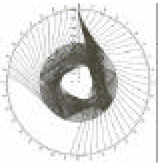
Things I learned from Chris (and friends): Anecdotes about Anisotropy

Douglas E. Miller

Schlumberger-Doll Research

Chapman Fest in Cambridge

4 May, 2007



Anecdotes about Anisotropy:

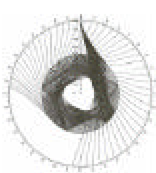
Occam's Razor Cuts Both Ways

Entities are not to be multiplied beyond necessity

- William of Ockham as paraphrased by John Ponce of Cork.

Entities must not be reduced to the point of inadequacy

- Walter of Chatton as paraphrased by Karl Menger.



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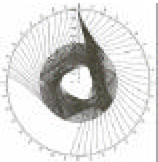
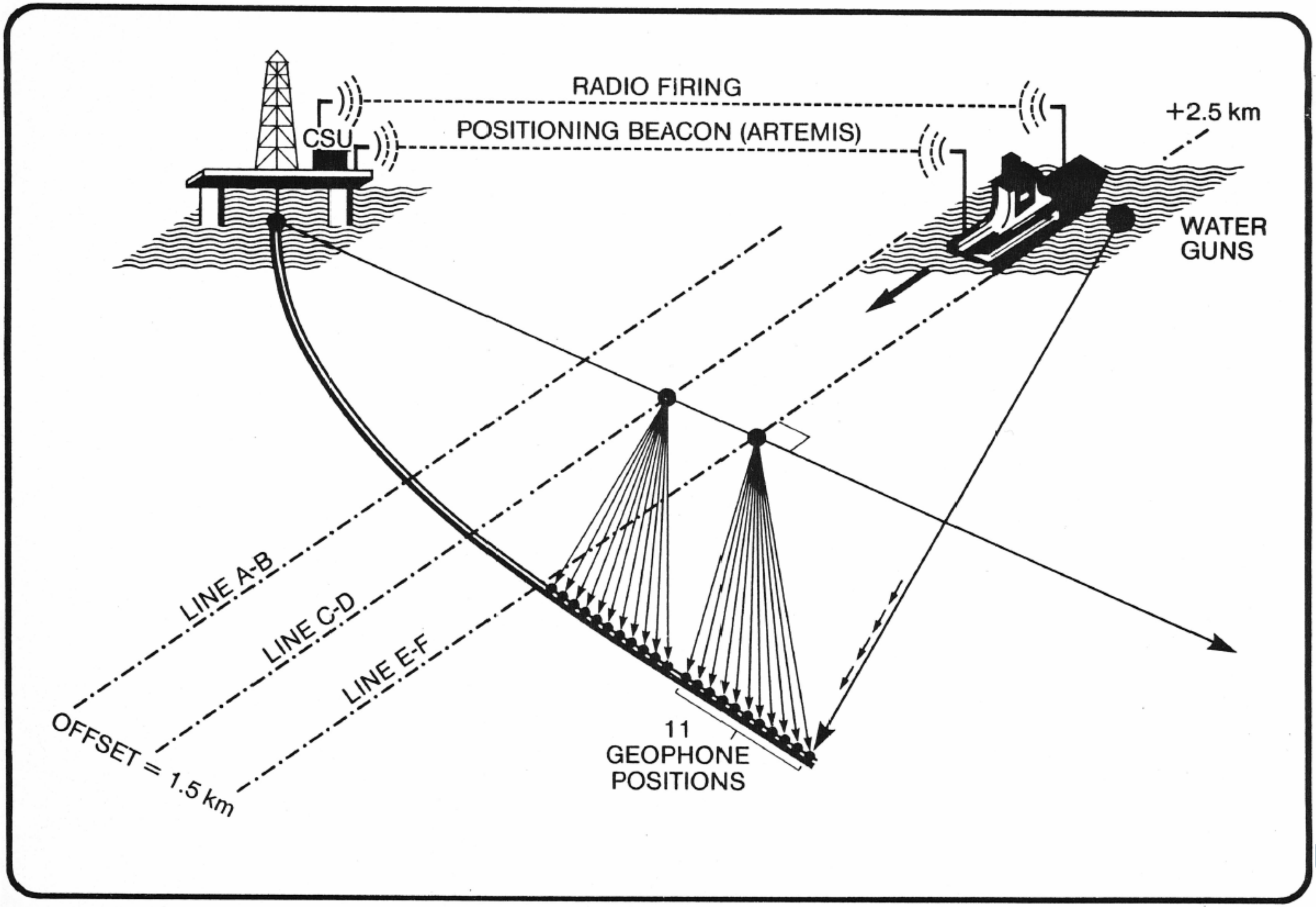
Shale Anisotropy - It's pretty obvious once you know where to look

- Prehistory: Traveltime Anomalies at Ekofisk 1983
- Crosswell Example
- Walkaway Vsp
- Alphabet Soup
- Exploding Reflectors and all that



Ekofisk Walkaway VSP - 1983



Something Fishy In FRAYTR ?

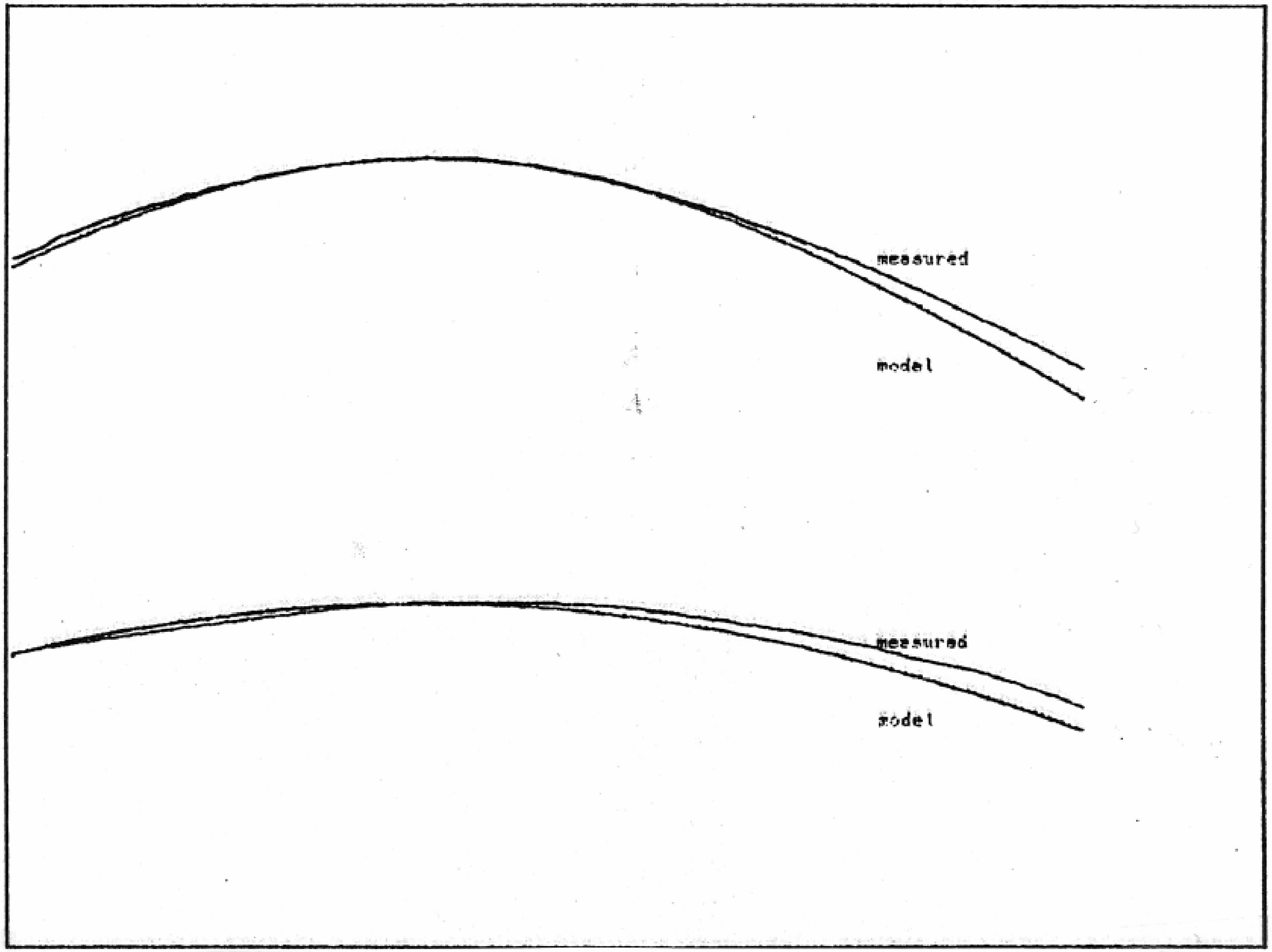
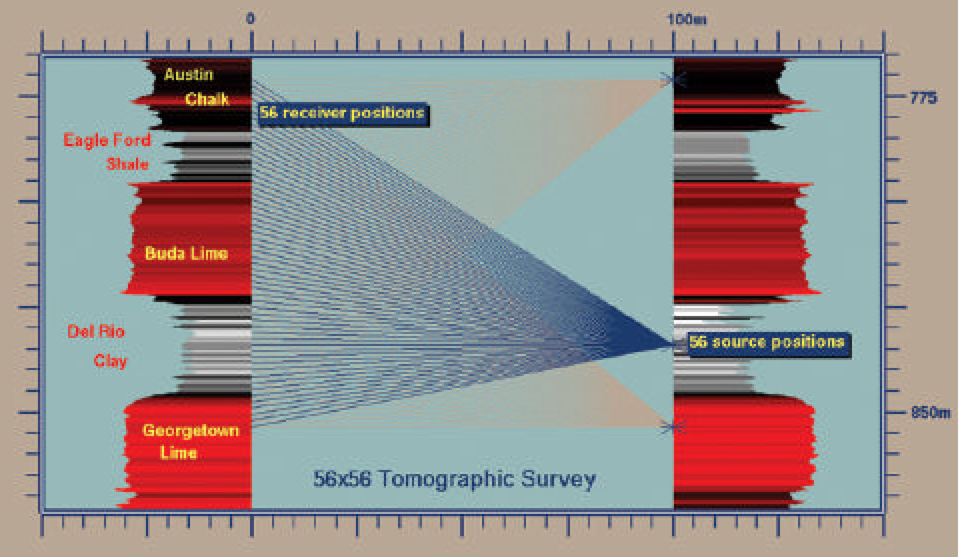


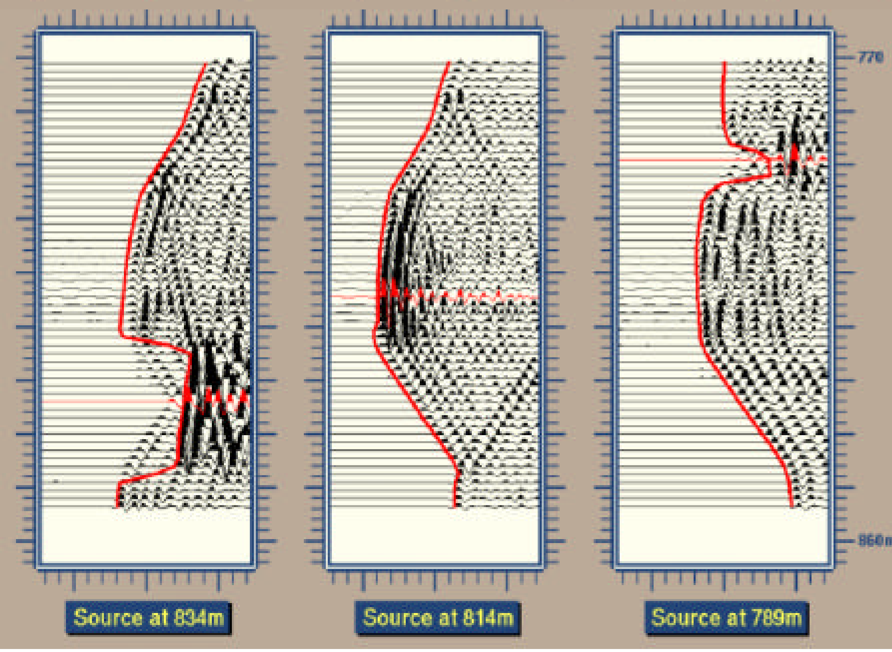
FIGURE 23. MEASURED AND MODELLED DIRECT AND REFLECTED ARRIVALS
ABSCISSA : OFFSET ORDINATE : RECORDED TIME.

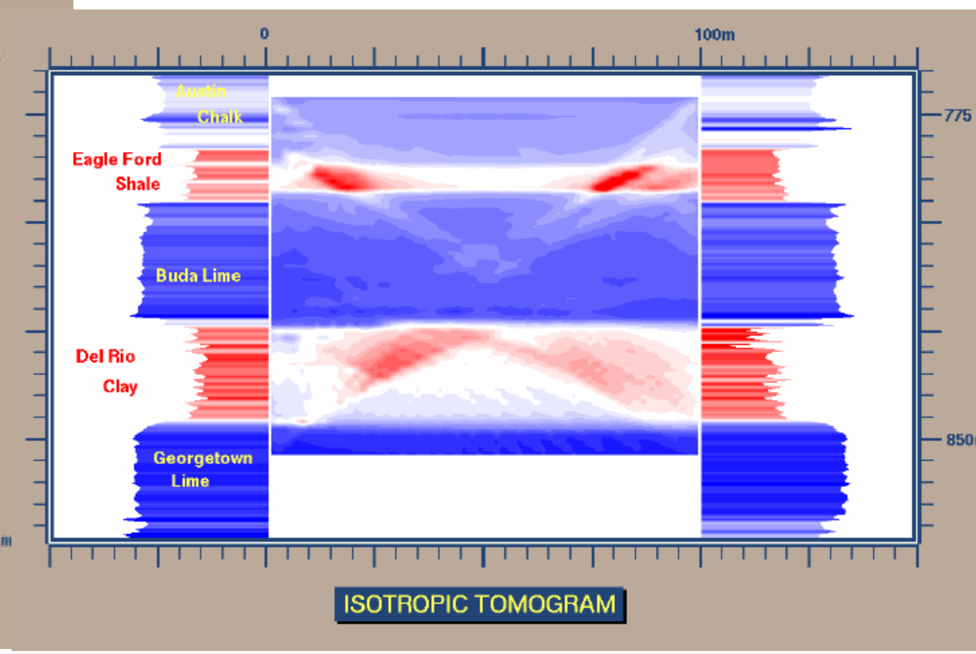
Crosswell Seismic Example

(Miller & Chapman 1991)

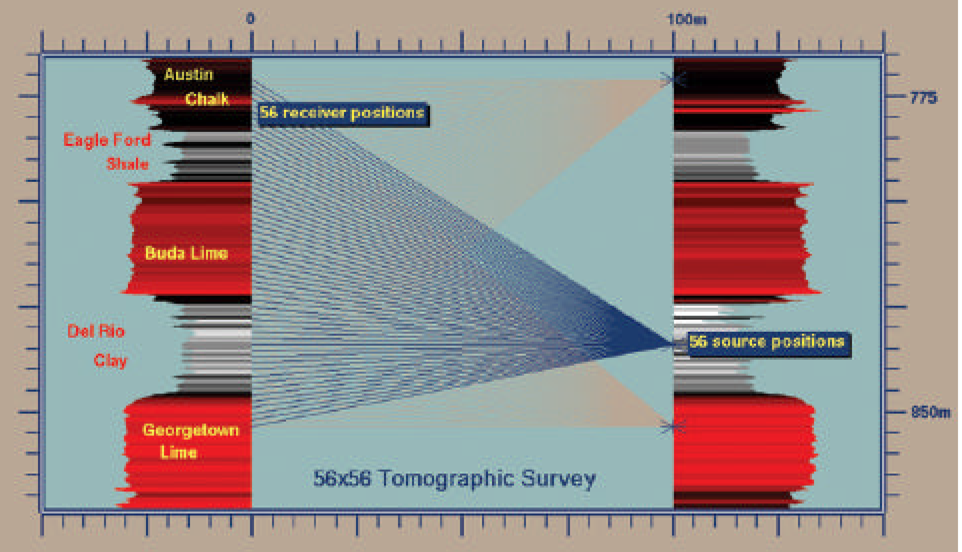


- **56 sources by 56 receivers**
- **Isotropic Solution with 56x56 parameters contains a big X (a.k.a. artifact)**

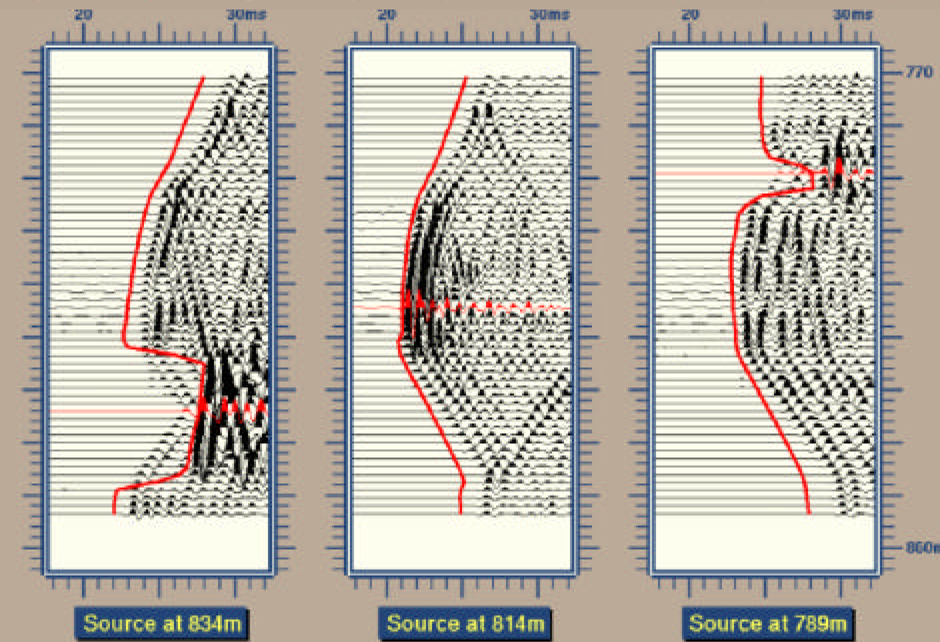


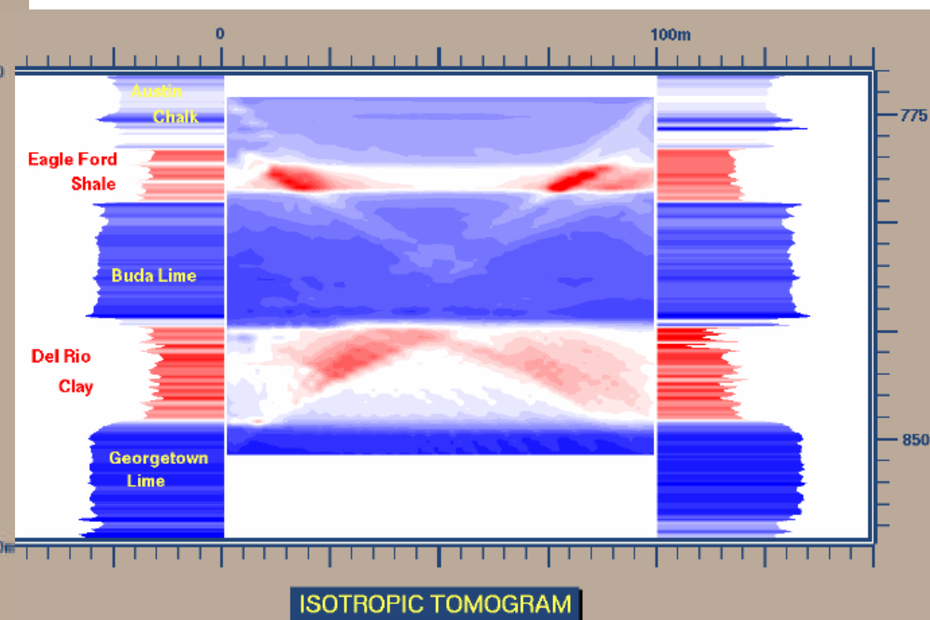


Crosswell Seismic Example (Miller & Chapman 1991)



- 56 sources by 56 receivers
- BP test site Devine, Texas

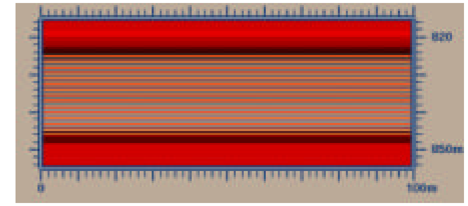


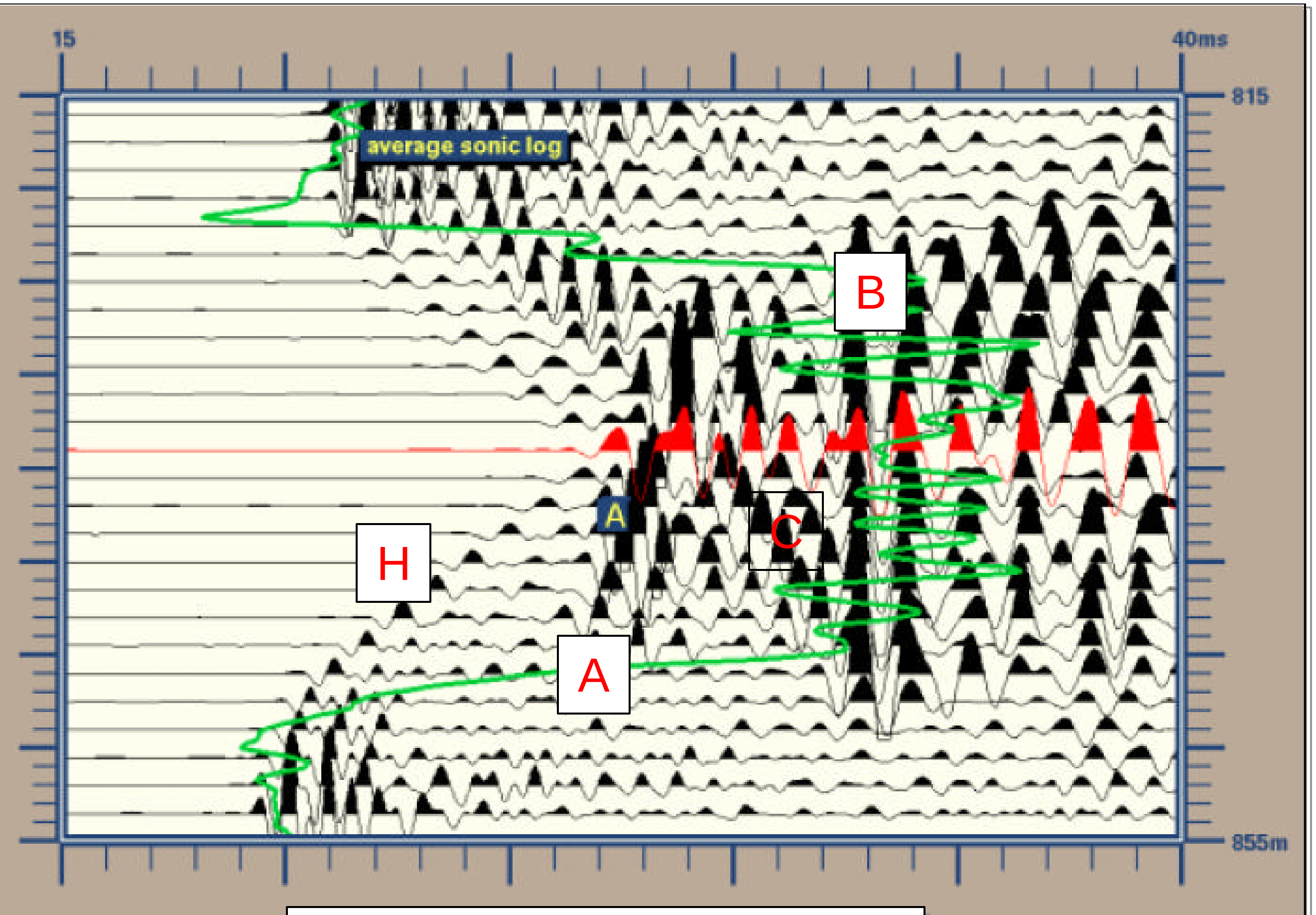


- **Isotropic Solution with 56 parameters (layers) has large residuals**

- **Isotropic Solution with 56x56 parameters contains a big X (a.k.a. artifact)**

