Multiparameter Inversion, Dip-Moveout, and the Generalized Radon Transform [1]

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Abstract

In a recent paper, G. Beylkin and R. Burridge developed an algorithm, based on the inverse Generalized Radon Transform, for multiparameter inversion of surface seismic reflection data. In their approach, elastic parameters (μ, λ, ρ) of the object medium are represented as linear combinations of three scalar potentials. These potentials can be separately recovered from three prestack GRT migrations in which the obliquity factor is varied. The material parameters are then obtained by solving a small linear system of equations at each object point.

This algorithm can be recast in terms of a GRT-based dip-movout operator. In this approach, zero-offset data for each scalar potential are synthesized from positive-offset data by application of a time-domain DMO operator with an appropriate obliquity dependence. Multiple copies of the synthesized zero-offset data are then stacked and migrated as in ordinary DMO processing. This reconstructs the scalar potentials, which can in turn be solved for the material parameters.

The GRT-DMO operator is obtained by applying a stationary-phase reduction of the cascaded operations of prestack GRT migration and forward zero-offset modelling. The operator differs from previously described time-domain DMO operators in the modification of stacking amplitudes and in the presence of one-dimensional filters that are applied before and after stacking.

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1. Introduction

Dip-moveout operators were introduced under various names ("DEVIL-ISH," Judson, *et.al.,* 1978), ("Partial prestack migration," Yilmaz and Claerbout, 1980), ("Dip-moveout, Bolondi, *et.al.,* 1982) as a remedy for the dip-filtering effects of the standard seismic processing chain (CDP stack + zero-offset migration). Deregowski and Rocca (1981) interpreted the problem in a form that is fundamental to the work reported here. In their view the goal for DMO processing is to convert input data, typically gathered into constant-offset files, into equivalent zero-offset data. Geometrically, the operation can be described as a stacking process in which every output point (corresponding to a midpoint and a time) is associated with a semicircular isochron curve centered at the midpoint and each input point (corresponding to a source, a receiver, and a time) is associated with an elliptical isochron curve with the source and receiver at the foci. The output value at each point is obtained as a weighted stack of all input values associated to ellipses that lie tangent to the circle associated with the output point. Formally, their analysis proceeded by cascading prestack migration with forward modelling and then eliminating an inner operator by means of a stationary-phase approximation. Hale (1983) derived a now popular version of the DMO operator from prestack Stolt migration.

More recently, an approach to seismic inversion has been developed from analysis of the Generalized Radon Transform (GRT). Originally derived as a solution to the scalar inversion problem of reconstructing a velocity perturbation in a constant-density acoustic medium (Miller, *et.al.,* 1984, 1987, Beylkin, 1985), the method has recently been extended to a solution of the vector inversion problem of reconstructing arbitrary perturbations of material parameters in an acoustic or elastic medium (Beylkin and Burridge, 1990).

Because of the highly geometric flavor of the GRT approach, it is amenable to a DMO analysis along the lines set forth by Deregowski and Rocca. Jorden (1987) analyzed the GRT velocity inversion and derived DMO operators for various experimental geometries (e.g. common shot, common offset). A question raised there (*cf.* Jorden, 1987 p.44) is the apparent dependence of the operator on the way in which the data is sorted — appying shot DMO to common source gathers and then stacking over source files gives a different result than applying constant-offset DMO and stacking over offset files. (Bleistein and Jorden, 1987) suggested that an averaging of shot weights and receiver weights be incorporated in the stacking process to symmetrize the prestack inversion operator, but the formal interpretation of the resultant operator was unclear (Jorden, 1987, pp. 57-58). Some related work on DMO and Born inversion may be found in (Liner, 1988).

In the present paper we show that the DMO analysis of the GRT multiparameter algorithm as described in (Beylkin and Burridge, 1990) can be obtained by a straightforward, geometrically motivated argument and that the resulting operator is independent of any data sorting. From the viewpoint of the multiparameter inversion algorithm, the possible benefit of such an approach is a speedup in processing analogous to the speedup obtained by using conventional DMO in place of conventional prestack migration. From the viewpoint of DMO algorithms, the possible benefit is a clearer picture of the theoretical basis, and a new view of "amplitude-preserving" DMO.

The paper will consist of a brief review of the GRT method, including its application to scalar problems other than constant density and to the multiparameter inversion problem, followed by a discussion of how to recast the algorithm in terms of a DMO operation. By the way, whenever we discuss "DMO" we mean "reduction to zero-offset" (that is, what has commonly been referred to as "NMO + DMO").

