Frequency Control for Multiple Microgrids in Energy Internet: A Stochastic H_∞ Approach

Haochen Hua,Yuchao Qin and Junwei Cao
Research Institute of Information Technology
Tsinghua University
Beijing, P. R. China
e-mail: jcao@tsinghua.edu.cn

***Abstract*—This paper deals with the frequency regulation problems for multiple AC MGs interconnected via energy routers (ERs). We assume electric power is transmitted between MGs with DC transmission technology, thus the frequencies in different MGs are independent. It is assumed that these multiple MGs are not connected with the main power grid. We consider the randomness power output from renewable distributed energy resources (DERs) and loads, and we formulate the system dynamics of each MG as a combination of ordinary differential equations (ODEs) and stochastic differential equations (SDEs). Then the frequency regulation problem regarding each MG's AC bus is formulated as a stochastic H_∞ control problem.A state feedback controller is designed,such that a prescribed H_∞ performance is satisfied. In case that obtained controller is over strong, certain constraint is implemented to ensure the rationality of the desired controller. A linear matrix inequality (LMI) and semi-definite programming (SDP) approach is developed to solve these problems. Simulations are provided to demonstrate the effectiveness of the proposed approach.**

Keywords- stochastic systems; H_∞ control; multiplemicrogrids; frequency control;linear matrix inequality, semi-definite programming

I. INTRODUCTION

Microgrid (MG) is becoming a solution to the challenges facing the traditional power systems with great promise. It systematically integratesdistributed energy resource (DER) units, energy storage (ES) systems and loads into the large powergrids [1]. The basic goal of theMG is to keep a balancebetweenpower supply and demand in an efficientand economical way[2], [3]. However, when renewable DER units such as wind power generators (WTGs), photovoltaic (PV) units are introduced into the MG, uncertain power generation from themwould make this goal challenging[4]. These uncertainties occur not only in supply side, but in demand side as well, since load power can be viewed as a stochastic process [5]. Especially fora small sizedMG, the smoothing effect of load aggregation is even weaker. In an AC MG the unbalance between power generation and usage results in frequency deviation, and large frequency deviation would lead to power blackout [6].When a MG is connected to the main power grid, the AC bus frequency can be regulated well by the grid. If a MG works individually (without being connected to the grid), it is named as islanded MG, or off-grid MG. For an islanded MG, power balance is relatively difficult to be achieved [7].

To solve the aforementioned challenge, the concept of multi-microgrids (multi-MGs) is proposed, in the sense that multiple MGs are interconnected; see, e.g., [8], [9] and the references therein. Multi-MGscan be connected to the main power grid, or simply work on their own [10]. Within the scenario of multi-MGs, each MG shall be able to exchange powerand share the capacity of ES devices withthe connectedones. If appropriate controllers are set in the whole multi-MGs system, such that each AC MG's local power balance is improved, then the frequency deviation of each AC MG is regulated, and better power quality can be ensured.

The frequency control problems have attracted much attention in the past decades and significant advances on this topic havebeen made; see, e.g., [6]. It has been shown that H_∞ control theory can be effectivelyapplied into power system frequency regulation problems[11].When the dynamics of an islanded MG are considered, authors in [7], [12] transform the power dynamic system into a state space control system, then H_∞ control theory is used to regulate the ACMG system's bus frequency. It is notable that deterministic system modeling approach is used in [7] and [12], without considering the objectively existing randomness involved by renewable DER units.In fact, the power output change from load, WTG and PV units shall be regarded as stochastic process [5], [13].For multi-MGs that are not connected with the main power grid, there have been few work focusing on each AC MG's bus frequency control, particularly when the dynamics of each MG components are taken into consideration.

Following the core router of network technology, the concept of energy router(ER)is proposed and its prototype is completed [14]. The ER can be used to dispatch electric power from one MG to the other, such that power balance for the whole multi-MGs is achieved [15].For example, if redundantenergy isgeneratedover the local load's requirement, the exceededenergycanbe transmitted into the other connected MGs via the ER, instead of being stored in the local BES system, if the neighbor MG is lack of energy. In previous literatures, ERs are also known as energy exchange devicesor energy hubs [16].

In this paper, we consider the scenario of multiple MGs interconnected viaERs.Electric power is assumed to be transmitted between MGs via DC transmission technology, thus the AC bus frequency in each MG can be different. We are concerned with the problem of frequency regulation within each AC MG which is composed of WTGs, PV units, battery energy storage (BES) systems, micro turbines (MTs) and loads.Considering the stochastic power output change from renewable power generation devices, we transform the power dynamics of the physical multi-MGs into a class of ordinary differential equations (ODEs) and stochastic differential equations (SDEs). Then the frequency regulation can be transformed into a H_∞ control problem for stochastic systems. The objectiveis to design a state feedbackcontroller which guarantees a prescribed disturbanceattenuation level for thestochastic closed loop system. It is worth pointingout that this is the very first time that dynamics of each component within multiple MGsare taken into consideration for the frequency control problem. A sufficientcondition for the solvability of the stochastic H_∞ control problem is obtained via thelinear matrix inequality (LMI) approach. We obtain an explicit expression of the desired controller. In addition, in case that the controller is over strong, we implement a constraint on the size of the controller, then we can obtain the desired controllers via semi-definite programming (SDP) approach.

