A Class of Optimal and Robust Controller Design for Energy Routers in Energy Internet

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***Abstract*—In this paper, a class of optimal and robust controller is designed for a typical energy router (ER) within the scenario of an energy Internet (EI) from the control perspective. As the core element of the EI, the considered ER is assumed to have access with photovoltaic (PV) units, wind turbine generators (WTGs), micro-turbines (MTs), fuel cells (FCs), battery energy storage (BES) devices, super capacities (SCs), loads and other ERs. Two control objectives are considered as follows. 1) Since the access of large-scale renewable energy sources (RESs) would lead to excessive DC bus voltage deviation within the ER system, a desired controller shall regulate the voltage deviation effectively. 2) According to the operational principle of EI, when multiple ERs are interconnected, an autonomous power balance of each ER is expected to be achieved with priority. Then the optimal energy dispatch problem shall be considered. When both two problems are considered simultaneously, a mixed robust H_2/H_∞ control problem is formulated and is solved analytically. Simulations show the usefulness and effectiveness of the proposed scheme.**

Keywords-Energy Internet; energy router; mixed H_2/H_∞ control; parameter uncertainty

I. INTRODUCTION

In the future, as a solution to the challenges facing the conventional power networks, microgrids (MGs) will be treated as significant infrastructures at the end of the prospective energy systems [1], [2]. According to the principle of smart grid, MGs are viewed as local small-scale autonomous power systems in which the balance of power generation and consumption is achieved [3]. When the energy management issues for large-scale energy systems are required to be considered, the concept of the energy Internet (EI) is proposed as a guideline for the future energy systems [4].

The EI can be considered as the upgraded version of smart grid [5]. In an EI scenario, energy flows from suppliers to customers in an open and peer-to-peer internet-style approach [6]. The core to realized such energy routing mechanism is the energy exchange device called the energy router (ER) [7], also known as the energy hub [8], [9]. The architecture of solid state transformer-based ER is introduced in [10]. A class of power circuit topology for ER is reported in [11]. A novel topology of ER with multiple HVAC ports for power distribution networks is proposed in [12]. Recently, research on energy routing strategies for ERs has attracted much attention and significant advances on this topic have been made; see, e.g., [13]-[16].

On the other hand, when energy management issues for ERs from the control perspective are taken into consideration, authors in [17] investigate the non-fragile H_∞ control problem for an islanded MG system, such that the DC bus voltage deviation is regulated. In [18], the coordinated frequency regulation problems for multiple AC MGs interconnected via ERs are solved via the stochastic H_∞ control approach. When the operation cost minimization problem is considered in EI, an optimal controller for the ER is developed in [19] where the optimization problem is formulated as an H_2 control problem.

It is notable that in the field of EI, there has been few work focusing on both problems of frequency/voltage regulation and optimal energy dispatch for ERs from the control perspective. In this paper, we consider the above two problems simultaneously for the ER which is designed based on the DC bus topology.

The considered ER is assumed to have access to photovoltaic (PV) units, wind turbine generators (WTGs), micro-turbines (MTs), super capacitors (SCs), and other ERs. Such assumption has been made in many works, e.g., [17]-[19]. Then, the dynamical system of ER is formulated as a class of ordinary differential equations (ODEs). In this paper, we take the ER system parameter uncertainty into consideration, due to the large-scale access of renewable energy sources (RESs). For the studied ER, the robust voltage regulation issue is formulated as a robust H_∞ control problem, whereas the optimal energy dispatch issue is formulated as a H_2 control problem. When both criterions are considered simultaneously, a mixed H_2/H_∞ control problem is formulated for the ER controller design and is solved analytically. We emphasis that this is the very first time that a mixed robust H_2/H_∞ controller is designed for ERs, considering norm bounded parameter uncertainties.

The rest of the paper is organized as follows: Section II introduces the ER system modelling. Problem formulation and solutions are given in Section III. In Section IV, numerical examples are illustrated. Finally, Section V concludes the paper.

