Inventory Management I: Economic Order Quantity (EOQ)

15.734 Intro to OM, Recitation 3

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June 5, 2014

Announcements

Questions?

- **Great job everyone on the PATA case!**
- **Process Improvement Analysis (individual)**
  - Due **Sat, June 14, 11pm (PDF on Stellar)**
  - Description and formatting guidelines in syllabus
  - Please put the appendix at the end
- **Sport Obermeyer Case (team)**
  - Due **Fri, June 20, beginning of class**
  - Hard copy & PDF on Stellar only; NO Excel sheets
  - Case on Study.Net
  - Questions and guidelines in syllabus
  - Q&A: submit questions by June 12 http://goo.gl/As7VEY
- **Recitation 4: next Thu, June 12, 8pm**
  - Inventory II: Newsvendor, (R,Q) policy

Recitation Outline

- **EOQ basics**
  - Cost trade-offs
  - EOQ computation
- **What-if analyses and robustness**
  - Parameter estimation error
  - Overestimation vs. underestimation
- **Applications**
  - Example 1: Cambridge Chowda Co.
  - Example 2: Textbook revisions
  - Example 3: New-hire orientation
  - Example 4: Building cell towers
  - Example 5: Sticky cement

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Cambridge Chowda Co.

Cambridge Chowda Co. consumes D = 60,000 cases of crackers per year. The crackers cost c = $4 per case, and each order incurs a delivery cost of K = $200. Money spent on crackers has an alternative investment with annual interest of 24%.

(Same numbers as Lecture 5, p.11)

- Demand: D = 60,000 cases per year
- Fixed ordering cost: K = $200 per order
- Variable ordering cost: c = $4 per case
- Holding cost: h = $0.96 per case per year

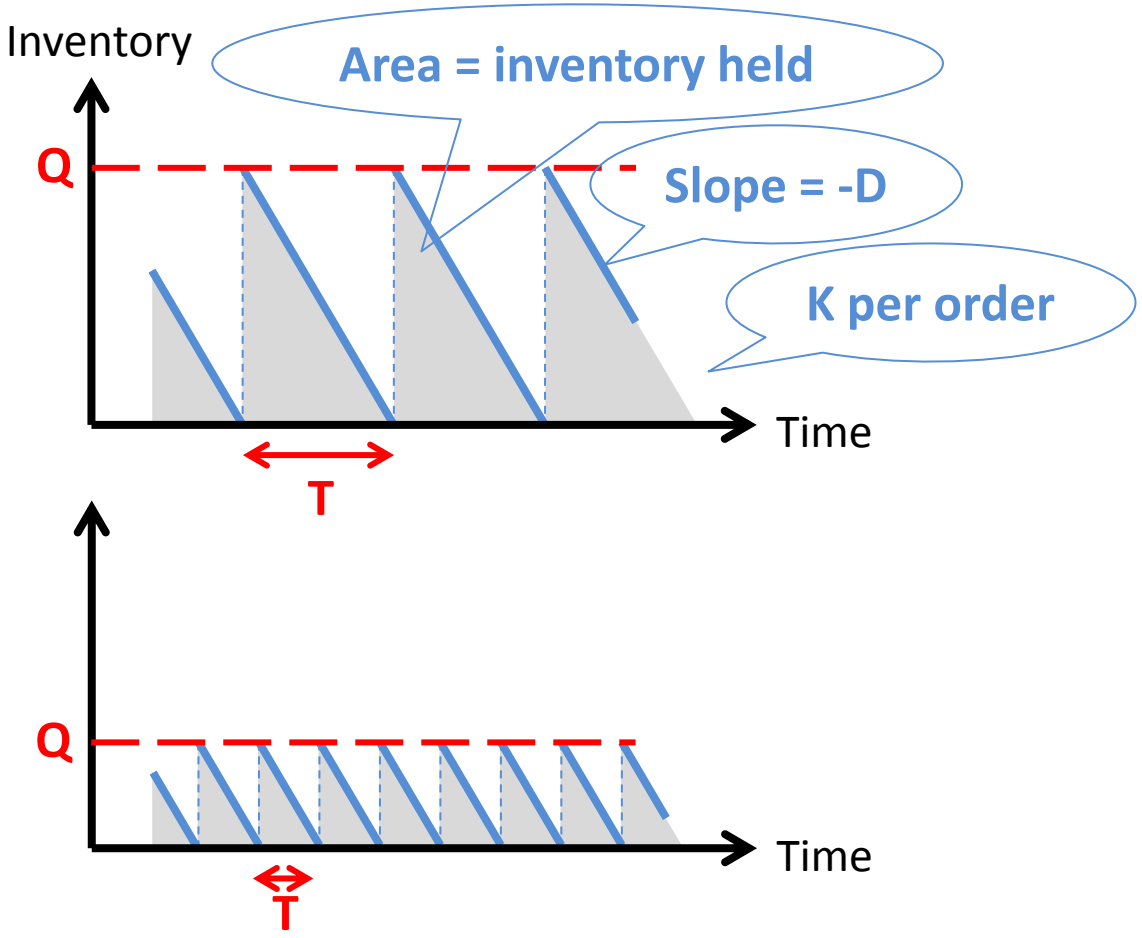
Opportunity cost: alternative investment with 24% annual interest