The rest of the paper is organized as follows. In Section Ⅱ system modeling of multiple MGs is described. Section Ⅲ formulates the frequency control problem for multiple MGs asastochastic H_∞ control problem and provides an analytical solution. Section Ⅳ illustrates some numerical simulations. Finally, we conclude this paper in Section Ⅴ.

II. System Modeling

With the penetration of renewable DERs in power grids, the control problems related to MG have received much attention[7], [8], [12], [17]. For the full utilization of these renewable resources, the cooperation among multiple MGs has become an important issue. In this paper, the coordinated controlfor the frequency regulation in multiple MGsis investigated. In this section Part A illustrates one kind of general modeling method for MG networks, andPart B shows the scalability of our proposed modeling approachfor thegeneralizedmulti-MGs.

A. System modeling for three interconnected MGs

As a typical case, the system modeling for threeAC MGs (MG_1 , MG_2 and MG_3) interconnected via two ERs are investigated in this subsection. The considered scenario is illustrated in Fig. 1 in which the power converters are omitted. We assume that each MG consists of local loads and one BES system. The BES system is assumedto be uncontrollable and passively responds to the MG's AC bus frequency deviation. For notation simplicity, we assume that MG_1 contains one WTG; MG_2 has one PV unit and one FC inside; and MG_3 comprises one WTG and one MT. As shown in Fig.1, ERs are used to transmit energy among MGs according to control signals.It is worth noting that such typical model can be extended into MGs with many components without essential difficulty. Here we assume the electric power are transmitted between MGs with DC transmission technology, thus the frequencies in different MGs are independent.

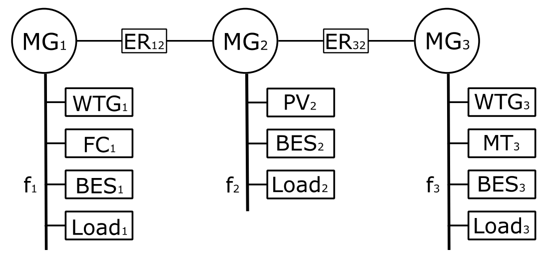

Fig. 1. The scenario of three interconnected MGs.

For simplicity, the power change (input/output) of the local load, BES system, WTG and FC in MG_1 are denoted as ΔP_{L_1}, ΔP_{BES_1} , ΔP_{WTG_1} and ΔP_{FC_1} , respectively; the power output change of local load, BES system and PV unit in MG_2 are denoted as ΔP_{L_2}, ΔP_{BES_2}, and ΔP_{PV_2}, respectively; the power output change of local load, BES system, WTG and DEG in MG_3 are denoted as ΔP_{L_3} , ΔP_{BES_3} , ΔP_{WTG_3} and ΔP_{MT_3} , respectively. The AC bus frequency deviations in MG_1, MG_2 and MG_3 are denoted as Δf_1, Δf_2 and Δf_3, respectively. We denote $\Delta P_{ER_{12}}$ as power changetransmittedfrom MG_1 to MG_2 and denote and $\Delta P_{ER_{32}}$ as the power change transmitted from MG_3 to MG_2. Referring to [7], [12], the power dynamics of allcomponents in the considered multi-MGs can be modeled by a group of linear ODEs.

In this paper, considering the stochastic nature of loads and DER units, Weiner process (also known as Brownian motion)is used to describe the detailed power deviations of the PV units, WTGs and loads.The dynamics for MG_1 , MG_2 and MG_3 are formulated with linear ODEs and SDEs presented in(1), (2) and (3), respectively (time t omitted).

$$
\begin{cases}
\mathrm{d}\Delta P_{L_1} = \dfrac{1}{T_{L_1}}\left(-\Delta P_{L_1} + v_{L_1}\right)\mathrm{d}t + r_{L_1}\Delta P_{L_1}\mathrm{d}W(t), \\
\Delta \dot{P}_{BES_1} = -\dfrac{1}{T_{BES_1}}\left(\Delta P_{BES_1} + r_{BES_1}\Delta f_1\right), \\
\mathrm{d}\Delta P_{WTG_1} = \dfrac{1}{T_{WTG_1}}\left(-\Delta P_{WTG_1} + v_{WTG_1}\right)\mathrm{d}t \\
\qquad + r_{WTG_1}\Delta P_{WTG_1}\mathrm{d}W(t), \\
\Delta \dot{P}_{FC_1} = \dfrac{1}{T_{FC_1}}\left(-\Delta P_{FC_1} + b_{FC_1}u_{FC_1}\right), \\
\Delta \dot{f}_1 = -\dfrac{2D_1}{M_1}\Delta f_1 + \dfrac{2}{M_1}\Delta P_1.
\end{cases} \tag{1}
$$

