Quantum Shannon Theory

Aram Harrow (MIT)

QIP 2016 tutorial
9–10 January, 2016
the prehistory of quantum information

ideas present in disconnected form

• 1927 Heisenberg uncertainty principle

• 1935 EPR paper / 1964 Bell’s theorem

• 1932 von Neumann entropy
  subadditivity (Araki–Lieb 1970)
  strong subadditivity (Lieb–Ruskai 1973)

• measurement theory
  (Helstrom, Holevo, Uhlmann, etc., 1970s)
relativity: a close relative

• Before Einstein, Maxwell’s equations were known to be incompatible with Galilean relativity.

• Lorentz proposed a mathematical fix, but without the right physical interpretation.

• Einstein’s solution redefined space/time, mass/momentum/energy, etc.

• Space and time had solid mathematical foundations (Descartes, etc.), unlike information and computing.
theory of information and computing

- 1948 Shannon created modern information theory (and to some extent cryptography) and justified entropy as a measure of information independent of physics. units of bits.

- Turing, Church, von Neumann, ..., Djikstra described a theory of computation, algorithms, complexity, etc.

- This made it possible to formulate questions such as:
  - how do “quantum effects” change the capacity? (→ Holevo bound)
  - what is the thermodynamic cost of computing? (Landauer principle, Bennett reversible computing)
  - what is the computational complexity of simulating QM? (→ DMRG/QMC, and also Feynman)
some wacky ideas

Feynman ’82: “Simulating Physics with Computers”
- Classical computers require exponential overhead to simulate quantum mechanics.
- But quantum systems obviously don’t need exp overhead to simulate themselves.
- Therefore they are doing something more computationally powerful than our existing computers.
- (Implicitly requires the idea of a universal Turing machine, and the strong Church-Turing thesis.)

Wiesner ’70: “Conjugate Coding”
- The uncertainty principle restricts possible measurements.
- In experiments, this is a disadvantage, but in crypto, limiting information is an advantage.
- (Requires crypto framework, notion of “adversary.”)
towards modern QIT

- Deutsch, Jozsa, Bernstein, Vazirani, Simon, etc. – impractical speedups
  required oracle model, precursors to Shor’s algorithm, following Feynman.

- quantum key distribution (BB84, B90, E91) – following Weisner.

- ca. 1995
  - Shor and Grover algorithms
  - quantum error-correcting codes
  - fault-tolerant quantum computing
  - teleportation, super-dense coding
  - Schumacher-Jozsa data compression
  - HSW coding theorem
  - resource theory of entanglement
modern QIT

**semdicalssical**

- **compression:** \( S(\rho) = -\text{tr} [\rho \log(\rho)] \)
- **CQ or QC channels:** \( \chi(\{p_x, \rho_x\}) = S(\sum_x p_x \rho_x) - \sum_x p_x S(\rho_x) \)
- **hypothesis testing:** \( D(\rho \parallel \sigma) = \text{tr} [\rho (\log(\rho) - \log(\sigma))] \)

**"fully quantum"**

- **complementary channel:** \( N(\rho) = \text{tr}_2 V \rho V^+, \quad N^c(\rho) := \text{tr}_1 V \rho V^+ \)
- **quantum capacity:** \( Q^{(1)}(N) = \max_{\rho} [S(N(\rho)) - S(N^c(\rho))] \)
  \( Q(N) = \lim_{n \to \infty} Q^{(1)}(N^\otimes n)/n \)
- **tools:** purifications (Stinespring), decoupling

**recent**

- **one-shot:** \( S_\alpha(\rho) := \log(\text{tr} \rho^\alpha)/(1-\alpha) \)
- **applications** to optimization, condensed matter, stat mech.
Relevant talks

- **Wed 9.** Omar Fawzi and Renato Renner. Quantum conditional mutual information and approximate Markov chains.

- **Wed 9:50.** Omar Fawzi, Marius Junge, Renato Renner, David Sutter, Mark Wilde and Andreas Winter. Universal recoverability in quantum information theory.