$\Rightarrow$ holding 1 case for 1 year costs 4*24%= **$0.96 = h**

Inventory Dynamics

Order period T = time it takes to consume Q

$\Rightarrow$ T = Q/D



Compared to the above:

- Lower holding cost

(smaller area under curve)

=> Opportunity cost

- Higher total fixed cost

(more frequent orders)

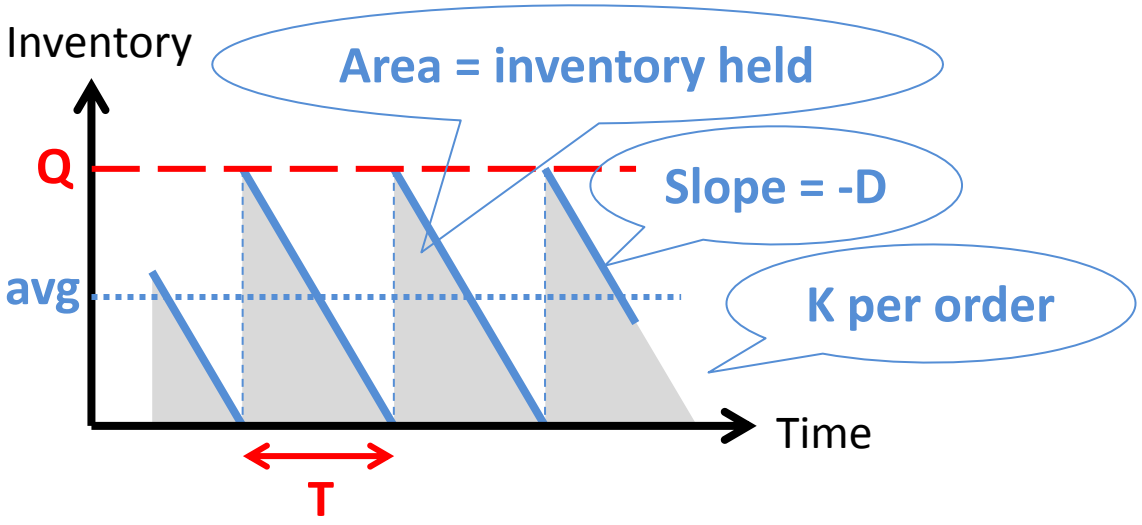
=> Economies of scale

The EOQ model quantifies the **trade-off** between **opportunity cost** and **economies of scale**

Trade-Off of Costs

Order period T = time it takes to consume Q

$\Rightarrow$ T = Q/D



What are the costs?

- Average fixed ordering cost = \$K per T years = K/T = KD/Q
- Average variable ordering cost = D per year * \$c each
- Average holding cost = Q/2 avg inventory * \$h

$\Rightarrow$ Total Average Cost:

$$TAC(Q) = \frac{KD}{Q} + cD + \frac{hQ}{2}$$

Note: Q does not effect cD (you have to spend this much \$ on the demand anyway)

Cambridge Chowda Co.