$$\begin{cases} \mathrm{d}\Delta P_{L_2} = \dfrac{1}{T_{L_2}}\left(-\Delta P_{L_2} + v_{L_2}\right)\mathrm{d}t + r_{L_2}\Delta P_{L_2}\mathrm{d}W(t) \\ \Delta \dot{P}_{BES_2} = -\dfrac{1}{T_{BES_2}}\left(\Delta P_{BES_2} + r_{BES_2}\Delta f_2\right), \\ \mathrm{d}\Delta P_{PV_2} = \dfrac{1}{T_{PV_2}}\left(-\Delta P_{PV_2} + v_{PV_2}\right)\mathrm{d}t \\ \qquad + r_{PV_2}\Delta P_{PV_2}\mathrm{d}W(t), \\ \Delta \dot{f}_2 = -\dfrac{2D_2}{M_2}\Delta f_2 + \dfrac{2}{M_2}\Delta P_2. \end{cases} \tag{2}$$

$$\begin{cases} \mathrm{d}\Delta P_{L_3} = \dfrac{1}{T_{L_3}}\left(-\Delta P_{L_3} + v_{L_3}\right)\mathrm{d}t + r_{L_3}\Delta P_{L_3}\mathrm{d}W(t), \\ \Delta \dot{P}_{BES_3} = -\dfrac{1}{T_{BES_3}}\left(\Delta P_{BES_3} + r_{BES_3}\Delta f_3\right), \\ \mathrm{d}\Delta P_{WTG_3} = \dfrac{1}{T_{WTG_3}}\left(-\Delta P_{WTG_3} + v_{WTG_3}\right)\mathrm{d}t \\ \qquad + r_{WTG_3}\Delta P_{WTG_3}\mathrm{d}W(t), \\ \Delta \dot{P}_{MT_3} = \dfrac{1}{T_{MT_3}}\left(-\Delta P_{MT_3} + b_{MT_3}u_{MT_3}\right), \\ \Delta \dot{f}_3 = -\dfrac{2D_3}{M_3}\Delta f_3 + \dfrac{2}{M_3}\Delta P_3. \end{cases} \tag{3}$$

$$\begin{cases} \Delta \dot{P}_{ER_{12}} = \dfrac{1}{T_{ER_{12}}}\left(-\Delta \dot{P}_{ER_{12}} + b_{ER_{12}}u_{ER_{12}}\right), \\ \Delta \dot{P}_{ER_{32}} = \dfrac{1}{T_{ER_{32}}}\left(-\Delta \dot{P}_{ER_{32}} + b_{ER_{32}}u_{ER_{32}}\right). \end{cases} \tag{4}$$

The total AC bus power changes inMG_1, MG_2 and MG_3 are denoted as ΔP_1, ΔP_2 and ΔP_3, respectively. We have

$$\begin{cases} \Delta P_1 = \Delta P_{WTG_1} + \Delta P_{BES_1} + \Delta P_{FC_1} - \Delta P_{L_1} - \Delta P_{ER_{12}}, \\ \Delta P_2 = \Delta P_{PV_2} + \Delta P_{BES_2} - \Delta P_{L_2} + \Delta P_{ER_{12}} + \Delta P_{ER_{32}}, \\ \Delta P_3 = \Delta P_{WTG_3} + \Delta P_{BES_3} + \Delta P_{MT_3} - \Delta P_{L_3} - \Delta P_{ER_{32}}. \end{cases} \tag{5}$$

The stochastic process $W(t)$ in (1)–(3) presents for a standard scalar Weiner process, which is used to describe the randomness involved by loads, WTGs and PV units. We denote v_{L_1}, v_{L_2}, v_{L_3}, v_{WTG_1}, v_{PV_2} and v_{WTG_3} as the disturbance inputs for power changes of the local loads in MG_1, MG_2, MG_3, the WTG in MG_1,the PV units in MG_2 and the WTG in MG_3, respectively. For the FC in MG_1 and the MT in MG_3, the control inputs are denoted asu_{FC_1} and u_{MT_3} in (1) and (3). In (4), the control inputs for the energy exchange devices ER_{12} and $ER_{32}$are denoted as $u_{ER_{12}}$ and $u_{ER_{32}}$.

