

Robust Frequency Control for Green Energy Powered Edge Computing System

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Abstract—Nowadays, in the face of the big data era and more and more task computing requirements, edge computing as a computing paradigm that offloads cloud computing and storage resources to the network edge side relieves the pressure of network bandwidth. With a large number of edge computing devices deployed to the network edge, it is an urgent problem to provide energy for these devices in an environmentally friendly and reliable way. In order to solve this problem, a feasible way is to replace part of the traditional energy with renewable energy, which has been widely studied. However, the stochastic characteristics of renewable energy and computing tasks and the delay of task scheduling normally have a negative impact on the power supply-demand balance in edge nodes. In this paper, we study the robust frequency control problem in the edge computing node powered by renewable energy considering task scheduling. Therefore, a stochastic differential equation with time delay is used to model the dynamics of task computing, energy supply and task scheduling of edge computing system, and the robust stability of AC bus frequency deviation of edge nodes is discussed. The linear matrix inequality (LMI) method is used to solve such control problem. Finally, numerical simulations show the effectiveness of the proposed method.

Index Terms—edge computing, robust stability, stochastic differential equation, task scheduling

I. INTRODUCTION

With the advent of the Internet era and the continuous development of wireless network, the number of devices on the edge of the network and the data generated are growing rapidly [1]. According to IDC, the international data company, the global data volume is expected to increase tenfold to 163ZB by 2025 compared with 16.1ZB in 2016 [2]. The cloud computing transfers all the data to the cloud server through the network

and solves the computing and storage problems by using the powerful computing and storage capacity of the cloud server [3]. However, in the context of the Internet of things, the traditional cloud computing cannot continue to effectively deal with such huge amount of data and tasks [4]. For example, transmitting all the data to the cloud computing center through the network may lead to insufficient network bandwidth, resulting in high service delay and low user experience.

In order to address the above challenges, edge computing (EC) has been widely concerned by both academia and industry in recent years. EC is a new computing paradigm, which can offload part or all of cloud resources to the edge side closer to users [5]. By computing tasks directly in the local area close to the user side, the transmission delay and bandwidth pressure of network are effectively alleviated, thus improving the user experience and quality of service [6]. At present, there exist a large number of research outputs on EC, focusing on improving the security, privacy and delay optimization of EC system [7], [8]. Considering that a large number of edge devices are installed in different locations in a geographically dispersed manner, the related energy consumption and supply has been a challenging issue.

Although EC can improve the response speed of task processing, it could lead to a sharp increase in energy consumption of edge nodes [9]. Because the tasks of edge nodes are diversified in different time and location, the energy consumption of EC system is uneven and difficult to predict [10]. Moreover, the world is facing the problems of energy shortage and environmental pollution, which could be aggravated by the huge energy consumption of EC system. Therefore, some research outputs proposed to rely on the geographical distribution of edge nodes to utilize different renewable energy around [15].

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Many studies have already mentioned the advantages of utilizing renewable energy in EC to reduce the traditional energy consumption; see, e.g., [11], [15]. In [11], the energy supply problem of microgrid for edge system is transformed into a mixed integer nonlinear optimization problem. Task scheduling between nodes can make full use of renewable energy and computing resources of different edge nodes, thus the internal power balance of nodes can be properly adjusted. The authors in [15] proposed to maintain the stability of renewable energy powered EC system by optimizing the power storage management strategy.

Considering the dynamic and stochastic nature of EC system, the authors in [13] proposed to use ordinary differential equations and stochastic differential equations to model the computing resource allocation of edge nodes. However, the existing research does not use stochastic differential equations to model the task scheduling and energy supply in edge computing systems. Besides, it is worth noting that although there exist many research outputs (e.g., [16], [17]) on the robust control of AC bus frequency in energy systems, literature on this kind of problem within the edge nodes can be hardly found.

Different from the methods proposed in [11], in order to better describe the dynamic and stochasticity of edge nodes and energy supply equipment, e.g., computing energy consumption of edge devices, computing task scheduling between different nodes and power of electricity generation equipment, both stochastic and ordinary differential equations are proposed to model the system in this paper. The investigated system includes several edge nodes, and each node includes EC devices, renewable energy generator, traditional power generator and energy storage equipment, so as to ensure the smooth progress of the task processing. The stochastic differential equation with time delay has been used to formulate the system model, and the robust frequency regulation problem is transformed into a stochastic H_∞ control problem in this paper.

