Stochastic Optimal and Robust Control Scheme for Islanded AC Microgrid

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***Abstract*—This paper presents a stochastic optimal and robust control scheme forislanded AC microgrid(MG). Such AC MG is composed of multiple wind turbine generators (WTGs), photovoltaic (PV) units, fuel cells (FCs), micro turbines(MTs), diesel engine generators (DEGs), battery energy storage (BES) devices, flywheel energy storage (FES) devices and loads. Our purpose is to design a controller such that two targets are achieved: 1)the rational utilization of MTs, FCs, DEGs, BESdevices and FES devices;2) the regulationof the AC bus frequency deviation. Firstly, we consider the modeling of the islanded MG system. Considering the stochastic power change within renewable power generation devices, energy storage (ES) devices and loads, we formulate the system via a group of linear stochastic differential equations (SDEs) and ordinary differential equations (ODEs). Secondly, we formulatethe problem of rationally utilizing MTs, FCs, DEGs and ES devices as the stochastic optimal control (H_2 control) problem, and we formulate the issue of frequency regulationas the stochasticH_∞ control problem. Thirdly, we present a mixed stochasticH_2/H_∞ controller via the Nash game approachsuch that bothour aims are achieved. Finally, numerical simulations are used to illustrate the effectiveness of the proposed design technique.**

Keywords—frequency regulation,islanded microgrid,mixed H_2/H_∞ control,Nash equilibrium,stochastic optimization

I. INTRODUCTION

At present, the challengesfacing the world include: global warming, energy crisis, environmental pollution, etc. To overcome these problems, distributed energy resources (DERs), such as wind power generation, solar power generation and hydropower generation, has drawn people's attention, and significant progress has been made over the last several decades[1].For the integration of smalland isolated DER units intothe power grid,microgrids (MGs)play a significant role [2]. Normally, a typical MG is composed ofmultiple distributed power generation devices, energy storage (ES) devices and loads.Some existing demonstration projects of MGs are introduced in [3], [4], and the references therein.

If a DC (or AC) MG works in thegrid-connectedmode, the main power grid shall be able to regulate its voltage (or frequency) deviation well. It is notable that control issues regardingoff-grid (also known as islanded)MG are more complicated than the ones regarding grid-connected MGs, since thestochastic fluctuation in renewable power generation and loads within anoff-grid MG must be compensated by some components ofthe MG [5]. Recently, systemmodeling and control problems regarding islanded MG have been popular. We list some of the interesting problems under investigation as follows.

Firstly, the renewable power generated by wind turbine generators (WTGs) and photovoltaic (PV) unitshas defects such as intermit, nonlinear, stochastic, unpredictable, etc. This will cause unbalance between power generation and usagewithout proper regulation, leading to an abnormal frequency and voltage deviation. Such results might give rise to thecollapse ofMG system [6]. In general, theunbalanced power deviation is absorbed by the ESdevices [7].Nowadays the battery energy storage (BES)and flywheel energy storage (FES) devices are used as two common ES devices. It is notable that the cost ofESdevices isexpensive, and such cost has been regarded as the one of the main costs for MG operation[8]. The lifetime of a ES device depends on how rationally it is used. For example, if consistent large-scale power charge/discharge is applied to a battery, its service time will be reduced notably. How to extend the service life of the ES devices has been popular recently [7], [8].

Secondly, the changeable wind power and solar irradiation would disturb the power output from WTGs and PVunits randomly. When a MG is working in the islanded mode,such varying power output would influence itsDC bus voltage (or AC bus frequency) significantly. To regulate the bus voltage (frequency), robust control methodsare normally implemented, such that system robustness and stability are achieved; see, e.g., [9], [10]. The robust H_∞ control strategiesfor MG frequency regulationhas beenstudied in [11], [12], etc.Specifically, when the problems of extending the service life of ES devices and regulating AC bus frequency are considered simultaneously within an off-grid MG, such mixed control targetsare formulated as adeterministic mixed H_2/H_∞ control problem solved in [13].The applications of H_2/H_∞ control techniquesto MG systems have attracted much attention; see, e.g., [14], [15].

