Frequency Control for Multiple Microgrids in Energy Internet: A Stochastic $H_\infty$ Approach

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Abstract—This paper deals with the frequency regulation problems for multiple AC MGs interconnected via energy routers (ERs). We assume electric power is transmitted between MGs with DC transmission technology, thus the frequencies in different MGs are independent. It is assumed that these multiple MGs are not connected with the main power grid. We consider the randomness power output from renewable distributed energy resources (DERs) and loads, and we formulate the system dynamics of each MG as a combination of ordinary differential equations (ODEs) and stochastic differential equations (SDEs). Then the frequency regulation problem regarding each MG’s AC bus is formulated as a stochastic $H_\infty$ control problem. A state feedback controller is designed, such that a prescribed $H_\infty$ performance is satisfied. In case that obtained controller is over strong, certain constraint is implemented to ensure the rationality of the desired controller. A linear matrix inequality (LMI) and semi-definite programming (SDP) approach is developed to solve these problems. Simulations are provided to demonstrate the effectiveness of the proposed approach.

Keywords—stochastic systems; $H_\infty$ control; multiple microgrids; frequency control; linear matrix inequality, semi-definite programming

I. INTRODUCTION

Microgrid (MG) is becoming a solution to the challenges facing the traditional power systems with great promise. It systematically integrates distributed energy resource (DER) units, energy storage (ES) systems and loads into the large power grids [1]. The basic goal of the MG is to keep a balance between power supply and demand in an efficient and economical way [2], [3]. However, when renewable DER units such as wind power generators (WTGs), photovoltaic (PV) units are introduced into the MG, uncertain power generation from them would make this goal challenging [4]. These uncertainties occur not only in supply side, but in demand side as well, since load power can be viewed as a stochastic process [5]. Especially for small sized MG, the smoothing effect of load aggregation is even weaker. In an AC MG the unbalance between power generation and usage results in frequency deviation, and large frequency deviation would lead to power blackout [6]. When a MG is connected to the main power grid, the AC bus frequency can be regulated well by the grid. If a MG works individually (without being connected to the grid), it is named as islanded MG, or off-grid MG. For an islanded MG, power balance is relatively difficult to be achieved [7].

To solve the aforementioned challenge, the concept of multimegrids (multi-MGs) is proposed, in the sense that multiple MGs are interconnected; see, e.g., [8], [9] and the references therein. Multi-MGs can be connected to the main power grid, or simply work on their own [10]. Within the scenario of multi-MGs, each MG shall be able to exchange power and share the capacity of ES devices with the connected ones. If appropriate controllers are set in the whole multi-MGs system, such that each AC MG’s local power balance is improved, then the frequency deviation of each AC MG is regulated, and better power quality can be assured.

The frequency control problems have attracted much attention in the past decades and significant advances on this topic have been made; see, e.g., [6]. It has been shown that $H_\infty$ control theory can be effectively applied into power system frequency regulation problems [11]. When the dynamics of an islanded MG are considered, authors in [7], [12] transform the power dynamic system into a state space control system, then $H_\infty$ control theory is used to regulate the AC MG system’s bus frequency. It is notable that deterministic system modeling approach is used in [7] and [12], without considering the objectively existing randomness involved by renewable DER units. In fact, the power output change from load, WTG and PV units shall be regarded as stochastic process [5], [13]. For multi-MGs that are not connected with the main power grid, there have been few work focusing on each AC MG’s bus frequency control, particularly when the dynamics of each MG components are taken into consideration.