The main contributions of this paper are outlined as follows:

- 1) In order to better describe the dynamic nature of the considered EC system, both stochastic and ordinary differential equations have been used to model the dynamical process of task scheduling between edge nodes, energy supply and energy storage in edge nodes. Notably, the dynamical task scheduling between edge nodes is modeled by differential equation for the very first time.
- 2) It is highlighted that, in order to achieve robust frequency stability, EC system stochasticity, transmission delay and external disturbance has been considered *simultaneously*, which is novel. In this sense, the modeled dynamical system is closer to the actual EC system in industry.
- 3) The problem of AC bus frequency regulation in each edge node is appropriately formulated as a robust H_∞ control problem and can be solved by existing mathematical techniques effectively, which can be viewed as an advantage of our proposed method.

The rest of the paper is organized as follows. Section II describes the system architecture and the modeling of

task scheduling, task computing, energy power components. Section III formulates the robust control problem and provides the solutions. In Section IV, some numerical simulations are provided. Finally, the conclusion is presented in Section V.

II. SYSTEM MODELING

In this section, we first introduce the system architecture. And the power models of EC devices, renewable energy generator, traditional energy generator and energy storage equipment are formulated, by which we obtain the mathematical model of the considered EC system.

A. Description of System Architecture

In this paper, it is assumed that there are n EC nodes, each EC node is connected with different traditional power generation devices, battery energy storage devices and renewable energy power generation equipment.

In order to better describe the task scheduling between different edge nodes, this paper describes the EC system by an undirected graph denoted as $G(V, \xi)$, where V denotes n edge nodes, ξ is the edges set of the undirected graph. Edge $e_l = (i, j) = (j, i)$ indicates that computing task can be scheduled between the i -th edge node ED_i and the j -th edge node ED_j through the l -th communication link. For example, if the amount of tasks of an edge node is too much, the local energy will be insufficient to make up for its consumption, then the edge node can schedule part of the computing tasks to other edge nodes. It is worth noting that considering the geographical location, time delay and other factors, there is not always communication between any two edge nodes in this paper.

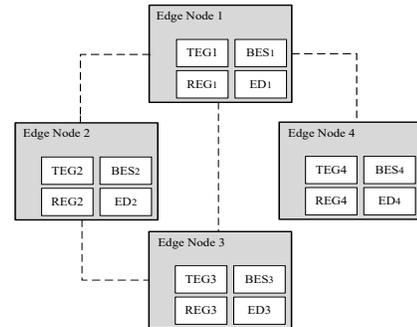


Fig. 1. The scenario of a typical EC system.

A typical EC system with four edge nodes concerned in this paper is shown in Fig. 1, in which TEG_i , BES_i , REG_i and ED_i represent the traditional energy generator, battery energy storage, renewable energy generator and edge device in the i -th edge node, respectively. The dotted line indicates that there is task scheduling between the two edge nodes.

B. Modeling for Task Scheduling

Task and resource scheduling in EC system is a dynamical process. In order to better represent such dynamic property, ordinary differential equation has been used to model resource scheduling in [13]. In many research outputs, e.g.,

[19], task scheduling is regarded as a controllable process. When a certain edge node is overloaded, a part of the task can be scheduled to other edge nodes through the control signal. However, there is no such research output, in which differential equations with control inputs are used to model task scheduling.

In addition, due to the time required for network transmission, the time delay must be considered in EC task scheduling modeling. Therefore, an ordinary differential equation with time delay and control inputs is used to model task scheduling in this paper.

$$\Delta \dot{S}_{ts}^l(t) = \frac{1}{T_{ts}^l} \left(-\Delta S_{ts}^l(t - \tau(t)) + b_{ts}^l u_{ts}^l(t) \right), \quad (1)$$

where $\Delta S_{ts}^l(t)$ denotes the change in the amount of tasks scheduled on the l -th communication line, and $u_{ts}^l(t)$ is the control input. T_{ts}^l and $\tau(t)$ denote the time constant of task scheduling and task transmission delay, respectively. The system coefficient b_{ts}^l can be obtained by parameter estimation method.

