Occam's Razor and Alphabet Soup:

What anisotropic elastic parameters can we reasonably measure in borehole seismic experiments?

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Schlumberger-Doll Research

MIT-ERL Friday Informal Seminar Hour

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Occam's Razor and Alphabet Soup:

What anisotropic elastic parameters can we reasonably measure in borehole seismic experiments?

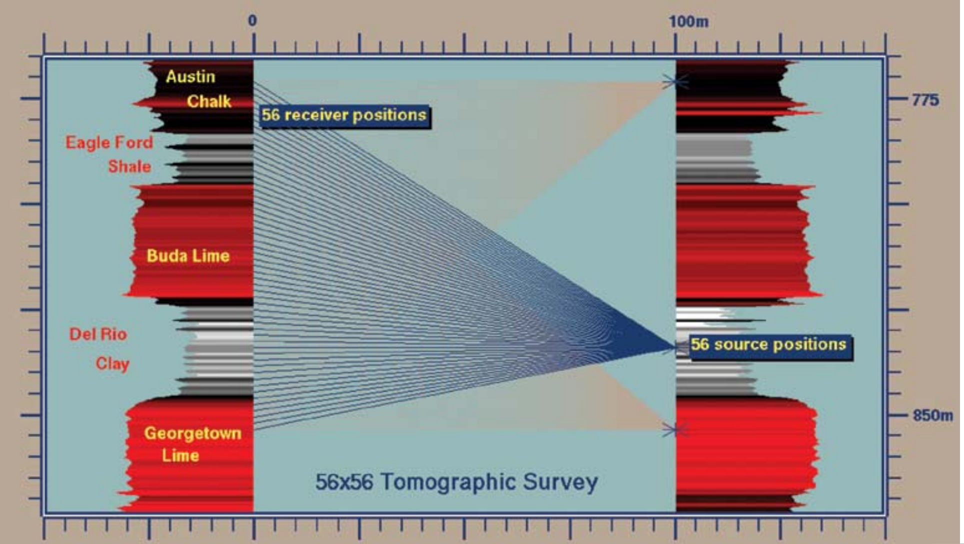
Entities are not to be multiplied beyond necessity

- William of Ockham as paraphrased by John Ponce of Cork.

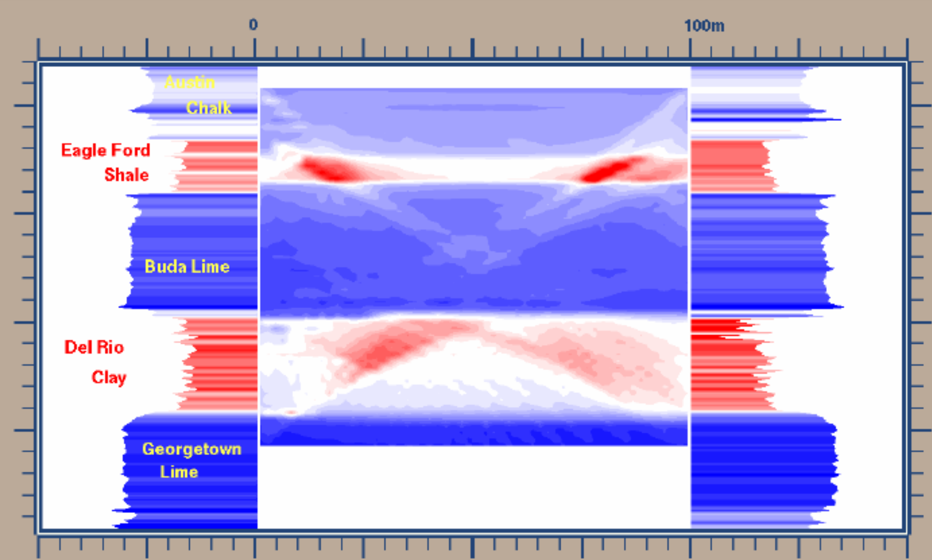
Entities must not be reduced to the point of inadequacy

- Walter of Chatton as paraphrased by Karl Menger.

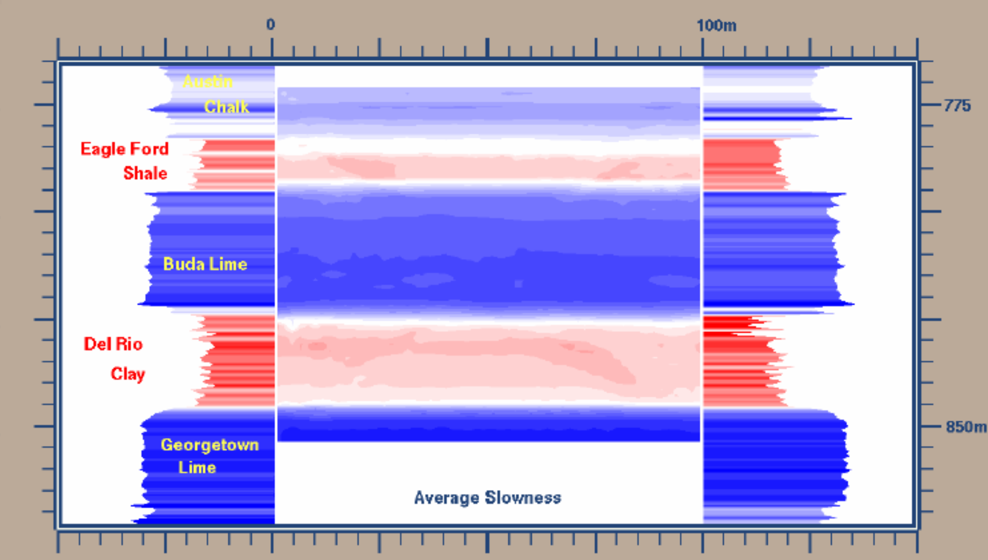
Crosswell Seismic Example



- Anisotropic Solution with 53x3 parameters
- Isotropic Solution with 56x56 parameters
- Similar (good) fit to data

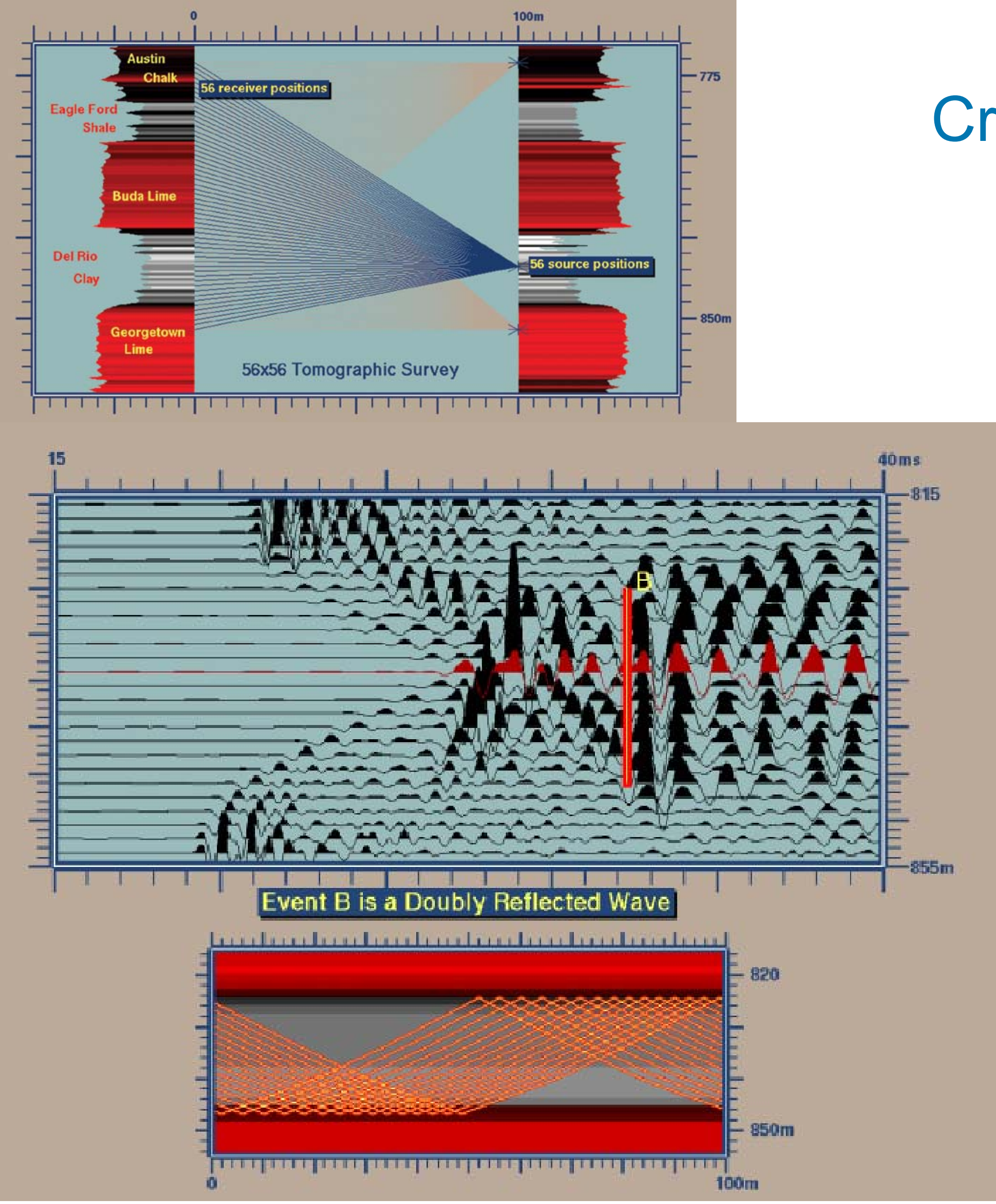


ISOTROPIC TOMOGRAM



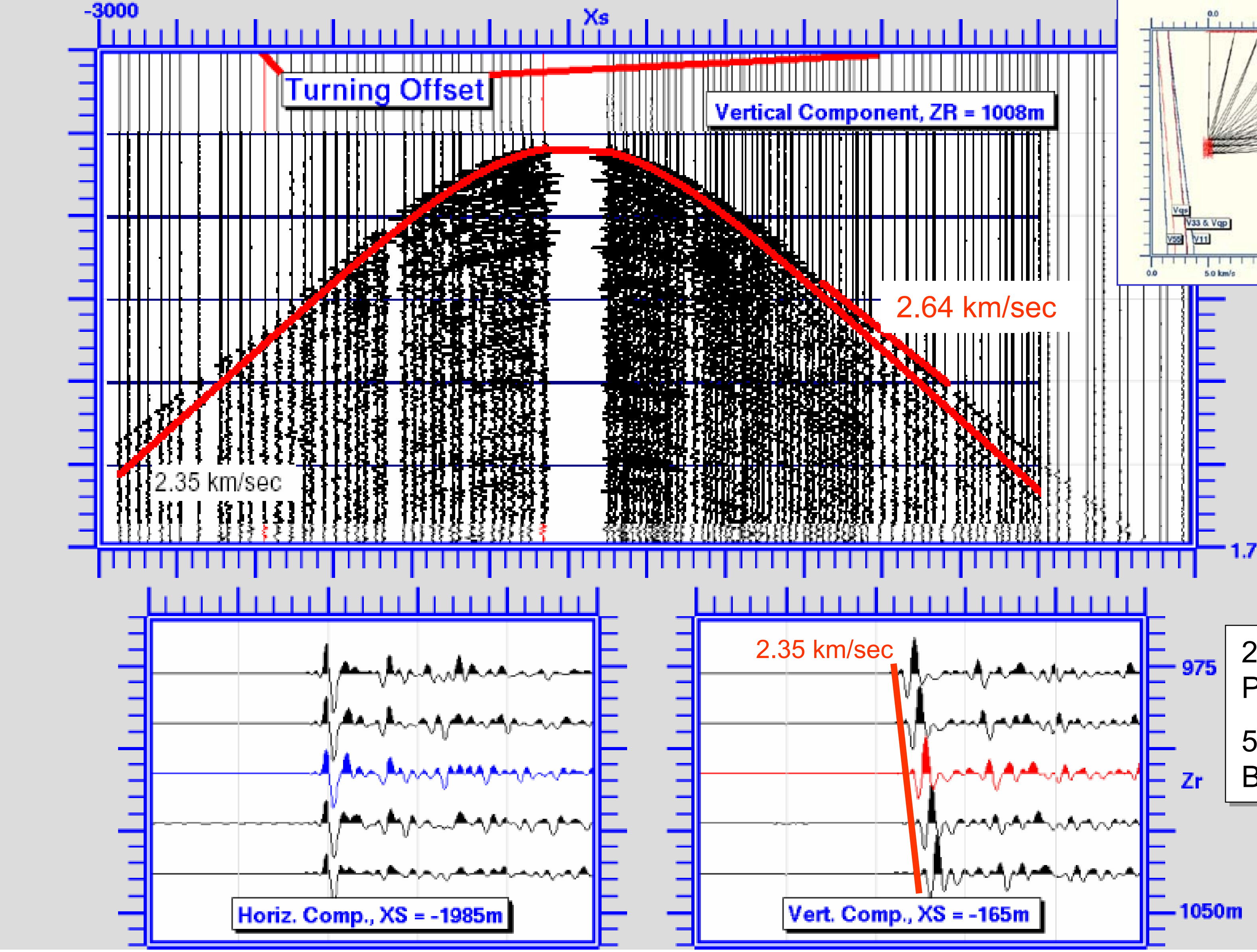
ANISOTROPIC TOMOGRAM

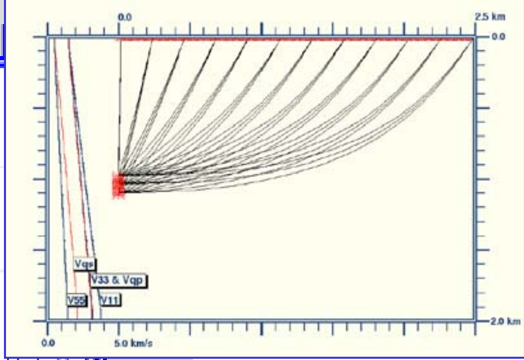
Crosswell Seismic Example



•The anisotropic solution is a good predictor of other coherent arrivals. The isotropic solution is not.