- Demand: D = 60,000 (cases per year)
- Fixed ordering cost: K = 200 ($ per order)
- Variable ordering cost: c = 4 ($ per case
- Holding cost: h = 0.96 ($ per case per year)

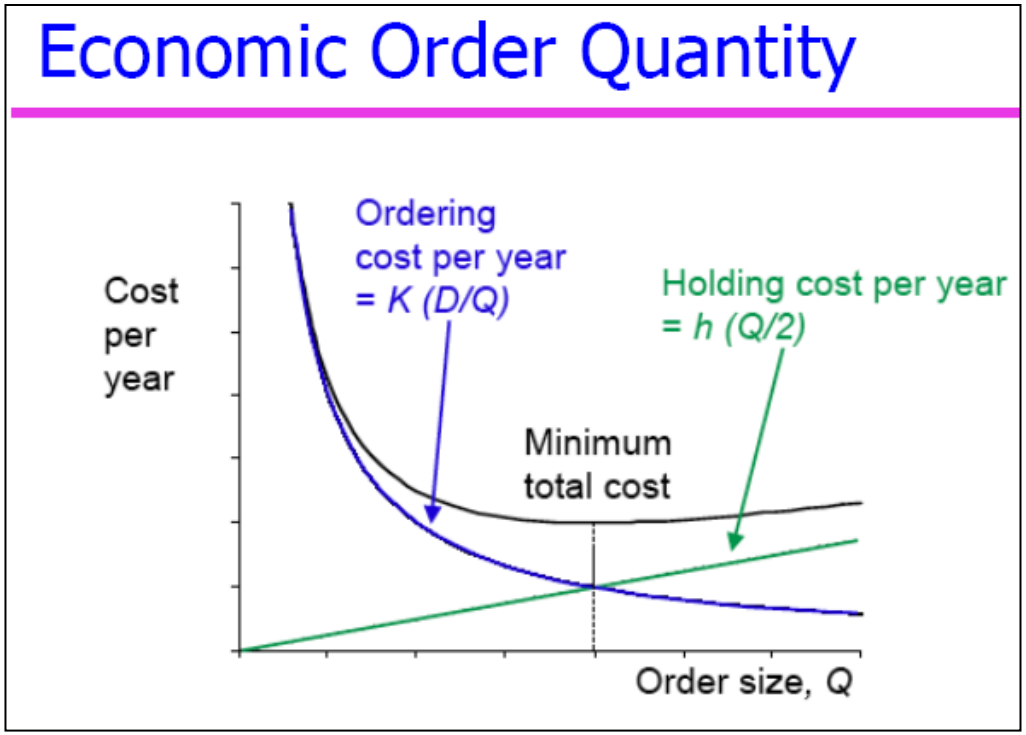
- What is the economic order quantity (EOQ) that minimizes

$$TAC(Q) = \frac{KD}{Q} + \frac{hQ}{2} + cD$$

(Excel demo)

Trade-Off of Costs

Lecture 5 handout, p.11



Observations:

1. At Q*, fixed cost = holding cost
2. Around Q*, cost curve is flat (robust)

Finding the Economic Order Quantity Q*

- **Numerical approach**

(Excel demo)

- **Analytical approach**

$$\min_Q \frac{KD}{Q} + cD + \frac{hQ}{2}$$

At the optimal Q*, the first derivative is 0:

$$\frac{d}{dQ}\left(\frac{KD}{Q} + cD + \frac{hQ}{2}\right)\Big|_{Q=Q^*} = -\frac{KD}{Q^{*2}} + \frac{h}{2} = 0 \quad \Rightarrow \quad Q^* = \sqrt{\frac{2KD}{h}}$$

Cambridge Chowda Co.

- Demand: D = 60,000 (cases per year)
- Fixed ordering cost: K = 200 ($ per order)
- Variable ordering cost: c = 4 ($ per case
- Holding cost: h = 0.96 ($ per case per year)

- EOQ formula:

$$Q^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2 \times 200 \times 60000}{0.96}} = 5,000 \text{ cases}$$

- If the demand expanded by 4 times (now 240,000 cases per year), how much would you order?

$$Q' = \sqrt{\frac{2KD'}{h}} = \sqrt{\frac{2 \times 200 \times (4)60000}{0.96}} = 10,000 \text{ cases}$$

$\Rightarrow$ 2x quantity per order, 2x as frequently

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Cambridge Chowda Co.

