By examining various philosophies involving knowledge, one can develop a “hierarchy of knowability”. Notably, at the very top of this hierarchy would lie *a priori* knowledge and below it would lie the gamut of *a posteriori* knowledge. Two major philosophies that regard themselves with this are Descartes, and Kant. Descartes defines “I think therefore I am” as the basis for all *a priori* knowledge, while Kant defines two classes of knowledge that relate to their knowability.

This paper is concerned not with the hierarchy itself, but only with where logic, in particular, fits in this hierarchy. At first glance, Descartes would appear to label logic *a posteriori* knowledge, as would Kant, though Ayer’s objection to Kant labels logic *a priori* knowledge.

It is [also] important to separate out two aspects of logic: the language, and the underlying meaning. It is easy to argue against the language of logic as *a priori* knowledge, because it does not arise necessarily. Indeed, the language of logic is arbitrary and could be replaced by any other isomorphic language. It just so happens that the language of logic that we use was developed to conveniently represent what was inductively discovered about the world. This precisely parallels the development and nature of arithmetic, as I will discuss later. However, the underlying concept expressed by the language of logic is where logic truly lies, and this is what must be analyzed with regards to logic’s nature as *a priori* or *a posteriori* knowledge.

Descartes stated in his *Meditations* that “I think therefore I am” is the only truly know-
able fact. Thus, given a hierarchy of “knowability”, Cartesians would place existence and nothing else at the very top of this hierarchy. This would be the only thing that is truly knowable \textit{a priori}, while everything else, including logic, would merely be \textit{a posteriori} knowledge, and further down the hierarchy.

Thus, logic must fit in with \textit{a posteriori} knowledge according to Descartes, and must be something garnered from experience and developed through induction. It is not \textit{necessary} that the nature of logic be as it is. However, if \textit{a posteriori} knowledge can only be gained by means of induction, then the rules of logic themselves would have to be arrived at inductively. Logic states that induction of this form is not valid. Thus, logic itself would say that it is false, and clearly this is a contradiction. However, this contradiction is rather meaningless, because the very concept of contradiction, as well as the logic used to conclude it, are defined by the rules of logic.

Taking another look at Descartes’ statement, “I think therefore I am”, reveals that the statement itself is in a logical form. This could be argued by saying the logical statement of it is not necessary for the concept itself, but is only a convenient means by which to present the underlying idea. However, as I pointed out before, it is necessary to separate the language of logic from the concept of logic. Because the accepted language of logic is used to present the idea, then there exists a logical representation of the underlying idea, and, even if the language were changed, this underlying idea would still be the same, and if the language were changed enough to make it no longer a statement of logic, then the underlying concept may no longer be within the realm of logic, but the idea would be fundamentally different.
Thus, by the fact that the underlying idea lies within the realm of logical representation, it becomes effectively inseparable from logic. If it were restated such that it were no longer within the realm of logic, it would not be the same idea. Therefore, logic necessarily exists.

Now, suppose that Descartes’ statement were only to imply the existence of itself within the logical realm (that is, it doesn’t imply the existence of all of logic, but only the subset of logic that is the statement itself). This would mean that logic is not fundamental, and that the remainder of it could be developed in a synthetic manner. However, logic is based on the fact that all logical statements are equivalent (everything can be resolved into Aristotle’s Law of Identity, $A = A$). Thus, every logical statement is equivalent to Descartes’ statement, and thus equally true, and equally knowable. Therefore, logic not only exists, but is complete.

Kant, on the other hand, took a different approach, and formalized a concept of analytical and synthetic judgments, in which analytical judgments were those that could be known *a priori*, and synthetic judgments were those that could be known *a posteriori*. Thus, Kantian thought would place analytical judgments at the very top of the hierarchy of “knowability”, and synthetic judgments would descend from there.

Interestingly, Kant defines analytic judgments as “adding nothing through the predicate to the concept of the subject, but merely breaking it up into those constituent concepts that have all along been thought in it”. Or, as Ayer puts it, “analytic judgments are devoid of factual content”. This has the interesting effect of making the only things that are at the very top of our hierarchy of knowability contain no factual information. []
We looked at the world and saw that it was conservative. Thus, we defined addition, the fundamental operator of conservation (and the operator from which all other arithmetic operators can be defined).

What is only knowable *a posteriori* is that the laws of conservation hold. If the laws of physics were different, $1 + 1$ would still be 2, but it may be of less interest to those seeking an *a posteriori* understanding of their universe.

Another interesting question that arises from this is the following: Is the statement “I think therefore I am”, simply a consequence of the nature of implication (and thus a Kantian analytical statement)? Or is it just another tautology; a way to “say in a round-about fashion $A = A$ ”, as Poincaré put it? [perhaps move with equivalence argument]

There is a fundamental flaw in any reasoning regarding the universality of logic, and any such reasoning will suffer from a fallacy similar to that of absolute relativism. Namely, if one believes that logic is not fundamental, but is, instead, a synthetic development, then they have just undermined any mechanism by which they could argue their point. On the other hand, if one believes that logic is fundamental, and argues that point, then they will
be assuming in their argument that logic is infallible.

[conclusion]