Thirdly, regarding the control issues with respect to (w.r.t.) islanded MG, the modeling of MG is very important. When DERs, distributed power generation devices, ES devices and loads are considered in MG control problems, linearized ordinary differential equations (ODEs) are often used to model the system dynamics, see, e.g., [12]–[18].However, systems

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formed by ODEs haveshortcomings, since the existing stochasticity within the power change of WTGs, PV units and loads cannot be described by ODEs precisely. To present the system randomness, the stochastic differential equation (SDE) is treated as an effective tool. In recent work [3] and [19], SDEs are used to model power system dynamics.

In this paper, we consider an off-grid AC MG and we assume it includes multiple WTGs, PV units, MTs, FCs, DEGs, BES devices, FES devices and loads. Our purpose is to design a series of controllers, such that two aims are achieved.First, the AC bus frequency is regulated; Second, the ES devices and MTs, FCs and DEGs are utilized rationally, in the sense that the service life of the BES and FES devices can be extended.

To achieve our target, we transform the dynamics of theoff-grid AC MG into a stochastic control system where SDEs are applied to describe the system randomness. It is notable that such stochastic MG system modeling is different from the existing literatures, e.g., [12]–[19]. The controllers are designed in MTs, FCs and DEGs only.Mathematically,the frequency regulation issue is formulated into a stochastic H_∞ control problem,whereasthe problem ofrational utilizationw.r.t. MTs, FCs, DEGs, BES devices and FES devices is formulatedas a stochastic optimization(stochastic H_2 control) problem. Since an over-strong controller might involve extra operation cost, restrictions w.r.t the sizeof thedesired controller is considered in the H_2control problem formulation. Since both H_∞ and H_2 performances are expected to be achieved, we consider suchsynthetical problem as the *stochastic*mixedH_2/H_∞ control problem. Then, itis solved viathe Nash game approach with an analytical solution obtained.By taking account of multiple components in MG, we emphasize that we are working on a *general* case and the novel controllers obtainedcan be viewed as *general* solutions to the islanded MG control problems.Finally, case studies are illustrated, indicating the usefulness of the proposedcontrol strategy.

The rest of the paper is organized as follows. In Section Ⅱ we describe the system modeling. Section Ⅲ introduces thestochasticH_2/H_∞ control problem formulation and provides an analytical solution. Section Ⅳ provides some numerical examples. Finally, we conclude this paper in Section Ⅴ.

II. System Modeling

In this paper, we consider an off-grid AC MG where the total numbers of loads, MTs, FCs, DEGs, WTGs, PV units, BES devices and FES devices are assumed to be n_1 , n_2,n_3,n_4,n_5,n_6.$n_7$and n_8, respectively. For the gth load, hth MT, ith FC, jth DEG, kth WTG, mth FES device and nth BES device, their power (output or input) changes are denoted as ΔP_{L_g} , ΔP_{MT_h} , ΔP_{FC_i} , ΔP_{DEG_j} , ΔP_{WTG_k} , ΔP_{BES_m} and ΔP_{FES_n} , respectively. The AC bus frequency deviation is denoted asΔf. When the dynamics of an islanded MG system are linearized into a group of differential equations, the time constants are always used to construct the mathematical systems; see, e.g., [12], [13]. Here, we denote T_{L_g},T_{MT_h},T_{FC_i},T_{DEG_j},T_{WTG_k},T_{PV_i},T_{BES_m} and T_{FES_n} as the time constants for the gth load device, hth MT, ith FC, jth DEG, kth WTG, mth BES device and nth FES device, respectively. Based on the stochastic nature of the dynamic power deviation within each WTG, PV unit and load, we consider a class of linear SDEs and ODEs to formulate the state space system of the islanded AC MG as follows (time t is omitted).

