered as a "cheap" reconfiguration, since 2 km/s of ΔV represents a fuel mass of approx 400 kg.

The Reconfiguration Map of inclination vs. altitude reveals interesting trends. First, reconfigurations from polar-to-polar seems to be feasible, except for the reconfiguration from MEO to LEO. The required ΔV for the polar-polar reconfigurations analyzed is around 1.8-1.9 km/s per satellite. The same trend appears for the Walker-Walker reconfigurations, where values can reach below 1.5 km/s. The most expensive reconfigurations are the reconfigurations requiring a high angle inclination change, in other words the reconfigurations from Walker-Polar or Polar-Walker. An exception is geosynchronous orbit (GEO), where the ΔV required is relatively low. The reconfigurations in MEO or from GEO to MEO are somewhat less expensive (between 2.7 km/s and 3.2 km/s) than the reconfigurations in LEO or from MEO to LEO, which require ΔVs above 3.5 km/s. The most expensive reconfiguration appeared to be the reconfiguration from a MEO Walker constellation with the following characteristics T/P/F = 16/8/5 to a LEO polar constellation with the characteristics 36/4/0. This reconfiguration requires an average ΔV per satellite of almost 4.5 km/s. This corresponds to an extra-fuel mass of 1.3 tons when executed with chemical propulsion. Reconfigurations coupling changes in altitude with changes in inclination are, not surprisingly, very expensive.

In short, constellation reconfigurations in altitude seem to be feasible. Inclination changes imply plane changes, which are very expensive in LEO. These results can be explained partly by the fact that polar constellations generally have fewer planes than Walker constellations and therefore appear to be more "reconfigurable" via the insertion of new planes, combined with partial transfers between planes as shown in Figure 11. To reconfigure a Polar into a Walker constellation or vice-versa is expensive, as this usually implies several plane changes with high angle increments.

Summary

The auction algorithm can be a useful method for determining optimal solutions to the orbital reconfiguration problem of satellite constellations. It helps in determining how to best assign each satellite of an existing constellation to a spot in a new constellation such that the total ΔV requirement is minimized. It was also shown that the auction algorithm is more efficient than Simulated Annealing or random assignments. Our analysis indicated that the auction algorithm was faster, and more reliable in its results. The auction algorithm was also used as part of a larger framework to study various types of reconfigurations involving change in inclination, altitude, and constellation type. A *Constellation Reconfiguration Map* was produced that shows the energy requirements for different kinds of reconfigurations. This map can be a useful tool in depicting which kind of reconfigurations comparatively require less fuel during conceptual constellation design. Future satellite constellations used for Earth Observation, National Defense and "hot spot" communications will likely benefit from such reconfigurability considerations.

Future Work

The metric adopted for the satellite assignment was to minimize the $\Delta V_{total} = \sum_{k=1}^{T_A} \Delta V_{k^{th} satellite}$. This metric is convenient, but since all satellites should be the same for commonality, manufacturing, and launch reasons, it would be more useful to minimize the variance of the required ΔV in the constellation. So, if the cost function minimizes $\sum_{k=1}^{T_A} (\Delta V_k - \frac{\Delta V_{total}}{T_A})^2$, the gap between the satellites propellant load could be lower, although the ΔV_{total} could be higher.

Some of the reconfiguration scenarios in this paper deal with ΔV requirements in the >2 km/s range, which appear prohibitive with low I_{sp} chemical systems. The benefit of using electric propulsion for constellation reconfiguration should be investigated in this context. This would reveal another interesting tradeoff, namely between the *time* required to achieve the reconfiguration process and the required propellant (ΔV) budget. With electric propulsion, the propellant cost will be lowered at the expense of service outage cost during configuration.

This paper deals mainly with a single reconfiguration from **A** to **B**. Multiple consecutive reconfigurations $\mathbf{A} \rightarrow \mathbf{B} \rightarrow \mathbf{C}$ would likely be very expensive or impossible by carrying all the fuel onboard for two or more future reconfigurations. It is unclear whether enough fuel could be carried for multiple reconfigurations. An alternative to this problem would be the exploration of fuel depots at strategic locations on orbit that could refuel satellites adaptively as needed.

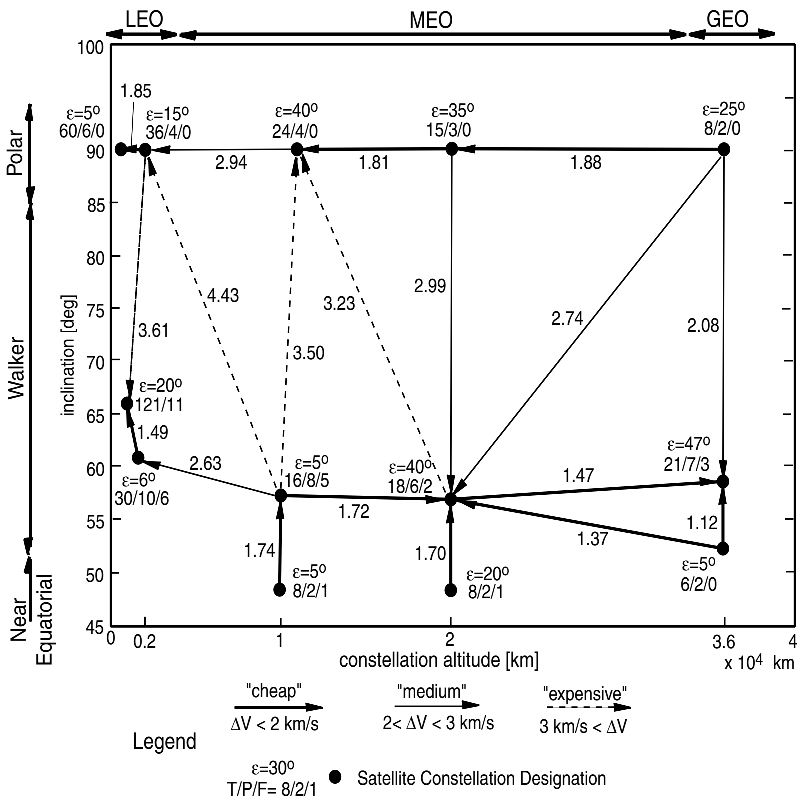

Fig. 12 Satellite Constellation Reconfiguration Map: Average $\Delta V/sat$

It could be also judicious to consider the utilization of a space tug as a "real option", instead of extra-fuel. The space tug would transport the satellites to be transferred from their spot in **A** to their optimally assigned slots in **B**.

There is another, perhaps more fundamental issue. The departure and arrival constellations **A,B** in this study were selected from the optimal set of constellations proposed by Adams, Rider, Lang and others as shown in Figure 5. Optimality was driven by the minimum number of satellites, T, required for a particular constellation type and Earth central half-angle $\theta = f(h, \epsilon)$. It is not obvious that such constellations are also optimal in the sense of *reconfigurability*. In other words, some amount of inefficiency or penalty in the initial constellation might be worthwhile, if it helps avoid downstream reconfiguration costs in terms of ΔV, number of launches or outage time. Reconfigurability might well become a more prominent objective function in future constellation design.

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