- **Thurs 11.** David Sutter, Volkher Scholz, Andreas Winter and Renato Renner. Approximate degradable quantum channels

- **Thurs 4:15.** Mario Berta, Joseph M. Renes, Marco Tomamichel, Mark Wilde and Andreas Winter. Strong Converse and Finite Resource Tradeoffs for Quantum Channels.
semi-relevant talks

- **Tues 11:50.** Ryan O'Donnell and John Wright. Efficient quantum tomography merged with Jeongwan Haah, Aram Harrow, Zhengfeng Ji, Xiaodi Wu and Nengkun Yu. Sample-optimal tomography of quantum states

- **Tues 3:35.** Ke Li. Discriminating quantum states: the multiple Chernoff distance

- **Thurs 10.** Mark Braverman, Ankit Garg, Young Kun Ko, Jiéming Mao and Dave Touchette. Near optimal bounds on bounded-round quantum communication complexity of disjointness

- **Thurs 3:35.** Fernando Brandao and Aram Harrow. Estimating operator norms using covering nets with applications to quantum information theory

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outline

- metrics
- compressing quantum ensembles (Schumacher coding)
- sending classical messages over q channels (HSW)
- remote state preparation (RSP)
- Schur duality
- RSP and the strong converse
- hypothesis testing
- merging
- quantum conditional mutual information and q Markov states
metrics

Trace distance \( T(\rho, \sigma) := \frac{1}{2} \| \rho - \sigma \|_1 \)
- Is a metric.
- **monotone:** \( T(\rho, \sigma) \geq T(N(\rho), N(\sigma)) \)
- and this is achieved by a measurement
  \( \rightarrow T = \max \text{ m'mt bias} \)

Fidelity \( F(\rho, \sigma) := \| \sqrt{\rho} \sqrt{\sigma} \|_1 = \text{tr} \sqrt{\sqrt{\sigma} \rho \sqrt{\sigma}} \)
- \( F=1 \) iff \( \rho = \sigma \) and \( F=0 \) iff \( \rho \perp \sigma \)
- **monotone** \( F(\rho, \sigma) \leq F(N(\rho), N(\sigma)) \)
- and this is achieved by a measurement!

Relation: \[ 1 - F \leq T \leq (1 - F^2)^{1/2} \]

**Pure states with angle \( \theta \):**
\[ F = \cos(\theta) \text{ and } T = \sin(\theta). \]
(exercise: which m'mts saturate?)
the case for fidelity

Uhlmann's theorem:
\[ F(\rho_A, \sigma_A) = \max_{\psi, \phi} F(\psi_{AB}, \phi_{AB}) \text{ s.t.} \]
\[ \psi = |\psi\rangle\langle\psi|, \quad \phi = |\phi\rangle\langle\phi|, \quad \psi_A = \rho_A, \quad \phi_A = \sigma_A. \]

Note:
1. \( \geq \) from monotonicity.
   = requires sweat
2. Can fix either \( \psi \) or \( \phi \) and max over the other.
3. \( F(\psi, \phi) = |\langle\psi|\phi\rangle|. \) (Some use different convention.)
4. Implies that \( (1-F)^{1/2} \) is a metric.

Also \( F \) is multiplicative.
Compression

\[ |\psi_x\rangle \in \mathbb{C}^d \]

with prob \( p_x \)

encoder \[ \text{dim} \ r < d \]

decoder \( \approx |\psi_x\rangle \)

**Average fidelity:**

\[ \Sigma_x p_x F(\psi_x, D(E(\psi_x))) \leq F(\rho, D(E(\rho))) \]

**Simplification:** use ensemble density matrix

\[ \rho = \Sigma_x p_x \psi_x \text{ with eigenvalues } \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_d \geq 0 \]

\[ \text{rank}(\sigma) = r \Rightarrow F(\rho, \sigma)^2 \leq \text{tr}[P_r \rho] = \lambda_1 + \ldots + \lambda_r \]

\( P_r \) projects onto top \( r \) eigenvectors

\[ \sigma = \frac{P_r \rho P_r}{\text{tr}[P_r \rho]} \]

Suggests optimal fidelity = \( (\lambda_1 + \ldots + \lambda_r)^{1/2} \).
Too good to be true!

Ensemble density matrix: $\rho = \sum_x p_x \psi_x$

Yes compression depends only on $\rho$.