$$\begin{cases} \mathrm{d}\Delta P_{L_g} = \frac{1}{T_{L_g}}\left(-\Delta P_{L_g} + v_{L_g}\right)\mathrm{d}t \\ \qquad + \frac{r_{Lg}}{T_{Lg}}\Delta P_{L_g}\mathrm{d}W(t), \\ \Delta \dot{P}_{MT_h} = \frac{1}{T_{MT_h}}\left(-\Delta P_{MT_h} + b_{MT_h}u_{MT_h}\right), \\ \Delta \dot{P}_{FC_i} = \frac{1}{T_{FC_i}}\left(-\Delta P_{FC_i} + b_{FC_i}u_{FC_i}\right), \\ \Delta \dot{P}_{DEG_j} = \frac{1}{T_{DEG_j}}\left(-\Delta P_{DEG_j} + b_{DEG_j}u_{DEG_j}\right), \\ \mathrm{d}\Delta P_{WTG_k} = \frac{1}{T_{WTG_k}}\left(-\Delta P_{WTG_k} + v_{WTG_k}\right)\mathrm{d}t \\ \qquad + \frac{r_{WTG_k}}{T_{WTG_k}}\Delta P_{WTG_k}\mathrm{d}W(t), \\ \mathrm{d}\Delta P_{PV_l} = \frac{1}{T_{PV_i}}\left(-\Delta P_{PV_l} + v_{PV_l}\right)\mathrm{d}t \\ \qquad + \frac{r_{PV_l}}{T_{PV_i}}\Delta P_{PV_l}\mathrm{d}W(t), \\ \Delta \dot{P}_{BES_m} = \frac{1}{T_{BES_m}}\left(-\Delta P_{BES_m} + \Delta f\right), \\ \Delta \dot{P}_{FES_n} = \frac{1}{T_{FES_n}}\left(-\Delta P_{FES_n} + \Delta f\right), \\ \Delta \dot{f} = -\frac{2\overline{D}}{\overline{M}}\Delta f + \frac{2}{\overline{M}}\Delta P, \end{cases} \tag{1}$$

where random variable $W(t)$ stands for a scalar Brownian motion, representing for the stochasticity involved by loads, WTGs and PV units. The definition of Brownian motion is introduced below.

Definition 1: [20] *Let* $\{\Omega, \mathcal{F}, \mathbb{P}\}$ *be a probability space with a filtration* $\{\mathcal{F}_t\}_{t\geq 0}$. *A (standard) one-dimensional Brownian motion is a real-valued* $\{\mathcal{F}_t\}$*-adapted process* $W(t), t > 0$*with the following three properties:*

1) $W(t) = 0$*(with probability 1);*

2) For $0 \leq s < t \leq T$, *the random variable given by the incement* $W(t) - W(s)$*is normally distributed with mean zero and variance* $t - s$*;*

3) For $0 \leq s < t < \sigma < \tau \leq T$, *the increments* $W(t) - W(s)$*and* $W(\tau) - W(\sigma)$*are independent.*

In (1) wedenote v_{L_g} , v_{WTG_k} and v_{PV_l} as load power disturbance, wind power and solar irradiation, respectively. For the modeling of AC bus frequency, constants$\overline{D}$ and $\overline{M}$ stand for the damping coefficient and the inertia constant, respectively. The controllers are set in distributed MTs, FCs and DEGs within the entire islanded MG. For the hth MT, ith FC and j th DEG, the control inputs are denoted as u_{MT_h}, u_{FC_i} and u_{DEG_j}, respectively. Constants r_{Lg}, r_{WTG_k} and r_{PV_l} are system parameters involved by the diffusion terms, and their values depend on real engineering scenarios. In (1) the power deviation between the total power input and output is denoted as ΔP and we have

$$\Delta P \triangleq -\sum_g \Delta P_{L_g} + \sum_h \Delta P_{MT_h} + \sum_i \Delta P_{FC_i} + \sum_j \Delta P_{DEG_j} + \sum_k \Delta P_{WTG_k} + \sum_l \Delta P_{PV_l} + \sum_m \Delta P_{BES_m} + \sum_n \Delta P_{FES_n}.$$

Since system (1) only focuses on the dynamics of each MG componentand we shall consider the dynamics of the whole islanded MG, an integrated systemdescribing the whole MG dynamics is required.

