MULTIPLE SIGNAL CLASSIFICATION FOR GRAVITATIONAL WAVE BURST SEARCH

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This work is mainly focused on the application of the multiple signal classification (MUSIC) algorithm for gravitational wave burst search. This algorithm extracts important gravitational wave characteristics from signals coming from detectors with arbitrary position, orientation and noise covariance.

In this paper, the MUSIC algorithm is described in detail along with the necessary adjustments required for gravitational wave burst search. The algorithm’s performance is measured using simulated signals and noise. MUSIC is compared with the Q-transform for signal triggering and with Bayesian analysis for direction of arrival (DOA) estimation, using the Ω-pipeline. Experimental results show that MUSIC has a lower resolution but is faster. MUSIC is a promising tool for real-time gravitational wave search for multi-messenger astronomy.

Keywords: MUSIC; gravitational wave burst; direction of arrival; multi-messenger astronomy.

1. Introduction

Gravitational wave (GW) astronomy is rapidly making progress, with several detectors planned to reach the necessary sensitivity in a few years, such as Advanced LIGO\footnote{Laser Interferometer Gravitational-wave Observatory} (Laser Interferometer Gravitational-wave Observatory), the Virgo\footnote{Large-scale Cryogenic Gravitational wave Telescope} GW detector, and LCGT\footnote{Large-scale Cryogenic Gravitational wave Telescope}. These detectors are essential for gravitational wave search.

Multiple signal classification\footnote{Multiple signal classification} (MUSIC) is a well-established method for processing signals coming from a series of sensors, devised by Ralph O. Schmidt in 1986. Given arbitrary locations and directions, with signal noises with arbitrary
covariances, MUSIC is capable of giving asymptotically unbiased estimations of:

- the number of signals,
- direction of arrivals (DOA),
- the strengths and cross correlations among the directional waveforms,
- wave polarization,
- strength of noise/interference.

The MUSIC method yields a rather high signal-to-noise (SNR) ratio and is not restricted to any specific waveform. MUSIC can be applied to the signals from the GW network. If a gravitational wave burst occurs and passes through earth, multiple detectors will record it simultaneously. A numerical method is required for the analysis of the signals. With the development of GW detection technology, the SNR ratio has been dramatically improved in the past decade; however, the current performance not yet perfect: this makes precise data analysis methods all the more useful. Although several types of GW forms can be modeled, many of them are yet unknown. Numerical signal detection methods must perform correctly despite this constraint, which is something that the MUSIC method can do.

The paper is organized as follows: The principle of the MUSIC algorithm and its extension to GW search are described in details in Section 2. In Section 3, Performance evaluation is carried out using simulated GW signals and noise. In Section 4, MUSIC is compared with the Q-transform\(^6\) approach to signal triggering, and to Bayesian\(^7\) analysis for DOA estimation, using the Ω-pipeline.\(^8\) Results show that MUSIC has a lower resolution, but is faster. MUSIC can usefully be applied to real-time GW search for multi-messenger astronomy.\(^9\)

2. The MUSIC Method

2.1. The MUSIC algorithm

The following represents the data used by MUSIC:

\[
X = AF + W
\]

\[
X = [X_1 \ X_2 \ \ldots \ X_M]^T
\]

\[
A = [a(\theta_1) \ a(\theta_2) \ \ldots \ a(\theta_D)]
\]

\[
W = [W_1 \ W_2 \ \ldots \ W_M]^T
\]

where \(M\) is the number of detectors and \(D\) the number of signals, with \(M>D\). \(X\) is the signal vector and \(A\) is the mode vector of the antenna response. \(F\) represents the source signal and \(W\) describes noises.

Considering the covariance matrix of \(X\) we get:

\[
S = X^*X = (AF + W)(AF + W)^* = (AF + W)(F^*A^* + W^*)
\]

\[
= AFF^*A^* + AFW^* + WF^*A + WW^* = AFF^*A^* + WW^*
\]

(2)
or

$$S = APA^* + \lambda S_0$$  \hspace{1cm} (3)

where $P$ is a $D \times D$ positive definite diagonal matrix and $A$ is a $M \times D$ full rank matrix. $APA^*$ is therefore nonnegative definite and of rank $D$. The number of non-zero eigenvalues is thus $D$ and the number of zero eigenvalues is $M-D$. With

$$|APA^*| = |S - \lambda S_0| = 0$$  \hspace{1cm} (4)

$\lambda$ is an eigenvalue of $S$ in the metric of $S_0$. In order to insure that $APA^*$ be non-negative definite, $\lambda$ must be the smallest eigenvalue of $S$.

