Real Options Signaling Games with Applications to Corporate Finance

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We study games in which the decision to exercise an option is a signal of private information to outsiders, whose beliefs affect the utility of the decision-maker. Signaling incentives distort the timing of exercise, and the direction of distortion depends on whether the decision-maker's utility increases or decreases in outsiders' belief about the payoff from exercise. In the former case, signaling incentives erode the value of the option to wait and speed up option exercise, while in the latter case option exercise is delayed. We demonstrate the model's implications through four corporate finance settings: investment under managerial myopia, venture capital grandstanding, investment under cash flow diversion, and product market competition. (*JEL* G31, D82)

The real options approach to investment and other corporate finance decisions has become an increasingly important area of research in financial economics. The main underlying concept is that an investment opportunity is valuable not only because of associated cash flows but also because the decision to invest can be postponed. As a result, when making the investment decision, one must take into account both the direct costs of investment and the indirect costs of foregoing the option to invest in the future. The applications of the real options framework have become quite broad.[1]

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[1] The early literature, started by Brennan and Schwartz (1985) and McDonald and Siegel (1986), is well summarized in Dixit and Pindyck (1994). Recently the real options framework has been extended to incorporate competition among several option holders (e.g., Grenadier 2002; Lambrecht and Perraudin 2003; Novy-Marx 2007) and agency conflicts (Grenadier and Wang 2005). Real options models have been applied to study specific industries such as real estate (Titman 1985; Williams 1991) and natural resources (Brennan and Schwartz 1985) and other corporate decisions such as defaults (e.g., Leland 1994) and mergers (Lambrecht 2004; Morellec and Zhdanov 2005; Hackbarth and Morellec 2008; Hackbarth and Miao 2011). See Leslie and Michaels (1997) for a discussion of how practitioners use real options ideas.

One aspect that is typically ignored in standard models is that most real options exercise decisions are made under asymmetric information: The decision-maker is better informed about the value of the option than outsiders. Given the importance of asymmetric information in corporate finance, it is useful to understand how it affects real options exercise decisions.[2] In this article, we explore this issue by incorporating information asymmetry into real options modeling. We consider a setting that is flexible enough to handle a variety of real-world examples, characterize the effects of asymmetric information, and then illustrate the model using four specific applications: investment under managerial myopia, venture capital grandstanding, investment under cash flow diversion by the manager, and product market entry decisions by two asymmetrically informed firms.

In the presence of asymmetric information, the exercise strategy of a real option is an important information transmission mechanism. Outsiders learn information about the decision-maker from observing the exercise (or lack of exercise) of the option, and thereby change their assessment of the decision-maker. In turn, because the decision-maker is aware of this information transmission effect, the option exercise strategy is shaped to take advantage of it. To provide further motivation for the study, consider two examples of options exercise decisions, where asymmetric information and signaling are likely to be especially important.

Example 1. Delegated investment decisions in corporations. In most modern corporations, the owners of the firm delegate investment decisions to the manager. There is substantial asymmetric information: The manager is typically much better informed about the underlying cash flows of the investment project than the shareholders. In this context, the manager's decision when to invest transmits information about the project's net present value (NPV). While in some agency settings the manager may want to signal higher project values to boost her future compensation, in other agency settings the manager may want to signal a lower project NPV to divert more value for her own private consumption. In either setting, however, the manager will take this information transmission effect into account when deciding when to invest.

Example 2. Exit decisions in the venture capital industry. In the venture capital (VC) industry, there is substantial asymmetric information about the value of the fund's portfolio companies, since the VC firm that manages the fund has much better information about the fund's portfolio companies than the fund's outside investors. In this context, the firm's decision when to sell a portfolio company transmits information about its value, and hence impacts outsiders' inferences of the firm's investment skill. Because investor inferences of the firm's investment skill impact the firm's future fund-raising ability, the

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[2] See Tirole (2006), Chapter 6, for a discussion of asymmetric information in corporate finance.

firm will take this information transmission effect into account when deciding when to sell a portfolio company.

We call such interactions real options signaling games, and study them in detail in this article. We begin our study with a general model of options exercise under asymmetric information. Specifically, we consider a decision-maker whose payoff from option exercise comprises two components. The first component is simply some fraction of the project's payoff. The second component, which we call the belief component, depends on outsiders' assessment of the decision-maker's type. The decision-maker's type determines the project's NPV and is the private information of the decision-maker. Our central interest is in separating equilibria—equilibria in which the decision-maker reveals her type through the options exercise strategy.[3] We characterize a separating equilibrium of the general model, and prove that under standard regularity conditions it exists and is unique. The equilibrium is determined by a differential equation given by local incentive compatibility.

We show that the implied options exercise behavior differs significantly from traditional real options models. The first-best (symmetric information) exercise threshold is never an equilibrium outcome, except for the most extreme type: Because the decision-maker's utility depends on outsiders' belief about the decision-maker's type, there is an incentive to deviate from the symmetric information threshold to mimic a different type and thereby take advantage of outsiders' incorrect belief. While information asymmetry distorts the timing of options exercise, the direction of the effect is ambiguous and depends on the nature of the interactions between the decision-maker and outsiders.

The first contribution of our article is the characterization of the direction of distortion. We show that the direction of distortion depends on a simple and intuitive characteristic: the derivative of the decision-maker's payoff with respect to the belief of outsiders about the decision-maker's type. If the decision-maker benefits from outsiders believing that the project's value is higher than in reality, then signaling incentives lead to earlier options exercise than in the case of symmetric information. In contrast, if the decision-maker benefits from outsiders believing that the project's value is lower than in reality, then the option is exercised later than in the case of symmetric information. The intuition underlying this result comes from the fact that earlier exercise is a signal of the better quality of the project. For example, other things equal, an oil-producing firm decides to drill an oil well at a lower oil price threshold when it believes that the quality of the oil well is higher. Because of this, if the decision-maker benefits from outsiders believing that the project's quality is higher (lower) than in reality, she has incentive to deviate from the first-best exercise threshold by exercising the option marginally earlier (later) and attempting to fool the

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[3] In fact, as we discuss in Section 2.4, any non-separating equilibrium can be ruled out using the D1 restriction on the out-of-equilibrium beliefs of outsiders.

market into believing that the project's quality is higher (lower) than in reality. In equilibrium, the exercise threshold will be lowered (raised) to the point at which the decision-maker's marginal costs of inefficiently early (late) exercise exactly offset her marginal benefits from fooling outsiders. Importantly, outsiders are rational. They are aware that the decision-maker shapes the exercise strategy to affect their belief. As a result, in equilibrium outsiders always correctly infer the private information of the agent. However, even though the private information is always revealed in equilibrium, signal-jamming occurs: The exercise thresholds of all types, except for the most extreme type, are different from the first-best case and are such that no type has an incentive to fool outsiders.

The second contribution of our article is illustrating the general model with four corporate finance applications that put additional structure on the belief component of the decision-maker's payoff: investment under managerial myopia, venture capital grandstanding, investment under cash flow diversion by the manager, and product market entry decisions by two asymmetrically informed firms. The first application we consider is a timing analogue to the myopia model of Stein (1989). We consider a public corporation, in which the investment decision is delegated to a manager, who has superior information about the project's NPV. As in Stein (1989), the manager is myopic in that she cares not only about the long-term performance of the company but also about the short-term stock price. The timing of investment reveals the manager's private information about the project and thereby affects the stock price. As a result, the manager invests inefficiently by exercising her investment option too early in an attempt to fool the market into overestimating the project's NPV and thereby inflating the current stock price.

The second application deals with the VC industry. As discussed in Gompers (1996), younger VC firms often take companies public earlier than older VC firms to establish a reputation and successfully raise capital for new funds. Gompers terms this phenomenon "grandstanding" and suggests that inexperienced VC firms employ early timing of initial public offerings (IPOs) as a signal of their ability to form higher-quality portfolios. We formalize this idea in a two-stage model of VC investment. An inexperienced VC firm invests limited partners' money in the first round and then decides when to take its portfolio company public. Limited partners update their estimate of the general partner's investment-picking ability by observing when the decision to take the portfolio company public is made and use this estimate when deciding how much to invest in the second round. Because the amount of second-round financing is positively related to the limited partners' estimate of the general partner's ability, the general partner has an incentive to fool the limited partners into believing that her ability is higher. Since an earlier IPO is a signal of better quality of the inexperienced general partner, signaling incentives lead to earlier than optimal exit timing of inexperienced general partners, consistent with the grandstanding phenomenon of Gompers (1996).

The other two applications belong to the case of the decision-maker benefiting more when outsiders believe that the project's NPV is lower than in reality, and thus imply an inefficiently delayed options exercise. Similar to the first application, the third application studies a delegated investment decision in a corporation. However, unlike the second application, the nature of the agency conflict is different. Specifically, we consider a setting in which a manager can divert a portion of the project's cash flows for private consumption, which makes the problem a timing analogue of the literature on agency, asymmetric information, and capital budgeting (e.g., Harris, Kriebel, and Raviv 1982; Stulz 1990; Bernardo, Cai, and Luo 2001). In this application, private information gives the manager an incentive to delay investment so that outside investors underestimate the true NPV of the project, which allows the manager to divert more without being caught. This creates incentives to fool outside investors by investing as if the project was worse than in reality and thereby leads to later investment than in the case of symmetric information. In equilibrium, outside investors correctly infer the NPV of the project, but still signal-jamming occurs: Investment is inefficiently delayed to prevent the manager from fooling outside investors.

Finally, the fourth application we consider is sequential entry into a product market in the duopoly framework outlined in Chapter 9 of Dixit and Pindyck (1994). The major distinction of our application is that we relax the assumption that both firms observe the potential NPV from launching the new product. Instead, we assume that the two firms are asymmetrically informed: One firm knows the project's NPV, while the other learns it from observing the investment (or lack of investment) of the better-informed firm. As a result, the better-informed firm has an incentive to delay investment to signal that the quality of the project is worse than in reality and thereby delay the entry of its competitor and enjoy monopoly power for a longer period of time. Thus, the timing of investment is inefficiently delayed. However, the underinformed firm rationally anticipates the delay of investment by the better-informed firm, so in equilibrium the timing of investment reveals the NPV of the product truthfully.

Our findings have a number of implications. First, the effect of information asymmetry on investment timing is far from straightforward. In fact, information asymmetry can both speed up and delay investment, thus leading to overinvestment and underinvestment, respectively. The direction of distortion depends on the nature of the agency conflict between the manager and shareholders. For example, both the first and the third applications deal with corporate investment under asymmetric information and agency, but have different implications for the effect of information asymmetry on investment. If the agency problem is in managerial short-termism, then asymmetric information leads to earlier investment. In contrast, if the agency problem is in the manager's ability to divert cash flows for personal consumption, then asymmetric information leads to later investment.

Second, because the degree of distortion depends on a simple and intuitive measure, one can evaluate the qualitative effect of asymmetric information on the timing of investment even in complicated settings with multiple agency conflicts of differing natures. Clearly, in the real world, there are many potential agency conflicts, including managerial short-termism and the ability to divert cash flows among others. One can obtain the resulting effect of asymmetric information by looking at the effect on the manager's payoff of a marginal change in the belief of outsiders. This characterization can be important for empirical research, as it implies a clear-cut relation between investment, on the one hand, and the complicated structure of managerial incentives, on the other hand.

Finally, regarding the last application, our findings demonstrate that competitive effects on investment can be significantly weakened if the competitors are asymmetrically informed about the value of the investment opportunity. A substantial literature on real options (e.g., Williams 1993; Grenadier 2002) argues that the fear of being preempted by a rival erodes the value of the option to wait and, as a consequence, speeds up investment. However, when the competitors are asymmetrically informed about the investment opportunity, better-informed firms have incentives to fool the uninformed firms into underestimating the investment opportunity and delaying their investment. The better-informed firms achieve this by investing later than in the symmetric information case. Thus, signaling incentives imply an additional value of waiting, and therefore greater delay in the firms' investment decisions.

Our article combines the traditional literature on real options with the extensive literature on signaling. It is most closely related to real options models with imperfect information. Grenadier (1999), Lambrecht and Perraudin (2003), and Hsu and Lambrecht (2007) study options exercise games with information imperfections, however, with very different equilibrium structures than in this article. In Grenadier (1999), each firm has an imperfect private signal about the true project value. In Lambrecht and Perraudin (2003), each firm knows its own investment cost but not the investment cost of the competitor. And in Hsu and Lambrecht (2007), an incumbent is uninformed about the challenger's investment cost. While these papers study options exercise with information imperfections of various forms, the beliefs of outsiders do not enter the payoff function of agents. Therefore, the models in these papers are not examples of real options signaling games: The informed decision-maker has no incentives to manipulate investment timing so as to alter the belief of outsiders.[4]

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[4] Our application on cash flow diversion is also related to Grenadier and Wang (2005) and Bouvard (2010), who study investment timing under asymmetric information between the manager and investors, where the timing of investment can be part of the contract between the parties. The major difference between their models and our diversion application is that theirs are screening models, while ours is a signaling model.

Notably, Morellec and Schürhoff (2011) and Bustamante (forthcoming) develop models that are examples of real options signaling games, and thus can be thought of in the context of our general model. Specifically, in Morellec and Schürhoff (2011), an informed firm, seeking external resources to finance an investment project, can choose both the timing of investment and the means of financing (debt or equity) of the project. In Bustamante (forthcoming), an informed firm can decide on both the timing of investment and whether to finance its investment project through an IPO or costlier private capital. Bustamante (forthcoming) and Morellec and Schürhoff (2011) find that asymmetric information speeds up investment as the firm attempts to signal better quality and thereby secure cheaper financing. Our contribution relative to these papers is the characterization of the distortion of investment in a general setting of real options signaling games, which allows for a wide range of environments where real options are common, such as public corporations, VC industry, or entrepreneurial firms. First, we show that whether asymmetric information speeds up or delays investment depends critically on the nature of the interactions between the decision-maker and outsiders. In fact, as we show in the applications, signaling incentives can often delay investment, unlike in Bustamante (forthcoming) and Morellec and Schürhoff (2011) where signaling incentives always speed up investment because of the specific nature of the interactions between the manager and outsiders. Second, we characterize the exact conditions when each of the two distortions is in place. This implies specific predictions for each particular institutional setting and shows when a distortion induced by one type of agency conflict (e.g., possibility of cash flow diversion) can be overturned by the presence of another agency conflict (e.g., managerial short-termism). Finally, Benmelech, Kandel, and Veronesi (2010) consider a dynamic model of investment with asymmetric information between the manager and outsiders and show that in the presence of stock-based compensation, asymmetric information creates incentives to conceal bad news about growth options. Unlike our article, they focus on a specific setting and do not model investment as a real option.

The remainder of the article is organized as follows. In Section 1, we formulate the general model of options exercise in a signaling equilibrium and consider the special case of symmetric information. In Section 2, we solve for the separating equilibrium of the model, prove its existence and uniqueness, and determine when asymmetric information leads to earlier or later options exercise. In Section 3, we consider two examples of real options signaling games in which signaling incentives speed up options exercise: investment in the presence of managerial myopia and VC grandstanding. In Section 4, we consider two examples of real options signaling games in which signaling incentives delay option exercise: investment under the opportunity to divert cash flows and strategic entry to the product market. Finally, we conclude in Section 5.

1. Model Setup

In this section, we present a general model of a real options signaling game. Then, as a useful benchmark, we provide the solution to the first-best case of symmetric information. For the ease of exposition, we discuss the model as if the real option is an option to invest. However, this is without loss of generality. For example, the real option can also be an option to penetrate a new market, make an acquisition, or sell a business.