•Conclusion: The shales are anisotropic.



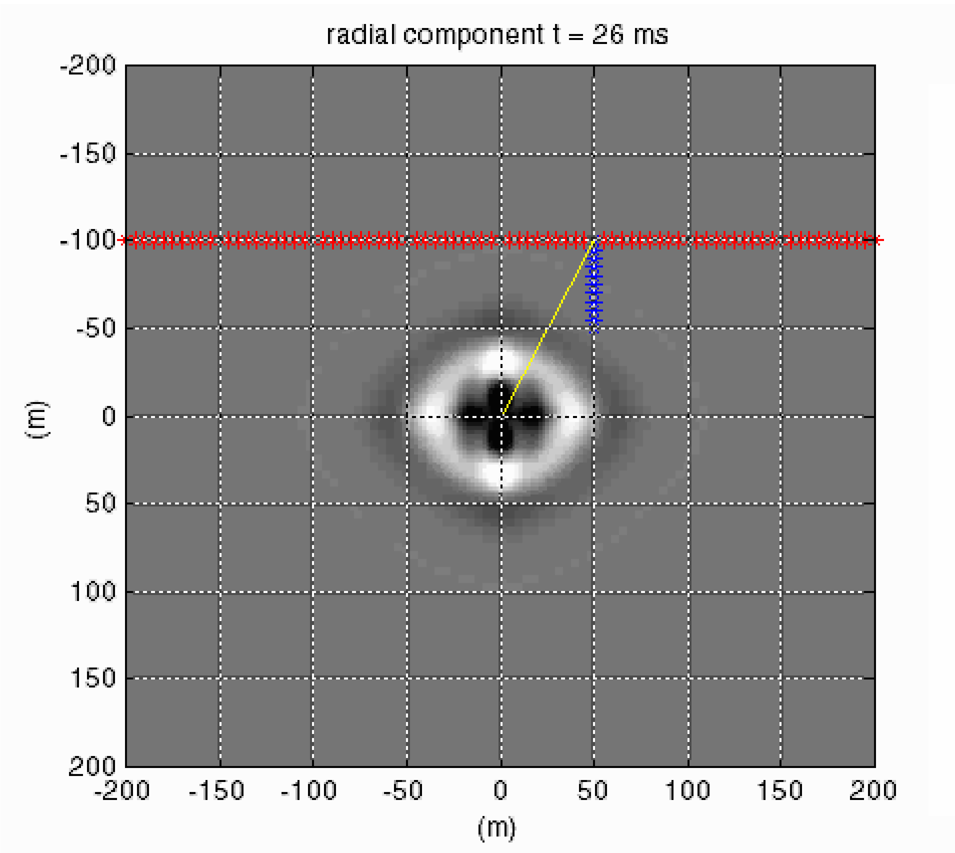


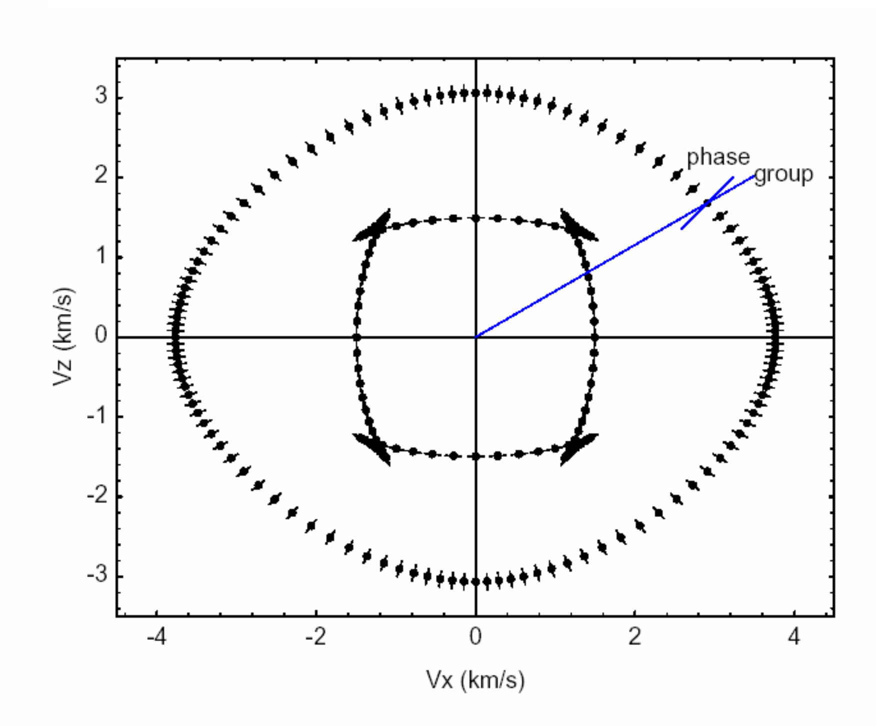
201 Source Positions

5 3-Component Borehole receivers

Walkaway VSP Example

Anisotropy 101





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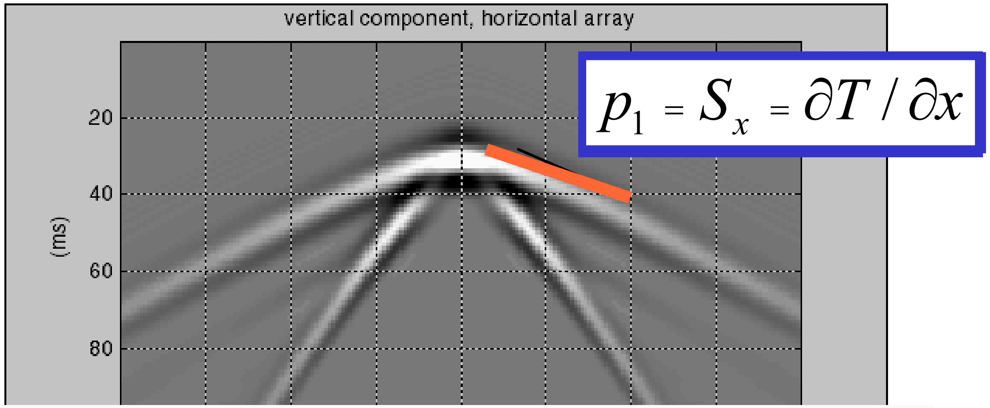
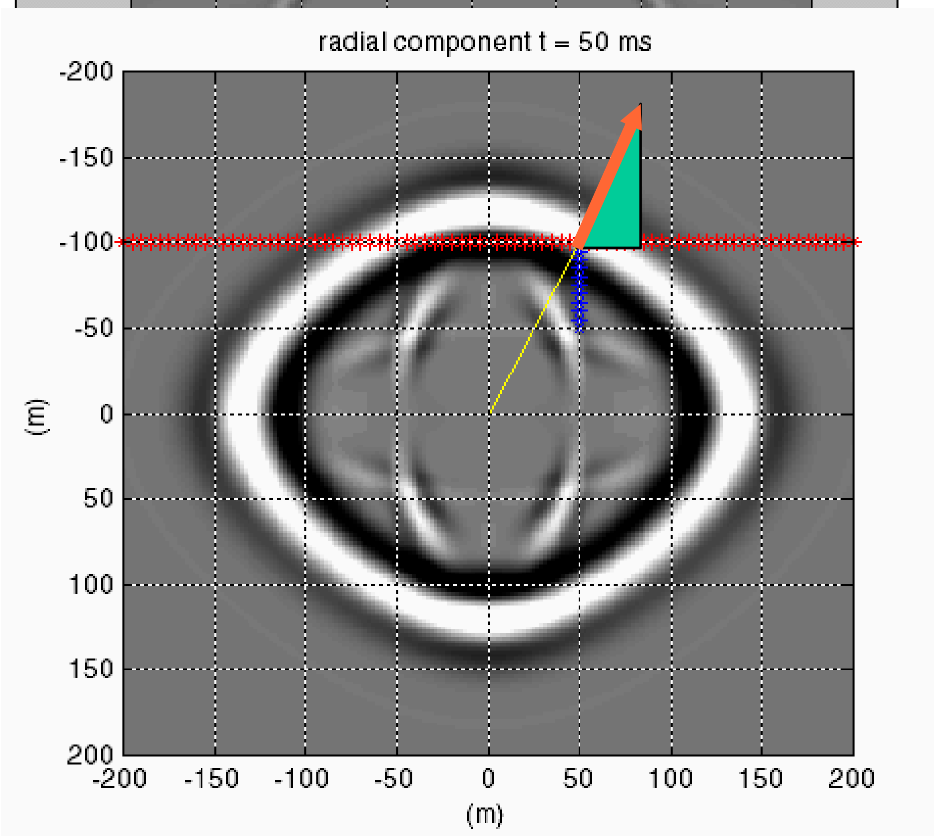
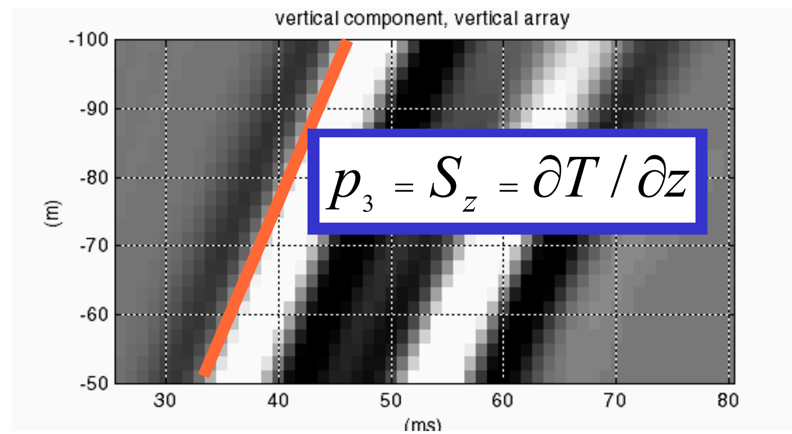




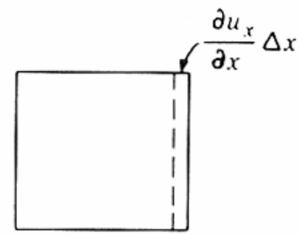
The spatial gradient of the traveltime function is the Phase Slowness Vector



Hooke's Law

- **isotropic**

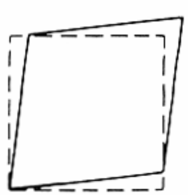
$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & & & \\ \lambda & \lambda + 2\mu & \lambda & & & \\ \lambda & \lambda & \lambda + 2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{12} \\ \epsilon_{13} \\ \epsilon_{23} \end{bmatrix}$$



(b) Simple extension

To achieve a unit of pure longitudinal strain along the 1-axis:

- Pull *left-right* with traction $\lambda + 2\mu$
- Pull *up-down, in-out* with traction λ


(e) Pure shear

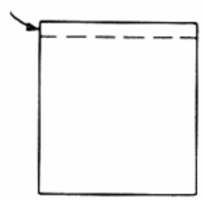
To achieve a unit of pure shear strain:

- Squeeze opposite corners with differential traction μ

Hooke's Law

- **TIV - rotational symmetry around 3-axis**

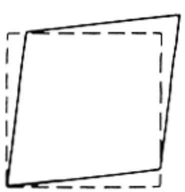
$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{13}\\ \sigma_{23}\end{bmatrix} = \begin{bmatrix} c_{1111} & c_{1111}-2c_{1212} & c_{1133} & & & \\ c_{1111}-2c_{1212} & c_{1111} & c_{1133} & & & \\ c_{1133} & c_{1133} & c_{3333} & & & \\ & & & c_{1313} & & \\ & & & & c_{1313} & \\ & & & & & c_{1212} \end{bmatrix} \begin{bmatrix}\epsilon_{11}\\ \epsilon_{22}\\ \epsilon_{33}\\ \epsilon_{12}\\ \epsilon_{13}\\ \epsilon_{23}\end{bmatrix}$$



(b) Simple extension

To achieve a unit of pure longitudinal strain along the 3-axis:

- Pull *up-down* with traction c_{3333}
- Pull *left-right, in-out* with traction c_{1133}



(e) Pure shear

To achieve a unit of pure 13 shear strain:

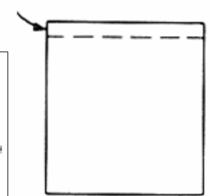
- Apply 13 traction c_{1313}

Hooke's Law: Reduced (Voigt) Notation

- **TIV - rotational symmetry around 3-axis**

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix} = \begin{bmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & & & \\ c_{11}-2c_{66} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{44} & \\ & & & & & c_{66}\end{bmatrix} \begin{bmatrix}\epsilon_1\\ \epsilon_2\\ \epsilon_3\\ \epsilon_4\\ \epsilon_5\\ \epsilon_6\end{bmatrix}$$

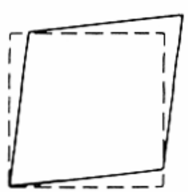
11	22	33	32 = 23	31 = 13	12 = 21
↓	↓	↓	↓	↓	↓
1	2	3	4	5	6



(b) Simple extension

To achieve a unit of pure longitudinal strain along the 3-axis:

- Pull *up-down* with traction c_{33}
- Pull *left-right, in-out* with traction c_{13}



(e) Pure shear

To achieve a unit of pure 13 shear strain:

- Apply 13 traction c_{55}

Christoffel (Dispersion) Relation

HL + $F = ma$ + $\mathbf{u} = \hat{\mathbf{g}} e^{(i\omega(\mathbf{p}\cdot\mathbf{x}-t))}$ gives the Christoffel (Eigenvalue) Equation:

$$[p_i p_l c_{ijkl} - \rho\delta_{jk}]\hat{g}_k = 0$$

- A solution exists when $Det(matrix) = 0$. This is the **Christoffel Relation** that implicitly defines the phase slowness surface:

$$\mathcal{S} = \{\mathbf{p} : |[p_i p_l c_{ijkl} - \rho\delta_{jk}]| = 0\}$$

-

$$a_{ijkl} = c_{ijkl}/\rho$$

$$A_{11} = a_{xxxx},\ A_{13} = a_{xxzz},\ A_{55} = a_{xzxz},\ \ldots$$

N.B.: Aij have units of velocity^2

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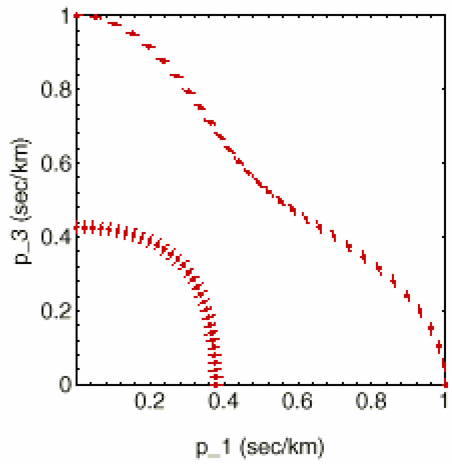
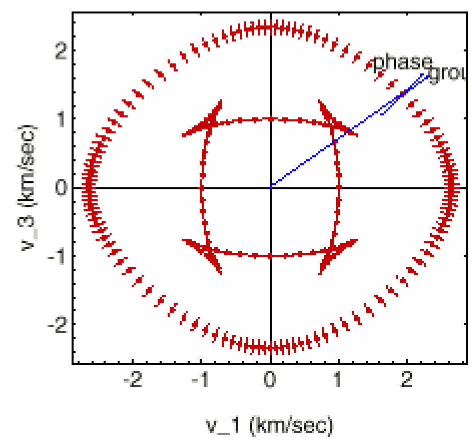




Dispersion Relation:

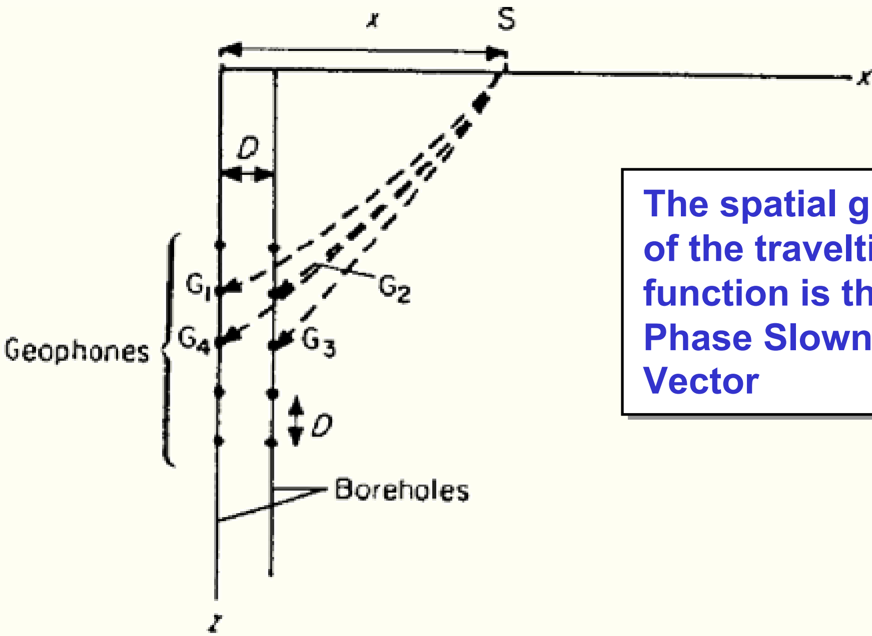
$$A_{11}A_{55}p_1^4 + A_{33}A_{55}p_3^4 + Ap_1^2p_3^2 - (A_{11} + A_{55})p_1^2 - (A_{33} + A_{55})p_3^2 + 1 = 0$$

$$A = A_{11}A_{33} + A_{55}^2 - (A_{13} + A_{55})^2$$

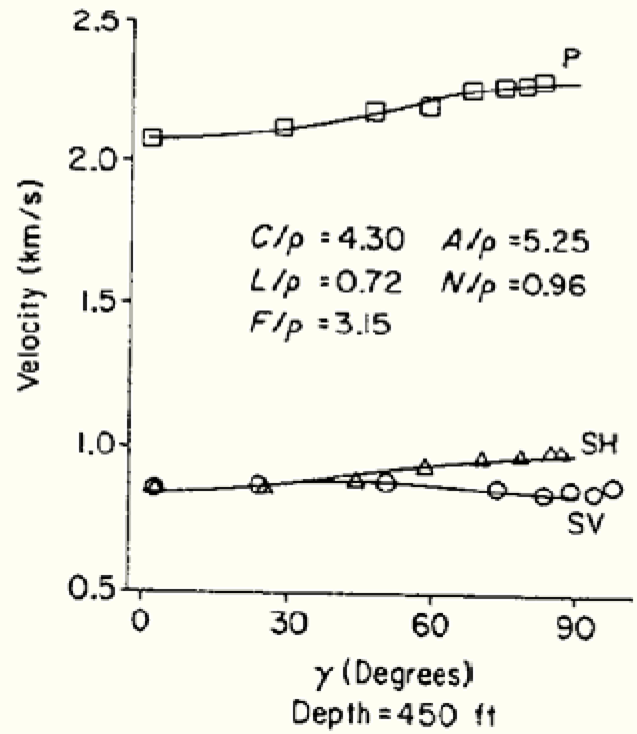
$$\mathbf{p} \cdot \mathbf{v} = 1$$

N.B.: Given A_{ij}'s and p_1, this yields a quadratic equation for $(p_3)^2$

White, et al., 1983



The spatial gradient of the traveltime function is the Phase Slowness Vector



$$\Delta t_h = (t_1 - t_2 + t_4 - t_3)/2,$$

$$\Delta t_v = (t_3 - t_2 + t_4 - t_1)/2.$$

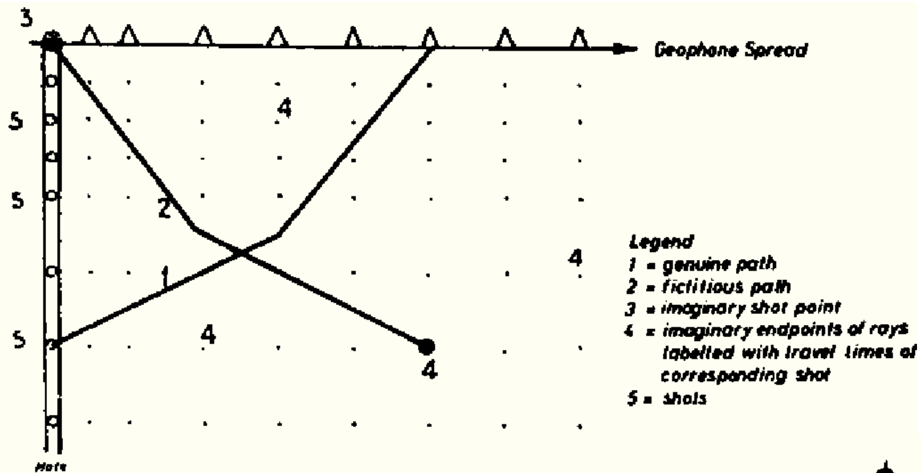
$$V_h = \frac{D}{\Delta t_h}$$

$$V_v = \frac{D}{\Delta t_v}$$

$$V_{PH} = [(1/V_h)^2 + (1/V_v)^2]^{-1/2},$$

$$\gamma = \tan^{-1}[V_v/V_h].$$

Meisner, 1961



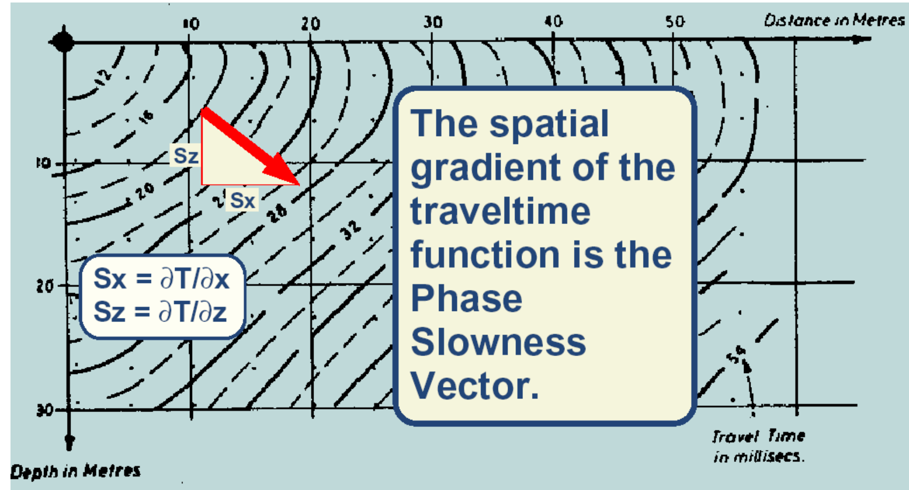


Fig. 2. Wave-front diagram.

J. Gaiser (1992) used this method to estimate phase slownesses which he inverted for TIV parameters.

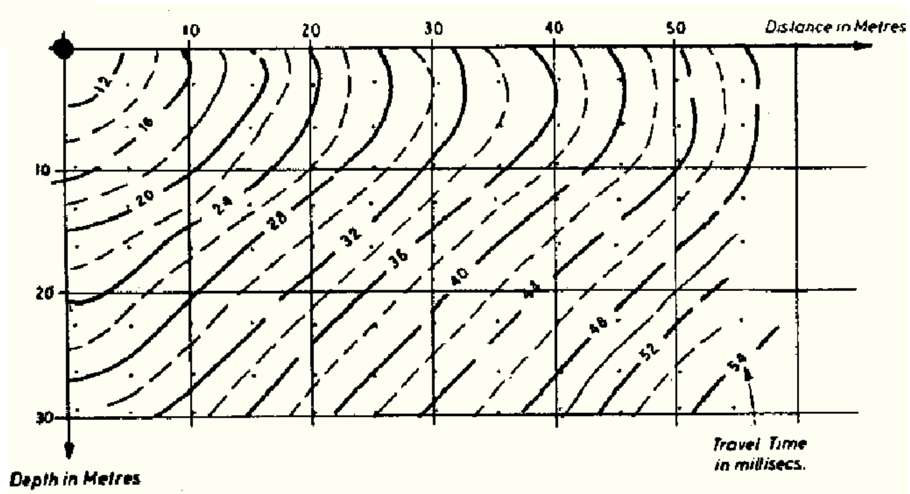
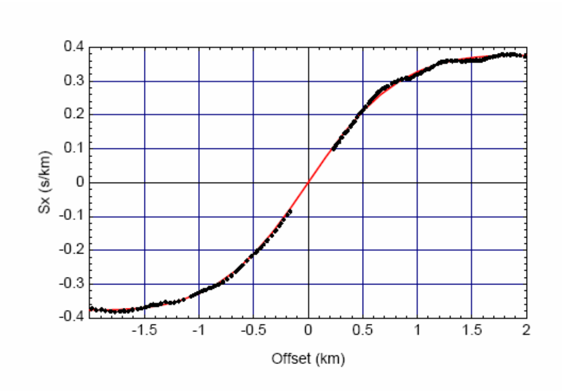
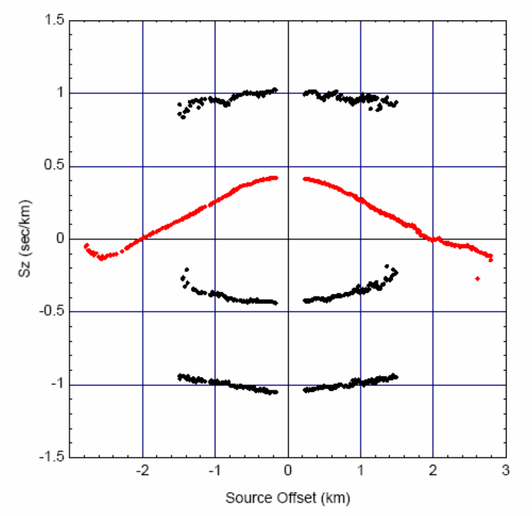
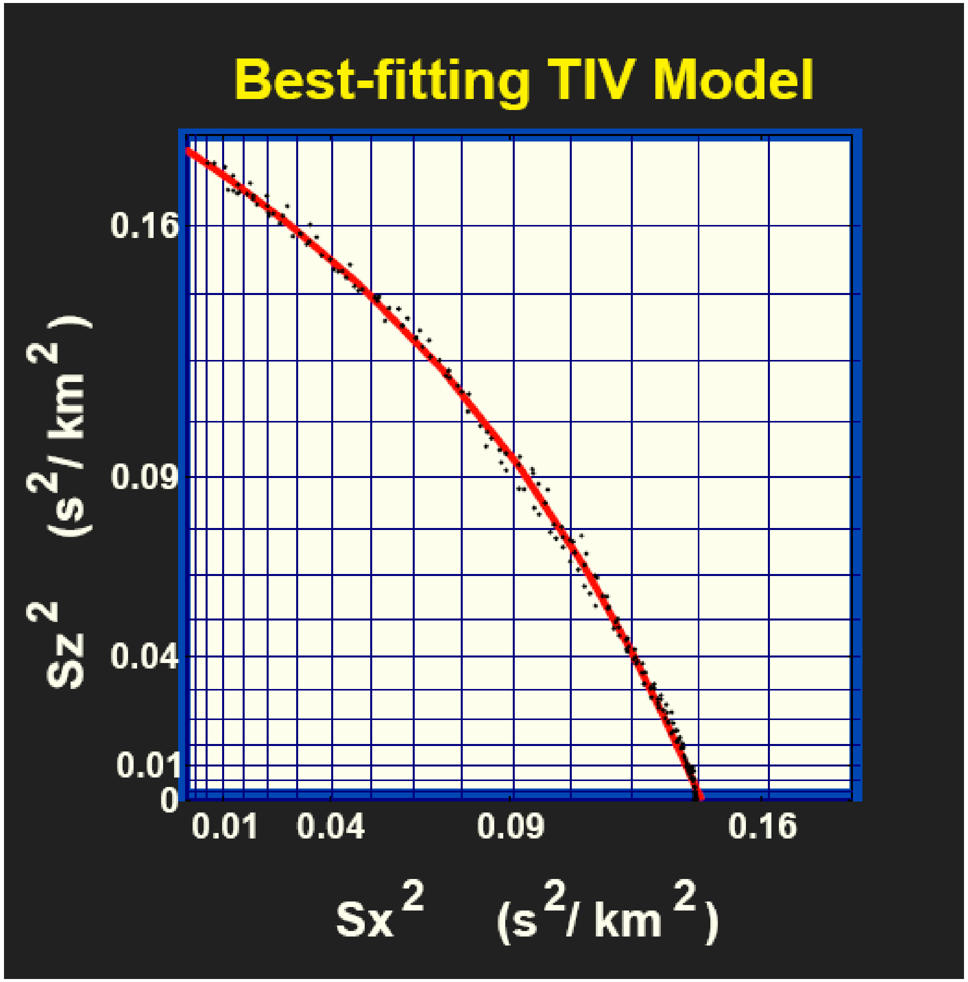