II. ER SYSTEM MODELLING

In this section, a mathematical model is built for a typical ER system whose structure is shown in Fig. 1. A group of ODEs are utilized to describe the dynamics of the studied ER system.

In Fig. 1, the considered ER, denoted as ER_1, is connected with loads, PV units, WTGs, MTs, SCs, BES devices and one more ER (denoted as ER_2). The main power input to ER_1 is assumed to be provided by PV units and WTGs. Power of PV units, WTGs and loads are affected by solar irradiation, power of wind and various conditions of loads, respectively, which may cause excessive the DC bus voltage deviation. Both BES devices and SCs are used to consume superfluous electric power and to provide energy when power generation is below the requirement of power usage in ER_1. Within the considered time period (normally within 10 minutes), our targets are 1) to regulate the DC bus power deviation; 2) to make sure that power transmitted between ER_1 and ER_2 tend to be constant. The main reasons for our control targets shall be discussed in Section III. The controllable MTs are utilized to provide changeable energy. We are focusing on designing a controller, such that the above two targets are achieved.

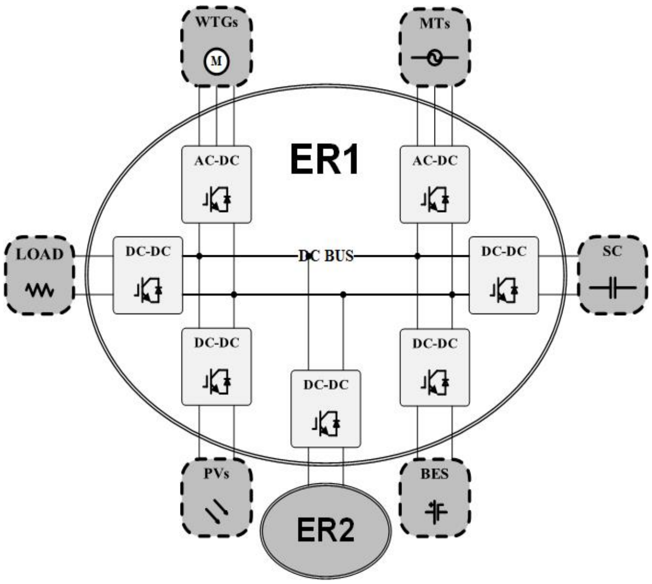

Figure 1. The scenario of a typical ER.

In this paper, we assume that the BES devices and SCs quickly respond to the DC bus voltage deviation. Note that such assumption has been used in many works; see, e.g., [17], [18]. The DC bus voltage deviation is denoted as ΔV. To further model each component of the studied ER system, we denote the power change of PVs, WTGs, loads, MTs, BES devices and SCs as ΔP_{PV}, ΔP_{WTG}, ΔP_L, ΔP_{MT}, ΔP_{BES} and ΔP_{SC}, respectively. The change of power transmited between ER_1 and ER_2 is denoted as ΔP_{AER}. Power balance relaionship can be expressed as: (time t omitted)

$$\Delta P_{AER} = -(\Delta P_{PV} + \Delta P_{WTG} + \Delta P_{MT} + \Delta P_{BES} + \Delta P_{SC} + \Delta P_L). \qquad (1)$$

Besides, we denote the time constants of PV units, WTGs, loads, MTs, BES devices and SCs, as T_{PV}, T_{WTG}, T_L, T_{MT}, T_{BES} and T_{SC}, respectively. Then, the power dynamics of the whole studied ER system can be obtained with a group of ODEs, as is shown in (2) (time t omitted). For similar power dynamics in forms of (2), readers can refer to [17]-[19], and the references therein.