1.1 The real option

The firm possesses a real option of the standard form: At any time t, the firm can spend a cost $\theta > 0$ to install an investment project. The project has a present value $P(t)$, representing the discounted expected cash flows. Following the standard real options framework (e.g., McDonald and Siegel 1986; Dixit and Pindyck 1994), we assume that $P(t)$ evolves as a geometric Brownian motion:

$$dP(t) = \mu P(t)\, dt + \sigma P(t)\, dB(t), \tag{1}$$

where $\sigma > 0$ and $dB(t)$ is the increment of a standard Brownian motion. All agents in the economy are risk-neutral, with the risk-free rate of interest denoted by r. To ensure finite values, we assume $\mu < r$.[5] If the firm invests at time t, it gets the value of

$$P(t) - \theta + \varepsilon, \tag{2}$$

where ε is a zero-mean noise term, reflecting the difference between the realized value of the project and its expected value upon investment. It reflects uncertainty over the value of the project at the time of investment, which can stem from random realized cash flows or random installation costs.

The investment decision is made by the agent, who has superior information about the NPV of the project. Specifically, $P(t)$ is publicly observable and known to both the agent and outsiders. In contrast, θ is the private information of the agent, which we refer to as the agent's (or project's) type. Because the payoff of the project depends on θ negatively, higher types correspond to worse projects. Outsiders do not have any information about θ except for its ex-ante distribution, which is given by the cumulative distribution function $\Phi(\cdot)$ with positive density function $\phi(\cdot)$ defined on $[\underline{\theta}, \bar{\theta}]$, where $\bar{\theta} > \underline{\theta} > 0$.[6]

Thus, the payoff from investment comprises three components: the publicly observable component $P(t)$, the privately observable component θ, and the

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[5] See McDonald and Siegel (1986) and Chapter 5 of Dixit and Pindyck (1994) for a discussion of this restriction. Instead of risk neutrality, we could assume that $P(t)$ evolves as (1) under the risk-neutral measure.

[6] The assumption that the privately observable component of the project is the investment cost is without loss of generality. The model can also be formulated when the privately observable component θ corresponds to part of the project's present value rather than the investment cost (as in Grenadier and Wang 2005) or when it affects the present value of the project multiplicatively (as in Bustamante forthcoming and Morellec and Schürhoff 2011).

noise term ε. Outsiders will update their belief about the type of the agent by observing the timing of investment and its proceeds. The noise term ε ensures that proceeds from investment provide only an imperfect signal of the agent's private information.[7]

1.2 The agent's utility from exercise

Having characterized the project payoff, we move on to the utility that the agent receives from exercise. We assume that the agent's utility from exercise is the sum of two components. The first component is the direct effect of the proceeds from the project on the agent's compensation. This effect can be explicit, such as through the agent's stock ownership in the firm, or implicit, such as through future changes in the agent's compensation. For tractability reasons, we abstain from solving the optimal contracting problem, and instead simply assume that the agent receives a positive share α of the total payoff from the investment project.[8] The second component is the indirect effect of investment on the agent's utility due to its effect on outsiders' belief about the agent's type. Intuitively, the timing of investment can reveal information about the agent, such as an ability to generate profitable investment projects. Letting $\tilde{\theta}$ denote outsiders' inference about the type of the agent after the investment, the agent's utility from the option exercise is

$$\text{Agent's utility from exercise} = \text{share of project } + \text{ belief component}$$
$$= \alpha \left(P\left(\tau \right) - \theta + \varepsilon \right) + W\left(\tilde{\theta}, \theta \right). \qquad (3)$$

While standard real options models typically assume that the agent's utility is solely a function of the option payoff, in this case the agent also cares about the belief of outsiders, in that $\tilde{\theta}$ explicitly enters into the agent's payoff function. The form of the utility function is general enough to accommodate a variety of settings in which a real option is exercised by a better-informed party who cares about the belief of less-informed outsiders.[9]

Following Mailath (1987), we impose the following regularity conditions on $W\left(\tilde{\theta}, \theta \right)$:

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[7] We introduce the noise term to make the timing of exercise a meaningful signal of the agent's private information. If ε were always equal to zero, then outsiders would be able to learn the exact value of θ from observing the realized value of the project. As a consequence, the timing of exercise would have no information role. Because of risk neutrality, as long as there is some noise, its distribution is not important for our results, with the exception of the model in Section 4.1, where its distribution impacts the underlying costly state verification model.

[8] See Grenadier and Wang (2005) and Philippon and Sannikov (2007) for optimal contracting problems in the real options context.

[9] The form of the utility function from exercise in (3) is chosen to keep the model both tractable and sufficiently general. We have also solved the model for an even more general utility function, $\alpha \left(F\left(P\left(\tau \right) \right) - \theta + \varepsilon \right) + W\left(P\left(\tau \right), \tilde{\theta}, \theta \right)$. The results are very similar, as long as the utility function satisfies the regularity conditions in Mailath (1987).

Assumption 1. $W\left(\tilde{\theta}, \theta\right)$ is C^2 on $[\underline{\theta}, \bar{\theta}]^2$;

Assumption 2. $W\left(\theta, \theta\right) < \alpha\theta$;

Assumption 3. $W_{\tilde{\theta}}\left(\tilde{\theta}, \theta\right)$ never equals zero on $[\underline{\theta}, \bar{\theta}]^2$, and so is either positive or negative;

Assumption 4. $W\left(\tilde{\theta}, \theta\right)$ is such that $W_\theta\left(\tilde{\theta}, \theta\right) < \alpha \; \forall \left(\tilde{\theta}, \theta\right) \in [\underline{\theta}, \bar{\theta}]^2$ and $W_{\tilde{\theta}}\left(\theta, \theta\right) + W_\theta\left(\theta, \theta\right) < \alpha \; \forall\theta \in [\underline{\theta}, \bar{\theta}]$;

Assumption 5. Agent's utility from exercise satisfies the single-crossing condition, defined in Appendix A.

These conditions allow us to establish the existence and uniqueness of the separating equilibrium derived in the following section. Assumption 1 is a standard smoothness restriction. Assumption 2 states that under perfect information the effect of the belief component does not exceed the direct effect of θ. This ensures that the exercise decision is non-trivial, because otherwise the optimal exercise decision would be to invest immediately for any project's present value $P(t)$. Assumption 3 is the belief monotonicity condition, which requires the agent's payoff to be monotone in outsiders' belief about the agent's type. This defines two cases to be analyzed. If $W_{\tilde{\theta}} < 0$, then the agent benefits if outsiders believe that the project has a lower investment cost. Conversely, if $W_{\tilde{\theta}} > 0$, then the agent gains from belief of outsiders that the project has a higher investment cost. Assumption 4 means that the agent is better off from having a better project: $W_\theta\left(\tilde{\theta}, \theta\right) < \alpha$ implies that the agent's utility from exercise is decreasing in θ for any fixed level of the outsiders' belief; similarly, $W_{\tilde{\theta}}\left(\theta, \theta\right) + W_\theta\left(\theta, \theta\right) < \alpha$ implies that the agent's utility from exercise is decreasing in θ if both the agent and outsiders know θ. Finally, Assumption 5 ensures that if the agent does not make extra gains by misrepresenting θ slightly, then she cannot make extra gains from a large misrepresentation. It allows us to find the separating equilibrium by considering only the first-order condition.

1.3 Symmetric information benchmark

As a benchmark, we consider the case in which information is symmetric. Specifically, assume that both the agent and outsiders observe θ.[10] Let V^* (P, θ) denote the value of the investment option to the agent, if the type of the agent is θ and the current level of $P(t)$ is P. Using standard arguments (e.g., Dixit and Pindyck 1994), in the range prior to investment, $V^*(P, \theta)$ must solve the differential equation:

$$0 = \frac{1}{2}\sigma^2 P^2 V_{PP}^* + \mu P V_P^* - r V^*. \tag{4}$$

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[10] If neither the agent nor outsiders observe θ, then the model is analogous to the one in this section.

Suppose that the agent of type θ invests the first time when $P(t)$ crosses threshold $P^*(\theta)$ from below. Upon investment, the payoff of the agent is specified in (3), implying the boundary condition for the agent's expected payoff from exercise:

$$V^*\left(P^*(\theta),\theta\right) = \alpha\left(P^*(\theta) - \theta\right) + W(\theta,\theta). \tag{5}$$

Solving (4) subject to boundary condition (5) yields the following option value to the agent:[11]

$$V^*(P,\theta) = \begin{cases} \left(\frac{P}{P^*(\theta)}\right)^{\beta}\left(\alpha\left(P^*(\theta) - \theta\right) + W(\theta,\theta)\right), & \text{if } P \leq P^*(\theta), \\ \alpha\left(P - \theta\right) + W(\theta,\theta), & \text{if } P > P^*(\theta), \end{cases} \tag{6}$$

where β is the positive root of the fundamental quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$:

$$\beta = \frac{1}{\sigma^2}\left[-\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right] > 1. \tag{7}$$

The investment trigger $P^*(\theta)$ is chosen by the agent so as to maximize her value:

$$P^*(\theta) = \arg\max_{\hat{P} \in \mathbb{R}_+}\left\{\frac{1}{\hat{P}^{\beta}}\left(\alpha\left(\hat{P} - \theta\right) + W(\theta,\theta)\right)\right\}. \tag{8}$$

Taking the first-order condition, we conclude that $P^*(\theta)$ is given by

$$P^*(\theta) = \frac{\beta}{\beta - 1}\left(\theta - \frac{W(\theta,\theta)}{\alpha}\right). \tag{9}$$

In particular, if $W(\theta,\theta) = 0$, we get the standard solution (e.g., Dixit and Pindyck 1994): $P^*(\theta) = \theta\beta/(\beta - 1)$. Because the agent's utility from exercise is decreasing in θ by Assumption 4, the investment threshold $P^*(\theta)$ is increasing in θ, which means that the firm invests earlier if the project is better.

The results of the benchmark case can be summarized in Proposition 1:

Proposition 1. Suppose that θ is known both to the agent and to outsiders. Then, the investment threshold of type θ, $P^*(\theta)$, is given by (9) and is increasing in θ.

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[11] Since $P = 0$ is an absorbing barrier, $V^*(P,\theta)$ must also satisfy the condition $V^*(0,\theta) = 0$.

2. Analysis

In this section, we provide the solution to the general real options signaling game under asymmetric information between the agent and outsiders. First, we solve for the agent's optimal exercise strategy for a given inference function of outsiders. Then, we apply the rational expectations condition that the inference function must be consistent with the agent's exercise strategy. This gives us the equilibrium investment threshold. We present a heuristic analysis in this section and prove that it indeed yields the unique separating equilibrium in Proposition 2. Finally, we analyze properties of the equilibrium.

2.1 Optimal exercise

To solve for the separating equilibrium, consider the agent's optimal exercise strategy for a given outsiders' inference function. Specifically, suppose that outsiders believe that the agent of type θ exercises the option at trigger $\bar{P}(\theta)$, where $\bar{P}(\theta)$ is a monotonic and differentiable function of θ. Thus, if the agent exercises the option at trigger $\hat{P} \in \bar{P}\left(\left[\underline{\theta}, \bar{\theta}\right]\right)$, then upon exercise outsiders infer that the agent's type is $\bar{P}^{-1}\left(\hat{P}\right)$.[12]

Let $V\left(P, \tilde{\theta}, \theta\right)$ denote the value of the option to the agent, where P is the current value of $P(t)$, $\tilde{\theta}$ is a fixed outsiders' belief about the agent's type, and θ is the agent's true type. By the standard valuation arguments (e.g., Dixit and Pindyck 1994), in the region prior to exercise, $V(P, \tilde{\theta}, \theta)$ must satisfy the differential equation:

$$0 = \frac{1}{2}\sigma^2 P^2 V_{PP} + \mu P V_P - rV. \tag{10}$$

Suppose that the agent decides to invest at trigger $\hat{P}$. Upon investment, the payoff to the agent is equal to (3), implying the boundary condition

$$V(\hat{P}, \tilde{\theta}, \theta) = \alpha\left(\hat{P} - \theta\right) + W(\tilde{\theta}, \theta). \tag{11}$$

Solving differential equation (10) subject to boundary condition (11) yields the value of the option to the agent for a given investment threshold and the belief of outsiders:[13]

$$V(P, \tilde{\theta}, \theta, \hat{P}) = P^\beta U\left(\tilde{\theta}, \theta, \hat{P}\right), \tag{12}$$

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[12] Note that outsiders also learn from observing that the agent has not yet exercised the option. Specifically, whenever $P(t)$ hits a new maximum, outsiders update their belief of the agent's type. If $P_M(t) = \max_{s \leq t} P(s)$, outsiders' posterior belief is the prior belief truncated at $\bar{P}^{-1}(P_M(t))$ from below (above), if $\bar{P}(\theta)$ is increasing (decreasing) in θ. Once the agent exercises the option at $\hat{P}$, outsiders' posterior belief jumps to $\bar{P}^{-1}\left(\hat{P}\right)$. Because only outsiders' belief upon option exercise enters the payoff function of the agent, we can disregard the pre-exercise dynamics of outsiders' belief.

[13] As in the symmetric information case, the option value must satisfy the absorbing barrier condition $V(0, \tilde{\theta}, \theta) = 0$.

where

$$U\left(\tilde{\theta}, \theta, \hat{P}\right) = \frac{1}{\hat{P}^{\beta}}\left[\alpha\left(\hat{P} - \theta\right) + W(\tilde{\theta}, \theta)\right]. \qquad (13)$$

Given solution (12) and the hypothesized outsiders' inference function $\bar{P}$, the optimal choice of exercise threshold $\hat{P} \in \bar{P}\left(\left[\underline{\theta}, \bar{\theta}\right]\right)$ solves

$$\hat{P}\left(\theta; \bar{P}\right) \in \arg \max_{Y \in \bar{P}\left(\left[\underline{\theta}, \bar{\theta}\right]\right)} \left\{\frac{1}{Y^{\beta}}\left(\alpha\left(Y - \theta\right) + W\left(\bar{P}^{-1}\left(Y\right), \theta\right)\right)\right\}. \qquad (14)$$

Taking the first-order condition, we arrive at the optimality condition

$$\beta\left(\alpha\left(\hat{P} - \theta\right) + W\left(\bar{P}^{-1}\left(\hat{P}\right), \theta\right)\right) = \alpha\hat{P} + \hat{P}W_{\tilde{\theta}}\left(\bar{P}^{-1}\left(\hat{P}\right), \theta\right)\frac{d\bar{P}^{-1}\left(\hat{P}\right)}{d\hat{P}}. \qquad (15)$$

Equation (15) illustrates the fundamental trade-off between the costs and benefits of waiting in the model with asymmetric information between the agent and outsiders. On the one hand, a higher threshold leads to a longer waiting period and, hence, greater discounting of cash flows from the option exercise. This effect is captured by the expression on the left-hand side of (15). On the other hand, a higher threshold leads to a greater NPV at the exercise time and higher belief of outsiders. These effects are captured by the first and the second terms on the right-hand side of (15), respectively.

2.2 Equilibrium

In a separating equilibrium under rational expectations, the inference function $\bar{P}(\theta)$ must be a monotonic function that is perfectly revealing. Thus, $\bar{P}^{-1}(\hat{P}) = \theta$. Intuitively, this means that when the agent takes the inference function $\bar{P}(\theta)$ as given, her exercise behavior fully reveals the true type.

Conjecturing that a separating equilibrium exists, we can set $\bar{P}^{-1}(\hat{P}) = \theta$ in Equation (15) and simplify to derive the equilibrium differential equation:

$$\frac{d\bar{P}(\theta)}{d\theta} = \frac{W_{\tilde{\theta}}(\theta, \theta)\bar{P}(\theta)}{\alpha\left((\beta - 1)\bar{P}(\theta) - \beta\theta\right) + \beta W(\theta, \theta)}. \qquad (16)$$

The equilibrium differential equation (16) is solved subject to the appropriate initial value condition. By Assumption 3, there are two cases to consider.