Fig. 2. Wave-front diagram.

Squared Phase Slowness







N.B.: Isotropy would require a line at 45°

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TI Parameters from Phase Slowness:

Let $X = S_x^2$, $Z = S_z^2$, and

$$A = A_{11}A_{33} + A_{55}^2 - (A_{13} + A_{55})^2.$$

Assuming a TI medium, the Christoffel relation can be written:

$$\boxed{A_{11}(A_{55}X^2 - X) + A_{33}(A_{55}Z^2 - Z) + AXZ = A_{55}(X + Z) - 1}$$

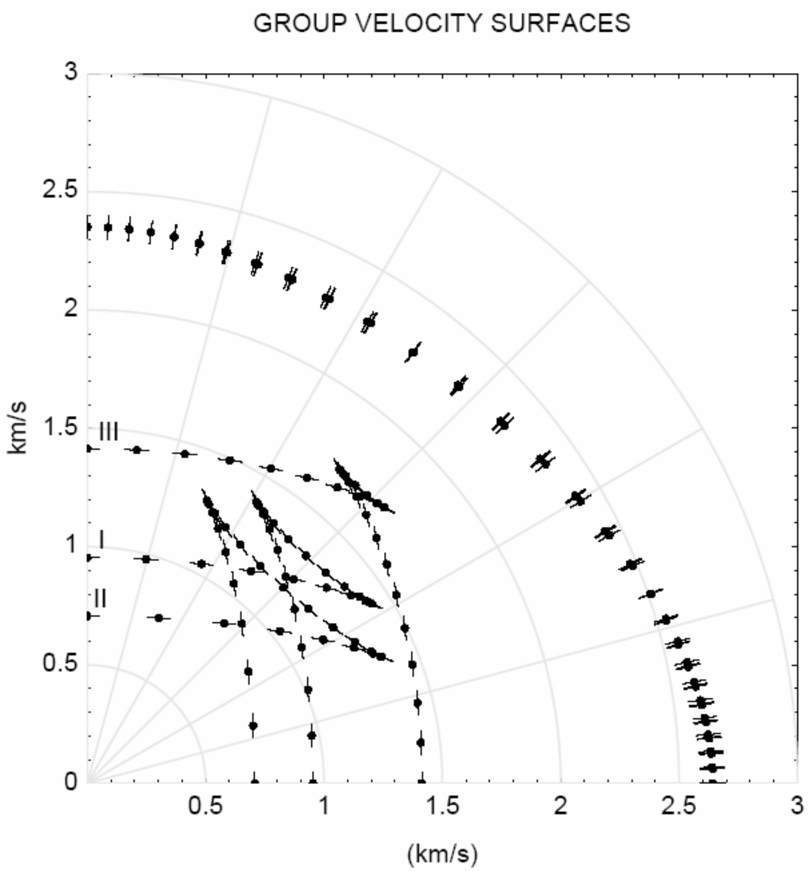
Given data points $\{X_i, Z_i\}$ and a choice of A_{55}, the above equation becomes a linear system to be solved for A_{11}, A_{33}, and A.

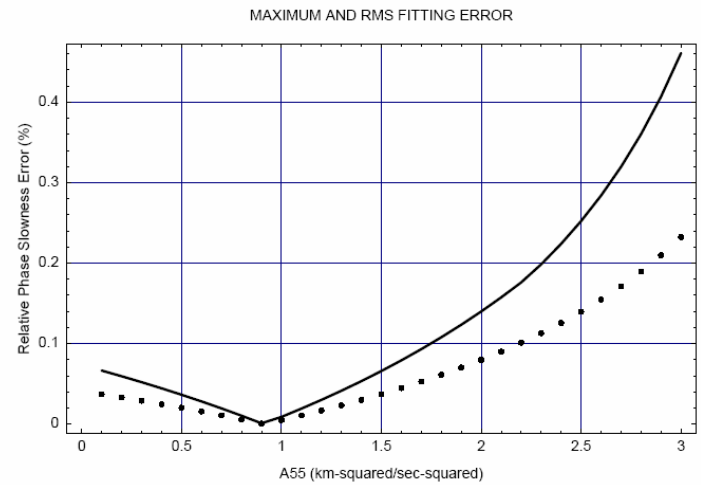
A is then solved for A_{13} assuming $A_{13} + A_{55} > 0$:

$$A_{13} = (A_{11}A_{33} + A_{55}^2 - A)^{.5} - A_{55}$$

Question: Can we optimize the fit as a function of A55 to determine all four "saggital" parameters from qP data only?

Question: Can we optimize the fit as a function of A_{55} to determine all four “saggital” parameters from qP data only?



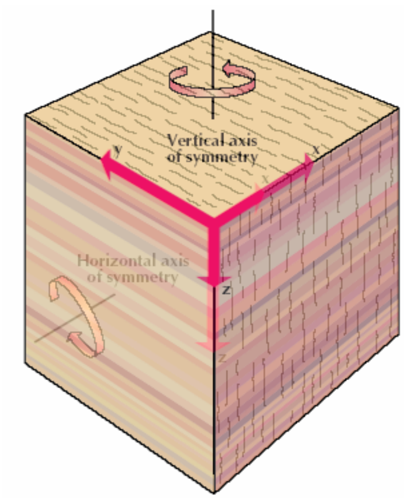


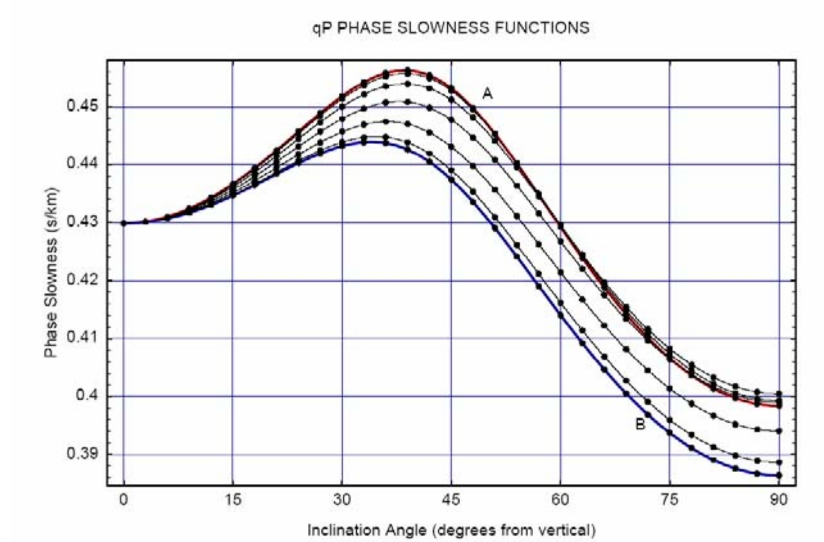
Answer: No

Question: Does a good TI fit to data from a single vertical plane imply that the medium has negligible azimuthal anisotropy?

- **Fractured TIV**

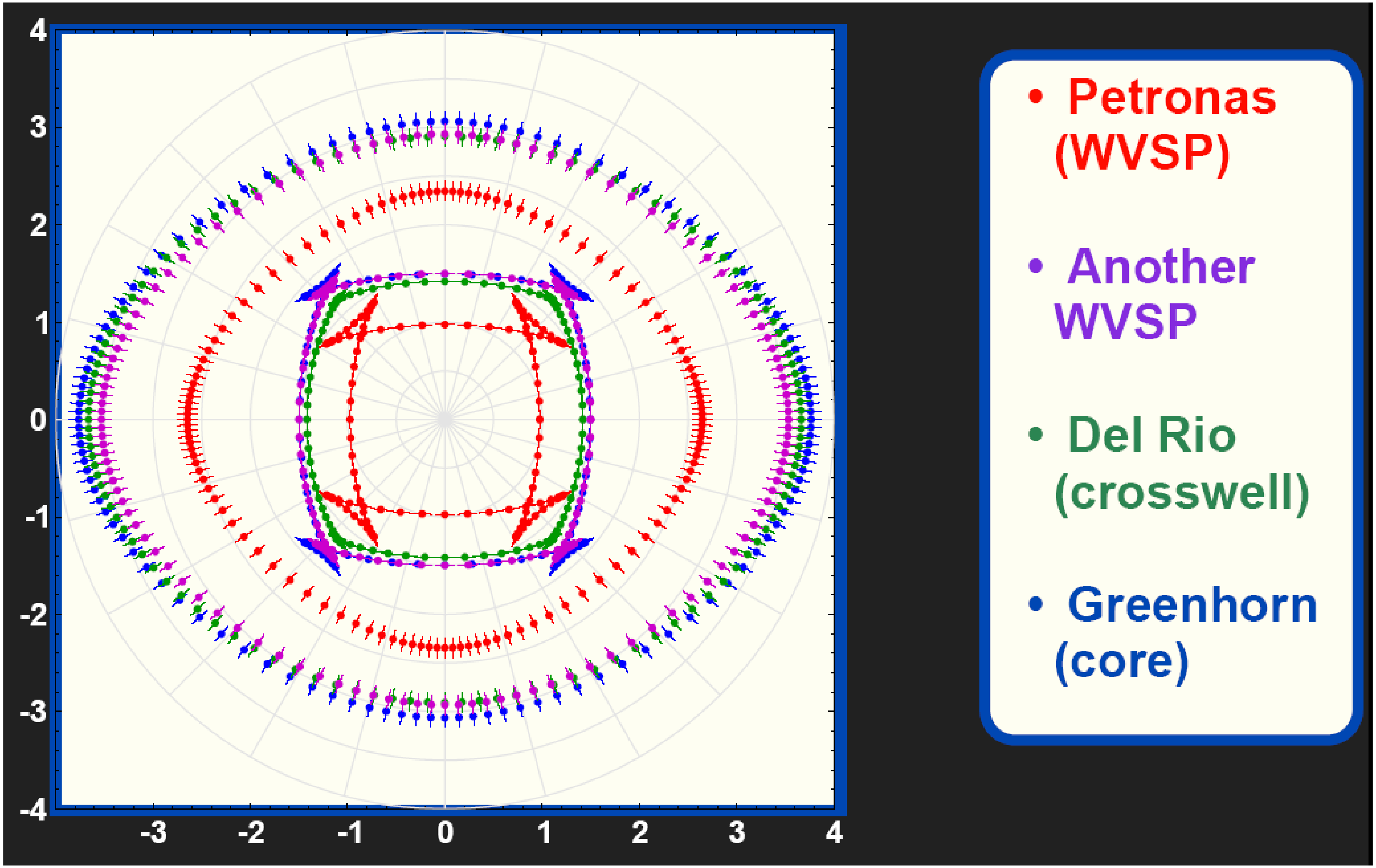
$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} c_{11} & \frac{c_{13}c_{22}-c_{23}c_{11}}{c_{23}-c_{13}} & c_{13} & & & \\ \frac{c_{13}c_{22}-c_{23}c_{11}}{c_{23}-c_{13}} & c_{22} & c_{23} & & & \\ c_{13} & c_{23} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{55} & \\ & & & & & c_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix}$$





