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A Stochastic Optimization Method for Energy-Based Path Planning

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Abstract. We present a novel stochastic optimization method to compute energy–optimal paths, among all time–optimal paths, for vehicles traveling in dynamic unsteady currents. The method defines a stochastic class of instantaneous nominal vehicle speeds and then obtains the energy–optimal paths within the class by minimizing the total time–integrated energy usage while still satisfying the strong–constraint time–optimal level set equation. This resulting stochastic level set equation is solved using a dynamically orthogonal decomposition and the energy–optimal paths are then selected for each arrival time, among all stochastic time–optimal paths. The first application computes energy–optimal paths for crossing a steady front. Results are validated using a semi–analytical solution obtained by solving a dual nonlinear energy–time optimization problem. The second application computes energy–optimal paths for a realistic mission in the Middle Atlantic Bight and New Jersey Shelf/Hudson Canyon region, using dynamic data–driven ocean field estimates.

Keywords: Energy–optimal · Time–optimal · Dynamically orthogonal equations · Level–set method · Autonomy · AUV · Dynamic data–driven

1 Introduction

Path planning refers to the navigation rules provided to autonomous mobile agents operating in a dynamic environment while optimizing an objective criterion. This criterion could be the travel time, energy utilized, quantity/quality of data collected, safety or a combination of these [12,16]. In the recent years, the growing usage of Autonomous Underwater Vehicles (AUVs) such as propelled vehicles and gliders in diverse applications (e.g. ocean exploration, security, conservation, and research) has led to increased research in path planning for underwater robotics [3,4,9,17,25,30]. As AUVs undertake complex tasks (e.g. cooperative exploration and sampling [19]), they are required to operate for long periods of time at sea by utilizing energy efficiently [4]. The dynamic environment in which these vehicles (and also other mobile agents such as land robots, drones, airplanes etc.) navigate can be utilized to reduce their energy consumption. In the case of AUVs, ocean currents can be comparable in magnitude to the

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average operational speed of propelled vehicles, and up to 2–3 times the typical speed of gliders [26,28,29]. As such, there is an opportunity to reduce the energy consumption by intelligently utilizing favorable currents while avoiding adverse currents. The availability of numerical ocean prediction systems enables agents to plan their motion using a forecast of the ocean currents (within predictability limits). Dynamic data–driven re–planning of these trajectories may be performed by utilizing open–loop planning algorithms which have short run–times. Using this as a motivation, our goal here is to develop a computationally efficient and rigorous path planning algorithm that computes energy–optimal paths, among all time–optimal paths, of a vehicle navigating between two points in a dynamic flow field. We show that this computation can be posed as a stochastic PDE–based design/optimization problem. In this paper, we focus on addressing the question of how to evaluate such a solution, and whether an analytical (or semi–analytical) benchmark exists for validation. We also illustrate the applicability of such an algorithm for path planning utilizing real ocean forecasts.

Most path planning algorithms for AUVs find their roots in robotics, e.g. [2,5]. The A* search algorithm, quite popular in robotic motion planning, has been applied to AUVs [6] to find near–optimal paths. These paths have been shown to utilize substantially lower energy compared to straight line paths when ocean currents are comparable to vehicle speeds [10]. Rapidly Exploring Random Trees (RRTs) [16], also popular in robotic path planning have been used for AUVs to obtain minimum work [14] and minimum energy (linear nominal relative speed) [24] paths. However, A* and RRTs do not work well for strong flows [24]. First, they are not well suited to computing exact solutions in strong dynamic flows [21,22]. Second, the heuristics reported are for a linear energy cost function and do not readily extend to nonlinear cost functions. [1] discusses a genetic algorithm to optimize the paths parameterized in space and time. They minimize an energy cost function which is a path integral of the cube of vehicle speed. In [15], the paths are computed using nonlinear optimization, where a weighted cost function accounts for the energy to overcome drag (proportional to square of nominal relative speed) and provide acceleration (proportional to rate of change of nominal relative speed). As the success of this optimization depends heavily on the parameterization, it cannot be easily generalized to all types of flows and domains. [32] discusses potential field techniques for obstacle avoidance and [34] reports a swarm optimization approach to minimize an energy cost function on the parameterized paths. Other algorithms utilize Lagrangian Coherent Structures (LCS) of the flow to design near–optimal navigation paths [13,35]. They illustrate that the optimal energy (quadratic nominal speed)-time-weighted paths computed using a heuristic receding–horizon nonlinear programming method are close to the ridges of the LCS. Other nonlinear optimization methods and evolutionary algorithms have also been used to obtain near–optimal paths by approximately solving the governing optimal control problem. We encourage the reader to refer to [23,31] for an in-depth literature survey.