$$\begin{cases} \Delta\dot{P}_{PV} = -\left(\frac{1}{T_{PV}} + \Delta o_{PV}\right)\Delta P_{PV} + \frac{1}{T_{PV}} v_{PV}, \\ \Delta\dot{P}_{WTG} = -\left(\frac{1}{T_{WTG}} + \Delta o_{WTG}\right)\Delta P_{WTG} + \frac{1}{T_{WTG}} v_{WTG}, \\ \Delta\dot{P}_{L} = -\left(\frac{1}{T_{L}} + \Delta o_{L}\right)\Delta P_{L} + \frac{1}{T_{L}} v_{L}, \\ \Delta\dot{P}_{MT} = -\frac{1}{T_{MT}}\Delta P_{MT} + \frac{1}{T_{MT}}(b_{MT} + \Delta b_{MT})u_{MT}, \\ \Delta\dot{P}_{BES} = -\frac{1}{T_{BES}}\Delta P_{BES} + \frac{1}{T_{BES}}(r_{BES} + \Delta r_{BES})\Delta V, \\ \Delta\dot{P}_{SC} = -\frac{1}{T_{SC}}\Delta P_{SC} + \frac{1}{T_{SC}}(r_{SC} + \Delta r_{SC})\Delta V, \\ \Delta\dot{V} = -\frac{1}{p}\Delta V + \frac{1}{q}\Delta P_{AER}. \end{cases} \qquad (2)$$

Power generation by PV units and WTGs heavily depends on time-varying environmental conditions such as solar irradiation and power of wind. Besides, the electric state of the demand side is changeable, resulting in the power consumption of load changing. In (2), v_{PV}, v_{WTG} and v_L stand for the change of solar irradiation, power of wind and the load power disturbances, respectively. We denote u_{MT} as the control input signal of MTs. Constants b_{MT}, r_{BES}, r_{SC}, p and q are system parameters which can be measured in engineering practices. Since the measurement error is unavoidable, for the power dynamic model of ER_1, parameter uncertainties are considered in this paper. We denote the uncertainties of factors b_{MT}, r_{BES} and r_{SC} as Δb_{MT}, Δr_{BES} and Δr_{SC}, respectively. In addition, we denote the uncertainties of the reciprocal of time constants T_{PV}, T_{WTG} and T_L as Δo_{PV}, o_{WTG} and Δo_L, respectively. Let

$$\Delta A = \begin{bmatrix} -\Delta O_{PV} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\Delta O_{WTG} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Delta O_L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\Delta r_{BES}}{T_{BES}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\Delta r_{SC}}{T_{SC}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and $\Delta B = [0 \quad 0 \quad 0 \quad \Delta b_{MT} \quad 0 \quad 0 \quad 0]$. The structure of the parameter uncertainties is given as follows.

$$[\Delta A(t)\ \Delta B(t)] = HF(t)[E_1\ E_2], \tag{3}$$

where H, E_1 and E_2 are known real constant matrices depending on real engineering situation, and $F(\cdot)$ is an unknown time-varying matrix satisfying

$$F(t)'F(t) \leq I. \tag{4}$$

The norm-bounded uncertainties in the form of (3) and (4) have been widely used; see, e.g., [17], [20].

Let $u = u_{MT}$, vector $v = [v_{PV}\quad v_{WTG}\quad v_L]'$ and vector $x = [\Delta P_{PV}\quad \Delta P_{WTG}\quad \Delta P_L\quad \Delta P_{MT}\quad \Delta P_{BES}\quad \Delta P_{SC}\quad \Delta V]'$. The ER system dynamical expressions in (2) can be rewritten into a mathematical state space control system (time t omitted):

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u + Cv. \tag{5}$$

In system (5), $u(\mathrm{t})$ is system control input, vector $x(t)$ is system state and $v(t)$ is system disturbance input. The coefficient matrices are as follows.

$$A = \begin{bmatrix} -\frac{1}{T_{PV}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{WTG}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_{MT}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T_{BES}} & 0 & \frac{r_{BES}}{T_{BES}} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{SC}} & \frac{r_{SC}}{T_{SC}} \\ -\frac{1}{q} & -\frac{1}{q} & -\frac{1}{q} & -\frac{1}{q} & -\frac{1}{q} & -\frac{1}{q} & -\frac{1}{p} \end{bmatrix},$$

$$B + \Delta B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_{MT} + \Delta b_{MT} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} \frac{1}{T_{PV}} & 0 & 0 \\ 0 & \frac{1}{T_{WTG}} & 0 \\ 0 & 0 & \frac{1}{T_L} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

So far, we have transformed the physical ER dynamical system into a mathematical model.

