

Conjugate Gradient for Domain Wall Fermions with 4-d EO preconditioning

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Abstract

This document presents an implementation of a conjugate gradient solver for the Domain Wall Fermion Dirac operator. The code targets SciDAC machines that implement QMP protocol.

This version is suboptimal in overlapping communication and computation because of deficiencies of the QMP implementation.

1 `<Version 1>≡`
`static const char *version = "Version 1.3.0";`

This code is used in chunk [14b](#).

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6 CHUNKS

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1 INTRODUCTION

The code below interfaces with a Chroma-like upper level environment to provide file access and machine initialization and configuration. In fact, this file is an implementation of a level 3 routine for solving the Dirac equation. There are some restrictions on input parameters imposed by the algorithm and a particular way the vector hardware is used by the implementation. There are the following restrictions on the lattice geometry:

- All four-dimensional extends of the lattice should be even. This is required for even-odd decomposition used in the preconditioner.
- The fifth-dimension extend should be a multiple of 4 or 2 depending on the implementation. It is needed for efficient use of vector registers and simplification of vector code.
- The implementation supports up to four dimensional tori as a network topology.

Because of many issues involved in optimizing the code, it is advantageous to put together some definitions and outline here the optimization strategy used.

1.1 Definitions

Lattice extend is the total size of the lattice in a given dimension.

Network is the logical topology of the network presented by QMP to the application.

Node is a computing element in the network which runs an execution thread. For this implementation we assume that there is one compute node per network location. If an SMP is used, it is the responsibility of QMP to provide a proper abstraction to the application.

Sublattice is the part of the lattice that resides on a compute node.

Site is a point on the lattice.

1.2 Optimization Strategy

For this code we assume that scarcity of resources makes us run the inverter on a small number of nodes compared to the number of sites. This is based on the observation that physics needs grow faster than SciDAC budget and computer deployment plans. We also assume, that the current trend in computer industry persists, namely, that the processors grow faster while memory speed and latency continues to lag in relative terms. We also want a solver whose performance would degrade gracefully when one moves out of the optimization domain. In particular, we impose no limitation on the size of sublattice. There is even no requirement that all sublattices should be of the same size.

For the optimization sweetspot, we assume that the typical problem is too large to fit into the cache hierarchy and mostly resides in main memory. This is true now for existing and proposed clusters and is like to remain true for the future, since large scale computations tend to use larger lattices most of the time.

2 PHYSICS

Here we give the fermion action and γ -matrix and other conventions.

2.1 Dirac Operator

The Domain Wall Fermion Dirac operator is

$$\begin{aligned}\chi_{s,x} = D\psi &= M_0\psi_{s,x} + \sum_{\mu} \left((1 + \gamma_{\mu})U_{x,\mu}\psi_{s,x+\hat{\mu}} + (1 - \gamma_{\mu})U_{x-\hat{\mu},\mu}^{\dagger}\psi_{s,x-\hat{\mu}} \right) \\ &+ (1 + \gamma_5)M_s^{(+)}\psi_{s+1,x} + (1 - \gamma_5)M_s^{(-)}\psi_{s-1,x}\end{aligned}$$

where

$$M_s^{(+)} = \begin{cases} 1, & \text{if } s < N_s - 1 \\ -m_f, & \text{if } s = N_s - 1 \end{cases}$$

and

$$M_s^{(-)} = \begin{cases} 1, & \text{if } s > 0 \\ -m_f, & \text{if } s = 0 \end{cases}$$

We also assume that $\psi_{N_s,x} = \psi_{0,x}$ and $\psi_{-1,x} = \psi_{N_s-1,x}$.

2.2 Gamma matrices

We use the same γ -matrix basis as Chroma to simplify conversion between two codes. The choice below could be changed with a few modifications to the rest of the code, if γ_5 is kept diagonal, and one of other γ -matrices has all nonzero entries equal to +1.

$$\gamma_0 = -\sigma_2 \otimes \sigma_1 = \begin{pmatrix} 0 & i\sigma_1 \\ -i\sigma_1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

First, the projector and the reconstructor for $(1 + \gamma_0)$:

6a $\langle \text{Build } (1 + \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 6a} \rangle \equiv$
`g->f[0][c].re = f->f[0][c].re - f->f[3][c].im;
g->f[0][c].im = f->f[0][c].im + f->f[3][c].re;
g->f[1][c].re = f->f[1][c].re - f->f[2][c].im;
g->f[1][c].im = f->f[1][c].im + f->f[2][c].re;`

This code is used in chunks 67b, 69d, 70a, 72c, and 73a.

6b $\langle \text{Unproject and accumulate } (1 + \gamma_0) \text{ link 6b} \rangle \equiv$
`qs->f[0][c].re += hh[k].f[0][c].re; qs->f[3][c].im -= hh[k].f[0][c].re;
qs->f[0][c].im += hh[k].f[0][c].im; qs->f[3][c].re += hh[k].f[0][c].im;
qs->f[1][c].re += hh[k].f[1][c].re; qs->f[2][c].im -= hh[k].f[1][c].re;
qs->f[1][c].im += hh[k].f[1][c].im; qs->f[2][c].re += hh[k].f[1][c].im;`

This code is used in chunks 70b, 73b, and 75b.

Now, same for $(1 - \gamma_0)$:

6c $\langle \text{Build } (1 - \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 6c} \rangle \equiv$
`g->f[0][c].re = f->f[0][c].re + f->f[3][c].im;
g->f[0][c].im = f->f[0][c].im - f->f[3][c].re;
g->f[1][c].re = f->f[1][c].re + f->f[2][c].im;
g->f[1][c].im = f->f[1][c].im - f->f[2][c].re;`

This code is used in chunks 67c, 69d, 70a, 72c, and 73a.

6d $\langle \text{Unproject and accumulate } (1 - \gamma_0) \text{ link 6d} \rangle \equiv$
`qs->f[0][c].re += hh[k].f[0][c].re; qs->f[3][c].im += hh[k].f[0][c].re;
qs->f[0][c].im += hh[k].f[0][c].im; qs->f[3][c].re -= hh[k].f[0][c].im;
qs->f[1][c].re += hh[k].f[1][c].re; qs->f[2][c].im += hh[k].f[1][c].re;
qs->f[1][c].im += hh[k].f[1][c].im; qs->f[2][c].re -= hh[k].f[1][c].im;`

This code is used in chunks 70b, 73b, and 75b.

$$\gamma_1 = \sigma_2 \otimes \sigma_2 = \begin{pmatrix} 0 & -i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

First, the projector and the reconstructor for $(1 + \gamma_1)$:

6e $\langle \text{Build } (1 + \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 6e} \rangle \equiv$
`g->f[0][c].re = f->f[0][c].re - f->f[3][c].re;
g->f[0][c].im = f->f[0][c].im - f->f[3][c].im;
g->f[1][c].re = f->f[1][c].re + f->f[2][c].re;
g->f[1][c].im = f->f[1][c].im + f->f[2][c].im;`

This code is used in chunks 67d, 69d, 70a, 72c, and 73a.

6f $\langle \text{Unproject and accumulate } (1 + \gamma_1) \text{ link 6f} \rangle \equiv$
`qs->f[0][c].re += hh[k].f[0][c].re; qs->f[3][c].re -= hh[k].f[0][c].re;
qs->f[0][c].im += hh[k].f[0][c].im; qs->f[3][c].im -= hh[k].f[0][c].im;
qs->f[1][c].re += hh[k].f[1][c].re; qs->f[2][c].re += hh[k].f[1][c].re;
qs->f[1][c].im += hh[k].f[1][c].im; qs->f[2][c].im += hh[k].f[1][c].im;`

This code is used in chunks 70b, 73b, and 75b.

Now, same for $(1 - \gamma_1)$:

6g $\langle \text{Build } (1 - \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 6g} \rangle \equiv$
`g->f[0][c].re = f->f[0][c].re + f->f[3][c].re;
g->f[0][c].im = f->f[0][c].im + f->f[3][c].im;
g->f[1][c].re = f->f[1][c].re - f->f[2][c].re;
g->f[1][c].im = f->f[1][c].im - f->f[2][c].im;`

This code is used in chunks 67e, 69d, 70a, 72c, and 73a.

7 $\langle \text{Unproject and accumulate } (1 - \gamma_1) \text{ link 7} \rangle \equiv$

```

qs->f[0][c].re += hh[k].f[0][c].re; qs->f[3][c].re += hh[k].f[0][c].re;
qs->f[0][c].im += hh[k].f[0][c].im; qs->f[3][c].im += hh[k].f[0][c].im;
qs->f[1][c].re += hh[k].f[1][c].re; qs->f[2][c].re -= hh[k].f[1][c].re;
qs->f[1][c].im += hh[k].f[1][c].im; qs->f[2][c].im -= hh[k].f[1][c].im;

```

This code is used in chunks [70b](#), [73b](#), and [75b](#).

$$\gamma_2 = -\sigma_2 \otimes \sigma_3 = \begin{pmatrix} 0 & i\sigma_3 \\ -i\sigma_3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

First, the projector and the reconstructor for $(1 + \gamma_2)$:

8a $\langle \text{Build } (1 + \gamma_2) \text{ projection of } *f \text{ in } *g \text{ 8a} \rangle \equiv$
`g->f[0][c].re = f->f[0][c].re - f->f[2][c].im;
g->f[0][c].im = f->f[0][c].im + f->f[2][c].re;
g->f[1][c].re = f->f[1][c].re + f->f[3][c].im;
g->f[1][c].im = f->f[1][c].im - f->f[3][c].re;`

This code is used in chunks 68–70, 72c, and 73a.

8b $\langle \text{Unproject and accumulate } (1 + \gamma_2) \text{ link 8b} \rangle \equiv$
`qs->f[0][c].re += hh[k].f[0][c].re; qs->f[2][c].im -= hh[k].f[0][c].re;
qs->f[0][c].im += hh[k].f[0][c].im; qs->f[2][c].re += hh[k].f[0][c].im;
qs->f[1][c].re += hh[k].f[1][c].re; qs->f[3][c].im += hh[k].f[1][c].re;
qs->f[1][c].im += hh[k].f[1][c].im; qs->f[3][c].re -= hh[k].f[1][c].im;`

This code is used in chunks 70b, 73b, and 75b.

Now, same for $(1 - \gamma_2)$:

8c $\langle \text{Build } (1 - \gamma_2) \text{ projection of } *f \text{ in } *g \text{ 8c} \rangle \equiv$
`g->f[0][c].re = f->f[0][c].re + f->f[2][c].im;
g->f[0][c].im = f->f[0][c].im - f->f[2][c].re;
g->f[1][c].re = f->f[1][c].re - f->f[3][c].im;
g->f[1][c].im = f->f[1][c].im + f->f[3][c].re;`

This code is used in chunks 68–70, 72c, and 73a.

8d $\langle \text{Unproject and accumulate } (1 - \gamma_2) \text{ link 8d} \rangle \equiv$
`qs->f[0][c].re += hh[k].f[0][c].re; qs->f[2][c].im += hh[k].f[0][c].re;
qs->f[0][c].im += hh[k].f[0][c].im; qs->f[2][c].re -= hh[k].f[0][c].im;
qs->f[1][c].re += hh[k].f[1][c].re; qs->f[3][c].im -= hh[k].f[1][c].re;
qs->f[1][c].im += hh[k].f[1][c].im; qs->f[3][c].re += hh[k].f[1][c].im;`

This code is used in chunks 70b, 73b, and 75b.

$$\gamma_3 = \sigma_1 \otimes 1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

First, the projector and the reconstructor for $(1 + \gamma_3)$:

8e $\langle \text{Build } (1 + \gamma_3) \text{ projection of } *f \text{ in } *g \text{ 8e} \rangle \equiv$
`g->f[0][c].re = f->f[0][c].re + f->f[2][c].re;
g->f[0][c].im = f->f[0][c].im + f->f[2][c].im;
g->f[1][c].re = f->f[1][c].re + f->f[3][c].re;
g->f[1][c].im = f->f[1][c].im + f->f[3][c].im;`

This code is used in chunks 68–70, 72c, and 73a.

8f $\langle \text{Unproject } (1 + \gamma_3) \text{ link 8f} \rangle \equiv$
`qs->f[0][c].re = hh[k].f[0][c].re; qs->f[2][c].re = hh[k].f[0][c].re;
qs->f[0][c].im = hh[k].f[0][c].im; qs->f[2][c].im = hh[k].f[0][c].im;
qs->f[1][c].re = hh[k].f[1][c].re; qs->f[3][c].re = hh[k].f[1][c].re;
qs->f[1][c].im = hh[k].f[1][c].im; qs->f[3][c].im = hh[k].f[1][c].im;`

This code is used in chunks 70b, 73b, and 75b.

Now, same for $(1 - \gamma_3)$:

9a $\langle \text{Build } (1 - \gamma_3) \text{ projection of } *f \text{ in } *g \text{ 9a} \rangle \equiv$
`g->f[0][c].re = f->f[0][c].re - f->f[2][c].re;
g->f[0][c].im = f->f[0][c].im - f->f[2][c].im;
g->f[1][c].re = f->f[1][c].re - f->f[3][c].re;
g->f[1][c].im = f->f[1][c].im - f->f[3][c].im;`

This code is used in chunks 68–70, 72c, and 73a.

9b $\langle \text{Unproject and accumulate } (1 - \gamma_3) \text{ link 9b} \rangle \equiv$
`qs->f[0][c].re += hh[k].f[0][c].re; qs->f[2][c].re -= hh[k].f[0][c].re;
qs->f[0][c].im += hh[k].f[0][c].im; qs->f[2][c].im -= hh[k].f[0][c].im;
qs->f[1][c].re += hh[k].f[1][c].re; qs->f[3][c].re -= hh[k].f[1][c].re;
qs->f[1][c].im += hh[k].f[1][c].im; qs->f[3][c].im -= hh[k].f[1][c].im;`

This code is used in chunks 70b, 73b, and 75b.

3 CONJUGATE GRADIENT

Here we develop the algorithm used in the solver.

3.1 Standard Algorithm

The basic conjugate gradient algorithm 1 is simple. Its only requirement is that matrix A is hermitian. Otherwise, it appears suited for DWF better than other iterative solvers.

```
Input:  $A$ , the matrix  
Input:  $b$ , the right hand side of the linear equation  
Input:  $x_0$ , an initial guess  
Input:  $n$ , the maximum number of iterations  
Input:  $\epsilon$ , required precision  
Output:  $x$ , approximate solution  
Output:  $\rho$ , final residue  
Output:  $k$ , number of iterations used  
begin  
   $x \leftarrow x_0$   
   $p \leftarrow r \leftarrow b - Ax$   
   $\rho \leftarrow \langle r, r \rangle$   
   $k \leftarrow 0$   
  while  $\rho > \epsilon$  or  $k < n$  do  
     $q \leftarrow Ap$   
     $\alpha \leftarrow \rho / \langle p, q \rangle$   
     $r \leftarrow r - \alpha q$   
     $x \leftarrow x + \alpha p$   
     $\gamma \leftarrow \langle r, r \rangle$   
    if  $\gamma < \epsilon$  then  
       $\rho \leftarrow \gamma$   
      break  
    end  
     $\beta \leftarrow \gamma / \rho$   
     $\rho \leftarrow \gamma$   
     $p \leftarrow r + \beta p$   
     $k \leftarrow k + 1$   
  end  
  return  $x, \rho, k$ .  
end
```

Algorithm 1: Generic Conjugate Gradient Solver

3.2 Overlap Opportunities

Our approach to overlapping computations with communications is to break the sublattice into boundary and inside pieces. After that, we first compute $(1 \pm \gamma_\mu)$ projections on the boundary and start send and receive operations. While communications are in progress, everything is computed on the inside nodes of the sublattice. Once receive is complete, we compute the operator on the boundary sites. Such an approach helps to improve temporal locality (and, therefore, cache utilization) at the expense of losing the ability of overlap if one of the sublattice dimensions is 2. However, it is unlikely that we could afford a large enough cluster to be forced into this corner of the parameter space.

3.3 Non-hermitial Matrix

Hermiticity of M is the only obstacle in applying algorithm 1 directly to our problem $M\psi = \eta$. This issue can be easily resolved by multiplying both sides by M^\dagger . However, instead of using algorithm 1 with $A = M^\dagger M$, it is better to keep M and M^\dagger separate—this makes it possible to hide one of the global sum computations, thus improving machine size scaling. Algorithm 2 is what we use in the solver.

Input: M , the matrix
Input: b , the right hand side of the linear equation
Input: x_0 , an initial guess
Input: n , the maximum number of iterations
Input: ϵ , required precision
Output: x , approximate solution
Output: ρ , final residue
Output: k , number of iterations used
begin
 $x \leftarrow x_0$
 $p \leftarrow r \leftarrow b - M^\dagger Mx$
 $\rho \leftarrow \langle r, r \rangle$
 $k \leftarrow 0$
while $\rho > \epsilon$ **or** $k < n$ **do**
 $z \leftarrow Mp$
 $q \leftarrow M^\dagger z$
 $\alpha \leftarrow \rho / \langle z, z \rangle$
 $r \leftarrow r - \alpha q$
 $x \leftarrow x + \alpha p$
 $\gamma \leftarrow \langle r, r \rangle$
if $\gamma < \epsilon$ **then**
 $\rho \leftarrow \gamma$
break
end
 $\beta \leftarrow \gamma / \rho$
 $\rho \leftarrow \gamma$
 $p \leftarrow r + \beta p$
 $k \leftarrow k + 1$
end
return x, ρ, k .
end

Algorithm 2: DWF-ready Gradient Solver.

4 PRECONDITIONING

We use four dimensional preconditioner to improve convergence of the CG. Following Kostas Orginos, let us color the lattice sites according to the parity of $x_0 + x_1 + x_2 + x_3$. Then we can rewrite $\chi = D\psi$ as follows:

$$\begin{pmatrix} \chi_e \\ \chi_o \end{pmatrix} = D\psi = \begin{pmatrix} Q_{ee} & Q_{eo} \\ Q_{oe} & Q_{oo} \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_o \end{pmatrix}$$

From the form of D it follows that all dependance on the gauge field is located in Q_{xy} , and that Q_{xx} does not depend on U . That, in turn, allows us to invert Q_{xx} easily. With this in mind, one writes:

$$\begin{pmatrix} Q_{ee} & Q_{eo} \\ Q_{oe} & Q_{oo} \end{pmatrix} = \begin{pmatrix} Q_{ee} & 0 \\ Q_{oe} & Q_{oo} \end{pmatrix} \begin{pmatrix} 1 & Q_{ee}^{-1}Q_{eo} \\ 0 & 1 - Q_{oo}^{-1}Q_{oe}Q_{ee}^{-1}Q_{eo} \end{pmatrix}$$

Now, to solve the equation

$$D\psi = \eta,$$

one needs to perform the following steps:

1. Compute

$$\phi_o = Q_{oo}^{-1}(\eta_o - Q_{oe}Q_{ee}^{-1}\eta_e)$$

2. Set $M = 1 - Q_{oo}^{-1}Q_{oe}Q_{ee}^{-1}Q_{eo}$ for the following.

3. Compute

$$\varphi_o = M^\dagger \phi_o$$

4. Solve for ψ_o the following equation using Algorithm 2

$$M^\dagger M \psi_o = \varphi_o$$

5. Compute

$$\psi_e = Q_{ee}^{-1}(\eta_e - Q_{eo}\psi_o)$$

Note, that $M^\dagger = 1 - (Q_{eo})^\dagger(Q_{ee}^{-1})^\dagger(Q_{oe})^\dagger(Q_{oo}^{-1})^\dagger = 1 - S_{oe}S_{ee}^{-1}S_{oe}S_{oo}^{-1}$, where

$$\begin{aligned} S_{ee} &= Q_{ee}[\gamma_5 \rightarrow -\gamma_5] \\ S_{oo} &= Q_{oo}[\gamma_5 \rightarrow -\gamma_5] \\ S_{oe} &= Q_{eo}[\gamma_\mu \rightarrow -\gamma_\mu] \\ S_{eo} &= Q_{oe}[\gamma_\mu \rightarrow -\gamma_\mu] \end{aligned}$$

4.1 Q_{xx} inversion

The previous section is based on a tacit assumption that Q_{ee} and Q_{oo} are easy to invert. Here we show that it is so. Let us rewrite

$$\chi_{s,x} = (Q_{ee}\psi)_{s,x} = M_0\psi_{s,x} + (1 + \gamma_5)M_s^{(+)}\psi_{s+1,x} + (1 + \gamma_5)M_s^{(-)}\psi_{s-1,x}$$

as follows:

$$(Q_{ee}\psi)_{s,x} = M_0 \left(\left(\frac{1 + \gamma_5}{2} \right) \left(\psi_{s,x} + \frac{2M_s^{(+)}}{M_0}\psi_{s+1,x} \right) + \left(\frac{1 - \gamma_5}{2} \right) \left(\psi_{s,x} + \frac{2M_s^{(-)}}{M_0}\psi_{s-1,x} \right) \right).$$

Thus,

$$Q_{ee} = \frac{1 + \gamma_5}{2} \begin{pmatrix} a & b & \cdots & 0 & 0 \\ 0 & a & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a & b \\ c & 0 & \cdots & 0 & a \end{pmatrix} + \frac{1 - \gamma_5}{2} \begin{pmatrix} a & 0 & \cdots & 0 & c \\ b & a & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a & 0 \\ 0 & 0 & \cdots & b & a \end{pmatrix} = P_+A + P_-B,$$

where $a = M_0$, $b = 2$, $c = -2m_f$. Computing $Q_{xx}\psi$ and $S_{xx}\psi$ is done below with the vector hardware. Here we compute constant values needed for effective use of vectors:

13a $\langle \text{Compute constant values for } Q_{xx} \text{ and } S_{xx} \text{ 13a} \rangle \equiv$

```

{
    double a = M_0;
    double b = 2.0;
    double c = -2 * m_f;

    va = vmk_1(a);
    vb = vmk_1(b);
    vcbn = vmk_1n(c, b); /* {c, b, ...} */
    vbnc = vmk_n1(b, c); /* {b, ..., c} */
}

```

This code is used in chunks 19c and 20a.

13b $\langle \text{Global variables 13b} \rangle \equiv$

```

static vReal vcbn;
static vReal vbnc;
static vReal vb;
static vReal va;

```

This definition is continued in chunks 14, 15, 19b, 24b, 26a, 30, 44–46, 53g, and 55a.
This code is used in chunk 83b.

Now, since P_{\pm} commute with A and B , $Q_{ee}^{-1} = P_+ A^{-1} + P_- B^{-1}$. Computing A^{-1} and B^{-1} is done by decomposition $A = L_A R_A$, $B = L_B R_B$, where

$$R_A = \begin{pmatrix} a & b & \cdots & 0 & 0 \\ 0 & a & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a & b \\ 0 & 0 & \cdots & 0 & a \end{pmatrix} \quad R_B = \begin{pmatrix} a & 0 & \cdots & 0 & 0 \\ b & a & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & a & 0 \\ 0 & 0 & \cdots & b & a \end{pmatrix},$$

and

$$L_A = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & & 0 \\ \vdots & & & & & \vdots \\ c/a & -bc/a^2 & b^2c/a^3 & -b^3c/a^4 & \cdots & 1 + (-b)^{n-1}c/a^n \end{pmatrix}$$

$$L_B = \begin{pmatrix} 1 + (-b)^{n-1}c/a^n & (-b)^{n-2}c/a^{n-1} & \cdots & b^2c/a^3 & -bc/a^2 & c/a \\ 0 & 1 & & 0 & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix}.$$

In these terms,

$$Q_{ee}^{-1} = \frac{1 + \gamma_5}{2} R_A^{-1} L_A^{-1} + \frac{1 - \gamma_5}{2} R_B^{-1} L_B^{-1}.$$

We will also need

$$S_{ee}^{-1} = \frac{1 - \gamma_5}{2} R_A^{-1} L_A^{-1} + \frac{1 + \gamma_5}{2} R_B^{-1} L_B^{-1}.$$

For further reference,

$$\gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

5 CODE

This section contains chunks of the source that go into `dwf.c` source file. We start with the interface functions and elaborate from there.

5.1 Interface Functions

We can not expect the user to call different parts of the interface in an appropriate order. Therefore, successful initialization allows the user to call other interface elements, as well as prevents repeated initializations.

14a *⟨Global variables 13b⟩* +=
static int init_p = 0;

5.1.1 DWF Initializer

14b *⟨Interface functions 14b⟩* ≡
int
L3(DWF_init)(const int lattice[DIM+1],
void *(*allocator)(size_t size),
void (*dealloc)(void *))
{
 ⟨Version 1⟩

 if (init_p)
 return 1; /* error: second init */

 ⟨Check lattice size 15d⟩
 ⟨Get network topology 25b⟩
 ⟨Setup heap management functions 15a⟩
 ⟨Initialize tables 29c⟩
 ⟨Allocate fields 44d⟩
 ⟨Initialize QMP 39e⟩
 ⟨Show DWF version 81c⟩
 init_p = 1;
 DEBUG_DWF("finished init, lattice=[%d %d %d %d %d]\n",
 lattice[0], lattice[1], lattice[2], lattice[3], lattice[4])
 return 0;

 ⟨Handle init errors 14c⟩
}

This definition is continued in chunks 15–21.

This code is used in chunk 83b.

If any error occurs during initialization, we simply unroll state and fail:

14c *⟨Handle init errors 14c⟩* ≡
error:
 L3(DWF_fini)();
 return 1;

This code is used in chunk 14b.