For AC bus frequency dynamics in (1)–(3), constants M_1, M_2, M_3 stand for the inertia constantsof MG_1, MG_2 and MG_3, respectively, and constants D_1, D_2, D_3 stand for the damping coefficients of MG_1, MG_2 and MG_3 ,respectively. The time constants T_{L_1}, T_{L_2}, T_{L_3}, T_{WTG_1}, T_{PV_2}, T_{WTG_3}, T_{BES_1}, T_{BES_2}, T_{BES_3}, T_{FC_1}, T_{MT_3}, T_{ER_1}, T_{ER_2} and factors r_{L_1}, r_{L_2}, r_{L_3}, r_{WTG_1}, r_{PV_2}, r_{WTG_3}, r_{BES_1}, r_{BES_2}, r_{BES_3}, b_{FC_1}, b_{MT_3}, $b_{ER_{12}}$, $b_{ER_{32}}$ in equation (1)-(4) depend on real engineering scenarios and can be measured by parameter estimation methods.

Let vector $x_L = [\Delta P_{L_1} \quad \Delta P_{L_2} \quad \Delta P_{L_3}]'$, $x_{WTG} = [\Delta P_{WTG_1} \quad \Delta P_{WTG_3}]'$, $x_{PV} = [\Delta P_{PV_2}]'$, $x_{BES} = [\Delta P_{BES_1} \quad \Delta P_{BES_2} \quad \Delta P_{BES_3}]'$, $x_{FC} = [\Delta P_{FC_1}]'$, $x_{MT} = [\Delta P_{MT_3}]'$, $x_{ER} = [\Delta P_{ER_1} \quad \Delta P_{ER_2}]'$, $x_f = [\Delta f_1 \quad \Delta f_2 \quad \Delta f_3]'$. We denote vector $x = [x_L \quad x_{WTG} \quad x_{PV} \quad x_{BES} \quad x_{FC} \quad x_{MT} \quad x_{ER} \quad x_f]'$. Similarly, let $u = [u_{FC_1} \quad u_{MT_3} \quad u_{ER_{12}} \quad u_{ER_{32}}]'$ and $v = [v_{L_1} \quad v_{L_2} \quad v_{L_3} \quad v_{WTG_1} \quad v_{PV_2} \quad v_{WTG_3}]'$.

By rewriting (1)–(5) with the notations defined above, we obtain a state space system as follows, (time t omitted)

$$\begin{cases} \mathrm{d}x = (Ax + Bu + Cv)\mathrm{d}t + Rx\mathrm{d}W(t), \\ z = Dx, \end{cases} \tag{6}$$

where D is a diagonal matrix, x, $u$and $v$represent for the system state, control input and disturbance input, respectively. If we denoteI_n as an $n \times n$ identity matrix and$O_{m\times n}$ as an $m \times n$ zero matrix, D can be expressed as follows,

$$D = \begin{bmatrix} O_{13\times 13} & O_{13\times 3} \\ O_{3\times 13} & I_3 \end{bmatrix}.$$

Here we have transformed the investigated multi-MGs system into amathematical control system.

B. Modeling for more general MG networks

By adding one more MG and two more ERs to the MG network shown in Fig. 1, we consider a generalizedmulti-MGs structure in Fig. 2 (converters and other electronic devices omitted).

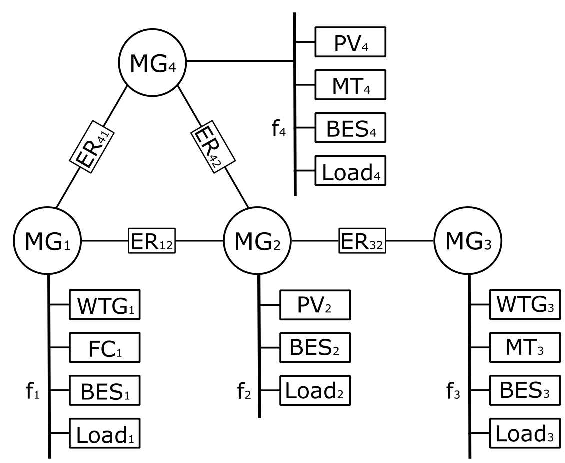

Fig. 2. A more general MG network.

The newly added MG_4 is composed of local load, one PV unit, one MT and a BES system. We denote ΔP_{L_4}, $\Delta P_{PV\,4}$, ΔP_{MT_4} and ΔP_{BES_4} as the power deviation of its local load, PV unit, MT and BES system, respectively. The AC bus frequency deviation and total AC bus power change of MG_4 are denoted as Δf_4 and ΔP_4 . For routers ER_{41} , ER_{42} in Fig. 2, $\Delta P_{ER_{41}}$ and $\Delta P_{ER_{42}}$represent for the deviations ofthe power transmitted over them.