Puzzling Observations:

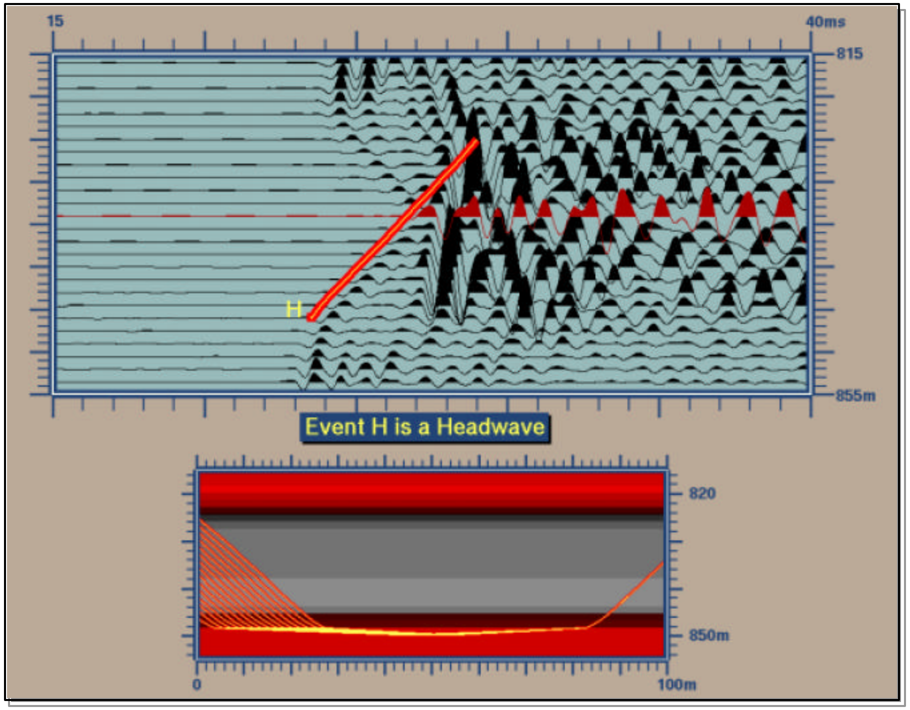


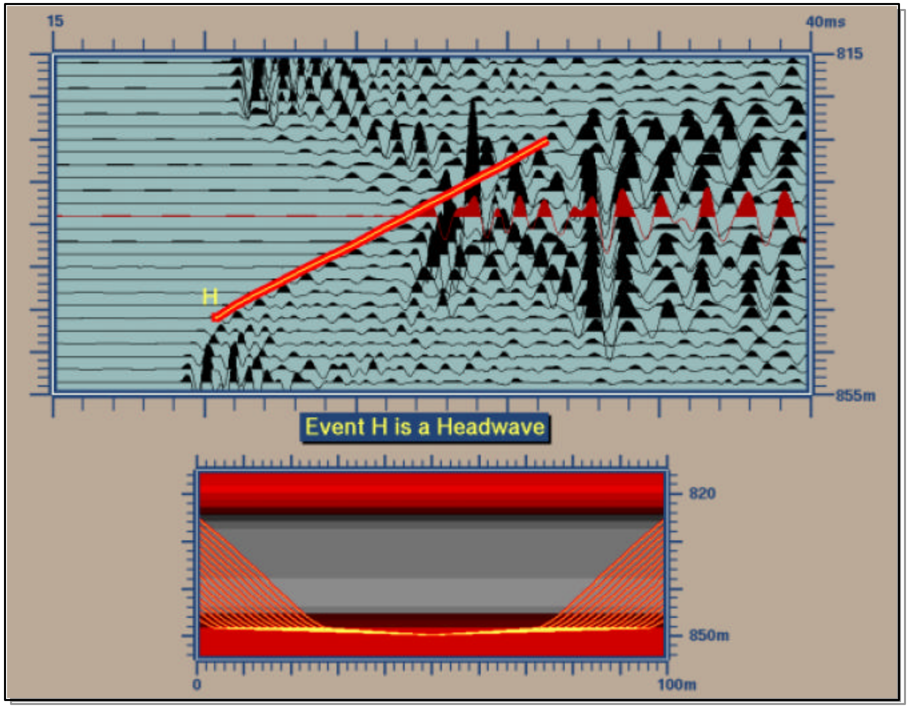


Common Depth Gather

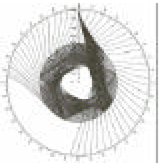
- **Event H is clearly a headwave**
- **Event B arrives at the time predicted by the log.**
- **If B is the direct arrival, what is A?**
- **If A is the direct arrival, what are B and C?**
- **Why is B so straight and so sharply terminated?**

Observations Clarified With Raytracing:

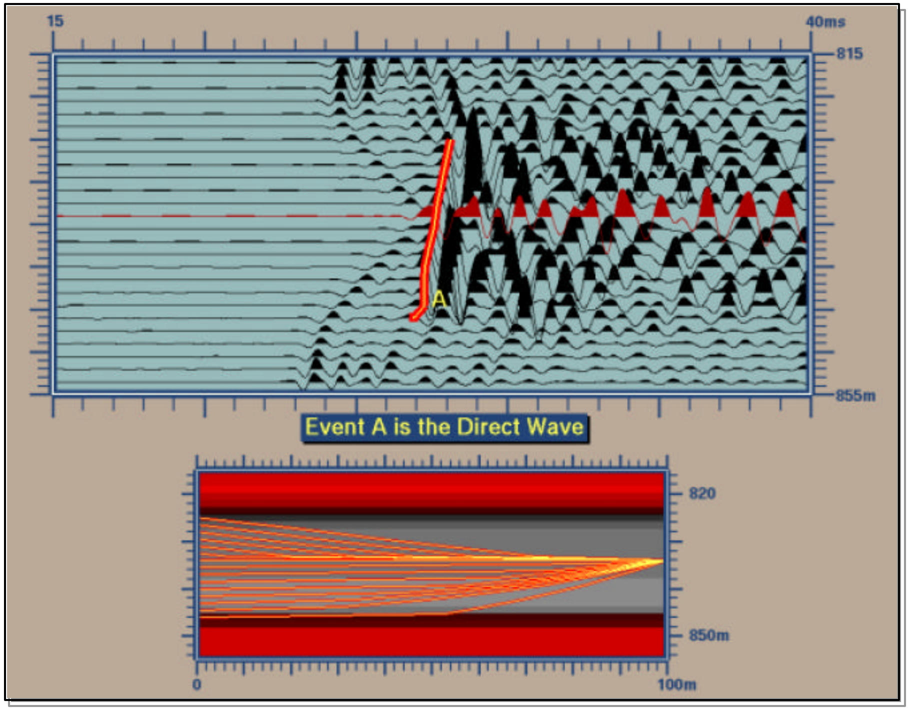


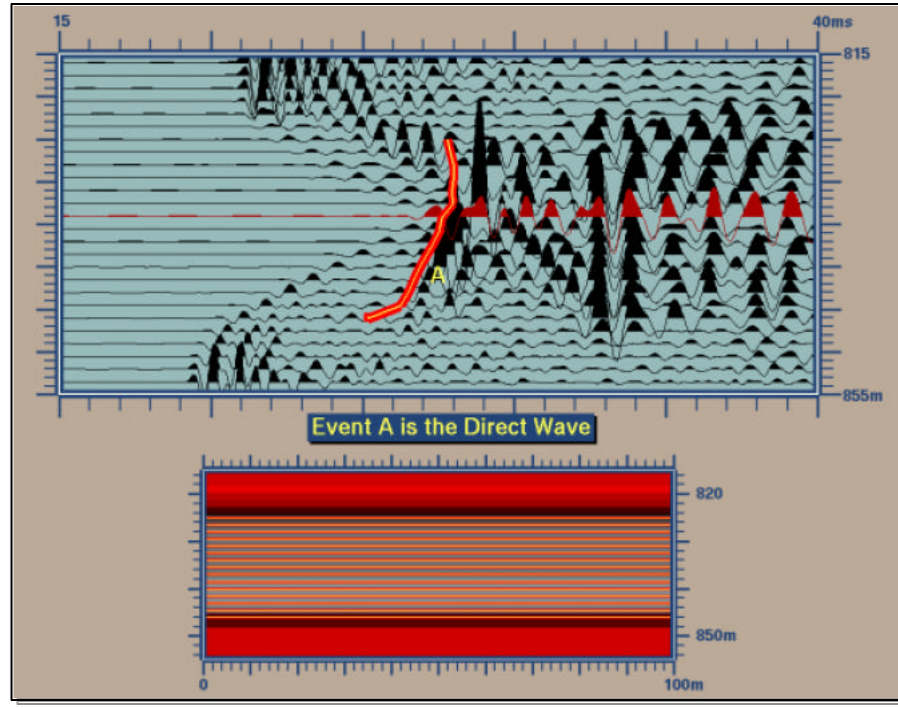


H is a headwave



Observations Clarified:





A is the direct wave

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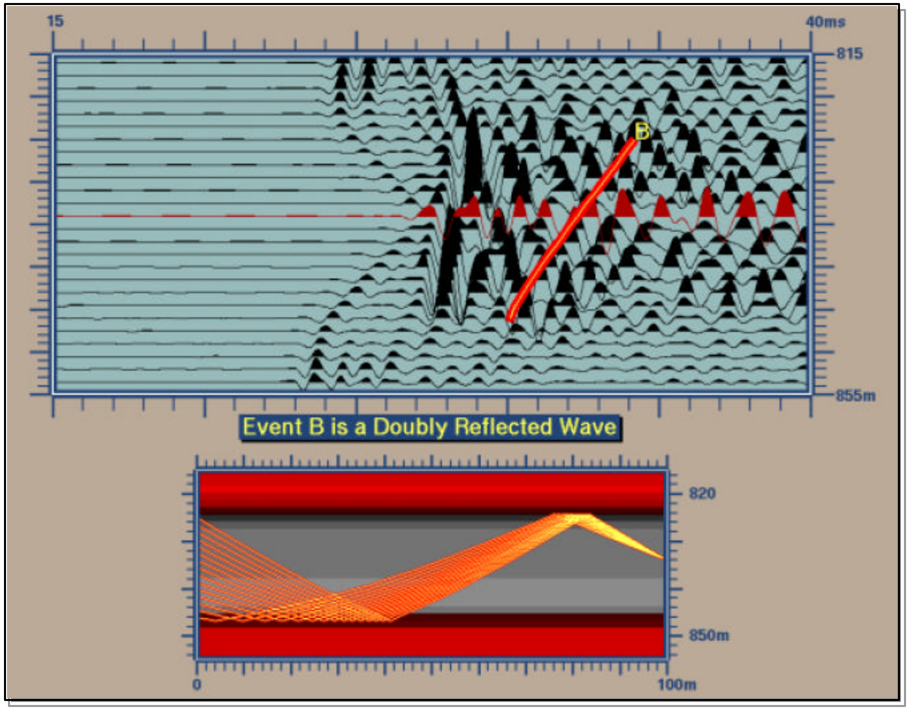
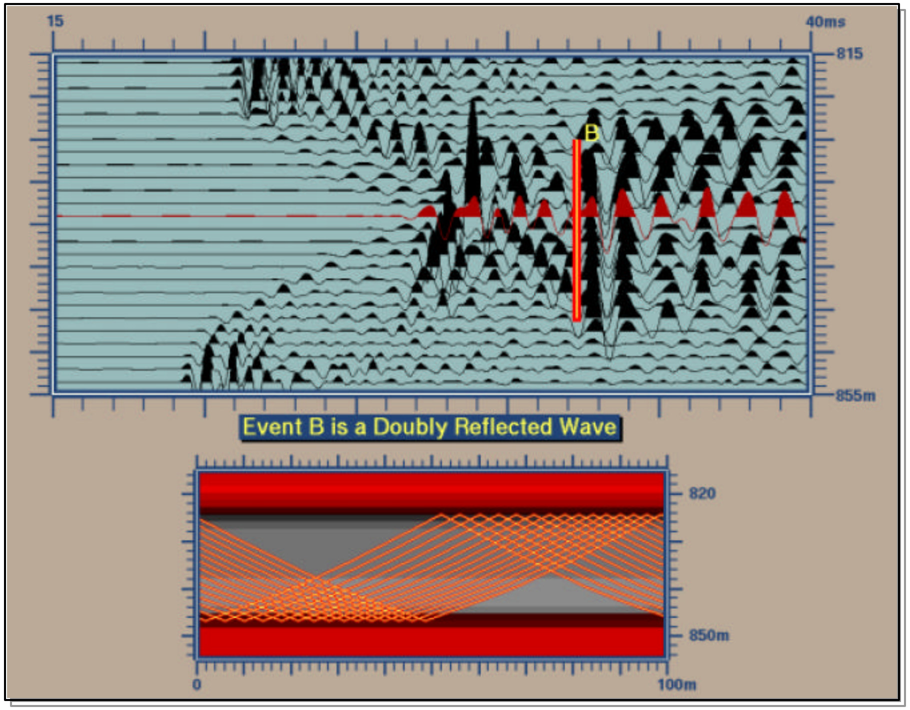
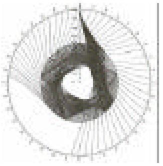
Observations Clarified:



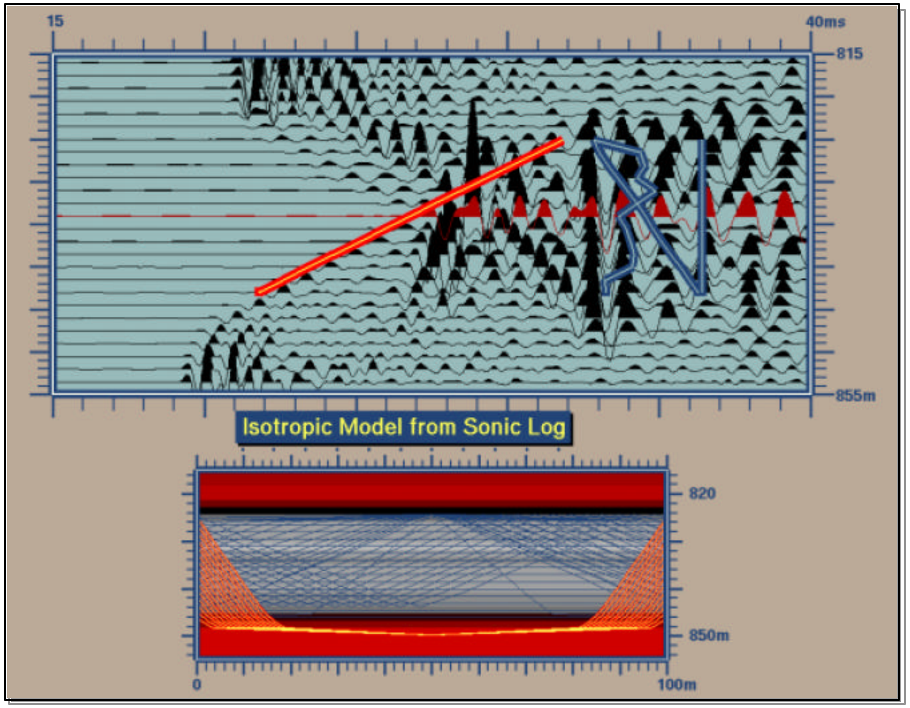


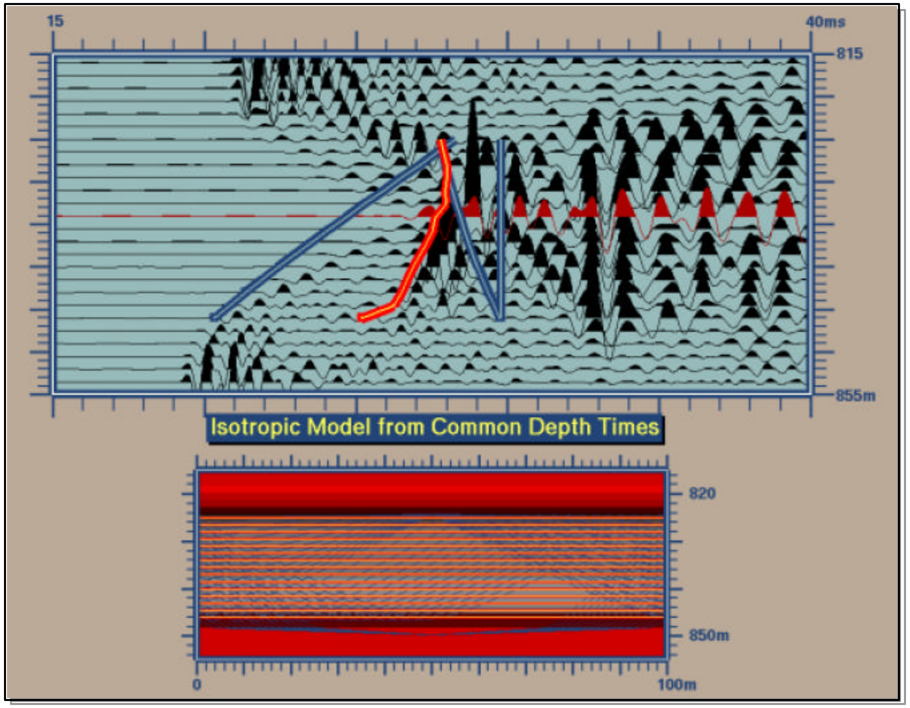
•**B is a doubly scattered wave.**

•**B is vertically uniform because the medium is laterally uniform!**

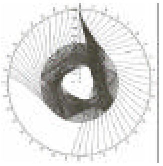


Incontrovertible Evidence of Anisotropy:





- **Event B demands a layered solution**
- **Each event samples a different angle and requires a different velocity**



Chapman & Pratt

Linearization of traveltime changes due to perturbation of Cij leads to linear system of equations in canonical slownesses with coefficients given by basic spherical harmonic functions.

Cf. Chapman and Pratt, 1992

2.1 Traveltime perturbations

The theory for tracing rays in anisotropic, inhomogeneous media is well known (Červený 1972). Červený (1982) and Červený & Jech (1982) have developed a theory for linearized perturbations to the traveltime in anisotropic media. This has been extended to cover the case of degenerate qS rays by Jech & Pšenčík (1989). In this section we summarize the results of those papers and clarify the rôle of the slowness perturbation (equations 13 and 18).

Defining the density normalized elastic parameters as

$$a_{ijkl} = c_{ijkl}/\rho, \tag{1}$$

the slowness as

$$\mathbf{p} = \nabla T, \tag{2}$$

and the Christoffel matrix as

$$\Gamma_{jk} = p_i p_l a_{ijkl}, \tag{3}$$

then

$$G(\mathbf{p}, \mathbf{x}) = 1 \tag{4}$$

defines the slowness surface, where G solves the eigenvalue equation (Červený 1972)

$$(\Gamma_{jk} - \delta_{jk} G)\hat{g}_k = 0, \tag{5}$$

with $\hat{\mathbf{g}}$ the normalized, i.e. unit, polarization vector. From this equation we have

$$G = \Gamma_{jk}\hat{g}_j\hat{g}_k = a_{ijkl}p_i p_l \hat{g}_j \hat{g}_k. \tag{6}$$

Chapman & Pratt

Linearization of traveltime changes due to perturbation of Cij leads to linear system of equations in canonical slownesses with coefficients given by basic spherical harmonic functions.

Cf. Chapman and Pratt, 1992

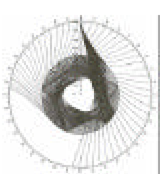
2.2 2-D tomography and weak anisotropy

If non-degenerate perturbation theory is valid (17), as it is for qP rays, then the traveltime perturbation is given by

$$\delta T^{(\mu)} = -\frac{1}{2}\int_{\mathscr{L}} p_i p_l \hat{g}_j^{(\mu)} \hat{g}_k^{(\mu)}\, \delta a_{ijkl}\, dT, \tag{31}$$

where in general the integral may contain 21 independent terms. If the unperturbed model is isotropic and we are considering qP rays, equation (31) is considerably simplified. In isotropic media, the P-wave polarization, the slowness direction and the ray direction are all parallel, so $\hat{\mathbf{g}}^{(1)} = \hat{\mathbf{p}} = \alpha\mathbf{p}$. Thus

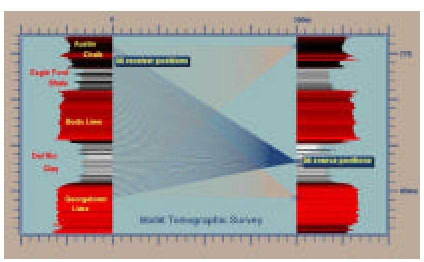
$$\delta T^{(1)} = -\frac{1}{2}\int_{\mathscr{L}} \alpha^2 p_i p_j p_k p_l\, \delta a_{ijkl}\, dT. \tag{32}$$



Miller & Chapman 1991

Linearization of traveltime changes due to perturbation of Cij leads to linear system of equations in canonical slownesses with coefficients given by basic spherical harmonic functions.