Case 1: $W_{\tilde{\theta}} < 0$

For this case, the appropriate initial value condition is that the highest type invests efficiently:

$$\bar{P}\left(\bar{\theta}\right) = P^*\left(\bar{\theta}\right). \qquad (17)$$

The intuition is as follows. Suppose you are the worst possible type, which is $\bar{\theta}$ for the case $W_{\tilde{\theta}} < 0$. Then, any exercise strategy in which (17) did not hold

would not be incentive-compatible. This is because type $\bar{\theta}$ could always deviate and choose the full-information trigger $P^*(\bar{\theta})$. Not only would this deviation improve the direct payoff from exercise, but the agent could do no worse in terms of reputation since the current belief is already as bad as possible.[14] Therefore, only when (17) holds does the worst possible type have no incentive to deviate.

Case 2: $W_{\tilde{\theta}} > 0$

For this case, the appropriate initial value condition is that the lowest type invests efficiently:

$$\bar{P}\left(\underline{\theta}\right) = P^*\left(\underline{\theta}\right). \tag{18}$$

The intuition for (18) is the same as for (17). However, with $W_{\tilde{\theta}} > 0$, $\underline{\theta}$ is now the worst type.

Proposition 2 shows that under regularity conditions, there exists a unique (up to the out-of-equilibrium beliefs) separating equilibrium, and it is given as a solution to Equation (16) subject to boundary condition (17) or (18). The proof appears in Appendix B.

Proposition 2. Let $\bar{P}(\theta)$ be the increasing function that solves differential equation (16), subject to the initial value condition (17) if $W_{\tilde{\theta}} < 0$, or (18) if $W_{\tilde{\theta}} > 0$, where Assumptions 1–5 are satisfied. Then, $\bar{P}(\theta)$ is the investment trigger of type θ in the unique (up to the out-of-equilibrium beliefs) separating equilibrium.

2.3 Properties of the equilibrium

To examine how asymmetric information affects the equilibrium timing of investment, we compare the separating equilibrium derived above with the symmetric information solution established in Section 1.3.

Proposition 3 shows that asymmetric information between the decision-maker and outsiders has an important effect on the timing of investment. Its direction depends on the sign of $W_{\tilde{\theta}}$. The proof appears in Appendix B.

Proposition 3. Asymmetric information between the decision-maker and outsiders affects the timing of investment. The direction of the effect depends on the sign of $W_{\tilde{\theta}}$:

(i) If $W_{\tilde{\theta}} < 0$, then the firm invests earlier than in the case of symmetric information:

$$\bar{P}(\theta) < P^*(\theta) \text{ for all } \theta < \bar{\theta}.$$

---

[14] Our model assumes that outsiders' actions impact the agent's payoff only through the belief component, $\bar{\theta}$. As discussed in Mailath (1987), this is the reduced-form specification that incorporates optimal (with respect to the given belief) actions of outsiders, which are taken after the agent exercises the option. Thus, the harshest punishment that can be inflicted on the agent is the belief that she is the worst possible type.

(ii) If $W_{\tilde{\theta}} > 0$, then the firm invests later than in the case of symmetric information:

$$\bar{P}(\theta) > P^*(\theta) \text{ for all } \theta > \underline{\theta}.$$

As we can see, information asymmetry has powerful consequences for the timing of investment. It can both increase and decrease the waiting period, and the direction of the effect depends on the sign of $W_{\tilde{\theta}}$. The intuition comes from traditional signal-jamming models (e.g., Fudenberg and Tirole 1986; Stein 1989; Holmstrom 1999). When θ is the agent's private information, outsiders try to infer it from observing when the firm invests. Knowing this, the agent has incentives to manipulate the timing of investment to confuse outsiders. For example, if $W_{\tilde{\theta}} > 0$, the agent has an interest in mimicking the investment strategy of the agent with a higher investment cost. Since higher types invest at higher investment thresholds, the agent will try to mimic that by investing later than in the case of symmetric information. In equilibrium, outsiders correctly infer the type of the agent from observing the timing of investment. However, signal-jamming occurs: Outsiders correctly conjecture that investment occurs at a higher threshold. The opposite happens when $W_{\tilde{\theta}} < 0$.

For concreteness, let us consider a particular parameterization of $W\left(\tilde{\theta}, \theta\right)$ that permits a simple analytical solution. Specifically, we set $W\left(\tilde{\theta}, \theta\right) = w\left(\tilde{\theta} - \theta\right)$, for some function $w\left(\cdot\right)$ with $w(0)$ being zero.[15] In this case, the agent's utility from misspecification of outsiders' belief about the agent's private information depends only on the degree of misspecification, $\tilde{\theta} - \theta$. For this special case, Equation (16) takes the following form:

$$\frac{d\bar{P}(\theta)}{d\theta} = \frac{\bar{P}(\theta) w'(0)}{\alpha\left((\beta - 1)\bar{P}(\theta) - \beta\theta\right)}. \tag{19}$$

The general solution to this equation is given implicitly by

$$\bar{P}(\theta) + C\bar{P}(\theta)^{-\frac{\beta\alpha}{w'(0)}} = \frac{\beta + w'(0)/\alpha}{\beta - 1}\theta, \tag{20}$$

where the constant C is determined by the appropriate boundary condition.

For the case in which $w' < 0$, we apply boundary condition (17) to show that the equilibrium solution $\bar{P}(\theta)$ satisfies:

$$\bar{P}(\theta)\left(1 + \frac{w'(0)}{\alpha\beta}\left(\frac{\bar{P}(\theta)}{P^*(\bar{\theta})}\right)^{-\frac{\beta\alpha}{w'(0)}-1}\right) = \frac{\beta + w'(0)/\alpha}{\beta - 1}\theta. \tag{21}$$

---

[15] An example of such function that satisfies Assumptions 1–5 is $w\left(\tilde{\theta} - \theta\right) = c_w \times \left(\tilde{\theta} - \theta\right)$, where c_w is any non-zero constant above $-\alpha$.

In the limit, if the highest type has an unboundedly large cost ($\bar{\theta} \to \infty$), then $\bar{P}(\theta)$ approaches the simple linear solution:[16]

$$\bar{P}(\theta) \approx \frac{\beta + w'(0)/\alpha}{\beta - 1}\theta. \tag{22}$$

For the case in which $w' > 0$, we apply boundary condition (18) to show that the equilibrium solution $\bar{P}(\theta)$ satisfies:

$$\bar{P}(\theta)\left(1 + \frac{w'(0)}{\alpha\beta}\left(\frac{\bar{P}(\theta)}{P^*(\underline{\theta})}\right)^{-\frac{\beta\alpha}{w'(0)}-1}\right) = \frac{\beta + w'(0)/\alpha}{\beta - 1}\theta. \tag{23}$$

If the lowest type can reach an infinitesimal cost ($\underline{\theta} \to 0$), then $\bar{P}(\theta)$ again approaches the simple linear solution (22).

2.4 Other equilibria

While our article focuses on the separating equilibrium, various forms of pooling equilibria are also possible. Here, we present a simple example of an equilibrium in which there are a range of types that pool, and a range of types that separate. Notably, the construction of this equilibrium with pooling requires much of the analysis presented for the construction of the separating equilibrium.

In this simple example, θ is distributed uniformly over $[\underline{\theta}, \bar{\theta}]$. We also assume a simple functional form for the belief component: $c_w\left(\tilde{\theta} - \theta\right)$, with $c_w < 0$, where $\tilde{\theta}$ now refers to the expected type of the agent according to the belief of outsiders.[17] Finally, in this simple example, we assume that proceeds from the project are not informative about the agent's type. Consider type $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$. We will show that there exists a P_{pool}, with $P_{pool} \leq \bar{P}\left(\hat{\theta}\right) < P^*\left(\hat{\theta}\right)$, such that all types θ in the range $[\underline{\theta}, \hat{\theta}]$ pool and exercise together at P_{pool}, while all types θ in the range $(\hat{\theta}, \bar{\theta}]$ separate and exercise at the trigger $\bar{P}(\theta)$.

Suppose that $P(t) = P_{pool}$ and consider the decision of the agent whether to exercise the option immediately and pool or wait and exercise the option in the future. For types that exercise immediately and pool, $\tilde{\theta} = \left(\underline{\theta} + \hat{\theta}\right)/2$. Thus, the immediate payoff from pooling is

$$\alpha\left(P_{pool} - \theta\right) + c_w\left(\frac{\underline{\theta} + \hat{\theta}}{2} - \theta\right). \tag{24}$$

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[16] To see this, note that Assumption 4 and $w'(0) < 0$ imply that $-\frac{\beta\alpha}{w'(0)} - 1 > 0$. Therefore, as $\bar{\theta} \to \infty$, the left-hand side converges to $\bar{P}(\theta)$.

[17] More generally, the belief component can be any function of the distribution of outsiders' posterior belief about the agent's type.

Types that wait and separate obtain

$$\left(\frac{P_{pool}}{\bar{P}(\theta)} \right)^{\beta} \alpha \left(\bar{P}(\theta) - \theta \right), \quad (25)$$

where $\bar{P}(\theta)$ is the threshold of type θ in the fully separating equilibrium, given by (16)–(17).[18] Type $\hat{\theta}$ is the one that is indifferent between pooling and separating:

$$\alpha \left(P_{pool} - \hat{\theta} \right) + c_w \left(\frac{\theta - \hat{\theta}}{2} \right) = \left(\frac{P_{pool}}{\bar{P}(\hat{\theta})} \right)^{\beta} \alpha \left(\bar{P}(\hat{\theta}) - \hat{\theta} \right). \quad (26)$$

As shown in Appendix B, for any $\hat{\theta}$, (26) determines the unique value of $P_{pool}\left(\hat{\theta}\right)$. All types $\theta < \hat{\theta}$ find it optimal to exercise at $P_{pool}\left(\hat{\theta}\right)$, while all types $\theta > \hat{\theta}$ find it optimal to separate and exercise at $\bar{P}(\theta)$. By varying $\hat{\theta}$, one can obtain a continuum of these equilibria. In addition, there may exist equilibria with higher types pooling and lower types separating, as well as equilibria with multiple pooling groups. In general, it is difficult to say whether the agent's utility in the separating equilibrium is higher or lower than in other equilibria. As shown in Appendix B, in this particular example the agent's utility in the semi-pooling equilibrium is the same as in the separating equilibria if $\theta \geq \hat{\theta}$ and higher than the utility in the separating equilibrium if $\theta < \hat{\theta}$.

Given multiplicity of equilibria, it is important to select the most reasonable one. A standard approach in signaling games to select between equilibria is to impose additional restrictions on out-of-equilibrium beliefs. One standard restriction is the D1 refinement, which has been applied to a wide range of signaling environments such as security design (e.g., Nachman and Noe 1994; DeMarzo, Kremer, and Skrzypacz 2005) and intercorporate asset sales (Hege et al. 2009). Intuitively, according to the D1 refinement, following an "unexpected" action of the informed party, the uninformed party is restricted to place zero posterior belief on type θ whenever there is another type θ' that has a stronger incentive to deviate.[19] As Cho and Sobel (1990) and Ramey (1996) show, only separating equilibria can satisfy the D1 refinement. A slight modification of Ramey's (1996) proof can be applied here to establish the same result in our model.[20] Thus, the separating equilibrium is in fact the unique equilibrium under the assumption that out-of-equilibrium beliefs must satisfy

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[18] This result holds because for any $\hat{\theta}$, the boundary condition is the same and is determined by type $\bar{\theta}$. Note that in the case of $W_{\bar{\theta}} > 0$, this result does not hold, because the boundary conditions are different: In this semi-separating equilibrium it is determined by type $\hat{\theta}$, while in the separating equilibrium it is determined by type $\underline{\theta}$.

[19] See Cho and Kreps (1987), Cho and Sobel (1990), or Ramey (1996) for a formal definition.

[20] Specifically, unlike in Ramey (1996), the space of actions in our model is bounded from below and the agent's payoff converges to zero as the action converges to infinity.

the restriction specified by the D1 refinement. In this regard, focusing on separating equilibria is without loss of generality.

3. Applications with Acceleration of Options Exercise

3.1 Managerial myopia

In this section, we present an application of the timing signaling equilibrium that is similar in spirit to Stein's (1989) article on managerial myopia. In Stein (1989), the manager cares about both the current stock price and long-run earnings. The manager invests inefficiently through earnings manipulation (by boosting current earnings at the expense of future earnings) to attempt to fool the stock market into overestimating future earnings in the stock valuation. Even though the equilibrium ensures that the market is not fooled, the manager behaves myopically and inefficiently sacrifices future earnings for short-term profits. Our version is an analogue of Stein (1989) that focuses on investment timing rather than earnings manipulation. Here, the manager invests inefficiently by exercising the investment option too early to attempt to fool the market into overestimating the project's NPV.

3.1.1 Manager utility. As in Stein (1989), the manager's utility comes from a combination of current stock value and long-run earnings value. Specifically, the manager's utility comes from holding $\alpha_1 > 0$ shares of stock that may be freely sold, plus $\alpha_2 > 0$ times the present value of future earnings. This can be viewed as a reduced form utility coming out of a more complicated model of incentive compensation.[21] Thus, the manager makes two decisions: when to invest in the project and when to sell holdings that may be freely sold.[22]

Let $S(t)$ denote the stock price and $P(t)$ the present value of the project's cash flows. At a chosen time of exercise τ, if the manager still holds α_1 shares of stock, her stock holdings will be worth $\alpha_1 S(\tau)$.[23] Similarly, her utility from her interest in the present value of all future earnings is $\alpha_2 (P(\tau) - \theta)$. In summary, the manager's utility from exercise at any time τ is

$$\text{manager's utility from exercise} = \alpha_1 S(\tau) + \alpha_2 (P(\tau) - \theta). \quad (27)$$

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[21] One can motivate this split between the current and long-term stock price as dealing with options vesting schedules, limits on stock sales of executives (either contractual, or determined by the informational costs of trading), or the expected tenure of the manager's affiliation with the firm.

[22] We assume that outsiders do not observe whether the manager sells the stock or not. We make this assumption to make the application simple and tractable. One can get similar results in a more realistic setting, in which outsiders observe the manager's sale decision, as long as it does not reveal the manager's private information perfectly; for example, if the manager sells stock with positive probability for an exogenous reason.

[23] As we shall see below, managers with sufficiently high θ choose to sell all shares prior to investment, in which case the stock component of utility disappears.

3.1.2 The stock price process. Let us now consider the valuation of the stock. The market will infer the value of θ by observing whether or not the manager has yet invested. We begin by valuing the stock for all moments prior to the investment in the project. During this time period, the market updates its belief every time the project value rises to a new historical maximum. Let $\bar{P}(\theta)$ denote the equilibrium investment threshold for type θ, a function increasing in θ to be determined below. Let $P_M(t)$ denote the historical maximum of $P(t)$ up to time t. Then, at any time t prior to investment, the stock price $S(t) = S(P(t), P_M(t))$ is given by

$$S(P(t), P_M(t)) = \mathbb{E}_\theta \left[\left(\frac{P(t)}{\bar{P}(\theta)} \right)^\beta (\bar{P}(\theta) - \theta) \,|\, \theta > \bar{P}^{-1}(P_M(t)) \right]. \quad (28)$$

Next, consider the value of the stock when the firm invests at threshold $\hat{P}$. At this moment, the market observes the investment trigger, and the stock price immediately is set using the imputed $\tilde{\theta} = \bar{P}^{-1}(\hat{P})$. Thus, the stock price immediately jumps to the value $\hat{P} - \bar{P}^{-1}(\hat{P})$. Finally, after the net proceeds from investment are realized, the stock price moves to $\hat{P} - \theta + \varepsilon$. Recall, however, that the market is unable to disentangle the true cost from $\theta - \varepsilon$, and its expectation of θ remains $\tilde{\theta}$.