Similar to part A, the linearized dynamic system of $MG_4$is shown in (7). The dynamics of ER_{41}, ER_{42} are given in (8). Based on the new connections in Fig. 2, the dynamics of total AC bus power deviation ΔP_1, $\Delta P_2 \Delta P_3$and ΔP_4 in MG_1, $MG_2 MG_3$ and MG_4 are provided in (9).

$$\begin{cases} \mathrm{d}\Delta P_{L_4} = \dfrac{1}{T_{L_4}}\left(-\Delta P_{L_4} + v_{L_4}\right)\mathrm{d}t + r_{L_4}\Delta P_{L_4}\mathrm{d}W(t) \\ \Delta \dot{P}_{BES_4} = -\dfrac{1}{T_{BES_4}}\left(\Delta P_{BES_4} + r_{BES_4}\Delta f_4\right), \\ \mathrm{d}\Delta P_{PV_4} = \dfrac{1}{T_{PV_4}}\left(-\Delta P_{PV_4} + v_{PV_4}\right)\mathrm{d}t \\ \qquad + r_{PV_4}\Delta P_{PV_4}\mathrm{d}W(t), \\ \Delta \dot{P}_{MT_4} = -\dfrac{1}{T_{MT_4}}\left(\Delta P_{MT_4} + b_{MT_4}u_{MT_4}\right), \\ \Delta \dot{f}_4 = -\dfrac{2D_4}{M_4}\Delta f_4 + \dfrac{2}{M_4}\Delta P_4. \end{cases} \tag{7}$$

$$\begin{cases} \Delta \dot{P}_{ER_{41}} = \dfrac{1}{T_{ER_{41}}}\left(-\Delta \dot{P}_{ER_{41}} + b_{ER_{41}}u_{ER_{41}}\right), \\ \Delta \dot{P}_{ER_{42}} = \dfrac{1}{T_{ER_{42}}}\left(-\Delta \dot{P}_{ER_{42}} + b_{ER_{42}}u_{ER_{42}}\right). \end{cases} \tag{8}$$

$$\begin{cases} \Delta P_1 = \Delta P_{WTG_1} + \Delta P_{BES_1} + \Delta P_{FC_1} - \Delta P_{L_1} \\ \qquad - \Delta P_{ER_{12}} + \Delta P_{ER_{41}}, \\ \Delta P_2 = \Delta P_{PV_2} + \Delta P_{BES_2} - \Delta P_{L_2} \\ \qquad + \Delta P_{ER_{12}} + \Delta P_{ER_{32}} + \Delta P_{ER_{42}}, \\ \Delta P_3 = \Delta P_{WTG_3} + \Delta P_{BES_3} + \Delta P_{MT_3} - \Delta P_{L_3} - \Delta P_{ER_{32}}, \\ \Delta P_4 = \Delta P_{PV_4} + \Delta P_{BES_4} + \Delta P_{MT_4} - \Delta P_{L_4} \\ \qquad - \Delta P_{ER_{41}} - \Delta P_{ER_{42}}. \end{cases} \tag{9}$$

Obviously, (7)–(9) are essentially identical to (1)–(5). Thus, we can simply combine (7)–(9)withthe multi-MGs system (1)–(5) and shall be able to obtain a new stochastic control system with the same form of (6).

As is shown in Fig. 2, such new structure has certain universality. For example, among the four MGs in total, MG_2has connections to three other MGs; $\mathrm{MG}_1$and$\mathrm{MG}_4$are both connected with two MGs; MG_3 is only connected to MG_2. By analogy, let us consider a generalized case in which the total number of MGs is n. Analogy to the scenario considered in Fig. 2 for which we have$n = 4$, we have certain MGs with $n-1, n-2$ororonly one connections to the other MGs. Such typical connection mode can be regarded and summarized as a generalized version of multi-MGs connection. Whenever there are finitely more MGs interconnected, we can always refer to the example in this subsectionandobtaina corresponding system model in the same form with (6).

III. STOCHASTICH_∞ CONTROL PROBLEM FORMULATION AND SOLUTION

To achieve the frequency regulation in for multiple MGs as well as to make full use of the renewable DERs, a coordinated control scheme is required. In this section, the frequency regulation target is formulated as a stochastic H_∞ control problem, and solved with related stochastic control theory and convex optimization approaches.

Firstly, we define the stochastic H_∞ performance problem and stochastic H_∞ cost functionalas follows.

Definition 1: [18]*Given a scalar* $\gamma > 0$, *the*H_∞ *performance for the frequency regulation problem is defined as* $||z(t)|| < \gamma ||v(t)||$ *.The norm* $\|\cdot\|$ *term is defined in (10), where* $\mathbb{E}$ *represents the mathematical expectation.*

$$\left\|z(t)\right\| \triangleq \left(\mathbb{E}\left\{\int_0^\infty |z(t)|^2 \mathrm{d}t\right\}\right)^{1/2}, \tag{10}$$

where scalar γ*is called disturbance attenuation. Based on the*H_∞ *performance above, the stochastic* H_∞ *cost functional is formulated in (11),*

$$J(u,v) \triangleq \mathbb{E}\left[\int_0^T (\gamma^2 v' v - z' z)\mathrm{d}t\right]. \tag{11}$$

The stochastic H_∞ control problem is to find a controller u^* for system (6), such that for all nonzero disturbance $v(t) \in L_2[0,\infty)$, $J(u^*, v) \le 0$ holds. Using the techniques in [19], we obtain the following theorem.