Let us denote vectors $x_L = [\Delta P_{L_1}, \cdots, \Delta P_{L_{n_1}}]'$, $x_{MT} = [\Delta P_{MT_1}, \cdots, \Delta P_{MT_{n_2}}]'$, $x_{FC} = [\Delta P_{FC_1}, \cdots, \Delta P_{FC_{n_3}}]'$, $x_{DEG} = [\Delta P_{DEG_1}, \cdots, \Delta P_{DEG_{n_4}}]'$, $x_{WTG} = [\Delta P_{WTG_1}, \cdots, \Delta P_{WTG_{n_5}}]'$, $x_{PV} = [\Delta P_{PV_1}, \cdots, \Delta P_{PV_{n_6}}]'$, $x_{BES} = [\Delta P_{BES_1}, \cdots, \Delta P_{BES_{n_7}}]'$ and $x_{FES} = [\Delta P_{FES_1}, \cdots, \Delta P_{FES_{n_8}}]'$ as the power change of all loads, MTs, FCs, DEGs, WTGs, PV units, BES devices and FES devices, respectively. Similarly, we denote vectors$u_{MT} = [u_{MT_1}, \cdots, u_{MT_{n_2}}]'$, $u_{FC} = [u_{FC_1}, \cdots, u_{FC_{n_3}}]'$ and $u_{DEG} = [u_{DEG_1}, \cdots, u_{DEG_{n_4}}]'$ as controllers designed in all MTs, FCs and DEGs, respectively. Vectors $v_L = [v_{L_1}, \cdots, v_{L_{n_1}}]'$, $v_{WTG} = [v_{WTG_1}, \cdots, v_{WTG_{n_5}}]'$ and $v_{PV} = [v_{PV_1}, \cdots, v_{PV_{n_6}}]'$ stand for system disturbance inputs.

Next, based on system (1) and the notations introduced above, we formulate the dynamic system for the off-grid MG as the following state space control system, (time t is omitted)

$$\begin{cases} \mathrm{d}x = (Ax + Bu + Cv)\mathrm{d}t + Dx\mathrm{d}W(t), \\ z = Fx, \end{cases} \quad (2)$$

where vector$x = [x_L \quad x_{MT} \quad x_{FC} \quad x_{DEG} \quad x_{WT} \quad x_{PV} \quad x_{BES} \quad x_{FES} \quad \Delta f]'$is state, vector$u = [u_{MT} \quad u_{FC} \quad u_{DEG}]'$is control input, vector $v = [v_L \quad v_{WTG} \quad v_{PV}]'$is disturbance input and $z = \Delta f$is controlled output. For simplicity, we define $N_r \triangleq \sum_{i=1}^{r} n_i$, $r = 1,2,\ldots,8$,and define the total dimension of vector $x$as $N$where$N \triangleq N_8 + 1$.In (2),$F_r$is denoted as the rth element of vector $F \in \mathbb{R}^N$ and is defined as follows,

$$F_r \triangleq \begin{cases} 0, & for\ 1 \le r < N, \\ 1, & for\ r = N. \end{cases}$$

Now we have transformed the whole dynamics of the considered AC MG into a mathematical control system.

III. Stochastic Mixed H_2/H_∞ Control Problem Formulation and Solution

Our aim is to design a class of controllers for the islanded AC MG, such that all the MTs, FCs and DEGs are utilized rationally and the service life of BES and FES devices is extended. Meanwhile, the AC bus frequency shall be regulated.

First, similar to the modelingtechniquesintroduced in [14],we formulate the frequency regulation issue into the stochastic H_∞ control problem.