3.1.3 The equilibrium investment decision. Consider the manager's investment timing decision, conditional on holding α_1 shares of tradable stock. Suppose that the manager has not sold the tradable shares prior to the investment date. If the market's belief about the type of the manager, $\tilde{\theta}$, is below θ, then the manager is better off selling shares immediately upon the investment date and gaining from the market's optimistic belief: She receives $\hat{P} - \tilde{\theta}$ from selling versus (an expected) $\hat{P} - \theta$ from holding. Alternatively, if $\tilde{\theta} \geq \theta$, the manager is better off holding the stock. Thus, given the equilibrium threshold function $\bar{P}(\theta)$, the problem of the manager who does not sell the stock before the investment date is

$$\max_{\hat{P}} \left\{ 1_{\tilde{\theta} < \theta} \left[\alpha_1 \frac{1}{\hat{P}^\beta} (\hat{P} - \tilde{\theta}) + \alpha_2 \frac{1}{\hat{P}^\beta} (\hat{P} - \theta) \right] \right.$$
$$\left. + 1_{\tilde{\theta} \geq \theta} (\alpha_1 + \alpha_2) \frac{1}{\hat{P}^\beta} (\hat{P} - \theta) \right\}$$
$$= \max_{\hat{P}} \left\{ (\alpha_1 + \alpha_2) \frac{1}{\hat{P}^\beta} (\hat{P} - \theta) - \frac{1}{\hat{P}^\beta} \alpha_1 \min(\tilde{\theta} - \theta, 0) \right\}. \quad (29)$$

We can thus see that this problem is a special case of the general model with

$$W(\tilde{\theta}, \theta) = -\alpha_1 \min(\tilde{\theta} - \theta, 0) \text{ and } \alpha = \alpha_1 + \alpha_2. \quad (30)$$

Moreover, because $W\left(\tilde{\theta}, \theta\right)$ is a function of $\tilde{\theta} - \theta$, the separating equilibrium function $\bar{P}(\theta)$ is given by (20):[24]

$$\bar{P}(\theta) + C\bar{P}(\theta)^{\frac{\beta(\alpha_1 + \alpha_2)}{\alpha_1}} = \frac{\beta - \frac{\alpha_1}{\alpha_1 + \alpha_2}}{\beta - 1}\theta. \qquad (31)$$

The boundary condition for Equation (31) is determined by noting that the manager may choose to sell shares prior to investment. In the separating equilibrium with all α_1 shares held, information is fully revealed, and thus the manager does not gain from selling overvalued stock at the time of investment. Therefore, the manager sells shares before investment if and only if they are overvalued by the market. As is apparent from the valuation function in (28), the overvaluation is decreasing over time, and thus the manager will either sell shares at the initial point or never. Thus, the appropriate boundary condition is that for the range of θ for which the stock is initially overvalued, the manager will choose to sell all of the liquid shares. This implies that for this range of θ, $\alpha_1 = 0$ in Equation (31), which means that $\bar{P}(\theta)$ equals the first-best trigger, $\bar{P}(\theta) = \frac{\beta}{\beta - 1}\theta$.

All that remains is to determine the range of θ at which immediate sale of stock is warranted. If the stock is sold immediately, the stock will be priced based on the market's prior on θ, or $\int_{\underline{\theta}}^{\bar{\theta}}\left(\frac{P(0)}{\bar{P}(\theta)}\right)^{\beta}\left(\bar{P}(\theta) - \theta\right)\phi(\theta)\,d\theta$.[25] If the stock is held, it is worth $\left(\frac{P(0)}{\bar{P}(\theta)}\right)^{\beta}\left(\bar{P}(\theta) - \theta\right)$. Therefore, the manager will sell stock immediately if and only if θ is above a fixed threshold $\hat{\theta}$, determined by

$$\int_{\underline{\theta}}^{\bar{\theta}}\frac{\bar{P}(\theta) - \theta}{\bar{P}(\theta)^{\beta}}\phi(\theta)\,d\theta = \frac{\bar{P}\left(\hat{\theta}\right) - \hat{\theta}}{\bar{P}\left(\hat{\theta}\right)^{\beta}}. \qquad (32)$$

We have now fully characterized the solution. For $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$, the investment threshold $\bar{P}(\theta)$ is given by (31), where C is given by

$$C = -\left(\frac{\beta}{\beta - 1}\hat{\theta}\right)^{-\frac{\beta(\alpha_1 + \alpha_2)}{\alpha_1}}\frac{\alpha_1}{(\alpha_1 + \alpha_2)(\beta - 1)}\hat{\theta}. \qquad (33)$$

For $\theta \in (\hat{\theta}, \bar{\theta}]$, $\bar{P}(\theta) = \frac{\beta}{\beta - 1}\theta$.

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[24] Note that $-\alpha_1 \min\left(\tilde{\theta} - \theta, 0\right)$ has a kink at $\tilde{\theta} = \theta$. However, this does not create problems, because only the region $\theta > \tilde{\theta}$ is important for the incentives: Clearly, no type wants to mimic a type above. Hence, the problem is equivalent to a problem with $W\left(\tilde{\theta}, \theta\right) = -\alpha_1\left(\tilde{\theta} - \theta\right)$. Note that this function $W\left(\tilde{\theta}, \theta\right)$ satisfies Assumptions 1–5, as argued in footnote 15.

[25] To ensure that none of the types invest immediately, we assume that $P(0) < \bar{P}(\underline{\theta})$.

3.1.4 Discussion. The equilibrium investment strategy is to invest according to strategy $\bar{P}(\theta)$ in (31), which implies earlier investment than in the case of symmetric information for all types below $\hat{\theta}$. For types above $\hat{\theta}$, however, investment occurs at the full-information threshold. Intuitively, if the private information of the manager is such that the stock is overvalued, then the manager sells the flexible part of her holdings before investment reveals the type of the project. Once the manager sells her tradable stock, the manager no longer has any short-term incentives, so she chooses the investment threshold to maximize the long-term firm value. On the contrary, if the project is sufficiently good, then the stock of the company is undervalued relative to the private information of the manager, so she does not sell the flexible part of her holdings. As a result, when deciding on the optimal time to invest, the manager cares not only about the long-term firm value but also about the short-term stock price. In an attempt to manipulate the stock price, the manager invests earlier than in the symmetric information case. In equilibrium, the market correctly predicts this myopic behavior and infers the private information correctly.

The left graph of Figure 1 shows the equilibrium investment threshold $\bar{P}(\theta)$ as a function of the investment cost θ for three different values of $\alpha_1/(\alpha_1 + \alpha_2)$. The equilibrium investment threshold $\bar{P}(\theta)$ has two interesting properties. First, it moves further away from the first-best investment threshold $P^*(\theta)$ as $\alpha_1/(\alpha_1 + \alpha_2)$ goes up. Intuitively, if the manager can freely sell a higher portion of her shares, she has a greater incentive to invest earlier to fool the market into overestimating the NPV of the project and thereby boost the current stock price. Even though the market correctly infers θ in equilibrium, the
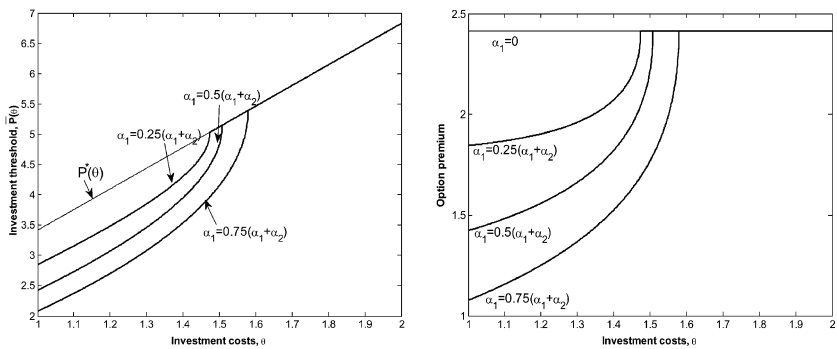

Figure 1
Equilibrium investment threshold of a myopic manager
The left graph shows the equilibrium investment trigger as a function of the investment cost θ for three different values of $\alpha_1/(\alpha_1 + \alpha_2)$, as well as the benchmark case. The top curve corresponds to the investment threshold $P^*(\theta)$ when there is no incentive for signaling. The other curves correspond (from top to bottom) to the cases when the manager can freely sell 25%, 50%, and 75% of her shares, respectively. The right graph shows the corresponding option premium as a function of the investment cost θ. The parameter values of the project value process are $r = 0.04$, $\mu = 0.02$, $\sigma = 0.2$. The investment costs are distributed uniformly over [1, 2].

equilibrium investment threshold goes down so that the manager has no incentives to deviate. Second, for each of the curves, the impact of asymmetric information is lower for projects with greater costs (lower types). Intuitively, incentive compatibility requires that the investment threshold of type θ be sufficiently below that of type $\theta + \varepsilon$ for an infinitesimal positive ε, so that type $\theta + \varepsilon$ has no incentive to mimic type θ. However, this lowers not only the investment threshold of type θ, but also investment thresholds of all types below θ, as they must have no incentive to mimic θ. In this way, the distortion accumulates, so the investment threshold of a lower type is closer to the zero NPV rule.

Another implication is that the investment option value can be significantly eroded through information asymmetry.[26] Analogously to Grenadier (2002), let the option premium define the NPV of investment at the moment of exercise divided by the investment cost:

$$OP\left(\theta, \frac{\alpha_1}{\alpha_1 + \alpha_2}\right) = \frac{\bar{P}(\theta) - \theta}{\theta}. \tag{34}$$

The right graph of Figure 1 quantifies the effect of asymmetric information on the option premium. In the case of symmetric information, equilibrium investment occurs only when the NPV of the project is more than 2.41 times its investment cost. Asymmetric information reduces the option premium, and the effect is greater for projects with lower investment costs and managers with greater incentives to boost the short-run stock price. For example, if the manager can freely sell 50% of her shares, the option premium of type $\theta = 1$ is 1.41, a greater than 40% decrease from the symmetric information case. Asymmetric information typically affects the option premium of the best projects the most and does not affect the option premium of sufficiently bad projects at all.

3.2 Venture capital grandstanding
In this section, we consider an application of our real options signaling model to VC firms. As shown in Gompers (1996), younger VC firms often take companies public earlier to establish a reputation and successfully raise capital for new funds. Gompers terms this phenomenon "grandstanding" and suggests that younger VC firms employ early timing of IPOs as a signal of their ability to form higher-quality portfolios.

We characterize experienced VC firms (the general partners) as those having a performance track record, and inexperienced VC firms as having no performance track record. For simplicity, we consider a two-stage model. An

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[26] Previous research (Williams 1993; Grenadier 2002) has demonstrated that the value of the option to invest can be significantly eroded because of competitive pressure in the industry. This application shows that if a portion of the manager's utility comes from the short-term stock price, then the value of the option to invest can be eroded even in monopolistic industries, as long as the manager is better informed about the investment project than the market.

inexperienced VC firm invests outsiders' (limited partners') money in the first round. The firm then chooses when to allow its first-round portfolio companies to go public. When such an IPO takes place, the firm becomes experienced and raises money for the second round. Notably, its ability to attract outsiders' funds in the second round will depend on the belief of outsiders of its skill, as inferred from the results of the first round.

We shall work backward and initially consider the second round (an experienced VC firm), to be followed by the first round (an inexperienced VC firm).

3.2.1 The experienced VC firm.

In the second round of financing, I_2 dollars are invested, where I_2 is endogenized below. The value of the fund, should it choose to go public at time t, is

$$(P_2(t) - \theta + \varepsilon_2) H(I_2), \qquad (35)$$

where $P_2(t)$ is the publicly observable component of value, θ is the privately observed value of the VC firm's skill, and ε_2 is a zero-mean shock, which corresponds to the contribution of luck. Only the VC firm knows the value of its skill θ (lower θ means higher skill); the outside investors must use an inferred value of $\tilde{\theta}$.[27] While outside investors cannot disentangle the mix of skill and luck, the VC firm learns the realization of luck, ε_2, upon investment. Finally, $H(\cdot)$ describes the nature of the returns to scale on investment. To account for declining returns to scale (that is, at some point the firm runs out of good project opportunities), we impose the Inada conditions: $H(0) = 0$, $H' > 0$, $H'' < 0$, $H'(0) = \infty$, and $H'(\infty) = 0$. In addition, we assume that H''' is continuous.

We assume that the VC firm receives as compensation a fraction α of the proceeds from an IPO (or a similar liquidity event).[28] The VC firm decides if and when to allow the portfolio to go public. Thus, the timing of the IPO is a standard option exercise problem where the expected payoff to the VC firm upon exercise is

$$\alpha (P_2(t) - \theta) H(I_2). \qquad (36)$$

The optimal second-round IPO exercise trigger is thus the first-best solution:

$$\bar{P}_2(\theta) = \frac{\beta}{\beta - 1}\theta. \qquad (37)$$

We now endogenize the second-round level of investment. At the beginning of the second round, the limited partners decide how much capital to contribute

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[27] The model can be extended to a more realistic, albeit less tractable, setting in which the firm has imperfect knowledge of its ability. This extended model has similar results and intuition, as long as the firm is better informed about its ability than investors.

[28] For purposes of this application, we take the compensation structure of the general partner as given. This structure is quite similar to the observed industry practice (e.g., Metrick and Yasuda 2010).

to the fund. We normalize the value of the publicly observable component upon the initiation of the second round, $P_2(0)$, to one, so that $P_2(t)$ represents the value growth over the initial cost.[29] The limited partners choose the level of investment I_2 so as to maximize the expected value of their net investment. Because the limited partners do not observe the VC firm's skill θ, they use inference $\tilde{\theta}$ based on the IPO signal from the first round. For a given $\tilde{\theta}$, the limited partners choose I_2 by solving the following optimization problem:

$$\max_{I_2} \left\{ (1 - \alpha) \frac{\bar{P}_2\left(\tilde{\theta}\right) - \tilde{\theta}}{\bar{P}_2\left(\tilde{\theta}\right)^\beta} H(I_2) - I_2 \right\}. \tag{38}$$

The Inada conditions guarantee that the optimal level of investment, $I_2\left(\tilde{\theta}\right)$, is given by the first-order condition:

$$I_2\left(\tilde{\theta}\right) = {H'}^{-1}\left[\left(\frac{\beta}{\beta - 1}\right)^\beta \frac{\beta - 1}{1 - \alpha} \tilde{\theta}^{\beta - 1} \right]. \tag{39}$$

$I_2\left(\tilde{\theta}\right)$ is strictly decreasing in $\tilde{\theta}$, meaning that the limited partners invest more if they believe that the general partner is more skilled.

Thus, for given $\tilde{\theta}$ and θ, the value of the second-round financing to the VC firm is

$$\alpha \frac{\bar{P}_2(\theta) - \theta}{\bar{P}_2(\theta)^\beta} H\left(I_2\left(\tilde{\theta}\right)\right). \tag{40}$$

Importantly, this value is a decreasing function of the inferred type $\tilde{\theta}$. Hence, the VC firm benefits from higher inferred skill.

3.2.2 The inexperienced VC firm.
Now, let us consider the first round.[30] The fund has I_1 invested, and the VC firm must choose if and when to allow its portfolio to go public. The payoff to the VC firm is the sum of their share of the proceeds from going public and the expected utility of the second-round financing. The proceeds from going public at time t are $(P_1(t) - \theta + \varepsilon_1) H(I_1)$, where ε_1 is a zero-mean shock, while the value of the second-round financing is given by (40). Thus, for an IPO trigger of $\hat{P}_1$, the expected payoff to the VC firm is

$$\alpha \left(\hat{P}_1 - \theta\right) H(I_1) + \alpha \frac{\bar{P}_2(\theta) - \theta}{\bar{P}_2(\theta)^\beta} H\left(I_2\left(\tilde{\theta}\right)\right), \tag{41}$$

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[29] Because of this normalization, we assume that the parameters of the model are such that $\bar{P}\left(\underline{\theta}\right) > 1$.