III. Robust Mixed H_2/H_∞ Problem Formulation and Solution

In this section, the problems of stabilizing the DC bus voltage and finding the optimal energy dispatching strategy of ER are formulated as the robust mixed H_2/H_∞ guaranteed cost control problem. Then, we solve such control problem analytically.

The robustness of ER_1 is on the premise of DC bus voltage stability. It has been shown that H_∞ control theory is closely associated with many robustness problems in the field of power systems such as voltage regulation [17] and frequency regulation [18]. In this paper, designing a controller in MTs to regulate the DC bus voltage deviation is considered as a robust H_∞ control problem. First, the definition of robust stable against system parameter uncertainties Δo_{PV}, Δo_{WTG}, Δo_L, Δb_{MT}, Δr_{BES} and Δr_{SC} is given as follows.

Definition 1: (see, e.g., [21]) *The system (5) with $u = 0$ and $v = 0$ is said to be mean-square stable if for any $\varepsilon > 0$, we have*

$$\mathbb{E}|x(t)|^2 < \varepsilon, \qquad t > 0,$$

If, in addition

$$\lim_{t\to\infty} \mathbb{E}|x(t)|^2 = 0,$$

for any initial conditions, then system (5) with $u = 0$ and $v = 0$ is said to be mean-square asymptotically stable. The uncertain system in (5) is said to be robustly stable if the system associated to (5) with $u = 0$ and $v = 0$ is mean-square asymptotically stable for all the system parameter uncertainties.

Let us denote the DC bus voltage deviation ΔV as the controlled output z_1. From (2), we have $z_1 = D_1 x$, where

$$D_1 = [0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 1].$$

Based on Definition 1, the definition of robust H_∞ performance for our considered voltage control problem is presented as follows.

Definition 2: (see, e.g., [21]) *Given a scalar $\gamma > 0$, the H_∞ performance of the studied ER system is defined as $||z_1(t)|| < \gamma||v(t)||$. Here, norm $\|\cdot\|$ is defined as*

$$||z_1(t)|| \triangleq \left(\mathbb{E}\left\{\int_0^\infty |z_1(t)|^2 \mathrm{d}t\right\}\right)^{1/2},$$

where scalar γ is disturbance attenuation, $\mathbb{E}$ is the mathematical expectation. *Based on the H_∞ performance introduced above, the H_∞ cost functional is formulated as follows,*

$$J_\infty(u,v) \triangleq \mathbb{E}\left[\int_0^T (z_1{}'z_1 - \gamma^2 v'v)\mathrm{d}t\right]. \qquad (6)$$

For a typical EI scenario, multiple ERs are interconnected such that the electric power dispatching network is formed. According to the operational principle of EI, when multiple ERs are interconnected, an autonomous power balance of each ER is expected to be achieved with priority [5], [16]. If the local ER's power balance cannot be realized autonomously, then power can be transmitted from/into the local ER, such that the whole EI scenario achieves power balance. In this sense, if the power change of the local ER is always varying, which means that the local power balance is not achieved, then power shall be transmited between the interconnected ER. It is notable that energy transmission cost between two interconnected ERs may be conspicuous, if these two ERs are far away from each other [19]. In addition, frequent power transmission between ERs is also costly [19]. So, our target is to make sure that power transmitted between ER_1 and ER_2 tend to be constant, which is equivalent to minimizing the value of ΔP_{AER}. Such optimal energy dispatch objective is formulated as the H_2 performance. Let us denote the other controlled output as $z_0 = \Delta P_{AER}$. Equivalently, $z_0 = D_0 x$, where

$$D_0 = [-1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad 0],$$

Then the H_2 cost functional is defined as follows.