2. Acoustic Inversion and the GRT

In this section we review the acoustic GRT inversion framework essentially as presented in (Miller, *et.al.,* 1987), but incorporating the extension to variable-density media suggested by the exposition in (Beylkin and Burridge, 1990). To simplify the discussion, we will assume throughout the paper that we are in a two-dimensional acoustic world in which scattered wavefields arise from perturbations from a homogeneous background medium with density $\rho_o = 1$, velocity $c_o = 1$. Let the true medium be defined by parameters $\sigma(\mathbf{x})$ and $\kappa(\mathbf{x})$ where $\sigma(\mathbf{x}) = (\frac{1}{\rho(\mathbf{x})} - 1)$ is the perturbation at $\mathbf{x} = (x_0, x_1)$ in specific volume and $\kappa(\mathbf{x}) = (\frac{1}{\rho(\mathbf{x})c(\mathbf{x})^2} - 1)$ is the perturbation in compliance.

Let $u(\mathbf{x}, \mathbf{s}, \omega)$ and $G(\mathbf{x}, \mathbf{s}, \omega)$ be the Green functions of the total and background media, respectively, and write $u_{sc} = u - G$ for their difference (the *scattered wavefield*). The acoustic wave equation can be written as an integral equation (*cf.* Stolt and Weglein, 1985, equation (44), or Beylkin and Burridge, 1990, equation (1-10))

$$u_{sc}(\mathbf{r}, \mathbf{s}, \omega) \;=\; \int d^2\mathbf{x}\, G(\mathbf{r}, \mathbf{x}, \omega)\, [\omega^2 \kappa + \nabla \cdot \sigma \nabla]\, u(\mathbf{x}, \mathbf{s}, \omega). \tag{1}$$

Here, ω is temporal frequency, $\mathbf{s} = (s_0, s_1)$ and $\mathbf{r} = (r_0, r_1)$ are source and receiver locations, and $\mathbf{x}$ ranges over the set of points where $|\kappa(\mathbf{x})| + |\sigma(\mathbf{x})| \neq 0$. Since we assume a homogeneous 2D medium with velocity 1, G has the approximate form (accurate for large $\omega|\mathbf{y} - \mathbf{x}|$)

$$G(\mathbf{x}, \mathbf{y}, \omega) \;=\; (-i\omega)^{-1/2} |\mathbf{y} - \mathbf{x}|^{-1/2}\, e^{i\omega|\mathbf{y}-\mathbf{x}|}.$$

Assuming weak scattering (Born approximation) and taking a Fourier transform over ω, equation (1) can be rewritten (*cf.* Stolt and Weglein, 1985, equation (54), or Beylkin and Burridge, 1990, equation (1-21))

$$u_{sc}(\mathbf{r}, \mathbf{s}, t) \;=\; -\frac{\partial}{\partial t} \int d^2\mathbf{x}\, A(\mathbf{r}, \mathbf{x}, \mathbf{s})\, [\kappa(\mathbf{x}) + \sigma(\mathbf{x}) \cos\theta]\, \delta\, [t - T(\mathbf{r}, \mathbf{x}, \mathbf{s})]\,. \tag{2}$$

Here,

$$A(\mathbf{r}, \mathbf{x}, \mathbf{s}) \;=\; |\mathbf{r} - \mathbf{x}|^{-1/2} |\mathbf{x} - \mathbf{s}|^{-1/2}$$

is the total geometrical spreading amplitude,

$$T(\mathbf{r}, \mathbf{x}, \mathbf{s}) \;=\; |\mathbf{r} - \mathbf{x}| + |\mathbf{x} - \mathbf{s}|$$

is the total traveltime, and $\theta = \theta(\mathbf{r}, \mathbf{x}, \mathbf{s})$ is the angle between rays connecting the scattering point $\mathbf{x}$ with the source $\mathbf{s}$ and receiver $\mathbf{r}$.

$u_{sc}(\mathbf{r}, \mathbf{s}, t)$ represents a Generalized Radon Transform (that is, an integral) over an elliptical isochron curve $I_{\mathbf{r},\mathbf{s},t} = \{\mathbf{x} : t = T(\mathbf{r}, \mathbf{x}, \mathbf{s})\}$ of an obliquity-dependent "scattering potential" $f = [\kappa + \sigma \cos\theta]$ (Figure 1). The GRT approach to acoustic inversion is based on the close analogy between the scattering equation (2), and the ordinary Radon transform.

Given a scalar potential $f(\mathbf{x})$, the Radon transform of f is defined for each angle ϕ and real number p by the equation

$$f^{\triangle}(\phi, p) \;=\; \int d^2\mathbf{x}\, f(\mathbf{x})\, \delta(p - (x_0 \sin\phi + x_1 \cos\phi)) \tag{3}$$

Thus, $f^{\triangle}(\phi, p)$ is the integral of f over the line p units from the origin, with dip angle ϕ. f can be recovered from $f^{\triangle}$ by the Radon Inversion Formula (e.g. Deans, 1983):

$$f(\mathbf{x}) \;=\; -\frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\phi\, \mathcal{H} \frac{\partial}{\partial p} f^{\triangle}(\phi, p = (x_0 \sin\phi + x_1 \cos\phi)), \tag{4}$$

where $\mathcal{H}$ denotes the Hilbert transform (principal-value integral),

$$\mathcal{H}u(p) \;=\; \frac{1}{\pi} \int_{-\infty}^{+\infty} dp' \, \frac{u(p')}{p - p'} \; .$$