[30] For simplicity, we assume that the skill parameter θ of the VC firm is the same in both rounds. The model can be extended to the case of different, but correlated, skill levels across rounds. In such a case, in equilibrium the timing of investment is an imperfect rather than perfect signal about the general partner's talent.

where $I_2\left(\tilde{\theta}\right)$ is given by (39). For simplicity, we normalize $H\left(I_1\right)$ to 1. We can thus see that this problem corresponds to the general model with[31]

$$W\left(\tilde{\theta},\theta\right) = \alpha \frac{\bar{P}_2\left(\theta\right) - \theta}{\bar{P}_2\left(\theta\right)^{\beta}} H\left(I_2\left(\tilde{\theta}\right)\right), \tag{42}$$

where $W_{\tilde{\theta}} < 0$. Assuming that the lowest possible type is not too low,

$$\underline{\theta} > \frac{\beta - 1}{\beta}\left(\bar{\theta} - \frac{W\left(\bar{\theta},\bar{\theta}\right)}{\alpha}\right), \tag{43}$$

the single-crossing condition is satisfied.[32] Thus, the separating equilibrium the investment trigger $\bar{P}_1\left(\theta\right)$ is given by

$$\frac{d\bar{P}_1\left(\theta\right)}{d\theta} = \frac{\bar{P}_1\left(\theta\right) I_2'\left(\theta\right) / \left(1 - \alpha\right)}{\left(\beta - 1\right)\bar{P}_1\left(\theta\right) - \beta\theta + \beta \frac{\bar{P}_2(\theta) - \theta}{\bar{P}_2(\theta)^{\beta}} H\left(I_2\left(\theta\right)\right)}, \tag{44}$$

solved subject to the boundary condition that type $\bar{\theta}$ invests at the full-information threshold:[33]

$$\bar{P}_1\left(\bar{\theta}\right) = \frac{\beta}{\beta - 1}\left(\bar{\theta} - \frac{\bar{P}_2\left(\bar{\theta}\right) - \bar{\theta}}{\bar{P}_2\left(\bar{\theta}\right)^{\beta}} H\left(I_2\left(\bar{\theta}\right)\right)\right). \tag{45}$$

3.2.3 Discussion. The timing of the IPO of the inexperienced firm characterized by (44)–(45) has several intuitive properties. First, the inexperienced firm takes the portfolio company public earlier than optimal. Because the inexperienced firm is better informed about its talent than the limited partners, the inexperienced firm has an incentive to manipulate the timing of the IPO to make the limited partners believe that its quality is higher. Because an earlier IPO is a signal of higher quality, it will go public earlier than in the case of symmetric information. In equilibrium, signal-jamming occurs: The limited partners correctly conjecture that the VC firm goes public earlier than optimal, so the type of the general partner is revealed. The degree of inefficient timing is

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[31] Note that if the limited partners observe the proceeds from the first round, then they may also use this information to infer θ. However, this does not affect the model, because the proceeds are a noisier signal of the firm's private information than the timing. Indeed, the proceeds reveal the value of $\theta - \varepsilon_1$, while the timing in a separating equilibrium reveals θ.

[32] It can be easily checked that function $W\left(\tilde{\theta},\theta\right)$ in this application also satisfies Assumptions 1–4, provided that the optimal IPO threshold in the first round in the case of symmetric information is finite.

[33] To ensure that none of the types does an IPO immediately, we make an assumption that the initial value $P\left(0\right)$ is below $\bar{P}_1\left(\underline{\theta}\right)$. Then, the unique separating equilibrium investment threshold is defined as an increasing function, which solves (44) subject to (45).
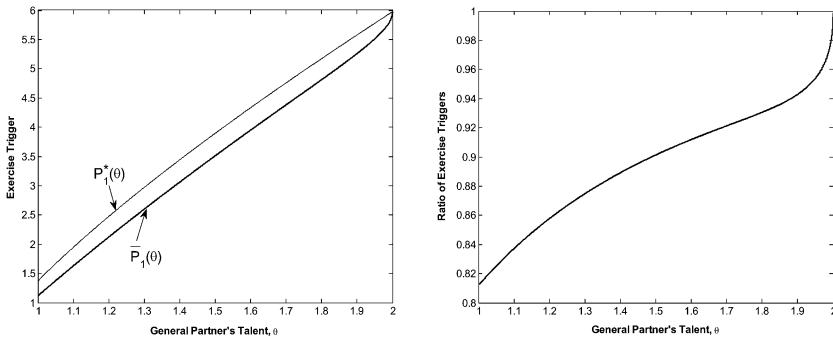
Figure 2
Exit strategies of the inexperienced general partner
The left graph shows the equilibrium trigger, $\bar{P}_1(\theta)$, and the symmetric information trigger, $P_1^*(\theta)$, as functions of θ (higher θ corresponds to lower talent). The right graph shows the ratio of the two triggers, $\bar{P}_1(\theta)/\bar{P}_1^*(\theta)$. The production function is the power function: $H(I) = AI^{2/3}$. The parameter values of the price process are $r = 0.04$, $\mu = 0.02$, $\sigma = 0.2$. The interval of possible types is $[\underline{\theta}, \bar{\theta}] = [1, 2]$. The share of the IPO proceeds that goes to the general partner is $\alpha = 0.2$. The value of A is calibrated at $A = 3.015$ so that for the middle type $\theta = 1.5$, the equilibrium investment into the second project equals the investment into the first project, i.e., $F(I_2(\theta)) = 1$.

illustrated in Figure 2. The left graph plots the equilibrium exercise threshold of the inexperienced firm, $\bar{P}_1(\theta)$, and the efficient exercise threshold, $P_1^*(\theta)$, which would be the equilibrium if the limited partners were fully informed about the general partner's talent.

Inefficient investment timing depends not only on the experience of the general partner, but also on the firm's talent. Specifically, (45) implies that the least talented firm takes the portfolio public at the efficient time even if it is inexperienced. At the same time, all other types take the portfolio public earlier than efficient. The right graph of Figure 2 illustrates the dependence of earlier than optimal IPO on the general partner's talent. The degree of inefficient investment increases in the general partner's talent from 0% for the least talented general partner ($\bar{\theta} = 2$) to 19% for the most talented general partner ($\underline{\theta} = 1$).

While the inexperienced firm takes the company public earlier than optimal, the experienced firm does so at the efficient threshold. Because the limited partners learn the true talent of the firm from observing its track record, the experienced firm does not have any incentive to manipulate the belief of the limited partners.

4. Applications with Delay of Options Exercise

4.1 Cash flow diversion
We consider a cash flow diversion model in which a manager (with a partial ownership interest) derives utility from diverting the owners' cash flow from

investment for personal consumption.[34] Thus, in this case, the manager would like shareholders to believe that the investment cost is higher than in reality. We begin by providing a costly state verification model to endogenize the manager's cash flow diversion utility. Then, conditional on the manager's diversion incentives, we move on to modeling the manager's optimal investment strategy.

The assumption that a portion of project value is observed only by the manager and not verifiable by the owners is common in the capital budgeting literature. This information asymmetry invites a host of agency issues. Harris, Kriebel, and Raviv (1982) posit that managers have incentives to understate project payoffs and to divert the free cash flow to themselves. In their model, such value diversion takes the form of the manager reducing her level of effort. Stulz (1990), Harris and Raviv (1996), Bernardo, Cai, and Luo (2001), and Malenko (2011) model the manager as having preferences for perquisite consumption or empire-building. In these models, the manager has incentives to divert free cash flows to inefficient investments or to excessive perquisites. Grenadier and Wang (2005) apply an optimal contracting approach to ensure against diversion and to provide an incentive for the manager to exercise optimally.

4.1.1 Costly state verification model.   Suppose that the manager can divert any amount d from the project value before the noise ε is realized.[35] As is standard in the literature (e.g., DeMarzo and Sannikov 2006), diversion is potentially wasteful, so that the manager receives only a fraction $\lambda \in [0, 1]$ of the diverted value. After the project cash flow of $P - \theta - d + \varepsilon$ is realized, the shareholders either verify whether the manager diverted or not. Verification costs $c > 0$. If the shareholders verify that the manager diverted funds d from the firm, the manager is required to return them to the firm.[36] Thus, the timing of the interactions is the following. First, the manager decides when to exercise the investment option. Then, after the investment has been made but before the cash flow is realized, the shareholders decide on the verification strategy.[37] As in traditional costly state verification models (Townsend 1979; Gale and Hellwig 1985), the investors (shareholders, in our case) can commit to the deterministic verification strategy. After that, but before observing the noise ε, the manager decides how much to divert. Finally, the project's cash flow of $P - \theta - d + \varepsilon$ is realized, and the shareholders either verify the manager or

---

[34] We take the structure of the manager's compensation contract as given. In a more general model, the manager's ownership stake could itself be endogenous.

[35] We make an assumption that the manager is not allowed to inject her own funds into the firm. This assumption simplifies the solution but is not critical, as long as injection is not too profitable.

[36] The model can be extended by allowing the shareholders to impose a non-pecuniary cost on the manager if diversion is verified.

[37] While we assume that the proceeds from the project realize an instant after the investment has been made, the model can be extended to include the time to build feature (e.g., as in Majd and Pindyck 1987).

not, according to the prespecified verification strategy. Let Ψ and ψ denote the cumulative distribution and density functions of ε, respectively. Assume that ψ is C^2.

In Appendix B, we demonstrate that any optimal verification strategy takes the form of verifying the manager if and only if the difference between the expected and the realized cash flows is greater than a particular threshold. In other words, for some v, verification occurs if and only if $P - \theta - d + \varepsilon - \left(P - \tilde{\theta}\right) < v$, or, equivalently, $\varepsilon < v - \tilde{\theta} + \theta + d$. Let us initially choose any verification parameter v and determine the manager's optimal diversion strategy in response. Then, conditional on this managerial response, we determine the shareholders' optimal choice of v.

Consider a manager of type θ that is inferred by the market as type $\tilde{\theta}$. If the manager diverts d, she expects to be verified with probability $\Psi\left(v - \tilde{\theta} + \theta + d\right)$, in which case there is no impact on her payoff, as the diverted cash flow is returned to the firm. However, she is not verified with probability $1 - \Psi\left(v - \tilde{\theta} + \theta + d\right)$, in which case she gains fraction λ of the diverted cash flow for her private benefit and loses fraction α due to her ownership position. Hence, the manager's problem is

$$\max_{d \geq 0} \left\{ (\lambda - \alpha) d \left(1 - \Psi\left(v - \tilde{\theta} + \theta + d\right)\right) \right\}. \tag{46}$$

Clearly, if $\alpha \geq \lambda$, then the manager does not divert anything: $d = 0$.[38] Now, consider the case $\alpha < \lambda$. Assuming that the hazard rate of the distribution of ε, $h_\psi(z) \equiv \psi(z) / (1 - \Psi(z))$, is increasing, (46) has a unique solution d^* that satisfies

$$d^* h_\psi\left(v - \tilde{\theta} + \theta + d^*\right) = 1. \tag{47}$$

The solution d^* is a decreasing function of $v - \tilde{\theta} + \theta$. Let us denote this functional dependence by $D\left(v - \tilde{\theta} + \theta\right)$.

Given the manager's response to verification rule v, we now solve for the shareholders' optimal choice of v. Under the shareholders' information set, they expect the manager to divert $D(v)$ and estimate the probability of verification at $\Psi(v + D(v))$. For any choice of v, the shareholders lose $1 - \alpha$ of the diverted cash flow when verification does not occur, and pay cost c when verification occurs. Thus, the optimal verification parameter v^* is

$$v^* = \arg\min_v \left\{ (1 - \alpha)(1 - \Psi(v + D(v))) D(v) + \Psi(v + D(v)) c \right\}. \tag{48}$$

---

[38] Technically, the manager is indifferent in her choice of d when $\alpha = \lambda$. However, if there is any infinitesimal but positive fixed cost of diversion, a zero level will be chosen.

In summary, we have determined the manager's diversion and shareholders' verification strategies. If $\alpha \geq \lambda$, then the manager does not divert cash flow and the shareholders do not verify the manager: $d = 0, v = -\infty$. If $\alpha < \lambda$, the manager diverts $D\left(v^* - \tilde{\theta} + \theta\right)$, and the shareholders verify the manager if and only if the project's realized cash flow falls below $v^* + P - \tilde{\theta}$.

4.1.2 Equilibrium investment timing.

Given the manager's diversion rule derived above, her payoff from exercising the option at threshold $\hat{P}$, when her type is θ and the shareholders' belief is $\tilde{\theta}$, equals

$$
\alpha\left(\hat{P} - \theta\right) + \max\left(\lambda - \alpha, 0\right) D\left(v^* - \tilde{\theta} + \theta\right)
$$
$$
\times \left(1 - \Psi\left(v^* - \tilde{\theta} + \theta + D\left(v^* - \tilde{\theta} + \theta\right)\right)\right). \tag{49}
$$

Thus, this problem is a special case of the general model with

$$
W\left(\tilde{\theta}, \theta\right) = \max\left(\lambda - \alpha, 0\right) D\left(v^* - \tilde{\theta} + \theta\right)
$$
$$
\times \left(1 - \Psi\left(v^* - \tilde{\theta} + \theta + D\left(v^* - \tilde{\theta} + \theta\right)\right)\right). \tag{50}
$$

Notice that for $\lambda > \alpha$, $W_{\tilde{\theta}}\left(\tilde{\theta}, \theta\right) > 0$, meaning that the application corresponds to Case 2 of the general model. Intuitively, as the shareholders become more pessimistic about the project's value, the manager diverts more and gets verified less frequently.

Using the solution analogous to (23), we can express $\bar{P}\left(\theta\right)$ implicitly as the solution to the following equation:[39]

$$
1 + \frac{\max\left(\lambda - \alpha, 0\right)\left(1 - \Psi\left(v^* + D\left(v^*\right)\right)\right)}{\alpha\beta}\left(\frac{\bar{P}\left(\theta\right)}{P^*\left(\underline{\theta}\right)}\right)^{-\frac{\beta\alpha}{\max\left(\lambda - \alpha, 0\right)\left(1 - \Psi\left(v^* + D\left(v^*\right)\right)\right)} - 1}
$$
$$
= \frac{\beta + \max\left(\frac{\lambda}{\alpha} - 1, 0\right)\left(1 - \Psi\left(v^* + D\left(v^*\right)\right)\right)}{\beta - 1}
$$
$$
\times \frac{\theta - \max\left(\frac{\lambda}{\alpha} - 1, 0\right) D\left(v^*\right)\left(1 - \Psi\left(v^* + D\left(v^*\right)\right)\right)}{\bar{P}\left(\theta\right)}, \tag{51}
$$

---

[39] For $\bar{P}\left(\theta\right)$ to correspond to the separating equilibrium, we need to ensure that the parameters of the application satisfy Assumption 5. A sufficient condition is $\alpha > \left[\max_{z \in \left[\underline{\theta} - \bar{\theta}, \bar{\theta} - \underline{\theta}\right]} \frac{w''(z)}{w'(z)}\right] \times \left(\frac{\beta - 1}{\beta}\alpha\bar{P}\left(\bar{\theta}\right) - \alpha\underline{\theta} + w\left(\bar{\theta} - \underline{\theta}\right)\right) - w'\left(\underline{\theta} - \bar{\theta}\right)$. Analogously to (43), this condition is always satisfied if the interval $\left[\underline{\theta}, \bar{\theta}\right]$ is not too wide. Assumptions 1–4 are always satisfied by $W\left(\tilde{\theta}, \theta\right)$ in this application, as long as $P^*\left(\underline{\theta}\right) > 0$, as given below.

where $P^*\left(\underline{\theta}\right)$ is the symmetric-information threshold of type $\underline{\theta}$:

$$P^*\left(\underline{\theta}\right) = \frac{\beta}{\beta-1}\left(\underline{\theta} - \max\left(\frac{\lambda}{\alpha}-1,0\right)D\left(v^*\right)\left(1-\Psi\left(v^*+D\left(v^*\right)\right)\right)\right).$$

(52)

Note that for the case of $\lambda \le \alpha$, $\bar{P}(\theta) = P^*(\theta) = \frac{\beta}{\beta-1}\theta$.

