

# Stochastic Optimal Energy Storage Management for Energy Routers via Compressive Sensing

**Abstract**—The functionality of energy routing among microgrids is becoming increasingly important with the progress of deploying smart power systems all over the world. For higher energy routing performance and better renewable energy integration, a new type of electrical device, called energy router (ER), is being developed as a part of the infrastructure of the future energy Internet (EI). Generally, the long-term operation of ERs requires an effective energy management scheme for the energy storage inside these devices. In this paper, considering the randomness of power generation by renewable energy sources and the stochastic power usage of loads in EI scenario, the compressive sensing is adopted for the solution to the nonlinear energy storage management problem which is essential for the design of ERs. The compressive sensing method used in this paper is proven to be more efficient than the conventional Monte Carlo methods and polynomial chaos expansion method, and the performance of the proposed method is evaluated with numerical examples.

**Index Terms**—Compressive sensing, energy Internet, energy router, energy storage, stochastic optimization.

## NOMENCLATURE

EI	Energy Internet.
ER	Energy router.
MG	Microgrid.
DG	Distributed controllable power generator.
RES	Renewable energy source.
$\Delta P_{Load}$	Power deviation of loads.
$\Delta P_{RES}$	Power deviation of RESs.
$\Delta P_{DG}$	Power deviation of DGs.
$\Delta P_{out}$	Aggregate power transmission request from external ERs.
$\Delta P$	Power input/output of the energy storage in ER.
$S_{ER}$	Energy storage status of the considered ER.
CS	Compressive sensing.
MC	Monte Carlo.
PCE	Polynomial chaos expansion.

## I. INTRODUCTION

The EI has changed the mode and the structure of energy supply and demand in the sense that the centralized top-down energy management mode is being transformed into a combination of an interactive centralized top-down mode and a distributed bottom-up mode, such that consumers are satisfied with the demand for power services, the contradiction between power peaks and valleys can be effectively solved, and the requirements of energy security and reliability can be met [1]. Various energy forms, such as cooling, heating, electricity, gas, are fully integrated in the generation, transmission and consumption side, making full use of the complementary characteristics of multiple energy sources, and increasing the energy utilization efficiency of the terminal through energy

cascade utilization. Through the free choice and interactive management of energy sources, the energy industry has also been influenced by the Internet; related industries have been risen, and the economic structure and even social habits are being reformed [2].

In future EI scenarios, demand side response based on big data and wide-area information platform would improve the efficiency of energy production and transmission [3]. Located at the connection points of EI, ERs are the key equipment to realize not only information transmission but also open and peer-to-peer transmission routing of energy [4]. The ER (also called energy hub, or electric router), an emerging device based on advanced power electronic techniques, shall be able to implement flexible and dynamic power distribution in EI analogous to the character of information routers in the Internet. With the support of ER, communication network and power network, the bottom-up infrastructure in the future EI can be built [5].

In recent years, the investigation of ER regarding different aspects has been popular. The architectures, functionalities and demonstration of ER have been introduced in [6]. Similar to the data caching function of routers in the Internet, ER needs the capability of fast and reliable energy storage to enable the flexible energy sharing in future EI scenarios. Although research regarding ER has been popular during the past five years, only prototypes of ERs are developed in laboratories, and there has been no commercial products so far. In this paper, the energy storage management for ERs is considered in a macro perspective, in which sense, control strategies shall be designed for all MGs in EI, rather than one single ER in [7]. In particular, the stochastic feature of the power generation by RESs and the power of loads are considered. We assume the controllers are set in DGs and ERs. Both ordinary and stochastic differential equations are adopted to describe the power dynamics of the considered system. Then, the energy storage management task for ER is formulated as a nonlinear stochastic optimization problem.

It is almost impossible to solve this problem analytically, and numerical methods shall be considered. To address this issue, adaptive dynamic programming (ADP), proposed by Werbos [8], has widely been applied to obtain the optimal energy management policies in EI situation. For example in [9], it proposes an ADP based approach for the economic dispatch of MG with DGs and a modified value function update strategy is included in this work. A lookup tables based ADP algorithm for the real-time energy management of the MG under uncertainties is proposed in [10], while the optimal operation of the MG is also formulated as a stochastic nonlinear programming problem. In [11], they use the ADP method to solve a smart home energy management

system and improve the performance of such problem by considering uncertainties of the system. A stochastic gas-power network constrained unit commitment model considering both combined-cycle units and gas Network is established in [12], in order to prevent the curse of dimensionality they also use ADP method to deal with the stochastic control problem. An action-dependent heuristic dynamic programming (ADHDP) method is applied to the optimal control of dynamic energy management systems for MGs [13]. Notably, ADHDP method also denoted as Q-learning. Moreover, due to the rapid development of artificial intelligence, reinforcement learning method becomes more attractive in solving model-free or model-based problems of EI, such as in [14], an artificial neural network-based reinforcement learning algorithm was proposed to manage an optimal multi-energy management-based energy routing design problem. Reference [15] concentrates on a model-free energy routing strategy problem, and the actor-critic reinforcement learning approach is applied to solve the optimal control problem. A multi-agent based model is used to study distributed energy management in a MG in [16], and a reinforcement learning algorithm was developed to derive the optimal strategies for energy management and load scheduling without prior information of the MG system.

An alternative approach is to combine MC method with heuristic optimization algorithm [17] or traditional iterative method such as gradient decent, Newton, et. al. Normally, the MC method has a relatively low convergence rate. Thus, in order to achieve a desirable computation accuracy, it often results in a prohibitively high computational complexity. Hence, aiming to overcome the low efficiency caused by the huge amount of computation, more efficient methods for solving stochastic equations should be applied instead of using MC, an alternative method is PCE [18] which has been proved efficient in solving stochastic equations, but unlike the MC method, it usually requires recoding the system into a much higher dimension problem which make it complex in programming. However, due to the sparsity of the solution for stochastic system, CS method which is emerged from the field of sparse signal recovery [19] can be used to overcome the disadvantages of both PCE and MC method, because it does not require recoding the system like PCE method, nor does it require a large number of samples like MC method. As a whole, the novelty aspects of this study are summarized as follows:

1. A novel gradient-based method with CS has been proposed to solve high-dimensional stochastic optimization problem. The main innovation of our proposed algorithm is to apply CS method instead of MC method to deal with stochastic system in the iteration of traditional optimization algorithm, and it has been shown effective.