The presence of the time derivative, the spatially variant nature of the $\cos\theta$ and $A(\mathbf{r}, \mathbf{x}, \mathbf{s})$ terms, and of the isochrons themselves combine to make the acoustic transform (2) less attractive for inversion than the simple Radon transform (3). However, the spatial variations are locally small, and the Radon inversion operator (4) is essentially local in nature. Furthermore, in the vicinity of each scattering point, we can match isochron curves to their tangent lines to obtain a correspondence between experimental variables $t, \mathbf{s}$, and $\mathbf{r}$, and geometric variables p, ϕ, and θ. In this correspondence, t maps to $2p\cos(\frac{\theta}{2})$, with partial derivative $\frac{\partial t}{\partial p} = 2\cos(\frac{\theta}{2})$. Exploiting the correspondance, one may perform an appropriate change of variables in the Radon inversion formula to obtain an approximate inversion of the acoustic GRT. Depending on assumptions about the experimental geometry and about the complexity of the scattering medium, this GRT inversion operator takes various concrete forms. In all cases, however, it has been shown (Beylkin, 1985, Beylkin and Burridge, 1990) that the operator correctly locates and quantifies the highest-order discontinuities in the scattering medium. Some specific examples follow.

3. Acoustic GRT Inversion (Scalar Case)

Constant Density ($\sigma = 0$). This is the case treated in the first references (Miller, *et.al.,* 1984, 1987, Beylkin, 1985). If we assume that the true and reference media share identical density values, then $[\kappa + \sigma \cos\theta] = \kappa$ and the scattering equation (2) simplifies to

$$u_{sc}(\mathbf{r}, \mathbf{s}, t) \quad = \quad -\frac{\partial}{\partial t} \int d^2\mathbf{x}\, A(\mathbf{r}, \mathbf{x}, \mathbf{s})\, [\kappa(\mathbf{x})]\, \delta\, [t - T(\mathbf{r}, \mathbf{x}, \mathbf{s})]\,. \qquad (5)$$

The GRT inversion formula for this case (*cf.* Miller, *et.al.,* 1987, (27a)) is

$$\langle \kappa(\mathbf{x}) \rangle \quad = \quad -\frac{2}{\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\phi\, \frac{\cos^2(\frac{\theta}{2})}{A(\mathbf{r}, \mathbf{x}, \mathbf{s})}\, \mathcal{H} u_{sc}(\mathbf{r}, \mathbf{s}, t = T(\mathbf{r}, \mathbf{x}, \mathbf{s})). \qquad (6)$$

where the angular brackets enclosing $\kappa(\mathbf{x})$ indicate the approximate nature of the reconstruction. Note that the partial derivative with respect to p in (4) has cancelled with the time derivative in (5) leaving a Jacobian term $\cos^2(\frac{\theta}{2})$ from the change from p to t. The additional Jacobian term $d\phi$ represents the rate of change of the dip angle ϕ with respect to a single variable (such as receiver offset in a single-source experiment) indexing input data traces. See (Miller, *et.al.,* 1987) for further discussion. The amplitude term $\frac{1}{A(\mathbf{r}, \mathbf{x}, \mathbf{s})}$ is just the reciprocal of the amplitude term in (5).

Constant Bulk Modulus ($\kappa = 0$). Other scalar inversion problems can be solved in the same way by treating additional obliquity terms from the scattering equation just as we have treated geometrical spreading. For example, if we assume that the true and reference media share identical compliance values, then $\kappa = 0$ and $[\kappa + \sigma \cos\theta] = \sigma \cos\theta$. Balancing the inversion equation by dividing out the extra obliquity term, we obtain the inversion equation

$$\langle\sigma(\mathbf{x})\rangle \;=\; -\frac{2}{\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\phi\, \frac{\cos^2(\frac{\theta}{2})\cos^{-1}(\theta)}{A(\mathbf{r},\mathbf{x},\mathbf{s})}\, \mathcal{H}u_{sc}(\mathbf{r},\mathbf{s},t=T(\mathbf{r},\mathbf{x},\mathbf{s})). \quad (7)$$

Constant Velocity ($\sigma = \kappa$). The case of constant velocity is interesting enough to include here explicitly. If the true and reference media share identical velocity values, then $\sigma = \kappa$ and $[\kappa+\sigma\cos\theta] = [\sigma+\sigma\cos\theta] = 2\sigma\cos^2(\frac{\theta}{2})$. The new obliquity term miraculously cancels the Jacobian obliquity term and we obtain the inversion equation

$$\langle\sigma(\mathbf{x})\rangle \;=\; -\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\phi\, \frac{1}{A(\mathbf{r},\mathbf{x},\mathbf{s})}\, \mathcal{H}u_{sc}(\mathbf{r},\mathbf{s},t=T(\mathbf{r},\mathbf{x},\mathbf{s})). \quad (8)$$

A similar constant-velocity inversion operator was derived in (Dillon, 1990).