Following the core router of network technology, the concept of energy router (ER) is proposed and its prototype is completed [14]. The ER can be used to dispatch electric power from one MG to the other, such that power balance for the whole multi-MGs is achieved [15]. For example, if redundant energy is generated over the local load’s requirement, the excessed energy can be transmitted into the other connected MGs via the ER, instead of being stored in the local BES system, if the neighbor MG is lack of energy. In previous literatures, ERs are also known as energy exchange devices or energy hubs [16].
In this paper, we consider the scenario of multiple MGs interconnected via ERs. Electric power is assumed to be transmitted between MGs via DC transmission technology, thus the AC bus frequency in each MG can be different. We are concerned with the problem of frequency regulation within each AC MG which is composed of WTGs, PV units, battery energy storage (BES) systems, micro turbines (MTs) and loads. Considering the stochastic power output change from renewable power generation devices, we transform the power dynamics of the physical multi-MGs into a class of ordinary differential equations (ODEs) and stochastic differential equations (SDEs). Then the frequency regulation can be transformed into an $H_\infty$ control problem for stochastic systems. The objective is to design a state feedback controller which guarantees a prescribed disturbance attenuation level for the stochastic closed-loop system. It is worth pointing out that this is the very first time that dynamics of each component within multiple MGs are taken into consideration for the frequency control problem. A sufficient condition for the solvability of the $H_\infty$ control problem is obtained via the linear matrix inequality (LMI) approach. We obtain an explicit expression of the controller. In addition, in case that the controller is over strong, we implement a constraint on the size of the controller, then we can obtain the desired controllers via semi-definite programming (SDP) approach.

The rest of the paper is organized as follows. In Section II, system modeling of multiple MGs is described. Section III formulates the frequency control problem for multiple MGs as a stochastic $H_\infty$ control problem and provides an analytical solution. Section IV illustrates some numerical simulations. Finally, we conclude this paper in Section V.

II. SYSTEM MODELING

With the penetration of renewable DERs in power grids, the control problems related to MG have received much attention[7], [8], [12], [17]. For the full utilization of these renewable resources, the cooperation among multiple MGs has become an important issue. In this paper, the coordinated control for the frequency regulation in multiple MGs is investigated. In this section Part A illustrates one kind of general modeling method for MG networks, and Part B shows the scalability of our proposed modeling approach for the generalized multi-MGs.

A. System modeling for three interconnected MGs

As a typical case, the system modeling for three AC MGs ($MG_1$, $MG_2$ and $MG_3$) interconnected via two ERs are investigated in this subsection. The considered scenario is illustrated in Fig. 1 in which the power converters are omitted. We assume that each MG consists of local loads and one BES system. The BES system is assumed to be uncontrollable and passively responds to the MG’s AC bus frequency deviation. For notation simplicity, we assume that $MG_1$ contains one WTG; $MG_2$ has one PV unit and one FC inside; and $MG_3$ comprises one WTG and one MT. As shown in Fig.1, ERs are used to transmit energy among MGs according to control signals. It is worth noting that such typical model can be extended into MGs with many components without essential difficulty. Here we assume the electric power are transmitted between MGs with DC transmission technology, thus the frequencies in different MGs are independent.

![Fig. 1. The scenario of three interconnected MGs.](image)

For simplicity, the power change (input/output) of the local load, BES system, WTG and FC in $MG_1$ are denoted as $\Delta P_L$, $\Delta P_{BES}$, $\Delta P_{WTG}$, and $\Delta P_{FC}$, respectively; the power output change of local load, BES system and PV unit in $MG_2$ are denoted as $\Delta P_L$, $\Delta P_{BES}$, $\Delta P_{WTG}$, and $\Delta P_{MT}$, respectively; the power output change of local load, BES system, WTG and DED in $MG_3$ are denoted as $\Delta P_L$, $\Delta P_{BES}$, $\Delta P_{WTG}$, and $\Delta P_{MT}$, respectively. The AC bus frequency deviations in $MG_1$, $MG_2$ and $MG_3$ are denoted as $\Delta f_1$, $\Delta f_2$ and $\Delta f_3$, respectively. We denote $\Delta P_{ER}$ as power change transmitted from $MG_1$ to $MG_2$ and denote $\Delta P_{ER}$ as the power change transmitted from $MG_3$ to $MG_2$. Referring to [7], [12], the power dynamics of all components in the considered multi-MG can be modeled by a group of linear ODEs.