2. In this paper, an energy storage management problem for the ER is formulated as an optimal control problem, taking the ER system's nonlinearity and stochasticity into consideration, which has not been fully investigated in previous works.

3. Simulation results show that when solving the aforementioned problem, CS method converges significantly faster than MC method, and is much more easy to be realized by programming than PCE method, which indicates that CS can

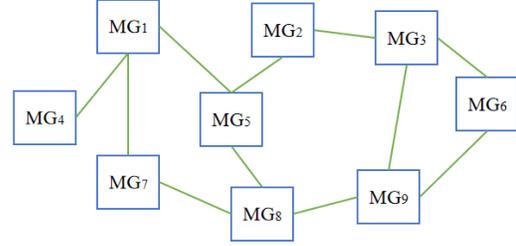


Fig. 1. Typical off-grid EI network.

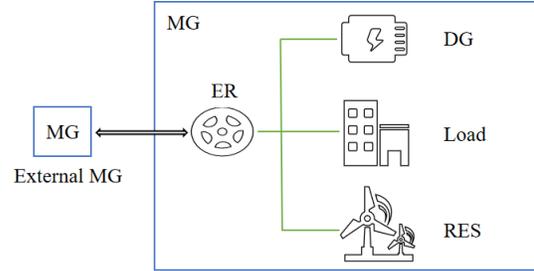


Fig. 2. A typical MG.

save computational cost dramatically when applied in such kind of stochastic optimization problem. It is notable that our proposed algorithm can be used not only in the problem of ER energy management, but also in a variety of fields where similar optimal control problems are expected to be solved.

The rest of the paper is organized as follows. System modelling and problem formulation are provided in Section II and Section III, respectively. The solution to the considered optimization problem is investigated in Section IV. Section V provides some simulations. Finally, some concluding remarks are given in Section VI.

## II. SYSTEM MODELLING

In this paper, we consider a special EI scenario where a variety of MGs are interconnected without access to the utility grid, in the sense that such EI is functioning in an off-grid mode. A simplified configuration is illustrated in Fig. 1. Such EI scenario, in which the main power generation relies on RESs, is suitable for some rural or remote areas; see, e.g., [20].

For the EI scenario in Fig. 1, our focus is put on one of the interconnected MGs. Each MG consists of multiple types of RESs, DGs and loads which are all connect with an ER, and such ER is connected with an external ER in the other MG; see, Fig. 2.

The linearised modelling approach is adopted to describe the power dynamics of each component in MG. To reveal the uncertainty and variation of power of load and RESs [21], stochastic differential equations driven by scalar Brownian motion are given as follows,

$$d\Delta P_{Load} = -\theta_l(\Delta P_{Load} - \mu_l)dt + \sigma_l dW_l, \quad (1)$$

$$d\Delta P_{RES} = -\theta_r(\Delta P_{RES} - \mu_r)dt + \sigma_r dW_r, \quad (2)$$

where  $W_l$  and  $W_r$  refer to scalar Brownian motions, scalars  $\theta_l$ ,  $\theta_r$ ,  $\mu_l$ ,  $\mu_r$ ,  $\sigma_l$  and  $\sigma_r$  are system parameters that can be measured by parameter estimation methods. For notation simplicity, time  $t$  of all variables throughout this context is omitted.

The DGs in our considered MG can be micro-turbines, diesel engine generators, fuel cells, etc. The following ordinary differential equation shows the generalized power dynamics of DGs.

$$d\Delta P_{DG} = -\frac{1}{T_{DG}}(\Delta P_{DG} - u_G)dt, \quad (3)$$

where  $T_{DG}$  stands for the time constant of DGs,  $u_G$  is the desired power adjustment for the DGs, which can be viewed as control input of DGs.

Power generation by RESs and power usage caused by loads flows into and out of the ER in Fig. 2, respectively. Thus, it is obvious that the power to be stored in ER is

$$\Delta P = \Delta P_{RES} + \Delta P_G - \Delta P_{Load} + \Delta P_{out}(1 - u_{ER}), \quad (4)$$

where  $u_{ER}$  denotes the average transmission rejection rate for other ERs connected to the considered ER. In this sense,  $u_{ER}$  is the control input for the investigated ER. For such ER, since the total power required from the accessed external ERs is also random, depending on various complex conditions, stochastic differential equation similar with (1) and (2) are used to describe  $\Delta P_{out}$ . Let us denote  $W_p$  as the stochastic process in the same probability space of  $W_l$  and  $W_r$ . Then, we have

$$d\Delta P_{out} = -\theta_p(\Delta P_{out} - \mu_p)dt + \sigma_p dW_p, \quad (5)$$

where scalars  $\theta_p$ ,  $\mu_p$ ,  $\sigma_p$  are system parameters.

The energy storage status of the considered ER, denoted as  $S_{ER}$ , is actually the state of charge of ER's energy storage component, and it is represented as

$$dS_{ER} = \eta \Delta P dt, \quad (6)$$

where  $\eta$  is a constant related to the energy efficiency and capacity of the energy storage attached in the ER, and the value of  $S_{ER}$  is naturally defined to be within  $[0, 100\%]$ .

In this sense, the linearized models from (1) to (6) fully describe the dynamics of one typical MG in the considered EI scenario. For similar modelling approaches, readers can refer to [22].