Check if the user requested special allocation mechanisms:

```
15a  <Setup heap management functions 15a>≡
      if (allocator)
          tmalloc = allocator;
      else
          tmalloc = malloc;

      if (deallocator)
          tfree = deallocator;
      else
          tfree = free;
```

This code is used in chunk 14b.

```
15b  <Global variables 13b>+≡
      static void *(*tmalloc)(size_t size);
      static void (*tfree)(void *ptr);
```

```
15c  <Include files 15c>≡
      #include <string.h>
      #include <stdlib.h>
```

This definition is continued in chunk 39d.

This code is used in chunk 83b.

For simplicity of s -slice operations, L_s should be a multiple of the vector size.

```
15d  <Check lattice size 15d>≡
      if (lattice[DIM] % Vs)
          goto error;
      tlattice[DIM] = lattice[DIM];
```

This definition is continued in chunk 15e.

This code is used in chunk 14b.

Otherwise, lattice sizes must be even to allow us to do red/black preconditioning:

```
15e  <Check lattice size 15d>+≡
      {
          int i;
          for (i = 0; i < DIM; i++) {
              if (lattice[i] & 1)
                  goto error;
              tlattice[i] = lattice[i];
          }
      }
```

```
15f  <Global variables 13b>+≡
      static int tlattice[DIM+1];
```

5.1.2 DWF Clean Up

The cleanup routine may be called from partially initialized context, we should be able to do a partial cleanup.

```
15g  <Interface functions 14b>+≡
      void
      L3(DWF_fini)(void)
      {
          <Cleanup QMP 43b>
          <Free fields 44e>
          <Free tables 38a>
          initied_p = 0;
          DEBUG_DWF("fini done\n")
      }
```

5.1.3 DWF Fermion Allocator

When one needs a DWF fermion, the allocator does the job. Remember, users are stupid enough to call this function in the uninitialized state. It is convenient to break all internal fermions into odd and even parts at this stage.

16a

```
⟨Interface functions 14b⟩+≡
L3(DWF_Fermion) *
L3(DWF_allocate_fermion)(void)
{
    L3(DWF_Fermion) *ptr;

    if (!inited_p)
        return 0;

    ptr = tmalloc(sizeof (*ptr));
    if (ptr == 0)
        return 0;

    ptr->even = allocate_even_fermion();
    if (ptr->even == 0)
        goto error1;

    ptr->odd = allocate_odd_fermion();
    if (ptr->odd == 0)
        goto error2;

    return ptr;
error2:
    free16(ptr->even);
error1:
    tfree(ptr);
    return 0;
}
```

5.1.4 DWF Fermion Exporter

When we need to create a DWF field and populate it from an outer environment, we use the following procedure

16b

```
⟨Interface functions 14b⟩+≡
L3(DWF_Fermion) *
L3(DWF_load_fermion)(const void *OuterFermion,
                    void *env,
                    L3(DWF_fermion_reader) reader)
{
    L3(DWF_Fermion) *ptr = L3(DWF_allocate_fermion)();

    DEBUG_DWF("OuterFermion=%p, reader=%p\n", OuterFermion, reader)

    /* Handle both lack of memory and missing initialization */
    if (ptr == 0)
        return 0;

    ⟨Read fermion 27d⟩

    return ptr;
}
```


5.1.5 DWF Fermion Importer

For moving data back to the outer environment, the following importer is used:

```
17a  <Interface functions 14b>+≡
      void
      L3(DWF_save_fermion)(void *OuterFermion,
                           void *env,
                           L3(DWF_fermion_writer) writer,
                           L3(DWF_Fermion) *CGfermion)
      {
        if (!initied_p)
          return;

        DEBUG_DWF("Outer fermion=%p, writer=%p\n", OuterFermion, writer)

        <Write fermion 28c>
      }
```

5.1.6 DWF Fermion Deallocator

We only free pointers that we allocated. The magic is in `free16()`—it knows about all heap objects allocated by `alloc16()`.

```
17b  <Interface functions 14b>+≡
      void
      L3(DWF_delete_fermion)(L3(DWF_Fermion) *ptr)
      {
        if (!initied_p)
          return;

        free16(ptr->even);
        free16(ptr->odd);
        tfree(ptr);
      }
```

5.1.7 DWF Gauge Exporter

Unlike fermions, gauge fields are 4-d in the solver. Though they are not loaded by vector memory operations, we still allocate 16-byte aligned memory for them (apparently for no good reason at all.)

18a *⟨Interface functions 14b⟩* +=

```

L3(DWF_Gauge) *
L3(DWF_load_gauge)(const void *OuterGauge_U,
                   const void *OuterGauge_V,
                   void *env,
                   L3(DWF_gauge_reader) reader)
{
    L3(DWF_Gauge) *g;

    if (!initied_p)
        return 0;

    DEBUG_DWF("U=%p, V=%p, reader=%p\n", OuterGauge_U, OuterGauge_V, reader)

    g = allocate_gauge_field();
    if (g == 0)
        return 0;

    ⟨Read gauge field 26b⟩
    return g;
}

```

Let us also define L3(DWF_Gauge) here. We do not need anything fancy for the gauge field:

18b *⟨Data types 18b⟩* =

```

struct L3(DWF_Gauge) {
    scalar_complex v[Nc][Nc];
};

```

This definition is continued in chunks 22–24 and 30d.
This code is used in chunk 83b.

5.1.8 DWF Gauge Deallocator

Gauge deallocator is very much like fermion deallocator. We only keep them separate to help the type system cope with a error making user.

18c *⟨Interface functions 14b⟩* +=

```

void
L3(DWF_delete_gauge)(L3(DWF_Gauge) *ptr)
{
    if (!initied_p)
        return;

    free16(ptr);
}

```

5.1.9 The Solver

Finally, the solver itself. Here we check if the system has been properly initialized and dispatch on the float size (but not now yet.)

```
19a  <Interface functions 14b>+≡
      int
      L3(DWF_cg_solver)(L3(DWF_Fermion)      *psi,
                        double                 *out_eps,
                        int                    *out_iter,
                        const L3(DWF_Gauge)    *gauge,
                        double                 M,
                        double                 m_f,
                        const L3(DWF_Fermion)  *x0,
                        const L3(DWF_Fermion)  *eta,
                        double                 eps,
                        int                    min_iter,
                        int                    max_iter)
      {
        int status;

        if (!inited_p)
          return 1;

        U = (SU3 *)gauge;
        <Compute constant values for  $Q_{xx}^{-1}$  and  $S_{xx}^{-1}$  80g>
        <Compute  $\varphi_o$  44b>
        <Solve  $M^\dagger M \psi_o = \varphi_o$  44f>
        <Compute  $\psi_e$  46c>
        return status;
      }
```

Save one argument in many functions:

```
19b  <Global variables 13b>+≡
      static SU3 *U;
```

5.1.10 Dirac Operator

It is convenient to have the Dirac operator and its conjugate as separate functions.

$$\chi \leftarrow D_{DW}\psi.$$

```
19c  <Interface functions 14b>+≡
      void
      L3(DWF_Dirac_Operator)(L3(DWF_Fermion)  *chi,
                             const L3(DWF_Gauge) *gauge,
                             double              M_0,
                             double              m_f,
                             const L3(DWF_Fermion) *psi)
      {
        if (!inited_p)
          return;

        U = (SU3 *)gauge;
        <Compute constant values for  $Q_{xx}$  and  $S_{xx}$  13a>
        compute_Do(chi->odd, psi->odd, psi->even);
        compute_De(chi->even, psi->even, psi->odd);
      }
```

$$\chi \leftarrow D_{DW}^\dagger \psi.$$

20a \langle Interface functions 14b $\rangle + \equiv$

```

void
L3(DWF_Dirac_Operator_conjugate) (L3(DWF_Fermion)      *chi,
                                   const L3(DWF_Gauge)    *gauge,
                                   double                  M_0,
                                   double                  m_f,
                                   const L3(DWF_Fermion)    *psi)
{
    if (!inited_p)
        return;

    U = (SU3 *)gauge;
     $\langle$ Compute constant values for  $Q_{xx}$  and  $S_{xx}$  13a $\rangle$ 
    compute_Dco(chi->odd, psi->odd, psi->even);
    compute_Dce(chi->even, psi->even, psi->odd);
}

```

5.1.11 Little Helpers

$$\psi \leftarrow \varphi + a\eta$$

20b \langle Interface functions 14b $\rangle + \equiv$

```

void
L3(DWF_Add_Fermion) (L3(DWF_Fermion)      *psi,
                    const L3(DWF_Fermion)  *phi,
                    double                  a,
                    const L3(DWF_Fermion)    *eta)
{
    collect_add(&psi->odd->f, &phi->odd->f, a, &eta->odd->f, odd_even.size * Sv);
    collect_add(&psi->even->f, &phi->even->f, a, &eta->even->f, even_odd.size * Sv);
}

```

21a \langle Static function prototypes 21a $\rangle \equiv$

```
static inline void
collect_add(vFermion *r,
            const vFermion *x,
            double A,
            const vFermion *y,
            int n)
{
    int i;
    vReal a = vmk_1(A);

    for (i = 0; i < n; i++) {
        r->f[0][0].re = x->f[0][0].re + a * y->f[0][0].re;
        r->f[0][0].im = x->f[0][0].im + a * y->f[0][0].im;
        r->f[0][1].re = x->f[0][1].re + a * y->f[0][1].re;
        r->f[0][1].im = x->f[0][1].im + a * y->f[0][1].im;
        r->f[0][2].re = x->f[0][2].re + a * y->f[0][2].re;
        r->f[0][2].im = x->f[0][2].im + a * y->f[0][2].im;

        r->f[1][0].re = x->f[1][0].re + a * y->f[1][0].re;
        r->f[1][0].im = x->f[1][0].im + a * y->f[1][0].im;
        r->f[1][1].re = x->f[1][1].re + a * y->f[1][1].re;
        r->f[1][1].im = x->f[1][1].im + a * y->f[1][1].im;
        r->f[1][2].re = x->f[1][2].re + a * y->f[1][2].re;
        r->f[1][2].im = x->f[1][2].im + a * y->f[1][2].im;

        r->f[2][0].re = x->f[2][0].re + a * y->f[2][0].re;
        r->f[2][0].im = x->f[2][0].im + a * y->f[2][0].im;
        r->f[2][1].re = x->f[2][1].re + a * y->f[2][1].re;
        r->f[2][1].im = x->f[2][1].im + a * y->f[2][1].im;
        r->f[2][2].re = x->f[2][2].re + a * y->f[2][2].re;
        r->f[2][2].im = x->f[2][2].im + a * y->f[2][2].im;

        r->f[3][0].re = x->f[3][0].re + a * y->f[3][0].re;
        r->f[3][0].im = x->f[3][0].im + a * y->f[3][0].im;
        r->f[3][1].re = x->f[3][1].re + a * y->f[3][1].re;
        r->f[3][1].im = x->f[3][1].im + a * y->f[3][1].im;
        r->f[3][2].re = x->f[3][2].re + a * y->f[3][2].re;
        r->f[3][2].im = x->f[3][2].im + a * y->f[3][2].im;
    }
}
```

This definition is continued in chunks 22a, 24d, 28b, 29b, 31, 34a, 35f, 37–43, 45–52, 61, 62a, 65, 66, 79c, 81, and 82.
This code is used in chunk 83b.

21b \langle Interface functions 14b $\rangle + \equiv$

```
void
L3(DWF_Fermion_Dot_Product)(double *r_re,
                             double *r_im,
                             const L3(DWF_Fermion) *psi,
                             const L3(DWF_Fermion) *phi)
{
    *r_re = *r_im = 0;
    collect_dot(r_re, r_im, &psi->odd->f, &phi->odd->f, odd_even.size * Sv);
    collect_dot(r_re, r_im, &psi->even->f, &phi->even->f, even_odd.size * Sv);
    QMP_sum_double(r_re);
    QMP_sum_double(r_im);
}
```

Running the dot product on one parity of fermions:

```

22a  <Static function prototypes 21a>+=
static inline void
collect_dot(double *r_re,
            double *r_im,
            const vFermion *a,
            const vFermion *b,
            int n)
{
    int i;
    vReal c0_re, c1_re, c2_re;
    vReal c0_im, c1_im, c2_im;

    for (i = 0; i < n; i++, a++, b++) {
        c0_re = a->f[0][0].re*b->f[0][0].re; c0_re += a->f[0][0].im*b->f[0][0].im;
        c0_im = a->f[0][0].im*b->f[0][0].re; c0_im += a->f[0][0].re*b->f[0][0].im;
        c1_re = a->f[0][1].re*b->f[0][1].re; c1_re += a->f[0][1].im*b->f[0][1].im;
        c1_im = a->f[0][1].im*b->f[0][1].re; c1_im += a->f[0][1].re*b->f[0][1].im;
        c2_re = a->f[0][2].re*b->f[0][2].re; c2_re += a->f[0][2].im*b->f[0][2].im;
        c2_im = a->f[0][2].im*b->f[0][2].re; c2_im += a->f[0][2].re*b->f[0][2].im;

        c0_re += a->f[1][0].re*b->f[1][0].re; c0_re += a->f[1][0].im*b->f[1][0].im;
        c0_im += a->f[1][0].im*b->f[1][0].re; c0_im += a->f[1][0].re*b->f[1][0].im;
        c1_re += a->f[1][1].re*b->f[1][1].re; c1_re += a->f[1][1].im*b->f[1][1].im;
        c1_im += a->f[1][1].im*b->f[1][1].re; c1_im += a->f[1][1].re*b->f[1][1].im;
        c2_re += a->f[1][2].re*b->f[1][2].re; c2_re += a->f[1][2].im*b->f[1][2].im;
        c2_im += a->f[1][2].im*b->f[1][2].re; c2_im += a->f[1][2].re*b->f[1][2].im;

        c0_re += a->f[2][0].re*b->f[2][0].re; c0_re += a->f[2][0].im*b->f[2][0].im;
        c0_im += a->f[2][0].im*b->f[2][0].re; c0_im += a->f[2][0].re*b->f[2][0].im;
        c1_re += a->f[2][1].re*b->f[2][1].re; c1_re += a->f[2][1].im*b->f[2][1].im;
        c1_im += a->f[2][1].im*b->f[2][1].re; c1_im += a->f[2][1].re*b->f[2][1].im;
        c2_re += a->f[2][2].re*b->f[2][2].re; c2_re += a->f[2][2].im*b->f[2][2].im;
        c2_im += a->f[2][2].im*b->f[2][2].re; c2_im += a->f[2][2].re*b->f[2][2].im;

        c0_re += a->f[3][0].re*b->f[3][0].re; c0_re += a->f[3][0].im*b->f[3][0].im;
        c0_im += a->f[3][0].im*b->f[3][0].re; c0_im += a->f[3][0].re*b->f[3][0].im;
        c1_re += a->f[3][1].re*b->f[3][1].re; c1_re += a->f[3][1].im*b->f[3][1].im;
        c1_im += a->f[3][1].im*b->f[3][1].re; c1_im += a->f[3][1].re*b->f[3][1].im;
        c2_re += a->f[3][2].re*b->f[3][2].re; c2_re += a->f[3][2].im*b->f[3][2].im;
        c2_im += a->f[3][2].im*b->f[3][2].re; c2_im += a->f[3][2].re*b->f[3][2].im;

        *r_re += vsum(c0_re + c1_re + c2_re);
        *r_im += vsum(c0_im + c1_im + c2_im);
    }
}

```

5.2 Internal Data Types

Given `scalar_complex` and `vector_complex` types, define QCD related data types. A vector of full Dirac fermions:

```

22b  <Data types 18b>+=
typedef struct {
    vector_complex f[Fd][Nc];
} vFermion;

```

The projected fermion vector. We do not distinguish different projections here:

```
23a <Data types 18b>+≡
    typedef struct {
        vector_complex f[Fd/2][Nc];
    } vHalfFermion;
```

Gauge field after conversion into DWF form:

```
23b <Data types 18b>+≡
    typedef struct {
        scalar_complex v[Nc][Nc];
    } SU3;
```

Small piece of the gauge field converted to a vector form:

```
23c <Data types 18b>+≡
    typedef struct {
        vector_complex v[Nc][Nc];
    } vSU3;
```

Even and odd sublattices of fermions:

```
23d <Data types 18b>+≡
    typedef struct {
        vFermion f;
    } vEvenFermion;

    typedef struct {
        vFermion f;
    } vOddFermion;
```

The interface fermion is a pair of even and odd sublattices:

```
23e <Data types 18b>+≡
    struct L3(DWF_Fermion) {
        vEvenFermion *even;
        vOddFermion *odd;
    };
```

5.3 Memory Allocation

Vector hardware does like properly aligned memory. While automatic variables are aligned by the compiler, extra care is needed when dealing with the heap. The code allocates all its own memory aligned on 16-byte boundary by calling `alloc16()`, and returns the memory through `free16()`.

```
23f <Static functions 23f>≡
    static void *
    alloc16(int size)
    {
        int xsize = PAD16(size + sizeof (struct memblock));
        struct memblock *p = tmalloc(xsize);

        if (p == 0)
            return p;

        p->data = ALIGN16(&p[1]);
        p->size = size;
        p->next = memblock.next;
        p->prev = &memblock;
        p->next->prev = p;
        p->prev->next = p;

        return p->data;
    }
```

This definition is continued in chunks [24a](#), [25a](#), [30a](#), [31c](#), [33d](#), [36a](#), [37b](#), [40–43](#), [45–52](#), and [62–65](#). This code is used in chunk [83b](#).

For deallocation we need to find an appropriate memory block:

```
24a <Static functions 23f>+≡
static void
free16(void *ptr)
{
    struct memblock *p;

    if (ptr == 0)
        return;

    for (p = memblock.next; p != &memblock; p = p->next) {
        if (p->data != ptr)
            continue;
        p->next->prev = p->prev;
        p->prev->next = p->next;
        tfree(p);
        return;
    }
    /* this is BAD: control should not reach here! */
}
```

The head of the memory list is stored in a static variable. Of course, such an implementation is not threadable, but let us worry about that when the time is right.

```
24b <Global variables 13b>+≡
static struct memblock memblock = {
    &memblock,
    &memblock,
    NULL,
    0
};
```

Finally, the datatype for the linked list:

```
24c <Data types 18b>+≡
struct memblock {
    struct memblock *next;
    struct memblock *prev;
    void *data;
    size_t size;
};
```

5.3.1 Field allocators

First, the prototypes:

```
24d <Static function prototypes 21a>+≡
static vEvenFermion *allocate_even_fermion(void);
static vOddFermion *allocate_odd_fermion(void);
static L3(DWF_Gauge) *allocate_gauge_field(void);
```


The only difference between even and odd fermions is (possibly) their size:

```
25a  <Static functions 23f>+≡
      vEvenFermion *
      allocate_even_fermion(void)
      {
          return alloc16(even_odd.size * Sv * sizeof (vFermion));
      }

      vOddFermion *
      allocate_odd_fermion(void)
      {
          return alloc16(odd_even.size * Sv * sizeof (vFermion));
      }

      L3(DWF_Gauge) *
      allocate_gauge_field(void)
      {
          return alloc16(gauge_XYZT * sizeof (L3(DWF_Gauge)));
      }
```

5.4 Probing Cluster Topology

There is no proper way to query QMP about lattice layout. We have to request the minimal meaningful information the library provides and try to repeat outer layer's partitioning of the lattice. There are good chances of success, but this is a potential danger spot.

Here we prepare compute where on the lattice this node is and to build up our understanding of neighbors. Maybe optimistically, we assume that once QMP is initialized, it reports logical dimensions and coordinates properly, so that we do not need to be paranoid about errors here.

```
25b  <Get network topology 25b>≡
      {
          int i, dn;
          const int *xn, *xc;

          if (!QMP_logical_topology_is_declared())
              /* The user must have declared logical topology before */
              goto error;
          dn = QMP_get_logical_number_of_dimensions();
          if (dn > DIM)
              /* Too high dimension of the logical network */
              goto error;

          xn = QMP_get_logical_dimensions();
          xc = QMP_get_logical_coordinates();
          for (i = 0; i < dn; i++) {
              network[i] = xn[i];
              coord[i] = xc[i];
          }

          for (; i < DIM; i++) {
              network[i] = 1;
              coord[i] = 0;
          }
      }
```

This code is used in chunk 14b.

Some global variables:

```
26a  ⟨Global variables 13b⟩+≡  
    static int network[DIM];  
    static int coord[DIM];
```

5.5 Moving Data

5.5.1 Reading the Gauge Field

Let us start with reading of the gauge field from the outer environment first. Here we assume that there is an address translation function to help us in talking to the outer layer.

```
26b  ⟨Read gauge field 26b⟩≡  
    {  
        int x[DIM], i, d, a, b, p1;  
  
        ⟨Start DIM-d sublattice scan 26d⟩  
        ⟨Load DIM gauge links from U at x 26c⟩  
        ⟨Advance DIM-d index for a sublattice scan 26e⟩  
  
        for (d = 0; d < DIM; d++)  
            ⟨Load gauge boundary in direction d 27b⟩  
    }
```

This code is used in chunk 18a.

At a given site, load DIM gauge elements:

```
26c  ⟨Load DIM gauge links from U at x 26c⟩≡  
    p1 = to_Ulinear(x, &bounds, -1);  
    for (d = 0; d < DIM; d++) {  
        for (a = 0; a < Nc; a++) {  
            for (b = 0; b < Nc; b++) {  
                g[p1 + d].v[a][b].re = reader(OuterGauge_U, env, x, d, a, b, 0);  
                g[p1 + d].v[a][b].im = reader(OuterGauge_U, env, x, d, a, b, 1);  
            }  
        }  
    }
```

This code is used in chunk 26b.

To start a scan over the lattice, initialize x and start the loop:

```
26d  ⟨Start DIM-d sublattice scan 26d⟩≡  
    for (i = 0; i < DIM; i++)  
        x[i] = bounds.lo[i];  
    for (i = 0; i < DIM;) {
```

This code is used in chunks 26–28 and 34c.

Once all is done with the site x, we are ready to advance the index:

```
26e  ⟨Advance DIM-d index for a sublattice scan 26e⟩≡  
    for (i = 0; i < DIM; i++) {  
        ⟨Advance x at i 27a⟩  
    }
```

This code is used in chunks 26–28 and 34c.

Since we are going to use a DIM-1 dimensional scan as well, let us write it down here:

```
26f  ⟨Advance DIM-d index for DIM-1-d scan 26f⟩≡  
    for (i = 0; i < DIM; i++) {  
        if (i == d)  
            continue;  
        ⟨Advance x at i 27a⟩  
    }
```

This code is used in chunk 27b.

Now we can scan DIM-dimensional indices:

```
27a  ⟨Advance x at i 27a⟩≡
      if (++x[i] == bounds.hi[i])
        x[i] = bounds.lo[i];
      else
        break;
```

This code is used in chunk 26.

DWF Dirac operator needs backward gauge links. We get them from OuterGauge_V. Here we only read the boundary links.

```
27b  ⟨Load gauge boundary in direction d 27b⟩≡
      {
        if (network[d] == 1)
          continue;

        ⟨Start DIM-d sublattice scan 26d⟩
        ⟨Load a d gauge link from V at x 27c⟩
        ⟨Advance DIM-d index for DIM-1-d scan 26f⟩
      }
```

This code is used in chunk 26b.

Now we read a boundary element:

```
27c  ⟨Load a d gauge link from V at x 27c⟩≡
      p1 = to_Ulinear(x, &bounds, d);
      for (a = 0; a < Nc; a++) {
        for (b = 0; b < Nc; b++) {
          g[p1].v[a][b].re = reader(OuterGauge_V, env, x, d, a, b, 0);
          g[p1].v[a][b].im = reader(OuterGauge_V, env, x, d, a, b, 1);
        }
      }
```

This code is used in chunk 27b.

5.5.2 Reading a Fermion

There are but two complications in reading the domain wall fermion. First, this is a good time to break the fermion into red and black pieces. In addition, here we construct DWF fermions.

```
27d  ⟨Read fermion 27d⟩≡
      {
        int x[DIM+1], i;

        ⟨Start DIM-d sublattice scan 26d⟩
        ⟨Load an s-line of fermion at x 28a⟩
        ⟨Advance DIM-d index for a sublattice scan 26e⟩
      }
```

This code is used in chunk 16b.

Data conversion is inherently inefficient. We do not try to optimize it here:

```

28a  <Load an s-line of fermion at x 28a>≡
    {
        int p = parity(x);
        int p1 = Sv * to_HFlinear(x, &bounds, -1, 0); /* p is taken care of! */
        vFermion *f = p? &ptr->odd[p1].f: &ptr->even[p1].f;

        for (x[DIM] = 0; x[DIM] < tlattice[DIM]; x[DIM] += Vs, f++) {
            int d;
            for (d = 0; d < Fd; d++) {
                int c;
                for (c = 0; c < Nc; c++) {
                    f->f[d][c].re = import_vector(OuterFermion, env, reader,
                                                    x, c, d, 0);
                    f->f[d][c].im = import_vector(OuterFermion, env, reader,
                                                    x, c, d, 1);
                }
            }
        }
    }

```

This code is used in chunk 27d.

A simple packer of Vs elements into a vector:

```

28b  <Static function prototypes 21a>+≡
    static inline vReal
    import_vector(const void *z, void *env, L3(DWF_fermion_reader) reader,
                  int x[DIM+1], int c, int d, int re_im)
    {
        vReal f;
        REAL *v = (REAL *)&f;
        int i, xs;

        for (xs = x[DIM], i = 0; i < Vs; i++, x[DIM]++) {
            *v++ = reader(z, env, x, c, d, re_im);
        }
        x[DIM] = xs;
        return f;
    }

```

5.5.3 Writing a Fermion

Writing a fermion is not much different:

```

28c  <Write fermion 28c>≡
    {
        int x[DIM+1], i;

        <Start DIM-d sublattice scan 26d>
        <Save an s-line of fermion at x 29a>
        <Advance DIM-d index for a sublattice scan 26e>
    }

```

This code is used in chunk 17a.