- Demand: D = 60,000 (cases per year)
- Fixed ordering cost: K = 200 ($ per order)
- Variable ordering cost: c = 4 ($ per case
- Holding cost: h = 0.96 ($ per case per year)

- EOQ: $$Q^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2 \times 200 \times 60000}{0.96}} = 5,000 \text{ cases}$$

- Estimating the above parameters is difficult. Suppose you overestimated D to be $72,600 (i.e. there were a +21% error in your estimation of D). How much would you lose?
  1. With D=72600, you would set Q = 5500 (+10% increase)
  2. This results in a cost of is TAC(Q) = 4821.82% (+0.45%)
  3. Compared with the optimal solution…

What if there is error in parameter estimation?

- Estimating input parameters D, K or h may sometimes be difficult.
- Estimation error in D, K or h

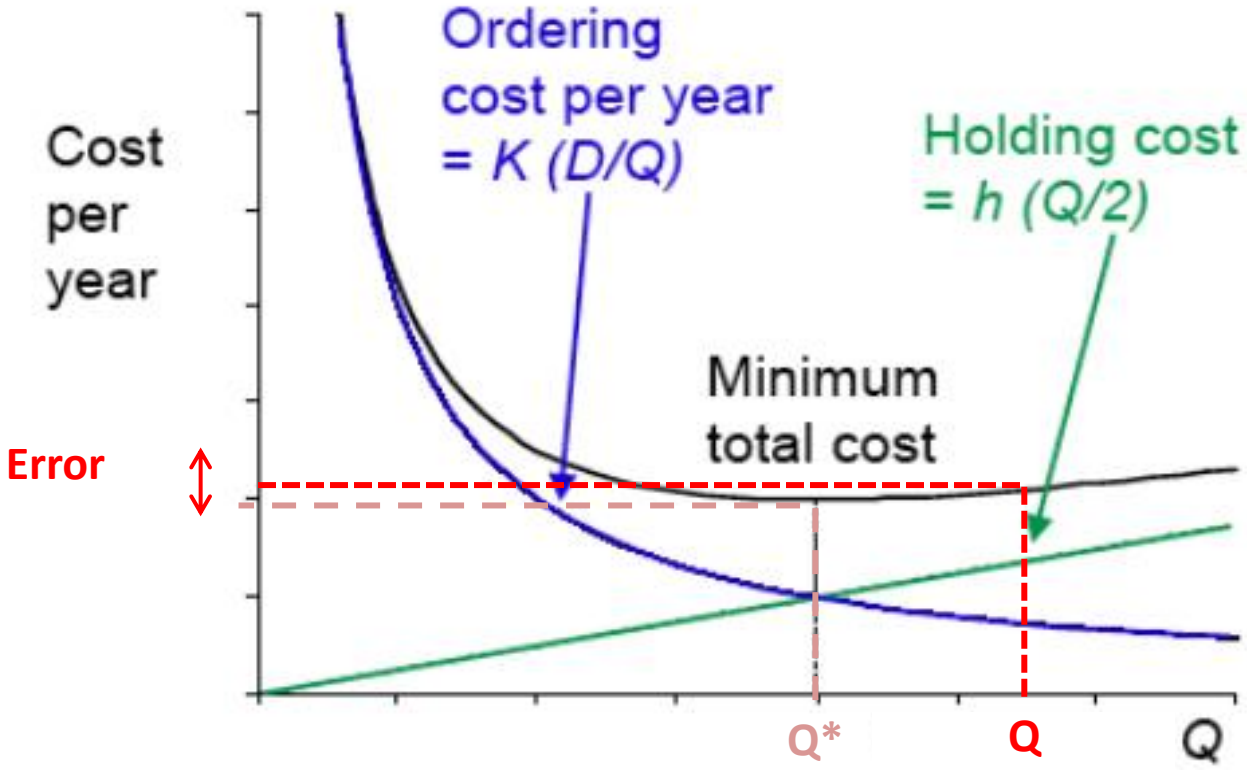
  → error in $Q^* = \sqrt{\frac{2KD}{h}}$

  → suboptimal TAC.
- The effect of error is not as bad:
  - E.g., error in D → less error in Q → even less error in TAC
  - Same story for K and h, except a higher h leads to a lower Q

⇒ **EOQ is robust!**

Error in D	Error in Q	Error in TAC
-75%	-50%	+25.00%
-64%	-40%	+13.33%
-51%	-30%	+6.43%
-36%	-20%	+2.50%
-19%	-10%	+0.56%
0%	0%	0.00%
+21%	+10%	+0.45%
+44%	+20%	+1.67%
+69%	+30%	+3.46%
+96%	+40%	+5.71%
+125%	+50%	+8.33%