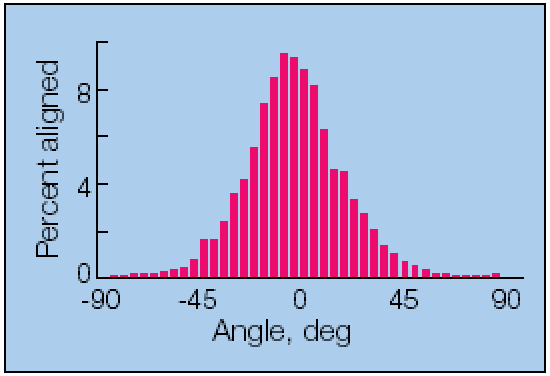
Answer: No

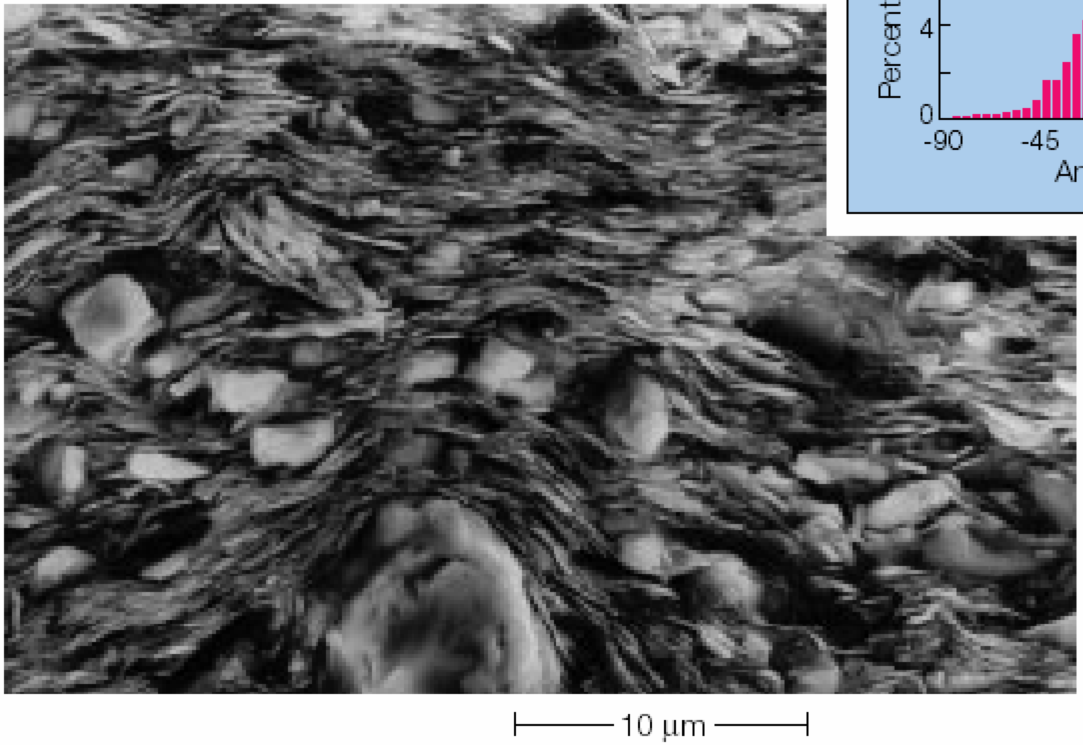
Question: Is this case typical?



Answer: It is not rare

Shale Morphology

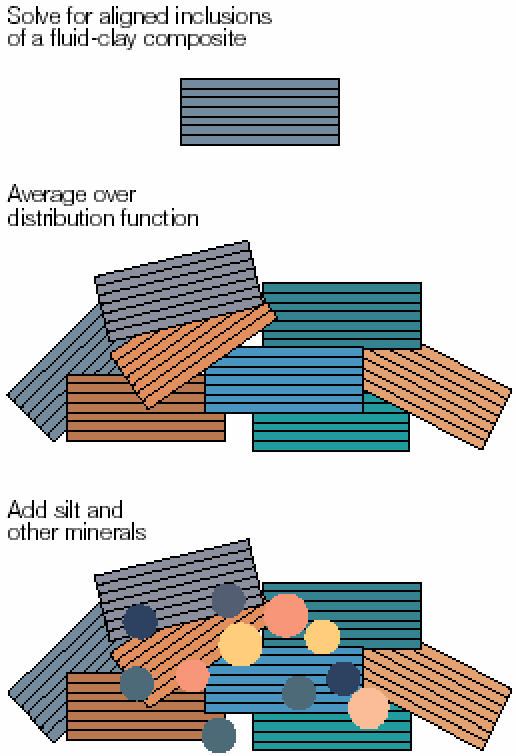


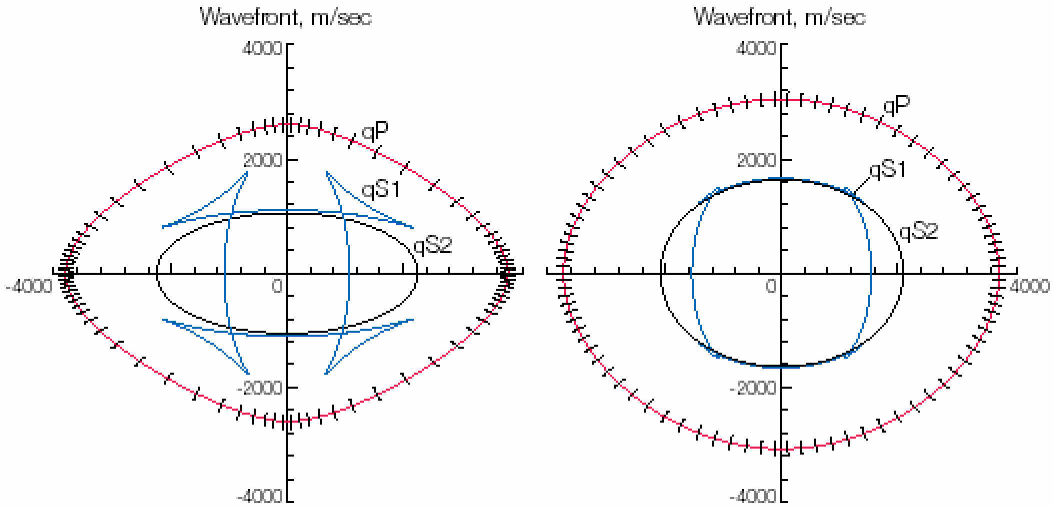


Photomicrograph of shale showing clay platelets distributed around the horizontal. Inset graph shows the distribution of the normal to the platelet, distributed around vertical.

Shale Model

N.B.: Think about excess horizontal shear compliance





□*Components of a shale model. Individual model clay platelets (top) are oriented according to the distribution measured in the shale photograph on previous page (middle). Silt particles are added (bottom) to resemble real shales.*

□*Wavefront velocities for synthetic shales. qP- and qS-wave velocities are computed for a shale with all clay platelets oriented horizontally (left). The shale synthesized with a realistic clay platelet distribution shows computed velocities (right) similar to those of the real shale depicted on the previous page.*

Hooke's Law Revisited

To get a unit of pure strain (assuming $\rho = 1$):

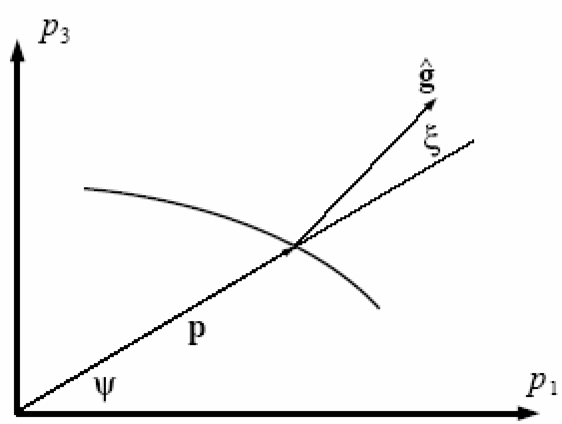
Mode	Direction	Stress
P	$0°$	A_{11}
P	$90°$	A_{33}
S	$0°$	A_{55}
S	$90°$	A_{55}
P	$45°$	$.25(A_{11} + A_{33} + 2(A_{13} + 2A_{55}))$
S	$45°$	$.25(A_{11} + A_{33} - 2A_{13})$

Perturbation Result (Chapman & Pratt, 1992)

$$\delta p \simeq -\frac{1}{2} p^3 \delta a_{ijkl}\, \hat{p}_i\, \hat{p}_l \hat{g}_j \hat{g}_k$$

$$= -\frac{1}{2} p^3 \left\{ \hat{p}_1^2 \hat{g}_1^2\, \delta A_{11} + 2 \hat{p}_1 \hat{p}_3 \hat{g}_1 \hat{g}_3\, \delta(A_{13} + 2A_{55}) + \hat{p}_3^2 \hat{g}_3^2\, \delta A_{33} + (\hat{p}_1 \hat{g}_3 - \hat{p}_3 \hat{g}_1)^2 \delta A_{55} \right\} .$$

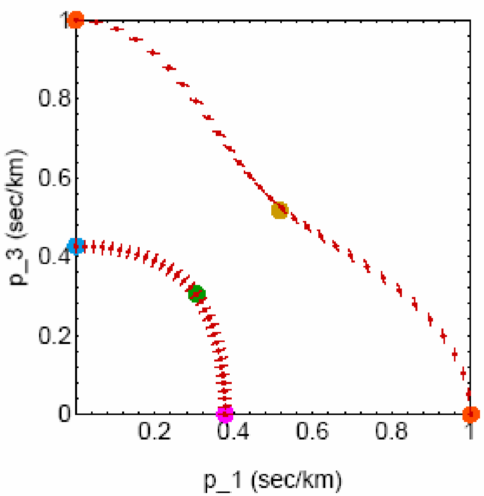
Analyze consequences of setting delta_ p = 0 under the approximation that phase and polarization vectors are parallel or orthogonal.

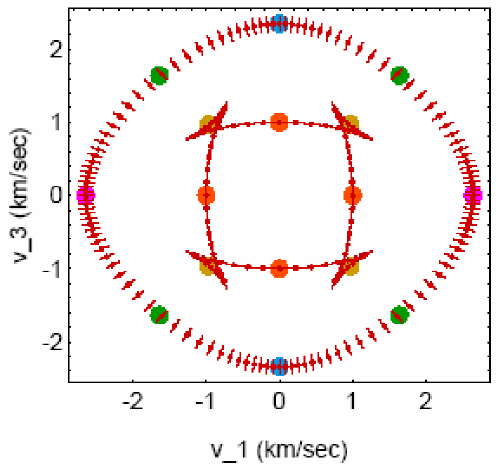


PushPin Parameters

P_{0°	A_{11}
P_{90°	A_{33}
S_{0°	A_{55}
S_{90°	A_{55}
P_{45°	$.25(A_{11} + A_{33} + 2(A_{13} + 2A_{55}))$
S_{45°	$.25(A_{11} + A_{33} - 2A_{13})$

If an arbitrary TI medium is perturbed in a way that preserves a given push-pin, then slowness points in the associated direction and mode will be approximately preserved in the new medium.





PushPin Parameters

P_{0°	A_{11}
P_{90°	A_{33}
S_{0°	A_{55}
S_{90°	A_{55}
P_{45°	$.25(A_{11} + A_{33} + 2(A_{13} + 2A_{55}))$
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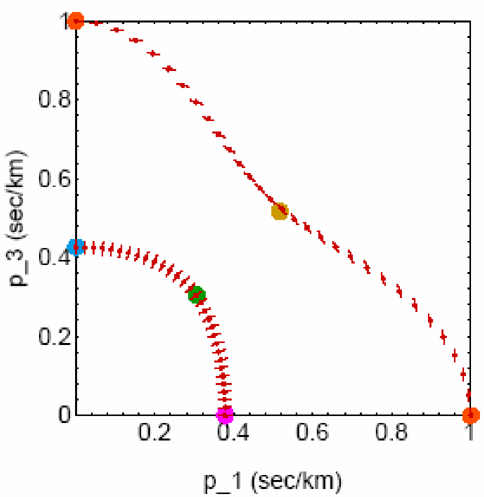

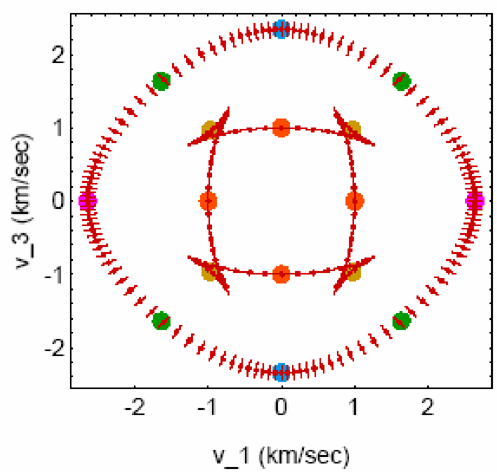

Thomsen Parameters

$$\varepsilon \equiv \frac{C_{11} - C_{33}}{2C_{33}};$$

$$\gamma \equiv \frac{C_{66} - C_{44}}{2C_{44}};$$

$$\delta \equiv \frac{1}{2}\left[\varepsilon + \frac{\delta^*}{(1 - \beta_0^2/\alpha_0^2)}\right]$$

$$= \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}.$$

TIV-Stressed Isotropic Medium (Bag of Marbles)

```
c11:=lambda+2 mu+e (2 nu (2 lambda+3 mu+A+4 B+2 C)-(lambda+2 B+2 C))

c33:=lambda+2 mu+e (2 nu (lambda+2 B+2 C)-(3 lambda+6 mu+2 A+6 B+2 C))

c55:=mu+e(2 nu (lambda+mu+A/4+B)-(lambda+2 mu+A/2+B))

c13:=lambda+mu+e (nu (lambda+mu+A/2+4 B+4 C)-(lambda+mu+A/2+3 B+2 C))-c55
c66:= mu + e (2 nu (lambda + 2 mu + A/2 + B) - (lambda + B))
```

```
Simplify[c11+c33-2(c13+2 c55)]
0
```