The present work is inspired from [21,22], where a modified level set methodology for *rigorous* time–optimal path planning is described. We extend this methodology to develop a novel energy optimal path planning algorithm, based on stochastic dynamically orthogonal level set equations [27]. In what follows, we state the problem and describe the new path planning method. We then consider a test case of a vehicle crossing a canonical steady front for a range of arrival times. We validate our results for a range of arrival times by comparing them to those of a dual energy and time optimization albeit for a single chosen arrival time. The latter is a semi–analytical solution for that front crossing problem, providing the energy and time optimal path(s) for a single arrival time. Finally, we apply our methodology to plan the time–dependent headings and energy usages of a vehicle undertaking a mission in the Middle Atlantic Bight region.

2 Problem Statement

Let $\Omega \subseteq \mathbb{R}^n$ be an open set. Consider a vehicle navigating from a start point ($\mathbf{x_s}$) to an end point ($\mathbf{x_f}$) with a specified instantaneous nominal speed $F(t) \geq 0$. The environmental flow is denoted by $\mathbf{v}(\mathbf{x}, t) : \Omega \times (0, \infty) \to \mathbb{R}^n$. The heading function is chosen such that when navigated at a relative *time–dependent* non–negative speed of $F(t)$, the vehicle reaches $\mathbf{x_f}$ in optimal time $T(\mathbf{x_f}; F(\bullet))$. Among all of these, we seek the $F(\bullet)$ that minimizes the energy cost function E, i.e.,

$$\min_{F(\bullet)} \qquad E(\bullet) = \int_0^{T(\mathbf{x_f}; F(\bullet))} p(t)\,\mathrm{d}t \tag{1a}$$

$$\text{s. t.} \quad \frac{\partial \phi(\mathbf{x}, t)}{\partial t} = -F(t)|\nabla\phi(\mathbf{x}, t)| - \mathbf{v}(\mathbf{x}, t) \cdot \nabla\phi(\mathbf{x}, t) \qquad \text{in } (\mathbf{x}, t) \in \Omega \times (0, \infty) \tag{1b}$$

$$T(\mathbf{x_f}; F(\bullet)) = \min_t \{t : \phi(\mathbf{x_f}, t) \leq 0\}\,, \tag{1c}$$

$$\phi(\mathbf{x}, 0) = |\mathbf{x} - \mathbf{x_s}|\,, \tag{1d}$$

$$p(t) = F(t)^n\,, \qquad \text{where } n \geq 1\,. \tag{1e}$$

Here, the scalar field $\phi(\mathbf{x}, t)$ is a reachability–front tracking level–set function [31]. For a given $F(\bullet)$, the viscosity solution of the level set Hamilton–Jacobi equation (1b) with initial conditions (1d) and the subsequent solution to the backtracking Eq. (2),

$$\frac{\mathrm{d}\mathbf{x}^*}{\mathrm{d}t} = -\mathbf{v}(\mathbf{x}^*, t) - F(t)\frac{\nabla\phi(\mathbf{x}^*, t)}{|\nabla\phi(\mathbf{x}^*, t)|}\,, \quad 0 \leq t \leq T(\mathbf{x_f}; F(\bullet)) \tag{2}$$

yield a continuous–time history of the time–optimal vehicle heading angles, $\theta^*(t)$ [21]. These headings guarantee time–optimality for the particular choice of the speed function $F(t)$ [31]. Then, among all such time–optimal paths which reach the target at relative speed $F(t)$, we seek to find the $F(t)$ that minimizes the energy required (1a). We reiterate that all of our paths are time–optimal: the optimization is on the total energy usage (1a). In contrast to multi–objective optimization formulations [1,15,34], in our method, the time–optimality is a strong constraint.