Cf. Chapman and Pratt, 1992



$$T = dl\ s_\theta$$

where dl is the length of the ray and s_θ is an angle-dependent (group) slowness of the form:

$$s_\theta = A\cos^4(\theta) + B\cos^2(\theta)\sin^2(\theta) + C\sin^4(\theta) \tag{1}$$

with

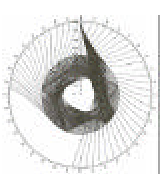
$$A = s_x, \quad C = s_z, \quad B = 4s_{45} - (s_x + s_z).$$

dl_{ijk} and θ_{ijk} respectively denote the length and angle in layer i of the ray connecting source j to receiver k. Then the approximate traveltime computations described above lead to a sparse, overdetermined system

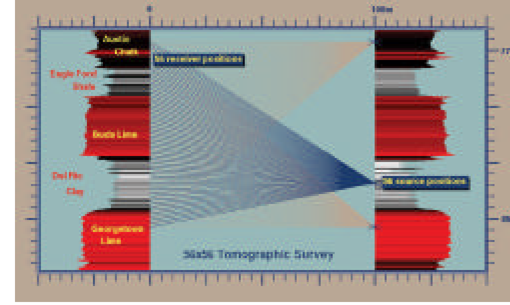
$$T_{jk} = \sum_i a_{ijk}s_x(i) + \sum_i b_{ijk}s_z(i) + \sum_i c_{ijk}s_{45}(i)$$

where T_{jk} is the measured traveltime from the jth receiver to the kth source, and

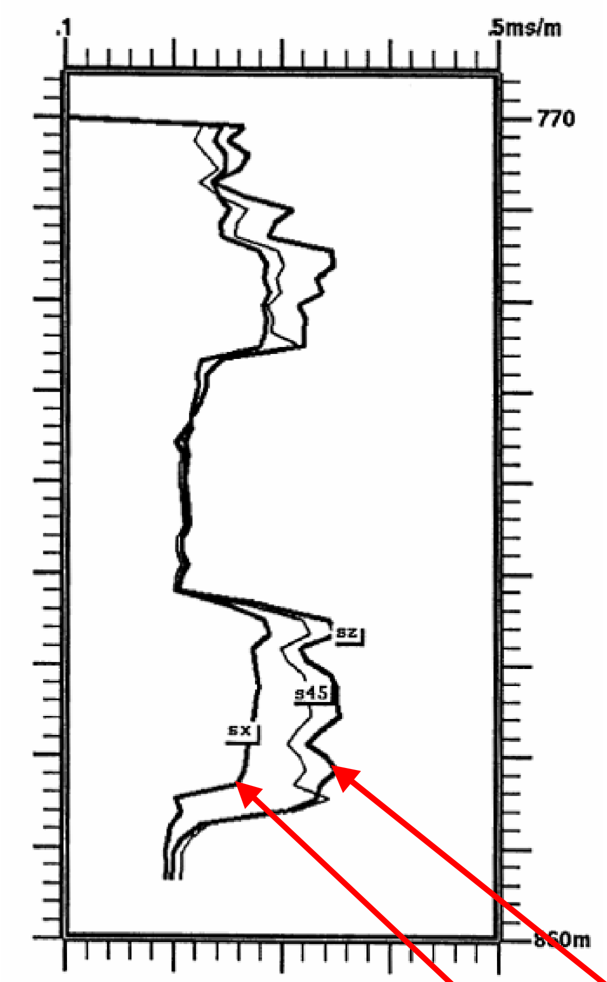
$$\begin{aligned} a_{ijk} &= dl_{ijk}(\cos^4(\theta_{ijk}) - \cos^2(\theta_{ijk})\sin^2(\theta_{ijk})) \\ b_{ijk} &= dl_{ijk}(\sin^4(\theta_{ijk}) - \cos^2(\theta_{ijk})\sin^2(\theta_{ijk})) \\ c_{ijk} &= dl_{ijk}(4\cos^2(\theta_{ijk})\sin^2(\theta_{ijk})) \end{aligned} \tag{2}$$



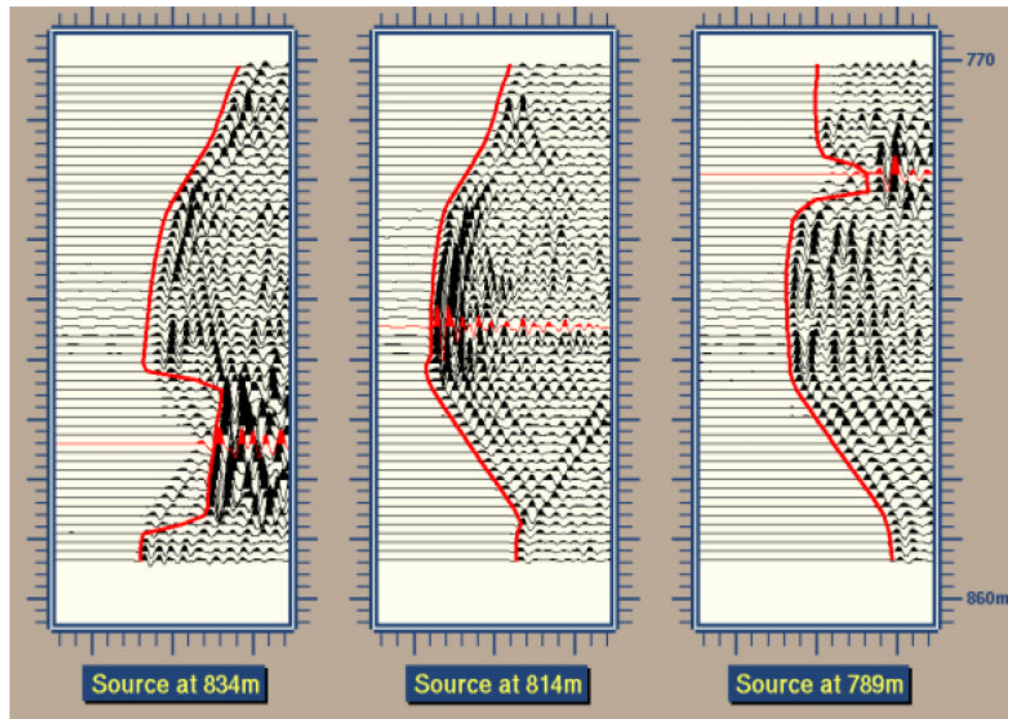
Miller & Chapman 1991



- Anisotropic Solution with 56x3 fits the data
- Limestones are isotropic
- Shales are anisotropic and anisotropy is anelliptic



Inverted slownesses Sx, S45, Sz



Fit

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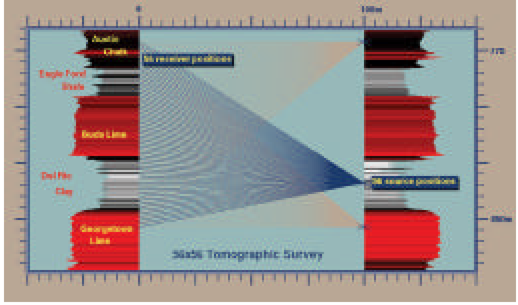
Miller & Chapman 1991



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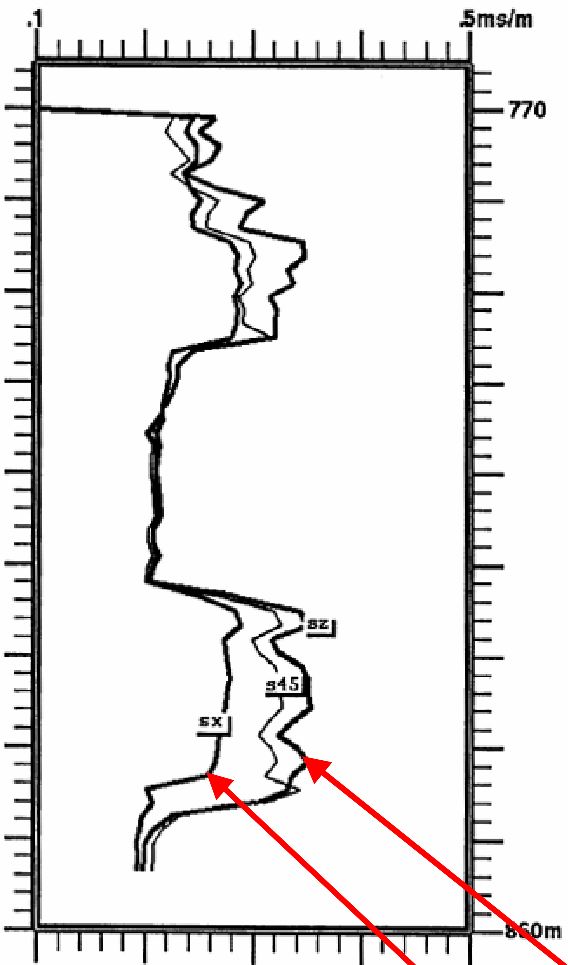

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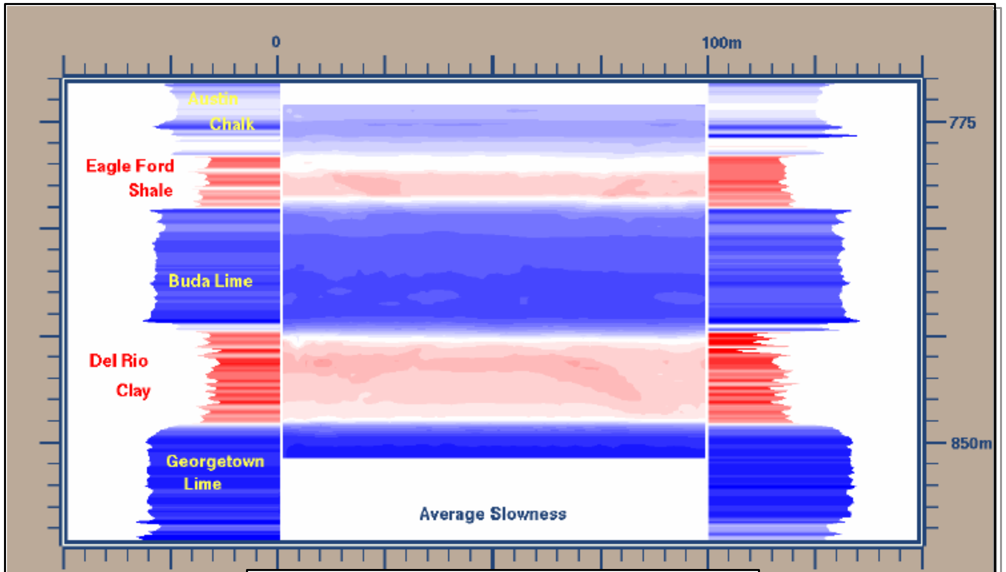
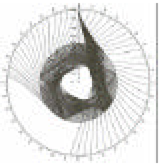


Anisotropic Tomogram



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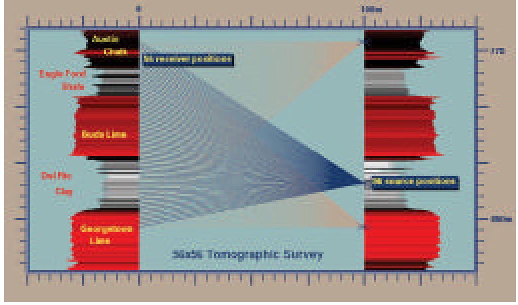
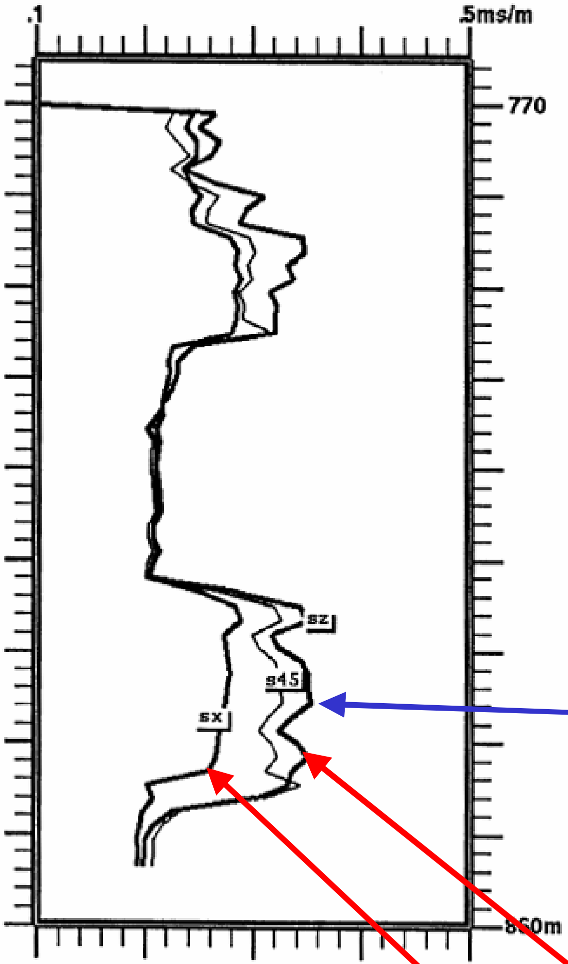
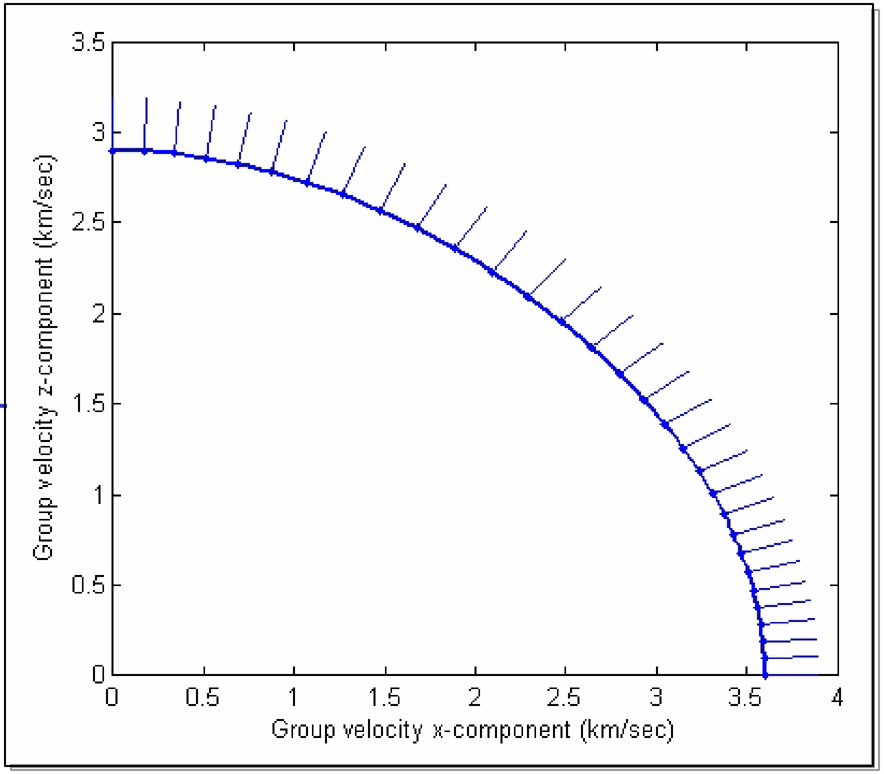




- Shales are anisotropic and anisotropy is anelliptic



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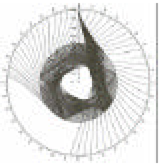

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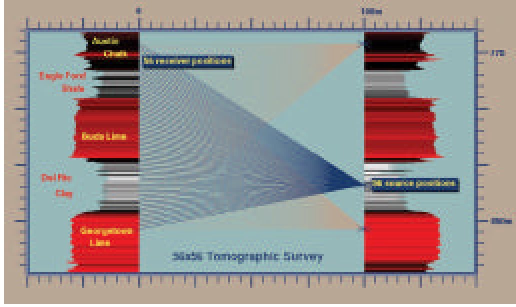
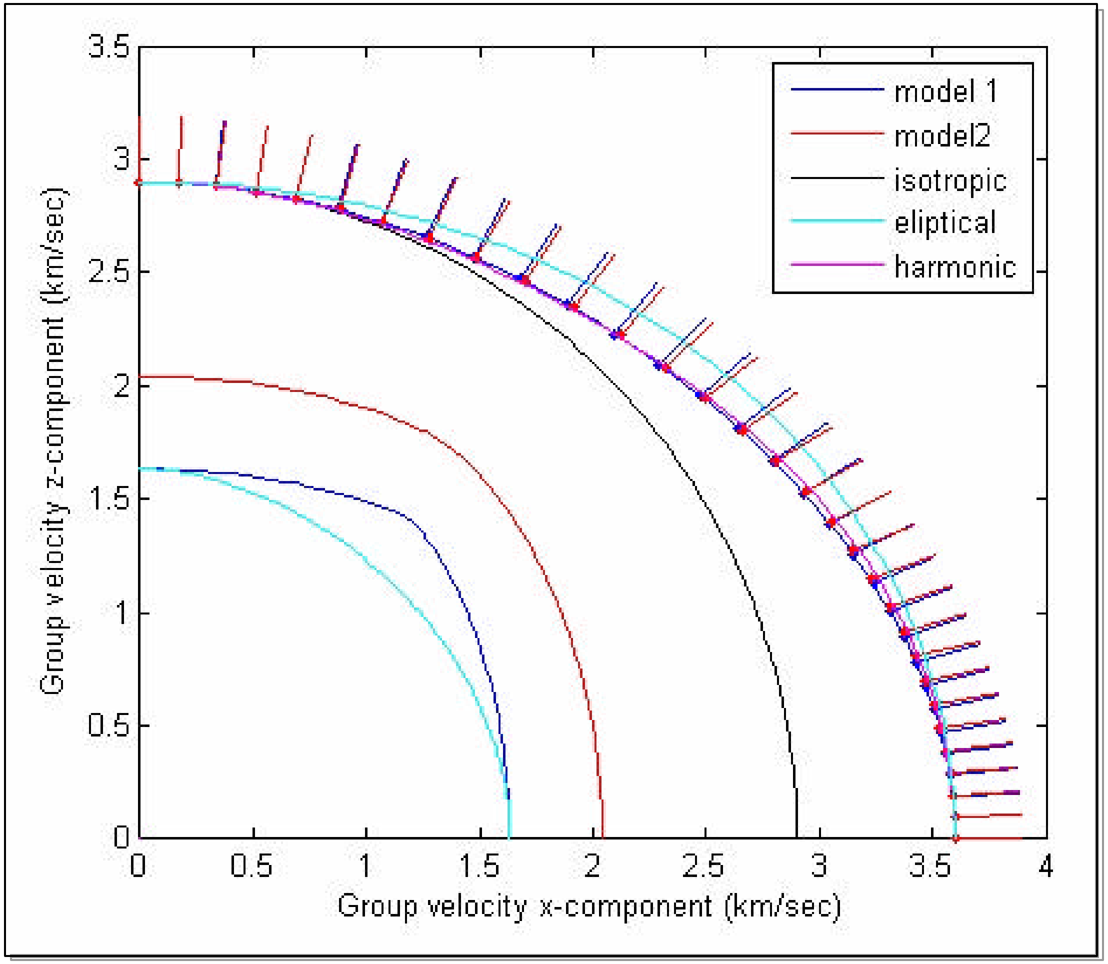
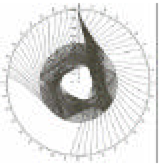
Remarks about the Model:

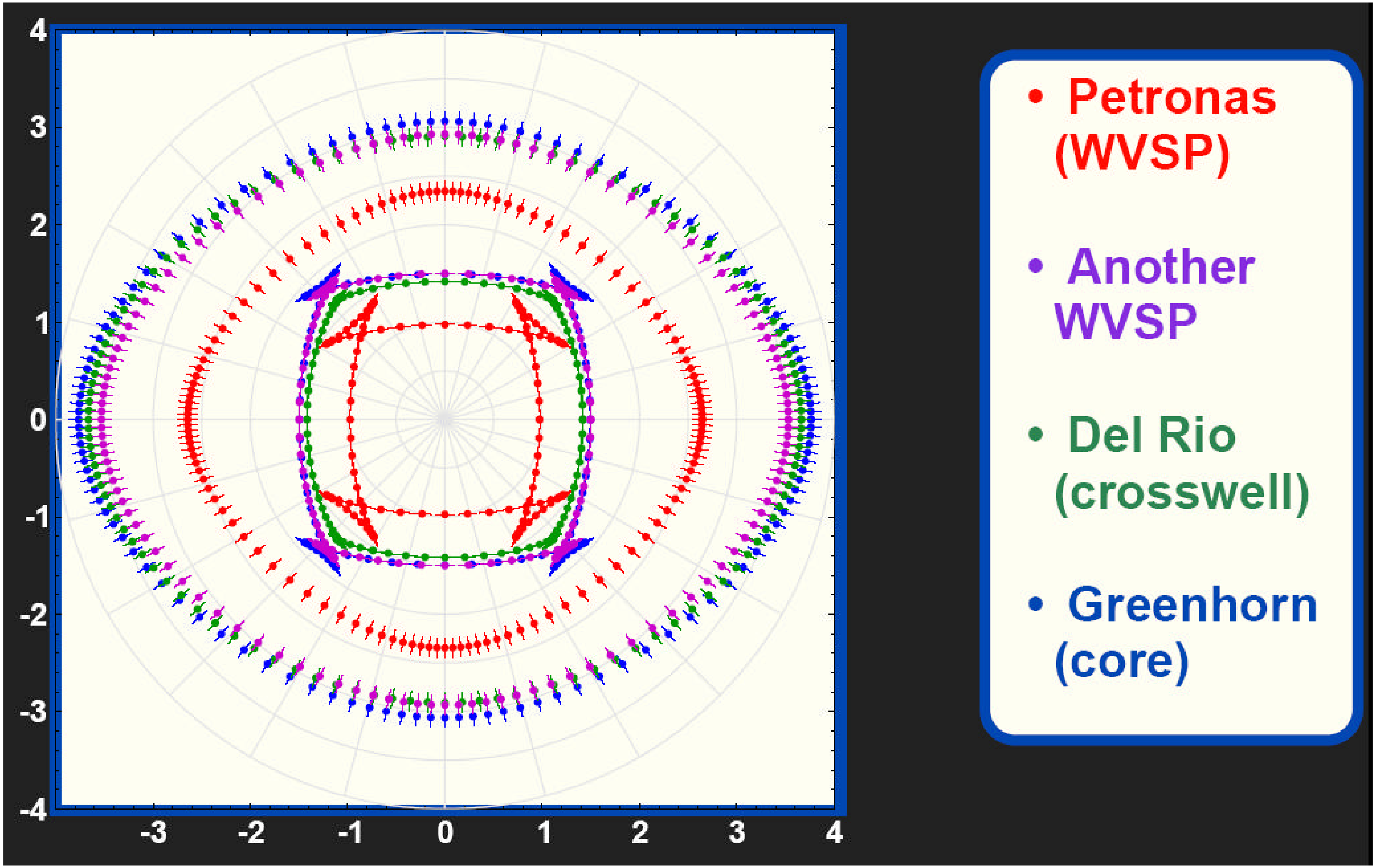


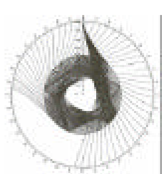
- **The isotropic approximation doesn't fit at all**
- **The elliptic approximation fits poorly**
- **The harmonic approximation fits well**
- **The fit is independent of shear modulus used**





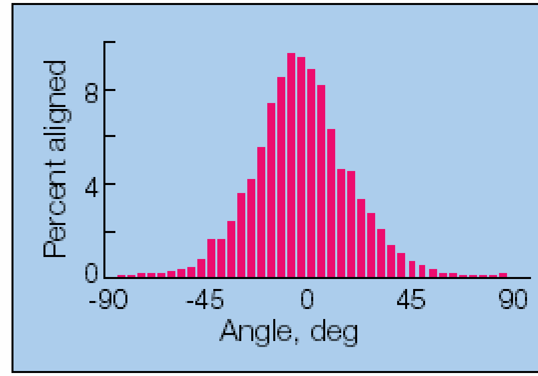
Question: Is this case typical?