Clearly there is something fundamentally unsatisfying about all of the operators described above. Each solves a well-posed scalar problem, but each scalar problem is created by placing unrealistic assumptions on the underlying vector problem of inverting for a medium with arbitrary variations in both κ and σ. A second, more subtle issue is the assumption that the dip variable ϕ translates uniquely into a single variable indexing input data traces. The scalar analysis gives no suggestion about how to combine data from multiple-shot, multiple-receiver experiments. Fortunately, the issue of multiparameter media and the issue of too much input data form a problem/antiproblem pair that is subject to energetic cancellation. (Beylkin and Burridge, 1990) treats in detail the GRT-based solution to this problem. The next section summarizes the main points of this analysis for the 2D acoustic case.

4. Acoustic GRT Inversion (Multiparameter Case)

A key motivating observation is that there are really two "natural" geometric variables, dip ϕ and obliquity θ, to map to experimental variables s and r. Assume a sufficient supply of sources and receivers (e.g. on a large circle surrounding the scattering medium, with a full circle of sources and a full circle of receivers for each source) so that for each point $\mathbf{x}$, and each pair of angles (ϕ, θ), there is a source-receiver pair s, r satisfying the geometric relationship of Figure 1. Write $\mathbf{d}(\mathbf{x}, \phi, \theta) = (\mathbf{r}, \mathbf{s}, T(\mathbf{r}, \mathbf{x}, \mathbf{s}))$ for the data point thereby associated with $\mathbf{x}, \phi, \theta$ and write $A(\mathbf{x}, \phi, \theta)$ for $A(\mathbf{r}, \mathbf{x}, \mathbf{s})$. For any fixed θ, write $f_\theta(\mathbf{x})$ for the scalar scattering potential at obliquity θ:

$$f_\theta(\mathbf{x}) \quad = \quad [\kappa(\mathbf{x}) + \sigma(\mathbf{x}) \cos\theta].$$

Then the basic GRT inversion operator (5) gives an approximate reconstruction of f_θ:

$$\langle f_\theta(\mathbf{x}) \rangle \quad = \quad -\frac{2}{\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\phi \, \frac{\cos^2(\frac{\theta}{2})}{A(\mathbf{x}, \phi, \theta)} \, \mathcal{H} u_{sc}(\mathbf{d}(\mathbf{x}, \phi, \theta)). \tag{9}$$

By integrating over θ , with and without an additional obliquity factor, we can recover the material parameters:

$$\int_0^\pi d\theta f_\theta \quad = \quad \pi\kappa, \qquad \int_0^\pi d\theta \cos(\theta) f_\theta = \frac{\pi}{2}\sigma \tag{10}$$

Combining (9) with (10) we obtain the basic GRT multiparameter operators:

$$\begin{aligned} \langle \kappa(x) \rangle \quad &= \quad -\frac{2}{\pi^2} \int_0^\pi d\theta \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\phi \, \frac{\cos^2(\frac{\theta}{2})}{A(\mathbf{x}, \phi, \theta)} \, \mathcal{H} u_{sc}(\mathbf{d}(\mathbf{x}, \phi, \theta)) \\ \langle \sigma(x) \rangle \quad &= \quad -\frac{4}{\pi^2} \int_0^\pi d\theta \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\phi \, \frac{\cos(\theta)\cos^2(\frac{\theta}{2})}{A(\mathbf{x}, \phi, \theta)} \, \mathcal{H} u_{sc}(\mathbf{d}(\mathbf{x}, \phi, \theta)) \end{aligned} \tag{11}$$

Some modification of these basic operators is required to take into account the limitations posed by practical experimental geometry. For the remainder of the paper, we take a mathematician's definition of "practical" and assume sources and receivers range over the x-axis, writing $\mathbf{s} = (s, 0)$, $\mathbf{r} = (r, 0)$. This imposes a limitation on the available range of values for θ and ϕ to

$$\mathcal{R}_0 \quad = \quad \{(\phi, \theta) : |\theta| < \pi \text{ and } |\phi| < \frac{\pi - |\theta|}{2}\}.$$

To insure uniformity in the inversion operators, we must further restrict our attention to a rectangular subset of $\mathcal{R}_0$

$$\mathcal{R}_1 \quad = \quad \{(\phi,\theta) : |\theta| < \theta_{max} \text{ and } |\phi| < \frac{\pi - \theta_{max}}{2}\}$$

where θ_{max} is chosen to make a compromise between competing requirements on θ and ϕ .