3 New Stochastic Dynamically Orthogonal Level Set Equations for Energy-Based Path Planning

Considering the nominal speed $F(t)$ as a random variable belonging to a stochastic class, i.e., $F(t) \to F(t;\omega)$, and a deterministic flow field $\mathbf{v}(\mathbf{x},t)$, we obtain a stochastic Langevin form of the level set equation (1b):

$$\frac{\partial \phi(\mathbf{x},t;\omega)}{\partial t} = -F(t;\omega)|\nabla \phi(\mathbf{x},t;\omega)| - \mathbf{v}(\mathbf{x},t) \cdot \nabla \phi(\mathbf{x},t;\omega)\,, \tag{3}$$

where $(\mathbf{x},t) \in \Omega \times (0,\infty)$ and ω denotes a random event. For $F(t;\omega) \geq 0$, we solve the SPDE (3) until the first time instant t such that $\phi(\mathbf{x_f},t;\omega) \leq 0$, starting from deterministic initial conditions $\phi(\mathbf{x},0;\omega) = |\mathbf{x} - \mathbf{x_s}|$ with boundary condition $\frac{\partial^2 \phi(\mathbf{x},t;\omega)}{\partial \mathbf{n}^2}\,|_{\delta\Omega} = 0$, where $\mathbf{n}$ denotes the outward normal to $\partial\Omega$. Such a stochastic simulation yields the distribution of the minimum time–to–reach $T(\mathbf{x_f};F(\bullet;\omega))$ for an externally forced distribution of $F(\bullet;\omega)$. Then, the distribution of energy utilized is computed from $F(\bullet;\omega)$ and $T(\mathbf{x_f};F(\bullet;\omega))$ as $E(\omega) = \int\limits_0^{T(\mathbf{x_f};F(\bullet;\omega))} p(t)\,\mathrm{d}t$. The function $p(t)$ can assume any power law dependence on $F(t)$. The power function $p(t)$ that has a linear dependence on $F(t)$ results in a constant drag optimal path (also known as fuel–optimal, e.g. [2]). It yields a linear drag optimal path when $p(t) \propto F(t)^2$, and a quadratic drag energy optimal path when $p(t) \propto F(t)^3$. Finally, for any choice of the time–to–reach (a particular time or a range of time), the speed function $F(\bullet;\omega)$ which minimizes the energy cost, $E(\omega)$, can be obtained by a search procedure. As we will see, the approach can operate on classes of stochastic functionals $F(\bullet;\omega)$ if these functionals can be efficiently represented by a reduced basis.

The most straightforward method to solve the SPDE (3) is through a Monte Carlo (MC) approach. The deterministic level set PDE (1b) can be solved for different realizations of $F(t;\omega)$ to yield a distribution of $T(\mathbf{x_f};F(\bullet;\omega))$. Unfortunately, the MC solution is expensive and the computational cost increases with number of realizations used. Since in (3), $\mathbf{v}(\mathbf{x},t)$ is the flow field velocity, and we consider ocean applications, an efficient solution method for solving (3) would be a methodology that exploits the nonlinearities of the flow, which tend to concentrate the scalar level set field, ϕ, responses into specific dynamic patterns. Such a methodology is offered by the Dynamically Orthogonal (DO) decomposition [27]. To the best of our knowledge, this approach has never been utilized to determine the stochastic viscosity solution of (3). A numerical challenge in obtaining the DO level set equations is the presence of the non–polynomial nonlinearity, $\gamma \equiv |\nabla\phi|$. We have considered several approaches for handling this term. One such approach [31] does not invoke a specific DO decomposition for γ, but evaluates it using an explicit Monte Carlo computation. This is the method we present in this paper. In what follows, the arguments $(\mathbf{x}, t)$ are dropped for the brevity of notation. The decompositions $F = \bar{F} + z\tilde{F}$, $\phi = \bar{\phi} + Y_i\tilde{\phi}_i$ are first substituted in (3). Enforcing the DO condition [27] then yields the following new equations for the mean $\bar{\phi}$, stochastic coefficients Y_i and modes $\tilde{\phi}_i$, in terms of the mean $(\bar{F})$, stochastic coefficients (z) and modes $(\tilde{F})$ of the nominal relative speed:

$$\frac{\partial \bar{\phi}}{\partial t} = -(\bar{F}\mathbb{E}[\gamma] + \mathbb{E}[z\gamma]\tilde{F}) - \mathbf{v} \cdot \nabla \bar{\phi}\,, \tag{4}$$

$$\frac{\mathrm{d}Y_i}{\mathrm{d}t} = -\left\langle \bar{F}(\gamma - \mathbb{E}[\gamma]) + \tilde{F}(z\gamma - \mathbb{E}[z\gamma]) + Y_k \mathbf{v} \cdot \nabla \tilde{\phi}_k, \tilde{\phi}_i \right\rangle, \tag{5}$$

$$\begin{aligned}\frac{\partial \tilde{\phi}_i}{\partial t} &= -C^{-1}_{Y_iY_j}(\bar{F}\mathbb{E}[Y_j\gamma] + \tilde{F}\mathbb{E}[zY_j\gamma]) + \mathbf{v} \cdot \nabla \tilde{\phi}_i \\ &\quad - \left\langle -C^{-1}_{Y_iY_j}(\bar{F}\mathbb{E}[Y_j\gamma] + \tilde{F}\mathbb{E}[zY_j\gamma]) + \mathbf{v} \cdot \nabla \tilde{\phi}_i, \tilde{\phi}_n \right\rangle \tilde{\phi}_n\,,\end{aligned} \tag{6}$$

where $\langle \bullet, \bullet \rangle$ denotes the inner product. We have also developed methods where a DO decomposition is considered for the non–polynomial nonlinearity γ. These methods along with their derivations are provided in [31]. We note that an equivalent formulation is possible through bi–orthogonal methods [8].

Algorithm. Our algorithm for energy optimal path planning has 5 main steps. (i) The first step is to obtain a comprehensive sampling of the stochastic class $F(t;\omega)$. In the DO sense, we obtain a comprehensive sample of $F_{\mathrm{DO}}(t;r)$, where r denotes realizations. (ii) Next, the new stochastic DO level set Eqs. (4)–(6) are solved using the chosen samples of $F_{\mathrm{DO}}(t;r)$. (iii) The energy utilized by each sample is computed as $E(r) = \int_0^{T(\mathbf{x_f};F_{\mathrm{DO}}(t;r))} p(t)\,\mathrm{d}t$. (iv) For a given time–to–reach, the sample $F^*_{\mathrm{DO}}(t;r)$ that leads to the minimum energy usage is identified using a sorting algorithm. This $F^*_{\mathrm{DO}}(t;r)$ is energy optimal within the class of $F_{\mathrm{DO}}(t;r)$ that reach $\mathbf{x_f}$ at a given time. (v) Finally, the sample class can be enriched and the algorithm iterated until no further refinement is required. The computational cost for direct Monte Carlo solution of the SPDE is $O(MN)$, where M is the number of samples and N is the total size of discrete computational domain utilized. Our DO algorithm has a computational cost of $O(SN)$, where S is the size of the DO–subspace, where S is often such that $S \ll M$.