Definition 3: *The H_2 cost functional is formulated as an upper bound of the worst case H_2 performance of the studied system, defined by*:

$$J_2(u,v) = \sup_{F(t)} \lim_{t\to\infty} E\{z_0{}^T(t) z_0(t)\}. \qquad (7)$$

After formulating the problems of both robust H_∞ control and H_2 control respectively, the control system of the studied ER can be rewritten as

$$\begin{cases} \dot{x} = (A+\Delta A)x + (B+\Delta B)u + Cv, \\ z_0 = D_0 x, \\ z_1 = D_1 x. \end{cases} \qquad (8)$$

When both H_∞ performance and H_2 performance are considered simultaneously, the definition of the mixed H_2/H_∞ guaranteed cost control problem is provided as follows.

Definition 4: *The mixed H_2/H_∞ guaranteed cost control problem is to find a controller $u(t) = Kx(t)$, such that the following three statements hold.*

(a) For all admissible parameter uncertainties, the controlled system is asymptotically stable;

(b) For H_∞ performance, $||z_1(t)|| < \gamma||v(t)||$ is achieved;

(c) For H_2 performance, $J_2(u,v)$ defined by (6) is minimized.

We have transformed the optimal and robust controller design problem for the considered ER system into a mixed H_2/H_∞ guaranteed cost control problem. The main results of this paper are given as two theorems below.

Theorem 1: (See, e.g., [20]) *For a given constant $\gamma > 0$ and the system (8), there exists a controller $u(t) = Kx(t)$ such that the controlled system is asymptotically stable and $||z_1(t)|| < \gamma||v(t)||$ are satisfied, if and only if there exists two scalars $\alpha > 0$, $\beta > 0$, a symmetric positive definite matrix X and a matrix V, such that the following linear matrix inequality (LMI) holds,*

$$\Gamma = \begin{bmatrix} W & W_1{}' & (D_1X)' & (D_0X)' \\ W_1 & -\alpha I & 0 & 0 \\ D_1X & 0 & -\beta I & 0 \\ D_0X & 0 & 0 & -I \end{bmatrix} < 0. \qquad (9)$$

where

$$W = (AX+BV)' + AX + BV + \alpha HH' + \beta\gamma^{-2}CC',$$

$$W_1 = E_1X + E_2V.$$

If (9) has a feasible solution (α,β,X,V), then the state feedback controller can be chosen by

$$u(t) = VX^{-1}x(t). \qquad (10)$$

And the H_2/H_∞ guaranteed cost bound of the closed loop system $J_2(u,v)$ can be calculated by $J_2(u,v) = tr(C'X^{-1}C)$. Here, $tr(\cdot)$ denotes the trace of the matrix $(\cdot)$.

Theorem 2: (See, e.g., [21]) *For a robustly stabilizable system, if there exist two scalars $\alpha > 0$, $\beta > 0$, two symmetric positive definite matrices X, N, and a matrix V such that the following optimization problem*

$$\min_{\alpha,\beta,X,V,N} z = tr(N),$$

$$s.t. \begin{cases} \Gamma < 0, \\ \begin{bmatrix} -N & C' \\ C & -X \end{bmatrix} < 0. \end{cases} \qquad (13)$$

has a solution (α,β,X,V,N), then $u(t) = VX^{-1}x(t)$ is the H_2/H_∞ guaranteed cost controller.

The proofs of Theorem 1 and Theorem 2 are similar to the ones in [20]. Here we omit the details.

IV. Simulation Results and Analysis

In this section, several numerical results are presented to demonstrate the feasibility of the optimal and robust controller designed for the considered ER.

The parameters of the ER system are shown in Table I. We assume that $\gamma = 10$, $E_1 = [0.58$ -0.74 0.73 0.70 0.01 -1.25 0.06], E_2 =0.62 and $H = [6.45$ -14.65 3.85 11.18 -0.05 -2.55 -11.69$]'$ in the simulation. *MATLAB LMI Control Toolbox* [22] is utilized to solve Theorem 1 and Theorem 2.