Definition 2: [12], [21]*Given a scalar $\gamma > 0$, we define the H_∞ performance of Δf as $||z(t)|| < \gamma||v(t)||$, where we define*

$$||z(t)|| \triangleq \left(\mathbb{E}\left\{\int_0^\infty |z(t)|^2 dt\right\}\right)^{1/2}.$$

Here,$\mathbb{E}$ stands for the mathematical expectation. Scalar $\gamma$is called disturbance attenuation.Thenwe define the stochastic H_∞ cost function in (3),

$$J_1(u,v) \triangleq \mathbb{E}\left[\int_0^T (\gamma^2 v'v - z'z)dt\right]. \quad (3)$$

Second, we formulate the cost function of the stochastic optimization (H_2 control)problem for the MG operation.The replacement cost of ES devices is relatively expensive and is regarded as the main MG operation cost [8]. To extend the lifetime of BES and FES devices,rapid power overcharge and over-dischargeshall be avoided, and the power deviation of the ES devices shall be controlled within a relatively small amount [8]. This goal can be achieved by settingdesired controllers in FCs, DEGs and WTGs.It is notable that over-strong controllers might bring extra cost to the system operation significantly. Thus, we also consider the additional cost caused by the controllers when formulating the cost functional of the stochastic H_2control problem.

Definition 3:*We define the criterion of the stochastic H_2 control problem for the islanded AC MG operation as follows.*

$$J_2(u,v) \triangleq \mathbb{E}\left[\int_0^T \left(\varepsilon_1 \sum_{m=1}^{n_7} \Delta P_{BES_m}^2 + \varepsilon_2 \sum_{n=1}^{n_8} \Delta P_{FES_n}^2 + \alpha u'u\right) dt\right], \quad (4)$$

where scalars ε_1,ε_2 and α are weighting coefficients and are determined according to real engineering scenarios. In this sense, as long as $J_2(u,v)$ is minimized, the rational utilization of ES devices, MTs, FCs and DEGs is achieved, and the MG operation cost can be reduced and kept within a relatively low level.

For notation simplicity, $J_2(u,v)$ in (4) is rewritten into

$$J_2(u,v) = \mathbb{E}\left[\int_0^T (x'Mx + \alpha u'u)\mathrm{d}t\right], \tag{5}$$

where diagonal matrix $M \triangleq diag(\lambda)$, $\lambda \in \mathbb{R}^{\mathrm{N}}$ and the r th component of λsatisfies

$$\lambda_r = \begin{cases} 0, & for\ 1 \le r \le N_6, \ \text{ or } \ r = N, \\ \varepsilon_1, & for\ N_6 < r \le N_7, \\ \varepsilon_2, & for\ N_7 < r \le N_8. \end{cases}$$

In this sense, such stochastic optimization problem has been transformed into the form of a stochastic liner quadratic (LQ) control problem.

Next, we consider bothstochasticH_∞andstochasticH_2performances simultaneously. Applying the game theoretic approach, we transform the above two control problems into a mixed *stochastic*H_2/H_∞ control problem which is defined as follows.

Definition 4: [22] *For given disturbance attenuation level* $\gamma > 0$*and terminal time*$0 < T < \infty$*, the finite horizon mixed stochastic*H_2/H_∞ *control problem is to find a state feedback controller* u^* *such that* $\mathbb{E}\left[\int_0^T z'zdt\right] < \mathbb{E}\left[\gamma^2 \int_0^T v'vdt\right]$*, and when the worst case disturbance* v^**, if existing, is implemented to system (5),* u^* *minimizes* $\mathbb{E}\left[\int_0^T (x'Mx + \alpha u'u)dt\right]$ *.Our mixed stochastic* H_2/H_∞ *control problem is equivalent to finding the Nash equilibrium* (u^*, v^*) *such that*$J_2(u^*, v^*) \le J_2(u^*, v)$ *and* $J_1(u^*, v^*) \le J_1(u, v^*)$*are satisfied.*

Now we have transformed the physical control problem for islanded MG with multiple MTs, FCs, DEGs, WTGs, PV units, load devices, BES and FES devices into a mathematical control problem. Supposing (u^*, v^*) is the Nash equilibrium of the mixed stochastic H_2/H_∞ control problem, we have the following theorem.