4 Applications

We first consider a simulated steady front test case and use it as a benchmark to test and validate our approach. Specifically, we compare our results to that of a nonlinear dual optimization approach that seeks a minimum energy path among time optimal paths for a fixed arrival time. We solve this problem using the iterated constrained nonlinear optimization toolbox of MATLAB. Next, we employ our methodology for path planning of a glider released from Buzzard's Bay (offshore from WHOI) to reach a target in the region of the Autonomous Wide Aperture Cluster for Surveillance (AWACS) experiment just south of the Hudson Canyon.

4.1 Energy Optimal Crossing of an Idealized Steady Front

Considering the test case of crossing a steady front, we first solve the energy optimal problem using a semi–analytical approach. This serves as a benchmark to test our new methodology. The schematic of the flow and the relevant notation is depicted in Fig. 1. The goal is to determine the optimal speed function $F(t)$, varying within limits $F_{\min}$ and $F_{\max}$ ($F_{\min} \leq F(t) \leq F_{\max}$ for all t), that minimizes the energy utilized while still reaching the end point in optimal time. In what follows, we provide arguments that allow us to formulate a dual minimization problem whose solution gives the energy–optimal trajectory in the sense defined in Sect. 2, but only for a specific arrival time.

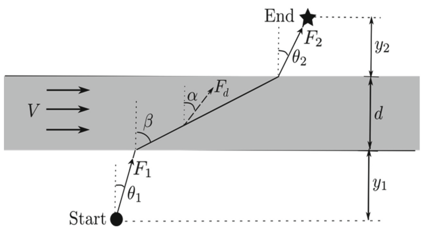


Fig. 1. Parameters involved in optimal crossing of a simulated steady front: steady front speed V and width d; start (circle), end (star), distances, vehicle nominal speed and headings are marked. Adapted from [23].

To start the arguments, we first consider the motion from the start point to the steady front. During this time, the vehicle remains unaffected by the environment and the corresponding zero–level–set expands radially outward at a rate equal to $F(t)$, the nominal instantaneous relative speed. The motion (e.g. the total displacement) achieved by any choice of the time series $F(t)$ over this period can be synthesized as the motion achieved by the mean nominal speed $\bar{F}(t)$ over the same time window. However, the energy consumed by the vehicle varies as a power function of $F(t)$, with $n \geq 1$ (see Sect. 3). As a result, that energy consumption will be different for each time series $F(t)$.

It can then be shown that the energy consumption is minimum when the mean speed is used over a given time interval. For instance, let us suppose that a vehicle (say v) travels at speed $F_{\min}$ for a total time of t_1 and at speed $F_{\max}$ for time t_2. The final position of this vehicle coincides with that of a different vehicle (say m), traveling at a uniform speed $\bar{F} = (F_{\min}t_1 + F_{\max}t_2)/(t_1 + t_2)$ for time $t_1 + t_2$. The energy expended by v is $E_v = F_{\min}^n t_1 + F_{\max}^n t_2$. On the other hand, m utilizes a total energy equal to $E_m = \bar{F}^n(t_1 + t_2)$. Hölder's inequality can now be used to show that $E_m \leq E_v$ [31]. In fact, it can be shown that the above result holds for any number of engine speed switches (≥ 2) and any $n \geq 1$. We also note that similar arguments can be made for the vehicle motion beyond the steady front to the end point.

To continue the arguments, we now consider the motion of the vehicle within the uniform and steady front proper. Inside this region, only the motion of vehicle in the x–direction is affected. Even here, it can be shown that using a single speed results in lower energy consumption, and that any time series $F(t)$ has an equivalent single time–mean speed [31]. Therefore, for a given travel time, the energy optimal path is executed by a vehicle that moves at a constant speed from the start point to the steady front at some to–be–determined point, then another speed (same or different) for the time optimal motion in the steady front and finally another speed from the steady front to the target. This completes the arguments that allow us to setup our dual minimization problem, 'energy–optimality subject to time–optimality'.