29a *<Save an s-line of fermion at x 29a>≡*

```

{
    int p = parity(x);
    int p1 = Sv * to_HFlinear(x, &bounds, -1, 0); /* p is taken care of! */
    vFermion *f = p? &CGfermion->odd[p1].f: &CGfermion->even[p1].f;

    for (x[DIM] = 0; x[DIM] < tlattice[DIM]; x[DIM] += Vs, f++) {
        int d;
        for (d = 0; d < Fd; d++) {
            int c;
            for (c = 0; c < Nc; c++) {
                save_vector(OuterFermion, env, writer, x, c, d, 0,
                           &f->f[d][c].re);
                save_vector(OuterFermion, env, writer, x, c, d, 1,
                           &f->f[d][c].im);
            }
        }
    }
}

```

This code is used in chunk 28c.

Here's another little helper good only for writing back the fermion from DWF to the outer environment:

29b *<Static function prototypes 21a>+≡*

```

static inline void
save_vector(void *z, void *env, L3(DWF_fermion_writer) writer,
            int x[DIM+1], int c, int d, int re_im, vReal *f)
{
    REAL *v = (REAL *)f;
    int i, xs;

    for (xs = x[DIM], i = 0; i < Vs; i++, x[DIM]++) {
        writer(z, env, x, c, d, re_im, *v++);
    }
    x[DIM] = xs;
}

```

5.6 Solver Initialization

Here are all pieces for setting up the structures needed to run the solver.

5.6.1 Constructing the neighbor tables

29c *<Initialize tables 29c>≡*

```

if (init_tables()) {
    /* Something went wrong in the table construction */
    goto error;
}

```

This code is used in chunk 14b.

The table initializer creates all tables necessary for communication and computation. Memory is allocated here for index arrays.

```
30a  <Static functions 23f>+≡
      static int
      init_tables(void)
      {
          struct neighbor tmp;
          int i, v;

          init_neighbor(&bounds, &neighbor);
          <Compute init sizes 30c>
          tmp = neighbor;
          build_neighbor(&even_odd, 0, &tmp);
          build_neighbor(&odd_even, 1, &tmp);

          return 0;
      }
```

First, we set global data:

```
30b  <Global variables 13b>+≡
      static struct bounds bounds;
      static int gauge_XYZT;
      static int Sv, Sv_1;

30c  <Compute init sizes 30c>≡
      Sv = tlattice[DIM] / Vs;
      Sv_1 = Sv - 1;
      for (v = 1, i = 0; i < DIM; i++) {
          v *= bounds.hi[i] - bounds.lo[i];
      }
      gauge_XYZT = DIM * v;
      for (i = 0; i < DIM; i++) {
          if (network[i] < 2)
              continue;
          gauge_XYZT += v / (bounds.hi[i] - bounds.lo[i]);
      }
```

This code is used in chunk 30a.

The `struct bounds` helps us to navigate through the local part of the lattice. It is used by the initialization code only.

```
30d  <Data types 18b>+≡
      struct bounds {
          int lo[DIM];
          int hi[DIM];
      };
```

We keep two `struct neighbor`, one for computation on the even sublattice, another—on the odd. In addition to `even_odd` and `odd_even`, we need one more `struct neighbor` to keep the allocated pointers in.

```
30e  <Global variables 13b>+≡
      static struct neighbor neighbor;
      #ifndef DEBUG_QMP
      static struct neighbor odd_even;
      static struct neighbor even_odd;
      #else
      struct neighbor odd_even;
      struct neighbor even_odd;
      #endif
```

Let us start with computing the boundary of the sublattice

```

31a  <Static function prototypes 21a>+=
static inline int
lattice_start(int lat, int net, int coord)
{
    int q = lat / net;
    int r = lat % net;

    return coord * q + ((coord < r)? coord: r);
}

static inline void
mk_sublattice(struct bounds *bounds,
              int coord[])
{
    int i;

    for (i = 0; i < DIM; i++) {
        bounds->lo[i] = lattice_start(tlattice[i], network[i], coord[i]);
        bounds->hi[i] = lattice_start(tlattice[i], network[i], coord[i] + 1);
    }
}

```

All dynamic data are allocated in `init_neighbor` and are stored in `neighbor`.

```

31b  <Static function prototypes 21a>+=
static void
init_neighbor(struct bounds *bounds, struct neighbor *neighbor);

31c  <Static functions 23f>+=
static void
init_neighbor(struct bounds *bounds, struct neighbor *neighbor)
{
    int i;

    mk_sublattice(bounds, coord);
#ifdef NO_DEBUG_QMP
    for (i = 0; i < DIM; i++)
        DEBUG_QMP("local: bounds[%d]: lo %d, hi %d\n",
                  i, bounds->lo[i], bounds->hi[i])
#endif /* defined(DEBUG_QMP) */
    neighbor->qmp_smask = 0;
    <Compute inside_size and boundary_size 32a>
    <Allocate inside table 32b>
    <Allocate boundary table 32c>
    <Compute send sizes and allocate index tables 32d>
}

```

32a \langle Compute inside_size and boundary_size 32a $\rangle \equiv$

```

for (neighbor->size = 1, neighbor->inside_size = 1, i = 0; i < DIM; i++) {
    int ext = bounds->hi[i] - bounds->lo[i];

    neighbor->size *= ext;
    if (network[i] > 1)
        neighbor->inside_size *= ext - 2;
    else
        neighbor->inside_size *= ext;
}
neighbor->boundary_size = neighbor->size - neighbor->inside_size;
neighbor->site = tmalloc(neighbor->size * sizeof (struct site));
#ifdef DEBUG_QMP
memset(neighbor->site, -1, neighbor->size * sizeof (struct site));
#endif

```

This code is used in chunk 31c.

32b \langle Allocate inside table 32b $\rangle \equiv$

```

if (neighbor->inside_size)
    neighbor->inside = tmalloc(neighbor->inside_size * sizeof (int));
else
    neighbor->inside = 0;

```

This code is used in chunk 31c.

32c \langle Allocate boundary table 32c $\rangle \equiv$

```

if (neighbor->boundary_size)
    neighbor->boundary = tmalloc(neighbor->boundary_size * sizeof (struct boundary));
else
    neighbor->boundary = 0;

```

This code is used in chunk 31c.

32d \langle Compute send sizes and allocate index tables 32d $\rangle \equiv$

```

for (i = 0; i < 2 * DIM; i++) {
    int d = i / 2;

    if (network[d] > 1) {
        neighbor->snd_size[i] = neighbor->size / (bounds->hi[d] - bounds->lo[d]);
        neighbor->snd[i] = tmalloc(neighbor->snd_size[i] * sizeof (int));
#ifdef DEBUG_QMP
memset(neighbor->snd[i], -1, neighbor->snd_size[i] * sizeof (int));
#endif
    } else {
        neighbor->snd_size[i] = 0;
        neighbor->snd[i] = 0;
    }
    DEBUG_QMP("Compute send sizes... snd_size[%d]=%d\n",
        i, neighbor->snd_size[i])
}

```

This code is used in chunk 31c.

Here is the definition of the neighbor table we spent soo much time initializing:

```
33a  <Neighbor tables 33a>≡
      struct neighbor {
          int          size;           /* size of site table */
          int          inside_size;    /* number of inside sites */
          int          boundary_size;  /* number of boundary sites */
          int          snd_size[2*DIM]; /* size of send buffers in 8 dirs */
          int          rcv_size[2*DIM]; /* size of receive buffers */
          int          *snd[2*DIM];    /* i->x translation for send buffers */
          int          *inside;        /* i->x translation for inside sites */
          struct boundary *boundary;   /* i->x,mask translation for boundary */
          struct site   *site;         /* x->site translation for sites */
          vHalfFermion  *snd_buf[2*DIM]; /* Send buffers */
          vHalfFermion  *rcv_buf[2*DIM]; /* Receive buffers */

          int          qmp_size[4*DIM]; /* sizes of QMP buffers */
          void          *qmp_xbuf[4*DIM]; /* QMP snd/rcv buffer addresses */
          vHalfFermion  *qmp_buf[4*DIM]; /* send and receive buffers for QMP */
          QMP_msgmem_t   qmp_mm[4*DIM]; /* msgmem's for send and receive */
          int          Nx;             /* number of msecs */

          int          qmp_smask;      /* send flags for qmp_sh[] */
          QMP_msghandle_t qmp_handle;  /* common send & receive handle */
      };
```

This definition is continued in chunk 33.

This code is used in chunk 83.

For boundary sites we only need 8 bits for the boundary indicators. However, allocating a whole `int` for `mask` is what the compiler does anyway.

```
33b  <Neighbor tables 33a>+=
      struct boundary {
          int  index; /* x-index of this boundary site */
          int  mask;  /* bitmask of the borders */
      };
```

In the following structure we keep information about links and neighbors of the site. Note, that there is one address for four forward links: they are packed in memory as defined in the comment.

```
33c  <Neighbor tables 33a>+=
      struct site {
          int Uup; /* up-links are Uup, Uup+1, Uup+2, Uup+3 */
          int Udown[DIM]; /* four down-links */
          int F[2*DIM]; /* eight neighboring fermions on the other sublattice */
      };
```

Now we can define `build_neighbor()`:

```
33d  <Static functions 23f>+=
      static void
      build_neighbor(struct neighbor *out,
                    int par,
                    struct neighbor *in)
      {
          int i,d, s, p, m;
          int x[DIM];

          <Initialize out 34b>
          <Walk through sublattice 34c>
          <Build outside indices 35e>
      }
```

```

34a  <Static function prototypes 21a>+=
      static void build_neighbor(struct neighbor *out,
                                int parity,
                                struct neighbor *in);

```

First part is easy: we start with copying in to out, resetting fields which will be computed shortly.

```

34b  <Initialize out 34b>≡
      *out = *in;
      out->size = 0;
      out->inside_size = 0;
      out->boundary_size = 0;
      for (d = 0; d < DIM; d++) {
          out->rcv_size[2*d] = out->snd_size[2*d] = 0;
          out->rcv_size[2*d+1] = out->snd_size[2*d+1] = 0;
      }

```

This code is used in chunk 33d.

This is a good place to reuse our lattice walking chunks.

```

34c  <Walk through sublattice 34c>≡
      <Start DIM-d sublattice scan 26d>
          <Select same parity 34e>
          <Compute p and m 34f>
          DEBUG_QMP("A: x[%d %d %d %d], (s,par)=(%d,%d), p=%d\n",
                    x[0], x[1], x[2], x[3], s, par, p)
          <Setup boundary or inside 35a>
          <Build local neighbors 35d>
          out->size++;
          in->site++;
      next:
          <Advance DIM-d index for a sublattice scan 26e>

```

This code is used in chunk 33d.

The lattice is broken into a “same” and an “opposite” parity pieces. Here are the selectors:

```

34d  <Select opposite parity 34d>≡
      s = parity(x);
      if (s == par)
          goto next;

```

This code is used in chunk 36a.

```

34e  <Select same parity 34e>≡
      s = parity(x);
      if (s != par)
          goto next;

```

This code is used in chunks 34c and 37b.

For p we use a function to compute it from x. As for m, its eight low bits encode if there is a boundary nearby. Note, that even bits corresponds to *step down* and odd bits correspond to *step up*.

```

34f  <Compute p and m 34f>≡
      p = to_HFlinear(x, &bounds, -1, 0);
      for (m = 0, d = 0; d < DIM; d++) {
          if (network[d] > 1) {
              if (x[d] == bounds.lo[d])
                  m |= 1 << (2 * d);
              if (x[d] + 1 == bounds.hi[d])
                  m |= 1 << (2 * d + 1);
          }
      }

```

This code is used in chunk 34c.

If no boundary was found near **p**, we put it into inside. Otherwise, **p** belongs to the boundary.

```
35a  <Setup boundary or inside 35a>≡
      if (m) {
          <Setup boundary 35c>
      } else {
          <Setup inside 35b>
      }
```

This code is used in chunk 34c.

For the inside, simply add **p** to the list of sites and advance pointers and counters:

```
35b  <Setup inside 35b>≡
      *in->inside++ = p;
      out->inside_size++;
```

This code is used in chunk 35a.

For the boundary, place **p** into **index** and **m** into **mask** and advance pointers. We also take the opportunity to place **p** into send buffers where bits of **m** are set

```
35c  <Setup boundary 35c>≡
      in->boundary->index = p;
      in->boundary->mask = m;
      in->boundary++;
      out->boundary_size++;
```

This code is used in chunk 35a.

We are ready now to build local neighbors. All gauge fields are local, and we still have **m** to tell if the other sublattice neighbor is local or not.

```
35d  <Build local neighbors 35d>≡
      in->site->Uup = to_Ulinear(x, &bounds, -1);
      for (d = 0; d < DIM; d++) {
          in->site->Udown[d] = to_Ulinear(x, &bounds, d);
          if ((m & (1 << (2 * d))) == 0)
              in->site->F[2*d] = Sv * to_HFlinear(x, &bounds, d, -1);
          if ((m & (1 << (2 * d + 1))) == 0)
              in->site->F[2*d + 1] = Sv * to_HFlinear(x, &bounds, d, +1);
      }
```

This code is used in chunk 34c.

The only piece left is the one dealing with outside indices. This is a tricky part, but we just happen to have almost enough machinery already to solve it:

```
35e  <Build outside indices 35e>≡
      for (d = 0; d < DIM; d++) {
          if (network[d] < 2)
              continue;
          construct_rec(out, par, &bounds, d, +1);
          construct_snd(out, in, par, &bounds, d, +1);
          construct_rec(out, par, &bounds, d, -1);
          construct_snd(out, in, par, &bounds, d, -1);
      }
```

This code is used in chunk 33d.

We also need a function that will walk through a boundary of a neighbor building the outside part of the **site[]**.F indices.

```
35f  <Static function prototypes 21a>+≡
      static void construct_rec(struct neighbor *out,
                              int par,
                              struct bounds *bounds,
                              int dir,
                              int step);
```

```

36a  <Static functions 23f>+=
      static void
      construct_rec(struct neighbor *out,
                    int par,
                    struct bounds *bounds,
                    int dir,
                    int step)
      {
          struct bounds xb;
          int xc[DIM], x[DIM];
          int s, d, p, k;
          int dz = dir * 2 + ((step>0)?1:0);

          <Construct the neighbor's network coordinates xc and bounds xb 36b>
          <Construct the initial point of the hypersurface 36c>
          <Start the hypersurface scan 37c>
              <Select opposite parity 34d>
              <Translate x to target p 36d>
              DEBUG_QMP("B: x[%d %d %d %d], (s,par)=(%d,%d), p=%d, k=%d\n",
                        x[0], x[1], x[2], x[3], s, par, p, k)
              <Insert k into site[p].F[dx] 36e>
              <Advance the hypersurface point 37d>
              out->rcv_size[dz^1] = k;
      }

```

Constructing the neighbor's network position is straightforward:

```

36b  <Construct the neighbor's network coordinates xc and bounds xb 36b>=
      for (d = 0; d < DIM; d++) {
          int v = coord[d] + ((d==dir)?step:0);

          if (v < 0)
              v += network[d];
          if (v >= network[d])
              v -= network[d];
          xc[d] = v;
      }
      mk_sublattice(&xb, xc);
      #ifndef NO_DEBUG_QMP
          DEBUG_QMP("par=%d, dir=%d, step=%d\n", par, dir, step)
          for (d = 0; d < DIM; d++)
              DEBUG_QMP("neighbor: xb[%d] lo %d, di %d\n", d, xb.lo[d], xb.hi[d])
      #endif /* defined(DEBUG_QMP) */

```

This code is used in chunks 36a and 37b.

The initial point should be on the surface we are walking:

```

36c  <Construct the initial point of the hypersurface 36c>=
      for (d = 0; d < DIM; d++)
          x[d] = ((d == dir) && (step < 0)) ? (xb.hi[d] - 1) : xb.lo[d];

```

This code is used in chunks 36a and 37b.

```

36d  <Translate x to target p 36d>=
      p = to_HFlinear(x, bounds, dir, -step);

```

This code is used in chunks 36a and 37b.

```

36e  <Insert k into site[p].F[dx] 36e>=
      out->site[p].F[dz] = Sv * k++;

```

This code is used in chunk 36a.

Constructing send buffer indices must match `construct_rec()`. Here we walk the same parity sublattice on the neighbor and index our local *opposite* parity sublattice.

```

37a  <Static function prototypes 21a>+=
      static void construct_snd(struct neighbor *out,
                              struct neighbor *in,
                              int par,
                              struct bounds *bounds,
                              int dir,
                              int step);

37b  <Static functions 23f>+=
      static void
      construct_snd(struct neighbor *out,
                    struct neighbor *in,
                    int par,
                    struct bounds *bounds,
                    int dir,
                    int step)
      {
          struct bounds xb;
          int xc[DIM], x[DIM];
          int s, d, p, k;
          int dz = dir * 2 + ((step>0)?1:0);

          <Construct the neighbor's network coordinates xc and bounds xb 36b>
          <Construct the initial point of the hypersurface 36c>
          <Start the hypersurface scan 37c>
              <Select same parity 34e>
              <Translate x to target p 36d>
              DEBUG_QMP("C: x[%d %d %d %d], (s,par)=(%d,%d), p=%d, k=%d\n",
                        x[0], x[1], x[2], x[3], s, par, p, k)
              *in->snd[dz]++ = p * Sv;
              k++;
          <Advance the hypersurface point 37d>
          out->snd_size[dz] = k;
      }

37c  <Start the hypersurface scan 37c>=
      for (k = 0, d = 0; d < DIM; ) {
This code is used in chunks 36a and 37b.

37d  <Advance the hypersurface point 37d>=
      next:
          for (d = 0; d < DIM; d++) {
              if (d == dir)
                  continue;
              if (++x[d] == xb.hi[d])
                  x[d] = xb.lo[d];
              else
                  break;
          }
      }

```

This code is used in chunks 36a and 37b.

Here we do the reverse, namely, free all memory allocated by `init_tables()`:

```
38a  <Free tables 38a>≡
    {
        int i;

        if (neighbor.site) {
            tfree(neighbor.site);
            neighbor.site = 0;
        }

        if (neighbor.inside) {
            tfree(neighbor.inside);
            neighbor.inside = 0;
        }

        if (neighbor.boundary) {
            tfree(neighbor.boundary);
            neighbor.boundary = 0;
        }

        for (i = 2 * DIM; i--;) {
            if (neighbor.snd[i] == 0)
                continue;
            tfree(neighbor.snd[i]);
            neighbor.snd[i] = 0;
        }
    }
```

This code is used in chunk [15g](#).

5.6.2 Address translation routines

Let us define a couple of functions for translating 4-d lattice positions into 1-d offsets.

Computing linear position on the sublattice is used often enough to be placed in a function. To avoid writing two very similar functions, we pass two arguments `q`, and `z` to specify that q -component of \mathbf{p} should adjusted by z . If $q < 0$, q and z are ignored.

```
38b  <Static function prototypes 21a>+≡
    static int
    to_HFlinear(int p[],
                struct bounds *b,
                int q,
                int z)
    {
        int x, d;
        for (x = 0, d = DIM; d--;) {
            int v = p[d] + ((d == q)?z:0);
            int s = b->hi[d] - b->lo[d];
            if (v < 0)
                v += tlattice[d];
            if (v >= tlattice[d])
                v -= tlattice[d];
            x = x * s + v - b->lo[d];
        }
        return x / 2;
    }
```

Computing the index of the gauge link is similar to `to_HFlinear`, except that the extra parameter `q` tells us which of `p` should be stepped down by one. If $q < 0$, we are computing forward link position.

```
39a <Static function prototypes 21a>+=
static int
to_Ulinear(int p[],
           struct bounds *b,
           int q)
{
    int x, d;

    if ((q < 0) || (p[q] > b->lo[q]) || (network[q] < 2)) {
        <Find index of a regular gauge link 39b>
    } else {
        <Find index of a borrowed gauge link 39c>
    }
}
```

Regular gauge links sits four per site and their indices are easy to compute:

```
39b <Find index of a regular gauge link 39b>=
for (x = 0, d = DIM; d--;) {
    int s = b->hi[d] - b->lo[d];
    int v = p[d] - ((q == d)?1:0);
    if (v < 0)
        v += tlattice[d];
    x = x * s + v - b->lo[d];
}
return DIM * x + ((q < 0)?0:q);
```

This code is used in chunk 39a.

For borrowed links we need first to skip all regulars and previous faces and then count position on the borrowed 3-face:

```
39c <Find index of a borrowed gauge link 39c>=
int s0, v0;
for (d = 0, v0 = 1; d < DIM; d++)
    v0 *= b->hi[d] - b->lo[d];
for (d = 0, s0 = DIM * v0; d < q; d++) {
    if (network[d] < 2)
        continue;
    s0 += v0 / (b->hi[d] - b->lo[d]);
}
for (d = DIM, x = 0; d--;) {
    int s = b->hi[d] - b->lo[d];
    int v = p[d];

    if (d == q)
        continue;
    x = x * s + v - b->lo[d];
}
return s0 + x;
```

This code is used in chunk 39a.

5.7 QMP Initialization

```
39d <Include files 15c>+=
#include <qmp.h>
```

Once the tables and sizes are known, allocate all send and receive buffers and register them with QMP.

```
39e <Initialize QMP 39e>=
if (build_buffers(&even_odd)) goto error;
if (build_buffers(&odd_even)) goto error;
```

This code is used in chunk 14b.

There are three cases we need to consider when preparing the communication handles. Note: return 1 if there was trouble.

```

40a  <Static function prototypes 21a>+=
      static int build_buffers(struct neighbor *nb);

40b  <Static functions 23f>+=
      static int
      build_buffers(struct neighbor *nb)
      {
          int i, k, Nh;
          QMP_msghandle_t SRh[4*DIM];

          DEBUG_QMP("-----\n")
          DEBUG_QMP("build buffers [%s]\n", nb == &even_odd? "even": "odd")
#ifdef NO_DEBUG_QMP
          {
              int i;

              DEBUG_QMP("nb->size          = %d\n", nb->size)
              DEBUG_QMP("nb->inside_size = %d\n", nb->inside_size)
              DEBUG_QMP("nb->boundary_size = %d\n", nb->boundary_size)
              for (i = 0; i < 2 * DIM; i++)
                  DEBUG_QMP("[%d]: snd=%d, rcv=%d\n",
                          i, nb->snd_size[i], nb->rcv_size[i])
          }
#endif /* defined(DEBUG_QMP) */
          Nh = nb->Nx = 0;
          for (i = 0; i < DIM; i++) {
              switch (network[i]) {
                  case 1: break;
                  case 2:
                      <Clump up and down directions 40c>
                      break;
                  default:
                      /* Order here is important */
                      <Allocate down buffers 41b>
                      <Allocate up buffers 41a>
                      break;
              }
          }
          <Construct the collective handle 42d>
          return 0;
      }

```

If there is only two nodes in a direction, we use only up link to communicate (because there is only one wire between the nodes.)

```

40c  <Clump up and down directions 40c>=
      DEBUG_QMP("Allocate up and down buffers, i=%d\n", i)
      k = make_buffer(nb, nb->snd_size[2*i] + nb->snd_size[2*i+1]);
      nb->snd_buf[2*i] = nb->qmp_buf[k];
      nb->snd_buf[2*i+1] = nb->snd_buf[2*i] + Sv * nb->snd_size[2*i];
      Nh = make_send(nb, k, i, +1, SRh, Nh);

      k = make_buffer(nb, nb->rcv_size[2*i] + nb->rcv_size[2*i+1]);
      nb->rcv_buf[2*i+1] = nb->qmp_buf[k]; /* should be opposite to snd_buf[] */
      nb->rcv_buf[2*i] = nb->rcv_buf[2*i+1] + Sv * nb->rcv_size[2*i+1];
      Nh = make_receive(nb, k, i, -1, SRh, Nh); /* -1 fixes a bug in GigE QMP */

```

This code is used in chunk 40b.

On a large machine, up and down buffers are separate:

```
41a  <Allocate up buffers 41a>≡
      DEBUG_QMP("Allocate up buffers, i=%d\n", i)
      k = make_buffer(nb, nb->snd_size[2*i+1]);
      nb->snd_buf[2*i+1] = nb->qmp_buf[k];
      Nh = make_send(nb, k, i, +1, SRh, Nh);

      k = make_buffer(nb, nb->rcv_size[2*i+1]);
      nb->rcv_buf[2*i+1] = nb->qmp_buf[k];
      Nh = make_receive(nb, k, i, +1, SRh, Nh);
```

This code is used in chunk 40b.

```
41b  <Allocate down buffers 41b>≡
      DEBUG_QMP("Allocate down buffers, i=%d\n", i)
      k = make_buffer(nb, nb->snd_size[2*i]);
      nb->snd_buf[2*i] = nb->qmp_buf[k];
      Nh = make_send(nb, k, i, -1, SRh, Nh);

      k = make_buffer(nb, nb->rcv_size[2*i]);
      nb->rcv_buf[2*i] = nb->qmp_buf[k];
      Nh = make_receive(nb, k, i, -1, SRh, Nh);
```

This code is used in chunk 40b.