Theorem 1: [22] *For system (2), supposing that the differential Riccatiequations in (6) have one solution* (P_1, P_2, K_1, K_2) *meeting the terminal condition* $P_1(T) = 0, P_2(T) = 0, P_1(0) \ge 0, P_2(0) \ge 0$*, then the solution to our mixed stochastic* H_2/H_∞ *control problem is* $u^* = K_2 x$ *and* $v^* = K_1 x$*.*

$$\begin{cases} \dot{P}_1 - F'F + D'P_1D + \gamma^2 K_1' K_1 + 2P_1 A \\ \qquad +2P_1 B K_2 + 2P_1 C K_1 = 0, \\ \dot{P}_2 + M + D'P_2 D + \alpha K_2' K_2 + 2P_2 A \\ \qquad +2P_2 B K_2 + 2P_2 C K_1 = 0, \\ K_1 = -\gamma^{-2} C' P_1', \\ K_2 = -\alpha^{-1} B' P_2'. \end{cases} \tag{6}$$

IV. Numerical Simulation

In this section, we give some numerical examples based on typical MG system coefficients in real-world scenarios. First, we show the advantages of system modeling using SDEs against using ODEs. Then the effectiveness of the H_2/H_∞ controller obtained in Theorem 1 isverified.*Python*is used to work out all the simulation results below.

For illustrative purposes, we consider anislanded ACMG system composed of two load devices, three MTs, two FCs, two DEGs, two WTGs, two PV units, two BES devices and two FES devices. In this sense,we have $n_1 = 2$,$n_2 = 3$,$n_3 = 2$,$n_4 = 2$, $n_5 = 2$, $n_6 = 2$, $n_7 = 2$ and$n_8 = 2$.We assume the initial values of the power change of loads, WTGs and PV units to be 0.001 pu, 0.002 pu and -0.002 pu, respectively. The initial frequency deviation is assumed to be -0.003 Hz and the initial values of the other elements in vector$x$ are assumed to be zero. The remaining MG system parameters are shown in Table I and Table II.

TABLE I. Time Constants of MG Model

Parameter	Value	Parameter	Value
$T_{L_1}(s)$	1.3	$T_{L_2}(s)$	1.5
$T_{MT_1}(s)$	1.1	$T_{MT_2}(s)$	1.4
$T_{MT_3}(s)$	1.2	$T_{FC_1}(s)$	1.1
$T_{FC_2}(s)$	1.7	$T_{DEG_1}(s)$	1.0
$T_{DEG_2}(s)$	1.2	$T_{WTG_1}(s)$	1.5
$T_{WTG_2}(s)$	1.8	$T_{PV_1}(s)$	1.8
$T_{PV_2}(s)$	2.3	$T_{BES_1}(s)$	0.2
$T_{BES_2}(s)$	0.3	$T_{FES_1}(s)$	0.05
$T_{FES_2}(s)$	0.07		

TABLE II. Parameters of MG Model

Parameter	Value	Parameter	Value
r_{L_1}	1.5	r_{L_2}	1.7
r_{WTG_1}	1.3	r_{WTG_2}	1.7
r_{PV_1}	1.8	r_{PV_2}	1.6
b_{MT_1}	2.9	b_{MT_2}	2.1
b_{MT_3}	2.8	b_{FC_1}	2.9
b_{FC_2}	2.1	b_{DEG_1}	2.5
b_{DEG_2}	2.4	$\bar{D}$(pu/Hz)	0.012
$\bar{M}$(pu/s)	0.2	α	0.3
ε_1	1.0	ε_2	3.0
γ	0.09		

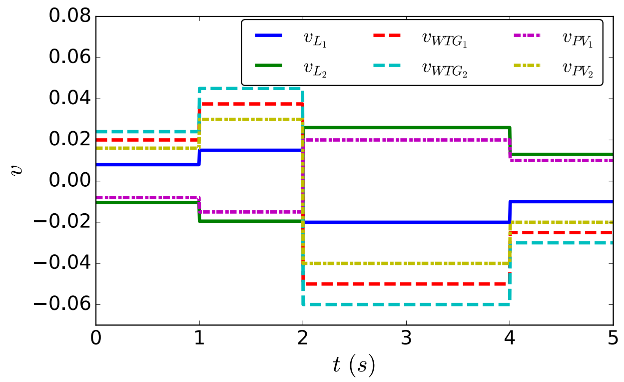

Fig. 1. Disturbance inputs of MG system.