Next, let  $x = [\Delta P_{Load}, \Delta P_{RES}, \Delta P_G, \Delta P_{out}, S_{ER}]'$ ,  $u = [u_G, u_{ER}]'$ , and  $W = [W_l, W_r, W_p]'$ . The system from (1) to (6) can be rewritten as:

$$dx = [A(u)x + Bu + C]dt + DdW, \quad (7)$$

where matrices

$$A(u) = \begin{pmatrix} -\theta_l & 0 & 0 & 0 & 0 \\ 0 & -\theta_r & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_g} & 0 & 0 \\ 0 & 0 & 0 & -\theta_p & 0 \\ -\eta & \eta & \eta & \eta(1 - u_{ER}) & 0 \end{pmatrix}, \quad (8)$$

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{T_g} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} \theta_l \mu_l \\ \theta_r \mu_r \\ 0 \\ \theta_p \mu_p \\ 0 \end{pmatrix}, \quad D = \begin{pmatrix} \sigma_l & 0 & 0 \\ 0 & \sigma_r & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_p \\ 0 & 0 & 0 \end{pmatrix}. \quad (9)$$

In this sense, our studied MG power dynamics have been transformed into a stochastic control system (7), where  $x(t)$  is system state,  $u(t)$  is system control input.

It is notable that matrix  $A(u)$  is not a time-invariant parameter matrix. Essentially, it is a function of  $u_{ER}(t)$ . The term  $A(u)x$  indicates that system (7) is indeed a *nonlinear* system.

### III. PROBLEM FORMULATION

In this section, the energy storage management for the considered ER is formulated as an optimal control problem.

Due to the energy management principle of EI, the autonomous operation of the power system shall be preferably achieved at the MG level [5]. As key devices providing the energy sharing functionality among MGs, ERs are expected to be of the ability to achieve long-term operations with high stability. To this end, one of the important aspects is to maintain the energy storage status of the considered ER at a stable level over time [7]. This target could be achieved by restricting the power throughput of the energy storage in the considered ER. However, over-restricted power throughput may impair the energy storage capability of the ER and reduce the stability of the entire EI system. In this sense, instead of utilizing regular quadratic cost functions, the objective function in (10) is adopted in this paper, whose value should be minimized by designing proper control strategies.

$$\mathcal{J}_0 = \mathbb{E} \int_0^T \{\varepsilon_1 \log(1 + e^{\Delta P^2}) + \varepsilon_2 u_G^2\} dt, \quad (10)$$

where  $\mathbb{E}$  is mathematical expectation,  $\varepsilon_1$  and  $\varepsilon_2$  are weight coefficients. The interpretation of each term in (10) is as follows.

For stochastic control problems, the traditional LQ problem only considers the mean of the cost and ignores the higher order momenta. In order to incorporate higher order momenta of the objective function, we made a little modification by adopting the term  $\log(1 + e^{\Delta P^2})$  in (10), the objective function (10) can consider the higher order momenta of the stochastic inputs according to the design of the exponential-quadratic control objective function in risk-sensitive control theory [23], and the energy storage capability of ERs could be better utilized compared to simply adopting  $\Delta P^2$  in (10). Given the same value of  $\Delta P$ , for instance, for a small  $\Delta P$ , we have  $\log(1 + e^{\Delta P^2}) \geq \Delta P^2$ . This means that the optimal controller for (10) would tend to address small power deviation  $\Delta P$  via the adjustment of DGs. Additionally, the second term in the integral of (10) is used to avoid the over-control of DGs.

Let  $C_{min}$  and  $C_{max}$  be the minimum and maximum values of  $S_{ER}$ ,  $\Delta P_{Load}^{max}$ ,  $\Delta P_{RES}^{max}$  and  $\Delta P_G^{max}$  be the maximum power variation constraints of  $\Delta P_{Load}$ ,  $\Delta P_{RES}$  and  $\Delta P_G$ ,

respectively. The optimal control problem is then formulated as (11).

$$\begin{aligned} \min_{u_G, u_{ER}} \quad & \mathcal{J}_0, \\ \text{s.t.} \quad & (1) - (6), \\ & \Delta P_{Load} \leq |\Delta P_{Load}^{max}|, \\ & \Delta P_{RES} \leq |\Delta P_{RES}^{max}|, \\ & \Delta P_G \leq |\Delta P_G^{max}|, \\ & C_{min} \leq S_{ER} \leq C_{max}, \\ & 0 \leq u_{ER} \leq 1. \end{aligned} \quad (11)$$

In order to deal with the inequality constraints in (11), two penalty terms  $\mathcal{P}_1$  and  $\mathcal{P}_2$  are introduced to the objective function. Specifically, the penalty term  $\mathcal{P}_1$  for  $S_{ER}$  and  $u_{ER}$  is designed in the following form to ensure  $S_{ER}$  and  $u_{ER}$  vary near the midpoints of the constraints as much as possible.

$$\mathcal{P}_1 = \left( \frac{S_{ER} - \frac{1}{2}(C_{max} + C_{min})}{C_{max} - C_{min}} \right)^2 + \left( u_{ER} - \frac{1}{2} \right)^2, \quad (12)$$

Moreover, to keep the power variations within the maximum constraints, the penalty term  $\mathcal{P}_2$  for  $\Delta P_{Load}$ ,  $\Delta P_{RES}$  and  $\Delta P_G$ , has the following form:

$$\mathcal{P}_2 = 10^5 (\mathbb{I}_{Load} + \mathbb{I}_{RES} + \mathbb{I}_G), \quad (13)$$

where  $\mathbb{I}_{Load}$  is the characteristic function defined by:

$$\mathbb{I}_{Load} = \begin{cases} 0 & \text{if } \Delta P_{Load} \leq |\Delta P_{Load}^{max}| \\ 1 & \text{otherwise,} \end{cases} \quad (14)$$

Similarly,  $\mathbb{I}_{RES}$  and  $\mathbb{I}_G$  are defined in the same way. Multiplying the characteristic function by  $10^5$  can prevent the power variations from exceeding the limits

With a large weight coefficient for the penalty term  $\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2$ , the optimal controller would presumably keep  $S_{ER}$  and  $u_{ER}$  around the midpoints of their allowed ranges and control the power variations within reasonable boundaries. In this manner, the optimal control problem with inequality constraints in (11) is transformed to be a relaxed problem (15).