Allocate a buffer of size vHalfFermion's fit for send and/or receive.

```
41c  <Static function prototypes 21a>+≡
      static int make_buffer(struct neighbor *nb, int size);

41d  <Static functions 23f>+≡
      static int
      make_buffer(struct neighbor *nb, int size)
      {
          int bcount = size * Sv * sizeof (vHalfFermion);
          int N = nb->Nx;

          nb->qmp_size[N] = size;
          sse_aligned_buffer(nb, N, bcount);
          nb->qmp_mm[N] = QMP_declare_msgmem(nb->qmp_buf[N], bcount);
          nb->Nx = N + 1;
          DEBUG_QMP("declare_msgmem(%p,%d)=0x%x\n",
                    nb->qmp_buf[N], bcount, (int)nb->qmp_mm[N])
          return N;
      }
```

Construct a send handle. This function places a send handle into SRh for future construction of the superhandle and sets a bit in qmp_smask.

```
41e  <Static function prototypes 21a>+≡
      static int make_send(struct neighbor *nb, int k, int i, int d,
                          QMP_msghandle_t SRh[4*DIM], int Nsr);
```

```

42a  <Static functions 23f>+≡
      static int
      make_send(struct neighbor *nb, int k, int i, int d,
                QMP_msghandle_t SRh[4*DIM], int Nsr)
      {
          int j = 2 * i + ((d < 0)? 0: 1);

          nb->qmp_smask |= (1 << j);
          SRh[Nsr] = QMP_declare_send_relative(nb->qmp_mm[k], i, d, 1);

          DEBUG_QMP("declare_send_relative(0x%x,%d,%d,1)=0x%x\n",
                    (int)nb->qmp_mm[k], i, d, (int)SRh[Nsr])

          return Nsr+1;
      }

```

Constructing a receive handle is similar. There is no need for a mask bit though.

```

42b  <Static function prototypes 21a>+≡
      static int make_receive(struct neighbor *nb, int k, int i, int d,
                             QMP_msghandle_t SRh[4*DIM], int Nsr);

42c  <Static functions 23f>+≡
      static int
      make_receive(struct neighbor *nb, int k, int i, int d,
                   QMP_msghandle_t SRh[4*DIM], int Nsr)
      {
          SRh[Nsr] = QMP_declare_receive_relative(nb->qmp_mm[k], i, d, 1);

          DEBUG_QMP("declare_receive_relative(0x%x,%d,%d,1)=0x%x\n",
                    (int)nb->qmp_mm[k], i, d, (int)SRh[Nsr])

          return Nsr+1;
      }

```

Finally, aggregate all receive handles:

```

42d  <Construct the collective handle 42d>≡
      if (nb->qmp_smask) {
          nb->qmp_handle = QMP_declare_multiple(SRh, Nh);
          #ifndef NO_DEBUG_QMP
          {
              int i;
              for (i = 0; i < Nh; i++) {
                  DEBUG_QMP("declare_multiple([%d]=0x%x\n", i, (int)SRh[i])
              }
              DEBUG_QMP("declare_multiple(..., %d)=0x%x\n", Nh, (int)nb->qmp_handle)
          }
          #endif
      }

```

This code is used in chunk 40b.

The Vector hardware likes its memory aligned at 16 bytes. We need to keep that in mind when asking for QMP memory. Note, that this function may be in violation of a strict interpretation of the QMP Specification, but on many SciDAC calls numerous assurances were given that such usage is permissible.

```

42e  <Static function prototypes 21a>+≡
      static void sse_aligned_buffer(struct neighbor *nb, int k, int size);

```

43a *<Static functions 23f>+≡*

```
static void
sse_aligned_buffer(struct neighbor *nb, int k, int size)
{
#ifdef USE_QMP2
    nb->qmp_xbuf[k] = QMP_allocate_aligned_memory(size, 128, 0);
    nb->qmp_buf[k] = QMP_get_memory_pointer(nb->qmp_xbuf[k]);
#else
    int xcount = size + 15;
    char *ptr = QMP_allocate_aligned_memory(xcount);
    nb->qmp_buf[k] = (void *)(&ptr[15] & (15 + (unsigned long)(ptr)));
    nb->qmp_xbuf[k] = ptr;
#endif
    DEBUG_QMP("(%p,%d,%d): allocate: 0x%x\n",
              nb, k, size, (int)nb->qmp_xbuf[k])
    DEBUG_QMP("(%p,%d,%d): ptr: %p\n",
              nb, k, size, (void *)nb->qmp_buf[k])
}
```

Freeing QMP structure does the reverse of the allocator:

43b *<Cleanup QMP 43b>≡*

```
free_buffers(&even_odd);
free_buffers(&odd_even);
```

This code is used in chunk 15g.

There are some unsettling omissions in the QMP specification. What follows is based on the tribal wisdom which was not codified.

43c *<Static function prototypes 21a>+≡*

```
static void free_buffers(struct neighbor *nb);
```

43d *<Static functions 23f>+≡*

```
static void
free_buffers(struct neighbor *nb)
{
    int i;

    <Free common handle 43e>
    <Free QMP buffers 44a>
}
```

Here we assume that `QMP_free_msghandle()` knows what to do with a bad handle returned from `QMP_declare_send...` and `QMP_declare_receive...`.

The first common wisdom is that `QMP_declare_multiple()` invalidates individual handles. We only need to free one handle in `nb->qmp_handle`:

43e *<Free common handle 43e>≡*

```
if (nb->qmp_handle) {
    QMP_free_msghandle(nb->qmp_handle);
    DEBUG_QMP("free_msghandle(0x%x) / common receive handle\n",
              (int)nb->qmp_handle)
}
```

This code is used in chunk 43d.

Two steps are needed to deallocate QMP memory:

```

44a  <Free QMP buffers 44a>≡
    for (i = nb->Nx; i--;) {
        if (nb->qmp_mm[i])
            QMP_free_msgmem(nb->qmp_mm[i]);
        DEBUG_QMP("free_msgmem(0x%x)\n", (int)nb->qmp_mm[i]);
    #ifdef USE_QMP2
        if (nb->qmp_xbuf[i])
            QMP_free_memory(nb->qmp_xbuf[i]);
    #else /* QMP 1.3 */
        if (nb->qmp_xbug[i])
            QMP_free_aligned_memory(nb->qmp_xbuf[i]);
    #endif
        DEBUG_QMP("free_memory(0x%x)\n", (int)nb->qmp_xbuf[i]);
    }

```

This code is used in chunk 43d.

5.8 Parts of the Solver

Here are three principal parts of the solver. First, we compute the right hand side of the equation to be solved by the CG. Next, there is a solver of a hermitian matrix. Finally, the second half of the solution is computed.

5.8.1 Compute the RHS

Here we perform steps 1–3 of the outline above.

```

44b  <Compute  $\varphi_o$  44b>≡
    compute_Qee1(auxA_e, eta->even);
    compute_Qoe(auxB_o, auxA_e);
    compute_sum_o(auxA_o, eta->odd, -1, auxB_o);
    compute_Qoo1(auxB_o, auxA_o);
    compute_Mx(Phi_o, auxB_o);

```

This code is used in chunk 19a.

```

44c  <Global variables 13b>+≡
    static vOddFermion *auxA_o, *auxB_o, *Phi_o;
    static vEvenFermion *auxA_e;

```

```

44d  <Allocate fields 44d>≡
    Phi_o = allocate_odd_fermion(); if (Phi_o == 0) goto error;
    auxA_o = allocate_odd_fermion(); if (auxA_o == 0) goto error;
    auxB_o = allocate_odd_fermion(); if (auxB_o == 0) goto error;
    auxA_e = allocate_even_fermion(); if (auxA_e == 0) goto error;

```

This definition is continued in chunk 46.

This code is used in chunk 14b.

```

44e  <Free fields 44e>≡
    if (auxA_e) free16(auxA_e); auxA_e = 0;
    if (auxB_o) free16(auxB_o); auxB_o = 0;
    if (auxA_o) free16(auxA_o); auxA_o = 0;
    if (Phi_o) free16(Phi_o); Phi_o = 0;

```

This definition is continued in chunk 46.

This code is used in chunk 15g.

5.9 Field Operations

Hermitian solver follows:

```

44f  <Solve  $M^\dagger M \psi_o = \varphi_o$  44f>≡
    status = cg(psi->odd, Phi_o, x0->odd, eps, min_iter, max_iter, out_eps, out_iter);

```

This code is used in chunk 19a.

```

45a  <Static function prototypes 21a>+=
      static int cg(vOddFermion *psi,
                   const vOddFermion *b,
                   const vOddFermion *x0,
                   double epsilon,
                   int min_iter,
                   int max_iter,
                   double *out_eps,
                   int *out_iter);

45b  <Static functions 23f>+=
      static int
      cg(vOddFermion *x_o,
         const vOddFermion *b,
         const vOddFermion *x0,
         double epsilon,
         int N0,
         int N,
         double *out_eps,
         int *out_N)
    {
      double rho, alpha, beta, gamma, norm_z;
      int status = 1;
      int k;

      copy_o(x_o, x0);
      compute_MxM(p_o, &norm_z, x_o);
      compute_sum_oN(r_o, &rho, b, -1, p_o);
      copy_o(p_o, r_o);
      <Finalize <r,r> computation 80h>

      for (k = 0; (k < N0) || ((rho > epsilon) && (k < N)); k++) {
        compute_MxM(q_o, &norm_z, p_o);
        <Finalize <r,r> computation 80h>
        alpha = rho / norm_z;
        compute_sum2_oN(r_o, &gamma, -alpha, q_o);
        compute_sum2_o(x_o, alpha, p_o);
        <Finalize <r,r> computation 80h>
        DEBUG_DWF("cg loop: k=%d, rho=%g, norm_z=%g, alpha=%g, gamma=%g\n",
                  k, rho, norm_z, alpha, gamma)
        if (gamma <= epsilon) {
          rho = gamma;
          status = 0;
          break;
        }
        beta = gamma / rho;
        rho = gamma;
        compute_sum2x_o(p_o, r_o, beta);
      }
      *out_N = k;
      *out_eps = rho;

      return status;
    }

Temporaries used by the CG

45c  <Global variables 13b>+=
      static vOddFermion *r_o, *p_o, *q_o;

```

```

46a  <Allocate fields 44d>+=
      r_o = allocate_odd_fermion(); if (r_o == 0) goto error;
      p_o = allocate_odd_fermion(); if (p_o == 0) goto error;
      q_o = allocate_odd_fermion(); if (q_o == 0) goto error;

46b  <Free fields 44e>+=
      if (r_o) free16(r_o); r_o = 0;
      if (p_o) free16(p_o); p_o = 0;
      if (q_o) free16(q_o); q_o = 0;

```

5.9.1 Computing the even part of the result

Again, this is simpling performing step 5 of the outline above:

```

46c  <Compute  $\psi_e$  46c>+=
      compute_Qeo(auxA_e, psi->odd);
      compute_sum_e(auxB_e, eta->even, -1, auxA_e);
      compute_Qee1(psi->even, auxB_e);
This code is used in chunk 19a.

46d  <Global variables 13b>+=
      static vEvenFermion *auxB_e;

46e  <Allocate fields 44d>+=
      auxB_e = allocate_even_fermion(); if (auxB_e == 0) goto error;

46f  <Free fields 44e>+=
      if (auxB_e) free16(auxB_e); auxB_e = 0;

```

5.9.2 copy_o(d, s) or $d \leftarrow s$

This is a copies $d \leftarrow s$. Since it is used outside of the cg loop, we do not worry too much about efficiency here. Hence, cache pollution.

```

46g  <Static function prototypes 21a>+=
      static void copy_o(vOddFermion *dst, const vOddFermion *src);

46h  <Static functions 23f>+=
      static void
      copy_o(vOddFermion *dst, const vOddFermion *src)
      {
          int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);
          vReal *d = (vReal *)dst;
          const vReal *s = (const vReal *)src;

          for ( ;i--;)
              *d++ = *s++;
      }

```

5.9.3 compute_sum2_o(d,alpha,s), or $d \leftarrow d + \alpha s$

This is a function we can not speedup much: too many bytes are needed per operation. In principle, one can play with uncached loads and stores, but let us leave that for later.

```

46i  <Static function prototypes 21a>+=
      static void compute_sum2_o(vOddFermion *dst, double alpha, const vOddFermion *src);

```

47a $\langle \text{Static functions 23f} \rangle + \equiv$

```

static void
compute_sum2_o(vOddFermion *dst, double alpha, const vOddFermion *src)
{
    int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);
    vReal a = vmk_1(alpha);
    vReal *d = (vReal *)dst;
    const vReal *s = (const vReal *)src;

    for ( ; i--;)
        *d++ += a * *s++;
}

```

5.9.4 compute_sum2x_o(d,s,alpha), or $d \leftarrow \alpha d + s$

Almost the same as the previous one, but scaling is applied to another summand.

47b $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```

static void compute_sum2x_o(vOddFermion *dst, const vOddFermion *src, double alpha);

```

47c $\langle \text{Static functions 23f} \rangle + \equiv$

```

static void
compute_sum2x_o(vOddFermion *dst, const vOddFermion *src, double alpha)
{
    int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);
    vReal a = vmk_1(alpha);
    vReal *d = (vReal *)dst;
    const vReal *s = (const vReal *)src;

    for ( ; i--; d++)
        *d = a * *d + *s++;
}

```

5.9.5 compute_sum_x(d,x,alpha,y) or $q \leftarrow x + \alpha y$

Next are a pair of general sums with the destination distinct from the sources. Do we really need separate functions for these?

47d $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```

static void compute_sum_e(vEvenFermion *d,
                        const vEvenFermion *x, double alpha, const vEvenFermion *y);
static void compute_sum_o(vOddFermion *d,
                        const vOddFermion *x, double alpha, const vOddFermion *y);

```

47e $\langle \text{Static functions 23f} \rangle + \equiv$

```

static void
compute_sum_e(vEvenFermion *d,
              const vEvenFermion *x, double alpha, const vEvenFermion *y)
{
    const vReal *X = (const vReal *)x;
    const vReal *Y = (const vReal *)y;
    vReal *D = (vReal *)d;
    vReal a = vmk_1(alpha);
    int i = even_odd.size * Sv * sizeof (vEvenFermion) / sizeof (vReal);

    for (; i--;)
        *D++ = *X++ + a * *Y++;
}

```

48a $\langle \text{Static functions 23f} \rangle + \equiv$

```

static void
compute_sum_o(vOddFermion *d,
              const vOddFermion *x, double alpha, const vOddFermion *y)
{
    const vReal *X = (const vReal *)x;
    const vReal *Y = (const vReal *)y;
    vReal *D = (vReal *)d;
    vReal a = vmk_1(alpha);
    int i = odd_even.size * Sv * sizeof (vOddFermion) / sizeof (vReal);

    for (; i--;)
        *D++ = *X++ + a * *Y++;
}

```

5.9.6 compute_sum_oN(d,norm,x,alpha,y), or $d \leftarrow x + \alpha y$ and $r \leftarrow \langle d, d \rangle$

There are two remaining sums which compute a sum of two fermions and the norm of the result at the same time.

48b $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```

static void compute_sum_oN(vOddFermion *d, double *norm,
                          const vOddFermion *x, double alpha, const vOddFermion *y);

```


49a

 $\langle \text{Static functions 23f} \rangle + \equiv$

```

static void
compute_sum_oN(vOddFermion *d, double *norm,
               const vOddFermion *x, double alpha, const vOddFermion *y)
{
    vFermion *D = &d->f;
    const vFermion *X = &x->f;
    const vFermion *Y = &y->f;
    int n = odd_even.size * Sv;
    vReal a = vmk_1(alpha);
    vReal q0r, q1r, q2r, q0i, q1i, q2i;
    vReal s0r, s1r, s2r, s0i, s1i, s2i;
    int i;

    *norm = 0;
    for (i = 0; i < n; i++, X++, Y++, D++) {
        q0r = D->f[0][0].re = X->f[0][0].re + a * Y->f[0][0].re; s0r = q0r * q0r;
        q0i = D->f[0][0].im = X->f[0][0].im + a * Y->f[0][0].im; s0i = q0i * q0i;
        q1r = D->f[0][1].re = X->f[0][1].re + a * Y->f[0][1].re; s1r = q1r * q1r;
        q1i = D->f[0][1].im = X->f[0][1].im + a * Y->f[0][1].im; s1i = q1i * q1i;
        q2r = D->f[0][2].re = X->f[0][2].re + a * Y->f[0][2].re; s2r = q2r * q2r;
        q2i = D->f[0][2].im = X->f[0][2].im + a * Y->f[0][2].im; s2i = q2i * q2i;

        q0r = D->f[1][0].re = X->f[1][0].re + a * Y->f[1][0].re; s0r += q0r * q0r;
        q0i = D->f[1][0].im = X->f[1][0].im + a * Y->f[1][0].im; s0i += q0i * q0i;
        q1r = D->f[1][1].re = X->f[1][1].re + a * Y->f[1][1].re; s1r += q1r * q1r;
        q1i = D->f[1][1].im = X->f[1][1].im + a * Y->f[1][1].im; s1i += q1i * q1i;
        q2r = D->f[1][2].re = X->f[1][2].re + a * Y->f[1][2].re; s2r += q2r * q2r;
        q2i = D->f[1][2].im = X->f[1][2].im + a * Y->f[1][2].im; s2i += q2i * q2i;

        q0r = D->f[2][0].re = X->f[2][0].re + a * Y->f[2][0].re; s0r += q0r * q0r;
        q0i = D->f[2][0].im = X->f[2][0].im + a * Y->f[2][0].im; s0i += q0i * q0i;
        q1r = D->f[2][1].re = X->f[2][1].re + a * Y->f[2][1].re; s1r += q1r * q1r;
        q1i = D->f[2][1].im = X->f[2][1].im + a * Y->f[2][1].im; s1i += q1i * q1i;
        q2r = D->f[2][2].re = X->f[2][2].re + a * Y->f[2][2].re; s2r += q2r * q2r;
        q2i = D->f[2][2].im = X->f[2][2].im + a * Y->f[2][2].im; s2i += q2i * q2i;

        q0r = D->f[3][0].re = X->f[3][0].re + a * Y->f[3][0].re; s0r += q0r * q0r;
        q0i = D->f[3][0].im = X->f[3][0].im + a * Y->f[3][0].im; s0i += q0i * q0i;
        q1r = D->f[3][1].re = X->f[3][1].re + a * Y->f[3][1].re; s1r += q1r * q1r;
        q1i = D->f[3][1].im = X->f[3][1].im + a * Y->f[3][1].im; s1i += q1i * q1i;
        q2r = D->f[3][2].re = X->f[3][2].re + a * Y->f[3][2].re; s2r += q2r * q2r;
        q2i = D->f[3][2].im = X->f[3][2].im + a * Y->f[3][2].im; s2i += q2i * q2i;

        *norm += vsum(s0r + s0i + s1r + s1i + s2r + s2i);
    }
     $\langle \text{Start } \langle r, r \rangle \text{ computation 81d} \rangle$ 
}

```

49b

 $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```

static void compute_sum2_oN(vOddFermion *d, double *norm,
                           double alpha, const vOddFermion *y);

```

50a $\langle \text{Static functions 23f} \rangle + \equiv$

```

static void
compute_sum2_oN(vOddFermion *d, double *norm,
                double alpha, const vOddFermion *y)
{
    vFermion *D = &d->f;
    const vFermion *Y = &y->f;
    int n = odd_even.size * Sv;
    vReal a = vmk_1(alpha);
    vReal q0r, q1r, q2r, q0i, q1i, q2i;
    vReal s0r, s1r, s2r, s0i, s1i, s2i;
    int i;

    *norm = 0;
    for (i = 0; i < n; i++, Y++, D++) {
        q0r = D->f[0][0].re += a * Y->f[0][0].re; s0r = q0r * q0r;
        q0i = D->f[0][0].im += a * Y->f[0][0].im; s0i = q0i * q0i;
        q1r = D->f[0][1].re += a * Y->f[0][1].re; s1r = q1r * q1r;
        q1i = D->f[0][1].im += a * Y->f[0][1].im; s1i = q1i * q1i;
        q2r = D->f[0][2].re += a * Y->f[0][2].re; s2r = q2r * q2r;
        q2i = D->f[0][2].im += a * Y->f[0][2].im; s2i = q2i * q2i;

        q0r = D->f[1][0].re += a * Y->f[1][0].re; s0r += q0r * q0r;
        q0i = D->f[1][0].im += a * Y->f[1][0].im; s0i += q0i * q0i;
        q1r = D->f[1][1].re += a * Y->f[1][1].re; s1r += q1r * q1r;
        q1i = D->f[1][1].im += a * Y->f[1][1].im; s1i += q1i * q1i;
        q2r = D->f[1][2].re += a * Y->f[1][2].re; s2r += q2r * q2r;
        q2i = D->f[1][2].im += a * Y->f[1][2].im; s2i += q2i * q2i;

        q0r = D->f[2][0].re += a * Y->f[2][0].re; s0r += q0r * q0r;
        q0i = D->f[2][0].im += a * Y->f[2][0].im; s0i += q0i * q0i;
        q1r = D->f[2][1].re += a * Y->f[2][1].re; s1r += q1r * q1r;
        q1i = D->f[2][1].im += a * Y->f[2][1].im; s1i += q1i * q1i;
        q2r = D->f[2][2].re += a * Y->f[2][2].re; s2r += q2r * q2r;
        q2i = D->f[2][2].im += a * Y->f[2][2].im; s2i += q2i * q2i;

        q0r = D->f[3][0].re += a * Y->f[3][0].re; s0r += q0r * q0r;
        q0i = D->f[3][0].im += a * Y->f[3][0].im; s0i += q0i * q0i;
        q1r = D->f[3][1].re += a * Y->f[3][1].re; s1r += q1r * q1r;
        q1i = D->f[3][1].im += a * Y->f[3][1].im; s1i += q1i * q1i;
        q2r = D->f[3][2].re += a * Y->f[3][2].re; s2r += q2r * q2r;
        q2i = D->f[3][2].im += a * Y->f[3][2].im; s2i += q2i * q2i;

        *norm += vsum(s0r + s0i + s1r + s1i + s2r + s2i);
    }
     $\langle \text{Start } \langle r, r \rangle \text{ computation 81d} \rangle$ 
}

```

5.9.7 compute_MxM(eta,norm,psi), or $\eta \leftarrow M^\dagger M \psi$ and friends

Last three easy pieces.