4.1.3 Discussion.

The effect of potential cash flow diversion on the timing of investment is illustrated in Figure 3. If diversion is sufficiently costly ($\lambda \le \alpha$), or, equivalently, managerial ownership is sufficiently high, then the interests of the manager and those of the outside shareholders are aligned. Because diversion is never optimal in this case, information asymmetry does not affect the investment strategy. If diversion is not costly enough ($\lambda > \alpha$), then information asymmetry leads to a delay in investment compared to the case of symmetric information. Interestingly, diversion also affects investment threshold under symmetric information about θ. When the manager expects to divert value from the project, she exercises the option at a lower threshold. Because the manager diverts the same amount for any exercise threshold, higher diversion is equivalent to a decrease in the investment cost from the manager's point of view. Consequently, the symmetric information threshold for the case $\lambda = 0.5$ is lower than that for the case $\lambda = 0$.
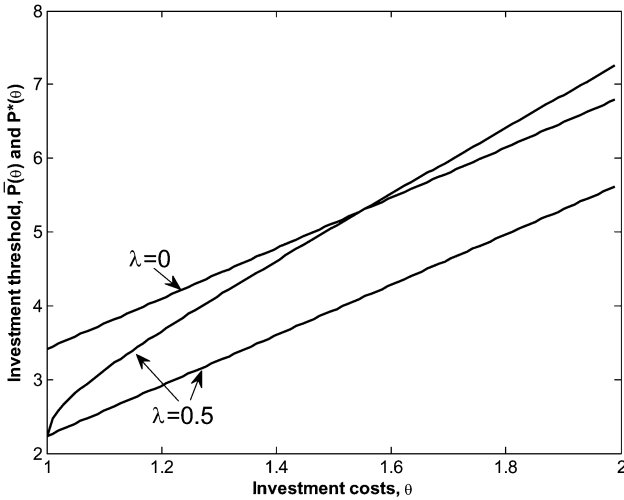

Figure 3
Investment threshold when the manager can divert cash flows from the project
The figure shows the equilibrium investment thresholds as a function of the investment cost θ for two different levels of the diversion parameter λ: 0 and 0.5. The bottom curve for this case corresponds to the investment threshold $P^*(\theta)$ when there is symmetric information between the manager and the market. The top curve for this case corresponds to the investment threshold $\bar{P}(\theta)$ in the unique separating equilibrium when there is asymmetric information between the manager and the market. The parameter values of the project value process are $r = 0.04$, $\mu = 0.02$, and $\sigma = 0.2$. The managerial ownership is $\alpha = 0.2$. The interval of possible investment costs is [1, 2]. The distribution of noise ε is $N(0, 1)$. The cost of verification is $c = 1$.

In Figure 3, distortion in the investment threshold due to information asymmetry is greater for projects of lower quality. This result contrasts with the results in the previous two applications. There, the exercise trigger is altered in a way that the manager has no incentives to mimic a lower type. As a result, distortion in the exercise timing does not exist for the highest types (worst projects) and exists for lower types. In contrast, now the exercise trigger is altered in a way that the manager has no incentives to mimic a higher type. As a result, distortion in exercise timing does not exist for the lowest types (best projects) and exists for higher types.

4.2 Strategic product market competition

Another example of a real options signaling game in which asymmetric information delays options exercise is the strategic entry into a product market. Specifically, consider the entry decisions of two firms that are asymmetrically informed about the value of a new product. Firm 1 knows the investment cost θ, while firm 2 does not. For example, firm 1 may have greater experience in similar product introductions or may be the industry's technology leader. When there is only one firm in the industry, it receives a monopoly profit flow of $P(t)$. When there are two firms in the industry, each receives a duopoly profit flow of $\lambda P(t)$, where $\lambda \in \left(1 - \frac{1}{\beta}, 1\right)$.[40] We derive the Bayes-Nash separating equilibrium in two different versions of the game: when firm 1 is the designated leader (the "Stackelberg equilibrium") and when the roles of the two firms are determined endogenously (the "Cournot equilibrium"). We focus on the limiting case $\underline{\theta} \to 0$ to obtain the closed-form solutions.

Product market competition in a real options framework has been frequently analyzed in the literature. Leahy (1993), Williams (1993), and Grenadier (2002) study simultaneous investment by symmetric firms in a competitive equilibrium. Novy-Marx (2007) looks at a similar problem with heterogeneous firms. Grenadier (1996), Weeds (2002), and Lambrecht and Perraudin (2003) study sequential investment in leader-follower games. We follow the simple duopoly framework outlined in Chapter 9 of Dixit and Pindyck (1994). The key distinction with the perfect information framework in Dixit and Pindyck (1994) is that one firm knows the investment cost, while the other attempts to infer it through the informed firm's investment decision. The main insight is that the informed firm will delay its investment to signal to the uniformed firm that the cost is higher than in reality, thereby attempting to delay the uniformed firm's entrance and enjoy monopoly profits for a longer period. In equilibrium, this effort to deceive will fail, but the informed firm's entry will still be delayed relative to the full-information entry time.[41]

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[40] Essentially, the assumption that $\lambda > 1 - \frac{1}{\beta}$ rules out any overwhelming influence of monopoly power.

[41] Lambrecht and Perraudin (2003) and Hsu and Lambrecht (2007) are also related to the model in this section. They study competition between two firms for an investment opportunity when the information structure is

The investment decision of firm 1 depends on the degree of pressure it feels due to firm 2's potential preemption. We will begin with the assumption of a Stackelberg equilibrium (where there is no potential preemption) and then show the extension to a Cournot equilibrium (where preemption by firm 2 is possible).

4.2.1 The Stackelberg equilibrium. Let us work backward and begin by considering the situation when firm 1 has already invested. Firm 2 has used firm 1's entry time to make an inference about θ, denoted by $\tilde{\theta}$. Given its inferred signal, firm 2 holds a standard real option whose expected payoff at exercise is $\frac{\lambda P}{r-\mu} - \tilde{\theta}$. Firm 2 will thus enter at the first instant when $P(t)$ equals or exceeds $\bar{P}_F\left(\tilde{\theta}\right)$, given by

$$\bar{P}_F\left(\tilde{\theta}\right) = \frac{\beta}{\beta-1}\frac{r-\mu}{\lambda}\tilde{\theta}. \tag{53}$$

Now, consider the entry of firm 1. Upon payment of θ at exercise, firm 1 begins receiving the monopoly profit flow of $P(t)$, which is then reduced to $\lambda P(t)$ once firm 2 enters. Thus, for a given type θ, firm 2's belief $\tilde{\theta}$, and the entry trigger $\hat{P}_L$, the payoff to firm 1 at the moment of entry is

$$\frac{\hat{P}_L}{r-\mu} - \theta - \left(\frac{\hat{P}_L}{\bar{P}_F\left(\tilde{\theta}\right)}\right)^\beta \frac{\bar{P}_F\left(\tilde{\theta}\right)(1-\lambda)}{r-\mu}, \text{ for } \hat{P}_L \leq \bar{P}_F\left(\tilde{\theta}\right),$$

$$\frac{\lambda\hat{P}_L}{r-\mu} - \theta, \qquad\qquad \text{for } \hat{P}_L > \bar{P}_F\left(\tilde{\theta}\right). \tag{54}$$

Let $\bar{P}_L(\theta)$ denote the equilibrium entry threshold of firm 1 in the Stackelberg case. Conjecture that $\bar{P}_L(\theta) \leq \bar{P}_F(\theta)$, which is verified in Appendix B. Then, from (54), we can see that the payoff from exercise can be written as

$$\alpha\left(\hat{P}_L - \theta\right) + W(\hat{P}_L, \tilde{\theta}, \theta), \tag{55}$$

where $\alpha = \frac{1}{r-\mu}$ and

$$W(\hat{P}_L, \tilde{\theta}, \theta) = -\hat{P}_L^\beta \alpha(1-\lambda)\bar{P}_F\left(\tilde{\theta}\right)^{1-\beta} - (1-\alpha)\theta. \tag{56}$$

Note that the belief component of this payoff is not a special case of the model outlined in Section 1, given that $\hat{P}_L$ is included as an argument. In Appendix B, we show that this slight difference in the functions can be easily

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imperfect. In Lambrecht and Perraudin (2003), each firm knows its own investment cost but not the cost of its competitor. In Hsu and Lambrecht (2007), the investment cost of one firm (the incumbent) is public knowledge, while the investment cost of the other firm is known only to itself.

handled, and that the separating equilibrium investment trigger satisfies differential equation (B15). The resulting leader's Stackelberg strategy thus satisfies

$$\frac{d\bar{P}_L(\theta)}{d\theta} = \frac{1-\lambda}{\lambda} \frac{\beta \bar{P}_L(\theta)}{\frac{\beta-1}{r-\mu}\bar{P}_L(\theta) - \beta\theta} \left(\frac{\bar{P}_L(\theta)}{\frac{\beta}{\beta-1}\frac{r-\mu}{\lambda}\theta}\right)^\beta. \quad (57)$$

Since the leader's payoff is decreasing in $\tilde{\theta}$, it is solved subject to the boundary condition that type $\underline{\theta}$ invests at the symmetric information threshold. In the limiting case $\underline{\theta} \to 0$, this boundary condition approaches

$$\bar{P}_L(0) = 0. \quad (58)$$

In Appendix B, we show that the solution is

$$\bar{P}_L(\theta) = \frac{\beta}{\beta-1}\frac{r-\mu}{\lambda}\theta = \bar{P}_F(\theta). \quad (59)$$

Thus, firm 1's Stackelberg strategy is to delay investment up to the point that firm 2's response will be to invest immediately thereafter.

It is instructive to compare the equilibrium investment threshold of the leader (59) with the full-information case studied in Chapter 9 in Dixit and Pindyck (1994), in which both the leader and the follower know θ. In that case, the full-information Stackelberg equilibrium investment threshold for firm 1 is equal to[42]

$$P_L^*(\theta) = \frac{\beta}{\beta-1}(r-\mu)\theta. \quad (60)$$

Since $\lambda < 1$, firm 1's investment occurs later than in the full-information setting. Intuitively, firm 1 has an incentive to invest later than in the case of symmetric information to fool firm 2 and thereby postpone its entry. As in the other applications, in equilibrium the informed player is unsuccessful in fooling the uninformed player: Firm 2 learns the true type of the leader, and invests at the same investment threshold as in the case of perfect information. Information asymmetry not only leads to later entry of firm 1 but also shortens the period of time when firm 1 is a monopolist.

4.2.2 The Cournot equilibrium.   Now, consider how the Stackelberg equilibrium of the previous section is affected by the potential preemption of firm 2. Let $\tilde{P}_L$ be the threshold at which firm 2 preempts firm 1 by entering first. In the event of being preempted, the optimal best response for firm 1 is to invest

---

[42] The intuition for this result is as follows. The leader never enters after $\bar{P}_F(\theta)$, since in this region there is always simultaneous entry, and $\bar{P}_F(\theta)$ is the optimal trigger for simultaneous entry. Since the leader knows that the follower enters at $\bar{P}_F(\theta)$ and that at that point it will lose the difference between the monopoly value and the duopoly value, its entry time choice will be the one that maximizes its monopoly value: $P_L^*(\theta)$.

at the first time when $P(t)$ equals or exceeds the optimal follower's threshold $\bar{P}_F(\theta)$.

Given any preemption threshold, $\tilde{P}_L$, we can compute the conditional expected value of firm 2 before either firm invests. Let $P_M(t)$ denote the historical maximum of $P(t)$ as of time t. If firm 1 has not invested before time t, firm 2 learns that θ is such that $\bar{P}_L(\theta) > P_M(t)$, i.e., $\theta > \frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}P_M(t)$. There are two ranges of θ: In the upper range, firm 2 enters first, and in the lower range, both firms enter simultaneously. For the case in which $\theta > \frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}\tilde{P}_L$, $\tilde{P}_L < \bar{P}_L(\theta)$ and thus firm 2 preempts firm 1 by investing at $\tilde{P}_L$. For the case in which $\theta \leq \frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}\tilde{P}_L$, $\tilde{P}_L \geq \bar{P}_L(\theta)$ and thus firm 1 enters at the Stackelberg trigger $\bar{P}_L(\theta)$, where firm 2 will then infer θ and immediately enter. Combining these cases, firm 2's value, conditional on P and P_M, is equal to

$$\int_{\frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}P_M}^{\frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}\tilde{P}_L}\left(\frac{P}{P_F(\theta)}\right)^\beta\left(\frac{\lambda P_F(\theta)}{r-\mu}-\theta\right)\frac{\phi(\theta)}{1-\Phi\left(\frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}P_M\right)}d\theta$$

$$+\int_{\frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}\tilde{P}_L}^{\bar{\theta}}\left(\frac{P}{\tilde{P}_L}\right)^\beta\left(\frac{\tilde{P}_L}{r-\mu}-\left(\frac{\tilde{P}_L}{P_F(\theta)}\right)^\beta\frac{(1-\lambda)P_F(\theta)}{r-\mu}-\theta\right)\frac{\phi(\theta)}{1-\Phi\left(\frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}P_M\right)}d\theta.$$

$$(61)$$

The optimal preemption strategy is to invest at the $\tilde{P}_L$ that maximizes (61). The corresponding first-order condition is[43]

$$\tilde{P}_L = \frac{\beta}{\beta-1}\frac{\mathbb{E}\left[\theta|\theta \geq \frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}\tilde{P}_L\right]}{r-\mu}. \qquad (62)$$

In other words, the equilibrium preemption trigger equals the expected full-information Stackelberg trigger, conditional on preemption. In Appendix B, we show that the optimal preemption threshold, $\tilde{P}_L$, is always between 0 and $\bar{P}_F(\bar{\theta})$. In particular, assuming that $\theta\phi(\theta)/(1-\Phi(\theta))$ is increasing in θ, Equation (62) has a unique solution, which determines $\tilde{P}_L$.[44]

We can now fully characterize the Cournot equilibrium outcome. Firm 2 attempts to preempt firm 1 by investing at trigger $\tilde{P}_L$, which is implicitly given by (62). If θ is such that $\tilde{P}_L < \bar{P}_L(\theta)$, then firm 2 invests first at $\tilde{P}_L$, and firm 1 invests later at $\bar{P}_F(\theta) = \bar{P}_L(\theta)$. Alternatively, if θ is such that $\tilde{P}_L \geq \bar{P}_L(\theta)$, then both firms invest simultaneously at trigger $\bar{P}_L(\theta)$. Thus, in all cases the informed firm invests later than it would in the case of full information. This delay is due to its strategic incentive to artificially inflate firm 1's inferred estimate of θ.

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[43] Note that if θ is such that $\bar{P}_L(\theta) > \tilde{P}_L$, it is not optimal for firm 1 to preempt firm 2 by investing at a threshold below $\tilde{P}_L$. Indeed, if firm 1 invested at $\hat{P} < \tilde{P}_L$, firm 2 would respond by investing immediately after firm 1, as it would perceive that θ is such that $\bar{P}_L(\theta) = \bar{P}_F(\theta) = \hat{P}$. As a result, firm 1 does not gain any monopoly power from investing below $\tilde{P}_L$, so its best response to the preemptive strategy of firm 2 is to invest at $\bar{P}_F(\theta)$.