TABLE I. SYSTEM PAEAMETERS

Parameter	Value	Parameter	Value
T_{PV}	1.2	b_{MT}	1.2
T_{WTG}	1.7	r_{BES}	1.1
T_L	0.8	r_{SC}	1.2
T_{MT}	0.05	p	0.01
T_{BES}	0.5	q	0.03
T_{SC}	0.3		

We choose a one-dimensional stochastic funtion $F(t)$, the waveform of which is shown in Fig. 2. The power deviations of PV units, WTGs and loads are shown in Fig. 3.

In conventional robust control methods, only Theorm 1 is used to obtian the H_∞ controller. When the conventional controller is applied to system (7), the power fluctuations of MTs, BES devices and SCs are shown in Fig. 4.

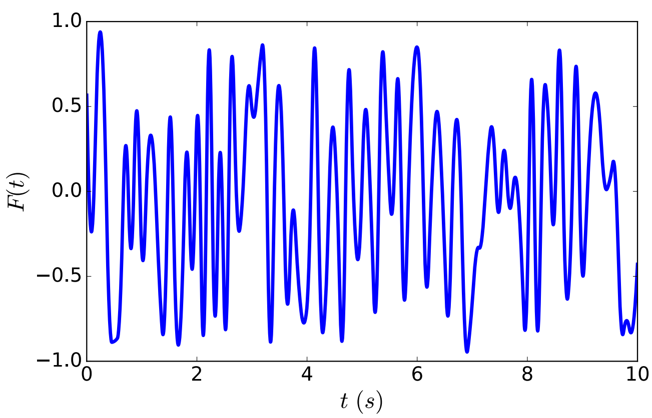

Figure 2. Waveform of $F(t)$.

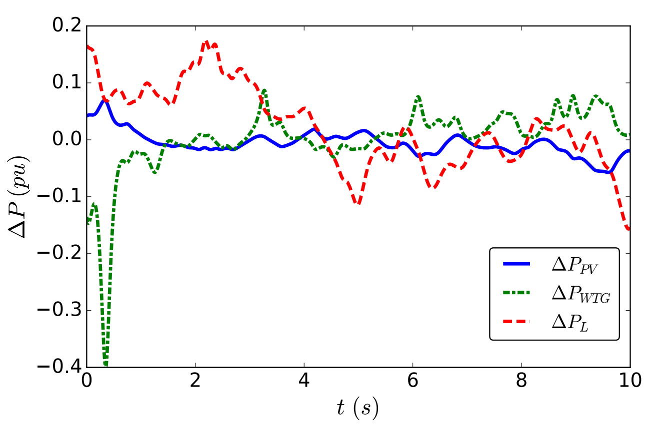

Figure 3. Power deviations of PVs, WTGs and Loads.

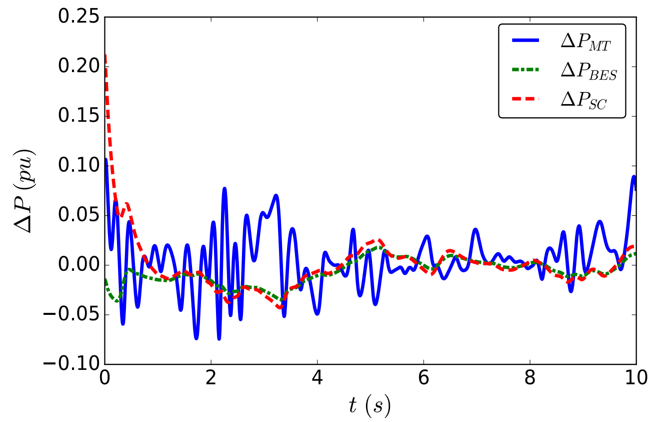

Figure 4. Power fluctuations of MTs, BES devices and SCs under conventional H_∞ controller.

The power fluctuations of MTs, BES devices and SCs under the proposed H_2/H_∞ guaranteed cost controller is shown in Fig. 5. It is obvious that the extent of MT power fluctuation under the proposed H_2/H_∞ controller is smaller than that under the conventional H_∞ controller. In this sense, the situation of over-control can be effectively avoided by our proposed method.