C. Modeling for Local Task Computing

For a single EC node, considering that the change of the amount of task requests generated by the terminal devices around it could not be smooth, the Wiener process $w(t)$ has been considered to simulate the stochasticity of the task quantity change in this paper. In addition, considering that some disturbances may have adverse effects on the bus frequency deviation, such as suddenly receiving a large amount of task requests, the variable $v_e^k(t)$ is used to represent such disturbances input. In this paper, the amount of local computing tasks received by each EC node is modeled by a stochastic differential equation with disturbance input. The modeling of local task computing is as follows.

$$d\Delta S_e^k(t) = \frac{1}{T_e^k} \left(-\Delta S_e^k(t) + v_e^k(t) \right) dt + r_e^k \Delta S_e^k(t) dw(t), \quad (2)$$

where T_e^k is the time constant of task computing, and r_e^k is the system coefficient. $\Delta S_e^k(t)$ represents the change of the computing task to be performed by the k -th edge node.

D. Modeling for Photovoltaic Unit

Facing the problem of energy consumption of edge nodes, renewable energy devices, such as photovoltaic panels or wind turbines, are deployed around the edge device according to geographical conditions, which can effectively reduce the consumption of traditional energy [11]. In this paper, the photovoltaic unit (PV) is used as the renewable energy generator. Considering the stochasticity of PV power and the possible external interference, such as the change of light intensity, a stochastic differential equation with disturbance $v_{PV}^k(t)$ is

used to model the power change of PV. The power change of PV in the k -th edge node is modeled as follows [17].

$$d\Delta P_{PV}^k(t) = \frac{1}{T_{PV}^k} \left(-\Delta P_{PV}^k(t) + v_{PV}^k(t) \right) dt + r_{PV}^k \Delta P_{PV}^k(t) dw(t), \quad (3)$$

where T_{PV}^k is the time constant of PV, and r_{PV}^k is the coefficient of the diffusion term.

E. Modeling for Controllable Power Generator

Due to the uncontrollability and strong stochasticity of renewable energy, it is necessary to make proper control strategy with respect to conventional generators, such that the normal operation of EC devices can be guaranteed. Without loss of generality, in this paper the traditional energy generator refers to micro turbine (MT). $u_{MT}^k(t)$ denotes the control input of the traditional power generator. The power changes of MT in the k -th edge node is modeled as follows [17].

$$\Delta \dot{P}_{MT}^k(t) = \frac{1}{T_{MT}^k} \left(-\Delta P_{MT}^k(t) + b_{MT}^k u_{MT}^k(t) \right), \quad (4)$$

where T_{MT}^k is the time constant of MT, and b_{MT}^k is the system coefficient related with the controller.

F. Modeling for Battery Energy Storage and Frequency Deviation

In order to further maintain the balance between energy consumption and generation, BES is considered in the energy supply system. The battery energy storage in the k -th edge node is modeled as follows:

$$\Delta \dot{P}_{BES}^k(t) = \frac{1}{T_{BES}^k} \left(-\Delta P_{BES}^k(t) - \Delta f^k(t) \right), \quad (5)$$

where T_{BES}^k is the time constant of BES, and $P_{BES}^k(t)$ represents the power changes of battery energy storage in the k -th edge node. Since the research focus of this paper is not on the BES itself, the state of charge (SOC) is not considered. The similar modeling approach without considering SOC has been reported in, e.g., [12] and [18].

In (5), $\Delta f^k(t)$ is the AC bus frequency deviation within the k -th edge node, which can be modeled as follows [20].

$$\Delta \dot{f}^k(t) = -\frac{2\bar{D}^k}{\bar{M}^k} \Delta f^k(t) + \frac{2}{\bar{M}^k} \Delta P^k(t), \quad (6)$$

where \bar{M}^k stands for the inertia constants, and \bar{D}^k stands for the damping coefficients of the k -th edge node. In (6), $\Delta P^k(t)$ denotes the bus power of the k -th edge node, which is modeled as follows.

$$\Delta P^k(t) = -G \left(S_e^k(t) - \sum_{l=1}^m f(k, l) \cdot S_{ts}^l(t) \right) + P_{PV}^k(t) + P_{MT}^k(t) + P_{BES}^k(t), \quad (7)$$

where G is the proportional relationship between task changes and power consumption changes. $S_e^k(t) - \sum_{l=1}^m f(k, l) \cdot S_{ts}^l(t)$ denotes the changes of tasks that the edge node needs to execute after considering task scheduling. The function $f(k, l)$

represents the direction of task scheduling. When $f(k, l) = 1$, the k -th edge node schedules tasks to other nodes through the transmission line l . When $f(k, l) = -1$, the k -th edge node receives the tasks scheduled from other nodes through the transmission line l .