So we get:

$$S = APA^* + \lambda_{\min} S_0$$  \hspace{1cm} (5)

In the case of white Gaussian noise:

$$\lambda_{\min} S_0 = \sigma^2 I$$  \hspace{1cm} (6)

Since the number of zero eigenvalues of $APA^*$ is $M-D$, $S$ must have its minimum eigenvalue repeated $M-D=N$ times. So:

$$\tilde{D} = M - \tilde{N}$$  \hspace{1cm} (7)

This is the estimation of the number of signals. Since the multiplicity of eigenvalue $\lambda_{\min}$ of $S$ in the metric of $S_0$ is $N$, there are $N$ linearly independent eigenvectors $e_i$ that satisfy:

$$Se_i = \lambda_{\min} S_0 e_i$$  \hspace{1cm} (8)

Since

$$Se_i = APA^* e_i + \lambda_{\min} S_0 e_i$$  \hspace{1cm} (9)

we have

$$APA^* e_i = 0$$  \hspace{1cm} (10)

and

$$A^* e_i = 0$$  \hspace{1cm} (11)

The vectors from $A$ form an RD signal subspace which is orthogonal to the noise subspace generated by the $N$ linearly independent eigenvectors $e_i$. Considering the Euclidean distance between them:

$$d^2 = Y^* E_N E_N^* Y$$  \hspace{1cm} (12)
we get
\[ P(\theta) = \frac{1}{a^*(\theta)E_N E^*_N a(\theta)} \] (13)

When \( \theta \) gets closer to DOA, \( P(\theta) \) becomes positive and infinite.

2.2. MUSIC extensions

The MUSIC method is widely used for periodic sine radio waves detection by a planar array of antennas. Several aspects of the algorithm were generalized before applying it to the detection of gravitational waves, through the use of

- spherical coordinates (generalization from a 2D to a 3D space);
- the concept of equal phase (to remove the limitation of regularly spaced detectors);
- a linear transfer in time domain (to extend the method to non-periodic signals).

The extended MUSIC algorithm was applied to GW detection through the following steps:

(i) Calculation of the covariance matrix \( S \) from collected data;
(ii) Calculation of the eigenvalues and eigenvectors of \( S \) in the metric of \( S_0 \);
(iii) Calculation of the \( M \)-1 eigenvectors of the noise subspace by assuming that there is a single, relatively long signal;
(iv) Calculation of \( P(\theta) \) and display of its graph;
(v) Detection of the signal peak;
(vi) Determination of the DOA and of other information of interest.

3. Performance Evaluation

A simulated signal containing some white Gaussian noise and a Gaussian pulse was used to evaluate the performance of MUSIC; this signal is shown in Fig. 1.
The results under different SNR ratios are shown in Fig. 2. The real signal position is $f_i=1.05$ and $s_i=1.05$. Figure 2a is the MUSIC spectrum with SNR is 7.5 dB and the calculated signal position is $f_i=1.08$ and $s_i=1.1$; Fig. 2b is the MUSIC spectrum with SNR is 1.5 dB and the calculated signal position is $f_i=1.1$ and $s_i=1.02$; Fig. 2c is the MUSIC spectrum with SNR is $-4$ dB and the calculated signal position is $f_i=1.02$ and $s_i=0.7$. In these scenarios, MUSIC performance is pretty good. Fig. 2d is the MUSIC spectrum with SNR is $-8.8$ dB and the calculated signal position is $f_i=0.5$ and $s_i=0.25$, which is pretty bad. So the SNR ratio limitation of MUSIC is around $-5$ dB.

4. Performance Comparison

MUSIC was compared with the $\Omega$-pipeline, with synthetic gravitational signals added on top of a simulated background. Two experiments were carried out for signal triggering and DOA estimation.
Table 1. Performance comparison results for signal triggering.

<table>
<thead>
<tr>
<th>Method</th>
<th>Lower limit for A</th>
<th>Calculation time</th>
<th>Time resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-transform</td>
<td>2</td>
<td>14s</td>
<td>0.015s</td>
</tr>
<tr>
<td>MUSIC</td>
<td>200</td>
<td>3200s</td>
<td>0.03s</td>
</tr>
</tbody>
</table>

Table 2. Performance comparison results for DOA estimation.

<table>
<thead>
<tr>
<th>Method</th>
<th>Lower limit for A</th>
<th>Calculation time</th>
<th>Angle resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bayesian</td>
<td>4</td>
<td>30s</td>
<td>0.019rad</td>
</tr>
<tr>
<td>MUSIC</td>
<td>1000</td>
<td>4.2s</td>
<td>Complicated</td>
</tr>
</tbody>
</table>

Fig. 3. Bayesian skymap when A=100.