With the restriction on θ , the integrals in (10) no longer give the material parameters directly. They do, however, give a pair of linear equations that can be solved for κ and σ. In matrix notation:

$$\begin{bmatrix} f^0 \\ \\ f^1 \end{bmatrix} = \begin{bmatrix} \int_0^{\theta_{max}} d\theta f_\theta \\ \\ \int_0^{\theta_{max}} d\theta \cos(\theta) f_\theta \end{bmatrix} = L \begin{bmatrix} \kappa \\ \\ \sigma \end{bmatrix} \tag{12}$$

where

$$L \quad = \quad \begin{bmatrix} \theta_{max} & \sin(\theta_{max}) \\ \\ \sin(\theta_{max}) & \dfrac{\theta_{max}}{2} + \sin(\theta_{max}) \end{bmatrix}.$$

The Jacobian of the transformation from variables of integration θ, ϕ to s, r has the simple form

$$[\theta\phi]_{s,r} \quad = \quad 4\frac{\partial\phi}{\partial s}\frac{\partial\phi}{\partial r} = 4\frac{(s - x_0)}{|\mathbf{s} - \mathbf{x}|^2}\frac{(r - x_0)}{|\mathbf{r} - \mathbf{x}|^2}. \tag{13}$$

Incorporating the change of variables and the resriction on θ, ϕ, into (11) and combining the Jacobian and geometrical spreading terms, we obtain the final form of the GRT multiparameter inversion operators:

$$\langle f^i(x) \rangle \quad = \quad -\frac{8}{\pi^2} \int ds \int dr \, W_i(\mathbf{r},\mathbf{x},\mathbf{s}) \, \mathcal{H}u_{sc}(\mathbf{r},\mathbf{s},t = T(\mathbf{r},\mathbf{x},\mathbf{s})) \tag{14}$$

where, for $i = 0, 1$,

$$W_i(\mathbf{r}, \mathbf{x}, \mathbf{s}) \quad = \quad \begin{cases} \frac{(s-x_0)}{|\mathbf{s}-\mathbf{x}|^{1.5}} \frac{(r-x_0)}{|\mathbf{r}-\mathbf{x}|^{1.5}} \cos^i(\theta) \cos^2(\frac{\theta}{2}) & \text{if } (\phi, \theta) \in \mathcal{R}_1 \\ \\ 0 & \text{otherwise.} \end{cases}$$

These operators (which can be computed simultaneously) provide estimates of the scalar potentials $f^0(\mathbf{x})$ and $f^1(\mathbf{x})$. The estimated material parameters are then obtained at each image point by matrx multiplication:

$$\begin{bmatrix} \langle \kappa(x) \rangle \\ \\ \langle \sigma(x) \rangle \end{bmatrix} \quad = \quad L^{-1} \begin{bmatrix} \langle f^0(x) \rangle \\ \\ \langle f^1(x) \rangle \end{bmatrix}. \tag{15}$$

Because of the restriction placed on the range of ϕ, these estimates are bandlimited in their angular spectrum.

5. The Multiparameter DMO Operators:

Recalling our introductory discussion, the DMO problem can be simply stated:

Find data fields $F^0(m, \tau), F^1(m, \tau)$ which represent zero-offset data for the scalar potentials $f^0(\mathbf{x}), f^1(\mathbf{x})$ (preferably using operators faster to compute than (14).)

Given such data fields, the scalar potentials can be recovered by zero-offset inversion (equation (9) with $\theta = 0$), and the material parameters can be obtained as before (equation (15)).

The outline of our approach is also simple to state: We substitute our estimated potential $\langle f^i(x) \rangle$ into the scattering equation (2) to obtain an expression for $F^i(m, \tau)$ as a triple integral. We then change the order of integration and collapse the innermost integral to a single term by means of a stationary-phase analysis.

We begin by restating (2) for the zero-offset case $\mathbf{s} = \mathbf{r} = (m, 0)$, $t = \tau$, $A(\mathbf{r}, \mathbf{x}, \mathbf{s}) = |\mathbf{r} - \mathbf{x}|^{-1} = \frac{2}{\tau}$, and with the integral over the semicircular isochron surface written explicitly in terms of an azimuthal angle ψ:

$$F^i(m, \tau) = \frac{\partial}{\partial \tau} \int d\psi \, \frac{2}{\tau} \, f^i(\mathbf{x} = (m + \frac{\tau}{2} \cos \psi, \frac{\tau}{2} \sin \psi)) \tag{16}$$

Substituting for f^i using (14) we have the expanded form (*cf.* Jorden):

$$F^i(m, \tau) = \frac{\partial}{\partial \tau} \int d\psi \int ds \int dr \, W_i(\mathbf{r}, \mathbf{x}, \mathbf{s}) \, \mathcal{H} u_{sc}(\mathbf{r}, \mathbf{s}, t = T(\mathbf{r}, \mathbf{x}, \mathbf{s})) \tag{17}$$

Here, and for the remainder of this section, $\mathbf{x} = (m + \frac{\tau}{2} \cos \psi, \frac{\tau}{2} \sin \psi)$, $\mathbf{s} = (s, 0)$, and $\mathbf{r} = (r, 0)$. Geometrically, (17) may be interpreted as a stack in which, for each point $\mathbf{x}$ on the semicircle with radius $\frac{\tau}{2}$ and center $(m, 0)$, all ellipses passing through $\mathbf{x}$ are included in the stack (Figure 2).