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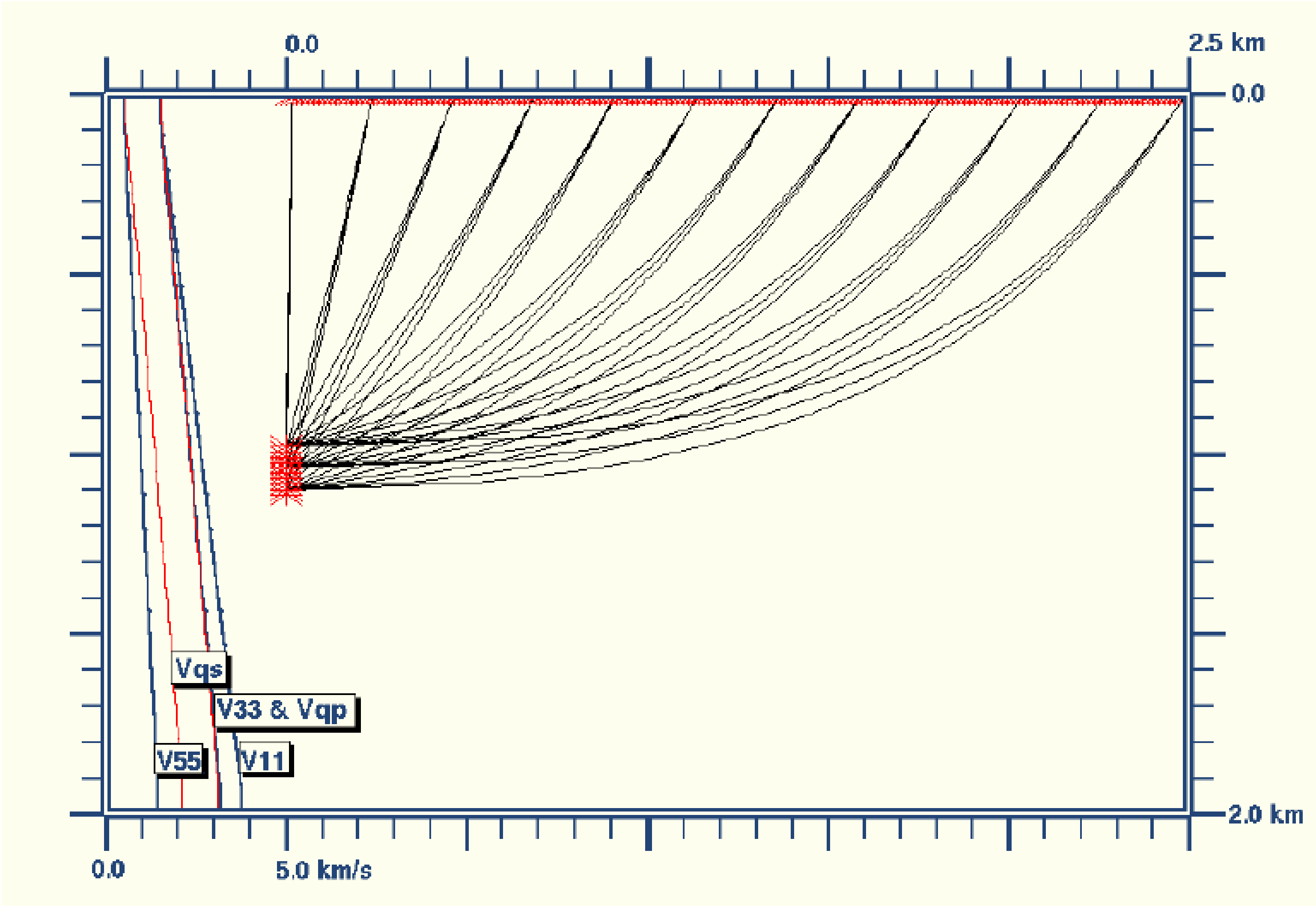
0.0
2.5 km
0.0
2.0 km
0.0
5.0 km/s
Vqs
V33 & Vqp
V55
V11

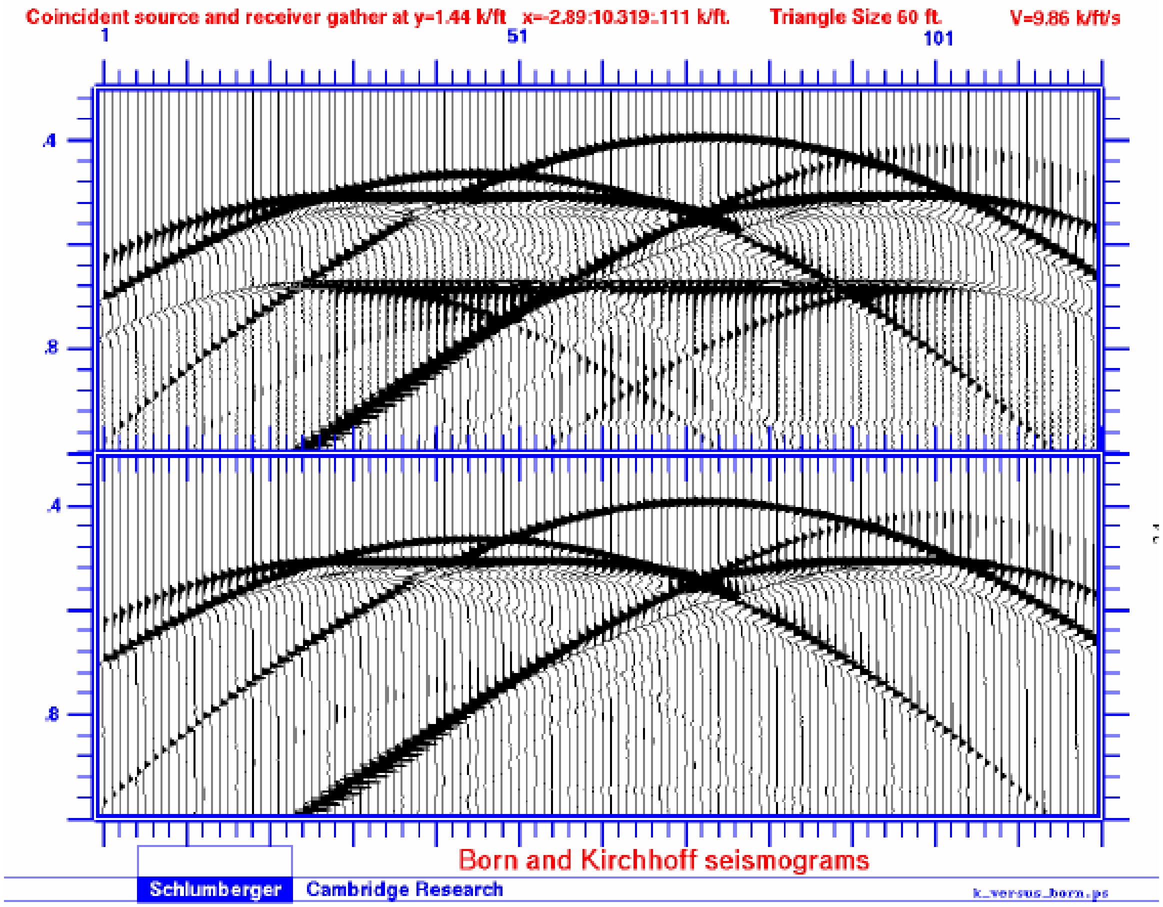

Figure 22: A coincident source and receiver profile over the French model calculated by Born volume integration (top) and Kirchhoff surface integration (bottom).

9 TI Zero-offset GRT Migration/Inversion

9.1 GRT inversion formula:

$$\langle f(\mathbf{x}_o)\rangle = \frac{1}{\pi^2}\int d^2\xi(\mathbf{s},\mathbf{x}_o)\,\frac{|\beta(\mathbf{s},\mathbf{x}_o)|^3}{A(\mathbf{s},\mathbf{x}_o)^2}\;u_{sc}(\mathbf{s},t=\tau_o).$$

9.2 Simplification

$$d^2\xi\frac{\beta^2}{A^2(\mathbf{s},\mathbf{x}_o)} = ds_1\,ds_2\;\cos(\alpha)$$

where α is the vertical phase angle at the surface.

9.3 What I Calculated:

$$\int ds_1\,ds_2\;\cos(\alpha)\,|\beta(\mathbf{s},\mathbf{x}_o)|\,u_{sc}(\mathbf{s},t=\tau_o)$$

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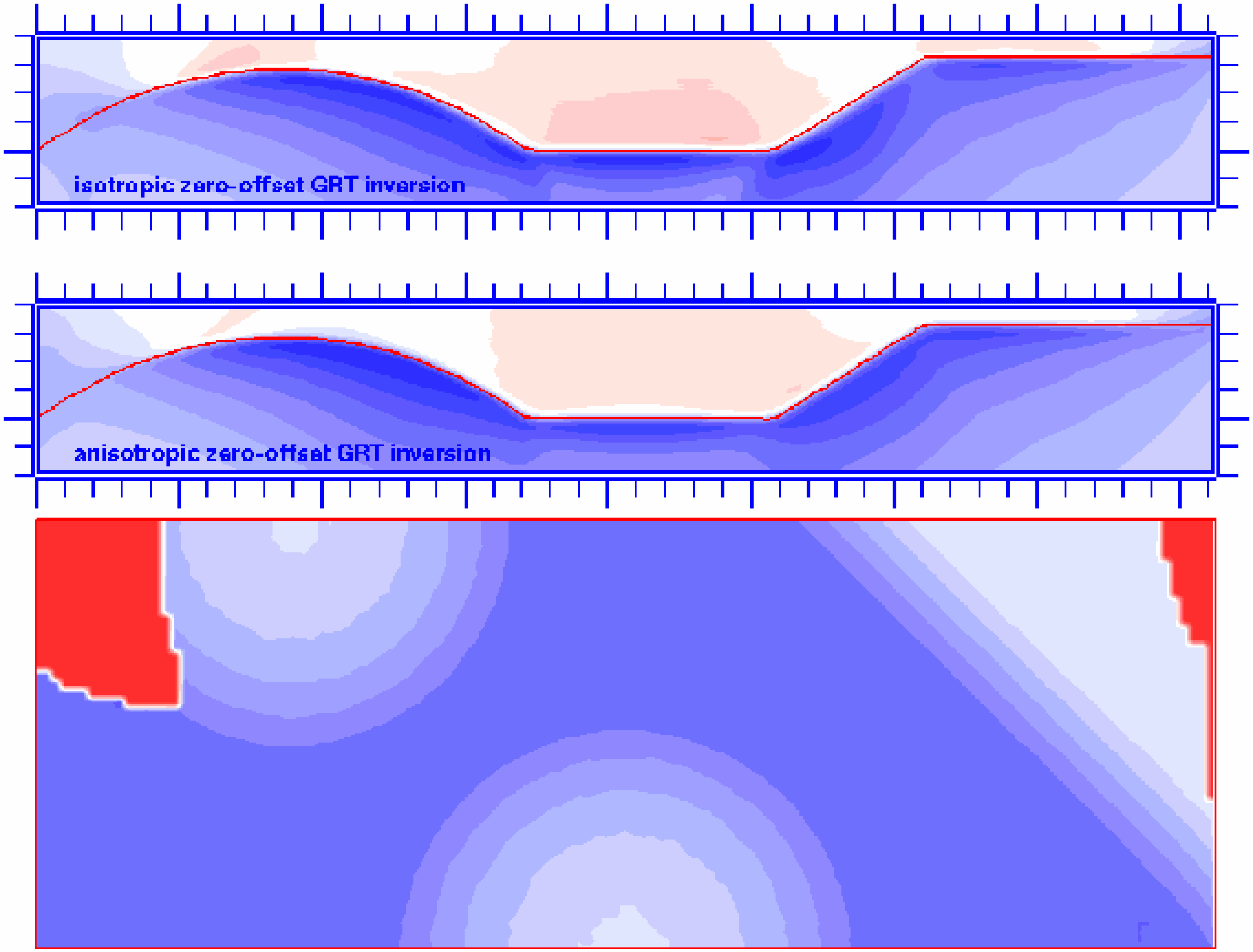
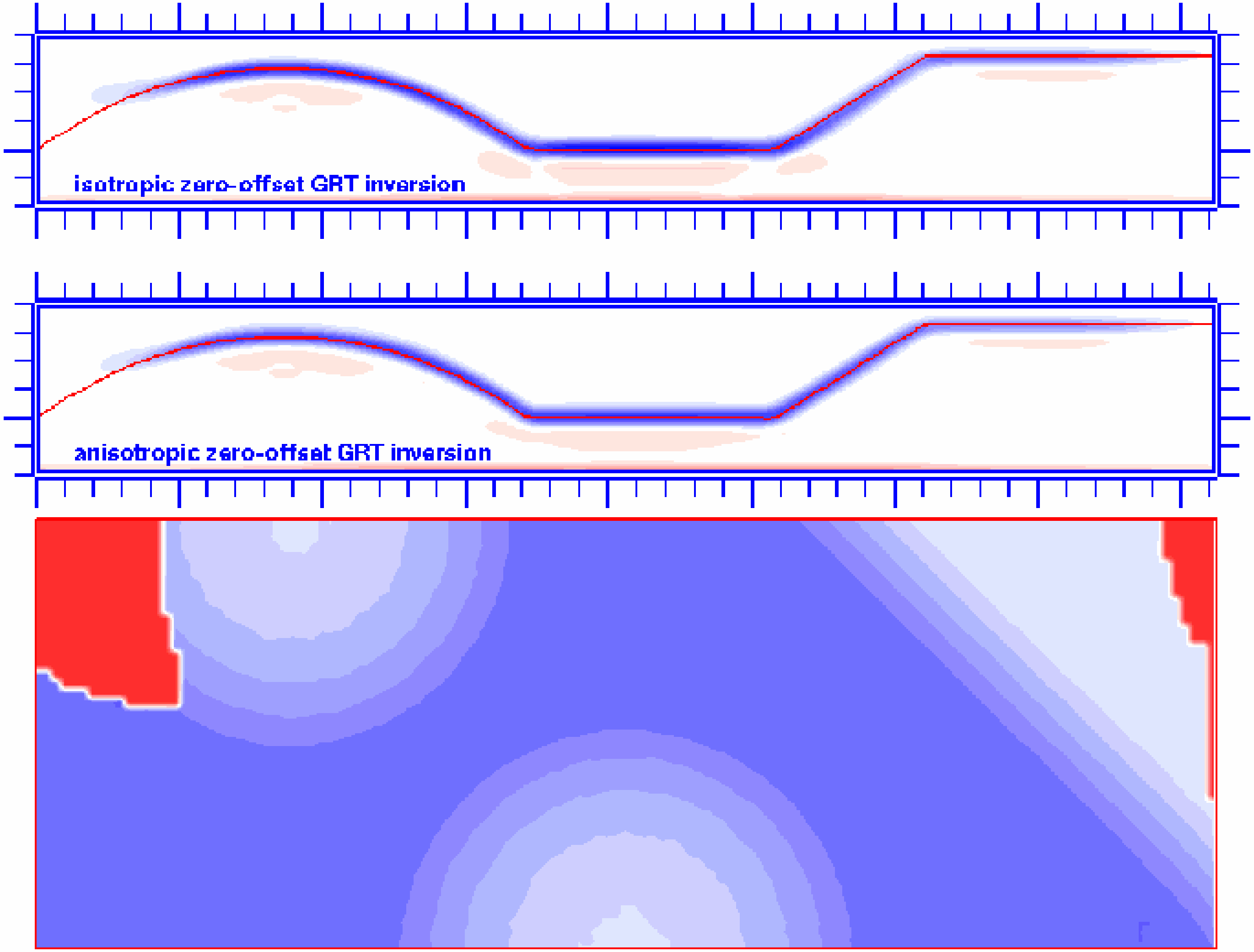
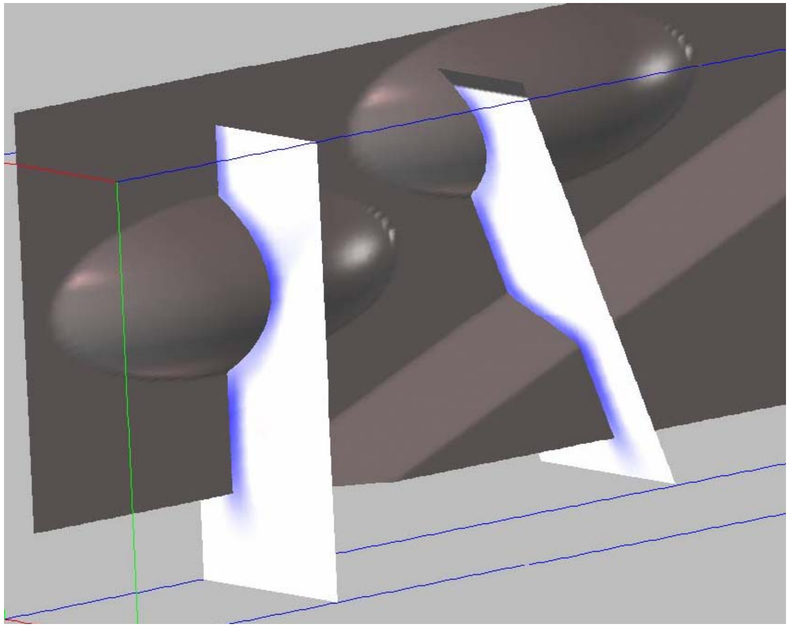
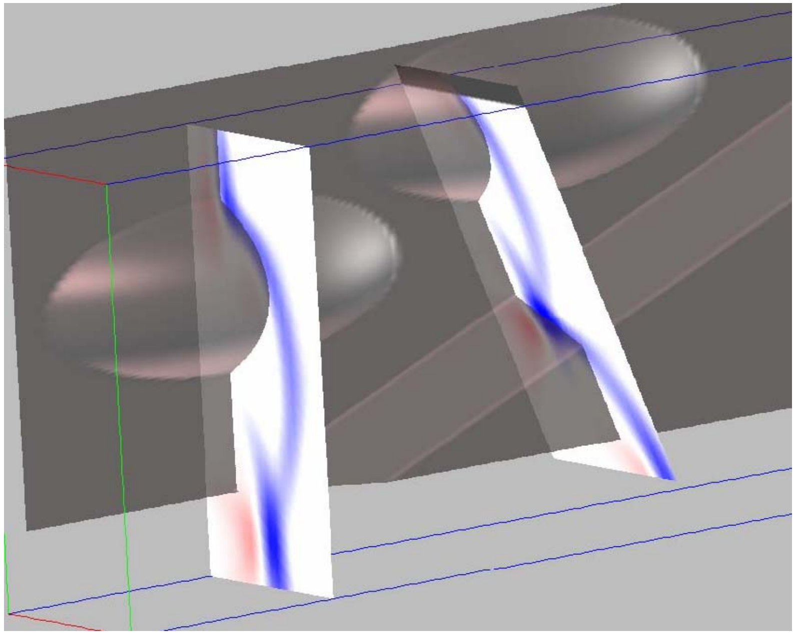