50b $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```

static void compute_MxM(vOddFermion *eta, double *norm,
                        const vOddFermion *psi);
static void compute_M(vOddFermion *eta, double *norm,
                        const vOddFermion *psi);
static void compute_Mx(vOddFermion *eta,
                        const vOddFermion *psi);

```

51a $\langle \text{Static functions 23f} \rangle + \equiv$

```
static void
compute_MxM(vOddFermion *eta, double *norm,
            const vOddFermion *psi)
{
    compute_M(auxB_o, norm, psi);
    compute_Mx(eta, auxB_o);
}
```

Computation of M starts the global sum which will be completed separately.

51b $\langle \text{Static functions 23f} \rangle + \equiv$

```
static void compute_M(vOddFermion *eta, double *norm,
                    const vOddFermion *psi)
{
    compute_Qee1Qeo(auxA_e, psi);
    compute_1Qoo1Qoe(eta, norm, psi, auxA_e);
}
```

For M^\dagger the order of factors differs from optimal. For now we have to live with the inefficiency here.

51c $\langle \text{Static functions 23f} \rangle + \equiv$

```
static void compute_Mx(vOddFermion *eta,
                    const vOddFermion *psi)
{
    compute_Soo1(auxA_o, psi);
    compute_See1Seo(auxA_e, auxA_o);
    compute_1Soe(eta, psi, auxA_e);
}
```

5.9.8 compute_Qee1(eta,psi), or $\eta \leftarrow Q_{ee}^{-1}\psi$

Some code savings are still possible, since compute_Qee1() may differ from compute_Qoo1() by the number of sites only.

51d $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```
static void compute_Qxx1(vFermion *eta, const vFermion *psi, int xyzt);
static void inline compute_Qee1(vEvenFermion *eta, const vEvenFermion *psi)
{
    compute_Qxx1(&eta->f, &psi->f, even_odd.size);
}
```

5.9.9 compute_Qoo1(eta,psi), or $\eta \leftarrow Q_{oo}^{-1}\psi$

51e $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```
static void inline compute_Qoo1(vOddFermion *eta, const vOddFermion *psi)
{
    compute_Qxx1(&eta->f, &psi->f, odd_even.size);
}
```

5.9.10 compute_Qxx1(eta,psi), or $\eta \leftarrow Q_{xx}^{-1}\psi$

52a $\langle \text{Static functions 23f} \rangle + \equiv$
static void
compute_Qxx1(vFermion *chi, const vFermion *psi, int size)
{
 const vFermion *qs, *qx5;
 $\langle Q \text{ common locals 80a} \rangle$
 $\langle Q_{xx} \text{ locals 56a} \rangle$

 for (i = 0; i < size; i++) {
 xyzt5 = i * Sv;
 $\langle \text{Compute rx5 80e} \rangle$
 $\langle \text{Compute qx5 80f} \rangle$
 $\langle \text{Compute } Q_{xx}^{-1} \text{ part on the s-chain 53a} \rangle$
 }
}

5.9.11 compute_Soo1(eta,psi), or $\eta \leftarrow S_{oo}^{-1}\psi$

52b $\langle \text{Static function prototypes 21a} \rangle + \equiv$
static void compute_Soo1(vOddFermion *eta, const vOddFermion *psi);

52c $\langle \text{Static functions 23f} \rangle + \equiv$
static void
compute_Soo1(vOddFermion *Chi, const vOddFermion *Psi)
{
 vFermion *chi = &Chi->f;
 const vFermion *psi = &Psi->f;
 int size = odd_even.size;
 const vFermion *qs, *qx5;
 $\langle Q \text{ common locals 80a} \rangle$
 $\langle Q_{xx} \text{ locals 56a} \rangle$

 for (i = 0; i < size; i++) {
 xyzt5 = i * Sv;
 $\langle \text{Compute rx5 80e} \rangle$
 $\langle \text{Compute qx5 80f} \rangle$
 $\langle \text{Compute } S_{xx}^{-1} \text{ part on the s-chain 53b} \rangle$
 }
}

5.9.12 Q_{xx}^{-1} and S_{xx}^{-1} on a single s -chain

Therefore,

53a $\langle \text{Compute } Q_{xx}^{-1} \text{ part on the } s\text{-chain 53a} \rangle \equiv$
 $\langle \text{Compute } A^{-1}\psi \text{ on the upper two components 53c} \rangle$
 $\langle \text{Compute } B^{-1}\psi \text{ on the lower two components 53f} \rangle$

This code is used in chunks 52a, 74, and 76a.

53b $\langle \text{Compute } S_{xx}^{-1} \text{ part on the } s\text{-chain 53b} \rangle \equiv$
 $\langle \text{Compute } A^{-1}\psi \text{ on the lower two components 53d} \rangle$
 $\langle \text{Compute } B^{-1}\psi \text{ on the upper two components 53e} \rangle$

This code is used in chunks 52c and 74.

And

53c $\langle \text{Compute } A^{-1}\psi \text{ on the upper two components 53c} \rangle \equiv$
 $\langle \text{Compute } L_A^{-1} \text{ on the upper components 56b} \rangle$
 $\langle \text{Compute } R_A^{-1} \text{ on the upper components 59b} \rangle$

This code is used in chunk 53a.

53d $\langle \text{Compute } A^{-1}\psi \text{ on the lower two components 53d} \rangle \equiv$
 $\langle \text{Compute } L_A^{-1} \text{ on the lower components 57a} \rangle$
 $\langle \text{Compute } R_A^{-1} \text{ on the lower components 59c} \rangle$

This code is used in chunk 53b.

53e $\langle \text{Compute } B^{-1}\psi \text{ on the upper two components 53e} \rangle \equiv$
 $\langle \text{Compute } L_B^{-1} \text{ on the upper components 58a} \rangle$
 $\langle \text{Compute } R_B^{-1} \text{ on the upper components 59d} \rangle$

This code is used in chunk 53b.

53f $\langle \text{Compute } B^{-1}\psi \text{ on the lower two components 53f} \rangle \equiv$
 $\langle \text{Compute } L_B^{-1} \text{ on the lower components 58b} \rangle$
 $\langle \text{Compute } R_B^{-1} \text{ on the lower components 59e} \rangle$

This code is used in chunk 53a.

For both Q_{xx}^{-1} and S_{xx}^{-1} we need to compute R_A and R_B . This can be done iteratively:

$$y_k^{(A)} = \begin{cases} \frac{1}{a}z_k, & \text{if } k = n - 1 \\ \frac{1}{a}z_k - \frac{b}{a}y_{k+1}^{(A)}, & \text{otherwise} \end{cases}$$

$$y_k^{(B)} = \begin{cases} \frac{1}{a}z_0, & \text{if } k = 0 \\ \frac{1}{a}z_k - \frac{b}{a}y_{k-1}^{(B)}, & \text{otherwise} \end{cases}$$

It turns out, that these computations are faster on the regular FP instructions than on vectors. For this reason corresponding parts for L_X^{-1} depend on the memory layout of `vReal`.

Let us compute constant pieces first. Division is slow, so we compute $1/a$ and $-b/a$ once and for all:

53g $\langle \text{Global variables 13b} \rangle + \equiv$
`static REAL inv_a;`
`static REAL b_over_a;`

53h $\langle \text{Compute values from } a, b \text{ and } c \text{ 53h} \rangle \equiv$
`inv_a = 1.0 / a;`
`b_over_a = -b * inv_a;`

This definition is continued in chunk 55b.

This code is used in chunk 80g.

Computing $z^{(A)} = L_A^{-1}x$ and $z^{(B)} = L_B^{-1}x$ is easy:

$$\begin{aligned} z_k^{(A)} &= \begin{cases} -\sum_{j=0}^{n-2} \frac{(-b)^j c/a^{j+1}}{1+(-b)^{n-1}c/a^n} x_j + \frac{1}{1+(-b)^{n-1}c/a^n} x_{n-1}, & \text{if } k = n-1 \\ x_k, & \text{otherwise} \end{cases} \\ z_k^{(B)} &= \begin{cases} \frac{1}{1+(-b)^{n-1}c/a^n} x_0 - \sum_{j=1}^{n-1} \frac{(-b)^{n-1-j} c/a^{n-j}}{1+(-b)^{n-1}c/a^n} x_j, & \text{if } k = 0 \\ x_k, & \text{otherwise} \end{cases} \end{aligned}$$

We need to rewrite these expressions in a form suitable for a vector processor.

If $Vs=4$, we can write

$$\begin{aligned} z_{n-1}^{(A)} &= z_a^{(A)} + z_b^{(A)} + z_c^{(A)} + z_d^{(A)} \\ z_0^{(B)} &= z_a^{(B)} + z_b^{(B)} + z_c^{(B)} + z_d^{(B)} \end{aligned}$$

where

$$\begin{aligned} z_a^{(A)} &= \sum_{j=0}^{n/4-1} \frac{-b^{4j} c/a^{4j+1}}{1+(-b)^{n-1}c/a^n} x_{4j} \\ z_b^{(A)} &= \sum_{j=0}^{n/4-1} \frac{b^{4j+1} c/a^{4j+2}}{1+(-b)^{n-1}c/a^n} x_{4j+1} \\ z_c^{(A)} &= \sum_{j=0}^{n/4-1} \frac{-b^{4j+2} c/a^{4j+3}}{1+(-b)^{n-1}c/a^n} x_{4j+2} \\ z_d^{(A)} &= \sum_{j=0}^{n/4-2} \frac{b^{4j+3} c/a^{4j+4}}{1+(-b)^{n-1}c/a^n} x_{4j+3} + \frac{1}{1+(-b)^{n-1}c/a^n} x_{n-1} \\ z_a^{(B)} &= \frac{1}{1+(-b)^{n-1}c/a^n} x_0 + \sum_{j=1}^{n/4-1} \frac{b^{n-1-4j} c/a^{n-4j}}{1+(-b)^{n-1}c/a^n} x_{4j} \\ z_b^{(B)} &= \sum_{j=0}^{n/4-1} \frac{-b^{n-2-4j} c/a^{n-4j-1}}{1+(-b)^{n-1}c/a^n} x_{4j+1} \\ z_c^{(B)} &= \sum_{j=0}^{n/4-1} \frac{b^{n-3-4j} c/a^{n-4j-2}}{1+(-b)^{n-1}c/a^n} x_{4j+2} \\ z_d^{(B)} &= \sum_{j=0}^{n/4-1} \frac{-b^{n-4-4j} c/a^{n-4j-3}}{1+(-b)^{n-1}c/a^n} x_{4j+3} \end{aligned}$$

For $Vs=2$, we can write

$$\begin{aligned} z_{n-1}^{(A)} &= z_e^{(A)} + z_f^{(A)} \\ z_0^{(B)} &= z_e^{(B)} + z_f^{(B)} \end{aligned}$$

where

$$\begin{aligned} z_e^{(A)} &= \sum_{j=0}^{n/2-1} \frac{-b^{2j}c/a^{2j+1}}{1 + (-b)^{n-1}c/a^n} x_{2j} \\ z_f^{(A)} &= \sum_{j=0}^{n/2-2} \frac{b^{2j+1}c/a^{2j+2}}{1 + (-b)^{n-1}c/a^n} x_{2j+1} + \frac{1}{1 + (-b)^{n-1}c/a^n} x_{n-1} \\ z_e^{(B)} &= \frac{1}{1 + (-b)^{n-1}c/a^n} x_0 + \sum_{j=1}^{n/2-1} \frac{b^{n-1-2j}c/a^{n-2j}}{1 + (-b)^{n-1}c/a^n} x_{2j} \\ z_f^{(B)} &= \sum_{j=0}^{n/2-1} \frac{-b^{n-2-2j}c/a^{n-2j-1}}{1 + (-b)^{n-1}c/a^n} x_{2j+1} \end{aligned}$$

These sums could be effectively computed with vector operations, because their structures match DWF memory layout. Here are constants needed to compute $z^{(A)}$ and $z^{(B)}$:

```
55a  <Global variables 13b>+=
      static vReal vfx_A;
      static vReal vfx_B;
      static vReal vab;
      static REAL c0;

55b  <Compute values from a, b and c 53h>+=
      c0 = 1./(1.-c/b*d_pow(b/a, Sv*Vs));
      vab = vmk_1(d_pow(b/a, Vs));
      vfx_A = vmk_fn(-c0*c/a, -b/a);
      vfx_B = vmk_bn(-b/a, -c0*c/a);
```

5.9.13 Compute L_A^{-1} and L_B^{-1}

There are two cases:

1. L_X^{-1} is computed as part of standalone diagonal piece. In this case, the computation is done from q to r and L_X^{-1} copies elements as needed.
2. Q_{xx}^{-1} is part of combined operator. In this case q is aliased to r , and no copy is performed.

Before spelling out the details, let us define a few handy macros:

```
55c  <Check xx-aliasing of q 55c>=
      #if defined(qs)
      #define QSETUP(s)
      #define Q2R(d,pt)
      #else
      #define QSETUP(s) qs = &qx5[s];
      #define Q2R(d,pt) rs->f[d][c].pt = qs->f[d][c].pt;
      #endif
```

This code is used in chunks 56–58.

```
55d  <End xx-aliasing of q 55d>=
      #undef QSETUP
      #undef Q2R
```

This code is used in chunks 56–58.

For completeness, here are definitions of variables used in the pieces bellow:

56a $\langle Q_{xx} \text{ locals } 56a \rangle \equiv$
`vReal fx;
vHalfFermion zV;
vector_complex zn;
scalar_complex zX[Fd/2][Nc];`

This definition is continued in chunk 59f.
This code is used in chunks 52, 63, and 64a.

56b $\langle \text{Compute } L_A^{-1} \text{ on the upper components } 56b \rangle \equiv$
`vhfzero(&zV);
fx = vfx_A;
<Check xx-aliasing of q 55c>
for (s = 0; s < Sv_1; s++, fx = fx * vab) {
 rs = &rx5[s];
 QSETUP(s)
<Compute zV ← zV + fx * qsup 56c>
}
rs = &rx5[Sv_1];
QSETUP(Sv_1)
fx = vput_n(fx, c0);
<Compute zV ← zV + fx * qsup 56c>
for (c = 0; c < Nc; c++) {
<Compute wall value in zX[c] 59a>

 zn.re = qs->f[0][c].re; zn.im = qs->f[0][c].im;
 zn.re = vput_n(zn.re, zX[0][c].re); zn.im = vput_n(zn.im, zX[0][c].im);
 rs->f[0][c].re = zn.re; rs->f[0][c].im = zn.im;

 zn.re = qs->f[1][c].re; zn.im = qs->f[1][c].im;
 zn.re = vput_n(zn.re, zX[1][c].re); zn.im = vput_n(zn.im, zX[1][c].im);
 rs->f[1][c].re = zn.re; rs->f[1][c].im = zn.im;
}
<End xx-aliasing of q 55d>`

This code is used in chunk 53c.

This piece is used twice: once in the loop over L_s , and the second time after correcting s_3 :

56c $\langle \text{Compute } zV \leftarrow zV + fx * qs^{up} 56c \rangle \equiv$
`for (c = 0; c < Nc; c++) {
 zV.f[0][c].re += fx * qs->f[0][c].re; Q2R(0,re)
 zV.f[0][c].im += fx * qs->f[0][c].im; Q2R(0,im)
 zV.f[1][c].re += fx * qs->f[1][c].re; Q2R(1,re)
 zV.f[1][c].im += fx * qs->f[1][c].im; Q2R(1,im)
}`

This code is used in chunks 56b and 58a.

The only difference between L_A^{-1} on lower components is the source of the data and the destination of the result. We have to repeat most of the above pieces though.

57a $\langle \text{Compute } L_A^{-1} \text{ on the lower components 57a} \rangle \equiv$

```

vhfzero(&zV);
fx = vfx_A;
 $\langle \text{Check } xx\text{-aliasing of } q \text{ 55c} \rangle$ 
for (s = 0; s < Sv_1; s++, fx = fx * vab) {
    rs = &rx5[s];
    QSETUP(s)
     $\langle \text{Compute } zV \leftarrow zV + fx * qs^{down} \text{ 57b} \rangle$ 
}
rs = &rx5[Sv_1];
QSETUP(Sv_1)
fx = vput_n(fx, c0);
 $\langle \text{Compute } zV \leftarrow zV + fx * qs^{down} \text{ 57b} \rangle$ 
for (c = 0; c < Nc; c++) {
     $\langle \text{Compute wall value in } zX[c] \text{ 59a} \rangle$ 

    zn.re = qs->f[2][c].re;          zn.im = qs->f[2][c].im;
    zn.re = vput_n(zn.re, zX[0][c].re); zn.im = vput_n(zn.im, zX[0][c].im);
    rs->f[2][c].re = zn.re;          rs->f[2][c].im = zn.im;

    zn.re = qs->f[3][c].re;          zn.im = qs->f[3][c].im;
    zn.re = vput_n(zn.re, zX[1][c].re); zn.im = vput_n(zn.im, zX[1][c].im);
    rs->f[3][c].re = zn.re;          rs->f[3][c].im = zn.im;
}
 $\langle \text{End } xx\text{-aliasing of } q \text{ 55d} \rangle$ 

```

This code is used in chunk 53d.

57b $\langle \text{Compute } zV \leftarrow zV + fx * qs^{down} \text{ 57b} \rangle \equiv$

```

for (c = 0; c < Nc; c++) {
    zV.f[0][c].re += fx * qs->f[2][c].re; Q2R(2,re)
    zV.f[0][c].im += fx * qs->f[2][c].im; Q2R(2,im)
    zV.f[1][c].re += fx * qs->f[3][c].re; Q2R(3,re)
    zV.f[1][c].im += fx * qs->f[3][c].im; Q2R(3,im)
}

```

This code is used in chunks 57a and 58b.

For L_B^{-1} the difference is in the direction of the sweep along the s -chain:

```

58a  <Compute  $L_B^{-1}$  on the upper components 58a>≡
      vhfzero(&zV);
      fx = vfx_B;
      <Check xx-aliasing of q 55c>
      for (s = Sv; --s; fx = fx * vab) {
          rs = &rx5[s];
          QSETUP(s)
          <Compute  $zV \leftarrow zV + fx * qs^{up}$  56c>
      }
      rs = &rx5[0];
      QSETUP(0)
      fx = vput_0(fx, c0);
      <Compute  $zV \leftarrow zV + fx * qs^{up}$  56c>
      for (c = 0; c < Nc; c++) {
          <Compute wall value in zX[c] 59a>

          zn.re = qs->f[0][c].re;          zn.im = qs->f[0][c].im;
          zn.re = vput_0(zn.re, zX[0][c].re); zn.im = vput_0(zn.im, zX[0][c].im);
          rs->f[0][c].re = zn.re;          rs->f[0][c].im = zn.im;

          zn.re = qs->f[1][c].re;          zn.im = qs->f[1][c].im;
          zn.re = vput_0(zn.re, zX[1][c].re); zn.im = vput_0(zn.im, zX[1][c].im);
          rs->f[1][c].re = zn.re;          rs->f[1][c].im = zn.im;
      }
      <End xx-aliasing of q 55d>

```

This code is used in chunk 53e.

Again, some repetition is needed for the lower component case:

```

58b  <Compute  $L_B^{-1}$  on the lower components 58b>≡
      vhfzero(&zV);
      fx = vfx_B;
      <Check xx-aliasing of q 55c>
      for (s = Sv; --s; fx = fx * vab) {
          rs = &rx5[s];
          QSETUP(s)
          <Compute  $zV \leftarrow zV + fx * qs^{down}$  57b>
      }
      rs = &rx5[0];
      QSETUP(0)
      fx = vput_0(fx, c0);
      <Compute  $zV \leftarrow zV + fx * qs^{down}$  57b>
      for (c = 0; c < Nc; c++) {
          <Compute wall value in zX[c] 59a>

          zn.re = qs->f[2][c].re;          zn.im = qs->f[2][c].im;
          zn.re = vput_0(zn.re, zX[0][c].re); zn.im = vput_0(zn.im, zX[0][c].im);
          rs->f[2][c].re = zn.re;          rs->f[2][c].im = zn.im;

          zn.re = qs->f[3][c].re;          zn.im = qs->f[3][c].im;
          zn.re = vput_0(zn.re, zX[1][c].re); zn.im = vput_0(zn.im, zX[1][c].im);
          rs->f[3][c].re = zn.re;          rs->f[3][c].im = zn.im;
      }
      <End xx-aliasing of q 55d>

```

This code is used in chunk 53f.

By now, we have four partial sums which must be combined into z_{n-1} :

```
59a  ⟨Compute wall value in zX[c] 59a⟩≡
      zX[0][c].re = vsum(zV.f[0][c].re);
      zX[0][c].im = vsum(zV.f[0][c].im);
      zX[1][c].re = vsum(zV.f[1][c].re);
      zX[1][c].im = vsum(zV.f[1][c].im);
```

This code is used in chunks 56–58.

5.9.14 Compute R_A^{-1} and R_B^{-1}

Since R_X^{-1} is always computed after L_X^{-1} , it takes its source from **rs** and places the result back into **rs**. For R_X^{-1} , again combinations of A and B and upper and lower parts require some cut, paste and edit.

```
59b  ⟨Compute  $R_A^{-1}$  on the upper components 59b⟩≡
      ⟨Init out of bound y 60a⟩
      for (s = Sv; s--;) {
          rs = &rx5[s];
          ⟨Compute  $y_{k,[0]}^{(A)}$  60b⟩
          ⟨Compute  $y_{k,[1]}^{(A)}$  60c⟩
      }
```

This code is used in chunk 53c.

```
59c  ⟨Compute  $R_A^{-1}$  on the lower components 59c⟩≡
      ⟨Init out of bound y 60a⟩
      for (s = Sv; s--;) {
          rs = &rx5[s];
          ⟨Compute  $y_{k,[2]}^{(A)}$  60d⟩
          ⟨Compute  $y_{k,[3]}^{(A)}$  60e⟩
      }
```

This code is used in chunk 53d.

```
59d  ⟨Compute  $R_B^{-1}$  on the upper components 59d⟩≡
      ⟨Init out of bound y 60a⟩
      for (s = 0; s < Sv; s++) {
          rs = &rx5[s];
          ⟨Compute  $y_{k,[0]}^{(B)}$  60f⟩
          ⟨Compute  $y_{k,[1]}^{(B)}$  60g⟩
      }
```

This code is used in chunk 53e.

```
59e  ⟨Compute  $R_B^{-1}$  on the lower components 59e⟩≡
      ⟨Init out of bound y 60a⟩
      for (s = 0; s < Sv; s++) {
          rs = &rx5[s];
          ⟨Compute  $y_{k,[2]}^{(B)}$  60h⟩
          ⟨Compute  $y_{k,[3]}^{(B)}$  60i⟩
      }
```

This code is used in chunk 53f.

We do not handle boundary cases in a special way. Instead, the previous value of y is stored in **yOut**:

```
59f  ⟨ $Q_{xx}$  locals 56a⟩+≡
      scalar_complex yOut[Fd/2][Nc];
```

60a $\langle \text{Init out of bound } y \text{ 60a} \rangle \equiv$
`yOut[0][0].re = yOut[0][0].im = 0;
yOut[0][1].re = yOut[0][1].im = 0;
yOut[0][2].re = yOut[0][2].im = 0;
yOut[1][0].re = yOut[1][0].im = 0;
yOut[1][1].re = yOut[1][1].im = 0;
yOut[1][2].re = yOut[1][2].im = 0;`

This code is used in chunk 59.

Now, the magic of copy paste. These are some of many places where we assume $N_c=3$:

60b $\langle \text{Compute } y_{k,[0]}^{(A)} \text{ 60b} \rangle \equiv$
`COMPUTE_YA(0,0,0) COMPUTE_YA(0,0,1) COMPUTE_YA(0,0,2)`

This code is used in chunk 59b.

60c $\langle \text{Compute } y_{k,[1]}^{(A)} \text{ 60c} \rangle \equiv$
`COMPUTE_YA(1,1,0) COMPUTE_YA(1,1,1) COMPUTE_YA(1,1,2)`

This code is used in chunk 59b.

60d $\langle \text{Compute } y_{k,[2]}^{(A)} \text{ 60d} \rangle \equiv$
`COMPUTE_YA(2,0,0) COMPUTE_YA(2,0,1) COMPUTE_YA(2,0,2)`

This code is used in chunk 59c.

60e $\langle \text{Compute } y_{k,[3]}^{(A)} \text{ 60e} \rangle \equiv$
`COMPUTE_YA(3,1,0) COMPUTE_YA(3,1,1) COMPUTE_YA(3,1,2)`

This code is used in chunk 59c.

60f $\langle \text{Compute } y_{k,[0]}^{(B)} \text{ 60f} \rangle \equiv$
`COMPUTE_YB(0,0,0) COMPUTE_YB(0,0,1) COMPUTE_YB(0,0,2)`

This code is used in chunk 59d.

60g $\langle \text{Compute } y_{k,[1]}^{(B)} \text{ 60g} \rangle \equiv$
`COMPUTE_YB(1,1,0) COMPUTE_YB(1,1,1) COMPUTE_YB(1,1,2)`

This code is used in chunk 59d.

60h $\langle \text{Compute } y_{k,[2]}^{(B)} \text{ 60h} \rangle \equiv$
`COMPUTE_YB(2,0,0) COMPUTE_YB(2,0,1) COMPUTE_YB(2,0,2)`

This code is used in chunk 59e.

60i $\langle \text{Compute } y_{k,[3]}^{(B)} \text{ 60i} \rangle \equiv$
`COMPUTE_YB(3,1,0) COMPUTE_YB(3,1,1) COMPUTE_YB(3,1,2)`

This code is used in chunk 59e.

Now, the computations for a single color and given in/out- and temp- spinor indices:

60j $\langle \text{Definitions 60j} \rangle \equiv$

```
#define COMPUTE_YA(j,t,c) {
    REAL *v_re = (REAL *)&rs->f[j][c].re;
    REAL *v_im = (REAL *)&rs->f[j][c].im;
    BLOCKOF_YA(j,t,c,re)
    BLOCKOF_YA(j,t,c,im) }
```

This definition is continued in chunks 60, 61, and 82.

This code is used in chunk 83b.

60k $\langle \text{Definitions 60j} \rangle + \equiv$

```
#define COMPUTE_YB(j,t,c) {
    REAL *v_re = (REAL *)&rs->f[j][c].re;
    REAL *v_im = (REAL *)&rs->f[j][c].im;
    BLOCKOF_YB(j,t,c,re)
    BLOCKOF_YB(j,t,c,im) }
```

Next, let us implement BLOCKOF_* for Vs==2 and Vs==4. The target definitions of vector operations should agree with the layout here, of course.

If Vs==4 then

```
61a  <Definitions 60j>+≡
      #define BLOCKOF2_YA(j,t,c,ri) \
      v_##ri[1] = inv_a * v_##ri[1] + b_over_a * yOut[t][c].ri; \
      yOut[t][c].ri = v_##ri[0] = inv_a * v_##ri[0] + b_over_a * v_##ri[1];
```

```
61b  <Definitions 60j>+≡
      #define BLOCKOF4_YA(j,t,c,ri) \
      v_##ri[3] = inv_a * v_##ri[3] + b_over_a * yOut[t][c].ri; \
      v_##ri[2] = inv_a * v_##ri[2] + b_over_a * v_##ri[3]; \
      v_##ri[1] = inv_a * v_##ri[1] + b_over_a * v_##ri[2]; \
      yOut[t][c].ri = v_##ri[0] = inv_a * v_##ri[0] + b_over_a * v_##ri[1];
```

```
61c  <Blocks for YA and YB of length four 61c>≡
      #define BLOCKOF_YA(j,t,c,ri) BLOCKOF4_YA(j,t,c,ri)
      #define BLOCKOF_YB(j,t,c,ri) BLOCKOF4_YB(j,t,c,ri)
```

This code is used in chunks 83e and 88c.

Else if Vs==2 then

```
61d  <Definitions 60j>+≡
      #define BLOCKOF2_YB(j,t,c,ri) \
      v_##ri[0] = inv_a * v_##ri[0] + b_over_a * yOut[t][c].ri; \
      yOut[t][c].ri = v_##ri[1] = inv_a * v_##ri[1] + b_over_a * v_##ri[0];
```

```
61e  <Definitions 60j>+≡
      #define BLOCKOF4_YB(j,t,c,ri) \
      v_##ri[0] = inv_a * v_##ri[0] + b_over_a * yOut[t][c].ri; \
      v_##ri[1] = inv_a * v_##ri[1] + b_over_a * v_##ri[0]; \
      v_##ri[2] = inv_a * v_##ri[2] + b_over_a * v_##ri[1]; \
      yOut[t][c].ri = v_##ri[3] = inv_a * v_##ri[3] + b_over_a * v_##ri[2];
```

```
61f  <Blocks for YA and YB of length two 61f>≡
      #define BLOCKOF_YA(j,t,c,ri) BLOCKOF2_YA(j,t,c,ri)
      #define BLOCKOF_YB(j,t,c,ri) BLOCKOF2_YB(j,t,c,ri)
```

This code is used in chunks 86b, 90f, and 92e.

Otherwise we will get an error. That is what we want.

5.9.15 Standalone off-diagonal pieces

First, simple off-diagonal parts.