Interchanging the order of integration, we may rewrite (17)

$$F^i(m, \tau) = \frac{\partial}{\partial \tau} \int ds \int dr \int d\psi \, W_i(\mathbf{r}, \mathbf{x}, \mathbf{s}) \, \mathcal{H} u_{sc}(\mathbf{r}, \mathbf{s}, t = T(\mathbf{r}, \mathbf{x}, \mathbf{s})) \tag{18}$$

and change variables from ψ to t:

$$F^i(m, \tau) = \frac{\partial}{\partial \tau} \int ds \int dr \int dt \, \frac{\partial \psi}{\partial t} \, W_i(\mathbf{r}, \mathbf{x}, \mathbf{s}) \, \mathcal{H} u_{sc}(\mathbf{r}, \mathbf{s}, t = T(\mathbf{r}, \mathbf{x}, \mathbf{s})) \tag{19}$$

Geometrically, (19) may be interpreted by fixing both the semicircle associated with m, τ and the foci $\mathbf{s}, \mathbf{r}$ for a family of ellipses associated with the data trace $\mathcal{H} u_{sc}(\mathbf{r}, \mathbf{s}, t)$ (Figure 3). Given such s, r, write $m' = .5(s + r)$. Then as ψ varies from $-\pi/2$ to $\pi/2$, t varies from $t_{min} = \tau - (m' - m)$ to t_{max} at the point $\mathbf{x}_0(m, \tau, s, r)$ where the circle and ellipse are tangent, and thence back to t_{min}. The Jacobian term $\frac{\partial \psi}{\partial t}$ has the form

$$\frac{\partial \psi}{\partial t} = \left(\tau \cos(\frac{\theta}{2}) \, \mathbf{v} \cdot \mathbf{w} \right)^{-1}$$

where $\mathbf{v}$ and $\mathbf{w}$ are, respectively, unit vectors tangent to the circle and normal to the ellipse. This term becomes singular at the point of tangency. As shown in the appendix, the inner integral can be reduced by means of a stationary phase analysis to a singular term at that point:

$$F^i(m,\tau) \;=\; \frac{\partial}{\partial \tau}\int ds \int dr\, \Omega\, W_i(\mathbf{r},\mathbf{x}_0,\mathbf{s})\, \mathcal{F}\mathcal{H}u_{sc}(\mathbf{r},\mathbf{s},t=t_{max}) \qquad (20)$$

where

$$\Omega \;=\; \left(2\tau^2 \cos(\frac{\theta}{2})\left(\frac{1}{\tau}-\frac{1}{\varrho}\right)\right)^{-\frac{1}{2}},$$

ϱ is twice the radius of curvature of the ellipse at the point of tangency, and $\mathcal{F}$ is the singular, one-sided, convolutional filter $t^{-\frac{1}{2}}$. Note that the obliquity term in Ω could be combined with the basic obliquity term in W_i to give a combined factor of $\cos^{\frac{3}{2}}(\frac{\theta}{2})$.

When $m < \min(s,r)$ or $m > \max(s,r)$, the point of tangency lies on the x-axis where $|\psi| = \frac{\pi}{2}$ and, hence, $W_i = 0$. Thus, the domain of integration can be restricted and a computational savings can be realized from the replacement of

$$\int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} dr$$

in (14) by

$$\int_{-\infty}^{\infty} ds \int_{s-2(s-x)}^{s+2(s-x)} dr$$

in (20).

6. Conclusions

Summary of the Algorithm:

The DMO formulation of the (2D acoustic) GRT multiparameter inversion algorithm may be summarized as follows.

First compute the integrals (20):

For each output data point (m,τ), for each input trace $\mathcal{F}\mathcal{H}u_{sc}(\mathbf{r},\mathbf{s},t)$, with $s<m<r$, find the contribution of the input trace to (20):

Let $h=.5|s-r|$, $\Delta m=m'-m$. Solve the standard DMO equations

$$\frac{\Delta m^2}{h^2}+\frac{\tau^2}{t_{nmo}^2}=1, \qquad t_{nmo}^2+4h^2=t_{max}^2 \tag{21}$$

for the "nmo-time" t_{nmo}, and the time of tangency t_{max}.

Solve

$$\frac{z(\Delta x-\Delta m)}{t_{max}^2}-\frac{z(\Delta x)}{t_{nmo}^2}=0, \qquad 4(\Delta x^2+z^2)=\tau^2 \tag{22}$$

for the point of tangency $\mathbf{x}_0=(m+\Delta x,z)$. ((22) is derived from the condition $\mathbf{v}\cdot\mathbf{w}=0$ on the unit tangent and normal vectors.) This gives all the geometry needed to compute the weighting terms in (20).

Second, procede as in ordinary DMO processing by applying zero-offset inversion (9) separately to the fields F^i to obtain the potentials f^i.

Finish as in prestack multiparameter inversion by applying (15).