The system disturbance inputsareassumed to be of the forms in Fig. 1.By eliminating the stochastic terms in system (2), we shall be able to obtain a conventional deterministic system formed by a group of ODEs. With the disturbance inputsshown in Fig. 1, the power deviations of load devices, WTGs and PV units in scenarios ofdeterministic and stochastic systems are presented in Fig. 2 and Fig. 3, respectively.Obviously, with the same disturbance inputs, the power dynamics in Fig. 3 with stochastic deviations is closer to the reality than the ones in Fig. 2.Weobserve that the stochastic MG system in Fig. 3 describes complex varying patterns that indeed exist in real-world power systems.

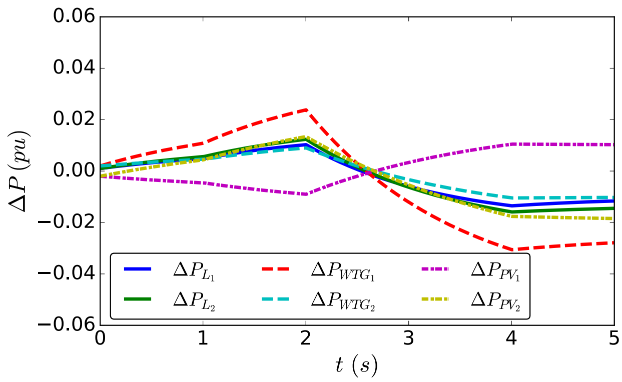

Fig. 2. Dynamics of load devices, WTGs and PV units (ODE system).

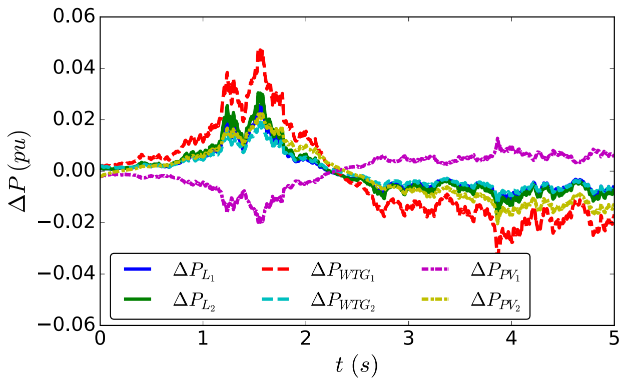

Fig. 3. Dynamics of load devices, WTGs and PV units (SDE system).

To show the usefulness of the proposed H_2/H_∞ control scheme, we present the simulation results when the Nash equilibrium(u^*, v^*) is applied.Fig. 4 illustrates the dynamics of frequency deviation with the obtainedH_2/H_∞ controller and without control, respectively. We see the H_2/H_∞ controller u^* successfully achieves the frequency regulation target under the worst-case disturbance input v^*.

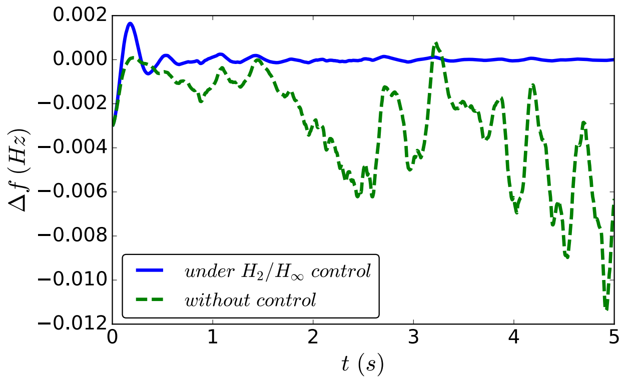

Fig. 4. Frequency deviations in MG system.