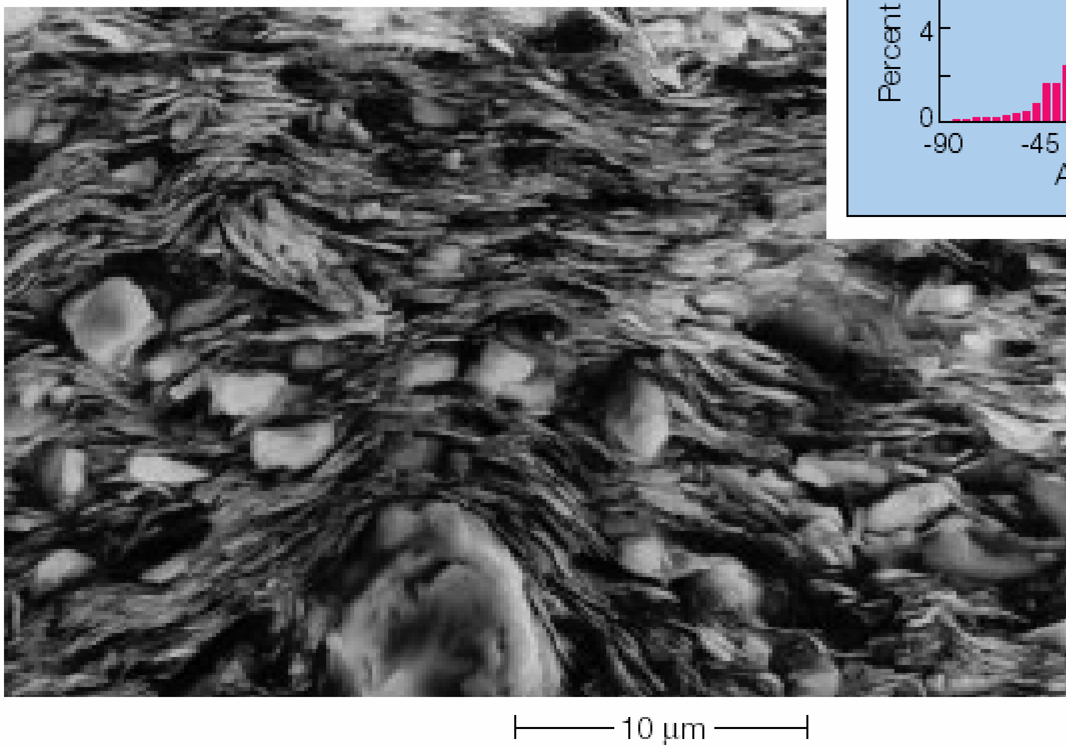




Answer: It is not rare

Shale Morphology





□*Photomicrograph of shale showing clay platelets distributed around the horizontal. Inset graph shows the distribution of the normal to the platelet, distributed around vertical.*

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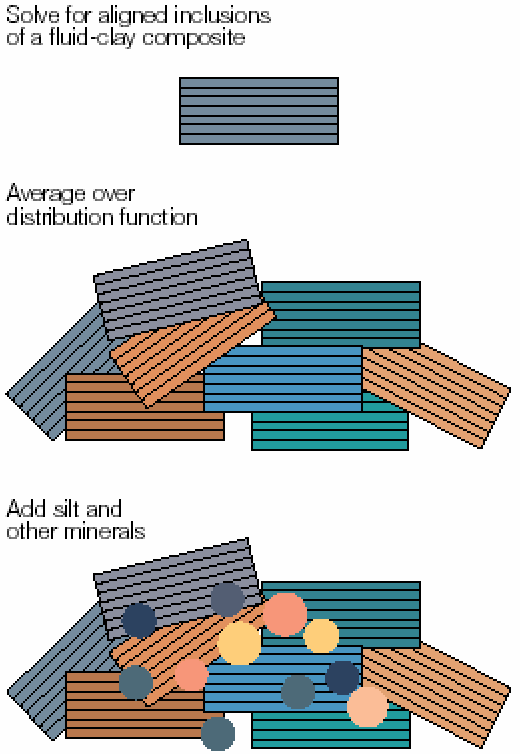
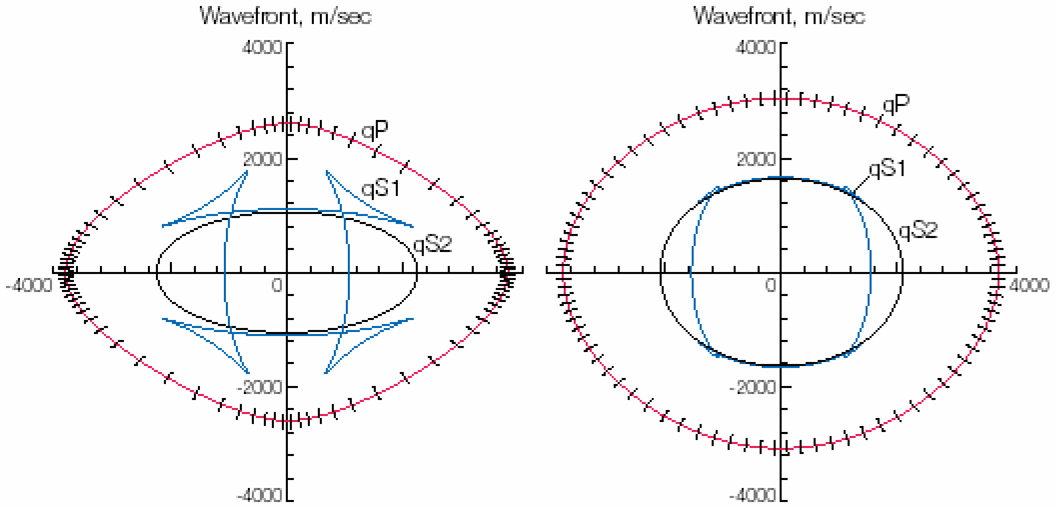
Shale Model

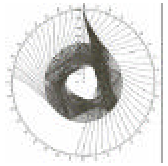
N.B.: Think about excess horizontal shear compliance





Components of a shale model. Individual model clay platelets (top) are oriented according to the distribution measured in the shale photograph on previous page (middle). Silt particles are added (bottom) to resemble real shales.

Wavefront velocities for synthetic shales. qP- and qS-wave velocities are computed for a shale with all clay platelets oriented horizontally (left). The shale synthesized with a realistic clay platelet distribution shows computed velocities (right) similar to those of the real shale depicted on the previous page.



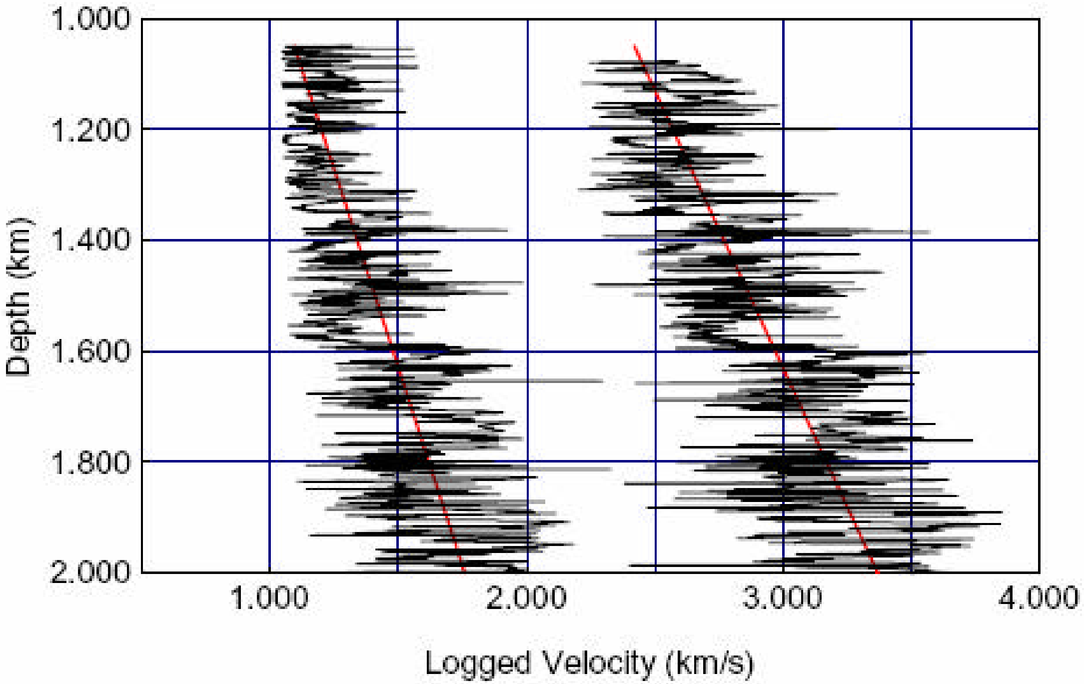
Compaction Process

Expectation:

As depth increases

•Porosity decreases so velocity increases

•Order increases so anisotropy increases (up to a point)



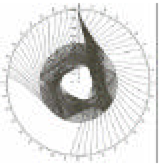
How does this look in Walkaway VSP Data?

5129 Sound velocities

AN *IN SITU* ESTIMATION OF ANISOTROPIC ELASTIC MODULI FOR A SUBMARINE SHALE.

Douglas E. Miller (Schlumberger-Doll Research, Old Quarry Road, Ridgefield CT 06977-4108, USA; (email: miller@sdr.slb.com))
Scott Leaney, Bill Borland

Direct arrival times and slownesses from wide aperture walkaway vertical seismic profile data acquired in a layered anisotropic medium can be processed to give a direct estimate of the phase slowness surface associated with the medium at the depth of the receivers. This slowness surface can, in turn, be fit by an estimated transversely isotropic medium with a vertical symmetry axis (a "TIV" medium). While the method requires that the medium between the receivers and the surface be horizontally stratified, no further measurement or knowledge of that medium is required. When applied to data acquired in a compacting shale sequence (here termed the "Petronas shale") encountered by a well in the South China Sea, the method yields an estimated TIV medium that fits the data extremely well over 180 degrees of propagation angles sampled by 201 source positions. The medium is strongly anisotropic. The anisotropy is significantly anelliptic and implies that the quasi-shear mode should be triplicated for off-axis propagation. Estimated density-normalized moduli (in units of km^2/s^2) for the Petronas shale are: $A_{11} = 6.99 \pm .21$; $A_{33} = 5.53 \pm .17$; $A_{55} = .91 \pm .05$; $A_{13} = 2.64 \pm .26$. Densities in the logged zone just below the survey lie in the range between 2200 and 2400 km^3/m^3 with an average value close to 2300 km^3/m^3.



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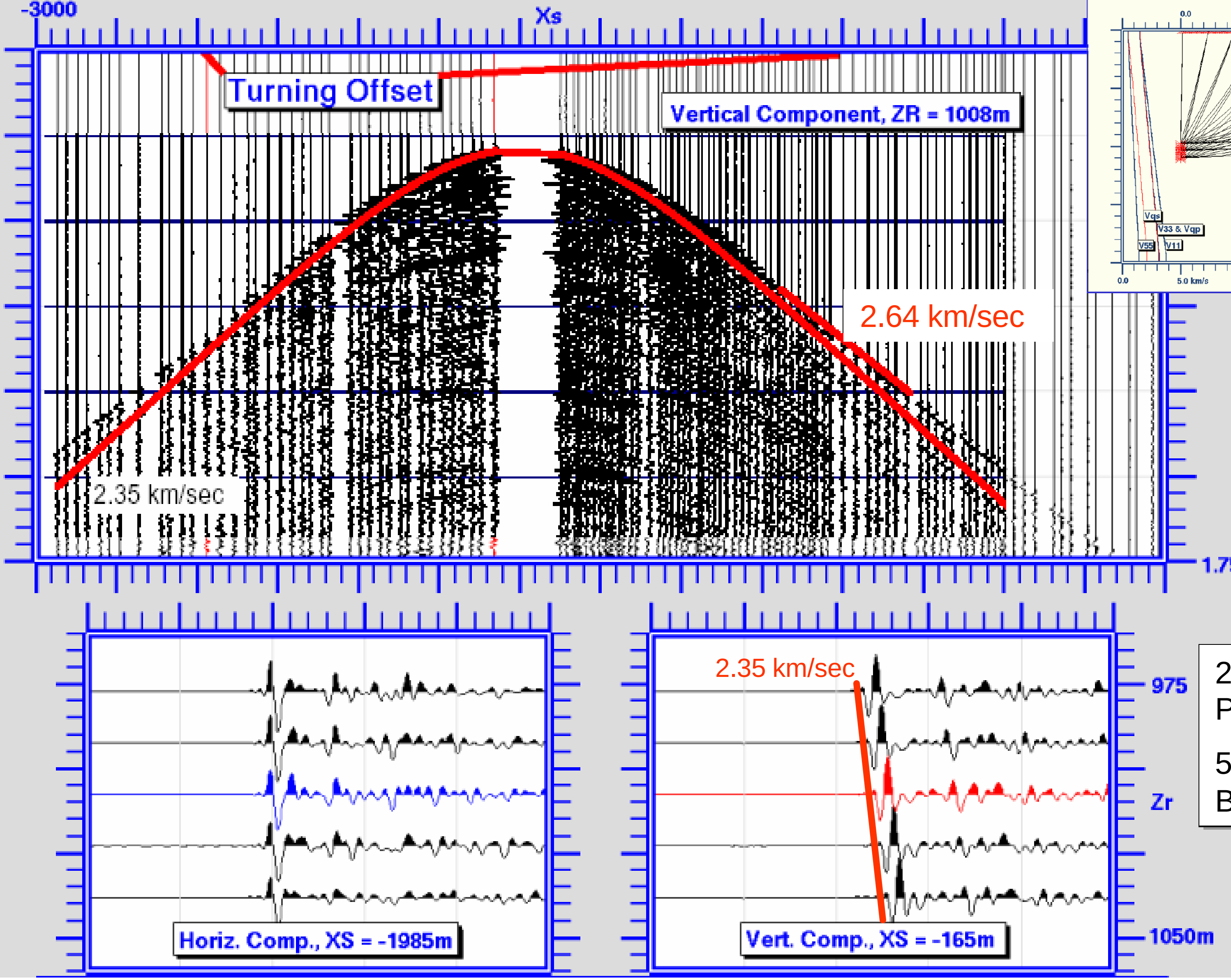

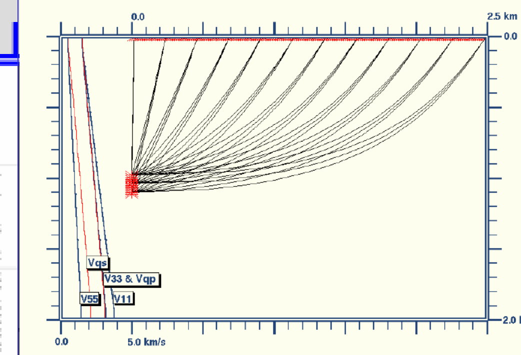

201 Source Positions

5 3-Component Borehole receivers

Walkaway VSP Example

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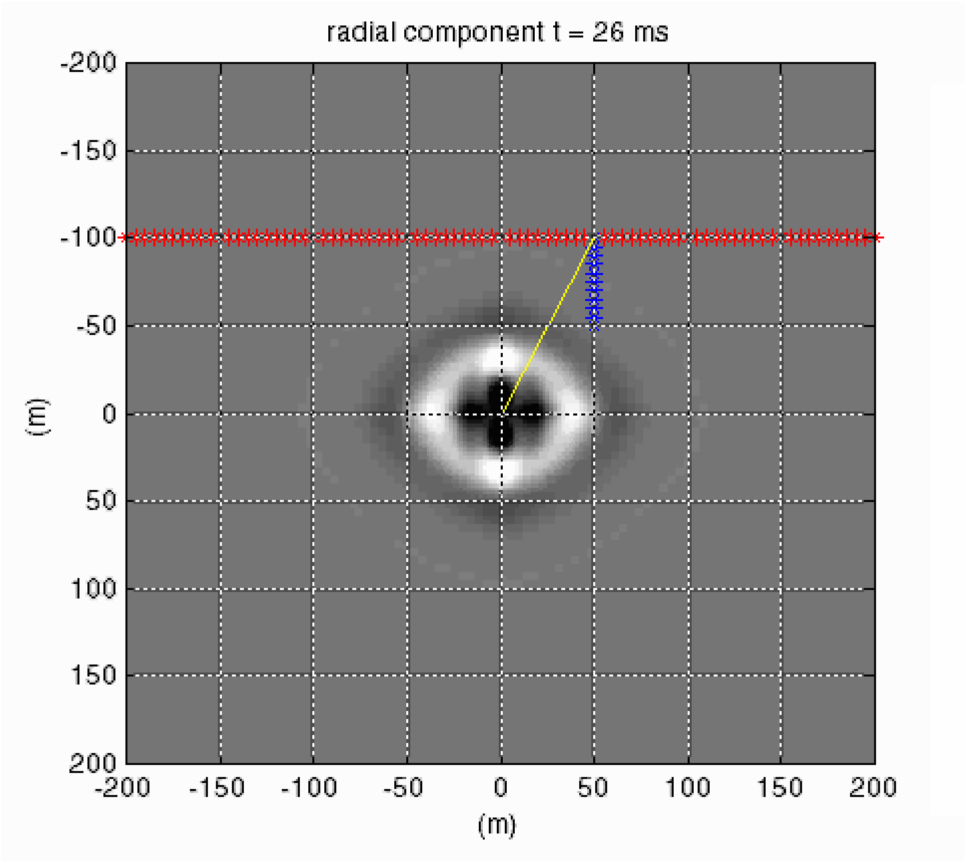
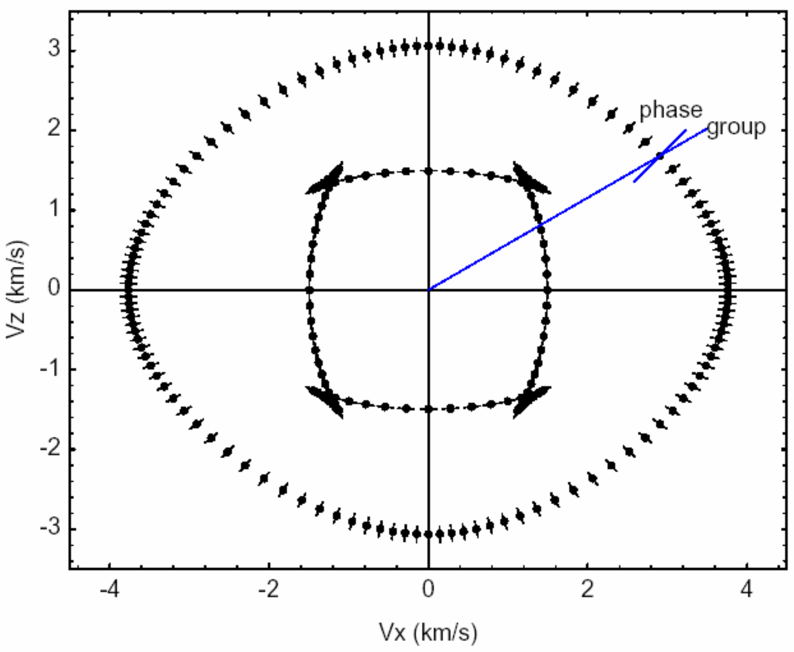
Anisotropy 101





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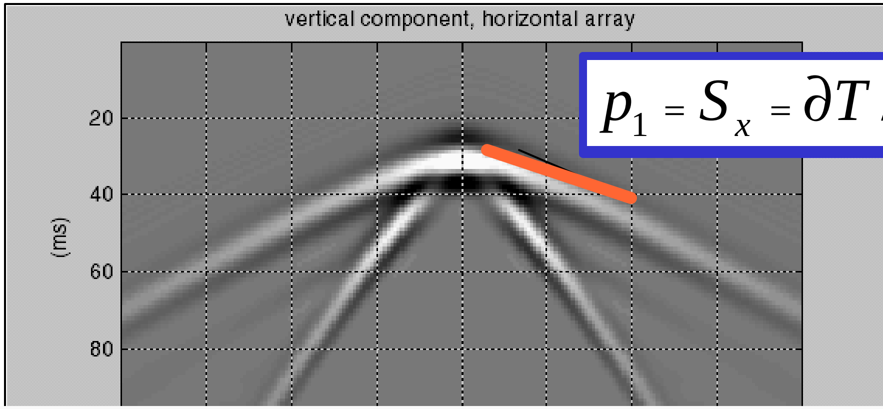
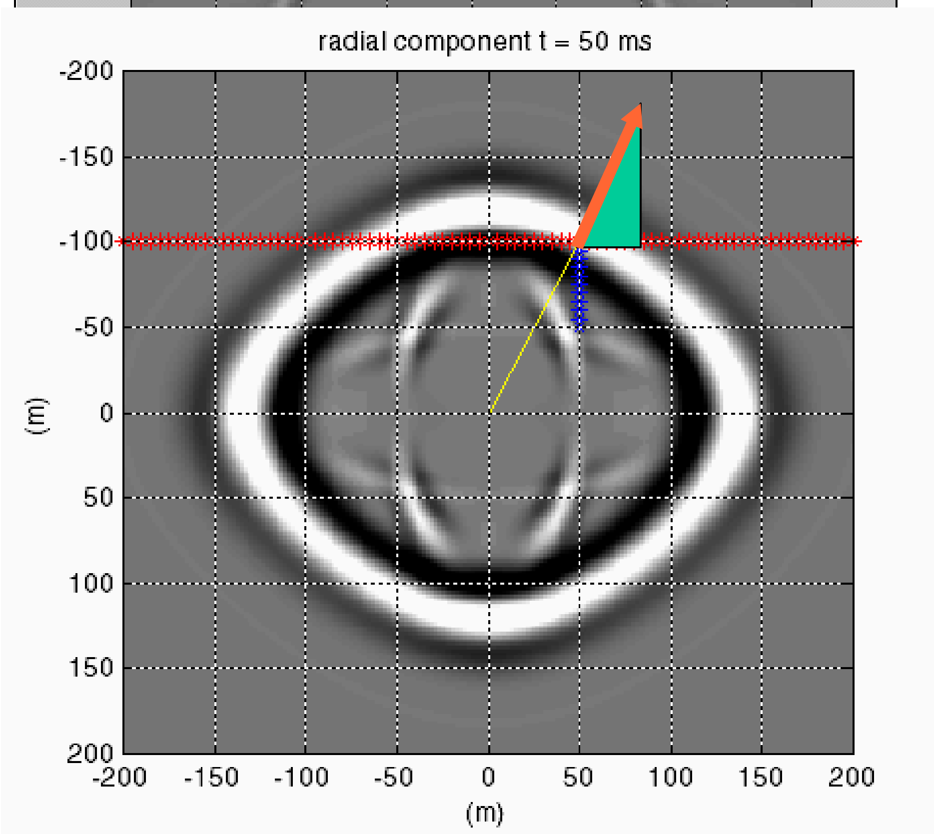
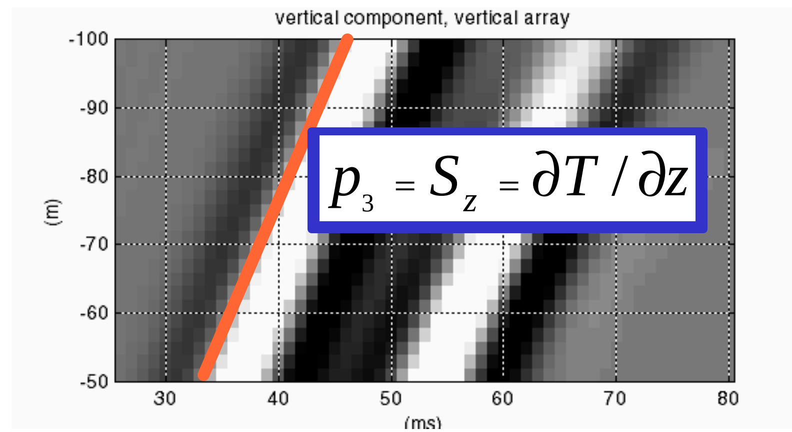
The spatial gradient of the traveltime function is the Phase Slowness Vector

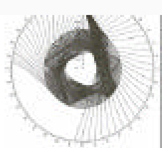


$$p_1 = S_x = \partial T / \partial x$$





$$p_3 = S_z = \partial T / \partial z$$



Key Observation

In a laterally invariant medium the horizontal component of slowness preserved along the ray and can be measured by estimating dT/dX at the source.

