Application of the Sobol’ Method to Large-Scale Aviation Environmental Policy-Making Tools

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The Fifth Summer School on Sensitivity Analysis
Venice, September 12, 2008
Aviation Environmental Tools Suite

APMT = Aviation environmental Portfolio Management Tool
AEDT = Aviation Environmental Design Tool
EDS = Environmental Design Space
Aircraft Emissions Model

- Aircraft Emissions Model computes emissions inventories of CO$_2$, CO, NO$_x$, SO$_x$, etc.
- Millions of inputs, six general groups: fuel burn, temperature, pressure, relative humidity, fuel flow, reference emissions index
- Analysis of a “representative day”
  - 68,343 operations
  - Single deterministic run of the full day takes ~16 sec
  - One 10,000-sample MCS takes ~44 hr
Surrogate Modeling: Aircraft Emissions Model

- Output of interest (e.g. total NO$_x$) is aggregate sum of emissions from a set of $N_o$ operations, each with $N_s$ segments

\[
y_{tot} = \sum_{l=1}^{N_o} y_l = \sum_{l=1}^{N_o} \sum_{k=1}^{N_s} (\text{fuel burn})_{kl} (\text{emissions index})_{kl}
\]

\[
y_{tot} = \sum_{l=1}^{N_o} y_l = \sum_{l=1}^{N_o} \sum_{k=1}^{N_s} q_{kl} g_{kl} (r_{kl}, s_{kl}, t_{kl}, u_{kl}, v_{kl})
\]

- Can exploit the additive structure of the model to create a hierarchical surrogate model for uncertainty analysis and sensitivity analysis

- Certifying uncertainty analysis results computed with a surrogate model is essential
Surrogate Modeling with Quantified Confidence Intervals for Sensitivity Analysis

- The global sensitivity analysis of the aircraft emissions model can be decomposed by operations

\[
\tau_j = 1 - \frac{\sum_{l=1}^{N_o} (1 - \tau_{jl}) D_{y_l}}{\sum_{l=1}^{N_o} D_{y_l}}
\]

\[
S_j = \frac{\sum_{l=1}^{N_o} S_{jl} D_{y_l}}{\sum_{l=1}^{N_o} D_{y_l}}.
\]

- Sensitivity indices estimated using a subset of \( n_o \) flights, with rigorous confidence intervals that narrow as \( n_o \) increases
Total and Main Effect Sensitivity Indices

Sensitivity indices and 95% confidence intervals using \( n_o = 5000 \).

Width of 95% confidence interval for total effect sensitivity index of temperature as a function of surrogate model size.