In this paper, considering the stochastic nature of loads and DER units, Weiner process (also known as Brownian motion) is used to describe the detailed power deviations of the PV units, WTGs and loads. The dynamics for $MG_1$, $MG_2$ and $MG_3$ are formulated with linear ODEs and SDEs presented in (1), (2) and (3), respectively (time $t$ omitted).

\[
\begin{align*}
\frac{d\Delta P_L}{dt} &= \frac{1}{L_1}(-\Delta P_L + v_L)dt + n_1\Delta P_L dW(t), \\
\Delta P_{BES} &= \frac{1}{T_{BES}}(\Delta P_{BES} + r_{BES})dW(t), \\
\frac{d\Delta P_{WTG}}{dt} &= \frac{1}{T_{WTG}}(-\Delta P_{WTG} + v_{WTG})dt + r_{WTG}\Delta P_{WTG}dW(t), \\
\Delta P_{FC} &= \frac{1}{T_{FC}}(-\Delta P_{FC} + b_{FC})dW(t), \\
\Delta f_1 &= -\frac{2D_1}{M_1}\Delta f_1 + \frac{2}{M_1}\Delta P_L.
\end{align*}
\]
\[
\begin{align*}
\text{(2)} & \quad \Delta P_{L_2} = \frac{1}{T_{L_2}} (-\Delta P_{L_2} + \nu_{L_2}) \, dt + n_{L_2} \Delta P_{L_2} \, dW(t), \\
\text{(3)} & \quad \Delta \dot{P}_{BES_2} = -\frac{1}{T_{BES_2}} (\Delta P_{BES_2} + r_{BES_2} \Delta f_2), \\
\text{(4)} & \quad \Delta P_{PV_2} = \frac{1}{T_{PV_2}} (-\Delta P_{PV_2} + \nu_{PV_2}) \, dt + r_{PV_2} \Delta P_{PV_2} \, dW(t), \\
\text{(5)} & \quad \Delta \dot{f}_2 = -\frac{2D_2}{M_2} \Delta f_2 + 2D_2 \Delta P_2.
\end{align*}
\]

The total AC bus power changes in MG1, MG2 and MG3 are denoted as \(\Delta P_1, \Delta P_2\) and \(\Delta P_3\), respectively. We have

\[
\begin{align*}
\Delta P_1 &= \Delta P_{W_{G_1}} + \Delta P_{BES_1} + \Delta P_{E_{FC_1}} - \Delta P_{L_1} - \Delta P_{E_{R12}}, \\
\Delta P_2 &= \Delta P_{PV_2} + \Delta P_{BES_2} - \Delta P_{L_2} - \Delta P_{E_{R12}} + \Delta P_{E_{R32}}, \\
\Delta P_3 &= \Delta P_{W_{G_3}} + \Delta P_{BES_3} + \Delta P_{MT_3} - \Delta P_{L_3} - \Delta P_{E_{R32}},
\end{align*}
\]

The stochastic process \(W(t)\) in (1)–(3) presents for a standard scalar Wiener process, which is used to describe the randomness involved by loads, WTGs and PV units. We denote \(\nu_{L_1}, \nu_{L_2}, \nu_{L_3}, \nu_{W_{G_1}}, \nu_{PV_2}\) and \(\nu_{W_{G_3}}\), as the disturbance inputs for power changes of the local loads in MG1, MG2, MG3, the WTG in MG1 the PV units in MG2 and the WTG in MG3, respectively. For the FC in MG1 and the MT in MG3, the control inputs are denoted as \(u_{E_{FC_1}}\) and \(u_{MT_3}\) in (1) and (3). In (4), the control inputs for the energy exchange devices \(E_{R12}\) and \(E_{R32}\) are denoted as \(u_{E_{R12}}\) and \(u_{E_{R32}}\).