Theorem 1.[18] *Given a disturbance attenuation* $\gamma > 0$, *the stochastic dynamic system (6) is said to satisfy the* H_∞ *performanceif there exist matrix* Y *and symmetric matrix* X *with appropriate dimensions, such that* $X \ge 0$ *and the LMI (12) holds, where* $\Gamma = AX + XA' + BY + Y'B'$. *The robust state feedback controller in this case can be obtained with* $u^* = Kx, K = YX^{-1}$.

$$\begin{bmatrix} \Gamma & XD' & XR' & C \\ DX & -I & 0 & 0 \\ RX & 0 & -X & 0 \\ C' & 0 & 0 & -\gamma^2 I \end{bmatrix} \le 0. \tag{12}$$

Remark 1. Generally, the solution to the LMI (12) is not unique. A strong controller can regulate the frequency well. However, an over strong might damage MT and FC.To obtain a rational controller, we transform the stochastic H_∞ control problem into a semi-definite programming (SDP)problem formulated in (13), where $\|\cdot\|_\infty$ stands for the ∞-norm of a matrix. In this paper, the SDP problem (13) can be solved with the MATLAB® convex optimization toolbox CVX[19].

$$\begin{aligned} &\text{minimize} \quad \|\Upsilon\|_\infty \\ &\text{s.t.} \quad \begin{bmatrix} \Gamma & XD' & XR' & C \\ DX & -I & 0 & 0 \\ RX & 0 & -X & 0 \\ C' & 0 & 0 & -\gamma^2 I \end{bmatrix} \leq 0. \end{aligned} \tag{13}$$

IV. Numerical Simulation

Based on the multiple MGs shown in Fig. 1, severalnumerical simulation results are presented in this section to illustrate the effectiveness of the proposed stochastic H_∞ control scheme for frequency regulation in each MG.

In the time domain simulation, the initial value of state variable x in (6) is assigned to be a zero vector. The parameters for the modelin Fig. 1 are shown in Table I and Table II.

TABLE I

TIME CONSTANTS OFMG NETWORK MODEL

Parameter	Value	Parameter	Value
$T_{L_1}(s)$	1.3	$T_{L_2}(s)$	2.1
$T_{L_3}(s)$	1.8	$T_{FC_1}(s)$	1.1
$T_{MT_3}(s)$	1.3	$T_{BES_1}(s)$	0.14
$T_{BES_2}(s)$	0.16	$T_{BES_3}(s)$	0.12
$T_{WTG_1}(s)$	2.1	$T_{PV_2}(s)$	1.4
$T_{WTG_3}(s)$	1.6	$T_{ER_{12}}(s)$	0.06
$T_{ER_{32}}(s)$	0.07		

TABLE II

PARAMETERS OF MG NETWORK MODEL

Parameter	Value	Parameter	Value
r_{L_1}	0.9	r_{L_2}	0.8
r_{L_3}	0.7	r_{WTG_1}	0.6
r_{PV_2}	0.8	r_{WTG_3}	0.9
r_{BES_1}	1.2	r_{BES_2}	1.1
r_{BES_3}	1.4	b_{FC_1}	2.6
b_{MT_3}	3.1	$b_{ER_{12}}$	2.1
$b_{ER_{32}}$	2.2	M_1(pu/s)	0.2
D_1(pu/Hz)	0.012	M_2(pu/s)	0.19
D_2(pu/Hz)	0.010	M_3(pu/s)	0.22
D_3(pu/Hz)	0.020	γ	0.4

For illustrative purpose, we assume that the disturbance input results in a relative significant fluctuation in the local load power of MG_2, andthe corresponding load power dynamics in MG_1, MG_2 and MG_3 are illustrated in Fig. 3. Considering the SDP problem in Remark 1, we plot the the AC bus frequency deviations in MG_1, MG_2 and MG_3 under the desired H_∞ control scheme and without control in Fig. 4. The frequency deviations of the AC bus without control are denoted as Δf_1^-, Δf_2^- and Δf_3^-, whereas Δf_1^*, Δf_2^* and Δf_3^* are used to represent the frequency deviations under the H_∞ control.