Using the above arguments, we set the time–variation of the unknown optimal relative speed to be the uniform speeds of F_1, F_d, and F_2 from start to the steady front, within the steady front, and from the exit of the steady front to the end point, respectively.

Let $\mathbf{U}$ denote the total effective velocity of the vehicle in the flow, as seen by a ground observer. Within the steady front, the component of $\mathbf{U}$ in the x–direction is $U_x = F_d \sin\alpha + V$ and in the y–direction, $U_y = F_d \cos\alpha$. The direction of resultant velocity and heading angle are related through the relation, $\tan\beta = \frac{U_x}{U_y} = \tan\alpha + \frac{V}{F_d}\sec\alpha$. Outside the steady front, the relations are the same, but with $V = 0$. Now, let X be the total downstream displacement of the vehicle, i.e. in the x direction. We have, from simple trigonometry, $X = y_1 \tan\theta_1 + d\tan\beta + y_2\tan\theta_2$. Finally, the total travel time T can be written as the sum of travel times in each individual region, $T = \frac{y_1}{F_1\cos\theta_1} + \frac{d}{F_d\cos\alpha} + \frac{y_2}{F_2\cos\theta_2}$. Now, we want to determine the energy optimal path, for each arrival time such that they are also time–optimal. Hence, assuming for now, a general energy cost over $\mathrm{d}t$, $\mathrm{d}E = p(t)\,\mathrm{d}t = F(t)^n\,\mathrm{d}t$ where $n \geq 1$, we obtain the total energy expended from the start point to the end point: $E = F_1^{n-1}\frac{y_1}{\cos\theta_1} + F_d^{n-1}\frac{d}{\cos\alpha} + F_2^{n-1}\frac{y_2}{\cos\theta_2}$. For a fixed time–to–reach the target, the double optimal energy–time problem is

$$\min_{F_1,F_d,F_2} E = F_1^{n-1}\frac{y_1}{\cos\theta_1} + F_d^{n-1}\frac{d}{\cos\alpha} + F_2^{n-1}\frac{y_2}{\cos\theta_2} \tag{7}$$

$$\text{s.t.} \qquad \mathrm{X} = y_1\tan\theta_1 + d\left(\tan\alpha + \frac{V}{F_d}\sec\alpha\right) + y_2\tan\theta_2 \tag{8}$$

$$T = \min_{\theta_1,\alpha,\theta_2} \frac{y_1}{F_1\cos\theta_1} + \frac{d}{F_d\cos\alpha} + \frac{y_2}{F_2\cos\theta_2} \tag{9}$$

$$\theta_1, \theta_2, \alpha \geq 0\,, \qquad F_{\min} \leq F_1, F_d, F_2 \leq F_{\max}\,, \qquad n \geq 1 \tag{10}$$

where X, T, $F_{\min}$, and $F_{\max}$ are inputs to the optimization problem. We note that the time constraint for the outer energy optimization is another inner optimization. This completes the derivation of a dual minimization problem whose solution provides the energy-optimal path in the sense defined in Sect. 2, but again, only for a fixed single arrival time at a time. For $y_1 = 0.2167$, $d = 0.2$, $y_2 = 0.2167$, $V = 3$, $F_{\min} = 2$, $F_{\max} = 3, X = 0.6334$, and fixing the single target T to be $T = 0.26$, we obtain the numerical solution of our dual optimization problem as presented in Table 1. The results shown in column 1 are computed using the iterative nonlinear optimization toolbox of MATLAB.

Now, we compare this 'semi–analytical' solution to that obtained by our new stochastic DO level–set optimization scheme. To do so, we need to select an

Table 1. Parameters of the energy optimal path that reaches the end point at time $T = 0.26$.