For AC bus frequency dynamics in (1)–(3), constants \(M_1, M_2, M_3\) stand for the inertia constants of MG1, MG2 and MG3, respectively, and constants \(D_1, D_2, D_3\) stand for the damping coefficients of MG1, MG2 and MG3, respectively. The time constants \(T_{L_1}, T_{L_2}, T_{L_3}, T_{W_{G_1}}, T_{PV_2}, T_{W_{G_3}}, T_{BES_1}, T_{BES_2}, T_{BES_3}, T_{E_{FC_1}}, T_{MT_3}, T_{E_{R1}}, T_{E_{R2}}\) and factors \(n_{L_1}, n_{L_2}, n_{L_3}, n_{W_{G_1}}, n_{PV_2}, n_{W_{G_3}}, n_{E_{FC_1}}, n_{MT_3}\) in equation (1)–(4) depend on real engineering scenarios and can be measured by parameter estimation methods.

Let vector \(x = [\Delta P_{L_1} \Delta P_{L_2} \Delta P_{L_3}]^T\), \(x_{WTG} = [\Delta P_{W_{G_1}} \Delta P_{W_{G_3}}]^T\), \(x_{BES} = [\Delta P_{BES_1} \Delta P_{BES_2} \Delta P_{BES_3}]^T\), \(x_{FC} = [\Delta P_{E_{FC_1}}]^T\), \(x_M = [\Delta P_{MT_3}]^T\), \(x_E = [\Delta P_{E_{R1}} \Delta P_{E_{R2}}]^T\),\( y = [\Delta f_1 \Delta f_2 \Delta f_3] \). We denote\( x = [x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8]^T\) and \(y = [x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8]^T\) as the disturbance inputs and \(u = [u_{E_{FC_1}} u_{MT_3} u_{E_{R12}} u_{E_{R32}}]^T\) and \(v = [v_{L_1} v_{L_2} v_{L_3} v_{W_{G_1}} v_{W_{G_3}} v_{E_{R12}} v_{E_{R32}}]^T\) as the disturbance inputs.

By rewriting (1)–(5) with the notations defined above, we obtain a state space system as follows, (time \(t\) omitted)

\[
\begin{align*}
\dot{x} &= (A x + B u + C v) \, dt + R x \, dW(t), \\
\dot{z} &= D x,
\end{align*}
\]

where \(D\) is a diagonal matrix, \(x, u, v\) represent for the system state, control input and disturbance input, respectively. If we denote \(D\) as an \(n \times n\) identity matrix and \(O_{m \times n}\) as an \(m \times n\) zero matrix, \(D\) can be expressed as follows:

\[
D = \begin{bmatrix} O_{13 \times 13} & O_{13 \times 3} \\ O_{3 \times 13} & I_3 \end{bmatrix}
\]

Here we have transformed the investigated multi-MGs system into a mathematical control system.

B. Modeling for more general MG networks

By adding one more MG and two more ERs to the MG network shown in Fig. 1, we consider a general multi-MGs structure in Fig. 2 (converters and other electronic devices omitted).

![Fig. 2. A more general MG network.](image-url)
and total AC bus power change of MG are denoted as $\Delta f_L$ and $\Delta P_4$. For routers ER41, ER42 in Fig. 2, $\Delta P_{ER41}$ and $\Delta P_{ER42}$ represent for the deviations of the power transmitted over them.