$$\begin{aligned} \min_{u_G, u_{ER}} \quad & \mathcal{J} = \mathbb{E} \int_0^T \{ \varepsilon_1 \log(1 + e^{x'Qx}) + \varepsilon_2 u' Ru + \varepsilon_3 \mathcal{P} \} dt. \\ \text{s.t.} \quad & (1) - (6), \end{aligned} \quad (15)$$

where  $\varepsilon_3$  is weight coefficients,

$$Q(u) = \begin{pmatrix} 1 & -1 & -1 & u_{ER} - 1 & 0 \\ -1 & 1 & 1 & (1 - u_{ER}) & 0 \\ -1 & 1 & 1 & (1 - u_{ER}) & 0 \\ u_{ER} - 1 & 1 - u_{ER} & 1 - u_{ER} & (1 - u_{ER})^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (16)$$

$$R = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad (17)$$

By solving the stochastic nonlinear optimal control problem (15), the optimal energy storage management strategy for the considered ER could be obtained. As mentioned in Section

I, the conventional methods, such as dynamic programming or the MC method, are limited in computational efficiency and may not be feasible for high dimensional problems. Since the polynomial expansion of stochastic process usually have sparsity [24], the CS method which has been developed rapidly in recent years is used to solve the aforementioned control problem. The detailed methodologies are given in next section.

#### IV. SOLVING THE OPTIMIZATION PROBLEM

In this section, an algorithm based on CS has been proposed to solve the optimization problem (15). First of all, we review the process of solving differential equations using the CS method. Then, by combing the CS with gradient descent method, we propose a new iterative algorithm, named as CS-GDM, to solve the optimization problem (15).

##### A. Compressive Sensing for Differential Equations

The process of using CS method to solve stochastic differential equations can be summarized as three steps:

- s1. Representation of Brownian motion via Fourier expansion;
- s2. Polynomial chaos expansion;
- s3. Sparse recovery using CS method.

For step s1, choosing an orthonormal basis  $\{\varphi_i\}_{i=1}^{\infty}$ , we have

$$W(s) = \sum_{i=1}^{\infty} \xi_i \int_0^s \varphi_i(\tau) d\tau, \quad (18)$$

where  $W(s)$  denotes Brownian motion,  $\{\xi_i\}_{i=1}^{\infty}$  are independent identically distributed random variables selected from the standard normal distribution, and the expansion (18) uniformly converges to  $W(s)$  in the mean square sense.

Expanding the Brownian motion is to discretize the original problem. Let  $i \in [1, d]$ , we truncate the expansion with  $d$  terms in (18). Then, the Brownian motion in the system (7) can be expressed via  $\xi = \{\xi_i\}_{i=1}^d$ , so the system state  $x(t)$  is actually related to  $\xi$ . More specifically,  $x(t)$  should be written as  $x(t, \xi)$ , then we can use a set of stochastic polynomials to expand the system state  $x(t, \xi)$ , which is described as step s2.

In this paper, since the random variables  $\xi$  comes from the standard normal distribution, we choose the generalized Hermite polynomial basis as the stochastic basis to expand  $x(t, \xi)$  [26]; that is:

$$x(t, \xi) = \sum_{j=1}^p c_j(t) \psi_j(\xi), \quad (19)$$

where  $\{\psi_j(\xi)\}_{j=1}^p$  are the Hermite polynomials,  $p$  is the number of polynomials needed for the expansion, and  $\{c_j(t)\}_{j=1}^p$  are the coefficients which need to be determined by CS method through step s3.

Once we have the polynomial chaos expansion (19) for  $x(t, \xi)$ , we can recover the sparse coefficients in (19) by solving the following basis pursuit de-noising (BPDN) problem.

$$\hat{c} = \arg \min \|c\|_1, \quad \text{subject to } \|X - \Psi c\| \leq \epsilon, \quad (20)$$

where  $X$  is the sample simulation results. If we use  $n$  as the number of sample simulation, then  $X$  is a  $n$  dimensional vector at each time  $t$ , and  $\Psi$  is a  $n \times p$  information matrix formed by inserting the stochastic sample points into Hermite polynomials.  $c = \{c_j\}_{j=1}^p$  is the coefficient vector to be determined. For theoretical analysis regarding this method, readers can refer to [25].

### B. Algorithm for the Stochastic Optimal Control Problem

To solve the optimal control problem (15), we need to introduce the Hamiltonian function  $H$  :

$$H = \mathcal{J} + \lambda'(t)\{A(u)x(t) + Bu + C + DdW\}, \quad (21)$$

where  $\lambda(t)$  is system co-state. Then, the equivalent Hamiltonian system for the optimal control problem are:

$$dx = \frac{\partial H}{\partial \lambda}, \quad d\lambda = -\frac{\partial H}{\partial x}, \quad 0 = \frac{\partial H}{\partial u}. \quad (22)$$

To be specific, the corresponding system for control problem (15) with respect to (7) are given as:

$$dx = A(u)x + Bu + C + DdW, \quad (23)$$

$$d\lambda = \frac{-2\varepsilon_1 Qx e^{x'Qx}}{1 + e^{x'Qx}} - A(u)' \lambda - 2\varepsilon_3 a_1(a_1 x - b_1), \quad (24)$$

$$0 = 2\varepsilon_2 Ru + 2\varepsilon_3 a_2(a_2 u - b_2) + \lambda(x \frac{\partial A(u)}{\partial u} + B). \quad (25)$$

Since system (23)-(25) is nonlinear, it is difficult to be solved directly. Thus, we tend to choose the iterative algorithm, such as gradient descent, conjugate direction, Newton, or quasi-Newton methods combining with CS method to solve this stochastic optimization problem numerically. In our proposed method, we choose gradient descent method as an example, and a new algorithm named as CS-GMD is proposed as Algorithm 1.

Instead of some conventional methods, e.g., the MC method, we use the CS method to calculate the system state  $x(t)$  and the system co-state  $\lambda(t)$ , which reduces computational cost dramatically and is more suitable for high-dimensional problems. This is regarded as one of the main contribution of this work. Now, we can numerically solve this complex stochastic optimal control problem. The obtained controller is indeed the control strategy for the management of energy storage in the considered ER. The relevant numerical results are given in Section V.