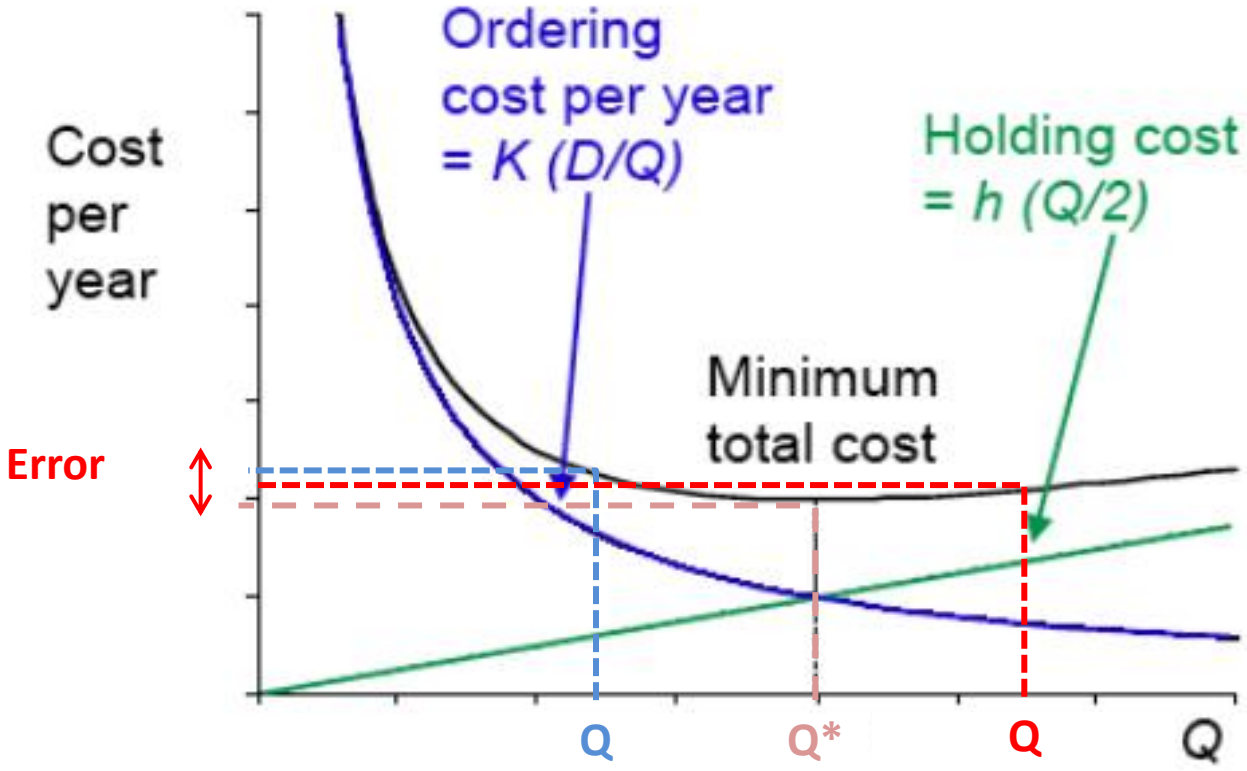
Robustness



Error in Q	Error in TAC
-50%	25.00%
-40%	13.33%
-30%	6.43%
-20%	2.50%
-10%	0.56%
0%	0.00%
+10%	+0.45%
+20%	+1.67%
+30%	+3.46%
+40%	+5.71%
+50%	+8.33%

⇒ **Because the cost curve is "flat" around Q*, the effect of estimation error is not as bad**

Overestimation vs. Underestimation



Error in Q	Error in TAC
-50%	25.00%
-40%	13.33%
-30%	6.43%
-20%	2.50%
-10%	0.56%
0%	0.00%
+10%	+0.45%
+20%	+1.67%
+30%	+3.46%
+40%	+5.71%
+50%	+8.33%

Would you rather overestimate Q, or underestimate?

⇒ **Overestimation causes less damage than underestimation!**

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Textbook Revisions

Pear & Sons publishes a textbook in the rapidly evolving field of operations management.

- Constant annual sales of 100,000 copies.
- Printing each copy costs $60.
- After a new edition is released, its sales value is discounted at the rate of $2 every 3 months.
- Each revision requires $250,000.