Similar to part A, the linearized dynamic system of $MG_n$ is shown in (7). The dynamics of ER41, ER42 are given in (8). Based on the new connections in Fig. 2, the dynamics of total AC bus power deviation $\Delta P_1, \Delta P_2, \Delta P_3, \Delta P_4$ in $MG_1, MG_2, MG_3$ and $MG_4$ are provided in (9).

\[
\begin{align*}
\frac{d\Delta P_{L_4}}{dt} &= \frac{1}{T_{L_4}} (-\Delta P_{L_4} + v_{L_4}) dt + r_{L_4} \Delta P_{L_4} dW(t), \\
\frac{d\Delta P_{BES_4}}{dt} &= -\frac{1}{T_{BES_4}} (\Delta P_{BES_4} + r_{BES_4} \Delta f_4), \\
\frac{d\Delta P_{PV_4}}{dt} &= \frac{1}{T_{PV_4}} (-\Delta P_{PV_4} + v_{PV_4}) dt + r_{PV_4} \Delta P_{PV_4} dW(t), \\
\frac{d\Delta P_{MT_4}}{dt} &= -\frac{1}{T_{MT_4}} (\Delta P_{MT_4} + b_{MT_4} \Delta f_4), \\
\Delta f_4 &= -\frac{2D_4}{M_4} \Delta f_4 + \frac{2}{M_4} \Delta P_4, \\
\Delta P_{ER41} &= \frac{1}{T_{ER41}} (-\Delta P_{ER41} + b_{ER41} \Delta f_4), \\
\Delta P_{ER42} &= \frac{1}{T_{ER42}} (-\Delta P_{ER42} + b_{ER42} \Delta f_4).
\end{align*}
\]

III. STOCHASTIC $H_\infty$ CONTROL PROBLEM FORMULATION AND SOLUTION

To achieve the frequency regulation in for multiple MGs as well as to make full use of the renewable DERs, a coordinated control scheme is required. In this section, the frequency regulation target is formulated as a stochastic $H_\infty$ control problem, and solved with related stochastic control theory and convex optimization approaches.

Firstly, we define the stochastic $H_\infty$ performance problem and stochastic $H_\infty$ cost functionals follows.

**Definition 1:** [18] Given a scalar $\gamma > 0$, the $H_\infty$ performance for the frequency regulation problem is defined as $||z(t)|| < \gamma ||v(t)||$. The norm $||.||$ term is defined in (10), where $E$ represents the mathematical expectation.

\[
||z(t)|| \triangleq \left( E \left( \int_0^T |z(t)|^2 dt \right) \right)^{1/2}
\]

where scalar $\gamma$ is called disturbance attenuation. Based on the $H_\infty$ performance above, the stochastic $H_\infty$ cost functional is formulated in (11),

\[
J(u, v) \triangleq E \left[ \int_0^T (y^2 v^2 - z^2 z) dt \right].
\]

The stochastic $H_\infty$ control problem is to find a controller $u^*$ for system (6), such that for all nonzero disturbance $v(t) \in L_2[0, \infty), J(u^*, v) \leq 0$ holds. Using the techniques in [19], we obtain the following theorem.

**Theorem 1:** [18] Given a disturbance attenuation $\gamma > 0$, the stochastic dynamic system (6) is said to satisfy the $H_\infty$ performance if there exist matrix $Y$ and symmetric matrix $X$ with appropriate dimensions, such that $X \succeq 0$ and the LMI (12) holds, where $\Gamma = AX + XA^T + BY + YB^T$. The robust state feedback controller in this case can be obtained with $u^* = Kx, X = KY^{-1}$.

\[
\begin{bmatrix}
\Gamma & XD' & XR' & C \\
DX & -I & 0 & 0 \\
RX & 0 & -X & 0 \\
C' & 0 & 0 & -\gamma^2 I
\end{bmatrix} \preceq 0.
\]

**Remark 1:** Generally, the solution to the LMI (12) is not unique. A strong controller can regulate the frequency well. However, an over strong might damage MT and FC. To obtain a rational controller, we transform the stochastic $H_\infty$ control problem into a semi-definite programming (SDP) problem formulated in (13), where $||.||_\infty$ stands for the $\infty$-norm of a matrix. In this paper, the SDP problem (13) can be solved with the MATLAB® convex optimization toolbox CVX [19].
minimize $\|Y\|_{\infty}$

s.t.