G. Modeling for System

To simplify the description of the system, we define the following vectors. For the k -th edge node, the system state vector is defined as $x^k(t) = [\Delta S_e^k(t), \Delta P_{PV}^k(t), \Delta P_{MT}^k(t), \Delta P_{BES}^k(t), \Delta f^k(t)]'$. The control input is defined as $u_{MT}^k(t)$. Disturbance input vector is $v^k(t) = [v_e^k(t), v_{PV}^k(t)]'$. Then, we define the state vector of task scheduling between edge nodes as $x_{ts}(t) = [\Delta S_{ts}^1(t), S_{ts}^2(t), \dots, S_{ts}^m(t)]'$. The control input vector of task scheduling is defined as $u_{ts}(t) = [u_{ts}^1(t), u_{ts}^2(t), \dots, u_{ts}^m(t)]'$, where m is the number of communication lines.

Then, for the whole EC system, we define the system state vector $x(t) = [x^1(t)', x^2(t)', \dots, x^n(t)', x_{ts}(t)']'$. The system control vector is $u(t) = [u_{MT}^1(t)', u_{MT}^2(t)', \dots, u_{MT}^n(t)', u_{ts}(t)']'$. The system controlled output vector is $z(t) = [\Delta f^1(t), \Delta f^2(t), \dots, \Delta f^n(t)]'$ where n means the total number of edge nodes. Finally, we transform the considered EC system into the following form:

$$\begin{cases} dx = (Ax + A_d x(t - \tau(t))) + Bu + Cv dt + RxdW(t) \\ z = Dx \end{cases} \quad (8)$$

From (1) to (8), we have transformed the EC system into a mathematical control system.

III. PROBLEM FORMULATION AND SOLUTION

In this section, a robust frequency control problem for edge nodes is formulated as a robust H_∞ control problem and is solved analytically.

Definition: Given a scalar $\gamma > 0$, the AC bus frequency regulation problem studied in this paper can be described by the H_∞ performance which is formulated as $\|z(t)\| < \gamma \|v(t)\|$, where $\|\cdot\|$ is defined as

$$\|z(t)\| \triangleq \left(\mathbb{E} \left\{ \int_0^\infty |z(t)|^2 dt \right\} \right)^{\frac{1}{2}}, \quad (9)$$

where γ denotes the disturbance attenuation. Then, the cost functional of the robust control problem is formulated in (10).

$$J(u, v) \triangleq \mathbb{E} \left[\int_0^T (\gamma^2 v'v - z'z) dt \right]. \quad (10)$$

In order to solve this stochastic robust control problem with time delay, we need to find a control u^* such that the cost function (10) is less than 0 for all non-zero disturbances.

Theorem 1. [?] Given a scalar $\gamma > 0$, if there exist symmetric matrices $X > 0$, $S > 0$ and matrix Y satisfying LMI (11), a control input u^* can be obtained to stabilize the stochastic robust control system with time delay, where $u^* = Kx$, $K = YX^{-1}$.

$$\begin{bmatrix} \Omega & A_d X & X D' & X R' & C \\ X A'_d & -(1-h)S & 0 & 0 & 0 \\ D X & 0 & -I & 0 & 0 \\ R X & 0 & 0 & -X & 0 \\ C' & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} \leq 0 \quad (11)$$

where $\Omega = AX + XA' + BY + Y'B' + S$.

The formulated control problem can be solved by the theorem in [14], so the detailed proof is omitted in this paper.

IV. NUMERICAL SIMULATION

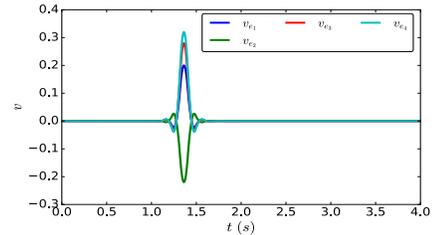
To verify the feasibility of the proposed method, numerical simulation results are provided in this section.

The detailed parameters, which can be measured by parameter estimation method [17], [18], are listed in the Table I. Two kinds of disturbances, illustrated in Fig. 2(a) and Fig. 3(a), have been used to test the feasibility of proposed control strategy. Without the implementation of any control strategy,

TABLE I
TIME CONSTANTS OF THE EC SYSTEM

Param.	Value	Param.	Value	Param.	Value
T_e^k	[1.4, 1.8]	T_{PV}^k	[1.1, 1.4]	T_{MT}^k	[1.2, 1.5]
T_{BES}^k	[0.12, 0.15]	T_{ts}^k	[0.11, 0.16]	r_e^k	[0.5, 0.9]
r_{PV}^k	[0.6, 0.9]	b_{MT}^k	[1.5, 2.4]	r_{BES}^k	[1.2, 1.5]
b_{ts}^k	[1.2, 1.5]	M^k	[0.18, 0.23]	D^k	[0.011, 0.016]
G	4	γ	0.35	h	0.7

the frequency deviation within the four edge nodes under the disturbance in Fig. 2(a) is shown in Fig. 2(b), whereas the frequency deviation within the four edge nodes under the proposed control strategy is shown in 2(c).]