It is notable that our proposed CS-GDM algorithm can be used not only in the problem of ER energy management, but also in a variety of fields such as fluid mechanics, computational biology, computational finance, etc., where similar optimal control problems are expected to be solved.

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### Algorithm 1 (CS-GDM optimization algorithm)

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1. Initialize the optimization.

The optimization loop is initialized as follows:

- 1) Select the number of Hermite polynomial basis  $p$  and the stochastic dimension  $d$ ,
- 2) Select a step size parameter  $z > 0$  and a tolerance parameter  $\beta$ ,
- 3) Select a time step  $\Delta t$ ,
- 4) Select an initial controller  $u^0$ ,
- 5) Using CS method to solve (23) to obtain the coefficients of expansion for system state  $x^0$ ,
- 6) Calculate the value  $\mathcal{J}^0$  of the objective function (15),

2. Optimization loop.

For  $l = 1, \dots$ ,

- 1) From  $x^{l-1}$  and  $u^{l-1}$ , using CS method to determine the solution  $\lambda^{l-1}$  of the co-state equation (24),
- 2) From  $\lambda^{l-1}$ ,  $u^{l-1}$  and  $x^{l-1}$ , determine the set of steps  $\frac{\partial H}{\partial u^{l-1}}$  from (25),
- 3) From  $u^{l-1}$  and  $\frac{\partial H}{\partial u^{l-1}}$ , determine the new value  $u^l$

$$u^l = u^{l-1} - z^{l-1} \frac{\partial H}{\partial u^{l-1}},$$

- 4) From  $u^l$ , using the CS method to calculate the coefficients of the expansion for system state  $x^l$ ,
  - 5) From  $u^l$  and  $x^l$ , determine the value of the functional  $J^l$  from (15),
  - 6) If  $J^l \leq J^{l-1}$  and  $(J^l - J^{l-1})/J^{l-1} \leq \beta$ , stop and accept the result,  
If  $J^l \leq J^{l-1}$  and  $(J^l - J^{l-1})/J^{l-1} \geq \beta$ , go to step 1), incrementing the iteration counter  $l$ ,  
If  $J^l \geq J^{l-1}$ , set  $z^l = z^{l-1}/2$  and go to step 2).
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## V. SIMULATION RESULTS

In this section, the simulation results derived by our proposed method are given. First, we present some results to illustrate the feasibility of compressive sensing method in solving the stochastic differential equations. Then, the simulation results derived by our proposed CS-DGM algorithm for storage management problem of single MG and multiple interconnected MGs are given in the second part and third part separately. All the solutions are obtained with the SPGL1 toolbox [27] in MATLAB (R2018b, Mathworks Inc., Natick, MA, USA) environment, and the hardware device is an Ubuntu 18.04 server with Intel Core i7-7700 CPU and one single GPU card with 2GB of graphic memory.

### A. Simulation for State Equations

Since the main innovation of our proposed Algorithm 1 is to apply the CS method instead of other method to deal with the stochastic state equation (7), and then combine it with optimization algorithm to solve optimization problems. So first of all, at this part, we verify the applicability and efficiency of CS method by comparing the simulation results with two traditional approaches, i.e. MC method and PCE method, for solving stochastic state equation (7). The values of all relevant parameters are given in Table I [5], [20].

TABLE I  
VALUE OF PARAMETERS

Parameter	value	Parameter	value	Parameter	value
$\theta_l$	0.4	$\mu_l$	0.7	$\sigma_l$	0.4
$\theta_r$	0.5	$\mu_r$	0.2	$\sigma_r$	0.12
$\theta_p$	0.1	$\mu_p$	0.1	$\sigma_p$	0.43
$T_{DG}$	0.4	$\eta$	0.05	-	-
$C_{max}$	0.8	$C_{min}$	0.3	-	-

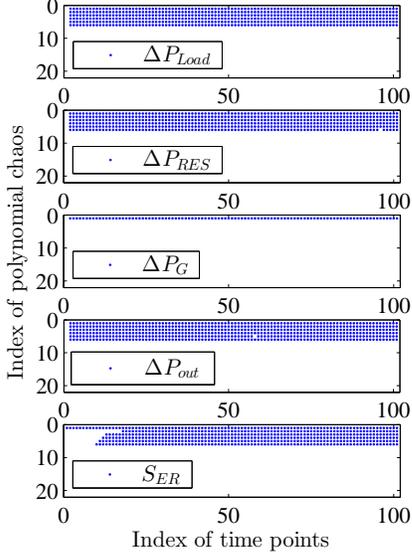


Fig. 3. Entries in coefficients  $c$  with absolute values larger than  $\tau = 1 \times 10^{-5}$  within time interval  $[0, 1]$ .

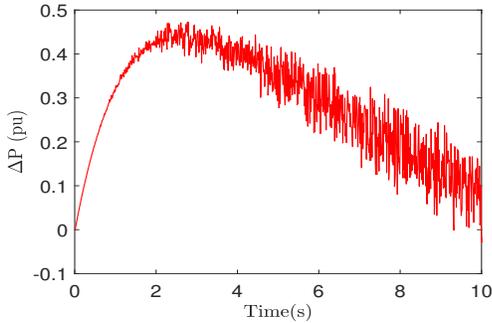


Fig. 4. Recovery of  $\Delta P$  using CS method.

Fig. 3 shows the sparsity of the coefficients  $c$ . We divide time interval  $[0, 1]$  into 100 sub-intervals, and the index are shown on the horizontal axis. The index of Hermite polynomial basis from (19) are shown on the vertical axis, which is from 0 to 20 in our numerical example. Each column in Fig. 3 corresponds to a coefficient vector  $c$  at different time nodes. To show the sparsity of the coefficients  $c$ , only elements that are greater than a threshold  $\tau$  are given as those dots. It can be seen that each component of  $x(t, \xi)$  is sparse with the

TABLE II  
ERROR ESTIMATES FOR EXPECTATION RECOVERY FOR  $S_{ER}$  USING MC AND CS METHODS

Number of Sample( $n$ )	Error <sup>MC</sup> $\times 10^{-2}$	Error <sup>CS</sup> $\times 10^{-2}$
10	7.83	12.71
20	5.49	2.20
30	4.69	0.72
50	3.35	0.006
100	2.41	0.006
150	2.05	0.006
200	1.69	0.006
1000	0.72	0.006

TABLE III  
ERROR ESTIMATES FOR  $S_{ER}$  USING CS AND PCE METHODS

	CS method	PCE method
Expectation error	$6.1 \times 10^{-5}$	$6.3 \times 10^{-5}$
Variance error	$1.9 \times 10^{-4}$	$2.3 \times 10^{-4}$
Sparsity(s)	0.2894	0.2579

expansion of Hermite polynomials in stochastic space, which suggests the feasibility of CS method.

