A Preconditioned Newton-Krylov Method for Computing Stationary Pulse Solutions of Mode-locked Lasers

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Abstract: We solve the periodic boundary value problem for a mode-locked laser cavity using a preconditioned matrix-implicit Newton-Krylov solver. Solutions are obtained two to three orders of magnitude faster than with standard tools.

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1. Overview

The effective numerical solution of the steady state solution to a mode-locked laser is essential to the design, optimization, and study of such lasers [1], as well as coherent pulse addition in passive cavities. The standard method for tackling such problems is to simply simulate the operation of the cavity in question, generally with a split-step Fourier method, until convergence is reached at some precision [2]. While this has the advantage of demonstrating self-starting and solution stability, simulation is rather poor when viewed as a numerical algorithm, exhibiting slow linear convergence. For example, a solid-state laser operating in the dispersion-managed soliton regime can take many tens of thousand round trips to converge to within one part in 10^4. This is especially true of cavities with relatively weak nonlinearity (i.e. high Q resonators).

In contrast, our algorithm converges quadratically to the stable solution, requiring the evaluation of only tens of round trips to converge to within numerical precision, typically two to three orders of magnitude faster than with simulation. This speed up is achieved by directly solving the periodic boundary value problem for the nonlinear cavity using a Newton-Raphson method. At each step we use a matrix-implicit, preconditioned Krylov subspace solver to compute the approximate solution. This means we never explicitly compute the Jacobian of our cavity, but rather send a short series of trial perturbations through the cavity to compute a very good approximation to the solution of the linearized subproblem. The preconditioning is critical to both the quadratic convergence of the algorithm, as well as its efficiency in terms of round-trip evaluations. Our method enables many new opportunities for design and analysis of mode-locked lasers, such as putting the laser model inside an optimization loop.

2. Algorithm

In our method, the cavity is simply treated as an arbitrary n-dimensional discrete nonlinear function $\mathbf{g}(\mathbf{u})$. Internally, it can be modeled in any way that is convenient, and it is not actually necessary for the vector $\mathbf{u}$ to correspond to direct physical quantities. In the case we consider here, a dispersion-managed soliton laser, the elements of $\mathbf{u}$ are the Fourier coefficients of the field. We seek an “eigenvector” $\mathbf{u}$ and “eigenvalue” $e^{i\phi}$ such that $\mathbf{g}(\mathbf{u}) = e^{i\phi}\mathbf{u}$. We solve this by casting it as the multidimensional root-finding problem

$$\mathbf{f}(\mathbf{u}) = e^{-i\phi(\mathbf{u})}\mathbf{g}(\mathbf{u}) - \mathbf{u} = \mathbf{0}, \quad (1)$$

where $\phi(\mathbf{u})$ is defined so as to set the phase of the first element of $e^{-i\phi(\mathbf{u})}\mathbf{g}(\mathbf{u})$ to zero. While this is consistent with solving the original problem, it is not necessarily optimal for convergence and was purely chosen for simplicity. It has been found to empirically work quite well, however.

Given a guess $\mathbf{u}_k$, we perform a standard Newton iteration by linearizing (1) around $\mathbf{u}_k$ and solving for $\mathbf{u}_{k+1}$. However, there are several problems with doing so directly. First, if we were to compute the full Jacobian $\mathbf{J_f}$ numerically, it would require n evaluations of the round trip model $\mathbf{g}$, largely negating the efficacy of the algorithm. Second, it turns out that, in general, the Jacobian is badly conditioned and has a non-sparse eigenspectrum. Thus, direct solution methods will be numerically unstable, and iterative solution methods will converge slowly.

To address this, we precondition with a diagonal matrix $\mathbf{B} = (\mathbf{D_g}(\mathbf{u}) - \mathbf{I})^{-1}$, where $\mathbf{D_g}(\mathbf{u})$ contains the sum of all diagonal terms in the model. For example, if $\mathbf{u}$ represents Fourier coefficients of our cavity field, then $\mathbf{D}$ will contain the gain and loss spectra, as well as dispersion phases. The better it estimates the actual Jacobian, the quicker the linear subproblems will converge. Thus, it pays to choose a basis where $\mathbf{J_g}$ is as diagonal as possible. In the case of our dispersion-managed soliton laser, the Fourier domain is optimal.