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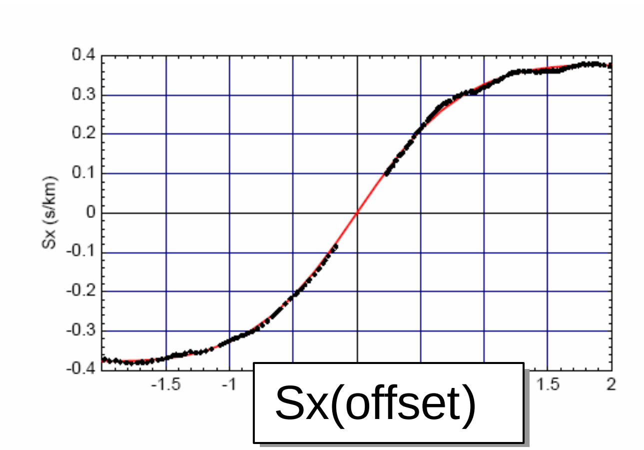
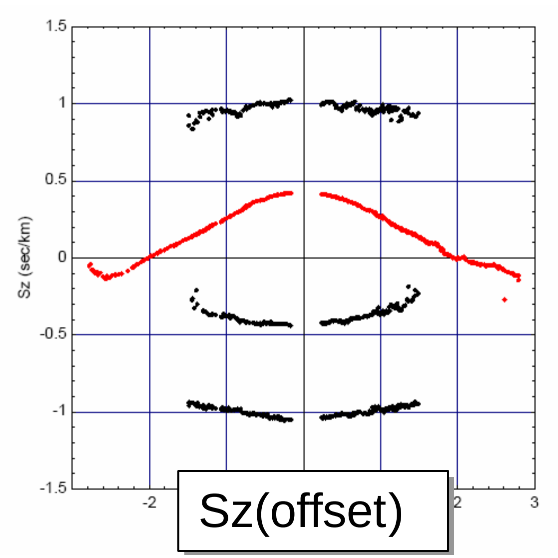
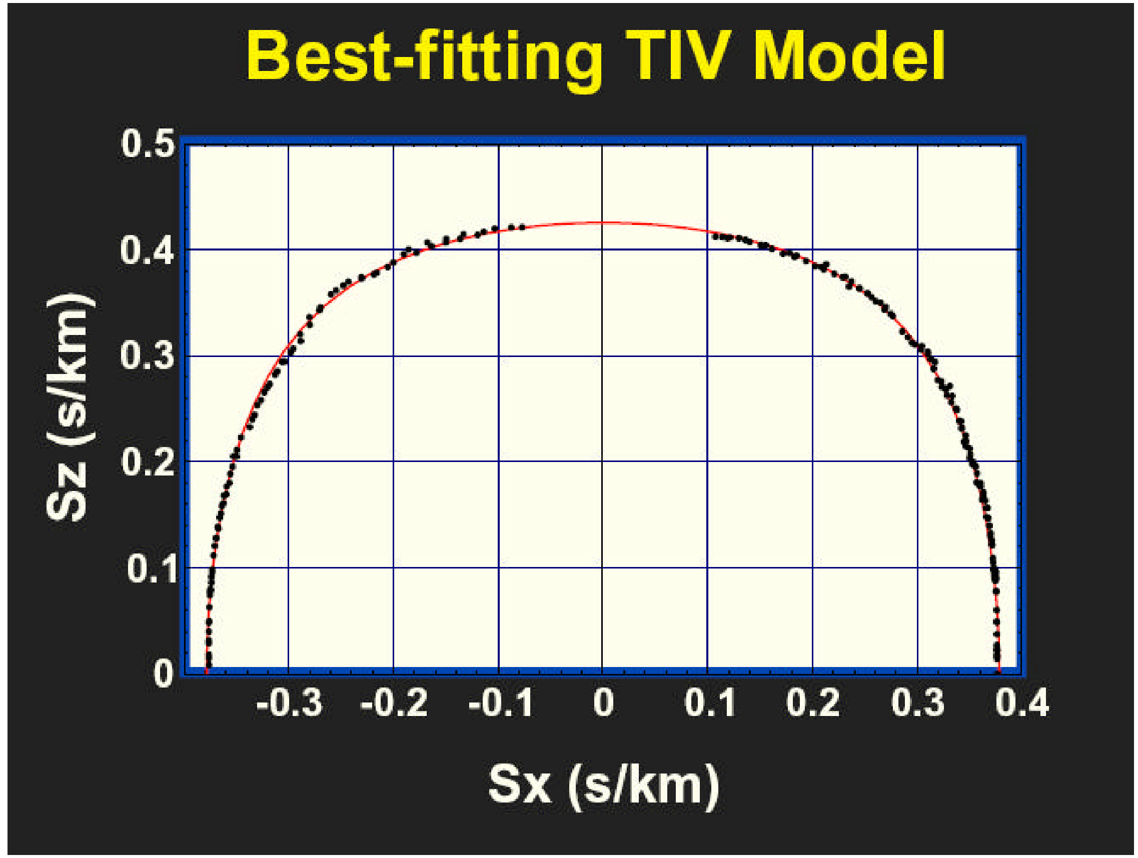
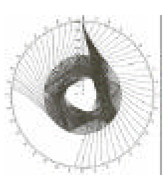
Squared Phase Slowness



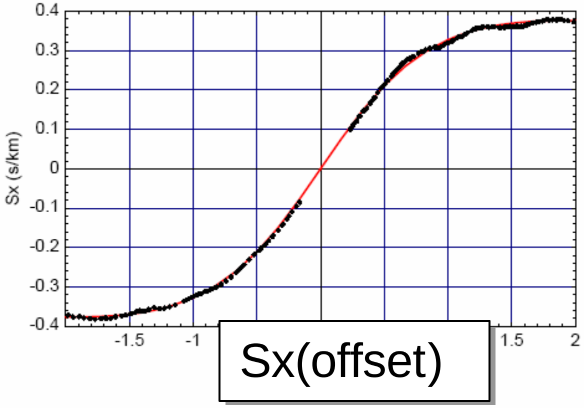


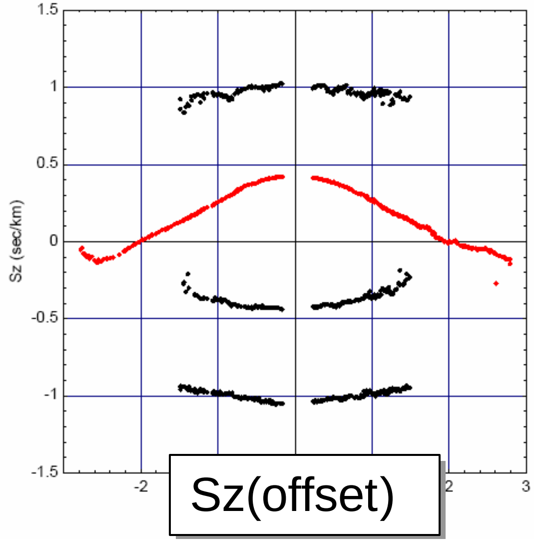


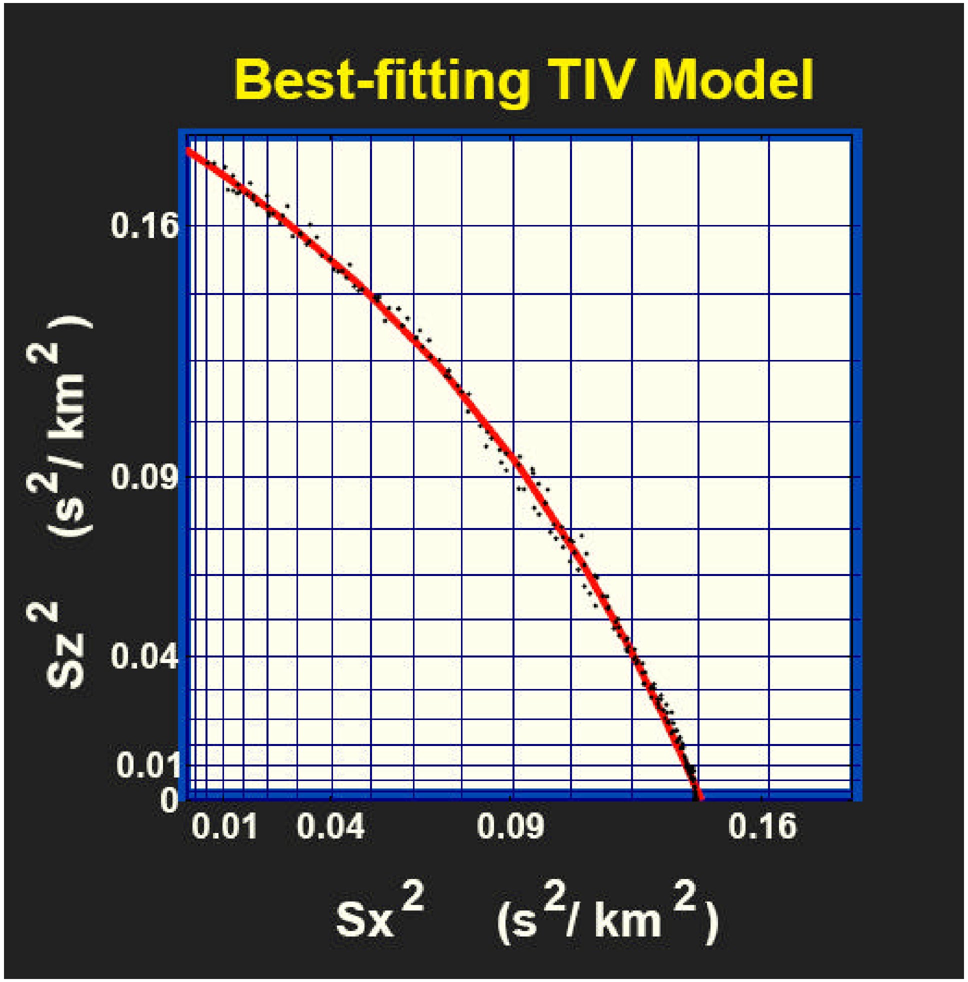
Crossplot of Sx and Sz gives phase slowness



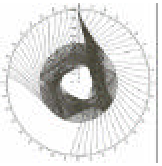
Squared Phase Slowness







N.B.: Isotropy would require a line at 45°



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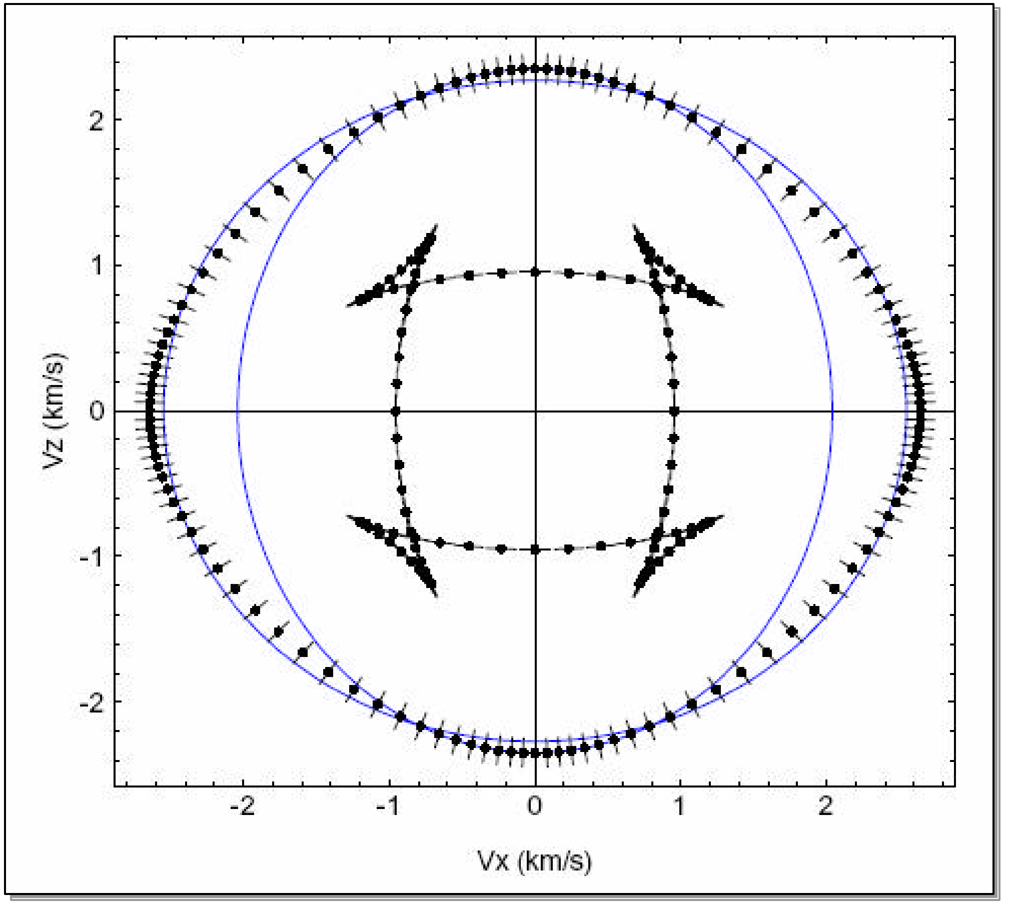
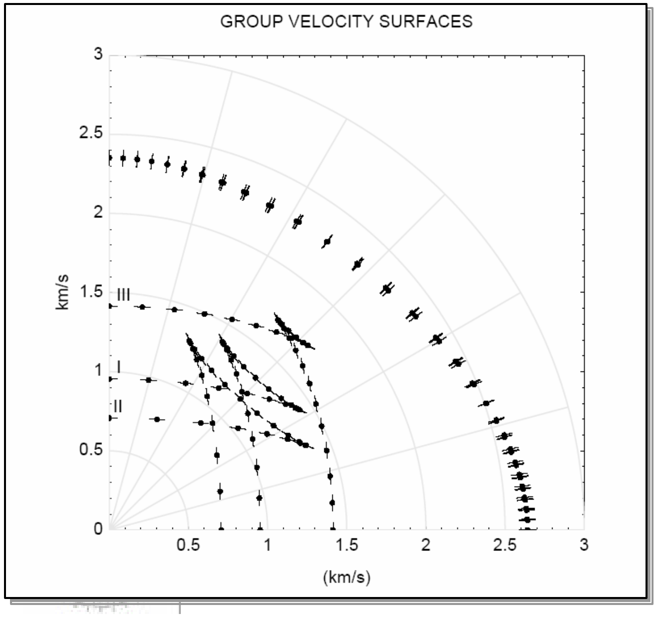
Remarks about the Model:

- **The isotropic approximation doesn't fit at all**
- **The elliptic approximation fits poorly**
- **The harmonic approximation fits well**
- **The fit is independent of shear modulus used**





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TI Sensitivities:

Questions:

- **Why is the fit independent of shear modulus used?**
- **What parameters are most important locally?**

Chris's Answers:

- **The Perturbation Theory yields the sensitivities.**
- **The coefficients can be recognized as static axial moduli in appropriately rotated coordinates.**

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Velocity Sensitivity in Transversely Isotropic Media

by

C.H. Chapman and D.E. Miller

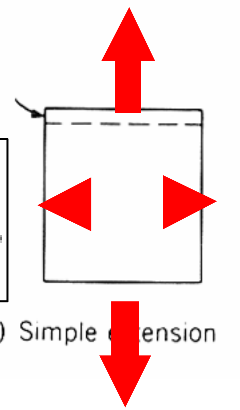
IUGG XXI, 1995

Hooke's Law: Reduced (Voigt) Notation

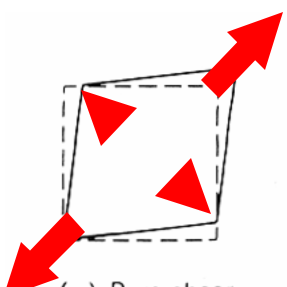
- **TIV - rotational symmetry around 3-axis**

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & & & \\ c_{11}-2c_{66} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{44} & \\ & & & & & c_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix}$$

11	22	33	32 = 23	31 = 13	12 = 21
↓	↓	↓	↓	↓	↓
1	2	3	4	5	6



(b) Simple tension



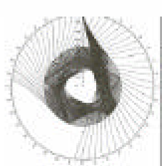
(e) Pure shear

To achieve a unit of pure longitudinal strain along the 3-axis:

- Pull *up-down* with traction c_{33}
- Pull *left-right, in-out* with traction c_{13}

To achieve a unit of pure 13 shear strain:

- Apply 13 traction c_{55}



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Important Remark (cf. Chapman p86)

86 *Review of continuum mechanics and elastic waves*

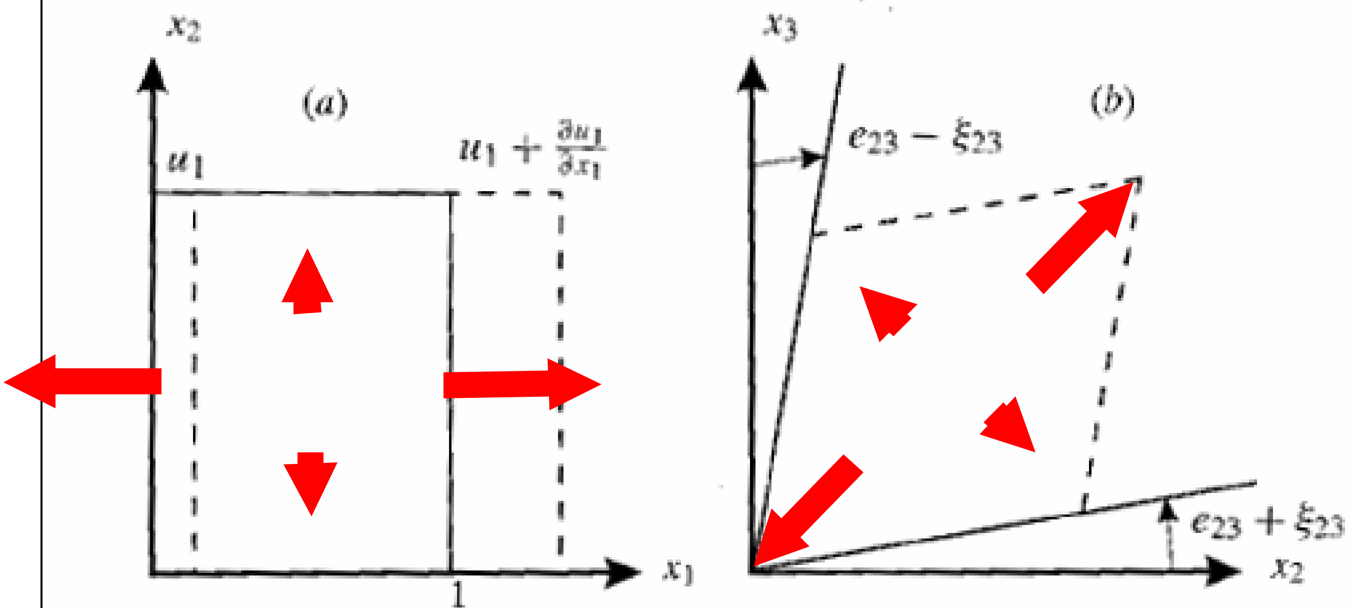


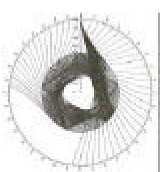
Fig. 4.8. The deformation of a unit square due to strain components (*a*) e_{11}, and (*b*) e_{23}.

the terms normal and shear strain are used, for Figures 4.8*a* and *b*, but they only make sense in a particular coordinate system. Normal strains of opposite signs along the x_1 and x_2 axes become shear strains, if the axes are rotated by $\pi/4$

Hooke's Law Revisited

To get a unit of pure strain (assuming $\rho = 1$):

Mode	Direction	Stress
P	$0°$	A_{11}
P	$90°$	A_{33}
S	$0°$	A_{55}
S	$90°$	A_{55}
P	$45°$	$.25(A_{11} + A_{33} + 2(A_{13} + 2A_{55}))$
S	$45°$	$.25(A_{11} + A_{33} - 2A_{13})$



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Perturbation Story

Červený (1982) and Červený and Jech (1982) published a perturbation theory for weak anisotropic media which gave the perturbed velocity as

$$2v\,\delta v = \delta c_{ijkl}\hat{p}_i\hat{p}_l\hat{g}_j\hat{g}_k\ . \tag{1}$$

where $\hat{\mathbf{p}}$ is the slowness direction and $\hat{\mathbf{g}}$ is the normalized polarization. This result was expanded by Jech and Pšenčík (1989) and used by various authors, e.g. Chapman and Pratt (1992), Chapman (1992), etc.

In the weak anisotropic approximation for qP rays we let $\hat{\mathbf{g}} = \hat{\mathbf{p}}$ and obtain

$$\begin{aligned} 2v\,dv = {} & \hat{p}_1^4\,\delta C_{11} + \hat{p}_2^4\,\delta C_{22} + \hat{p}_3^4\,\delta C_{33} \\ & + 4[\hat{p}_1^3\hat{p}_2\,\delta C_{16} + \hat{p}_1^3\hat{p}_3\,\delta C_{15} + \hat{p}_2^3\hat{p}_3\,\delta C_{24} + \hat{p}_2^3\hat{p}_1\,\delta C_{26} + \hat{p}_3^3\hat{p}_1\,\delta C_{35} + \hat{p}_3^3\hat{p}_2\,\delta C_{34}] \\ & + 2[\hat{p}_2^2\hat{p}_3^2(\delta C_{23} + 2\delta C_{44}) + \hat{p}_1^2\hat{p}_3^2(\delta C_{13} + 2\delta C_{55}) + \hat{p}_1^2\hat{p}_2^2(\delta C_{12} + 2\delta C_{66})] \\ & + 4[\hat{p}_1^2\hat{p}_2\hat{p}_3(\delta C_{14} + 2\delta C_{56}) + \hat{p}_1\hat{p}_2^2\hat{p}_3(\delta C_{25} + 2\delta C_{46}) + \hat{p}_1\hat{p}_2\hat{p}_3^2(\delta C_{36} + 2\delta C_{45})]\ . \end{aligned} \tag{2}$$

Substituting

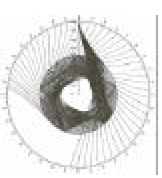
$$\hat{\mathbf{p}} = (\cos\eta\sin\chi,\ \sin\eta\sin\chi,\ \cos\chi)^{\mathrm{T}}\ , \tag{3}$$

the result can be converted into spherical harmonics.