The effectiveness of the proposed CS method is demonstrated with Fig. 4 in which the power input/output of the energy storage  $\Delta P$  integrated in the ER within 10 seconds is recovered from the solution to the BPDN problem (20) under the setting of  $u_{ER} = 0.1$  and  $u_G = 1$ .

Next, to demonstrate the computational efficiency of the proposed method, we give the numerical error estimates for expectation recovery between the MC method and the CS method in Table II. It can be seen that when the same computation accuracy is achieved, 30 sample points are used via CS method, while 1000 sample points shall be used via the MC method. In addition, the CS method converges much faster than MC method, and increasing the number of samples will not provide additional information. Therefore, in real-world engineering practice, the CS method can save significant computational resource.

To further verify the accuracy of the CS method, the comparison results with the typical PCE method [18] are given in Table III. The main principle of the PCE method is to take expansion (19) into the state equations, and then determine the expansion coefficients  $c$  by solving a high dimensional equations system(26).

$$d \sum_{j=1}^p c_j(t) \psi_j(\xi) = \sum_{j=1}^p [A c_j(t) \psi_j(\xi) + B u + C + D dW], \quad (26)$$

The dimension of equation system (26) is  $p$  times of the original system (7), which makes the simulation of the PCE method more complicated. The error estimation of the expectation and variance for CS and PCE are obtained by comparing with 3000 sample points MC method. The sparsity is the ratio of the

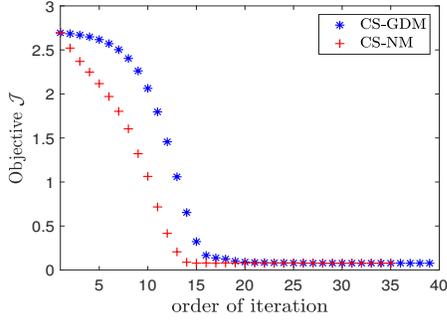


Fig. 5. Convergence of the objective function obtained by CS-GDM and CS-NM method

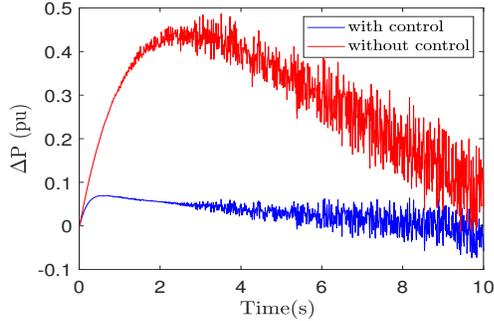


Fig. 6. Optimal  $\Delta P$  with control vs initial  $\Delta P$  without control.

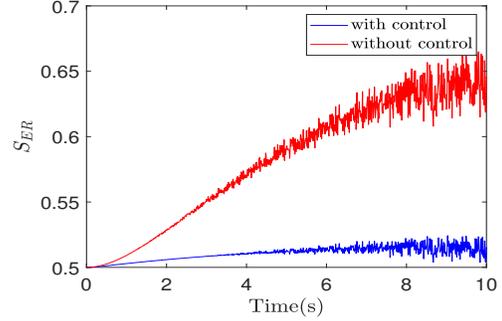


Fig. 7. Optimal  $S_{ER}$  with control vs initial  $S_{ER}$  without control.

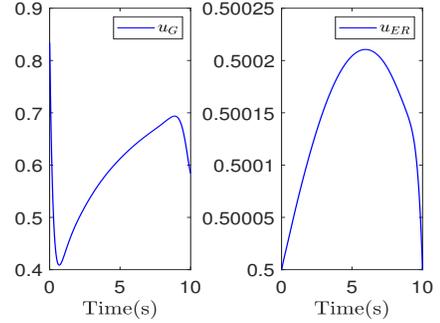


Fig. 8. Optimal Controller  $u_G$  and  $u_{ER}$ .

entries greater than  $\tau$  in  $\mathbf{c}$  to the total number of entries in  $\mathbf{c}$ . It can be seen that the CS method can achieve the same accuracy and sparsity with PCE method and does not need to rewrite the state equation solver, which is nevertheless necessary for PCE method.

In summary, the CS method not only overcomes the slow convergence of MC method, but also overcomes the complicated programming problem of PCE method, so it is more suitable to be combined with iterative method to solve stochastic optimization problems. In the following simulation parts, we verify the effectiveness of the algorithm1 in the scenarios of single MG and multiple interconnected MGs separately.

### B. Simulation results for single MG

To demonstrate the solution to the considered control problem, we give the simulation results within time period  $t \in [0, T]$ ,  $T = 10s$  as an illustrative example. If the simulation results for a longer time scale is required, there is no essential difficulty to perform further simulations in different time periods. To be specific, in this case study, the weighting coefficients are set as  $\varepsilon_1 = 1, \varepsilon_2 = 0.01, \varepsilon_3 = 1$ . The initial values  $x_0 = (0, 0, 0, 0, 0.5)$ , and the initial control variable  $u_G$  and  $u_{ER}$  are random numbers from  $(0,1)$ . The corresponding numerical results are given as follows.