But reproducing $\rho$ is not enough!

consider:

$E(\cdot) = |0\rangle\langle 0|$

$D(\cdot) = \rho$

Gets the average right but not the correlations.
Average fidelity:
$$
\begin{aligned}
\sum_x p_x \ F(\psi_x, E(D(\psi_x))) \\
= F(\sum_x p_x |x\rangle\langle x| \otimes \psi_x, \sum_x p_x |x\rangle\langle x| \otimes E(D(\psi_x)))
\end{aligned}
$$

Not so easy to analyze.
Instead follow the Church of the Larger Hilbert Space.

\[
|\varphi\rangle_{RQ} := \sum_x \sqrt{p_x} |x\rangle_R |\psi_x\rangle_Q
\]

Avg fidelity $\geq F(\varphi, (id_R \otimes D \circ E_Q)(\varphi))$

(pf: monotonicity under map that measures R.)

Protocol: $E(\omega) = P_r \omega P_r$. \quad D = id.
Achieves $F = \langle \varphi | (I \otimes P_r) |\varphi\rangle = \text{tr} [\rho P_r] = \lambda_1 + ... + \lambda_r$
Optimality

Complication: E, D might be noisy.

Solution: purify!

1. Write $D(E(\omega)) = \text{tr}_G V \omega V^+$ where $V$ is an isometry from $Q \rightarrow Q \otimes G$.

2. Uhlmann $\Rightarrow$
   
   $F(\varphi, \text{tr}_G V \varphi V^+) = |\langle \varphi |_{RQ} \langle 0 |_G V | \varphi \rangle_{RQ}|$

3. a little linear algebra $\Rightarrow$
   
   $F \leq \text{tr}[\rho P]$ for $P$ rank-$r$ and $\|P\| \leq 1$
   
   $\leq \lambda_1 + \ldots + \lambda_r$
compressing i.i.d. sources

Quantum story \( \approx \) classical story

\( \rho \otimes^n \) has eigenvalues \( \lambda_{x_1} \lambda_{x_2} \cdots \lambda_{x_n} \) for \( X=(x_1,\ldots,x_n) \in [d]^n \).

Typically this is \( \approx \lambda_1^{n_1} \cdots \lambda_d^{n_d} = \exp(-nH(\lambda)) \)

\( H(\lambda) = -\sum_x \lambda_x \log(\lambda_x) = S(\rho) = -\text{tr}[\rho \log(\rho)] \)

distribution of \( -\log(\lambda_{x_1} \lambda_{x_2} \cdots \lambda_{x_n}) \)

<table>
<thead>
<tr>
<th>qubits</th>
<th>fidelity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( nH(\lambda) + 2\sigma n^{1/2} )</td>
<td>0.98</td>
</tr>
<tr>
<td>( nH(\lambda) - 2\sigma n^{1/2} )</td>
<td>0.02</td>
</tr>
<tr>
<td>( n(H(\lambda)+\delta) )</td>
<td>( 1-\exp(-n \delta^2/2\sigma^2) )</td>
</tr>
<tr>
<td>( n(H(\lambda)-\delta) )</td>
<td>( \exp(-n \delta^2/2\sigma^2) )</td>
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typicality

Definitions:
An eigenvector of $\rho^\otimes n$ is \textbf{k-typical} if its eigenvalue is in the range $\exp(-nS(\rho) \pm k\sigma n^{1/2})$.

Typical subspace $V = \text{span of typical eigenvectors}$
Typical projector $P = \text{projector onto } V$

Structure theorem for iid states: “asymptotic equipartition”
\[ \text{tr } [P \rho^\otimes n] \geq 1 - k^{-2} \]
\[ \exp(-nS(\rho) - k\sigma n^{1/2}) \leq P \rho^\otimes n P \leq \exp(-nS(\rho) + k\sigma n^{1/2}) \leq \text{tr}[P] \approx \exp(nS(\rho) + k\sigma n^{1/2}) \]

Almost flat spectrum.
Plausible because of permutation symmetry.
Quantum Shannon Theory

Aram Harrow (MIT)

QIP 2016 tutorial day 2
10 January, 2016
**entropy**

\[ S(\rho) = -\text{tr} [\rho \log \rho] \]

- **range:** \( 0 \leq S(\rho) \leq \log(d) \)
- **symmetry:** \( S(\rho) = S(U\rho U^+) \)
- **multiplicative:** \( S(\rho \otimes \sigma) = S(\rho) + S(\sigma) \)
- **continuity** (Fannes–Audenaert):
  \[ |S(\rho) - S(\sigma)| \leq \varepsilon \log(d) + H(\varepsilon, 1-\varepsilon) \]
  \( \varepsilon := ||\rho - \sigma||_1 / 2 \)

- **multipartite systems:** \( \rho_{AB} \)
  \( S(A) = S(\rho_A), S(B) = S(\rho_B), \) etc.