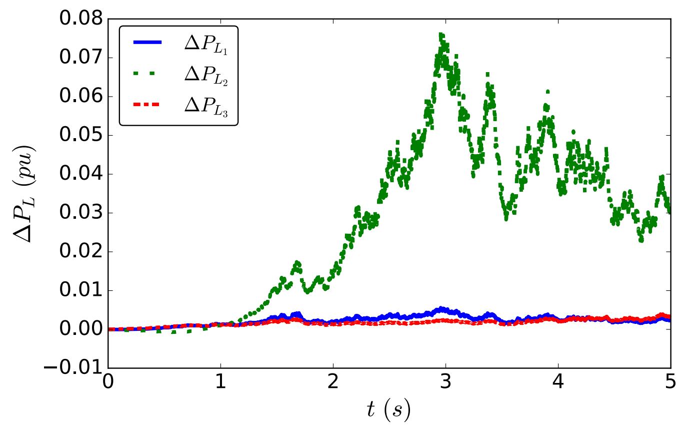

Fig. 3. Local load power dynamics in different MGs.

From Fig. 4, we see that the frequency is distinctly affected by the power deviations in MG system without control. However, when the H_∞ controller is performed,the frequency deviation of each MG is strictly limited to a small range, indicating the notable effectiveness of the proposed stochasticH_∞ control scheme.

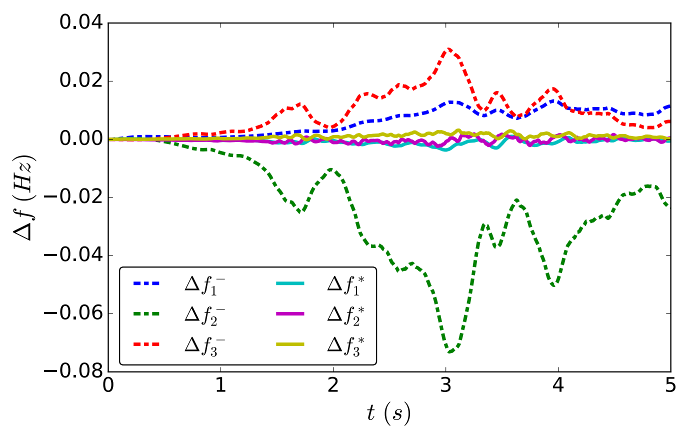

Fig. 4. Frequency deviations in different MGs inwith/without control.

Additionally, we show the proposed control scheme effectively stabilizes the frequencies without over-control in Fig. 5.During the simulation period, the power deviations

ΔP_{FC_1},ΔP_{MT_3},$\Delta P_{ER_{12}}$ and $\Delta P_{ER_{32}}$ fluctuates within a certain range without going towards infinitely large.

Noticing that MG_2 in Fig. 1 is composed of only one PV unit, one BES system and its local load. The frequency regulation of MG_2 can only rely on the energy transmitted from MG_1 and MG_3, which is illustrated in Fig. 5 as both values of $\Delta P_{ER_{12}}$ and $\Delta P_{ER_{32}}$ are positive. In this sense, the abundant energy in one MG could be consumed in other MGs instead of being stored in the BES system, which is conducive to improving energy efficiency.

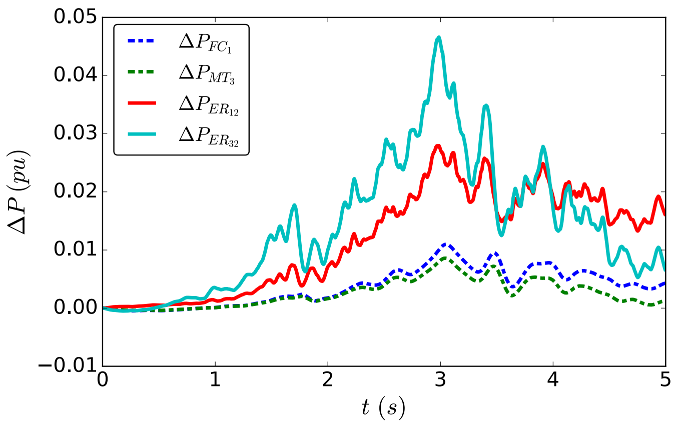

Fig. 5. Power deviations of the controlled devices under H_∞ control.

The simulation results provided above illustrate the usefulness of our proposed stochastic robust control method for the coordinated frequency regulation for multiple MGs. The stochastic process introduced in the dynamic system (6) makes our result more realistic.

V. Conclusion

In this paper the coordinated frequency regulation for multiple AC MGs is investigated. Multiple MGs are designed to be interconnected with ERs. Both ODEs and SDEs are used to model the dynamics of multi-MGs, including the power change of PVs, WTGs, MTs, FCs, BES systems, local loads, ERs and AC bus frequency deviations.We formulate the problem of frequency regulation formultiple MGs as a stochastic H_∞ control problem. The problem is solved with stochastic control theory and related optimization methods. An appropriate analytical solution to the problem is obtained, and typical numerical simulation results are presentedto show the effectiveness of the proposed control scheme.