In Fig. 5, the blue spot is the value of objective function  $\mathcal{J}$  derived by our Algorithm 1 at each step of the iteration, while the red spot represents the value of objective function derived by combining CS with Newton method. They illustrate the effectiveness of combining CS method with iterative methods

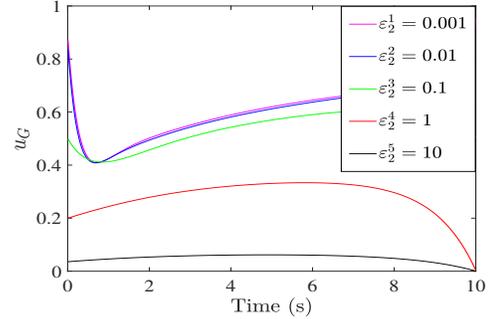


Fig. 9. Range of  $u_G$  for different control weight  $\varepsilon_2$ .

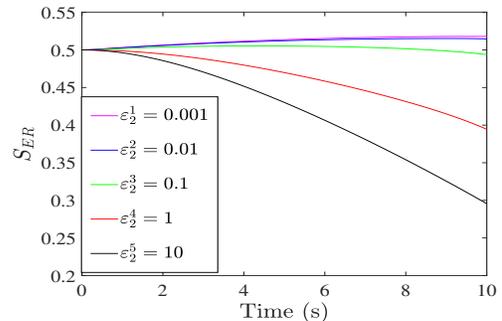


Fig. 10. Expectation of optimal  $S_{ER}$  for different control weight  $\varepsilon_2$ .

in solving the stochastic optimization problems. Although Newton method converges faster than our Algorithm 1, which is reasonable since Newton method has been proved to have

TABLE IV  
COMPARISON OF THE COMPUTATIONAL COSTS BETWEEN MC AND CS METHODS

Optimal Objective	Time Cost <sup>MC</sup> (s)	Time Cost <sup>CS</sup> (s)
0.0106( $\varepsilon_1 = 1, \varepsilon_2 = 0.01, \varepsilon_3 = 0.1$ )	4672.84	423.91
0.0776( $\varepsilon_1 = 1, \varepsilon_2 = 0.01, \varepsilon_3 = 1$ )	3833.15	444.81
1.7280( $\varepsilon_1 = 1, \varepsilon_2 = 1, \varepsilon_3 = 1$ )	2934.98	278.64

higher convergence rate than gradient descent method for optimization problems, we still use gradient descent method as example to do simulation since it is easier to be coded.

Fig. 6 shows the behaviours of the energy storage in ER under two ER configurations, i.e., with/without active energy storage management. It can be seen that with the proposed control scheme, the power deviation of the considered energy storage in ER is significantly smaller than that when there is no active energy management scheme applied.

Similar to Fig. 6, Fig. 7 demonstrates the efficacy of the proposed optimal controller for the energy storage in ER. It can be seen that the proposed method is of the capability to achieve a better energy storage stability, which would be essentially important in most of practical EI scenarios.

The optimal controllers  $u_G$  and  $u_{ER}$  are given in Fig. 8. As expected, due to the penalty term (12), the optimal average transmission rejection rate  $u_{ER}$  for ERs has been properly kept within the constrained range. The optimal power adjustment  $u_G$  controls the power output of DGs with time to meet the stability of the system.

Next, to illustrate the impact of the emphasis for controller  $u_G$  on the optimal control scheme, the effectiveness of energy management schemes corresponding to different values of weight coefficients  $\varepsilon_2$  for generators are evaluated. Here, we use the symbol  $\varepsilon_2^i, i = 1, 2, \dots$  to represent for different  $\varepsilon_2$ . Fig. 9 gives the optimal controller  $u_G$  for each  $\varepsilon_2$ , which suggests that a smaller value of  $\varepsilon_2$  would lead to a control signal  $u_G$  with larger variance. Therefore, in order to avoid over-control and ineffective control, we should properly choose the weight coefficients in the proposed optimal control problem (11).

Fig. 10 gives the expectation of energy storage level in the considered ER under a class of control schemes with different weight coefficient  $\varepsilon_2$ . It indicates that when the cost for the adjustment of generators is relatively small, i.e., with small value of  $\varepsilon_2$ , the energy storage level tend to be better stabilized.

In the final, we compare the time cost and accuracy for the MC method and our proposed CS-GDM algorithm by solving the same optimal control problem (15) with different sets of weight coefficients. In Table V, the error estimations are derived by comparing the results with 2000 points MC method. From Table IV and Table V it can be seen that when the accuracies are under the same level, CS-GDM method can improve the computational performance significantly. Apparently, the advantages of our proposed method in improving the efficiency of such numerical computation has been demonstrated.

TABLE V  
COMPARISON OF THE ERROR BETWEEN MC AND CS METHODS

Optimal Objective	Err <sup>MC</sup> $\times 10^{-5}$	Err <sup>CS</sup> $\times 10^{-5}$
0.0106( $\varepsilon_1 = 1, \varepsilon_2 = 0.01, \varepsilon_3 = 0.1$ )	2.34	4.87
0.0776( $\varepsilon_1 = 1, \varepsilon_2 = 0.01, \varepsilon_3 = 1$ )	1.56	2.35
1.7280( $\varepsilon_1 = 1, \varepsilon_2 = 1, \varepsilon_3 = 1$ )	2.14	3.91

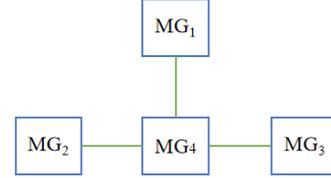


Fig. 11. The interconnected MGs topology for simulation

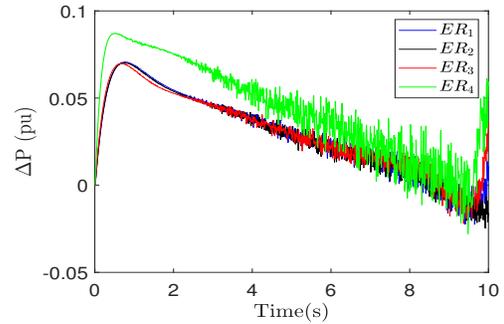


Fig. 12. Optimal  $\Delta P$  for four  $ER_s$ .