- **conditional entropy:** \( S(A|B) := S(AB) - S(B), \) can be < 0

- **mutual information:** \( I(A:B) = S(A) + S(B) - S(AB) = S(A) - S(A|B) = S(B) - S(B|A) \geq 0 \) “subadditivity”
CQ channel coding

\[ \text{CQ} = \text{Classical input, Quantum output} \]

\[ |x\rangle\langle x| \xrightarrow{N} \rho_x = N(|x\rangle\langle x|) \]

Given \( n \) uses of \( N \), how many bits can we send?

Allow error that \( \to 0 \) as \( n \to \infty \).

**HSW theorem:** Capacity = \( \max \chi \)

\[ \chi \left( \{ p_x, \rho_x \} \right) = S(\Sigma_x p_x \rho_x) - \Sigma_x p_x S(\rho_x) \]

\[ \omega_{XQ} = \Sigma_x p_x |x\rangle\langle x| \otimes \rho_x \]

\[ \chi = I(X;Q)_\omega = S(Q) - S(Q|X) \]
HSW coding

\[
\rho = \sum_x p_x \rho_x \\
\chi = S(\rho) - \sum_x p_x S(\rho_x) \\
= S(Q) - S(Q|X)
\]

ambiguity in each message

total information

typical subspace of \( \rho \otimes^n \) has dim \( \approx \exp(n S(\rho)) \)

If \( x=(x_1,\ldots,x_n) \) is \( p \)-typical then \( \rho_{x_1} \otimes \rho_{x_2} \otimes \ldots \otimes \rho_{x_n} \) has typical subspace of dim \( \approx \exp(n \sum_x p_x S(\rho_x)) \)

“Packing lemma”
Can fit \( \approx \exp(n \chi) \) messages.
Packing lemma

Classically: random coding and maximum-likelihood decoding
Quantumly: messages do not commute with each other

Suppose \( \sigma = \sum_x p_x \sigma_x \) and there exist \( \Pi, \{ \Pi_x \} \) s.t.
1. \( \text{tr}[\Pi \sigma_x] \geq 1 - \varepsilon \)
2. \( \text{tr}[\Pi_x \sigma_x] \geq 1 - \varepsilon \)
3. \( \text{tr}[\Pi_x] \leq d \)
4. \( \Pi \sigma \Pi \leq \Pi / D \)

For HSW:
\( \sigma = \rho \otimes^n \) with typ proj \( \Pi \).
\( \sigma_x = \rho_{x_1} \otimes \ldots \otimes \rho_{x_n} \) with typ proj \( \Pi_x \).
\( D \approx \exp(n S(Q)) \)
\( d \approx \exp(n S(Q|X)). \)
Upper bound

\[ X \in \{X_1, \ldots, X^M\} \]

\[ N \otimes_n \]

\[ \rho_X = \rho_{x_1} \otimes \cdots \otimes \rho_{x_n} \]

\[ D \]

\[ Q \]

\[ \Sigma_Y D_Y = I \]

\[ \Pr[Y|X] = \text{tr}[\rho_X D_Y] \]

proof: \[ n \chi \geq I(X;Q) \geq I(X;Y) \geq (1-O(\varepsilon)) \log(M) \]

additivity
Wed 10:50
Cross-Li-Smith.
also Shannon 1948

continuity

data-processing inequality

\[ I(X:Q) = I(X:YQ') \geq I(X:Y) \]

isometry
conditional mutual information

Claim that $I(A:BC) - I(A:B) \geq 0$.