[44] Intuitively, this assumption means that the density of distribution does not have abrupt kinks. It is satisfied for most standard distributions.
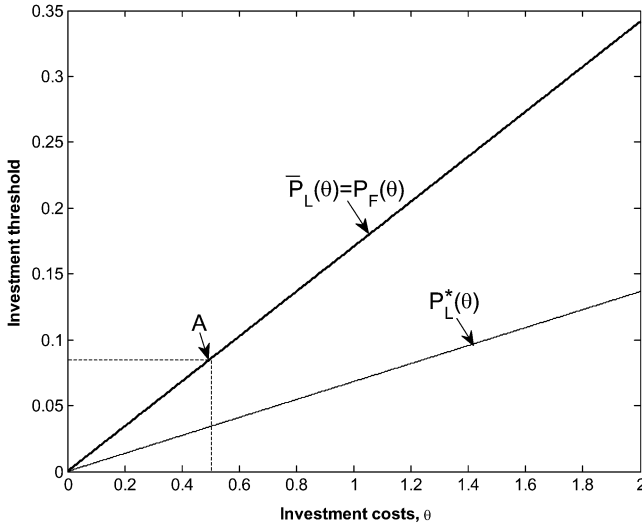
Figure 4
Equilibrium investment thresholds in the Stackelberg and Cournot equilibria
The figure shows the equilibrium investment triggers of firm 1 and firm 2 in the Stackelberg and Cournot equilibria. The lower line corresponds to the investment threshold of the leader (firm 1) in the Stackelberg equilibrium when both the leader and the follower know θ. The upper line corresponds to the investment thresholds of both firm 1 and firm 2 in the Stackelberg equilibrium when only the leader knows θ. Point A corresponds to the preemption threshold in the Cournot equilibrium. If $\theta \leq 0.5$, then the outcome in the Cournot equilibrium is the same as in the Stackelberg equilibrium. If $\theta > 0.5$, then in the Cournot equilibrium firm 2 invests first. The parameter values of the project value process are $r = 0.04$, $\mu = 0.02$, and $\sigma = 0.2$. The competition parameter is $\lambda = 0.4$. The investment costs, θ, are distributed uniformly over [0, 2].

The Cournot equilibrium is illustrated in Figure 4, for the case in which θ is distributed uniformly over [0, 2]. The preemption threshold, $\tilde{P}_L$, is equal to 0.085. At the point designated A where $\theta = 0.5$, the Stackelberg trigger $\bar{P}_F(\theta)$ is equal to $\tilde{P}_L$. Thus, for all $\theta > 0.5$, the equilibrium outcome is for firm 2 to invest first at trigger $\tilde{P}_L$ and for firm 1 to invest later at the Stackelberg trigger $\bar{P}_F(\theta)$. Conversely, for all $\theta \leq 0.5$, there will be simultaneous entry at the Stackelberg trigger $\bar{P}_F(\theta)$.

5. Conclusion

This article studies real options signaling games. These are games in which the decision to exercise an option is a signal of private information to outsiders, whose beliefs affect the payoff of the decision-maker. The decision-maker attempts to fool outsiders by altering the timing of options exercise. In equilibrium, signal-jamming occurs: Outsiders infer private information of the decision-maker correctly, but the timing of the options exercise is significantly distorted. The distortion can go in both directions. If the decision-maker's payoff increases in outsiders' belief about the value of the asset, then signaling

incentives speed up options exercise. Conversely, if the decision-maker's pay-off decreases in outsiders' belief about the value of the asset, then signaling incentives delay options exercise.

We illustrate the findings of the general model using four corporate finance applications: investment under managerial myopia, venture capital grandstanding, investment under cash flow diversion by the manager, and product market entry decisions by two asymmetrically informed firms. The first two applications provide examples in which signaling erodes the value of the option to wait and speeds up investment. In the first application, the manager cares not only about the long-term performance of the company but also about the short-term stock price. In attempt to boost the short-term stock price, the manager invests too early, attempting to fool the market into overestimating the project's NPV. In the second application, we consider the decision when to take the company public by a venture capitalist, who is better informed about its value than are outside investors. Here, a venture capitalist with a short track record takes his portfolio companies public earlier in an attempt to establish a reputation and raise more capital for new funds. The last two applications provide examples in which signaling incentives delay investment. First, signaling can significantly delay investment if the agent can divert cash flows from the project for her own private benefit. In this case, investment is delayed as the agent tries to signal that the NPV of the project is lower than in reality, thereby diverting more for her personal consumption. Second, we illustrate how signaling delays investment in a duopoly, where the firms are asymmetrically informed about the value of a new product. In this case, the informed firm's decision when to launch the new product reveals information about its value to the uninformed firm and thereby potentially impacting future competition. The informed firm delays the decision to launch the product in an attempt to fool the rival into underestimating the value of the product.

Irrespective of the application, the main message of the article is the same: Signaling incentives have an important role in distorting major timing decisions of firms such as investment in large projects, IPOs, and developing new products. This gives rise to several interesting questions that are left outside the scope of this article. For example, to what extent do the existing contracts provide incentives to make the timing decisions optimally? As another example, in what applications do signaling incentives work for or against social welfare? While signaling incentives reduce the decision-maker's utility due to an inefficient timing of options exercise, their effect on the social welfare is unclear.

Appendix

Appendix A: Single-crossing Condition

We list the single-crossing condition, which is used to obtain existence and uniqueness of the separating equilibrium.

Assumption 5 (Single-crossing condition). Function $W\left(\tilde{\theta}, \theta\right)$ satisfies

$$W_{\tilde{\theta}}\left(\tilde{\theta}, \theta\right) \times \frac{\partial \left[\dfrac{U_{\hat{P}}\left(\tilde{\theta}, \theta, \hat{P}\right)}{U_{\tilde{\theta}}\left(\tilde{\theta}, \theta, \hat{P}\right)} \right]}{\partial \theta} > 0 \tag{A1}$$

for $\left(\tilde{\theta}, \hat{P}\right)$ in the graph of $\bar{P}$, where $U\left(\tilde{\theta}, \theta, \hat{P}\right)$ is given by (13), β is given by (7), and $\bar{P}\left(\theta\right)$ is the unique increasing solution of the differential equation (16), subject to the boundary condition (17), if $W_{\tilde{\theta}}\left(\tilde{\theta}, \theta\right) < 0$, or (18), if $W_{\tilde{\theta}}\left(\tilde{\theta}, \theta\right) > 0$.

The single-crossing condition ensures that if the agent does not make extra gains by misrepresenting θ slightly, then extra gains cannot be made from a large misrepresentation. It is standard for games of asymmetric information, both signaling and screening. Importantly, it is enough that the single-crossing condition is satisfied for $\left(\tilde{\theta}, \hat{P}\right)$ in the graph of $\bar{P}$.

The single-crossing condition holds in all applications that we consider. As an example, below we verify the single-crossing condition for the venture capital grandstanding example of Section 3.2. For this application, we have from (42):

$$W\left(\tilde{\theta}, \theta\right) = \alpha \frac{\bar{P}_2\left(\theta\right) - \theta}{\bar{P}_2\left(\theta\right)^{\beta}} H\left(I_2\left(\tilde{\theta}\right)\right), \tag{A2}$$

where $H' > 0$, $I_2' < 0$, and $\bar{P}_2\left(\theta\right)$ is the full-information trigger in (37). Simplifying, we have

$$W\left(\tilde{\theta}, \theta\right) = k\theta^{1-\beta} H\left(I_2\left(\tilde{\theta}\right)\right), \tag{A3}$$

where $k = \frac{\alpha}{\beta-1}\left(\frac{\beta}{\beta-1}\right)^{-\beta} > 0$. Taking derivatives, we obtain that Assumption 5 requires that

$$\frac{(\beta - 1)\,\hat{P}\theta^{\beta-1} - \beta\theta^{\beta} + \left(\frac{\beta-1}{\beta}\right)^{\beta-1} H\left(I_2\left(\tilde{\theta}\right)\right)}{\hat{P}\tilde{\theta}^{\beta-1}\frac{1}{1-\alpha} I_2'\left(\tilde{\theta}\right)} \tag{A4}$$

is a strictly decreasing function of θ for $\left(\tilde{\theta}, \hat{P}\right)$ in the graph of $\bar{P}_1$. Taking the derivative with respect to θ, we get the following requirement:

$$(\beta - 1)^2\,\hat{P} - \beta^2\theta < 0 \text{ for } \hat{P} \text{ in the graph of } \bar{P}_1. \tag{A5}$$

In the graph of $\bar{P}_1$, the highest value of $\hat{P}$ is $\bar{P}_1\left(\bar{\theta}\right)$, which is given by (45). Therefore, a sufficient condition for (A5) is

$$(\beta - 1)^2\,\bar{P}_1\left(\bar{\theta}\right) - \beta^2\underline{\theta} < 0. \tag{A6}$$

This is equivalent to (43) in Section 3.2. Hence, the single-crossing condition is verified.

Appendix B: Proofs

Proof of Proposition 2. We apply Theorems 1–3 from Mailath (1987) to prove the proposition. We need to show that function $U\left(\tilde{\theta}, \theta, \hat{P}\right)$ satisfies Mailath's (1987) regularity conditions:

- Smoothness: $U\left(\tilde{\theta}, \theta, \hat{P}\right)$ is C^2 on $\left[\underline{\theta}, \bar{\theta}\right]^2 \times \mathbb{R}_+$;

- Belief monotonicity: $U_{\tilde{\theta}}$ never equals zero, and so is either positive or negative;

- Type monotonicity: $U_{\theta\hat{P}}$ never equals zero, and so is either positive or negative;

- "Strict"quasiconcavity: $U_{\hat{P}}\left(\theta, \theta, \hat{P}\right) = 0$ has a unique solution in $\hat{P}$, which maximizes $U\left(\theta, \theta, \hat{P}\right)$, and $U_{\hat{P}\hat{P}}\left(\theta, \theta, \hat{P}\right) < 0$ at this solution;

- Boundedness: There exists $\delta > 0$ such that for all $\left(\theta, \hat{P}\right) \in [\underline{\theta}, \bar{\theta}] \times \mathbb{R}_+$ $U_{\hat{P}\hat{P}}\left(\theta, \theta, \hat{P}\right) \geq 0 \Rightarrow \left|U_{\hat{P}}\left(\theta, \theta, \hat{P}\right)\right| > \delta.$

Let us check that these conditions are satisfied for our problem. The smoothness condition is satisfied, because $W\left(\tilde{\theta}, \theta\right)$ is C^2 on $[\underline{\theta}, \bar{\theta}]^2$. The belief monotonicity condition is satisfied, because $W_{\tilde{\theta}}$ is either always positive or always negative. The type monotonicity condition is satisfied, because

$$U_{\theta\hat{P}}\left(\tilde{\theta}, \theta, \hat{P}\right) = \frac{\beta\left(\alpha - W_\theta\left(\tilde{\theta}, \theta\right)\right)}{\hat{P}^{\beta+1}} > 0, \tag{B1}$$

as $\alpha > W_\theta\left(\tilde{\theta}, \theta\right)$ by Assumption 4. As we show in Section 1.3, $U_{\hat{P}}\left(\theta, \theta, \hat{P}\right) = 0$ has a unique solution in $\hat{P}$, denoted by $P^*(\theta)$, that maximizes $U\left(\theta, \theta, \hat{P}\right)$. Also,

$$U_{\hat{P}\hat{P}}\left(\theta, \theta, P^*(\theta)\right) = \frac{\beta}{P^*(\theta)^{\beta+2}}\left[\alpha(\beta-1)P^*(\theta) - (\beta+1)(\alpha\theta - W(\theta, \theta))\right] \tag{B2}$$

$$= -\frac{\beta(\alpha\theta - W(\theta, \theta))}{P^*(\theta)^{\beta+2}} < 0.$$

Hence, the "strict"quasiconcavity condition is satisfied. Finally, to ensure that the boundedness condition is satisfied, we restrict the set of potential investment thresholds to be bounded by k from above, where k can be arbitrarily large. We will later show that extending the set of actions to $\hat{P} \in (0, \infty)$ neither destroys the separating equilibrium nor creates additional separating equilibria. Notice that $U_{\hat{P}\hat{P}}\left(\theta, \theta, \hat{P}\right) \geq 0$ implies that $\alpha\theta - W(\theta, \theta) \leq \hat{P}\alpha(\beta-1)/(\beta+1)$. Hence, for any $\left(\theta, \hat{P}\right) \in [\underline{\theta}, \bar{\theta}] \times [0, k]$ such that $U_{\hat{P}\hat{P}}\left(\theta, \theta, \hat{P}\right) \geq 0$,

$$\left|U_{\hat{P}}\left(\theta, \theta, \hat{P}\right)\right| = \frac{\alpha(\beta-1)\hat{P} - \beta(\alpha\theta - W(\theta, \theta))}{\hat{P}^{\beta+1}} \geq \frac{\alpha(\beta-1)\hat{P} - \frac{\beta\alpha(\beta-1)}{\beta+1}\hat{P}}{\hat{P}^{\beta+1}} = \frac{\alpha(\beta-1)}{(\beta+1)\hat{P}^\beta} \geq \frac{\alpha(\beta-1)}{(\beta+1)k^\beta} > 0 \tag{B3}$$

for any arbitrarily large k. Then, the boundedness condition is satisfied.

By Mailath's (1987) Theorems 1 and 2, any separating equilibrium $\bar{P}(\theta)$ is continuous, differentiable, satisfies Equation (16), and $d\bar{P}/d\theta$ has the same sign as $U_{\theta\hat{P}}$. Because $U_{\theta\hat{P}} > 0$, $\bar{P}(\theta)$ is an increasing function of θ. Let $\tilde{P}$ denote the solution to the following restricted initial value problem: Equation (16), subject to (17), if $W_{\tilde{\theta}} < 0$, or (18), if $W_{\tilde{\theta}} > 0$. Because $\left|W_{\tilde{\theta}}(\theta, \theta)\right|$ is bounded above by $\max_{\theta \in [\underline{\theta}, \bar{\theta}]}\left|W_{\tilde{\theta}}(\theta, \theta)\right|$, $\tilde{P}$ is unique by Mailath's (1987) Proposition 5. Hence, if a separating equilibrium exists, it is unique and is given by $\tilde{P}$. By Mailath's (1987) Theorem 3, the single-crossing condition guarantees existence of the separating equilibrium.