$$
\begin{bmatrix}
\Gamma & XD' & XR' & C \\
DX & -I & 0 & 0 \\
RX & 0 & -X & 0 \\
C' & 0 & 0 & -\gamma^2 I
\end{bmatrix} \leq 0.
$$

(13)

IV. NUMERICAL SIMULATION

Based on the multiple MGs shown in Fig. 1, several numerical simulation results are presented in this section to illustrate the effectiveness of the proposed stochastic $H_{\infty}$ control scheme for frequency regulation in each MG.

In the time domain simulation, the initial value of state variable $x$ in (6) is assigned to be a zero vector. The parameters for the model in Fig. 1 are shown in Table I and Table II.

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For illustrative purpose, we assume that the disturbance input results in a relative significant fluctuation in the local load power of MG$_2$, and the corresponding load power dynamics in MG$_1$, MG$_2$ and MG$_3$ are illustrated in Fig. 3. Considering the SDP problem in Remark 1, we plot the AC bus frequency deviations in MG$_1$, MG$_2$ and MG$_3$ under the desired $H_{\infty}$ control scheme and without control in Fig. 4. The frequency deviations under the $H_{\infty}$ control.

![Fig. 3. Local load power dynamics in different MGs.](image)

From Fig. 4, we see that the frequency is distinctly affected by the power deviations in MG system without control. However, when the $H_{\infty}$ controller is performed, the frequency deviation of each MG is strictly limited to a small range, indicating the notable effectiveness of the proposed stochastic $H_{\infty}$ control scheme.

![Fig. 4. Frequency deviations in different MGs in with/without control.](image)

Additionally, we show the proposed control scheme effectively stabilizes the frequencies without over-control in Fig. 5. During the simulation period, the power deviations
\[ \Delta P_{PC1}, \Delta P_{PT1}, \Delta P_{ER12} \text{ and } \Delta P_{ER32} \text{ fluctuates within a certain range without going towards infinitely large.} \]

Noticing that MG2 in Fig. 1 is composed of only one PV unit, one BES system and its local load. The frequency regulation of MG2 can only rely on the energy transmitted from MG1 and MG3, which is illustrated in Fig. 5 as both values of \( \Delta P_{ER12} \) and \( \Delta P_{ER32} \) are positive. In this sense, the abundant energy in one MG could be consumed in other MGs instead of being stored in the BES system, which is conducive to improving energy efficiency.

\[ \begin{align*}
\Delta P &\text{ (p.u.)} \\
0.05 &\leq \Delta P \\
0.04 &< \Delta P < 0.03 \\
0.02 &< \Delta P < 0.01 \\
0 &\leq \Delta P \\
0 &< \Delta P < 0.01 \\
0 &< \Delta P < 0.02 \\
0 &< \Delta P < 0.03 \\
0 &< \Delta P < 0.04 \\
0 &< \Delta P < 0.05
\end{align*} \]

Fig. 5. Power deviations of the controlled devices under \( H_{\infty} \) control.

The simulation results provided above illustrate the usefulness of our proposed stochastic robust control method for the coordinated frequency regulation for multiple MGs. The stochastic process introduced in the dynamic system (6) makes our result more realistic.

V. CONCLUSION

In this paper the coordinated frequency regulation for multiple AC MGs is investigated. Multiple MGs are designed to be interconnected with ERs. Both ODEs and SDEs are used to model the dynamics of multi-MGs, including the power change of PVs, WTGs, MTs, FCs, BES systems, local loads, ERs and AC bus frequency deviations. We formulate the problem of frequency regulation from multiple MGs as a stochastic \( H_{\infty} \) control problem. The problem is solved with stochastic control theory and related optimization methods. An appropriate analytical solution to the problem is obtained, and typical numerical simulation results are presented to show the effectiveness of the proposed control scheme.

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