How frequently should new editions be published?

$$\Rightarrow \quad D = 100,000, \quad h = 2 \times \tfrac{12}{3} = \$8 \text{ per year}, \quad K = 250,000$$

$$T = Q^*/D = \sqrt{\tfrac{2KD}{h}}/D = \sqrt{\tfrac{2\times 250000\times 100000}{8}}/100000 = 0.79 \text{ years}$$

New-Hire Orientation

Sweet Software Services is hiring!

- Expected growth requires 5 new engineers per month to sustain.
- Every new hire starts with a monthly salary of $7,000.
- New-hire orientation costs $8,500.

How frequently are the orientations? For how many new-hires?

$$\Rightarrow \quad D = 5, \quad h = 7,000, \quad K = 8,500$$

$$Q^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2\times 8500\times 5}{7000}} = 3.485, \quad T^* = \frac{Q}{D} = 0.697 \text{ months}$$

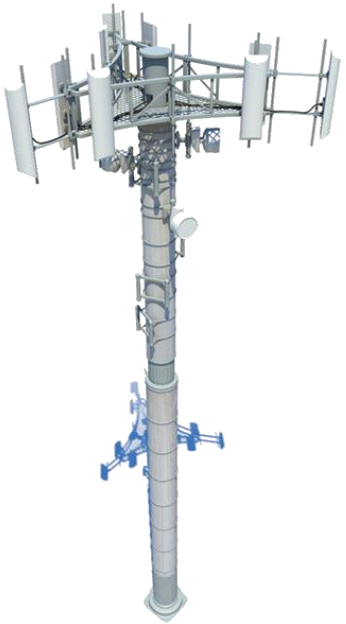
What if you can't hire 3.485 engineers at a time?

$$\Rightarrow \text{TotalAvgCost(Q)} = \frac{KD}{Q} + \frac{hQ}{2}$$

$$TAC(Q^* = 3.485) = 24,392, \quad TAC(3) = 24,666, \quad TAC(4) = 24,625$$

Note: check both rounding down and rounding up!

Building Cell Towers



5G wireless is here! A-Mobile plans to build new cell towers for its growing adoption.

- Marketing Dept predicts demand for coverage will grow at rate of 10 towers per year.
- Starting a new construction costs $500k.
- Each tower costs $50k annually to maintain.

How many towers to build, and how frequently?

$$\Rightarrow \quad D = 10, \quad K = 500, \quad h = 50$$

$$Q^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2\times 500\times 10}{50}} = 14.1 \text{ towers}, \quad T^* = \frac{Q}{D} = 1.41 \text{ years}$$

$$\text{Take } Q = 14 \text{ or } 15? \Rightarrow \text{Take } Q = 14, \quad T = \frac{Q}{D} = 1.4 \text{ years}$$

Sticky Cement

You are in charge of mixing cement and cleaning the extra-large caldron it was mixed in.

- Cement is used at 12 gallons/hr.
- It takes 20 minutes to make a caldron of any size.
- Because the cement is sticky, it takes an extra 2 minute to clean the caldron for every minute that one gallon of cement sits inside it.

How big should each batch be?

$\Rightarrow \quad D = 0.2 \text{ gal/min}, \quad K = 20, \quad h = 2$

$$Q^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2\times 20\times 0.2}{2}} = 2 \text{ gallons}, \quad T^* = \frac{Q}{D} = \frac{2}{0.2} = 10 \text{ minutes}$$

When should you start the next caldron?

Demand during lead time = D*L = 0.2*20 = 4 gal

$\Rightarrow$ Need to consider the effect of lead time!

Applications of EOQ

- "Inventory" and "cost" can mean many things:

Example	Inventory	Holding cost	Fixed cost
Cambridge Chowda Co.	Cases of crackers	Opportunity cost	Delivery
Textbook revision	Books	Depreciation	Revision
New-hire orientation	Software engineers	Salary	Orientation
Building cell towers	Capacity	Maintenance	Expansion
Sticky cement	Soup	Clean-up time	Set-up time

Many sources of holding & fixed costs

Inventory can be people, capacity, …

Cost can be time, effort, …

- Same principle of quantifying cost trade-offs between economies of scale vs. opportunity cost

Main Takeaways

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Questions?

EOQ:

EOQ is **robust** to estimation errors

Overestimation is less damaging

Applications everywhere! Identify inventory & costs