The power deviations of BES devices and FES devices under the H_2/H_∞ control and without control are shown in Fig. 5 and Fig. 6, respectively. In Fig. 7, the power changes of MTs, FCs and DEGs under theH_2/H_∞ control scheme are presented.

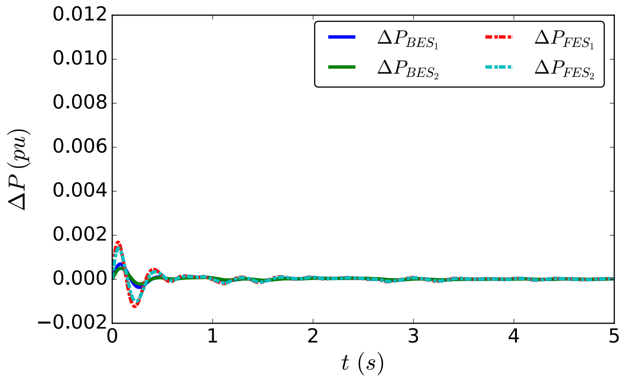

Fig. 5. Dynamics of BES and FES devices under H_2/H_∞control.

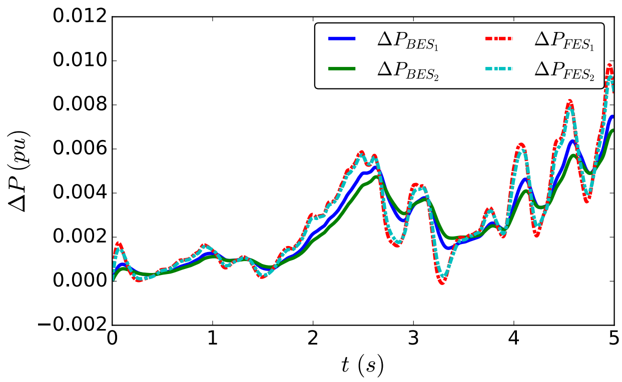

Fig. 6. Dynamics of BES and FES devices withoutcontrol.

Fig.4 and Fig. 6 indicate that without an appropriate control scheme, the frequency of the MG system is unstable and the power deviations of BES and FES devices become significantly large, which may lead to blackout of the whole islanded MG system. In Fig.5 and Fig.7, it can be seen that the rational utilization of the power generation devices and ES devices is realized.

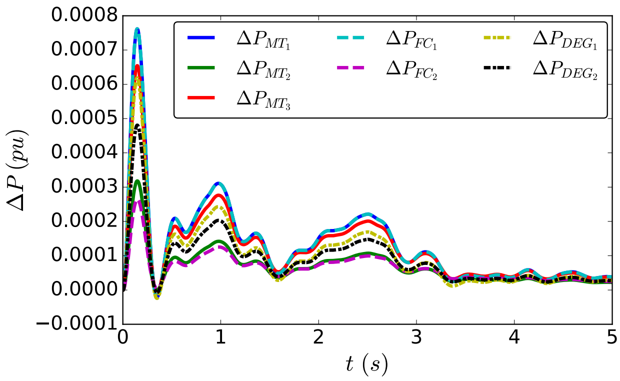

Fig. 7. Dynamics of MTs, FCs and DEGs under H_2/H_∞ control.

The effect of the proposed H_2/H_∞ controller is evaluated by the above numerical simulations which also indicate the advantage of stochastic systems over deterministic ones for islanded MG modeling.

V. Conclusion

In this paper, a typical islanded AC MG containing multiple WTGs, PV units, MTs, FCs, DEGs, BES devices, FES devices and load devices is considered. Both ODEs and SDEs are used for the modeling of the MG dynamics.We formulate a stochastic mixed H_2/H_∞ control problem for the islanded MG. Minimizing the cost of power generation devices and ES devices is formulated as the stochastic H_2 control problem; whereas the AC busfrequency regulation issue is considered as the stochasticH_∞control problem. The mixed stochasticH_2/H_∞ control problem is solved via stochastic control theory and game theoretic approaches. We obtain an analytical solution to suchproblem and alsoprovide several numerical simulations to illustrate the effectiveness of the proposed method.

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