This argument suggests that $\tilde{P}$ is the unique separating equilibrium in a problem in which the set of investment thresholds is $\hat{P} \in (0, k)$ for any sufficiently large finite k. Finally, it remains to show that considering the space of investment thresholds bounded by k is not restrictive. First, we argue that $\tilde{P}$ is a separating equilibrium in a problem with $\hat{P} \in (0, +\infty)$. To show this, note that the single-crossing condition holds for all $\hat{P} \in (0, +\infty)$. Therefore, local incentive compatibility guarantees global incentive compatibility for all $\hat{P} \in (0, +\infty)$, not only for

$\hat{P} \in (0, k)$. Hence, $\tilde{P}$ is a separating equilibrium in a problem with $\hat{P} \in (0, +\infty)$. Second, we argue that there are no other separating equilibria in a problem with $\hat{P} \in (0, +\infty)$. By contradiction, suppose that there is an additional separating equilibrium $\tilde{P}_2$, other than $\tilde{P}$. It must be the case that for some θ, $\tilde{P}_2(\theta)$ is infinite. Otherwise, it would be a separating equilibrium in the restricted problem for a sufficiently large k. However, if $\tilde{P}_2(\theta)$ is infinite for some θ, then the equilibrium expected payoff of type θ is zero. Hence, it would be optimal for this type to deviate to any finite $\hat{P} > \theta - \max_{\tilde{\theta}} W(\tilde{\theta}, \theta)/\alpha$. Thus, there are no other separating equilibria in a problem with $\hat{P} \in (0, +\infty)$. ∎

Proof of Proposition 3. We can rewrite Equation (16) in the following form:

$$\alpha(\beta - 1)\bar{P}(\theta) - \beta(\alpha\theta - W(\theta, \theta)) = \frac{\bar{P}(\theta)W_{\tilde{\theta}}(\theta, \theta)}{\bar{P}'(\theta)}. \tag{B4}$$

From the proof of Proposition 2, we know that $\bar{P}'(\theta) > 0$. Hence, if $W_{\tilde{\theta}} < 0$, then the right-hand side of (B4) is negative. Thus, (B4) implies that $\bar{P}(\theta) < P^*(\theta)$ except the point $\theta = \bar{\theta}$ in which the initial value condition holds. Analogously, if $W_{\tilde{\theta}} > 0$, then the right-hand side of (B4) is positive, so (B4) implies that $\bar{P}(\theta) > P^*(\theta)$ except the point $\theta = \underline{\theta}$ in which the initial value condition holds. ∎

Derivation of the semi-pooling equilibrium. First, we show that for any $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$, Equation (26) has the unique solution denoted $P_{pool}(\hat{\theta})$. Consider function

$$f(P; \hat{\theta}) = \left(\frac{P}{\bar{P}(\hat{\theta})}\right)^{\beta} \alpha(\bar{P}(\hat{\theta}) - \hat{\theta}) - \alpha(P - \hat{\theta}) + c_w \frac{\hat{\theta} - \underline{\theta}}{2}, \tag{B5}$$

defined over $P \in \left[0, \bar{P}(\hat{\theta})\right], \hat{\theta} \in (\underline{\theta}, \bar{\theta})$. Note that

$$f\left(\bar{P}(\hat{\theta}); \hat{\theta}\right) = c_w \frac{\hat{\theta} - \underline{\theta}}{2} < 0, \tag{B6}$$

$$f(0; \hat{\theta}) = \alpha\hat{\theta} + c_w \frac{\hat{\theta} - \underline{\theta}}{2} > 0, \tag{B7}$$

where the first inequality holds by $c_w < 0$ and the second inequality holds by Assumption 2. Consider the derivative of $f(P; \hat{\theta})$ with respect to P:

$$f_P(P; \hat{\theta}) = \beta\left(\frac{P}{\bar{P}(\hat{\theta})}\right)^{\beta} \alpha\frac{\bar{P}(\hat{\theta}) - \hat{\theta}}{P} - \alpha$$

$$\leq \beta\alpha\frac{\bar{P}(\hat{\theta}) - \hat{\theta}}{\bar{P}(\hat{\theta})} - \alpha \tag{B8}$$

$$< \alpha(\beta - 1) - \beta\alpha\frac{\hat{\theta}}{P^*(\hat{\theta})} = 0.$$

The first inequality follows from $P \leq \bar{P}(\hat{\theta})$ and $f_{PP}(P; \hat{\theta}) > 0$. The second inequality follows from $\bar{P}(\hat{\theta}) < P^*(\hat{\theta})$. By continuity of $f(P; \hat{\theta})$, for any $\hat{\theta}$ there exists a unique point in $P_{pool}(\hat{\theta})$ at which $f(P_{pool}(\hat{\theta}); \hat{\theta}) = 0$.

Second, we demonstrate that each type $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$ indeed finds it optimal to exercise at $P_{pool}\left(\hat{\theta}\right)$ and each type $\theta \in \left[\hat{\theta}, \bar{\theta}\right]$ finds it optimal to exercise at $\bar{P}\left(\theta\right)$. Consider type $\theta \in \left[\hat{\theta}, \bar{\theta}\right]$. The difference of the utilities from separating at $\bar{P}\left(\theta\right)$ and pooling is equal to

$$\left(\frac{P_{pool}}{\bar{P}\left(\theta\right)}\right)^{\beta} \alpha\left(\bar{P}\left(\theta\right) - \theta\right) - \alpha\left(P_{pool} - \theta\right) + c_w \frac{\hat{\theta} - \theta}{2} \tag{B9}$$

$$= P_{pool}^{\beta} \max_{Y \in \mathbb{R}_+} \left\{\frac{1}{Y^{\beta}}\left(\alpha\left(Y - \theta\right) + W\left(\bar{P}^{-1}\left(Y\right), \theta\right)\right)\right\} - \alpha\left(P_{pool} - \theta\right) + c_w \frac{\hat{\theta} - \theta}{2}.$$

By the envelope theorem, the derivative with respect to θ is

$$\left(\frac{P_{pool}}{\bar{P}\left(\theta\right)}\right)^{\beta}\left(-\alpha - c_w\right) + \alpha \geq 0, \tag{B10}$$

because $c_w < 0$ and $P_{pool} \leq \bar{P}\left(\theta\right)$. Because type $\hat{\theta}$ is indifferent between separating and pooling, any type θ above $\hat{\theta}$ does not have an incentive to deviate to P_{pool}. By the single-crossing condition, any deviation to a threshold that is different from P_{pool} is also not optimal for any type $\theta \in \left[\hat{\theta}, \bar{\theta}\right]$. Consider type $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$. From (B9), the payoff of type θ from pooling and investing at P_{pool} is higher than $P_{pool}^{\beta} U\left(\tilde{\theta}, \theta, \hat{P}\right)$. By the single-crossing condition, $U\left(\theta, \theta, \bar{P}\left(\theta\right)\right) \geq U\left(\tilde{\theta}, \theta, \bar{P}\left(\tilde{\theta}\right)\right)$. Therefore, under the worst-possible out-of-equilibrium beliefs, no type $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$ finds it optimal to deviate from P_{pool}. ∎

Proof that the form of the optimal verification threshold, $v\left(P, \tilde{\theta}\right)$, is $P - \tilde{\theta} - v$ for some constant v. Suppose that the manager's type is θ, and the shareholders' belief is $\tilde{\theta}$. Let $v\left(P, \tilde{\theta}\right)$ denote the more general verification threshold of the shareholders such that shareholders verify the manager if and only if the realized value is below $v\left(P, \tilde{\theta}\right)$. Then, if the manager of type θ diverts d, she expects to be verified with probability $\Psi\left(v\left(P, \tilde{\theta}\right) - P + \theta + d\right)$. Hence, the manager's problem is

$$\max_{d \geq 0}\left\{\left(\lambda - \alpha\right) d\left(1 - \Psi\left(v\left(P, \tilde{\theta}\right) - P + \theta + d\right)\right)\right\}. \tag{B11}$$

The solution is a function of $P - \theta - v\left(P, \tilde{\theta}\right)$, denoted by $D\left(P - \theta - v\left(P, \tilde{\theta}\right)\right)$.

Given the manager's response to a verification rule $v\left(P, \tilde{\theta}\right)$, we now derive the optimal $v\left(P, \tilde{\theta}\right)$. The shareholders expect the manager to divert $D\left(P - \tilde{\theta} - v\right)$, so they estimate the probability of verification at $\Psi\left(v - P + \tilde{\theta} + D\left(P - \tilde{\theta} - v\right)\right)$. Hence, for each P and $\tilde{\theta}$, $v\left(P, \tilde{\theta}\right)$ must minimize

$$\left(1 - \alpha\right) D\left(P - \tilde{\theta} - v\right)\left(1 - \Psi\left(v - P + \tilde{\theta} + D\left(P - \tilde{\theta} - v\right)\right)\right)$$
$$+ c\Psi\left(v - P + \tilde{\theta} + D\left(P - \tilde{\theta} - v\right)\right). \tag{B12}$$

Since the value function depends on v, P, and $\tilde{\theta}$ only through $v - P + \tilde{\theta}$, any optimal verification threshold is of the form $v\left(P, \tilde{\theta}\right) = P - \tilde{\theta} - v$ for some constant v. ∎

Verification of $\bar{P}_L(\theta) \leq \bar{P}_F(\theta)$. By contradiction, suppose that in equilibrium $\bar{P}_L(\theta) > \bar{P}_F(\theta)$ for some θ. If firm 1 invests at $\hat{P}_L \geq \bar{P}_F(\tilde{\theta})$, firm 2 will invest immediately after firm 1. Hence, in the range $\hat{P}_L \geq \bar{P}_F(\tilde{\theta})$, $P < \hat{P}_L$, $V_L(P, \theta, \tilde{\theta}; \hat{P}_L)$ is equal to

$$V_L\left(P, \theta, \tilde{\theta}; \hat{P}_L\right) = \left(\frac{P}{\hat{P}_L}\right)^\beta \left(\frac{\lambda \hat{P}_L}{r - \mu} - \theta\right). \tag{B13}$$

Irrespective of $\tilde{\theta}$, this value function is maximized at $\hat{P}_L = \bar{P}_F(\theta)$. Hence, any $\bar{P}_L(\theta) > \bar{P}_F(\theta)$ is inconsistent with equilibrium. ∎

Generalizing the payoff function to $W(P, \tilde{\theta}, \theta)$. The equilibrium differential equation in (16) can be generalized to the case in which the belief function also includes P as an argument. Provided that the payoff function satisfies the regularity condition in Mailath (1987), as does the particular function in (55)–(56), the equilibrium derivation can proceed as follows. Analogous to (15), the agent's first-order condition for the optimal selection of the trigger $\hat{P}$ is

$$\frac{\beta\left(\alpha\left(\hat{P} - \theta\right) + W\left(\hat{P}, \bar{P}^{-1}\left(\hat{P}\right), \theta\right)\right)}{\hat{P}}$$

$$= \alpha + W_P\left(\hat{P}, \bar{P}^{-1}\left(\hat{P}\right), \theta\right) + W_{\tilde{\theta}}\left(\hat{P}, \bar{P}^{-1}\left(\hat{P}\right), \theta\right) \frac{d\bar{P}^{-1}\left(\hat{P}\right)}{d\hat{P}}. \tag{B14}$$

In the separating equilibrium, we can set $\bar{P}^{-1}\left(\hat{P}\right) = \theta$ and obtain the equilibrium differential equation

$$\frac{d\bar{P}(\theta)}{d\theta} = \frac{\bar{P}(\theta) W_{\tilde{\theta}}\left(\bar{P}(\theta), \theta, \theta\right)}{\alpha\left((\beta - 1)\bar{P}(\theta) - \beta\theta\right) + \beta W\left(\bar{P}(\theta), \theta, \theta\right) - \bar{P}(\theta) W_P\left(\bar{P}(\theta), \theta, \theta\right)}. \tag{B15}$$

∎

Solution to differential equation (57) subject to boundary condition (58). Let us look for a solution in the form $\bar{P}_L(\theta) = A\theta$. Notice that this solution will satisfy the boundary condition (58) since $\bar{P}_L(0) = 0$. Equation (57) becomes

$$A\frac{\beta - 1}{\beta(r - \mu)} - \left(\frac{A(\beta - 1)\lambda}{\beta(r - \mu)}\right)^\beta \frac{1 - \lambda}{\lambda} = 1. \tag{B16}$$

Letting $v \equiv A\frac{(\beta - 1)\lambda}{\beta(r - \mu)}$, we get

$$v - v^\beta - \lambda\left(1 - v^\beta\right) = 0. \tag{B17}$$

Let $\kappa(v) = v - v^\beta - \lambda\left(1 - v^\beta\right)$. It is clear that $v = 1$ is a root of $\kappa(v)$. Since $\kappa''(v) = \beta(\beta - 1)(\lambda - 1)v^{\beta - 2} < 0$ and $\kappa(0) = -\lambda < 0$, $\kappa(v)$ has at most one other root. We have $\lim_{v \to \infty} \kappa(v) = -\infty$, and since $\lambda > 1 - \frac{1}{\beta}$, $\kappa'(1) > 0$. Thus, there exists the second root, and it exceeds 1. The upper root cannot yield the separating equilibrium since it implies the investment threshold above $\bar{P}_F(\theta)$, which is inconsistent with the separating equilibrium, as shown above. Hence, (57)–(58) is solved by

$$\bar{P}_L(\theta) = \frac{\beta}{\beta - 1}\frac{r - \mu}{\lambda}\theta. \tag{B18}$$

∎

Proof of properties of $\tilde{P}_L$. The first derivative of (61) with respect to $\tilde{P}_L$ equals

$$-\mathbb{E}_\theta\left[\left(\frac{P}{\tilde{P}_L}\right)^\beta \frac{1}{\tilde{P}_L}\left(\frac{(\beta-1)\,\tilde{P}_L}{r-\mu} - \beta\theta\right)|\theta \geq \eta\tilde{P}_L\right]$$

$$= \left(\frac{P}{\tilde{P}_L}\right)^\beta \frac{1}{\tilde{P}_L}\left(\beta\mathbb{E}_\theta\left[\theta|\theta \geq \eta\tilde{P}_L\right] - \frac{(\beta-1)\,\tilde{P}_L}{r-\mu}\right), \qquad (B19)$$

where $\eta = \frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}$. It is strictly positive for all $\tilde{P}_L$ sufficiently close to 0 and strictly negative for all $\tilde{P}_L$ sufficiently close to $\bar{P}_F\left(\bar{\theta}\right)$. Hence, (61) is maximized at $\tilde{P}_L \in \left(0, \bar{P}_F\left(\bar{\theta}\right)\right)$. Therefore, the sets $\left\{\theta : P_F(\theta) > \tilde{P}_L\right\}$ and $\left\{\theta : P_F(\theta) < \tilde{P}_L\right\}$ are nonempty.

Consider the case when $\theta\phi(\theta)/(1-\Phi(\theta))$ is increasing in θ. First, we show that $e\left(\theta^*\right) \equiv E_\theta\left[\frac{\theta}{\theta^*}|\theta \geq \theta^*\right]$ is strictly decreasing in θ^*. Taking the derivative,

$$\theta^* e'\left(\theta^*\right) = e\left(\theta^*\right)\left(\frac{\phi\left(\theta^*\right)\theta^*}{1-\Phi\left(\theta^*\right)} - 1\right) - \frac{\phi\left(\theta^*\right)\theta^*}{1-\Phi\left(\theta^*\right)}. \qquad (B20)$$

Clearly, $e\left(\theta^*\right)$ is strictly decreasing in θ^* for all points below the point at which $\frac{\theta^*\phi(\theta^*)}{1-\Phi(\theta^*)} = 1$. Consider the range above this point. If $e'\left(\theta^*\right) > 0$ for some θ^*, then it must be the case that $e'\left(\theta^*\right) > 0$ for all θ^* above. This implies $1 = e\left(\bar{\theta}\right) > e\left(\theta^*\right)$, which is a contradiction with $e\left(\theta^*\right)$ for all $\theta^* < \bar{\theta}$. Hence, $e\left(\theta^*\right)$ is strictly decreasing in θ^*. Now, consider (62). We can rewrite it as

$$\mathbb{E}_\theta\left[\frac{\theta}{\eta\tilde{P}_L}|\theta \geq \eta\tilde{P}_L\right] = \frac{1}{\lambda}, \qquad (B21)$$

where $\eta \equiv \frac{\beta-1}{\beta}\frac{\lambda}{r-\mu}$. Notice that the left-hand side approaches infinity when $\tilde{P}_L$ approaches zero and equals $1 < \frac{1}{\lambda}$ when $\tilde{P}_L = \bar{\theta}/\eta = \bar{P}_F\left(\bar{\theta}\right)$. Because $E_\theta\left[\frac{\theta}{\eta\tilde{P}_L}|\theta \geq \eta\tilde{P}_L\right]$ is decreasing in $\tilde{P}_L$, there exists the unique $\tilde{P}_L \in \left(0, \bar{P}_F\left(\bar{\theta}\right)\right)$ at which (62) is satisfied. ∎

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