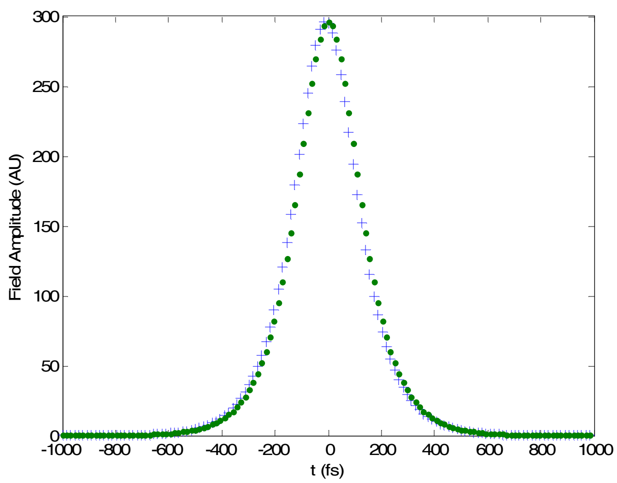

Fig. 2: Comparison of solution found by our method (crosses) and that found by laser dynamics simulation (dots).

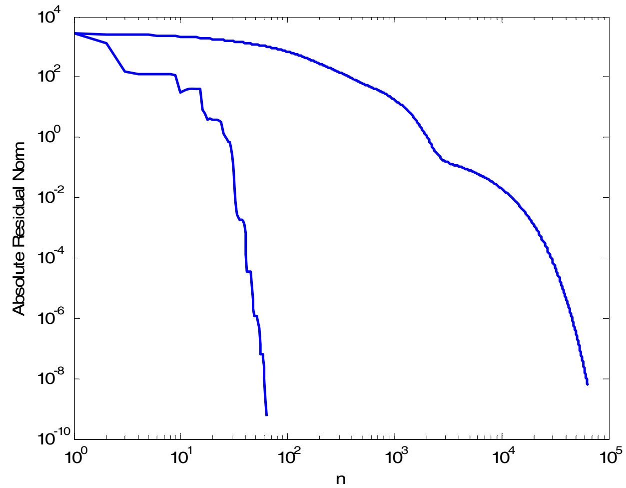

Fig. 1: Log-log convergence plot of our direct method (left) compared to standard simulation (right) as a function of roundtrip evaluation. The same noise initial conditions were used for both.

The linear subproblem $\mathbf{B}(\mathbf{J_g} - \mathbf{I})\Delta\mathbf{u}_k = \mathbf{Bf}(\mathbf{u}_k)$ generally becomes better conditioned by roughly an order of magnitude, and perhaps most importantly, it greatly simplifies the eigenspectrum of the system, allowing each subproblem to converge in only a few round trips. To solve the linear subsystem, we use a generalized minimal residual method (GMRES), a conjugate gradient-like method that can handle asymmetric systems [3]. A common feature of Krylov subspace solvers is that they only need information about how the matrix in question operates on a series of vectors. Thus, we never need to explicitly compute the Jacobian, but simply approximate its action on a given trial vector using first-order finite differences.

3. Numerical Results

We tested our algorithm on a dispersion-managed soliton laser model producing roughly 100 fs pulses, shown schematically in Fig. 3, below. This example was chosen due to the relatively slow evolution. A comparison of the pulse found by our method and that found by simulation is shown in Fig. 1. While there is a small, meaningless time shift between the two solutions, they are otherwise identical. Starting from noise, our method needed nine Newton steps to converge to an absolute error of 10^{-8}, with each step taking 6 GMRES iterations (7 round trip evaluations). Thus, a total of only 64 cavity round trip evaluations were needed. In comparison, over 70,000 round trips were required to converge to the same accuracy by simply simulating the laser dynamics (see Fig. 2).

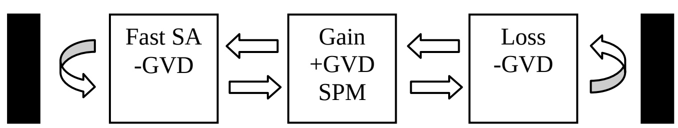

Fig. 3: A schematic of the laser cavity used as our model problem.

4. Applications and Future Work

There are several interesting avenues to pursue. One is the optimization of laser components (such as dispersion compensating mirrors) by directly minimizing the simulated pulse width. In addition, there are many theoretical studies which are enabled by the ability to rapidly compute a series of perturbations, such as an exploration of the relation between pulse energy and carrier envelope phase slip. Finally, the use of reduced basis sets (i.e. parameter vectors **u** which are much smaller in dimension than the underlying simulation) could allow for significant further speed gains with little sacrifice in accuracy. We hope this could allow the algorithm to be eventually applied towards a full spatio-temporal model of a laser cavity, facilitating the quantitative study of Kerr lens mode locking.

References

[1] F. Kärtner, *Few-cycle Laser Pulse Generation and its Applications*, Springer (2004).
[2] G. Agrawal, *Nonlinear Fiber Optics*, Academic Press (2001).
[3] Y. Saad, and M Schultz, “GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems.” SIAM J. Sci. Statist. Comput. **7**, 856-869 (1986).