isotropic zero-offset GRT inversion
anisotropic zero-offset GRT inversion


isotropic zero-offset GRT inversion
anisotropic zero-offset GRT inversion

Turning-ray migration of Vertical Object



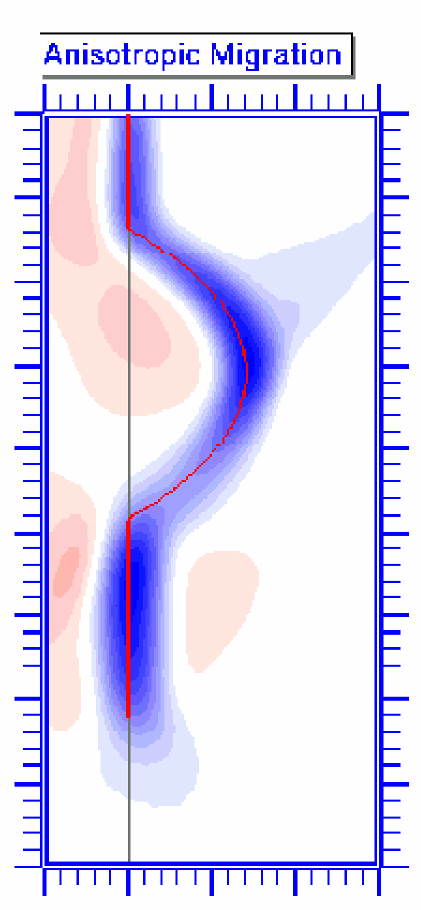
Anisotropic

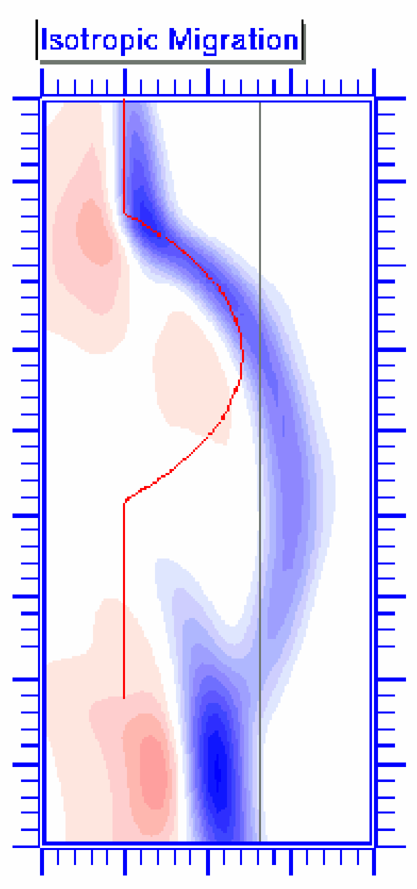


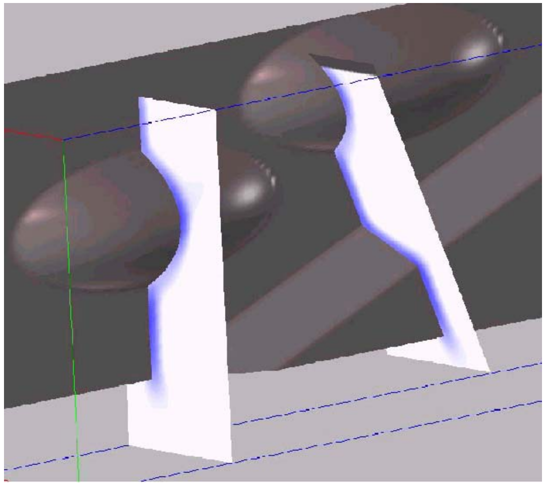
Isotropic

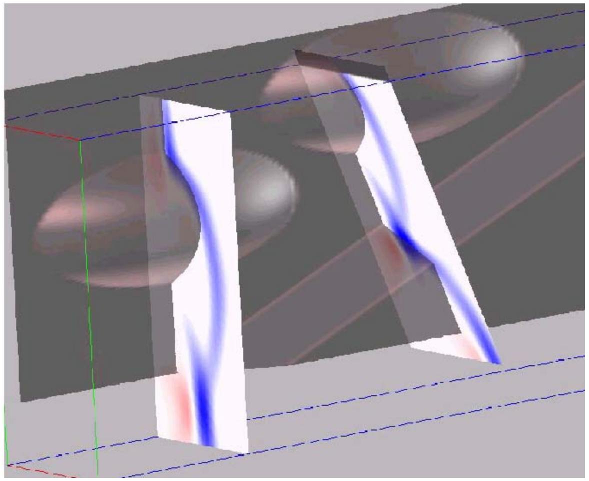
(vertical velocities)

Turning Ray Images









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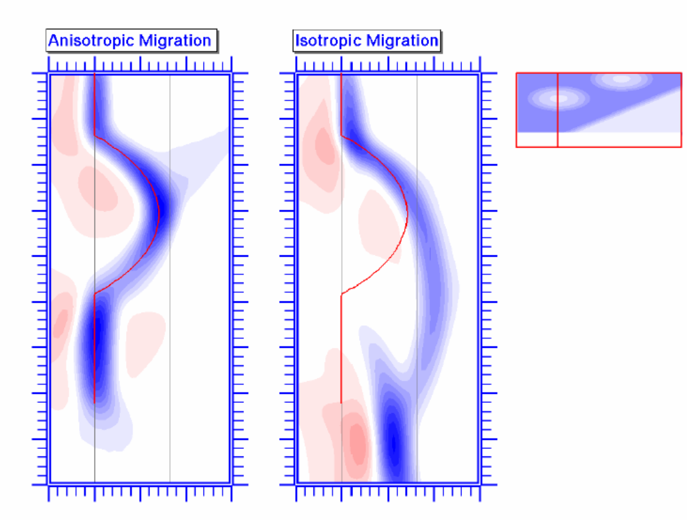
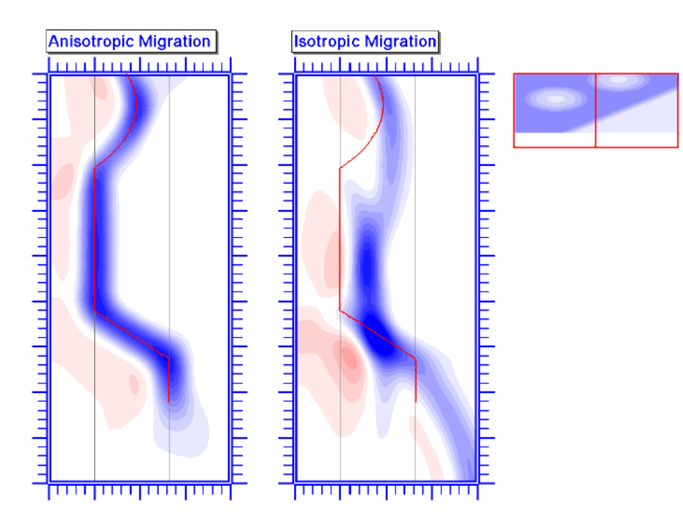
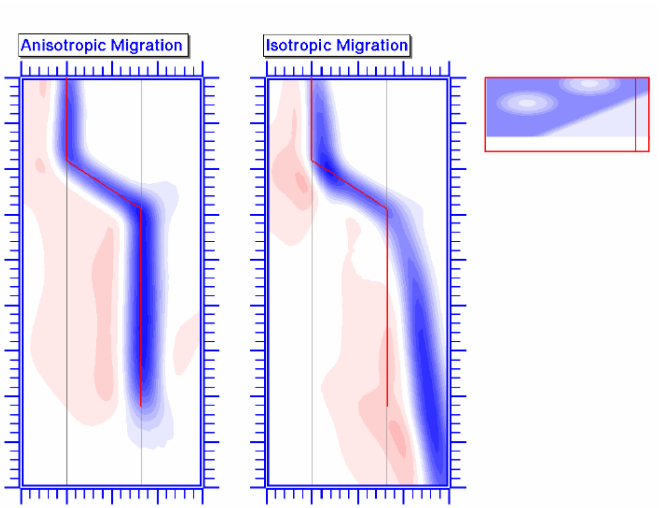
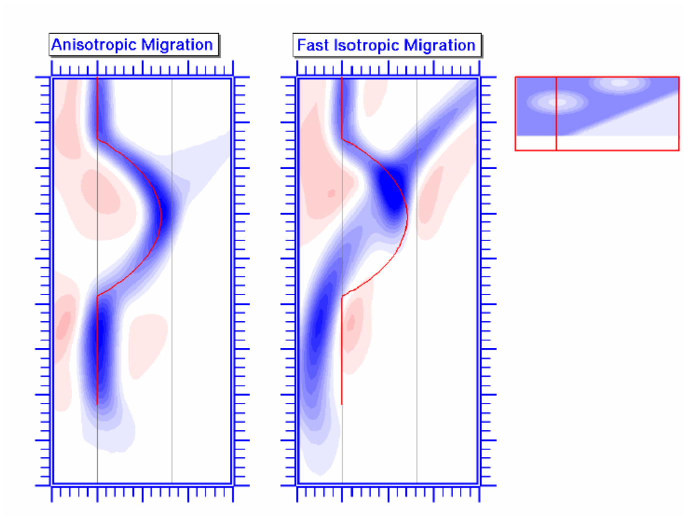
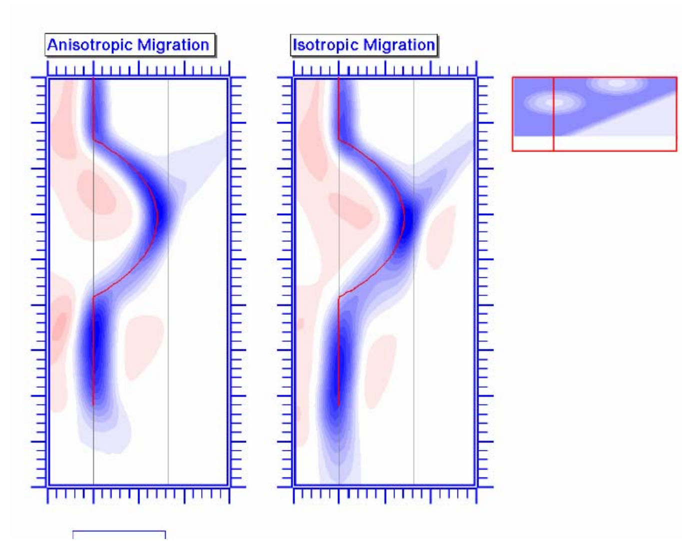