In the following, the relative signal strength is denoted by A (parameter Factor of LogFile of the injection part). A typical gravitational wave has a strength A=1.

The performance of MUSIC for signal triggering are shown in Table 1, where it is compared with the Q-transform approach. MUSIC does not appear to be suitable for signal triggering.

The performance of MUSIC for DOA estimation are shown in Table 2, where it is compared with the Bayesian approach. MUSIC is processed faster for DOA estimation, but has lower resolution.

The skymap of the Bayesian approach when A=100 is illustrated in Fig. 3. Skymaps for MUSIC when A=1000 and A=5000 are shown in Fig. 4. Skymaps show that the sensitivity, stability and quality of MUSIC detection is far below that of the Bayesian approach, even when the signal strength for MUSIC is tens of times larger.
5. Conclusions

According to our research, the MUSIC algorithm is an useful tool for gravitational wave burst search, as it is not restricted to any specific signal form. It can quickly obtain trigger and DOA.

Compared with existing tools such as those of the Ω-pipeline, MUSIC has the advantage of being faster; however, its high minimum SNR ratio, its time resolution and its stability make MUSIC a better choice only in cases where processing speed is critical to DOA determination, such as in real-time GW search for multi-messenger astronomy.\(^9\)

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Appendix A. Performance Data for the Bayesian Analysis

Table 3. The performance data for the Bayesian analysis.

<table>
<thead>
<tr>
<th>N</th>
<th>A</th>
<th>$\phi_0$</th>
<th>$\theta_0$</th>
<th>$\phi_b$</th>
<th>$\theta_b$</th>
<th>$\Delta \phi$</th>
<th>$\Delta \theta$</th>
<th>$T_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.236</td>
<td>1.677</td>
<td>0.754</td>
<td>2.813</td>
<td>0.518</td>
<td>1.146</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3.110</td>
<td>1.993</td>
<td>3.130</td>
<td>2.070</td>
<td>0.020</td>
<td>0.077</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.871</td>
<td>2.380</td>
<td>0.770</td>
<td>2.192</td>
<td>0.101</td>
<td>0.188</td>
<td>—</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2.885</td>
<td>1.647</td>
<td>2.884</td>
<td>1.616</td>
<td>0.001</td>
<td>0.031</td>
<td>—</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>4.880</td>
<td>3.000</td>
<td>4.95</td>
<td>2.91</td>
<td>0.07</td>
<td>0.09</td>
<td>—</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>1.143</td>
<td>1.989</td>
<td>1.110</td>
<td>1.952</td>
<td>0.033</td>
<td>0.037</td>
<td>23.3s</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>0.637</td>
<td>2.655</td>
<td>0.627</td>
<td>2.647</td>
<td>0.010</td>
<td>0.008</td>
<td>49.8s</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>1.652</td>
<td>1.706</td>
<td>1.650</td>
<td>1.727</td>
<td>0.002</td>
<td>0.021</td>
<td>16.0s</td>
</tr>
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</table>

Appendix B. Performance Data for MUSIC

Table 4. The performance data for MUSIC.

<table>
<thead>
<tr>
<th>N</th>
<th>A</th>
<th>$\phi_0$</th>
<th>$\theta_0$</th>
<th>$\phi_b$</th>
<th>$\theta_b$</th>
<th>$\Delta \phi$</th>
<th>$\Delta \theta$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>0.351</td>
<td>1.811</td>
<td>0</td>
<td>1.355</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>0.054</td>
<td>1.518</td>
<td>0</td>
<td>1.316</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>1.698</td>
<td>1.425</td>
<td>0.864</td>
<td>0.490</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>500</td>
<td>4.800</td>
<td>1.012</td>
<td>0.864</td>
<td>0.668</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>2.572</td>
<td>1.292</td>
<td>2.435</td>
<td>1.355</td>
<td>0.063</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>4.927</td>
<td>1.145</td>
<td>1.963</td>
<td>1.414</td>
<td>0.269</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1000</td>
<td>3.764</td>
<td>1.387</td>
<td>2.513</td>
<td>1.237</td>
<td>0.150</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>5000</td>
<td>4.184</td>
<td>1.104</td>
<td>0.314</td>
<td>1.119</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>5000</td>
<td>5.588</td>
<td>1.160</td>
<td>2.435</td>
<td>1.355</td>
<td>0.195</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5000</td>
<td>1.377</td>
<td>0.512</td>
<td>6.283</td>
<td>0.511</td>
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<td></td>
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References