```
61g  <Static function prototypes 21a>+≡
      static void compute_Qxy(vFermion *d, const vFermion *s, struct neighbor *nb);
```

5.9.16 compute_Qoe(d,s) or $d \leftarrow Q_{eo}s$

```
61h  <Static function prototypes 21a>+≡
      static void inline compute_Qoe(vOddFermion *d, const vEvenFermion *s)
      {
          compute_Qxy(&d->f, &s->f, &odd_even);
      }
```

5.9.17 compute_Qeo(d,s) or $d \leftarrow Q_{oe}s$

```
61i  <Static function prototypes 21a>+≡
      static void inline compute_Qeo(vEvenFermion *d, const vOddFermion *s)
      {
          compute_Qxy(&d->f, &s->f, &even_odd);
      }
```

5.9.18 compute_1Soe(d,q,s) or $d \leftarrow q - S_{eo}s$

62a $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```
static void compute_1Sxy(vFermion *d,
                        const vFermion *q,
                        const vFermion *s,
                        struct neighbor *nb);
static void inline compute_1Soe(vOddFermion *d,
                               const vOddFermion *q,
                               const vEvenFermion *s)
{
    compute_1Sxy(&d->f, &q->f, &s->f, &odd_even);
}
```

5.9.19 compute_Qxy(chi,psi), or $\chi \leftarrow Q_{xy}\psi$

62b $\langle \text{Static functions 23f} \rangle + \equiv$

```
static void
compute_Qxy(vFermion *chi,
            const vFermion *psi,
            struct neighbor *nb)
{
     $\langle Q \text{ common locals 80a} \rangle$ 
     $\langle Q_{xy} \text{ locals 80b} \rangle$ 

     $\langle \text{Setup xy-aliasing of } q \text{ 65b} \rangle$ 
     $\langle \text{Compute projections for } Q \text{ send 66c} \rangle$ 
     $\langle \text{Start sends and receives 81a} \rangle$ 
     $\langle \text{Compute inside part for } Q_{xy} \text{ 68e} \rangle$ 
     $\langle \text{Finish sends and receives 81b} \rangle$ 
     $\langle \text{Compute boundary part for } Q_{xy} \text{ 69a} \rangle$ 
     $\langle \text{Finish xy-aliasing of } q \text{ 65c} \rangle$ 
}
```

5.9.20 compute_1Sxy(chi,eta,psi), or $\chi \leftarrow \eta - S_{xy}\psi$

For other functions, little need to be changes at this granularity. E.g., here is the final part of M^\dagger :

62c $\langle \text{Static functions 23f} \rangle + \equiv$

```
static void
compute_1Sxy(vFermion *chi,
            const vFermion *eta,
            const vFermion *psi,
            struct neighbor *nb)
{
     $\langle Q \text{ common locals 80a} \rangle$ 
     $\langle Q_{xy} \text{ locals 80b} \rangle$ 

     $\langle \text{Setup xy-aliasing of } q \text{ 65b} \rangle$ 
     $\langle \text{Compute projections for } S \text{ send 67a} \rangle$ 
     $\langle \text{Start sends and receives 81a} \rangle$ 
     $\langle \text{Compute inside part for } 1 - S_{xy} \text{ 71c} \rangle$ 
     $\langle \text{Finish sends and receives 81b} \rangle$ 
     $\langle \text{Compute boundary part for } 1 - S_{xy} \text{ 71d} \rangle$ 
     $\langle \text{Finish xy-aliasing of } q \text{ 65c} \rangle$ 
}
```

5.9.21 `compute_Qxx1Qxy(chi,psi)`, or $\chi \leftarrow Q_{xx}^{-1}Q_{xy}\psi$

63a

```

⟨Static functions 23f⟩+=
static void
compute_Qxx1Qxy(vFermion *chi,
                const vFermion *psi,
                struct neighbor *nb)
{
    ⟨Q common locals 80a⟩
    ⟨Qxy locals 80b⟩
    ⟨Qxx locals 56a⟩

    ⟨Setup xy-aliasing of q 65b⟩
    ⟨Compute projections for Q send 66c⟩
    ⟨Start sends and receives 81a⟩
    ⟨Compute inside part for Qxx-1Qxy 74a⟩
    ⟨Finish sends and receives 81b⟩
    ⟨Compute boundary part for Qxx-1Qxy 74b⟩
    ⟨Finish xy-aliasing of q 65c⟩
}

```

5.9.22 `compute_Sxx1Sxy(chi,psi)`, or $\chi \leftarrow S_{xx}^{-1}S_{xy}\psi$

63b

```

⟨Static functions 23f⟩+=
static void
compute_Sxx1Sxy(vFermion *chi,
                const vFermion *psi,
                struct neighbor *nb)
{
    ⟨Q common locals 80a⟩
    ⟨Qxy locals 80b⟩
    ⟨Qxx locals 56a⟩

    ⟨Setup xy-aliasing of q 65b⟩
    ⟨Compute projections for S send 67a⟩
    ⟨Start sends and receives 81a⟩
    ⟨Compute inside part for Sxx-1Sxy 74c⟩
    ⟨Finish sends and receives 81b⟩
    ⟨Compute boundary part for Sxx-1Sxy 74d⟩
    ⟨Finish xy-aliasing of q 65c⟩
}

```

5.9.23 `compute_1Qxx1Qxy(chi,norm,eta,psi)`, or $\chi \leftarrow \eta - Q_{xx}^{-1}Q_{xy}\psi$ and $r \leftarrow \langle \chi, \chi \rangle$

64a

```

⟨Static functions 23f⟩+=
static void
compute_1Qxx1Qxy(vFermion *chi,
                  double *norm,
                  const vFermion *eta,
                  const vFermion *psi,
                  struct neighbor *nb)
{
    ⟨Q common locals 80a⟩
    ⟨Qxy locals 80b⟩
    ⟨Qxx locals 56a⟩
    vReal vv;
    vReal nv;
    *norm = 0;

    ⟨Setup xy-aliasing of q 65b⟩
    ⟨Compute projections for Q send 66c⟩
    ⟨Start sends and receives 81a⟩
    ⟨Compute inside part for 1 - Qxx-1Qxy 75c⟩
    ⟨Finish sends and receives 81b⟩
    ⟨Compute boundary part for 1 - Qxx-1Qxy 75d⟩
    ⟨Start ⟨r,r⟩ computation 81d⟩
    ⟨Finish xy-aliasing of q 65c⟩
}

```

5.9.24 `compute_Dx(chi,eta,psi)`, or $\chi_x \leftarrow Q_{xx}\eta_x + Q_{xy}\psi_y$

64b

```

⟨Static functions 23f⟩+=
static void
compute_Dx(vFermion *chi,
            const vFermion *eta,
            const vFermion *psi,
            struct neighbor *nb)
{
    ⟨Q common locals 80a⟩
    ⟨Qxy locals 80b⟩
    ⟨Dxx locals 78d⟩

    ⟨Setup xy-aliasing of q 65b⟩
    ⟨Compute projections for Q send 66c⟩
    ⟨Start sends and receives 81a⟩
    ⟨Compute inside part for Qxxη + Qxyψ 76c⟩
    ⟨Finish sends and receives 81b⟩
    ⟨Compute boundary part for Qxxη + Qxyψ 76d⟩
    ⟨Finish xy-aliasing of q 65c⟩
}

```


5.9.25 compute_Dcx(chi,eta,psi), or $\chi_x \leftarrow S_{xx}\eta_x + S_{xy}\psi_y$

65a $\langle \text{Static functions 23f} \rangle + \equiv$

```
static void
compute_Dcx(vFermion *chi,
            const vFermion *eta,
            const vFermion *psi,
            struct neighbor *nb)
{
     $\langle Q \text{ common locals 80a} \rangle$ 
     $\langle Q_{xy} \text{ locals 80b} \rangle$ 
     $\langle D_{xx} \text{ locals 78d} \rangle$ 

     $\langle \text{Setup xy-aliasing of } q \text{ 65b} \rangle$ 
     $\langle \text{Compute projections for } S \text{ send 67a} \rangle$ 
     $\langle \text{Start sends and receives 81a} \rangle$ 
     $\langle \text{Compute inside part for } S_{xx}\eta + S_{xy}\psi \text{ 77a} \rangle$ 
     $\langle \text{Finish sends and receives 81b} \rangle$ 
     $\langle \text{Compute boundary part for } S_{xx}\eta + S_{xy}\psi \text{ 77b} \rangle$ 
     $\langle \text{Finish xy-aliasing of } q \text{ 65c} \rangle$ 
}
```

5.9.26 Aliasing macros

Remember, that Z_{xy} always puts result into q. For standalone diagonal pieces a couple of `define`'s help to manage `__restrict__` pointers properly.

65b $\langle \text{Setup xy-aliasing of } q \text{ 65b} \rangle + \equiv$

```
#define qx5 rx5
#define qs rs
```

This code is used in chunks 62–65.

65c $\langle \text{Finish xy-aliasing of } q \text{ 65c} \rangle + \equiv$

```
#undef qs
#undef qx5
```

This code is used in chunks 62–65.

5.9.27 compute_De(chi,eta,psi), or $\chi \leftarrow Q_{ee}\eta + Q_{eo}\psi$

65d $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```
static void compute_Dx(vFermion *chi,
                      const vFermion *eta,
                      const vFermion *psi,
                      struct neighbor *nb);
static void inline compute_De(vEvenFermion *chi,
                             const vEvenFermion *eta,
                             const vOddFermion *psi)
{
    compute_Dx(&chi->f, &eta->f, &psi->f, &even_odd);
}
```

5.9.28 compute_Do(chi,eta,psi), or $\chi \leftarrow Q_{oo}\eta + Q_{oe}\psi$

65e $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```
static void inline compute_Do(vOddFermion *chi,
                             const vOddFermion *eta,
                             const vEvenFermion *psi)
{
    compute_Dx(&chi->f, &eta->f, &psi->f, &odd_even);
}
```

5.9.29 `compute_Dce(chi,eta,psi)`, or $\chi \leftarrow S_{ee}\eta + S_{eo}\psi$

66a $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```
static void compute_Dcx(vFermion *chi,
                        const vFermion *eta,
                        const vFermion *psi,
                        struct neighbor *nb);
static void inline compute_Dce(vEvenFermion *chi,
                              const vEvenFermion *eta,
                              const vOddFermion *psi)
{
    compute_Dcx(&chi->f, &eta->f, &psi->f, &even_odd);
}
```

5.9.30 `compute_Dco(chi,eta,psi)`, or $\chi \leftarrow S_{oo}\eta + S_{oe}\psi$

66b $\langle \text{Static function prototypes 21a} \rangle + \equiv$

```
static void inline compute_Dco(vOddFermion *chi,
                              const vOddFermion *eta,
                              const vEvenFermion *psi)
{
    compute_Dcx(&chi->f, &eta->f, &psi->f, &odd_even);
}
```

5.9.31 Projections to be sent

Next we compute $(1 \pm \gamma_\mu)$ projections to be sent to our neighbors. There are two cases here, one of Q_{xy} and another for S_{xy} . In principle, we might have handled both of them with some jungling of the `struct neighbor` tables, but let us go a simple if extensive way for now.

Notice that projections are done in the opposite direction than for inside sites. This is because the receiving node looks backward where we send forward.

66c $\langle \text{Compute projections for } Q \text{ send 66c} \rangle \equiv$

```
{
    int k, i, s, c, *src;
    const vFermion *f;
    vHalfFermion *g;

    k = 0;  $\langle \text{Construct } (1 + \gamma_0) \text{ send } k\text{-buffer 67b} \rangle$ 
    k = 1;  $\langle \text{Construct } (1 - \gamma_0) \text{ send } k\text{-buffer 67c} \rangle$ 
    k = 2;  $\langle \text{Construct } (1 + \gamma_1) \text{ send } k\text{-buffer 67d} \rangle$ 
    k = 3;  $\langle \text{Construct } (1 - \gamma_1) \text{ send } k\text{-buffer 67e} \rangle$ 
    k = 4;  $\langle \text{Construct } (1 + \gamma_2) \text{ send } k\text{-buffer 68a} \rangle$ 
    k = 5;  $\langle \text{Construct } (1 - \gamma_2) \text{ send } k\text{-buffer 68b} \rangle$ 
    k = 6;  $\langle \text{Construct } (1 + \gamma_3) \text{ send } k\text{-buffer 68c} \rangle$ 
    k = 7;  $\langle \text{Construct } (1 - \gamma_3) \text{ send } k\text{-buffer 68d} \rangle$ 
}
```

This code is used in chunks 62–64.

67a $\langle \text{Compute projections for } S \text{ send 67a} \rangle \equiv$

```

{
    int k, i, s, c, *src;
    const vFermion *f;
    vHalfFermion *g;

    k = 0;  $\langle \text{Construct } (1 - \gamma_0) \text{ send } k\text{-buffer 67c} \rangle$ 
    k = 1;  $\langle \text{Construct } (1 + \gamma_0) \text{ send } k\text{-buffer 67b} \rangle$ 
    k = 2;  $\langle \text{Construct } (1 - \gamma_1) \text{ send } k\text{-buffer 67e} \rangle$ 
    k = 3;  $\langle \text{Construct } (1 + \gamma_1) \text{ send } k\text{-buffer 67d} \rangle$ 
    k = 4;  $\langle \text{Construct } (1 - \gamma_2) \text{ send } k\text{-buffer 68b} \rangle$ 
    k = 5;  $\langle \text{Construct } (1 + \gamma_2) \text{ send } k\text{-buffer 68a} \rangle$ 
    k = 6;  $\langle \text{Construct } (1 - \gamma_3) \text{ send } k\text{-buffer 68d} \rangle$ 
    k = 7;  $\langle \text{Construct } (1 + \gamma_3) \text{ send } k\text{-buffer 68c} \rangle$ 
}

```

This code is used in chunks 62c, 63b, and 65a.

67b $\langle \text{Construct } (1 + \gamma_0) \text{ send } k\text{-buffer 67b} \rangle \equiv$

```

for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
    for (s = Sv, f = &psi[*src]; s--; g++, f++) {
        for (c = 0; c < Nc; c++) {
             $\langle \text{Build } (1 + \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 6a} \rangle$ 
        }
    }
}

```

This code is used in chunks 66c and 67a.

67c $\langle \text{Construct } (1 - \gamma_0) \text{ send } k\text{-buffer 67c} \rangle \equiv$

```

for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
    for (s = Sv, f = &psi[*src]; s--; g++, f++) {
        for (c = 0; c < Nc; c++) {
             $\langle \text{Build } (1 - \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 6c} \rangle$ 
        }
    }
}

```

This code is used in chunks 66c and 67a.

67d $\langle \text{Construct } (1 + \gamma_1) \text{ send } k\text{-buffer 67d} \rangle \equiv$

```

for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
    for (s = Sv, f = &psi[*src]; s--; g++, f++) {
        for (c = 0; c < Nc; c++) {
             $\langle \text{Build } (1 + \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 6e} \rangle$ 
        }
    }
}

```

This code is used in chunks 66c and 67a.

67e $\langle \text{Construct } (1 - \gamma_1) \text{ send } k\text{-buffer 67e} \rangle \equiv$

```

for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
    for (s = Sv, f = &psi[*src]; s--; g++, f++) {
        for (c = 0; c < Nc; c++) {
             $\langle \text{Build } (1 - \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 6g} \rangle$ 
        }
    }
}

```

This code is used in chunks 66c and 67a.

68a $\langle \text{Construct } (1 + \gamma_2) \text{ send } k\text{-buffer } 68a \rangle \equiv$

```

for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
    for (s = Sv, f = &psi[*src]; s--; g++, f++) {
        for (c = 0; c < Nc; c++) {
             $\langle \text{Build } (1 + \gamma_2) \text{ projection of } *f \text{ in } *g \text{ } 8a \rangle$ 
        }
    }
}

```

This code is used in chunks 66c and 67a.

68b $\langle \text{Construct } (1 - \gamma_2) \text{ send } k\text{-buffer } 68b \rangle \equiv$

```

for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
    for (s = Sv, f = &psi[*src]; s--; g++, f++) {
        for (c = 0; c < Nc; c++) {
             $\langle \text{Build } (1 - \gamma_2) \text{ projection of } *f \text{ in } *g \text{ } 8c \rangle$ 
        }
    }
}

```

This code is used in chunks 66c and 67a.

68c $\langle \text{Construct } (1 + \gamma_3) \text{ send } k\text{-buffer } 68c \rangle \equiv$

```

for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
    for (s = Sv, f = &psi[*src]; s--; g++, f++) {
        for (c = 0; c < Nc; c++) {
             $\langle \text{Build } (1 + \gamma_3) \text{ projection of } *f \text{ in } *g \text{ } 8e \rangle$ 
        }
    }
}

```

This code is used in chunks 66c and 67a.

68d $\langle \text{Construct } (1 - \gamma_3) \text{ send } k\text{-buffer } 68d \rangle \equiv$

```

for (i = nb->snd_size[k], g = nb->snd_buf[k], src = nb->snd[k]; i--; src++) {
    for (s = Sv, f = &psi[*src]; s--; g++, f++) {
        for (c = 0; c < Nc; c++) {
             $\langle \text{Build } (1 - \gamma_3) \text{ projection of } *f \text{ in } *g \text{ } 9a \rangle$ 
        }
    }
}

```

This code is used in chunks 66c and 67a.

5.9.32 Parts of $Q_{xy}\psi$

Let us start with the simplest of the five operators we need.

68e $\langle \text{Compute inside part for } Q_{xy} \text{ } 68e \rangle \equiv$

```

for (i = 0; i < nb->inside_size; i++) {
    xyz_t = nb->inside[i];
    xyz_t5 = xyz_t * Sv;
     $\langle \text{Extract 1-d addresses } 80d \rangle$ 
     $\langle \text{Build vector } SU(3) \text{ objects } 79a \rangle$ 
     $\langle \text{Compute } Q_{xy} \text{ part on the inside } s\text{-chain } 69b \rangle$ 
}

```

This code is used in chunk 62b.

```

69a  <Compute boundary part for  $Q_{xy}$  69a>≡
      for (i = 0; i < nb->boundary_size; i++) {
          int m = nb->boundary[i].mask;

          xyzt = nb->boundary[i].index;
          xyzt5 = xyzt * Sv;
          <Extract 1-d addresses 80d>
          <Build vector  $SU(3)$  objects 79a>
          <Compute  $Q_{xy}$  part on the boundary s-chain 69c>
      }

```

This code is used in chunk 62b.

```

69b  <Compute  $Q_{xy}$  part on the inside s-chain 69b>≡
      for (s = 0; s < Sv; s++) {
          <Compute  $Q$  inside  $\gamma$ -projections 69d>
          <Inside multiply by  $V$ s 70c>
          <Compute  $Q$   $\gamma$ -unprojections and sum the results 70b>
      }

```

This code is used in chunks 68e and 74–76.

```

69c  <Compute  $Q_{xy}$  part on the boundary s-chain 69c>≡
      for (s = 0; s < Sv; s++) {
          <Compute  $Q$  boundary  $\gamma$ -projections 70a>
          <Boundary multiply by  $V$ s 71a>
          <Compute  $Q$   $\gamma$ -unprojections and sum the results 70b>
      }

```

This code is used in chunks 69a and 74–76.

```

69d  <Compute  $Q$  inside  $\gamma$ -projections 69d>≡
      <Construct neighbor pointers 79b>
      for (c = 0; c < Nc; c++) {
          k=0; f=&psi[ps[0]]; g=&gg[0]; <Build  $(1 - \gamma_0)$  projection of *f in *g 6c>
          k=1; f=&psi[ps[1]]; g=&gg[1]; <Build  $(1 + \gamma_0)$  projection of *f in *g 6a>
          k=2; f=&psi[ps[2]]; g=&gg[2]; <Build  $(1 - \gamma_1)$  projection of *f in *g 6g>
          k=3; f=&psi[ps[3]]; g=&gg[3]; <Build  $(1 + \gamma_1)$  projection of *f in *g 6e>
          k=4; f=&psi[ps[4]]; g=&gg[4]; <Build  $(1 - \gamma_2)$  projection of *f in *g 8c>
          k=5; f=&psi[ps[5]]; g=&gg[5]; <Build  $(1 + \gamma_2)$  projection of *f in *g 8a>
          k=6; f=&psi[ps[6]]; g=&gg[6]; <Build  $(1 - \gamma_3)$  projection of *f in *g 9a>
          k=7; f=&psi[ps[7]]; g=&gg[7]; <Build  $(1 + \gamma_3)$  projection of *f in *g 8e>
      }

```

This code is used in chunk 69b.

70a $\langle \text{Compute } Q \text{ boundary } \gamma\text{-projections 70a} \rangle \equiv$
 $\langle \text{Construct neighbor pointers 79b} \rangle$

```

for (c = 0; c < 3; c++) {
    if ((m & 0x01) == 0) {
        k=0; f=&psi[ps[0]]; g=&gg[0];  $\langle \text{Build } (1 - \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 6c} \rangle$ 
    }
    if ((m & 0x02) == 0) {
        k=1; f=&psi[ps[1]]; g=&gg[1];  $\langle \text{Build } (1 + \gamma_0) \text{ projection of } *f \text{ in } *g \text{ 6a} \rangle$ 
    }
    if ((m & 0x04) == 0) {
        k=2; f=&psi[ps[2]]; g=&gg[2];  $\langle \text{Build } (1 - \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 6g} \rangle$ 
    }
    if ((m & 0x08) == 0) {
        k=3; f=&psi[ps[3]]; g=&gg[3];  $\langle \text{Build } (1 + \gamma_1) \text{ projection of } *f \text{ in } *g \text{ 6e} \rangle$ 
    }
    if ((m & 0x10) == 0) {
        k=4; f=&psi[ps[4]]; g=&gg[4];  $\langle \text{Build } (1 - \gamma_2) \text{ projection of } *f \text{ in } *g \text{ 8c} \rangle$ 
    }
    if ((m & 0x20) == 0) {
        k=5; f=&psi[ps[5]]; g=&gg[5];  $\langle \text{Build } (1 + \gamma_2) \text{ projection of } *f \text{ in } *g \text{ 8a} \rangle$ 
    }
    if ((m & 0x40) == 0) {
        k=6; f=&psi[ps[6]]; g=&gg[6];  $\langle \text{Build } (1 - \gamma_3) \text{ projection of } *f \text{ in } *g \text{ 9a} \rangle$ 
    }
    if ((m & 0x80) == 0) {
        k=7; f=&psi[ps[7]]; g=&gg[7];  $\langle \text{Build } (1 + \gamma_3) \text{ projection of } *f \text{ in } *g \text{ 8e} \rangle$ 
    }
}

```

This code is used in chunk 69c.

70b $\langle \text{Compute } Q \gamma\text{-unprojections and sum the results 70b} \rangle \equiv$

```

rs = &rx5[s];
for (c = 0; c < Nc; c++) {
    k = 7;  $\langle \text{Unproject } (1 + \gamma_3) \text{ link 8f} \rangle$ 
    k = 6;  $\langle \text{Unproject and accumulate } (1 - \gamma_3) \text{ link 9b} \rangle$ 
    k = 3;  $\langle \text{Unproject and accumulate } (1 + \gamma_1) \text{ link 6f} \rangle$ 
    k = 2;  $\langle \text{Unproject and accumulate } (1 - \gamma_1) \text{ link 7} \rangle$ 
    k = 1;  $\langle \text{Unproject and accumulate } (1 + \gamma_0) \text{ link 6b} \rangle$ 
    k = 0;  $\langle \text{Unproject and accumulate } (1 - \gamma_0) \text{ link 6d} \rangle$ 
    k = 5;  $\langle \text{Unproject and accumulate } (1 + \gamma_2) \text{ link 8b} \rangle$ 
    k = 4;  $\langle \text{Unproject and accumulate } (1 - \gamma_2) \text{ link 8d} \rangle$ 
}

```

This code is used in chunk 69.

Now we have everything we need to compute $U(1 \pm \gamma_\mu)\psi$ pieces:

70c $\langle \text{Inside multiply by } V s \text{ 70c} \rangle \equiv$

```

for (d = 0; d < 2*DIM; d++) {
    vHalfFermion * __restrict__ h = &hh[d];
    vSU3 *u = &V[d];
    g = &gg[d];
     $\langle \text{Multiply } *u \text{ by } *g \text{ and store the result in } *h \text{ 71b} \rangle$ 
}

```

This code is used in chunks 69b, 72a, and 74e.

If the neighbor is on another node, it is in the receive buffer by now.

```
71a  <Boundary multiply by Vs 71a>≡
      for (d = 0; d < 2*DIM; d++) {
          vHalfFermion * __restrict__ h = &hh[d];
          vSU3 *u = &V[d];
          g = (m & (1 << d))? &nb->rcv_buf[d][ps[d]]: &gg[d];
          <Multiply *u by *g and store the result in *h 71b>
      }
```

This code is used in chunks 69c, 72b, and 75a.