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References

[1] D. Olivares, A. Mehrizi-Sani, A. Etemadi, C. Canizares, R. Iravani, M. Kazerani, *et al.*,"Trends in microgrid control", *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1905–1919, May 2014.

[2] G.Venkataramanan and C. Marnay, "A larger role for microgrids,"*IEEEPower Energy Mag.*, vol. 6, no. 3, pp. 78–82, May 2008.

[3] A. Chaouachi, R. Kamel, R. Andoulsi, and K. Nagasaka, "Multiobjectiveintelligent energy management for a microgrid,"*IEEE Trans. Ind. Electron.*, vol. 60, no. 4, pp. 1688–1699, Apr. 2013.

[4] J. Pascual,J. Barricarte,P. Sanchis,and L. Marroyo,"Energy managementstrategy for a renewable-based residential microgrid with generation anddemand forecasting," *Appl. Energy*, vol. 158, no. 11, pp. 12–25, Nov. 2015

[5] F. Cingoz,A. Elrayyah, and Y. Sozer, "Optimized Settings of Droop Parameters Using Stochastic Load Modeling for Effective DC Microgrids Operation", *IEEE Trans. Ind. Appl.*, vol. 53, no. 2, pp. 1358–1371, Mar./Apr. 2017.

[6] H. Bevrani, *Robust Power System Frequency Control*, 2nd ed.Gewerbestrasse, Switzerland: Springer, 2014.

[7] H. Bevrani, M. R. Feizi, and S. Ataee, "Robust frequency control in an islanded microgrid:$H_\infty$and μ-Synthesis Approaches," *IEEE Trans. Smart Grid*, vol. 7, no. 2, pp. 706–717, Mar. 2016.

[8] P. Kou, D. Liang, and L. Gao, "Distributed EMPC of multiple microgrids for coordinated stochasticenergy management,"*Appl. Energy*, vol. 185, pp. 939–952, Jan. 2017.

[9] J.Cao and M.Yang, "Energy Internet - towards smart grid 2.0," Fourth Int.Conf. Networking & Distributed Computing, vol. 31,no. 3, pp. 105–110, 2013.

[10] X. Fang, Q. Yang, J. Wang, and W.Yan,"Coordinated dispatch in multiple cooperative autonomous islanded microgrids,"*Appl. Energy*, vol. 162 no. 1, pp. 40–48, Jan. 2016.

[11] V. P. Singh, S. R. Mohanty, N. Kishor, and P. K. Ray, "Robust *H*-infinity load frequency control in hybrid distributed generation system," *Int. J. Elect. Power Energy Syst.*, vol. 46, pp. 294–305,Mar. 2013.

[12] H. Hua, Y. Qin, and J. Cao, "A class of optimal and robust controller design for islanded microgrid, " 2017 IEEE 7th Int. Conf. on Power and Energy Systems, Toronto, Canada, Nov. 2017, pp. 111–116.

[13] J.G.Vlachogiannis, "Probabilistic constrained load flow considering integration of wind power generation and electric vehicles,"*IEEE Trans. PowerSyst.*, vol. 24, no. 4, pp. 1808–1817, Sept. 2009.

[14] Y. Xu, J. Zhang, W. Wang, A. Juneja, and S. Bhattacharya, "Energy router: architectures and functionalities toward energy internet," *Proc. 2011 IEEE Int. Conf. on Smart Grid Communitions*, pp. 31–36. Oct. 2011.

[15] J. Zhang, W. Wang, and S. Bhattacharya, "Architecture of solid state transformer-based energy router and models of energy traffic," *Proc. 2012 IEEE PES Innovative Smart Grid Technologies*, pp. 1–8. Jan. 2012.

[16] M.Geidl, G.Koeppel, P.Favre-Perrod, andB. Klockl,"Energy hubs for the future,"*IEEEPower Energy Mag.*, vol. 5, no. 1, pp. 24–30, Jan./Feb.2007.

[17] Y. Zhang, T. Zhang, R. Wang, Y. Liu, and B. Guo, "An innovative real-time price based distributed optimal energy management ofmulti-microgrids in a smart distribution system," 2016 IEEE Innovative Smart Grid Technologies – Asia, Melbourne, Australia, Nov./Dec.pp. 341–346, 2016.

[18] S. Xu and T. Chen, "Robust H_∞ control for uncertain stochastic systemswith state delay," *IEEE Trans. Automat. Contr.*, vol. 47, no. 12, pp. 2089–2094, Dec. 2002.

[19] M. Grant and S. Boyd, "Graph implementations for nonsmooth convex programs," *Recent Advances in Learning and Control*, Springer-Verlag Limited, pp. 95–110, 2008.