Remarks

It is clear that the issue of dependence on data sorting in the amplitudes of the inversion operators is completely independent of the DMO problem. The point is that when the problem is formulated in terms of scalar inversion as in section 3, assumptions are made on the obliquity dependence of the scattering that allow us to convert the double integral over ϕ and θ to a diagonal integral over ϕ. Given multifold data, one can gather the data in various ways, solve the scalar problem for each gather and then stack the results arbitrarily (i.e. integrate against an abitrary differential) because they are all estimating the same scalar (more or less "independently"). When transformed to a family of DMO operators (shot DMO, offset DMO, etc.), this arbitrary stacking weight shows up as a difference in stacking weights at each input trace. When treated as a vector problem, the arbitrariness disappears because we integrate two experimental variables against a differential that includes the full Jacobian of the transformation from the experimental variables (which could be source and receiver or midpoint and offset or ...) to dip and obliquity. In the transformation to zero offset, this Jacobian term just comes along as a passenger in the term W_i.

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Appendix: Derivation of Equation (20)

Consider the problem of approximating an integral

$$
\begin{aligned}
I &= \int dt \int d\psi\, V(\psi) U(t)\, \delta(t - T(\psi)) \\
&= \int dt\, U(t) \int d\psi\, V(\psi)\, \delta(t - T(\psi)) \\
&= \int dt\, U(t)\, I_t
\end{aligned}
\tag{A.1}
$$

where there exists a unique singular point ψ_0 satisfying

$$T'(\psi_0) = T'_0 = 0.$$

Here and below, we use the notation T' indicates differentiation with respect to ψ and the subscript 0 to denote evaluation at the point $\psi = \psi_0$.

Approximating T by the leading terms in a Taylor series at ψ_0, writing

$$g = \frac{-T''_0}{2}(\psi - \psi_0)^2$$

and changing the variable of integration from ψ to g, I_t can be written

$$
\begin{aligned}
I_t &= \int dg\, \frac{\partial \psi}{\partial g} V(\psi)\, \delta(t - T_0 + g) \\
&= \int dg\, \frac{V(\psi)}{(-2gT''_0)^{\frac{1}{2}}}\, \delta(t - T_0 + g).
\end{aligned}
\tag{A.2}
$$

The stationary-phase approximation to I is obtained by replacing $V(\psi)$ by its value at the stationary point and then evaluating the integrals:

$$I_t = \frac{V_0}{(-2T''_0(T_0 - t))^{\frac{1}{2}}}. \tag{A.3}$$

$$I = \frac{V_0}{(-2T_0'')^{\frac{1}{2}}} \int dt \, \frac{U(t)}{(T_0 - t)^{\frac{1}{2}}}. \tag{A.4}$$

Now consider equation (18). It can be rewritten

$$F^i(m, \tau) = \frac{\partial}{\partial \tau} \int ds \int dr \, J(m, \tau, s, r) \tag{A.5}$$

with

$$J = \int dt \int d\psi \, W_i(\mathbf{r}, \mathbf{x}(\psi), \mathbf{s}) \, \mathcal{H}u_{sc}(\mathbf{r}, \mathbf{s}, t) \, \delta(t - T(\mathbf{r}, \mathbf{x}(\psi), \mathbf{s})). \tag{A.6}$$

For fixed m, τ, s, r, (A.6) has the form of (A.1) where

$$\begin{aligned} V(\psi) &= W_i(\mathbf{r}, \mathbf{x}(\psi), \mathbf{s}) \\ U(t) &= \mathcal{H}u_{sc}(\mathbf{r}, \mathbf{s}, t). \end{aligned} \tag{A.7}$$

To complete the derivation of equation (20), we need an explicit representation of T_0''. Fix m, τ, s, r and let $\mathbf{x}_0 = (x_0, z_0)$ be the point of tangency. Let

$$\mathbf{w} = \frac{\mathbf{m} - \mathbf{x}_0}{|\mathbf{m} - \mathbf{x}_0|} = (u, v), \quad \mathbf{v} = (v, -u)$$

be unit normal and tangent vectors at $\mathbf{x}_0$, and let

$$\xi = \mathbf{v} \cdot (\mathbf{x} - \mathbf{x}_0), \quad \eta = \mathbf{w} \cdot (\mathbf{x} - \mathbf{x}_0)$$

be coordinates in a rotated system with origin at $\mathbf{x}_0$. For points lying on our semicircle,

$$\xi = -\frac{\tau}{2} \sin(\Delta\psi), \quad \eta = \frac{\tau}{2}(1 - \cos(\Delta\psi)) \tag{A.8}$$

where $\Delta\psi$ abbreviates $\psi - \psi_0$. Write T^ξ for the partial derivative of T with respect to ξ, etc. (so, for example, we write $T_0^{\psi\psi}$ instead of T_0''). Then

$$\begin{aligned} T^\psi &= \xi^\psi T^\xi + \eta^\psi T^\eta \\ &= -\frac{\tau}{2} \cos(\Delta\psi) T^\xi + \frac{\tau}{2} \sin(\Delta\psi) T^\eta. \end{aligned} \tag{A.9}$$

$$T^{\psi\psi} = (\frac{\tau}{2})^2[\cos^2(\Delta\psi)T^{\xi\xi} - \sin^2(\Delta\psi)T^{\eta\eta}]$$
$$+ \frac{\tau}{2}[\sin(\Delta\psi)T^{\xi} + \cos(\Delta\psi)T^{\eta}]. \quad \text{(A.10)}$$