From (2) with the background isotropic velocity denoted v_0 and making substitutions $v_0^2 + \delta C_{11} = C_{11}$, etc:

$$\begin{aligned} v^2 = v_0 + 2v_0\,dv = {} & \hat{p}_1^4\,C_{11} + \hat{p}_2^4\,C_{22} + \hat{p}_3^4\,C_{33} \\ & + 4[\hat{p}_1^3\hat{p}_2\,C_{16} + \hat{p}_1^3\hat{p}_3\,C_{15} + \hat{p}_2^3\hat{p}_3\,C_{24} + \hat{p}_2^3\hat{p}_1\,C_{26} + \hat{p}_3^3\hat{p}_1\,C_{35} + \hat{p}_3^3\hat{p}_2\,C_{34}] \\ & + 2[\hat{p}_2^2\hat{p}_3^2(C_{23} + 2C_{44}) + \hat{p}_1^2\hat{p}_3^2(C_{13} + 2C_{55}) + \hat{p}_1^2\hat{p}_2^2(C_{12} + 2C_{66})] \\ & + 4[\hat{p}_1^2\hat{p}_2\hat{p}_3(C_{14} + 2C_{56}) + \hat{p}_1\hat{p}_2^2\hat{p}_3(C_{25} + 2C_{46}) + \hat{p}_1\hat{p}_2\hat{p}_3^2(C_{36} + 2C_{45})]\ . \end{aligned} \tag{4}$$

This is Equation (11) of Every and Sachse (1992).

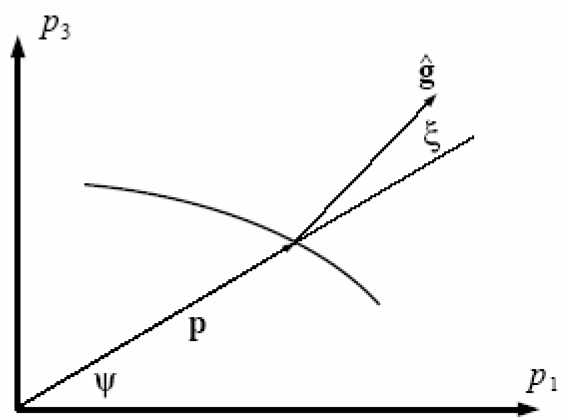


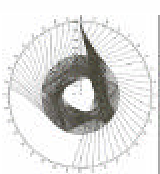
Perturbation Result (Chapman & Pratt, 1992)

$$\delta p \simeq -\frac{1}{2} p^3 \delta a_{ijkl}\, \hat{p}_i\, \hat{p}_l \hat{g}_j \hat{g}_k$$

$$= -\frac{1}{2} p^3 \left\{ \hat{p}_1^2 \hat{g}_1^2\, \delta A_{11} + 2\hat{p}_1 \hat{p}_3 \hat{g}_1 \hat{g}_3\, \delta(A_{13} + 2A_{55}) + \hat{p}_3^2 \hat{g}_3^2\, \delta A_{33} + (\hat{p}_1 \hat{g}_3 - \hat{p}_3 \hat{g}_1)^2 \delta A_{55} \right\} .$$

Analyze consequences of setting delta_ p = 0 under the approximation that phase and polarization vectors are parallel or orthogonal.





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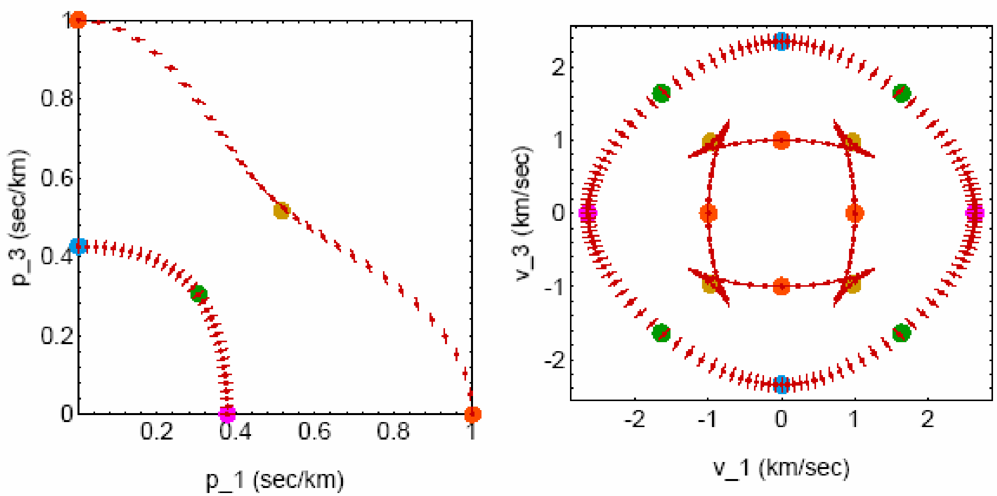
PushPin Parameters

P_{0°	A_{11}
P_{90°	A_{33}
S_{0°	A_{55}
S_{90°	A_{55}
P_{45°	$.25(A_{11} + A_{33} + 2(A_{13} + 2A_{55}))$
S_{45°	$.25(A_{11} + A_{33} - 2A_{13})$

If an arbitrary TI medium is perturbed in a way that preserves a given push-pin, then slowness points in the associated direction and mode will be approximately preserved in the new medium.



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PushPin Parameters

If an arbitrary TI medium is perturbed in a way that preserves a given push-pin, then slowness points in the associated direction and mode will be approximately preserved in the new medium.

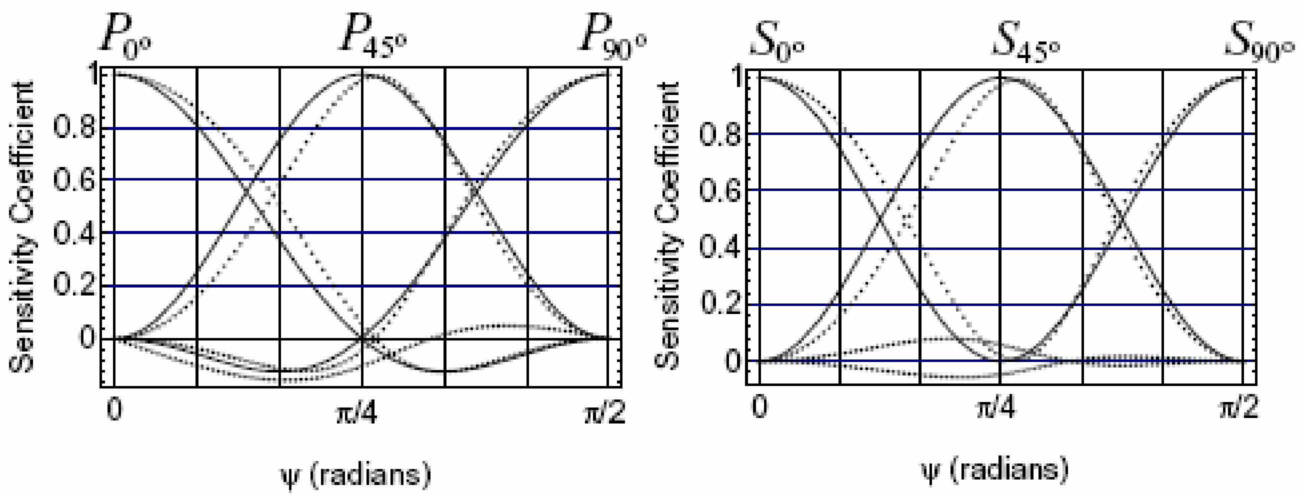

Figure 2: The exact (dotted line) and approximate (solid line) sensitivity functions for a representative shale medium. In units of km^2/s^2, the medium has density-normalized moduli $\{A^0_{11}, A^0_{13}, A^0_{33}, A^0_{55}\} = \{7., 2.5, 5.5, 1.0\}$. In the qP case, they are the coefficients in (2.13) and (3.3), respectively. In the qSV case, they are the coefficients in (2.14) and (3.4).

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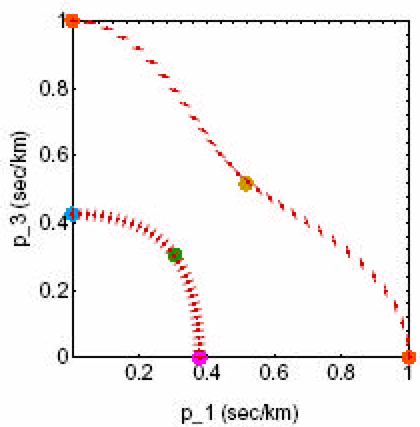
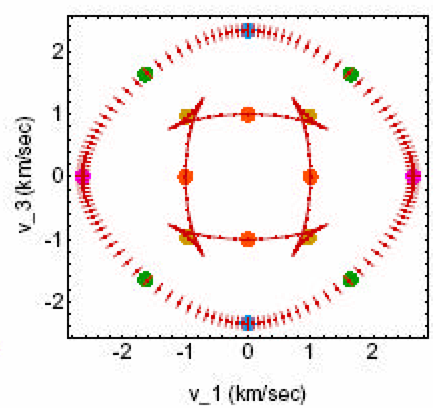
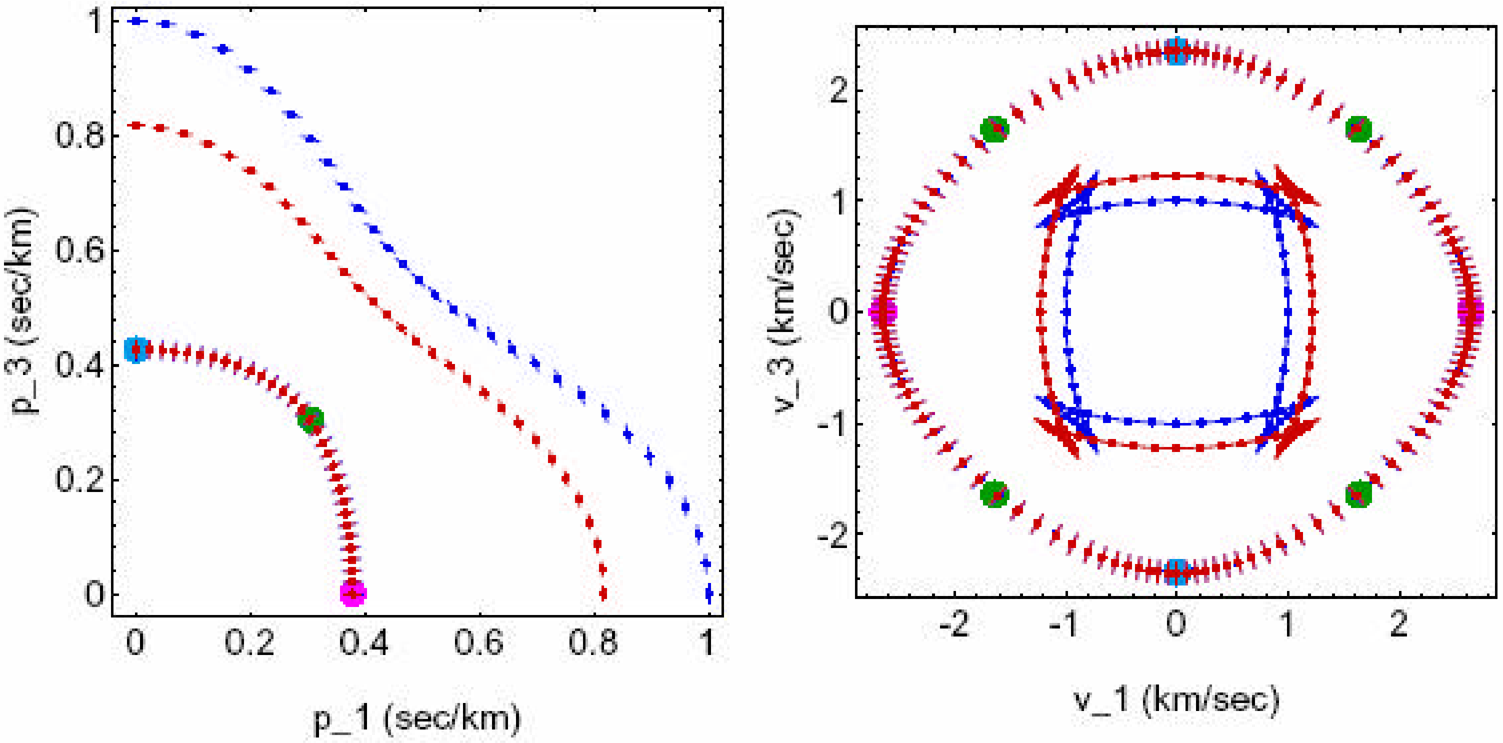
THE MAIN POINT

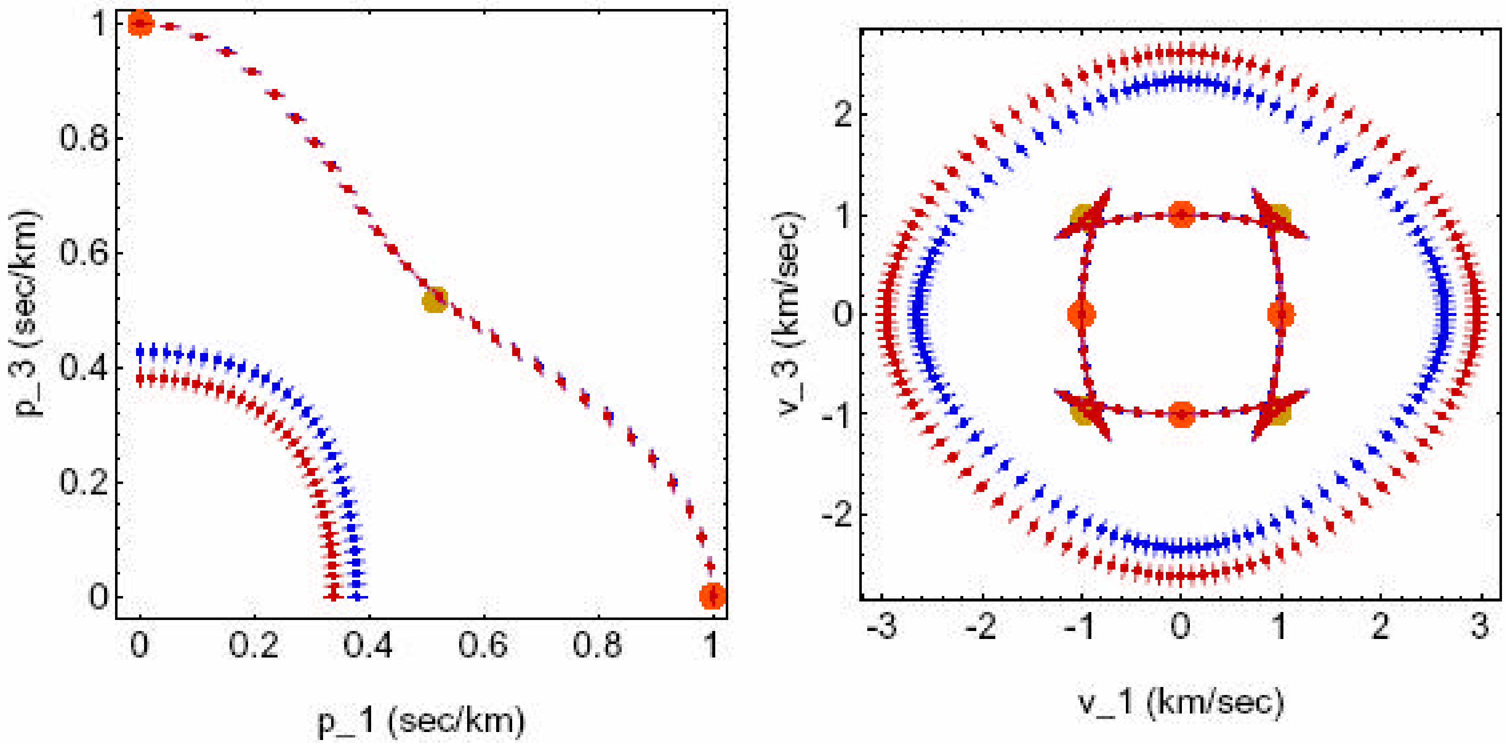
If an arbitrary TI medium is perturbed in a way that preserves a given push-pin, then slowness points in the associated direction and mode will be approximately preserved in the new medium.



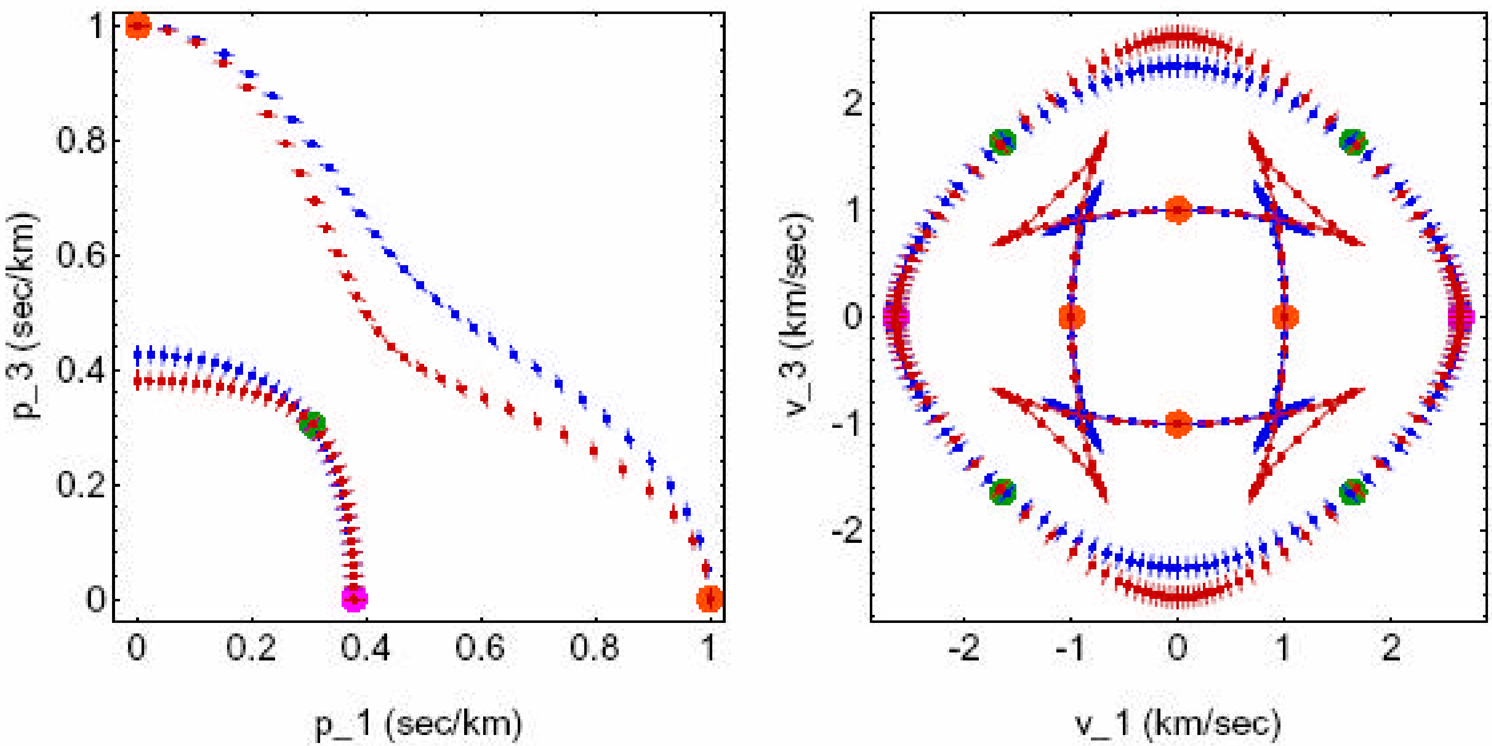




50% increase in S_{0°, keeping P_{0°, P_{45° and P_{90° fixed.



25% increase in P_{0°, keeping S_{0°, S_{45° and P_{0°/P_{90° fixed.



25% increase in $P_{90°}$, keeping $P_{0°}$, $P_{45°}$ and $S_{0°}$ fixed.

How does this look in Sonic Log Data?

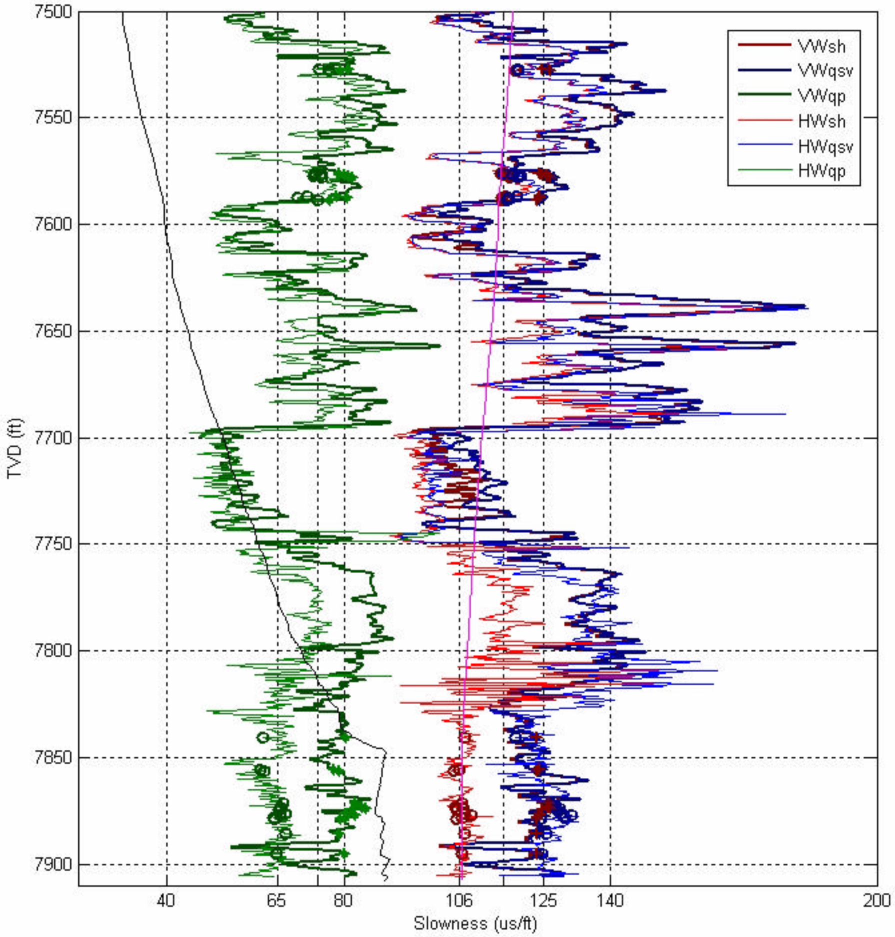
Derivation of anisotropy parameters in a shale using borehole sonic data

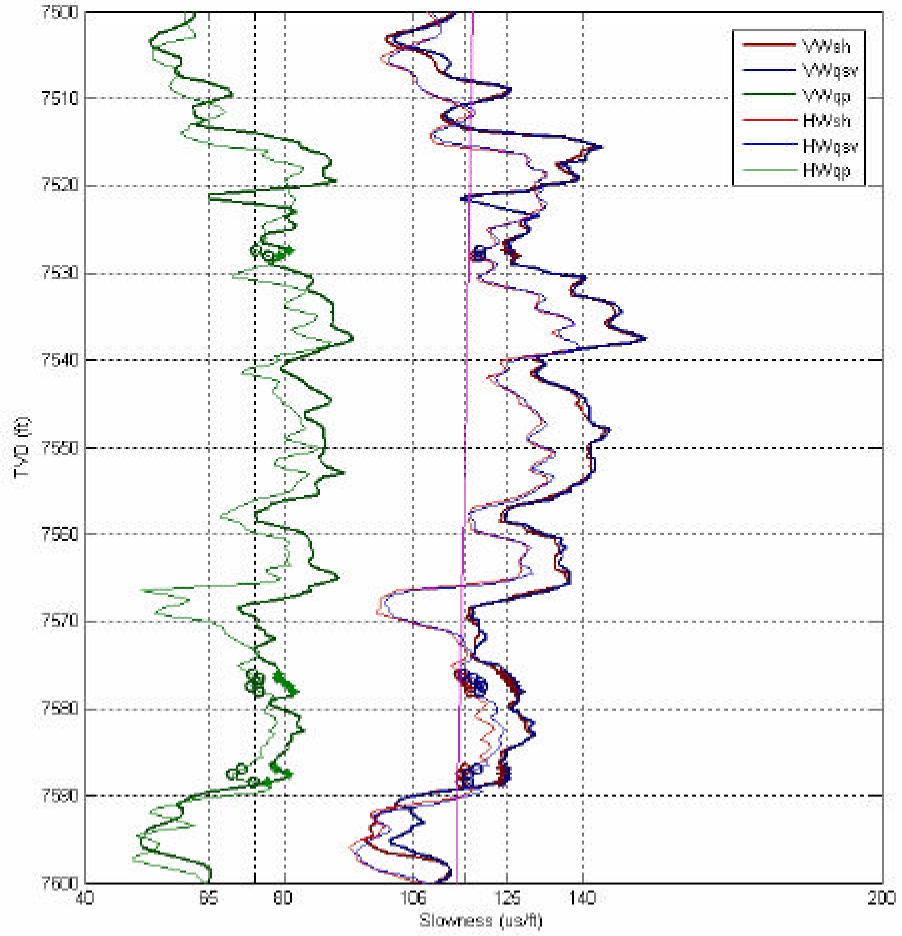
John Walsh, Bikash Sinha, Tom Plona and Doug Miller, Doug Bentley Schlumberger Oilfield Services*

Mike Ammerman, Devon Energy, Inc.