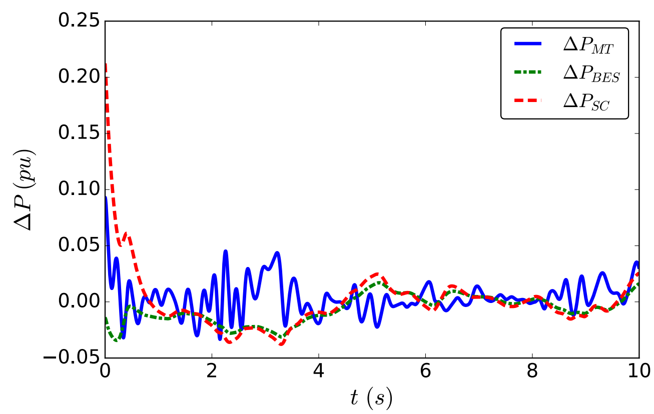

Figure 5. Power fluctuations of MTs, BES devices and SCs under the proposed H_2/H_∞ controller.

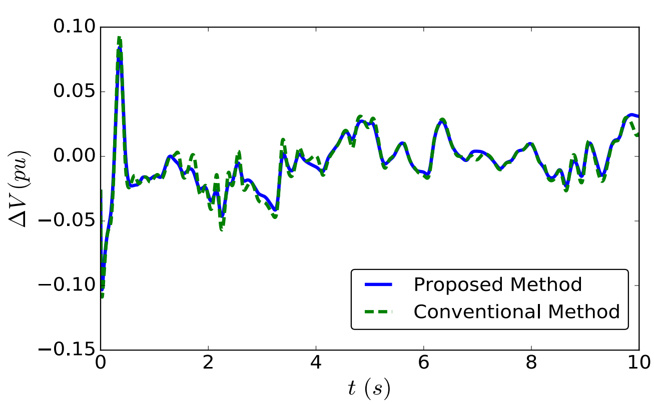

Figure 6. Power deviations under the proposed method and the conventional method.

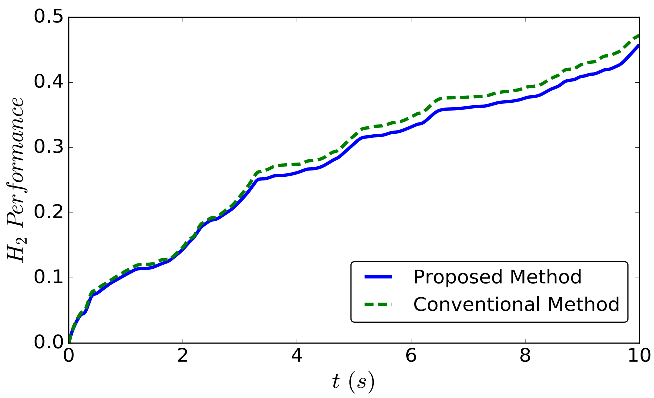

Figure 7. H_2 performances under the proposed method and the conventional method.

In Fig. 6, the power deviation of the DC bus under the mixed H_2/H_∞ controller and H_∞ controller are illustrated. The H_∞ control effects under the two methods are quiet close according to the simulation results. However, when considering the H_2 performance, our proposed method shows its superiority over the conventional one. The H_2 performance under the mixed H_2/H_∞ controller is better than that with the conventional method, as is shown in Fig. 7.

V. CONCLUSION

This paper investigates a typical ER within the scenario of an EI. The energy management problem is studied for the ER from the control perspective. ODEs are applied to describe the dynamic system of the considered ER. A mixed robust H_2/H_∞ control problem is formulated and solved. Simulations demonstrates the usefulness and effectiveness of the proposed scheme. For the future research, in order to represent the system stochasticity involved by power of PV units, WTGs, and loads, stochastic differential equations shall be implemented to formulate the ER dynamical system, and a stochastic mixed robust H_2/H_∞ control problem shall be investigated.

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