Evaluating (A.10) at $\psi = \psi_0$,

$$\begin{aligned} T_0^{\psi\psi} &= (\frac{\tau}{2})^2 T_0^{\xi\xi} + \frac{\tau}{2} T_0^{\eta} \\ &= (\frac{\tau}{2})^2 T_0^{\eta}\left[\frac{2}{\tau} + \frac{T_0^{\xi\xi}}{T_0^{\eta}}\right]. \end{aligned} \quad \text{(A.11)}$$

T_0^{η} is the familiar obliquity factor $-2\cos(\frac{\theta_0}{2})$, occurring here with a minus sign since T decreases as η increases. The bracketed term is recognizable as the difference between the curvatures of the circle $(\frac{2}{\tau})$ and the ellipse $(\frac{-T_0^{\xi\xi}}{T_0^{\eta}})$ at the point of tangency. Denoting the radius of curvature of the ellipse as $\frac{\varrho}{2}$, we can rewrite (A.11)

$$T_0^{\psi\psi} = -\tau^2\cos(\frac{\theta}{2})\left[\frac{1}{\tau} - \frac{1}{\varrho}\right]. \quad \text{(A.12)}$$

Substitution from (A.12) and (A.7) into (A.4) gives

$$J(m,\tau,s,r) = \frac{W_i(\mathbf{r},\mathbf{x}_0,\mathbf{s})}{(2\tau^2\cos(\frac{\theta}{2})\left[\frac{1}{\tau} - \frac{1}{\varrho}\right])^{\frac{1}{2}}}\int dt\, \frac{\mathcal{H}u_{sc}(\mathbf{r},\mathbf{s},t)}{(T_0 - t)^{\frac{1}{2}}}. \quad \text{(A.13)}$$

Substitution from (A.13) into (A.5) yields equation (20) as promised.

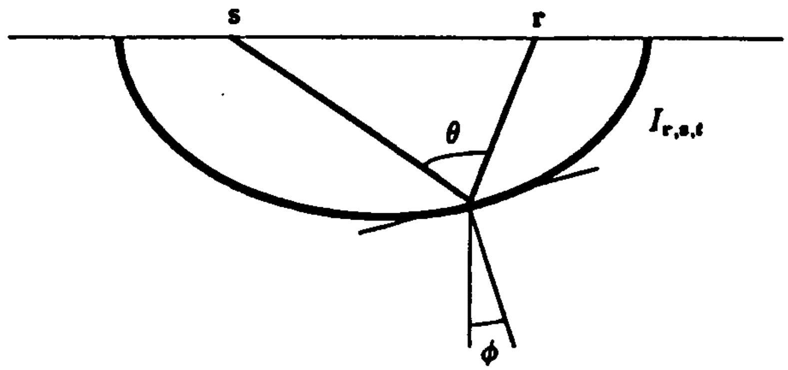

FIG. 1. *Geometry of equation (2).*

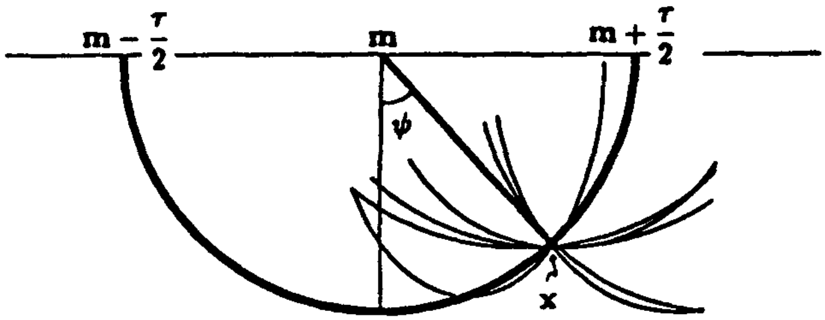

FIG. 2. *Geometry of equation (17).*

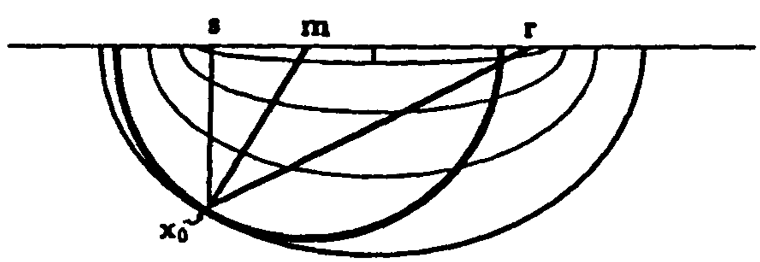

FIG. 3. *Geometry of equation (19).*