Parameter	Using NonLinear Optimization	Using new stochastic DO level-set optimization
θ_1	23.5^o	22.4^o
θ_2	23.5^o	20.7^o
β	65.8^o	65.9^o
F_1	2.9	2.8
F_d	2.6	2.5
F_2	2.9	3.0

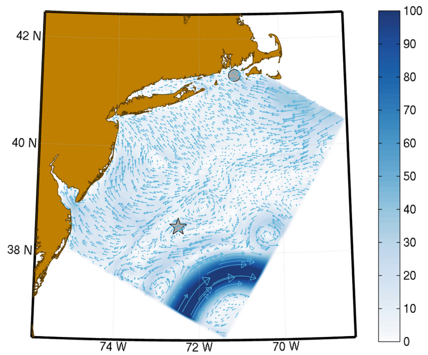
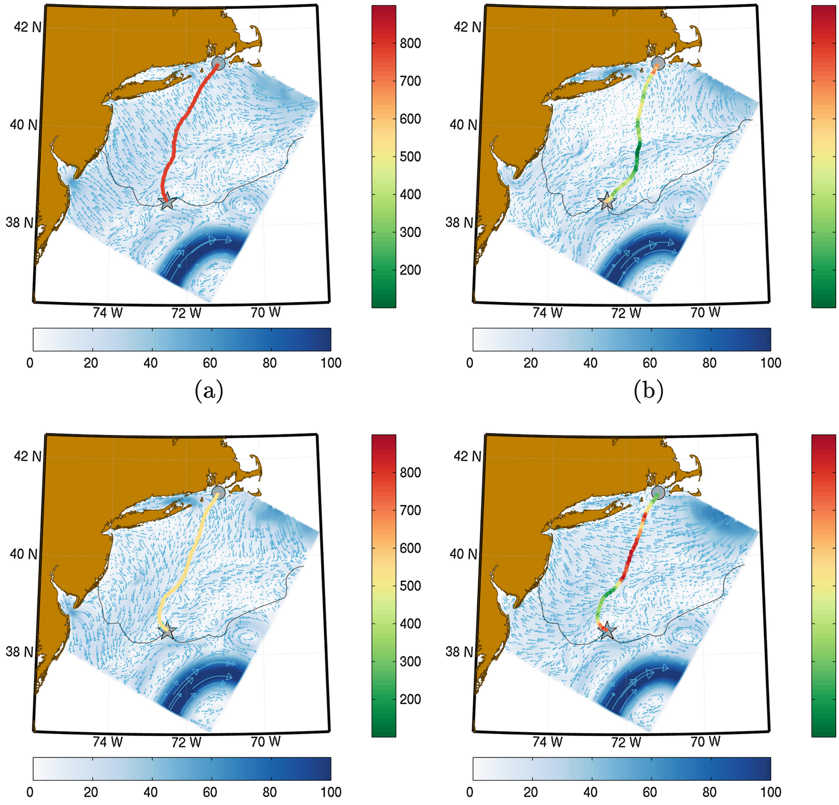


Fig. 2. The start point is marked as a circle and the end point is marked as a star. The initial flow on Aug 28, 00 UTC is shown on the color axis in cm/s.

adequate stochastic class of $F(t;\omega)$. First, we remark that all vehicles will reach faster than a vehicle which travels throughout the distance at $F_{\min}$. Hence, the total time required will at most be the time required by this slowest vehicle. Let this be denoted as $T_{\max}$. The number of $F(t;\omega)$ samples (i.e. $F_{\mathrm{DO}}(t;r)$, see Sect. 3) required grows with the resolution in time axis in an exponential manner, i.e., even if only two engine speed choices are allowed, and if the time axis that ranges from 0 to $T_{\max}$ is divided into n intervals, a total of 2^n $F_{\mathrm{DO}}(t;r)$ samples are required for an exhaustive search (in the bang–bang control sense). With the available computing resources and reasonable run–time, we choose to resolve the time axis into $n = 26$ intervals. The energy optimal path planning is then performed using our new stochastic DO level set equations with this choice of $F_{\mathrm{DO}}(t;r)$, i.e. an exhaustive sample space but only for those two speeds and 26 time–intervals (25 speed switches). The result of this stochastic DO level–set optimization with the same parameters as above is presented in Tabel 1. Critically, we note that our stochastic solution provides answers for a wide range of arrival times (instead of the single fixed time T).