### C. Simulation results for interconnected MGs

In this subsection, the results for a more complex EI scenario are provided to demonstrate the efficacy of our proposed method. We consider four MGs in the network and they are functioning in the off-grid mode. The topology for this scenario is shown in Fig. 11. Without loss of generality, we assume that all MGs have the same components as is presented in the previous subsection, and the parameter settings are the same as before except for  $\theta_p, \mu_p$  in equation (5) which influence  $\Delta P_{out}$ , the total power required from the other MGs. To be specific, let us set  $\theta_p^1 = 0.1, \theta_p^2 = 0.2, \theta_p^3 = 0.5, \theta_p^4 = 0.8$  and  $\mu_p^1 = 0.1, \mu_p^2 = 0.2, \mu_p^3 = 0.5, \mu_p^4 = 0.8$  for each MG. To ensure the balance of power transmission between MGs, the following constraint (27) need to be added to the system. The weighting coefficients are set as  $\varepsilon_1 = 1, \varepsilon_2 = 0.01, \varepsilon_3 = 1$ . The corresponding numerical results are given as follows.

$$\sum_{i=1}^4 \Delta P_{out}^i = 0, \quad (27)$$

Fig. 12 shows the power dynamics of the energy storage of the four ERs with active energy storage management. It can be seen that with the proposed control scheme, the power deviation of the energy storage in  $ER_1, ER_2, ER_3$  are almost the same, while more power needs to be stored in  $ER_4$  than

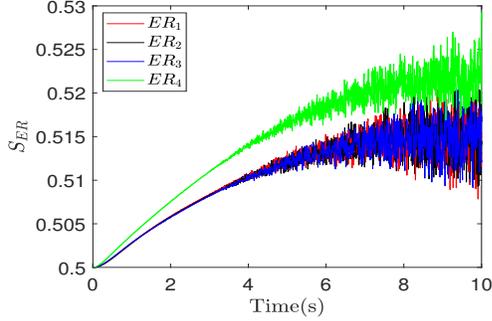


Fig. 13. Optimal  $S_{ER}$  for four  $ER_s$

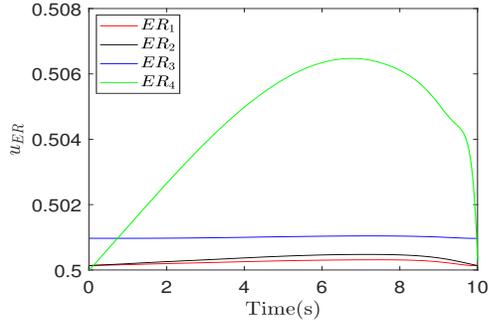


Fig. 14. Optimal controller  $u_{ER}$  for four  $ER_s$ .

TABLE VI  
ERROR ESTIMATES FOR EXPECTATION RECOVERY FOR  $S_{ER}$  USING MC  
AND CS METHODS

Number of Sample( $n$ )	Error <sup>MC</sup> $\times 10^{-2}$	Error <sup>CS</sup> $\times 10^{-2}$
30	10.45	23.59
50	8.18	10.42
80	6.46	4.21
100	5.29	1.12
500	2.31	0.054
1000	1.67	0.054
2000	1.39	0.054
3000	1.13	0.054

the others, since  $ER_4$  needs to transfer power with the other  $ER_s$ .

Similar to Fig. 12, Fig. 13 demonstrates the energy storage stability of the four ERs under control. It shows that the proposed method is of the capability to achieve a better energy storage stability. It also can be seen that the energy storage in  $ER_1, ER_2, ER_3$  are more stable than that in  $ER_4$ .

The optimal controllers  $u_{ER}$  are given in Fig. 14. Similar to results of the single MG, the optimal average transmission rejection rate  $u_{ER}$  for all the ERs have been properly kept within the constrained range. The adjustment of the transmission rejection rate of the  $ER_4$  plays a major role in controlling the stability of this EI system.

At the end of this part, the comparison results for MC method and our CS-GDM method for the interconnected MGs

TABLE VII  
TIME COSTS BETWEEN MC AND CS METHODS VS THE NUMBER OF MGs

Number of MGs	Time Cost <sup>MC</sup> (s)	Time Cost <sup>CS</sup> (s)
1	4672.84	423.91
2	5463.32	607.11
3	7566.14	772.84
4	11426.35	901.76
5	12648.73	1240.51
6	15971.54	1426.23

are illustrated. First, Table VI shows the numerical error estimates for expectation recovery of  $S_{ER}$  for the interconnected state equations system. It can be seen that when the same computation accuracy is achieved, only 100 sample points are used via CS method, while 3000 sample points shall be used via the MC method. Therefore, the CS method can save significantly computational cost when solving the state equations. However, the key step to solve the control problem with gradient descent method is to solve the state equations iteratively. Therefore, it is obvious that the CS-GDM method is much more efficient than the MC method to solve the same control problem. To be specific, as is shown in Table VII, for different number of MGs, the computational time cost is almost linear increases of both MC method and CS-GDM method, and MC method is nearly 10 times that of CS-GDM method. These comparison results fully show the advantages of CS-GDM method in computational efficiency.

To summarize, this section demonstrates the feasibility and effectiveness of our proposed compressive sensing based stochastic optimal control method. Analyses of simulation results shows the advantages of the CS-GDM approach we put forward in this work.

## VI. CONCLUSION

In this paper, an energy storage management problem for a typical ER is investigated. By modelling the MG system mathematically, this issue can be transformed into a nonlinear stochastic control problem. Then, by properly controlling desired power adjustment for DGs and the average transmission rejection for the studied ER, we can maintain the energy storage status at a stable level over time. The CS-GDM method is applied to solve such complex control problem. In the simulation part, the feasibility and validity of our proposed algorithm are verified. The comparison with the MC method in computational costs shows our proposed algorithm can save computation time significantly, which means that such method is more suitable for complex and high-dimensional problems.

There are two main research directions in our future work, one way is to apply our algorithm to energy management problems for ERs considering game-theoretic multi-objective control problems. Another way is to combine CS method with other existing methods such as ADP or reinforcement learning method to further improve the computation efficiency.

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