$=: I(A:C|B)$

$= S(A|B) + S(C|B) - S(AC|B)$

$= S(AB) + S(BC) - S(ABC) - S(B)$

If $B$ is classical, $\rho = \sum_b p(b) |b\rangle\langle b| \otimes \sigma(b)_{AC}$ then $I(A:C|B) = \sum_b p(b) I(A:C)_{\sigma(b)} \geq 0$ from subadditivity

$I(A:C|B) \geq 0$ is strong subadditivity [Lieb-Ruskai ’73].

$I(A:C|B) = 0$ for “quantum Markov states”

Wed morning you will hear $I(A:C|B) \geq “non-Markovianity”$
capacity of QQ channels

Additional degree of freedom: channel inputs $|\psi_x\rangle$.

$C^{(1)}(N) = \max\{p_x, \psi_x\} \chi\{p_x, \psi_x\}$

NP-hard optimization problem [Beigi-Shor, H.-Montanaro]

Worse: $C(N) = \lim_{n \to \infty} C^{(1)}(N^{\otimes n})/n$. and $\exists$ channels where $C(N) > C^{(1)}(N)$.

Open questions: Non-trivial upper bounds on capacity. Strong converse ($p_{\text{succ}} \to 0$ when sending $n(C+\delta)$ bits.) (see Berta et al, Thurs 4:15pm).
quantum capacity

How many qubits can be sent through a noisy channel?

\[ Q^{(1)}(N) := \max S(B) - S(E) = \max S(B) - S(RB) = \max -S(R|B) \]

“coherent information”

\[ Q(N) = \lim_{n \to \infty} \frac{Q^{(1)}(N^\otimes n)}{n} \]

not known when \( > 0 \).
sometimes \( Q^{(1)}(N) = 0 < Q(N) \).
Alice and Bob share unlimited free EPR pairs.

\[ C_E(N) = \max \ I(R:B) \]
\[ Q_E(N) = \frac{C_E(N)}{2} \]

1) additive
2) concave in input
\[ \rightarrow \] efficiently computable
covering lemma

Suppose $\sigma = \sum_x p_x \sigma_x$ and there exist $\Pi, \{\Pi_x\}$ s.t.
1. $\text{tr}[\Pi \sigma_x] \geq 1 - \varepsilon$
2. $\text{tr}[\Pi_x \sigma_x] \geq 1 - \varepsilon$
3. $\text{tr}[\Pi] \leq D$
4. $\Pi_x \sigma_x \Pi_x \leq \Pi_x / d$

Covering lemma:
If $x_1, \ldots, x_M$ are sampled randomly from $p$ and $M \gg (D/d) \log(D)/\varepsilon^3$ then with high probability

$$\sigma \approx O(\varepsilon^{1/4}) \quad \sigma x_1 + \cdots + \sigma x_M \leq M$$
Thm: Alice can send secret bits to Bob at rate $I(X:B) - I(X:E)$.  

Proof: packing lemma -> coding $\approx nI(X:B)$ bits for Bob 
covering lemma -> sacrifice $\approx nI(X:E)$ bits to decouple Eve
remote state preparation (RSP)

Q: Cost to transmit n qubits?
A: 2n cbits, n ebits using teleportation.

Cost is optimal given super-dense coding and entanglement distribution.

visible coding: What if the sender knows the state?

We want to simulate the map: “$\psi$” $\rightarrow$ $|\psi\rangle$.
Requires $\geq n$ cbits, but above optimal arguments break.
RSP via covering

Consider the ensemble \( \{U \psi U^\dagger\} \) for random \( U \). Average state is \( I/2^n \).

Covering-type arguments [Aubrun arXiv:0805.2900] \( \rightarrow \)
If we choose \( U_1, \ldots, U_M \) randomly with \( M \gg 2^n / \varepsilon^2 \) then

with high probability, \( \forall \psi \)

\[
\left\| \frac{1}{M} \sum_{i=1}^{M} U_i \psi U_i^\dagger - \frac{I}{2^n} \right\| \leq \varepsilon \frac{2^n}{2^n}
\]

Set \( E_i := \frac{2^n}{M(1 + \varepsilon)} U_i \psi U_i^\dagger \)

Then \( (1- \varepsilon) I \leq \sum_i E_i \leq I \)

So \( \{E_i\} \) is \( \approx \) a valid measurement. So what?
Lemma: \((A \otimes I)\ket{\Phi_d} = (I \otimes A^T)\ket{\Phi_d}\)