```
71b  <Multiply *u by *g and store the result in *h 71b>≡
      for (c = 0; c < Nc; c++) {
          h->f[0][c].re=u->v[c][0].re*g->f[0][0].re-u->v[c][0].im*g->f[0][0].im
              +u->v[c][1].re*g->f[0][1].re-u->v[c][1].im*g->f[0][1].im
              +u->v[c][2].re*g->f[0][2].re-u->v[c][2].im*g->f[0][2].im;
          h->f[0][c].im=u->v[c][0].im*g->f[0][0].re+u->v[c][0].re*g->f[0][0].im
              +u->v[c][1].im*g->f[0][1].re+u->v[c][1].re*g->f[0][1].im
              +u->v[c][2].im*g->f[0][2].re+u->v[c][2].re*g->f[0][2].im;
          h->f[1][c].re=u->v[c][0].re*g->f[1][0].re-u->v[c][0].im*g->f[1][0].im
              +u->v[c][1].re*g->f[1][1].re-u->v[c][1].im*g->f[1][1].im
              +u->v[c][2].re*g->f[1][2].re-u->v[c][2].im*g->f[1][2].im;
          h->f[1][c].im=u->v[c][0].im*g->f[1][0].re+u->v[c][0].re*g->f[1][0].im
              +u->v[c][1].im*g->f[1][1].re+u->v[c][1].re*g->f[1][1].im
              +u->v[c][2].im*g->f[1][2].re+u->v[c][2].re*g->f[1][2].im;
      }
```

This code is used in chunks 70c and 71a.

5.9.33 Parts of $\eta - S_{xy}\psi$

```
71c  <Compute inside part for  $1 - S_{xy}$  71c>≡
      for (i = 0; i < nb->inside_size; i++) {
          const vFermion *ex5, *es;

          xyzt = nb->inside[i];
          xyzt5 = xyzt * Sv;
          <Extract 1-d addresses 80d>
          ex5 = &eta[xyzt5];
          <Build vector SU(3) objects 79a>
          <Compute  $1 - S_{xy}$  part on the inside s-chain 72a>
      }
```

This code is used in chunk 62c.

```
71d  <Compute boundary part for  $1 - S_{xy}$  71d>≡
      for (i = 0; i < nb->boundary_size; i++) {
          const vFermion *ex5, *es;
          int m = nb->boundary[i].mask;

          xyzt = nb->boundary[i].index;
          xyzt5 = xyzt * Sv;
          <Extract 1-d addresses 80d>
          ex5 = &eta[xyzt5];
          <Build vector SU(3) objects 79a>
          <Compute  $1 - S_{xy}$  part on the boundary s-chain 72b>
      }
```

This code is used in chunk 62c.

72a $\langle \text{Compute } 1 - S_{xy} \text{ part on the inside } s\text{-chain } 72a \rangle \equiv$
 for (s = 0; s < Sv; s++) {
 $\langle \text{Compute } S \text{ inside } \gamma\text{-projections } 72c \rangle$
 $\langle \text{Inside multiply by } Vs \text{ } 70c \rangle$
 $\langle \text{Compute } 1 - S \text{ } \gamma\text{-unprojections and sum the results } 73b \rangle$
 }

This code is used in chunk 71c.

72b $\langle \text{Compute } 1 - S_{xy} \text{ part on the boundary } s\text{-chain } 72b \rangle \equiv$
 for (s = 0; s < Sv; s++) {
 $\langle \text{Compute } S \text{ boundary } \gamma\text{-projections } 73a \rangle$
 $\langle \text{Boundary multiply by } Vs \text{ } 71a \rangle$
 $\langle \text{Compute } 1 - S \text{ } \gamma\text{-unprojections and sum the results } 73b \rangle$
 }

This code is used in chunk 71d.

72c $\langle \text{Compute } S \text{ inside } \gamma\text{-projections } 72c \rangle \equiv$
 $\langle \text{Construct neighbor pointers } 79b \rangle$
 for (c = 0; c < Nc; c++) {
 k=0; f=&psi[ps[0]]; g=&gg[0]; $\langle \text{Build } (1 + \gamma_0) \text{ projection of } *f \text{ in } *g \text{ } 6a \rangle$
 k=1; f=&psi[ps[1]]; g=&gg[1]; $\langle \text{Build } (1 - \gamma_0) \text{ projection of } *f \text{ in } *g \text{ } 6c \rangle$
 k=2; f=&psi[ps[2]]; g=&gg[2]; $\langle \text{Build } (1 + \gamma_1) \text{ projection of } *f \text{ in } *g \text{ } 6e \rangle$
 k=3; f=&psi[ps[3]]; g=&gg[3]; $\langle \text{Build } (1 - \gamma_1) \text{ projection of } *f \text{ in } *g \text{ } 6g \rangle$
 k=4; f=&psi[ps[4]]; g=&gg[4]; $\langle \text{Build } (1 + \gamma_2) \text{ projection of } *f \text{ in } *g \text{ } 8a \rangle$
 k=5; f=&psi[ps[5]]; g=&gg[5]; $\langle \text{Build } (1 - \gamma_2) \text{ projection of } *f \text{ in } *g \text{ } 8c \rangle$
 k=6; f=&psi[ps[6]]; g=&gg[6]; $\langle \text{Build } (1 + \gamma_3) \text{ projection of } *f \text{ in } *g \text{ } 8e \rangle$
 k=7; f=&psi[ps[7]]; g=&gg[7]; $\langle \text{Build } (1 - \gamma_3) \text{ projection of } *f \text{ in } *g \text{ } 9a \rangle$
 }

This code is used in chunks 72a and 74e.

73a \langle Compute S boundary γ -projections 73a $\rangle \equiv$

```

 $\langle$  Construct neighbor pointers 79b  $\rangle$ 
for (c = 0; c < Nc; c++) {
    if ((m & 0x01) == 0) {
        k=0; f=&psi[ps[0]]; g=&gg[0];  $\langle$  Build  $(1 + \gamma_0)$  projection of *f in *g 6a  $\rangle$ 
    }
    if ((m & 0x02) == 0) {
        k=1; f=&psi[ps[1]]; g=&gg[1];  $\langle$  Build  $(1 - \gamma_0)$  projection of *f in *g 6c  $\rangle$ 
    }
    if ((m & 0x04) == 0) {
        k=2; f=&psi[ps[2]]; g=&gg[2];  $\langle$  Build  $(1 + \gamma_1)$  projection of *f in *g 6e  $\rangle$ 
    }
    if ((m & 0x08) == 0) {
        k=3; f=&psi[ps[3]]; g=&gg[3];  $\langle$  Build  $(1 - \gamma_1)$  projection of *f in *g 6g  $\rangle$ 
    }
    if ((m & 0x10) == 0) {
        k=4; f=&psi[ps[4]]; g=&gg[4];  $\langle$  Build  $(1 + \gamma_2)$  projection of *f in *g 8a  $\rangle$ 
    }
    if ((m & 0x20) == 0) {
        k=5; f=&psi[ps[5]]; g=&gg[5];  $\langle$  Build  $(1 - \gamma_2)$  projection of *f in *g 8c  $\rangle$ 
    }
    if ((m & 0x40) == 0) {
        k=6; f=&psi[ps[6]]; g=&gg[6];  $\langle$  Build  $(1 + \gamma_3)$  projection of *f in *g 8e  $\rangle$ 
    }
    if ((m & 0x80) == 0) {
        k=7; f=&psi[ps[7]]; g=&gg[7];  $\langle$  Build  $(1 - \gamma_3)$  projection of *f in *g 9a  $\rangle$ 
    }
}

```

This code is used in chunks 72b and 75a.

73b \langle Compute $1 - S$ γ -unprojections and sum the results 73b $\rangle \equiv$

```

rs = &rx5[s];
es = &ex5[s];
for (c = 0; c < Nc; c++) {
    k = 6;  $\langle$  Unproject  $(1 + \gamma_3)$  link 8f  $\rangle$ 
    k = 7;  $\langle$  Unproject and accumulate  $(1 - \gamma_3)$  link 9b  $\rangle$ 
    k = 2;  $\langle$  Unproject and accumulate  $(1 + \gamma_1)$  link 6f  $\rangle$ 
    k = 3;  $\langle$  Unproject and accumulate  $(1 - \gamma_1)$  link 7  $\rangle$ 
    k = 1;  $\langle$  Unproject and accumulate  $(1 - \gamma_0)$  link 6d  $\rangle$ 
    k = 0;  $\langle$  Unproject and accumulate  $(1 + \gamma_0)$  link 6b  $\rangle$ 
    k = 5;  $\langle$  Unproject and accumulate  $(1 - \gamma_2)$  link 8d  $\rangle$ 
    k = 4;  $\langle$  Unproject and accumulate  $(1 + \gamma_2)$  link 8b  $\rangle$ 
     $\langle$  Compute  $(*rs) \leftarrow \eta - (*rs)$  for color c 73c  $\rangle$ 
}

```

This code is used in chunk 72.

73c \langle Compute $(*rs) \leftarrow \eta - (*rs)$ for color c 73c $\rangle \equiv$

```

rs->f[0][c].re = es->f[0][c].re - rs->f[0][c].re;
rs->f[0][c].im = es->f[0][c].im - rs->f[0][c].im;
rs->f[1][c].re = es->f[1][c].re - rs->f[1][c].re;
rs->f[1][c].im = es->f[1][c].im - rs->f[1][c].im;
rs->f[2][c].re = es->f[2][c].re - rs->f[2][c].re;
rs->f[2][c].im = es->f[2][c].im - rs->f[2][c].im;
rs->f[3][c].re = es->f[3][c].re - rs->f[3][c].re;
rs->f[3][c].im = es->f[3][c].im - rs->f[3][c].im;

```

This code is used in chunk 73b.

5.9.34 Parts of $Q_{xx}^{-1}Q_{xy}\psi$

```

74a  ⟨Compute inside part for  $Q_{xx}^{-1}Q_{xy}$  74a⟩≡
      for (i = 0; i < nb->inside_size; i++) {
          xyzt = nb->inside[i];
          xyzt5 = xyzt * Sv;
          ⟨Extract 1-d addresses 80d⟩
          ⟨Build vector  $SU(3)$  objects 79a⟩
          ⟨Compute  $Q_{xy}$  part on the inside s-chain 69b⟩
          ⟨Compute  $Q_{xx}^{-1}$  part on the s-chain 53a⟩
      }

```

This code is used in chunk 63a.

```

74b  ⟨Compute boundary part for  $Q_{xx}^{-1}Q_{xy}$  74b⟩≡
      for (i = 0; i < nb->boundary_size; i++) {
          int m = nb->boundary[i].mask;

          xyzt = nb->boundary[i].index;
          xyzt5 = xyzt * Sv;
          ⟨Extract 1-d addresses 80d⟩
          ⟨Build vector  $SU(3)$  objects 79a⟩
          ⟨Compute  $Q_{xy}$  part on the boundary s-chain 69c⟩
          ⟨Compute  $Q_{xx}^{-1}$  part on the s-chain 53a⟩
      }

```

This code is used in chunk 63a.

5.9.35 Parts of $S_{xx}^{-1}S_{xy}\psi$

```

74c  ⟨Compute inside part for  $S_{xx}^{-1}S_{xy}$  74c⟩≡
      for (i = 0; i < nb->inside_size; i++) {
          xyzt = nb->inside[i];
          xyzt5 = xyzt * Sv;
          ⟨Extract 1-d addresses 80d⟩
          ⟨Build vector  $SU(3)$  objects 79a⟩
          ⟨Compute  $S_{xy}$  part on the inside s-chain 74e⟩
          ⟨Compute  $S_{xx}^{-1}$  part on the s-chain 53b⟩
      }

```

This code is used in chunk 63b.

```

74d  ⟨Compute boundary part for  $S_{xx}^{-1}S_{xy}$  74d⟩≡
      for (i = 0; i < nb->boundary_size; i++) {
          int m = nb->boundary[i].mask;

          xyzt = nb->boundary[i].index;
          xyzt5 = xyzt * Sv;
          ⟨Extract 1-d addresses 80d⟩
          ⟨Build vector  $SU(3)$  objects 79a⟩
          ⟨Compute  $S_{xy}$  part on the boundary s-chain 75a⟩
          ⟨Compute  $S_{xx}^{-1}$  part on the s-chain 53b⟩
      }

```

This code is used in chunk 63b.

```

74e  ⟨Compute  $S_{xy}$  part on the inside s-chain 74e⟩≡
      for (s = 0; s < Sv; s++) {
          ⟨Compute  $S$  inside  $\gamma$ -projections 72c⟩
          ⟨Inside multiply by  $V$ s 70c⟩
          ⟨Compute  $S$   $\gamma$ -unprojections and sum the results 75b⟩
      }

```

This code is used in chunks 74c and 77a.

```

75a  <Compute  $S_{xy}$  part on the boundary  $s$ -chain 75a>≡
      for (s = 0; s < Sv; s++) {
        <Compute  $S$  boundary  $\gamma$ -projections 73a>
        <Boundary multiply by  $V$ s 71a>
        <Compute  $S$   $\gamma$ -unprojections and sum the results 75b>
      }

```

This code is used in chunks 74d and 77b.

```

75b  <Compute  $S$   $\gamma$ -unprojections and sum the results 75b>≡
      rs = &rx5[s];
      for (c = 0; c < Nc; c++) {
        k = 6; <Unproject  $(1 + \gamma_3)$  link 8f>
        k = 7; <Unproject and accumulate  $(1 - \gamma_3)$  link 9b>
        k = 2; <Unproject and accumulate  $(1 + \gamma_1)$  link 6f>
        k = 3; <Unproject and accumulate  $(1 - \gamma_1)$  link 7>
        k = 1; <Unproject and accumulate  $(1 - \gamma_0)$  link 6d>
        k = 0; <Unproject and accumulate  $(1 + \gamma_0)$  link 6b>
        k = 5; <Unproject and accumulate  $(1 - \gamma_2)$  link 8d>
        k = 4; <Unproject and accumulate  $(1 + \gamma_2)$  link 8b>
      }

```

This code is used in chunks 74e and 75a.

5.9.36 Parts of $\eta - Q_{xx}^{-1}Q_{xy}\psi$

```

75c  <Compute inside part for  $1 - Q_{xx}^{-1}Q_{xy}$  75c>≡
      for (i = 0; i < nb->inside_size; i++) {
        const vFermion *ex5, *es;

        xyz5 = nb->inside[i];
        xyz5 = xyz5 * Sv;
        <Extract 1-d addresses 80d>
        ex5 = &eta[xyz5];
        <Build vector  $SU(3)$  objects 79a>
        <Compute  $Q_{xy}$  part on the inside  $s$ -chain 69b>
        <Compute  $1 - Q_{xx}^{-1}$  part on the  $s$ -chain 76a>
      }

```

This code is used in chunk 64a.

```

75d  <Compute boundary part for  $1 - Q_{xx}^{-1}Q_{xy}$  75d>≡
      for (i = 0; i < nb->boundary_size; i++) {
        const vFermion *ex5, *es;
        int m = nb->boundary[i].mask;

        xyz5 = nb->boundary[i].index;
        xyz5 = xyz5 * Sv;
        <Extract 1-d addresses 80d>
        ex5 = &eta[xyz5];
        <Build vector  $SU(3)$  objects 79a>
        <Compute  $Q_{xy}$  part on the boundary  $s$ -chain 69c>
        <Compute  $1 - Q_{xx}^{-1}$  part on the  $s$ -chain 76a>
      }

```

This code is used in chunk 64a.

76a $\langle \text{Compute } 1 - Q_{xx}^{-1} \text{ part on the } s\text{-chain } 76a \rangle \equiv$
 $\langle \text{Compute } Q_{xx}^{-1} \text{ part on the } s\text{-chain } 53a \rangle$
for (s = 0; s < Sv; s++) {
 rs = &rx5[s];
 es = &ex5[s];
 nv = vmk_1(0.0);
 for (c = 0; c < Nc; c++) {
 $\langle \text{Compute } (*rs) \leftarrow \eta - (*rs) \text{ and collect } \langle r, r \rangle 76b \rangle$
 }
 *norm += vsum(nv);
}

This code is used in chunk 75.

76b $\langle \text{Compute } (*rs) \leftarrow \eta - (*rs) \text{ and collect } \langle r, r \rangle 76b \rangle \equiv$
vv = es->f[0][c].re - rs->f[0][c].re; rs->f[0][c].re = vv; nv += vv * vv;
vv = es->f[0][c].im - rs->f[0][c].im; rs->f[0][c].im = vv; nv += vv * vv;
vv = es->f[1][c].re - rs->f[1][c].re; rs->f[1][c].re = vv; nv += vv * vv;
vv = es->f[1][c].im - rs->f[1][c].im; rs->f[1][c].im = vv; nv += vv * vv;
vv = es->f[2][c].re - rs->f[2][c].re; rs->f[2][c].re = vv; nv += vv * vv;
vv = es->f[2][c].im - rs->f[2][c].im; rs->f[2][c].im = vv; nv += vv * vv;
vv = es->f[3][c].re - rs->f[3][c].re; rs->f[3][c].re = vv; nv += vv * vv;
vv = es->f[3][c].im - rs->f[3][c].im; rs->f[3][c].im = vv; nv += vv * vv;

This code is used in chunk 76a.

5.9.37 Parts of $Q_{xx}\eta + Q_{xy}\psi$

76c $\langle \text{Compute inside part for } Q_{xx}\eta + Q_{xy}\psi 76c \rangle \equiv$
for (i = 0; i < nb->inside_size; i++) {
 const vFermion *ex5, *es;

 xyzt = nb->inside[i];
 xyzt5 = xyzt * Sv;
 $\langle \text{Extract 1-d addresses } 80d \rangle$
 ex5 = &eta[xyzt5];
 $\langle \text{Build vector } SU(3) \text{ objects } 79a \rangle$
 $\langle \text{Compute } Q_{xy} \text{ part on the inside } s\text{-chain } 69b \rangle$
 $\langle \text{Compute } Q_{xx}\eta + \chi \text{ part on the } s\text{-chain } 76e \rangle$
}

This code is used in chunk 64b.

76d $\langle \text{Compute boundary part for } Q_{xx}\eta + Q_{xy}\psi 76d \rangle \equiv$
for (i = 0; i < nb->boundary_size; i++) {
 const vFermion *ex5, *es;
 int m = nb->boundary[i].mask;

 xyzt = nb->boundary[i].index;
 xyzt5 = xyzt * Sv;
 $\langle \text{Extract 1-d addresses } 80d \rangle$
 ex5 = &eta[xyzt5];
 $\langle \text{Build vector } SU(3) \text{ objects } 79a \rangle$
 $\langle \text{Compute } Q_{xy} \text{ part on the boundary } s\text{-chain } 69c \rangle$
 $\langle \text{Compute } Q_{xx}\eta + \chi \text{ part on the } s\text{-chain } 76e \rangle$
}

This code is used in chunk 64b.

76e $\langle \text{Compute } Q_{xx}\eta + \chi \text{ part on the } s\text{-chain } 76e \rangle \equiv$
 $\langle \text{Compute } \chi + A\eta \text{ on the upper components } 77d \rangle$
 $\langle \text{Compute } \chi + B\eta \text{ on the lower components } 78c \rangle$

This code is used in chunk 76.

5.9.38 Parts of $S_{xx}\eta + S_{xy}\psi$

77a \langle Compute inside part for $S_{xx}\eta + S_{xy}\psi$ 77a $\rangle \equiv$

```

for (i = 0; i < nb->inside_size; i++) {
    const vFermion *ex5, *es;

    xyzt = nb->inside[i];
    xyzt5 = xyzt * Sv;
     $\langle$  Extract 1-d addresses 80d  $\rangle$ 
    ex5 = &eta[xyzt5];
     $\langle$  Build vector  $SU(3)$  objects 79a  $\rangle$ 
     $\langle$  Compute  $S_{xy}$  part on the inside s-chain 74e  $\rangle$ 
     $\langle$  Compute  $S_{xx}\eta + \chi$  part on the s-chain 77c  $\rangle$ 
}

```

This code is used in chunk 65a.

77b \langle Compute boundary part for $S_{xx}\eta + S_{xy}\psi$ 77b $\rangle \equiv$

```

for (i = 0; i < nb->boundary_size; i++) {
    const vFermion *ex5, *es;
    int m = nb->boundary[i].mask;

    xyzt = nb->boundary[i].index;
    xyzt5 = xyzt * Sv;
     $\langle$  Extract 1-d addresses 80d  $\rangle$ 
    ex5 = &eta[xyzt5];
     $\langle$  Build vector  $SU(3)$  objects 79a  $\rangle$ 
     $\langle$  Compute  $S_{xy}$  part on the boundary s-chain 75a  $\rangle$ 
     $\langle$  Compute  $S_{xx}\eta + \chi$  part on the s-chain 77c  $\rangle$ 
}

```

This code is used in chunk 65a.

77c \langle Compute $S_{xx}\eta + \chi$ part on the s-chain 77c $\rangle \equiv$

```

 $\langle$  Compute  $\chi + B\eta$  on the upper components 78b  $\rangle$ 
 $\langle$  Compute  $\chi + A\eta$  on the lower components 78a  $\rangle$ 

```

This code is used in chunk 77.

5.9.39 Computing A and B

77d \langle Compute $\chi + A\eta$ on the upper components 77d $\rangle \equiv$

```

for (s = Sv, vbc = vbnc, es1 = &ex5[0]; s--; vbc = vb) {
    es = &ex5[s];
    rs = &rx5[s];

#define QXX(d,c,r) rs->f[d][c].r += va * es->f[d][c].r \
                + vbc * shift_up1(es->f[d][c].r, es1->f[d][c].r)
    QXX(0,0,re); QXX(0,0,im);
    QXX(0,1,re); QXX(0,1,im);
    QXX(0,2,re); QXX(0,2,im);
    QXX(1,0,re); QXX(1,0,im);
    QXX(1,1,re); QXX(1,1,im);
    QXX(1,2,re); QXX(1,2,im);
#undef QXX
    es1 = es;
}

```

This code is used in chunk 76e.

78a $\langle \text{Compute } \chi + A\eta \text{ on the lower components 78a} \rangle \equiv$

```

for (s = Sv, vbc = vbnc, es1 = &ex5[0]; s--; vbc = vb) {
    es = &ex5[s];
    rs = &rx5[s];

#define QXX(d,c,r) rs->f[d][c].r += va * es->f[d][c].r \
                    + vbc * shift_up1(es->f[d][c].r, es1->f[d][c].r)
    QXX(2,0,re); QXX(2,0,im);
    QXX(2,1,re); QXX(2,1,im);
    QXX(2,2,re); QXX(2,2,im);
    QXX(3,0,re); QXX(3,0,im);
    QXX(3,1,re); QXX(3,1,im);
    QXX(3,2,re); QXX(3,2,im);
#undef QXX
    es1 = es;
}

```

This code is used in chunk 77c.

78b $\langle \text{Compute } \chi + B\eta \text{ on the upper components 78b} \rangle \equiv$

```

for (s = 0, vbc = vbcn, es1 = &ex5[Sv_1]; s < Sv; s++, vbc = vb) {
    es = &ex5[s];
    rs = &rx5[s];

#define QXX(d,c,r) rs->f[d][c].r += va * es->f[d][c].r \
                    + vbc * shift_upN(es1->f[d][c].r, es->f[d][c].r)
    QXX(0,0,re); QXX(0,0,im);
    QXX(0,1,re); QXX(0,1,im);
    QXX(0,2,re); QXX(0,2,im);
    QXX(1,0,re); QXX(1,0,im);
    QXX(1,1,re); QXX(1,1,im);
    QXX(1,2,re); QXX(1,2,im);
#undef QXX
    es1 = es;
}

```

This code is used in chunk 77c.

78c $\langle \text{Compute } \chi + B\eta \text{ on the lower components 78c} \rangle \equiv$

```

for (s = 0, vbc = vbcn, es1 = &ex5[Sv_1]; s < Sv; s++, vbc = vb) {
    es = &ex5[s];
    rs = &rx5[s];

#define QXX(d,c,r) rs->f[d][c].r += va * es->f[d][c].r \
                    + vbc * shift_upN(es1->f[d][c].r, es->f[d][c].r)
    QXX(2,0,re); QXX(2,0,im);
    QXX(2,1,re); QXX(2,1,im);
    QXX(2,2,re); QXX(2,2,im);
    QXX(3,0,re); QXX(3,0,im);
    QXX(3,1,re); QXX(3,1,im);
    QXX(3,2,re); QXX(3,2,im);
#undef QXX
    es1 = es;
}

```

This code is used in chunk 76e.

78d $\langle D_{xx} \text{ locals 78d} \rangle \equiv$

```

const vFermion *es1;
vReal vbc;

```

This code is used in chunks 64b and 65a.

5.9.40 Miscallenious

We also need to uplift the gauge fields

```
79a <Build vector SU(3) objects 79a>≡
    Uup = &U[nb->site[xyzt].Uup];
    for (d = 0; d < DIM; d++, Uup++) {
        Udown = &U[nb->site[xyzt].Udown[d]];
        for (c1 = 0; c1 < Nc; c1++) {
            for (c2 = 0; c2 < Nc; c2++) {
                /* conjugate down-link */
                V[d*2+0].v[c1][c2].re = vmk_1( Udown->v[c2][c1].re);
                V[d*2+0].v[c1][c2].im = vmk_1(-Udown->v[c2][c1].im);
                /* normal up-link */
                V[d*2+1].v[c1][c2].re = vmk_1(Uup->v[c1][c2].re);
                V[d*2+1].v[c1][c2].im = vmk_1(Uup->v[c1][c2].im);
            }
        }
    }
```

This code is used in chunks 68e, 69a, 71, and 74–77.

We want to keep code small, so computing the neighbors is done in a loop:

```
79b <Construct neighbor pointers 79b>≡
    for (d = 0; d < 2*DIM; d++) {
        ps[d] = p5[d] + s;
    }
```

This code is used in chunks 69d, 70a, 72c, and 73a.

5.9.41 Combined pieces

In these cases, Q_{xx}^{-1} is applied to the result of Q_{xy}