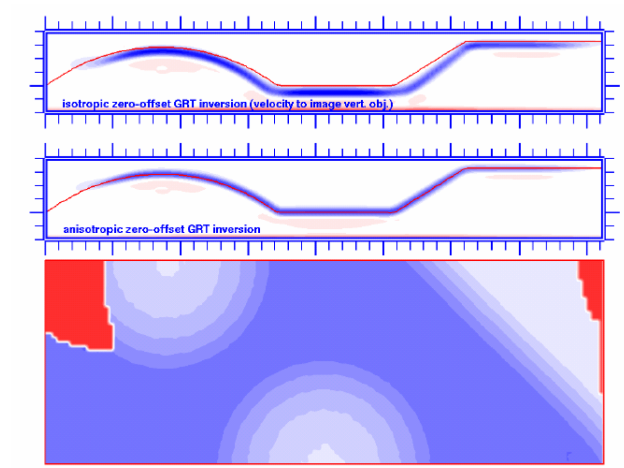


Isotropic Migration using vertical velocity profile systematically defocuses and mislocates vertical object









Isotropic Migration using a velocity profile that focuses the vertical object mislocates the horizontal object.

Local, interval VTI estimation

Phase method (Gaiser, 1990; Miller and Spencer, 1993)

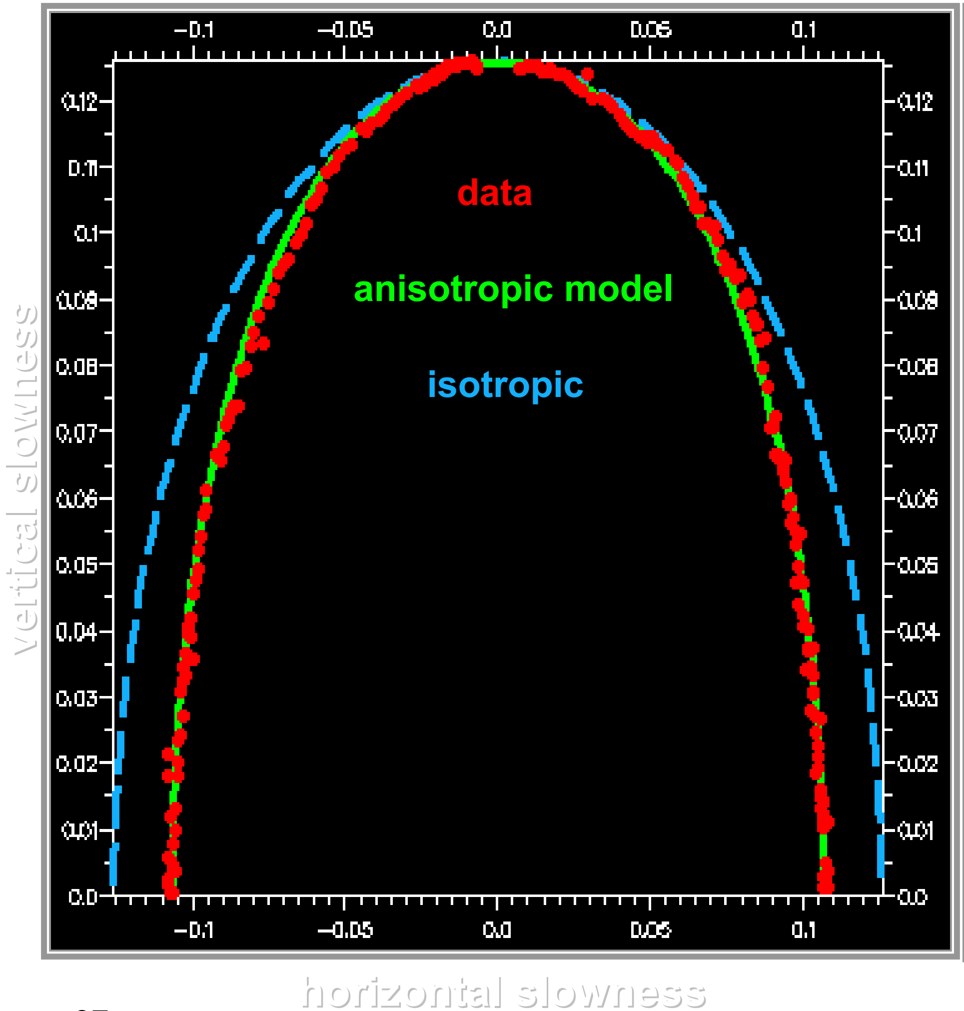
- Vertical and horizontal direct time derivatives yield phase slowness crossplot, fitting yields moduli
- Assumptions about overburden simplicity

Apparent Slowness + polarization method (de Parscau and Nicoletis, 1987; Hsu and Schoenberg, 1989; Horne and Leaney, 2000)

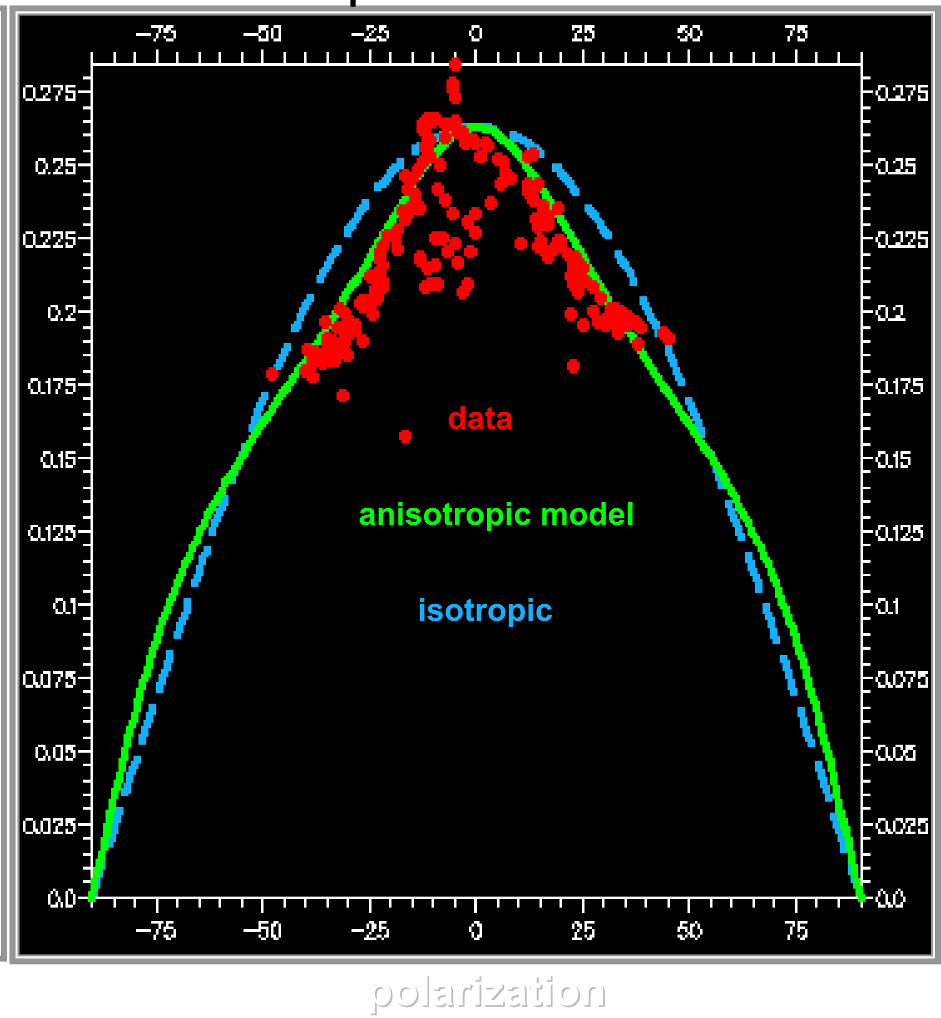
- Extraction of Sv and reflected parameters required picking
- Parametric waveform inversion (Leaney and Esmersoy, 1989) and downhole tools with sufficient vector fidelity have made it a commercially viable method.

Local VTI anisotropy:

Phase method



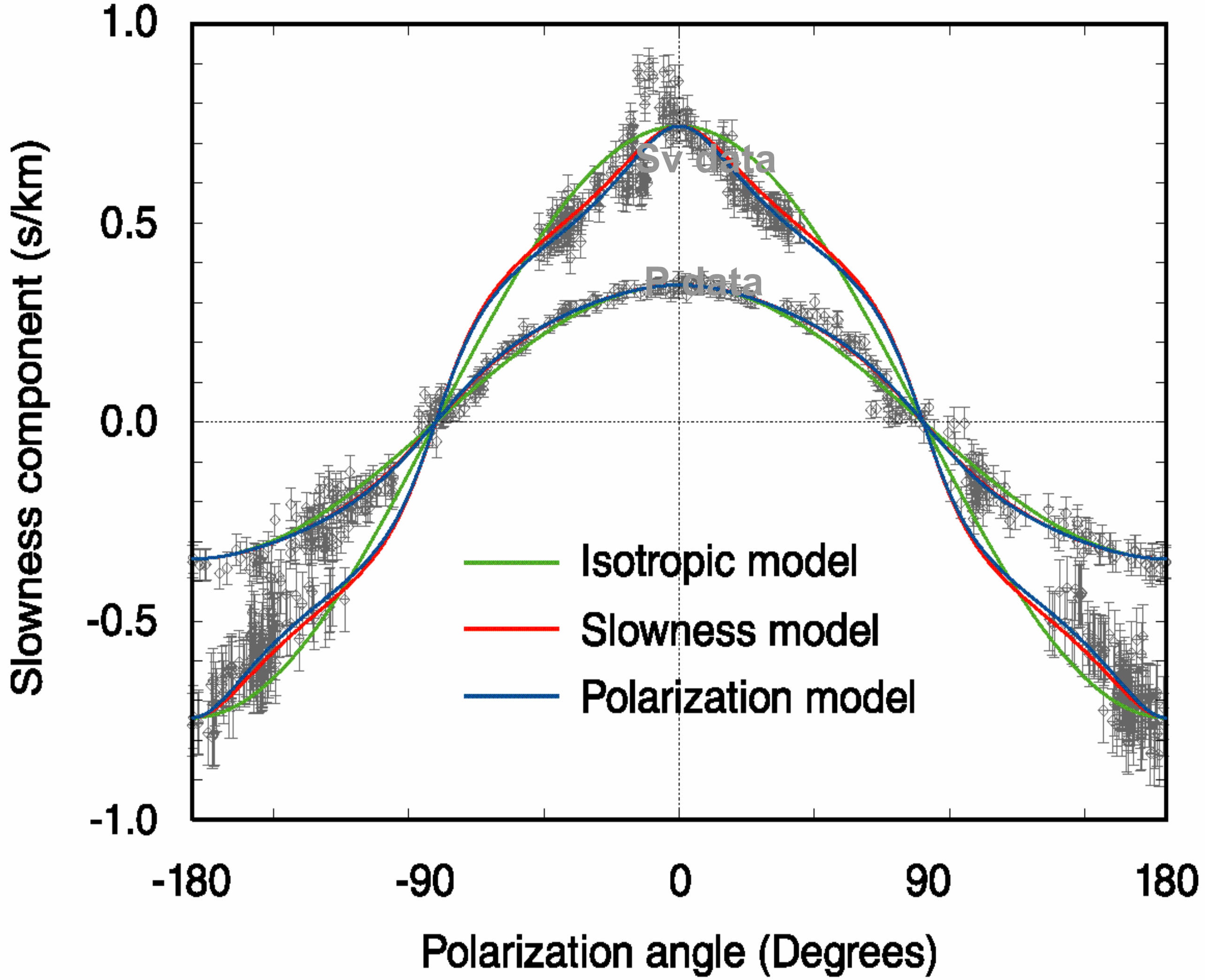
Slowness+polarization method



Better sensitivity to ε, ellip.

Better sensitivity to η, σ, anellip.

Comparison: phase slowness versus slowness+polarization



DM
21 Oct 05
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