Recall:
\[
E_i := \frac{2^n}{M(1 + \epsilon)} U_i \psi U_i^\dagger
\]

Protocol:
\[
|\Phi_{2^n}\rangle_{AB} \xrightarrow{\{E_i^T\}} |\psi\rangle \\
\alpha E_i^T \otimes E_i \propto (U_i \psi U_i^\dagger)^T \otimes (U_i \psi U_i^\dagger)
\]

Cost \(\approx n\) cbits + \(n\) ebits.
RSP of ensembles

can simulate \( x \rightarrow \rho_x \) with cost \( \chi \)

\[ \approx n \chi \text{ cbits} + \text{some ebits} \geq N^\otimes n \geq \approx n \chi \text{ cbits} \]

**Lemma:** Converting \( n(C-\delta) \) cbits + \( \infty \) ebits into \( nC \) cbits will have success probability \( \leq \exp(-n\delta) \).

implies **strong converse:**

sending \( n(\chi + \delta) \) bits through \( N^\otimes n \)
has \( \exp(-n\delta') \) success prob
Let $N$ be a general $q$ channel.

<table>
<thead>
<tr>
<th>Type of simulation</th>
<th>cbit simulation cost</th>
<th>also needs</th>
</tr>
</thead>
<tbody>
<tr>
<td>visible product input</td>
<td>$\chi$</td>
<td>EPR</td>
</tr>
<tr>
<td>visible arbitrary input</td>
<td>$R$</td>
<td>EPR</td>
</tr>
<tr>
<td>arbitrary quantum input</td>
<td>$C_E$</td>
<td>embezzling</td>
</tr>
</tbody>
</table>

$R$ is “strong converse rate”; i.e. $\min$ s.t. sending $n(R+\delta)$ bits has success prob $\leq \exp(-n\delta')$

$$\chi \leq C \leq R \leq C_E$$
merging and decoupling

Reference

R

Alice

\[ |\psi\rangle_{RAB} \]

Bob

A

B

M

AB

A'

B'

Let \[ |\omega\rangle = U|\psi\rangle. \]

Claim: All we need is \[ \omega_{RA'} \approx \omega_R \otimes \omega_{A'}. \]

Pf: The LHS is purified by \[ |\omega\rangle \] and the RHS by \[ |\psi\rangle_{RAB}|\Phi\rangle_{A'B'} \].

Uhlmann's theorem says \( \exists V: MB \rightarrow ABB' \) making these close.
state redistribution

\[ | \psi \rangle_{RABC} \]

Alice

Bob

Reference

\[ | \psi \rangle_{RABC} \]

\[ | \Phi \rangle \]

\[ | \psi \rangle_{RABC} \]

\[ |M| = \frac{1}{2} I(C:R|B) = \frac{1}{2} I(C:R|A) \] qubits communicated

entanglement consumed/created = \[ H(C|RB) \]

[Luo-Devetak, Devetak-Yard]
quantum Markov states

Bob can “redistribute” C to E with $\frac{1}{2} I(A:C|B)$ qubits. If $I(A:C|B)=0$ then this is reversible!

Implies recovery map $R : B \rightarrow BC$ such that $(\text{id}_A \otimes R_{B \rightarrow BC})(\rho_{AB}) = \rho_{ABC}$

structure theorem: $I(A:C|B)=0$ iff

$$B = \bigoplus_i B_i^L \otimes B_i^R$$

$$\rho_{ABC} = \bigoplus_i \rho_i \rho_{AB_i^L}^i \otimes \rho_{B_i^R}^i C$$
approximate Markov states

towards a structure thm: [Fawzi–Renner 1410.0664, others]
If $I(A:C|B) \approx 0$ then $\exists$ approximate recovery map $R$, i.e.
$(\text{id}_A \otimes R_{B \rightarrow BC})(\rho_{AB}) \approx \rho_{ABC}$

states with low CMI appear in condensed matter, optimization, communication complexity, ...

structure theorem: $I(A:C|B)=0$ iff

$$B = \bigoplus_i B^L_i \otimes B^R_i$$

$$\rho_{ABC} = \bigoplus_i \rho_i \rho_{AB^L_i} \otimes \rho_{B^R_i C}$$
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