(a) Disturbance inputs for the EC system.

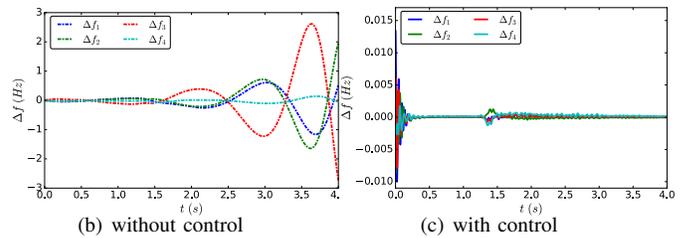


Fig. 2. Frequency deviations under the disturbance

By comparing Fig. 2(b) and 2(c), it can be seen that the frequency deviation within the four edge nodes is effectively stabilized under the proposed control strategy. It is notable that the disturbance at about 1.3 second has caused large frequency deviation when no controller has been implemented, which would probably cause system blackout, whereas the

same disturbance has only causes a small fluctuation under the proposed controller.

Comparing Fig. 3(b) and Fig. 3(c), it can be seen that the proposed method makes the frequency deviation stable under the continuous disturbance. Compared with Fig. 2(c), the control effect of frequency deviation in Fig. 3(c) is slightly worse due to the long-term negative impact of continuous disturbance.

In order to better show the influence of delay on system stability, let us set the delay to be a relatively large value. Fig. 4(a) shows the control effect when the disturbance is zero and the delay is set to be 600 ms. It can be seen that at 600 ms, the time delay causes the frequency deviation to fluctuate

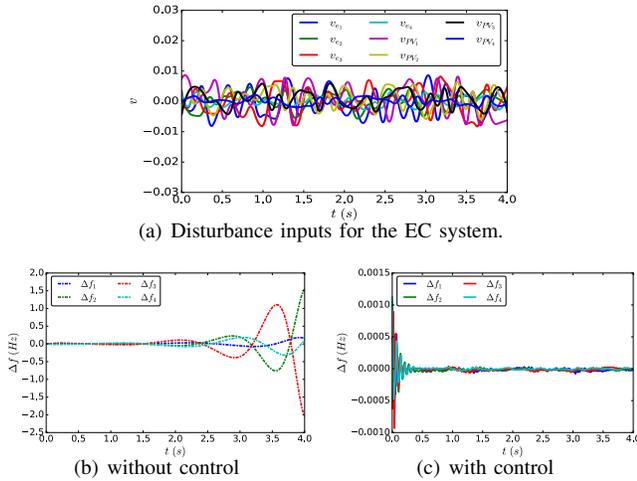


Fig. 3. Frequency deviations under the continuous disturbance temporarily. In Fig. 4(b), when the delay is set to be 900ms, the frequency deviation fluctuates slightly at that moment.

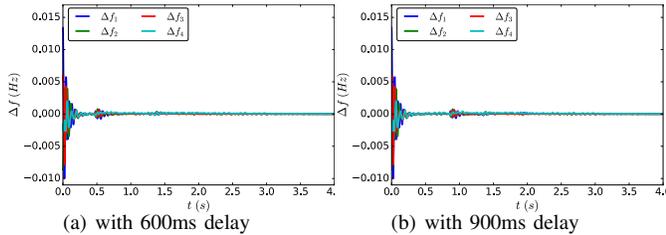


Fig. 4. Frequency deviations under control with different delay

V. CONCLUSION

In this paper, the bus frequency regulation of each edge node in EC system is discussed. Considering the dynamics of the system, the delay of task scheduling and external disturbance, a stochastic differential equation with time delay is used to model task scheduling, task processing and energy supply equipments. Finally, we transform the frequency regulation problem into a stochastic control problem, and the problem is solved by LMI approach. The effectiveness and feasibility of the proposed method are successfully verified by numerical simulations.

Since the energy scheduling in EC has not been fully considered in this work, our future research shall focus on the integration of information flow and energy flow in EC system.

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