23 points in upper and lower Barnett.

10 points in upper Barnett. Inclination near 35 deg.





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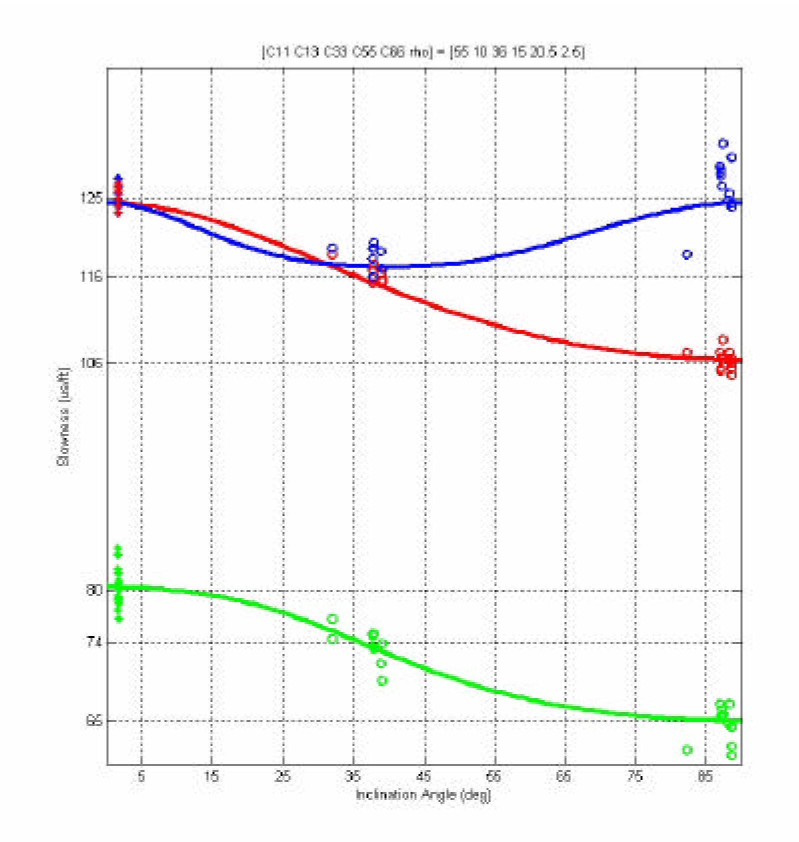
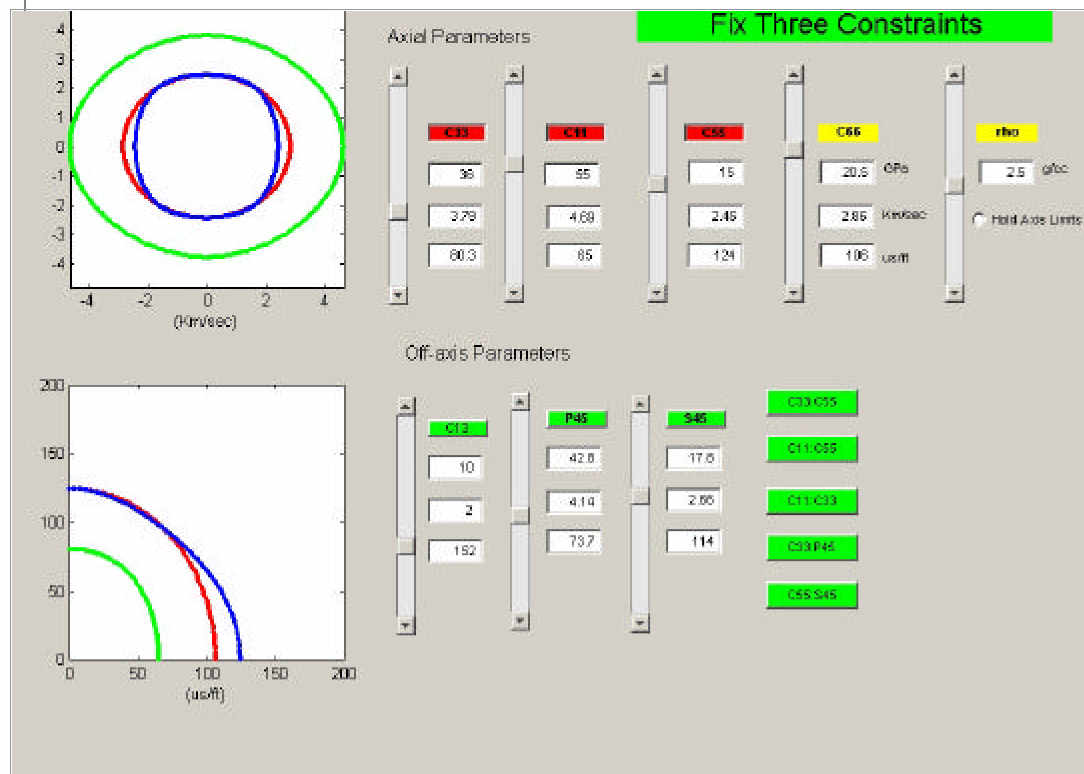
The Answer: Mild Anellipticity

- Good Fit with C13 = 10 Gpa
- So Thomsen's Delta = .12; (Epsilon = .26; Gamma = .18)





[C11,C13,C33,C55,C66]=[55,10,36,15,20.5]

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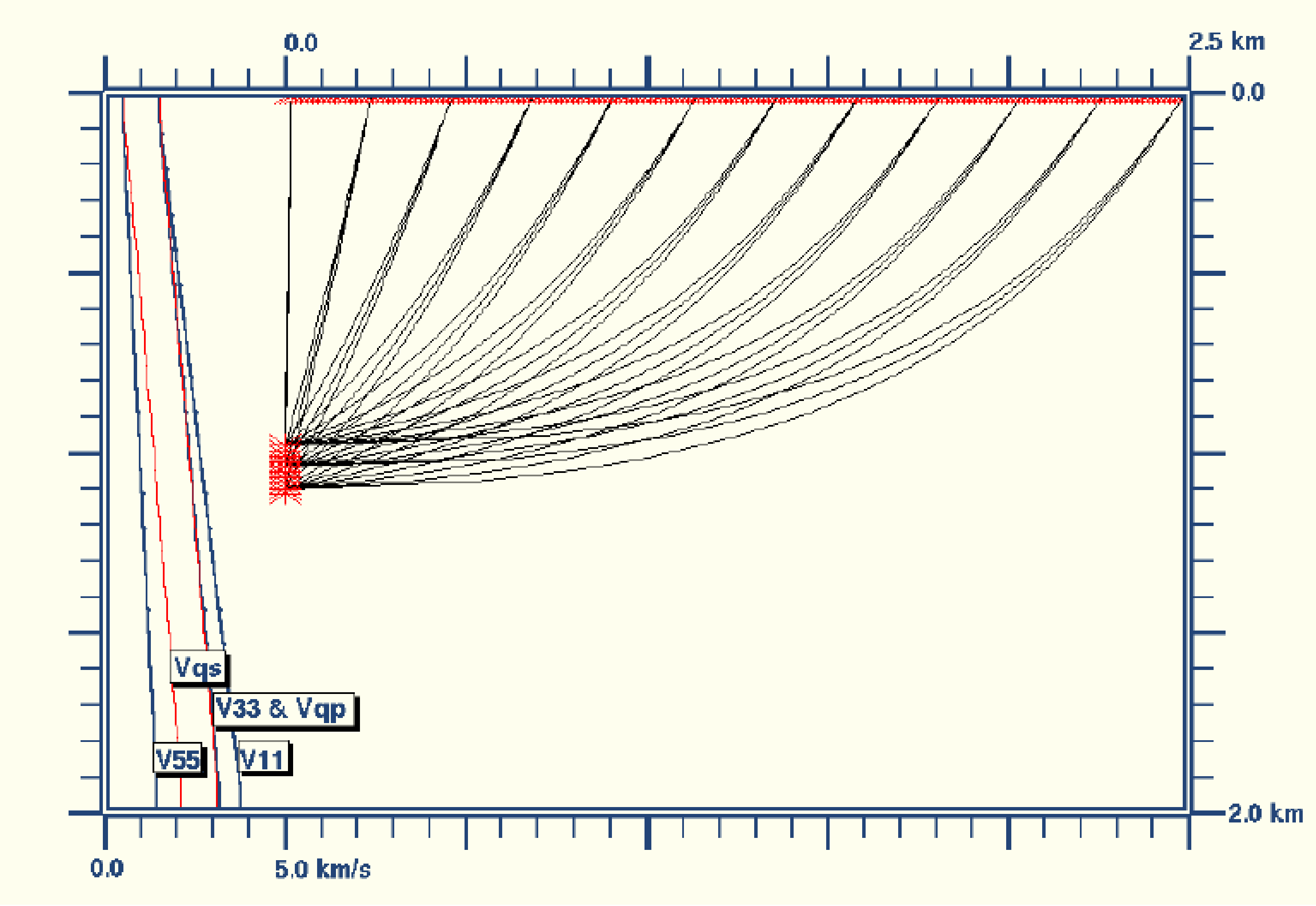

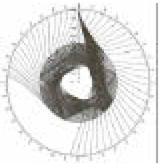

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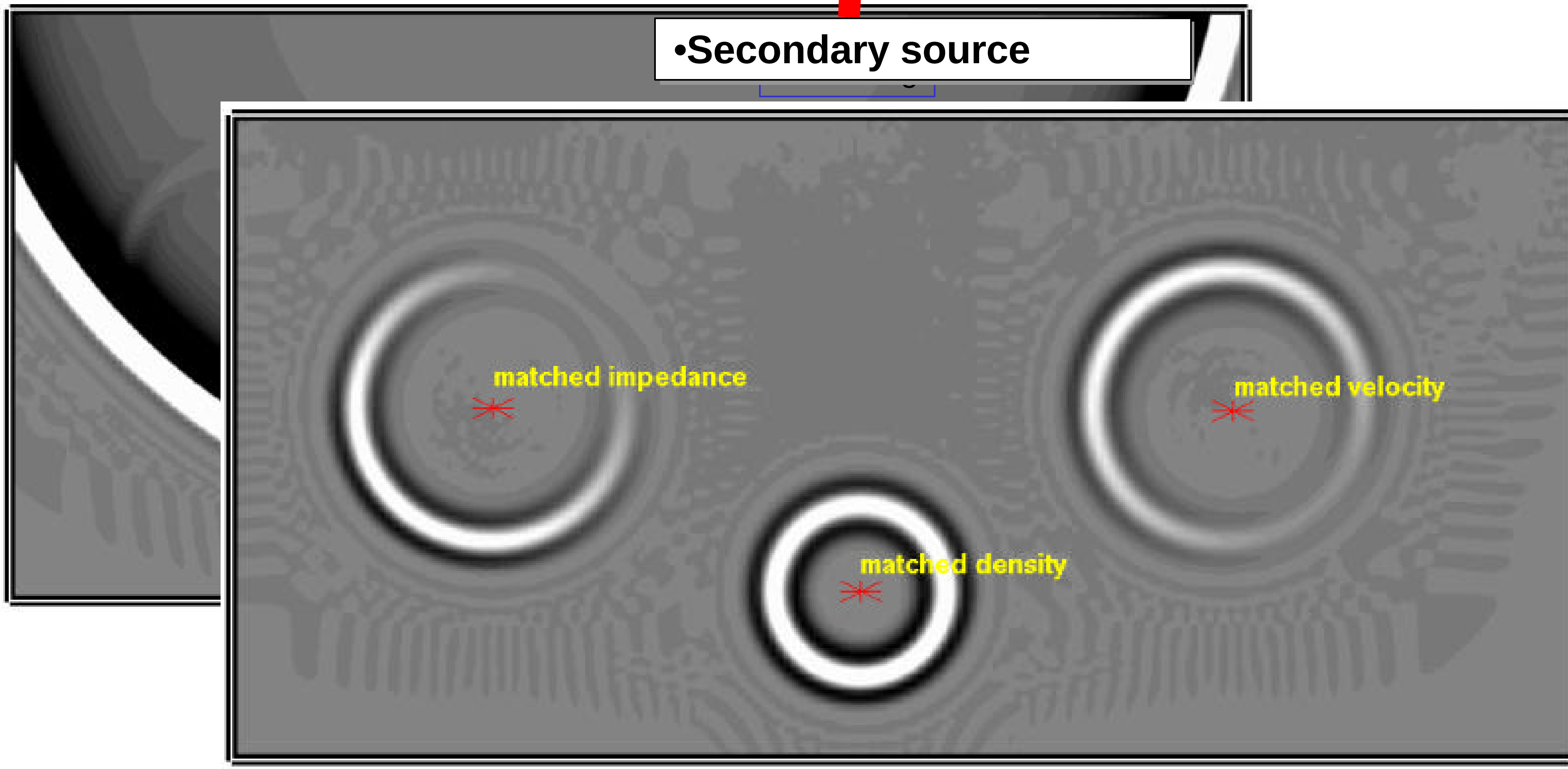
$$u_{sc}(\mathbf{r}, \mathbf{s}, \omega) = \int d^3\mathbf{x}\, G(\mathbf{r}, \mathbf{x}, \omega)\, [\omega^2 \kappa + \nabla \sigma \nabla]\, u(\mathbf{x}, \mathbf{s}, \omega)$$

Propagation

Incident field

•**Secondary source**

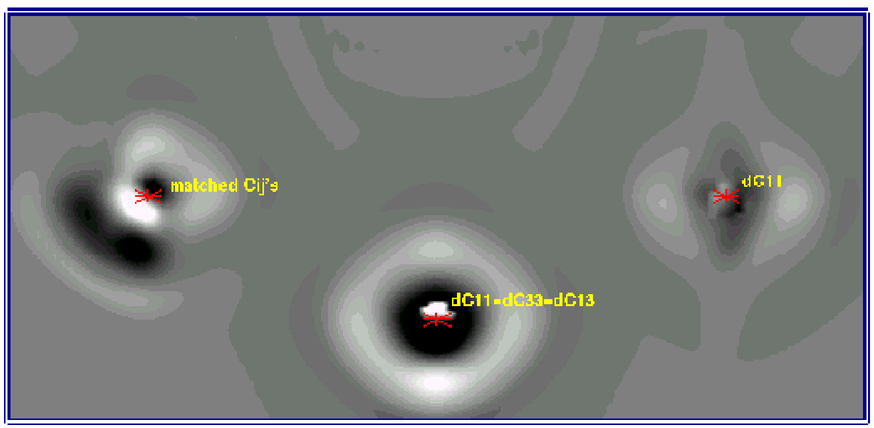


Exploding Reflectors

Exploding Reflectors

$$u_{pq}^{(1)(NM)}(\boldsymbol{r},\boldsymbol{s},t) = -\int_{\mathcal{D}} \grave{A}(\boldsymbol{x})\acute{A}(\boldsymbol{x})\grave{\xi}_k(\boldsymbol{x})\grave{\xi}_q(\boldsymbol{s})\acute{\xi}_p(\boldsymbol{r})\acute{\xi}_i(\boldsymbol{x})$$

$$\times\, \delta''(t-\grave{\tau}(\boldsymbol{x})-\acute{\tau}(\boldsymbol{x}))\,[\rho^{(1)}(\boldsymbol{x})\delta_{ki} + c^{(1)}_{ijk\ell}(\boldsymbol{x})\grave{\gamma}_\ell(\boldsymbol{x})\acute{\gamma}_j(\boldsymbol{x})]\,\mathrm{d}\boldsymbol{x}.$$



•Secondary source moment tensor

$$\frac{1}{A^{(N)}}\frac{\mathrm{d}A^{(N)}}{\mathrm{d}\tau^{(N)}} = -\tfrac{1}{2}\boldsymbol{\nabla}\boldsymbol{\cdot}(\rho^{(0)}\boldsymbol{v}^{(N)}).$$

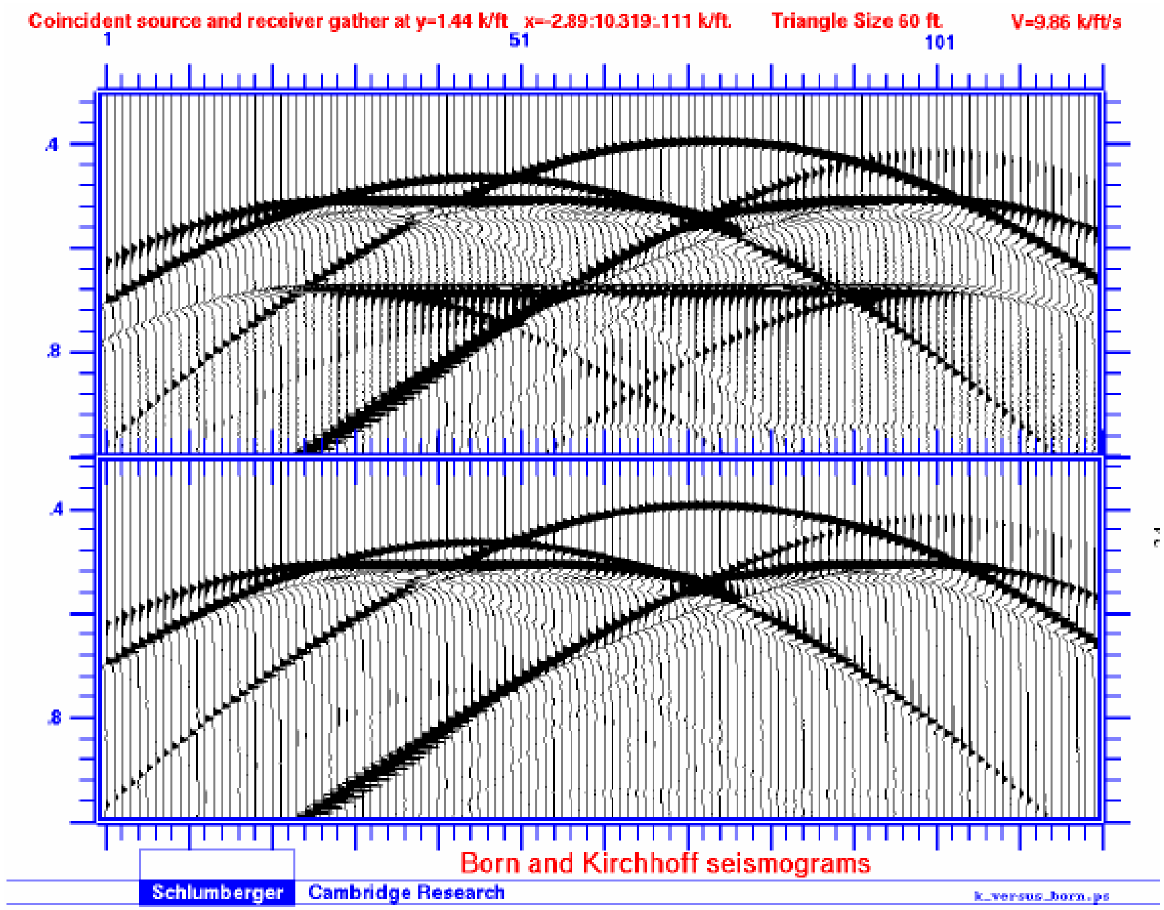


Figure 22: A coincident source and receiver profile over the French model calculated by Born volume integration (top) and Kirchhoff surface integration (bottom).

9 TI Zero-offset GRT Migration/Inversion

9.1 GRT inversion formula:

$$\langle f(\mathbf{x}_o) \rangle = \frac{1}{\pi^2} \int d^2\xi(\mathbf{s},\mathbf{x}_o)\, \frac{|\beta(\mathbf{s},\mathbf{x}_o)|^3}{A(\mathbf{s},\mathbf{x}_o)^2}\, u_{sc}(\mathbf{s}, t=\tau_o).$$

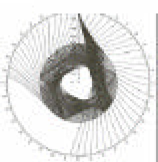
9.2 Simplification

$$d^2\xi \frac{\beta^2}{A^2(\mathbf{s},\mathbf{x}_o)} = ds_1\, ds_2\ \cos(\alpha)$$

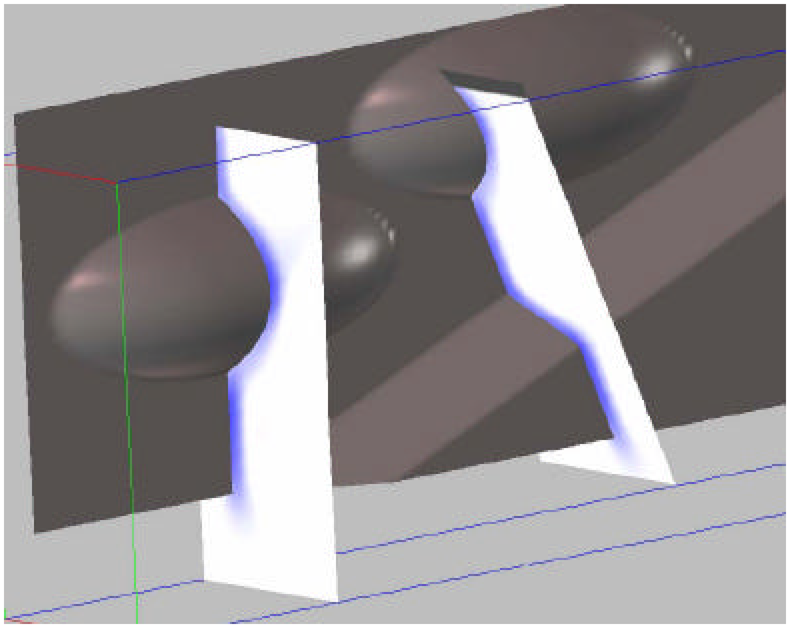
where α is the vertical phase angle at the surface.