(a) (b)

(c) (d)

Fig. 3. (a) Path that reaches in the shortest time, 12.96 days, but consumes the highest energy. (b) Path that takes 6 more days to reach the end point (18.78 days), but utilizes 40 % less energy. (c) Path that reaches in 16 days using a constant speed. (d) Path that also takes 16 days but is energy optimal: it utilizes about 10 % less energy than the path at constant speed. The energy utilized by the vehicle along the path is plotted in color. Flow field at arrival time is shown in blue. All paths are time optimal for the $F(t)$ utilized (Color figure online).

4.2 Realistic Dynamic Data–Driven Ocean Simulation

In this section we explore the application of our approach in realistic dynamic data–driven ocean simulations. The mission is to start just offshore of Buzzard's Bay near WHOI and reach a target in the AWACS region, as shown in Fig. 2. A glider that can travel at relative horizontal velocities between $F = 10$ cm/s and 30 cm/s is assumed to be released on Aug 28, 2006 at 00 UTC. The flow data is obtained from the MSEAS free–surface primitive–equation model utilized in an implicit two–way nested computational domain set–up [11], with both tidal and atmospheric forcings. These simulated ocean flows assimilate real ocean data and correspond to a reanalysis of the real–time AWACS and SW06 exercises (Aug.-Sep. 2006) in the Middle Atlantic Bight and shelfbreak front region [7,18,20,33].

All gliders are assumed to follow the same yo–yo pattern in the vertical and the effects of the small vertical ocean velocities are assumed to be accounted for in the forward motions of the vehicles. We consider yo–yo patterns from the near surface to either the local near bottom or 400 m depth, whichever is shallower (for the mission considered, a large portion of the paths occurs on the shelf, within about 20 to 100 m). The horizontal currents that a glider encounters during its yo–yo motion are then the horizontal currents integrated along its path. Of course, it is the path to–be–determined that specifies the currents that are actually encountered.

The new stochastic DO level–set based energy optimal path planning method is employed to determine the time–optimal level sets for the class of relative glider speeds $F_{\mathrm{DO}}(t;r)$ considered. Within that class, the evolution of the level sets corresponding to the minimum energy is obtained by sorting and the energy–optimal paths are computed by backtracking. We note that our method computes a large set of energy optimal paths, for a range of arrival times. Only a few of such paths are shown in Fig. 3, three of which are energy–optimal.

We first show the path that reaches the end point in shortest time on Fig. 3(a), corresponding to the glider with relative horizontal speed of $F = 30$ cm/s. Based on [21], the fastest glider indeed travels at the largest relative speed considered. The second path selected on Fig. 3(b) is one that takes 18.78 days to complete, but utilizes 40 % less energy. The third path on Fig. 3(c) is a constant speed path that is not energy–optimal and reaches in 16 days. The fourth path in Fig. 3(d) also takes 16 days but is the result of our stochastic optimization and utilizes about 10 % less energy than the path at constant speed.

5 Conclusion

A novel method for energy optimal path planning based on new stochastic dynamically orthogonal level set equations was introduced. It was first used to obtain an energy optimal path among time–optimal paths for crossing a steady front. We showed that the results agreed with those of a semi–analytical path obtained by solving a dual nonlinear optimization problem that minimizes energy and time. We also applied our methodology to realistic dynamic data–driven ocean flows and obtained promising results that illustrate that open–loop energy–time optimal paths can be computed quickly. This opens up the possibility to use our methodology for dynamic data–driven re–planning. Environment uncertainty was considered in [19] and this can be utilized in the future. Future studies can investigate the methodology in greater detail, providing derivations and algorithms for handling non–polynomial nonlinearities. Capabilities will also be illustrated in a wider range of idealized and realistic scenarios.

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