```
79c <Static function prototypes 21a>+≡
    static void compute_Qxx1Qxy(vFermion *d,
                                const vFermion *s,
                                struct neighbor *nb);
    static void inline compute_Qee1Qeo(vEvenFermion *d, const vOddFermion *s)
    {
        compute_Qxx1Qxy(&d->f, &s->f, &even_odd);
    }

    static void compute_Sxx1Sxy(vFermion *d,
                                const vFermion *s,
                                struct neighbor *nb);
    static void inline compute_See1Seo(vEvenFermion *d, const vOddFermion *s)
    {
        compute_Sxx1Sxy(&d->f, &s->f, &even_odd);
    }

    static void compute_1Qxx1Qxy(vFermion *d,
                                double *norm,
                                const vFermion *q,
                                const vFermion *s,
                                struct neighbor *nb);
    static void inline compute_1Qoo1Qoe(vOddFermion *d,
                                double *norm,
                                const vOddFermion *q,
                                const vEvenFermion *s)
    {
        compute_1Qxx1Qxy(&d->f, norm, &q->f, &s->f, &odd_even);
    }
```

5.9.42 Common locals

Some local bindings are used by all parts above. Let us collect them together.

80a $\langle Q \text{ common locals } 80a \rangle \equiv$

```
int i, xyzt5, s, c;
vFermion * __restrict__ rx5, * __restrict__ rs;
```

This code is used in chunks 52 and 62–65.

Others are used only in Z_{xy} parts:

80b $\langle Q_{xy} \text{ locals } 80b \rangle \equiv$

```
int xyzt, k, d;
const vFermion *f;
vHalfFermion *g;
vHalfFermion gg[2*DIM], hh[2*DIM];
vSU3 V[2*DIM];
int ps[2*DIM], p5[2*DIM];
```

This definition is continued in chunk 80c.

This code is used in chunks 62–65.

80c $\langle Q_{xy} \text{ locals } 80b \rangle + \equiv$

```
const SU3 *Uup, *Udown;
int c1, c2;
```

For the inside sites, compute the s -chain address of the neighbor. For the boundary sites, the address of the s -chain in the receive buffer is used instead:

80d $\langle \text{Extract 1-d addresses } 80d \rangle \equiv$

```
for (d = 0; d < 2*DIM; d++)
    p5[d] = nb->site[xyzt].F[d];
 $\langle \text{Compute rx5 } 80e \rangle$ 
```

This code is used in chunks 68e, 69a, 71, and 74–77.

80e $\langle \text{Compute rx5 } 80e \rangle \equiv$

```
rx5 = &chi[xyzt5];
```

This code is used in chunks 52 and 80d.

80f $\langle \text{Compute qx5 } 80f \rangle \equiv$

```
qx5 = &psi[xyzt5];
```

This code is used in chunk 52.

5.9.43 Common globals

Some of these values depend of m_f and M . Here we compute their values:

80g $\langle \text{Compute constant values for } Q_{xx}^{-1} \text{ and } S_{xx}^{-1} \text{ } 80g \rangle \equiv$

```
{
    double a = M;
    double b = 2.;
    double c = -2*m_f;

     $\langle \text{Compute values from a, b and c } 53h \rangle$ 
}
```

This code is used in chunk 19a.

5.10 QMP Pieces

Here are miscellaneous piece of QMP:

We are ready to use the result of the global sum. Check that it has been computed.

80h $\langle \text{Finalize } \langle r, r \rangle \text{ computation } 80h \rangle \equiv$

```
/* relax, QMP does not support split reductions yet. */
```

This code is used in chunk 45b.

Because there is no confirming QMP implementation, we have to deal with esoteric restrictions that are imposed rhyme or reason and are not documented either. Otherwise, they would be not esoteric, ain't they?

Rrestriction one says that deadlocks happen if there no than one active handle at a time. We had already packed all communication into a single superhandle, now it is a time to use it. To be extra paranoid, we check if there is a handle before using it.

```
81a <Start sends and receives 81a>≡
    if (nb->qmp_smask) {
        QMP_start(nb->qmp_handle);
        DEBUG_QMP("start sends and receives (0x%x)\n", (int)nb->qmp_handle)
    }
```

This code is used in chunks 62–65.

When it is a time to use the received data, we wait for the sends as well. It is a waste, but we have to work with software that we have.

```
81b <Finish sends and receives 81b>≡
    if (nb->qmp_smask) {
        QMP_wait(nb->qmp_handle);
        DEBUG_QMP("waiting for sends and receives (0x%x)\n",
            (int)nb->qmp_handle)
    }
```

This code is used in chunks 62–65.

We need to print the version of the code from node 0.

```
81c <Show DWF version 81c>≡
{
    if (QMP_get_node_number() == 0) {
        QMP_printf("DWF init: %s (" MACHINE ")\n", version);
    }
}
```

This code is used in chunk 14b.

5.10.1 Global sums

Until the split global sums are implemented in QMP, everything is done at the beginning, when ***norm** contains the local part of the sum. Start the global operation which will distribute the pieces, compute the sum, and provide the result to each node.

```
81d <Start <r,r> computation 81d>≡
    QMP_sum_double(norm);
    DEBUG_QMP("sum_double(%p)\n", norm)
```

This code is used in chunks 49a, 50a, and 64a.

5.11 Generally Useful Functions

Here is a collection of simple functions that are useful throughout the code:

```
81e <Static function prototypes 21a>+≡
    static inline int
    parity(const int x[DIM])
    {
        int i, v;
        for (i = v = 0; i < DIM; i++)
            v += x[i];
        return v & 1;
    }
```

We need some powers, but `pow()` is too generic. Here's a simple version

82a *<Static function prototypes 21a>+≡*

```
static double
d_pow(double x, unsigned int n)
{
    double v = 1;

    while (n) {
        if (n & 1)
            v *= x;
        x *= x;
        n /= 2;
    }
    return v;
}
```

Clear a half fermion:

82b *<Static function prototypes 21a>+≡*

```
static inline void
vhfzero(vHalfFermion *v)
{
    vReal z = vmk_1(0.0);

    v->f[0][0].re = v->f[0][0].im =
    v->f[0][1].re = v->f[0][1].im =
    v->f[0][2].re = v->f[0][2].im =
    v->f[1][0].re = v->f[1][0].im =
    v->f[1][1].re = v->f[1][1].im =
    v->f[1][2].re = v->f[1][2].im = z;
}
```

5.12 Debug Aids

Here are macros for debugging QMP.

82c *<Definitions 60j>+≡*

```
#ifndef DEBUG_QMP
#undef DEBUG_QMP
#define DEBUG_QMP(msg,a ...) do \
    printf("[%05d] %s:%d:QMP/%s(): " msg, QMP_get_node_number(), \
        __FILE__,__LINE__,__FUNCTION__,##a); \
    while(0);
#else /* !defined(DEBUG_QMP) */
#define DEBUG_QMP(msg,a ...)
#define NO_DEBUG_QMP
#endif /* defined(DEBUG_QMP) */
```

Another debug aid for other pieces of the code:

82d *<Definitions 60j>+≡*

```
#ifndef DEBUG_DWF
#undef DEBUG_DWF
#define DEBUG_DWF(msg,a ...) do \
    printf("[%05d] %s: %d: DWF/%s(): " msg, QMP_get_node_number(), \
        __FILE__,__LINE__,__FUNCTION__,##a); \
    while(0);
#else /* !defined(DEBUG_DWF) */
#define DEBUG_DWF(msg, a ...)
#define NO_DEBUG_DWF
#endif /* defined(DEBUG_DWF) */
```

5.12.1 Neighbor table debugging

83a `<dwf-tables.h 83a>≡`
 <Neighbor tables 33a>
 extern struct neighbor even_odd;
 extern struct neighbor odd_even;
Root chunk (not used in this document).

5.13 Source Files

The main body of the code resides in a single file:

83b `<dwf.c 83b>≡`
 <Include files 15c>
 <Definitions 60j>
 <Data types 18b>
 <Neighbor tables 33a>
 <Global variables 13b>
 <Static function prototypes 21a>
 <Static functions 23f>
 <Interface functions 14b>

Root chunk (not used in this document).

This file is included into target-specific files to produce an object file. For each target we build its own C file in corresponding subsections below.

For some constants it is better to have symbolic names even if one can not easily change their values.

83c `<Macro definitions 83c>≡`
 #define Nc 3 /* Number of colors */
 #define DIM 4 /* number of dimensions */
 #define Fd 4 /* Fermion representation dimension */

This code is used in chunks 83e, 86b, 88c, 90f, and 92e.

Strictly speaking, there is no need to have separate types for even/odd sublattices. But, while writing the CG, the compiler caught quite a few logic errors because of these two tiny structures.

When complex types are constructed in a trivial way from real types, this chunk is handy:

83d `<Usual complex types 83d>≡`
 typedef struct {
 float re, im;
 } scalar_complex;

 typedef struct {
 vReal re, im;
 } vector_complex;

This code is used in chunks 84a, 86c, 88d, 91a, and 92f.

5.13.1 Single Precision SSE version

83e `<dwf-ssef.c 83e>≡`
 <Macro definitions 83c>
 <SSE single precision types 84a>
 <SSE single precision functions 84b>
 #include "dwf-ssef.h"
 #define MACHINE "sse float"
 #define L3(n) MIT_ssef_##n
 #define PAD16(size) (15+(size))
 #define ALIGN16(addr) ((void *) (~15 & (15 + (size_t)(addr))))
 <Blocks for YA and YB of length four 61c>
 #include "dwf.c"

Root chunk (not used in this document).

For single precision SSE, vectors are of length 4. Special gcc construct is needed to access SSE units.

```
84a  <SSE single precision types 84a>≡
      #define Vs 4
      typedef float REAL;
      typedef REAL vReal __attribute__((mode(V4SF),aligned(16)));
      <Usual complex types 83d>
```

This code is used in chunk 83e.

Ten functions dealing with vectors follow. First, propagate a scalar value into all four components of the SSE vector.

```
84b  <SSE single precision functions 84b>≡
      static inline vReal
      vmk_1(double a)                                /* return (a a ... a) */
      {
          float b = a;
          vReal v = __builtin_ia32_loadss((float *)&b);
          asm("shufps\t$0,%0,%0" : "+x" (v));
          return v;
      }
```

This definition is continued in chunks 84-86.

This code is used in chunk 83e.

Next, packing Vs-1 copies of a and one b:

```
84c  <SSE single precision functions 84b>+≡
      static inline vReal
      vmk_n1(double a, double b)                     /* return (a ... a b) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = a; r[2] = a; r[3] = b;
          return v;
      }
```

Also, packing one a and Vs-1 copies of b:

```
84d  <SSE single precision functions 84b>+≡
      static inline vReal
      vmk_1n(double a, double b)                     /* return (a b ... b) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = b; r[2] = b; r[3] = b;
          return v;
      }
```

And two more constructors:

```
84e  <SSE single precision functions 84b>+≡
      static inline vReal
      vmk_fn(double a, double b)                     /* return (a a*b ... a*b^(Vs-1)) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = a*b; r[2] = a*b*b; r[3] = a*b*b*b;
          return v;
      }
```

85a \langle SSE single precision functions 84b $\rangle + \equiv$

```
static inline vReal
vmk_bn(double a, double b)                /* return (a^(Vs-1)*b ... a*b b) */
{
    vReal v;
    REAL *r = (REAL *)&v;
    r[0] = a*a*a*b; r[1] = a*a*b; r[2] = a*b; r[3] = b;
    return v;
}
```

Add all components of the vector together:

85b \langle SSE single precision functions 84b $\rangle + \equiv$

```
static inline double
vsum(vReal v)                            /* return sum(i, [i]v) */
{
    REAL *r = (REAL *)&v;
    return r[0] + r[1] + r[2] + r[3];
}
```

We need also to change the first element of the vector.

85c \langle SSE single precision functions 84b $\rangle + \equiv$

```
static inline vReal
vput_0(vReal a, double b)                /* return (b [1]a ... [Vs-1]a) */
{
    REAL *v = (REAL *)&a;
    v[0] = b;
    return a;
}
```

and the last element:

85d \langle SSE single precision functions 84b $\rangle + \equiv$

```
static inline vReal
vput_n(vReal a, double b)                /* return ([0]a ... [Vs-2]a b) */
{
    REAL *v = (REAL *)&a;
    v[3] = b;
    return a;
}
```

Shift to the left:

85e \langle SSE single precision functions 84b $\rangle + \equiv$

```
static inline vReal
shift_up1(vReal a, vReal b)              /* return ([1]a ... [Vs-1]a [0]b) */
{
    vReal x = a;
    vReal y = b;
    asm("shufps\t$0x30,%0,%1\n\t"
        "shufps\t$0x29,%1,%0"
        : "+x" (x), "+x" (y));
    return x;
}
```

And to the right:

```
86a  <SSE single precision functions 84b>+≡
      static inline vReal
      shift_upN(vReal a, vReal b)          /* return ([Vs-1]a [0]b ... [Vs-2]b) */
      {
          vReal x = a;
          asm("shufps\t$0x03,%1,%0\n\t"
              "shufps\t$0x9c,%1,%0"
              : "+x" (x): "x" (b));
          return x;
      }
```

5.13.2 Double Precision SSE version

```
86b  <dwf-ssed.c 86b>≡
      <Macro definitions 83c>
      <SSE double precision types 86c>
      <SSE double precision functions 86d>
      #include "dwf-ssed.h"
      #define MACHINE "sse double"
      #define L3(n) MIT_ssed_##n
      #define PAD16(size) (15+(size))
      #define ALIGN16(addr) ((void *) (~15 & (15 + (size_t)(addr))))
      <Blocks for YA and YB of length two 61f>
      #include "dwf.c"
```

Root chunk (not used in this document).

For double precision SSE, vectors are of length 4. Special gcc construct is needed to access SSE units.

```
86c  <SSE double precision types 86c>≡
      #define Vs 2
      typedef double REAL;
      typedef double __attribute__((mode(V2DF),aligned(16))) vReal;
      <Usual complex types 83d>
```

This code is used in chunk 86b.

Ten functions dealing with vectors follow. First, propagate a scalar value into all two components of the SSE vector.

```
86d  <SSE double precision functions 86d>≡
      static inline vReal
      vmk_1(double a)                      /* return (a a ... a) */
      {
          vReal v;
          REAL *w = (REAL *)&v;
          w[0] = w[1] = a;
          return v;
      }
```

This definition is continued in chunks 86-88.

This code is used in chunk 86b.

Next, packing Vs-1 copies of a and one b:

```
86e  <SSE double precision functions 86d>+≡
      static inline vReal
      vmk_n1(double a, double b)          /* return (a ... a b) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = b;
          return v;
      }
```

Also, packing one a and Vs-1 copies of b:

```
87a  <SSE double precision functions 86d>+≡
      static inline vReal
      vmk_1n(double a, double b)          /* return (a b ... b) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = b;
          return v;
      }
```

And two more constructors:

```
87b  <SSE double precision functions 86d>+≡
      static inline vReal
      vmk_fn(double a, double b)          /* return (a a*b ... a*b^(Vs-1)) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = a*b;
          return v;
      }
```

```
87c  <SSE double precision functions 86d>+≡
      static inline vReal
      vmk_bn(double a, double b)          /* return (a^(Vs-1)*b ... a*b b) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a*b; r[1] = b;
          return v;
      }
```

Add all components of the vector together:

```
87d  <SSE double precision functions 86d>+≡
      static inline double
      vsum(vReal v)                      /* return sum(i, [i]v) */
      {
          REAL *r = (REAL *)&v;
          return r[0] + r[1];
      }
```

We need also to change the first element of the vector.

```
87e  <SSE double precision functions 86d>+≡
      static inline vReal
      vput_0(vReal a, double b)          /* return (b [1]a ... [Vs-1]a) */
      {
          REAL *v = (REAL *)&a;
          v[0] = b;
          return a;
      }
```

and the last element:

```
87f  <SSE double precision functions 86d>+≡
      static inline vReal
      vput_n(vReal a, double b)          /* return ([0]a ... [Vs-2]a b) */
      {
          REAL *v = (REAL *)&a;
          v[1] = b;
          return a;
      }
```

Shift to the left:

```
88a  <SSE double precision functions 86d>+≡
      static inline vReal
      shift_up1(vReal a, vReal b)                /* return ([1]a ... [Vs-1]a [0]b) */
      {
          vReal r;
          REAL *x = (REAL *)&a;
          REAL *y = (REAL *)&b;
          REAL *z = (REAL *)&r;
          z[0] = x[1];
          z[1] = y[0];

          return r;
      }
```

And to the right:

```
88b  <SSE double precision functions 86d>+≡
      static inline vReal
      shift_upN(vReal a, vReal b)                /* return ([Vs-1]a [0]b ... [Vs-2]b) */
      {
          vReal r;
          REAL *x = (REAL *)&a;
          REAL *y = (REAL *)&b;
          REAL *z = (REAL *)&r;
          z[0] = x[1];
          z[1] = y[0];

          return r;
      }
```

5.13.3 Single Precision AltiVec version

```
88c  <dwf-altivec.c 88c>≡
      <Macro definitions 83c>
      <AltiVec single precision types 88d>
      <AltiVec single precision functions 89a>
      #include "dwf-altivec.h"
      #define MACHINE "altivec float"
      #define L3(n) MIT_altivec_##n
      #define PAD16(size) (15+(size))
      #define ALIGN16(addr) ((void *) (~15 & (15 + (size_t)(addr))))
      <Blocks for YA and YB of length four 61c>
      #include "dwf.c"
```

Root chunk (not used in this document).

The AltiVec extension of the PPC architecture is quite similar to SSE. It requires the `-maltivec` flag for gcc.

```
88d  <AltiVec single precision types 88d>≡
      #include <altivec.h>
      typedef float REAL;
      typedef vector float vReal;
      #define Vs 4
      <Usual complex types 83d>
```

This code is used in chunk 88c.

Now, the ten vector functions.

First, propagate a scalar value into all four components of the SSE vector.

```
89a  <Altivec single precision functions 89a>≡
      static inline vReal
      vmk_1(double a)                      /* return (a a ... a) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = a; r[2] = a; r[3] = a;
          return v;
      }
```

This definition is continued in chunks 89 and 90.

This code is used in chunk 88c.

Next, packing Vs-1 copies of a and one b:

```
89b  <Altivec single precision functions 89a>+≡
      static inline vReal
      vmk_n1(double a, double b)           /* return (a ... a b) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = a; r[2] = a; r[3] = b;
          return v;
      }
```

Also, packing one a and Vs-1 copies of b:

```
89c  <Altivec single precision functions 89a>+≡
      static inline vReal
      vmk_1n(double a, double b)           /* return (a b ... b) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = b; r[2] = b; r[3] = b;
          return v;
      }
```

And two more constructors:

```
89d  <Altivec single precision functions 89a>+≡
      static inline vReal
      vmk_fn(double a, double b)           /* return (a a*b ... a*b^(Vs-1)) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a; r[1] = a*b; r[2] = a*b*b; r[3] = a*b*b*b;
          return v;
      }
```

```
89e  <Altivec single precision functions 89a>+≡
      static inline vReal
      vmk_bn(double a, double b)           /* return (a^(Vs-1)*b ... a*b b) */
      {
          vReal v;
          REAL *r = (REAL *)&v;
          r[0] = a*a*a*b; r[1] = a*a*b; r[2] = a*b; r[3] = b;
          return v;
      }
```

Add all components of the vector together:

```
90a <AltiVec single precision functions 89a>+≡
    static inline double
    vsum(vReal v)                                /* return sum(i, [i]v) */
    {
        REAL *r = (REAL *)&v;
        return r[0] + r[1] + r[2] + r[3];
    }
```

We need also to change the first element of the vector.

```
90b <AltiVec single precision functions 89a>+≡
    static inline vReal
    vput_0(vReal a, double b)                    /* return (b [1]a ... [Vs-1]a) */
    {
        REAL *v = (REAL *)&a;
        v[0] = b;
        return a;
    }
```

and the last element:

```
90c <AltiVec single precision functions 89a>+≡
    static inline vReal
    vput_n(vReal a, double b)                    /* return ([0]a ... [Vs-2]a b) */
    {
        REAL *v = (REAL *)&a;
        v[3] = b;
        return a;
    }
```

Shift to the left:

```
90d <AltiVec single precision functions 89a>+≡
    static inline vReal
    shift_up1(vReal a, vReal b)                  /* return ([1]a ... [Vs-1]a [0]b) */
    {
        return vec_sld(a, b, 4);
    }
```

And to the right:

```
90e <AltiVec single precision functions 89a>+≡
    static inline vReal
    shift_upN(vReal a, vReal b)                  /* return ([Vs-1]a [0]b ... [Vs-2]b) */
    {
        return vec_sld(a, b, 12);
    }
```

5.13.4 Single Precision BlueLight version

```
90f <dwf-bluelightf.c 90f>≡
    <Macro definitions 83c>
    <BlueLight single precision types 91a>
    <BlueLight functions 91b>
    #include "dwf-bluelightf.h"
    #define MACHINE "bluelight float"
    #define L3(n) MIT_bluelightf_##n
    #define PAD16(size) (15+(size))
    #define ALIGN16(addr) ((void *) (~15 & (15 + (size_t)(addr))))
    <Blocks for YA and YB of length two 61f>
    #include "dwf.c"
```

Root chunk (not used in this document).

The BlueLight extension of the PPC architecture is for the BlueGene/L. It requires the `-mbluelight` flag for gcc.

91a `<BlueLight single precision types 91a>≡`
`#include <bluelight.h>`
`typedef REAL float;`
`typedef vector float vReal;`
`#define Vs 2`
`<Usual complex types 83d>`

This code is used in chunk 90f.

Now, the ten vector functions.

First, propagate a scalar value into all four components of the SSE vector.

91b `<BlueLight functions 91b>≡`
`static inline vReal`
`vmk_1(double a) /* return (a a ... a) */`
`{`
`return (vReal)(vec_mk2d(a, a));`
`}`

This definition is continued in chunks 91 and 92.

This code is used in chunks 90f and 92e.

Next, packing Vs-1 copies of a and one b:

91c `<BlueLight functions 91b>+≡`
`static inline vReal`
`vmk_n1(double a, double b) /* return (a ... a b) */`
`{`
`return (vReal)(vec_mk2d(a, b));`
`}`

Also, packing one a and Vs-1 copies of b:

91d `<BlueLight functions 91b>+≡`
`static inline vReal`
`vmk_1n(double a, double b) /* return (a b ... b) */`
`{`
`return (vReal)(vec_mk2d(a, b));`
`}`

And two more constructors:

91e `<BlueLight functions 91b>+≡`
`static inline vReal`
`vmk_fn(double a, double b) /* return (a a*b ... a*b^(Vs-1)) */`
`{`
`return (vReal)(vec_mk2d(a, a*b));`
`}`

91f `<BlueLight functions 91b>+≡`
`static inline vReal`
`vmk_bn(double a, double b) /* return (a^(Vs-1)*b ... a*b b) */`
`{`
`return (vReal)(vec_mk2d(a*b, b));`
`}`

Add all components of the vector together:

91g `<BlueLight functions 91b>+≡`
`static inline double`
`vsum(vReal v) /* return sum(i, [i]v) */`
`{`
`return vec_get0((vector double)v) + vec_get1((vector double)v);`
`}`

We need also to change the first element of the vector.

```
92a <BlueLight functions 91b>+≡
    static inline vReal
    vput_0(vReal a, double b)                /* return (b [1]a ... [Vs-1]a) */
    {
        return (vReal)(vec_mk2d(b, vec_get1((vector double)a));
    }
```

and the last element:

```
92b <BlueLight functions 91b>+≡
    static inline vReal
    vput_n(vReal a, double b)                /* return ([0]a ... [Vs-2]a b) */
    {
        return (vReal)(vec_mk2d(vec_get0((vector double)a), b));
    }
```

Shift to the left:

```
92c <BlueLight functions 91b>+≡
    static inline vReal
    shift_up1(vReal a, vReal b)              /* return ([1]a ... [Vs-1]a [0]b) */
    {
        return (vReal)(vec_mk10((vector double)a, (vector double)b));
    }
```

And to the right:

```
92d <BlueLight functions 91b>+≡
    static inline vReal
    shift_upN(vReal a, vReal b)              /* return ([Vs-1]a [0]b ... [Vs-2]b) */
    {
        return (vReal)(vec_mk10((vector double)a, (vector double)b));
    }
```

5.13.5 Double Precision BlueLight version

There is very little difference between single and double precision BlueLight implementations.

```
92e <dwf-bluelightd.c 92e>≡
    <Macro definitions 83c>
    <BlueLight double precision types 92f>
    <BlueLight functions 91b>
    #include "dwf-bluelightd.h"
    #define MACHINE "bluelight double"
    #define L3(n) MIT_bluelightd_##n
    #define PAD16(size) (15+(size))
    #define ALIGN16(addr) ((void *) (~15 & (15 + (size_t)(addr))))
    <Blocks for YA and YB of length two 61f>
    #include "dwf.c"
```

Root chunk (not used in this document).

The BlueLight extension requires the `-mbuelight` flag for gcc.

```
92f <BlueLight double precision types 92f>≡
    #include <bluelight.h>
    typedef double REAL;
    typedef vector double vReal;
    #define Vs 2
    <Usual complex types 83d>
```

This code is used in chunk 92e.

6 CHUNKS

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 ⟨ Compute $S_{xx}\eta + \chi$ part on the s -chain 77c⟩
 ⟨ Compute $1 - S_{xy}$ part on the boundary s -chain 72b⟩
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