9.3 What I Calculated:

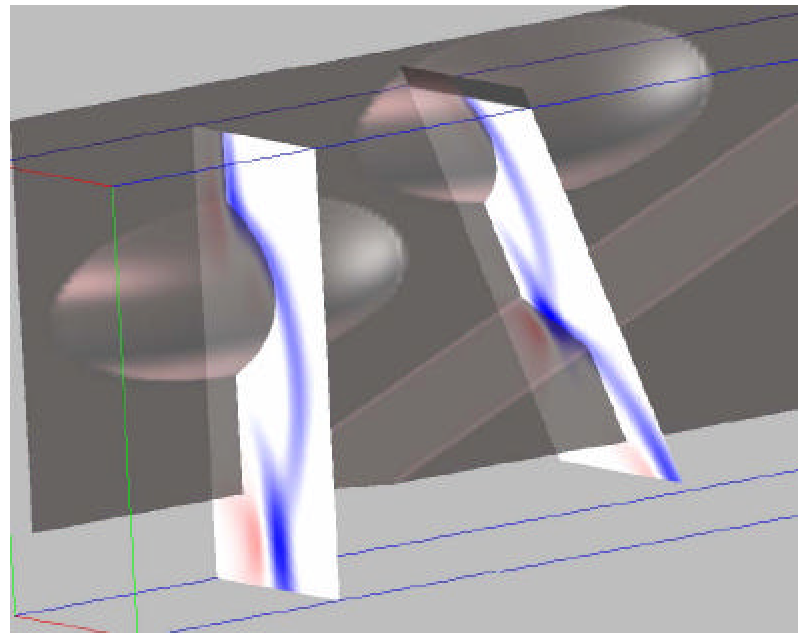
$$\int ds_1\, ds_2\ \cos(\alpha)\, |\beta(\mathbf{s},\mathbf{x}_o)|\, u_{sc}(\mathbf{s}, t=\tau_o)$$



Turning-ray migration of Vertical Object

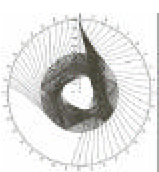


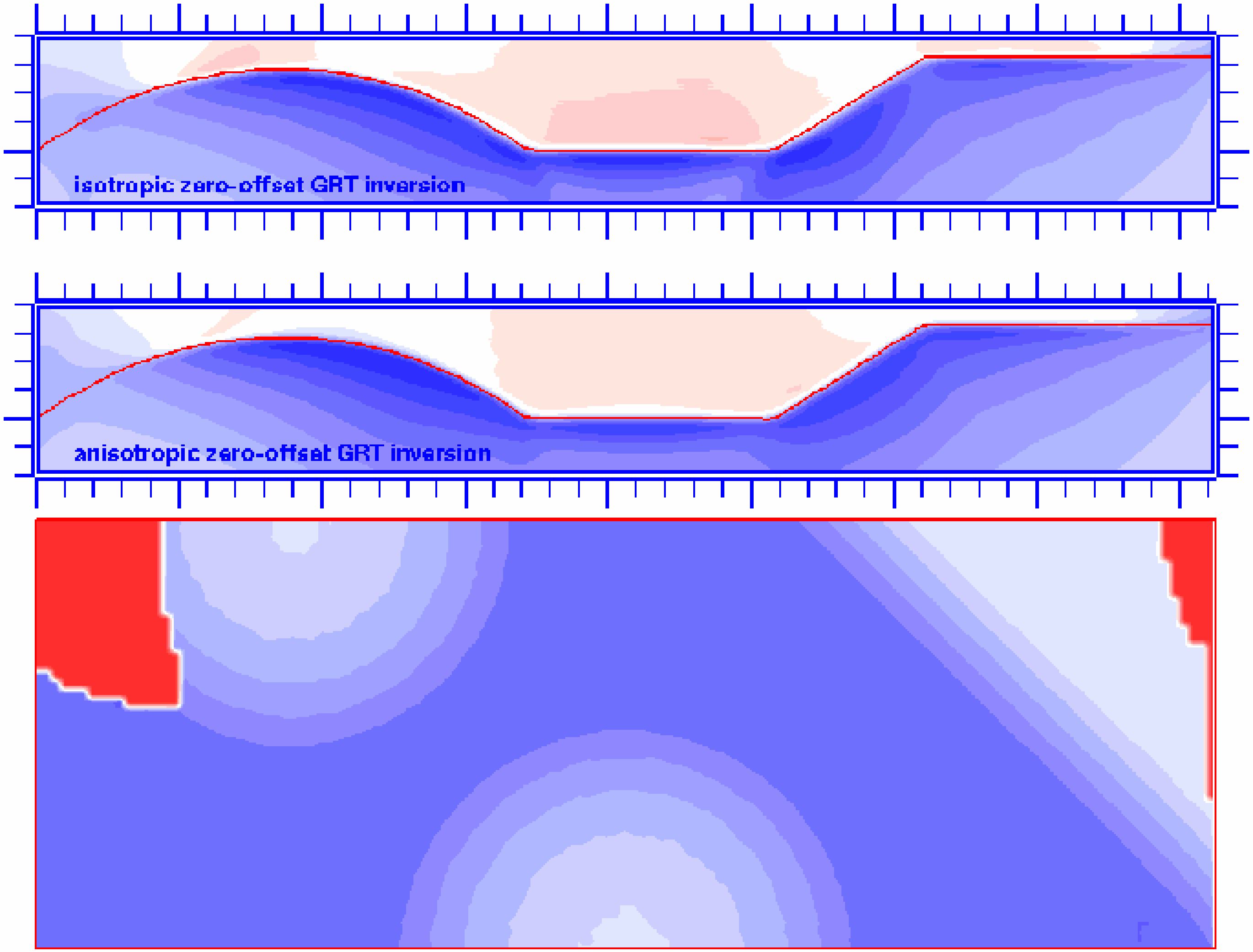
Anisotropic

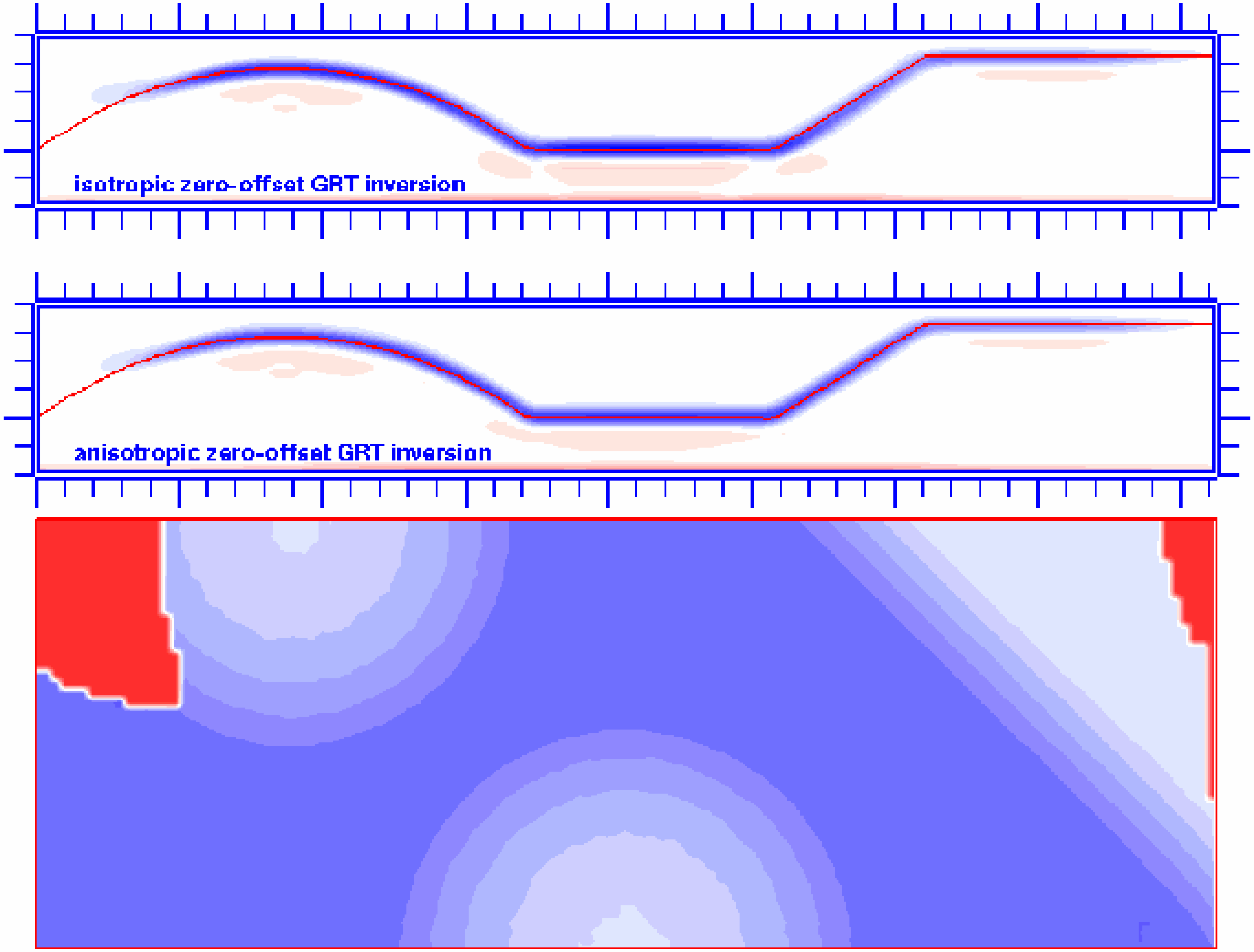


Isotropic

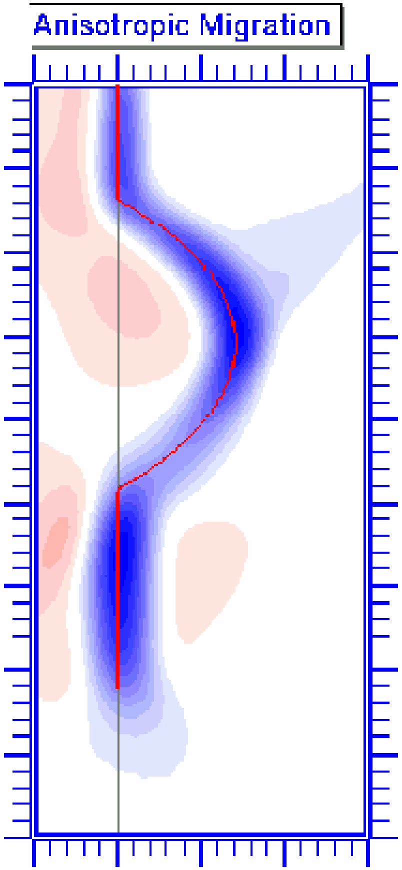
(vertical velocities)

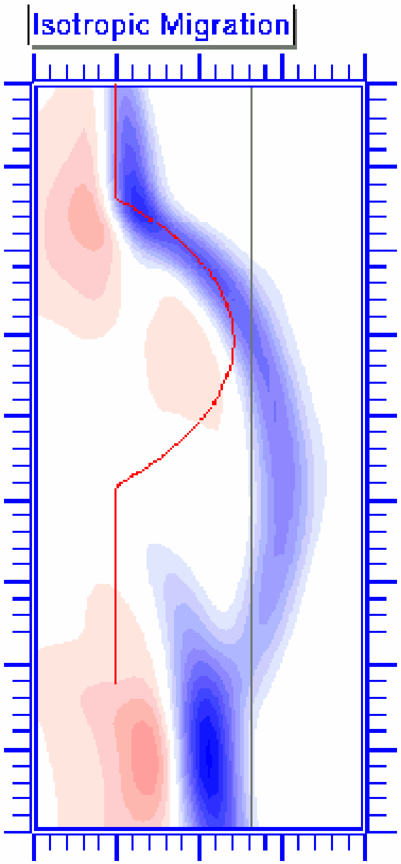


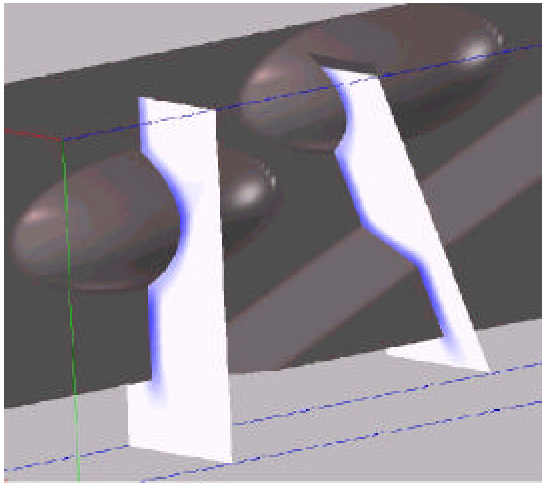

isotropic zero-offset GRT inversion
anisotropic zero-offset GRT inversion

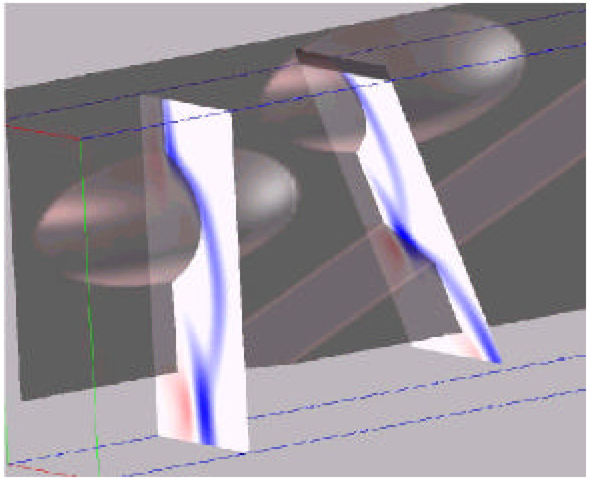

isotropic zero-offset GRT inversion
anisotropic zero-offset GRT inversion

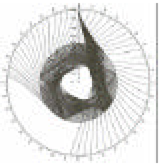
Turning Ray Images



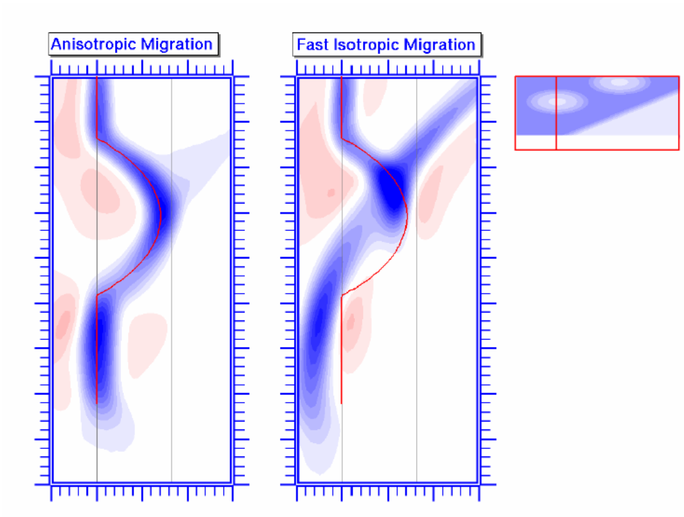


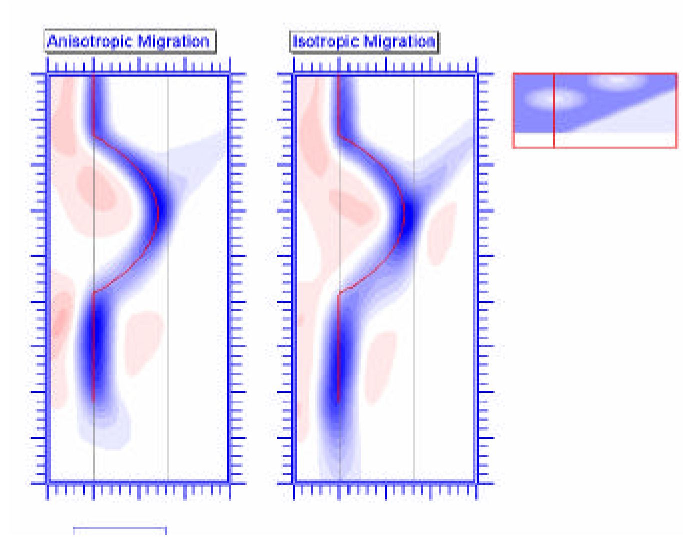


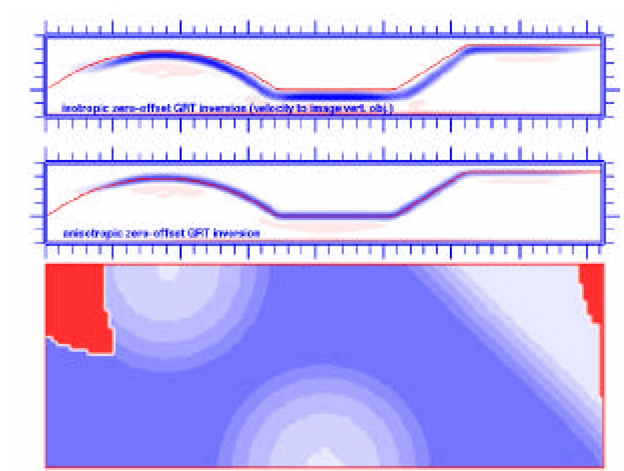




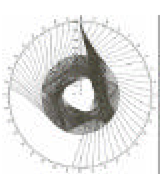
Schlumberger

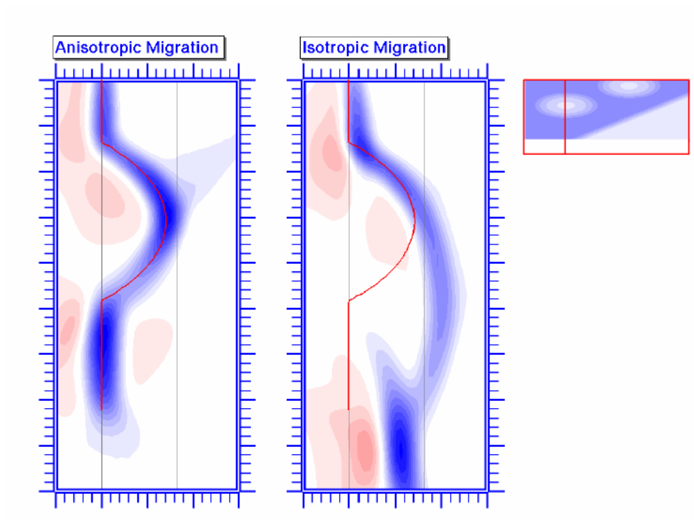


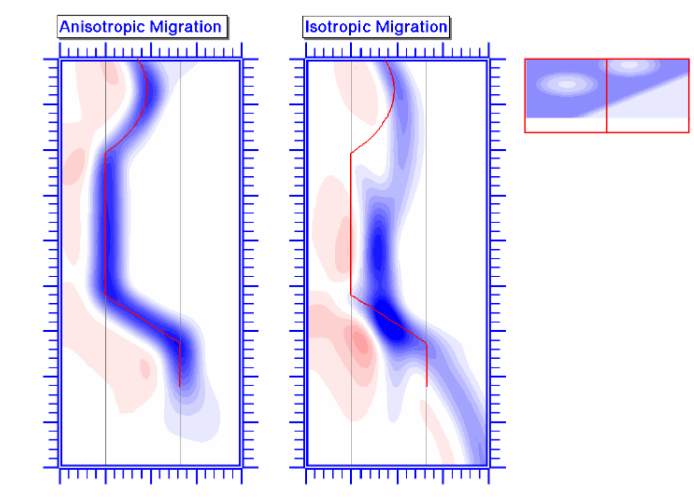




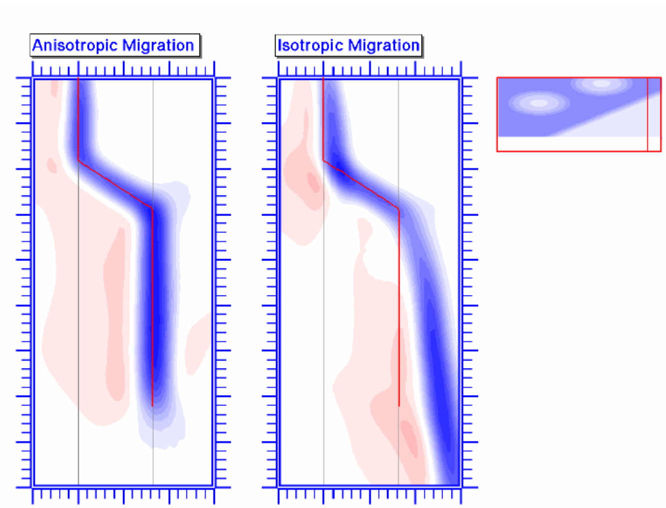
Isotropic Migration using a velocity profile that focuses the vertical object mislocates the horizontal object.







Isotropic Migration using vertical velocity profile systematically defocuses and mislocates vertical object



Anecdotes about Anisotropy:

Occam's Razor Cuts Both Ways

Entities are not to be multiplied beyond necessity

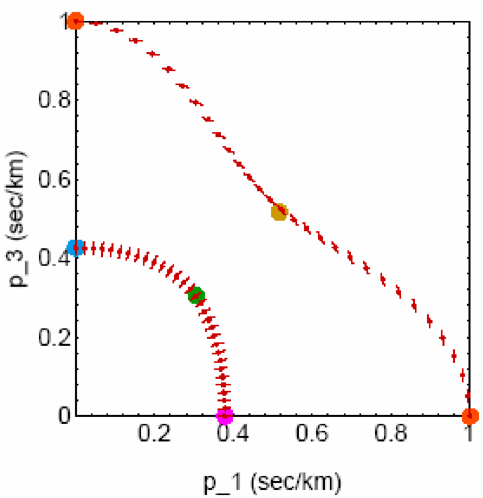
- William of Ockham as paraphrased by John Ponce of Cork.

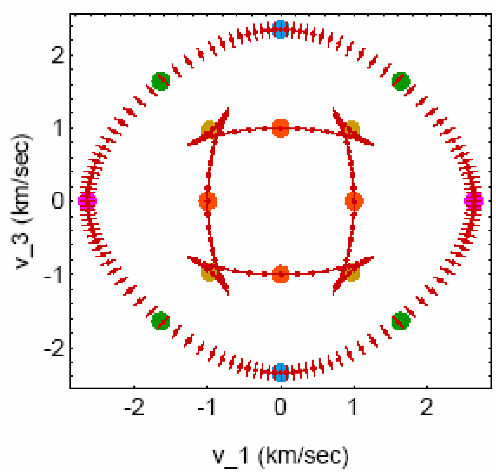
Entities must not be reduced to the point of inadequacy

- Walter of Chatton as paraphrased by Karl Menger.

PushPin Parameters

P_{0°	A_{11}
P_{90°	A_{33}
S_{0°	A_{55}
S_{90°	A_{55}
P_{45°	$.25(A_{11}+A_{33}+2(A_{13}+2A_{55}))$
S_{45°	$.25(A_{11}+A_{33}-2A_{13})$





Thomsen Parameters

$$\varepsilon \equiv \frac{C_{11}-C_{33}}{2C_{33}};$$

$$\gamma \equiv \frac{C_{66}-C_{44}}{2C_{44}};$$

$$\delta \equiv \frac{1}{2}\left[\varepsilon + \frac{\delta^*}{(1-\beta_0^2/\alpha_0^2)}\right]$$

$$= \frac{(C_{13}+C_{44})^2-(C_{33}-C_{44})^2}{2C_{33}(C_{33}-C_{44})}.$$