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Optimal Dynamic Capital Budgeting

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I study optimal design of a dynamic capital allocation process in an organization in which the division manager with empire-building preferences privately observes the arrival and properties of investment projects, and headquarters can audit projects at a cost. Under certain conditions, a budgeting mechanism with threshold separation of financing is optimal. Headquarters: (1) allocate a spending account to the manager and replenish it over time; (2) set a threshold, such that projects below it are financed from the account, while projects above are financed fully by headquarters upon an audit. Further analysis studies when co-financing of projects is optimal and how the size of the account depends on past performance of projects.

Key words: Principal agent, Capital budgeting, Internal capital markets, Repeated interactions.

JEL Codes: G31, D82, D86

Internal capital allocation is fundamental to any organization. An important concern is that investment projects are often conceived on lower levels of the organization, whose managers may have different incentives from upper-level managers, let alone shareholders.[1] In particular, an important misalignment is that division managers want to spend too much. When aligning incentives through performance-based contracts is not fully feasible, headquarters have to rely on the internal capital allocation process—a collection of rules specifying how managers at various levels share information about potential investments and how investment decisions are made. An important question in organizational economics and corporate finance is how this process should be designed.

While there is a large literature on internal capital allocation, with rare exceptions, it focuses on one-shot settings. This literature concludes that the agency friction justifies a funding restriction (*e.g.* a budget) on the manager that can be relaxed after audit by headquarters.[2] However, the internal capital allocation problem in real-world organizations is dynamic in nature: headquarters and division managers interact repeatedly and over multiple projects. For example, consider a loan officer evaluating a loan application: the loan officer has superior information about loan quality and often has a bias in favour of approving too many loan applications (*e.g.*

1. According to survey evidence in Petty *et al.* (1975), in a typical Fortune 500 firm, less than 20% of projects are originated at the central office level. Akalu (2003) provides similar evidence for European firms.

2. See Harris and Raviv (1996) and the follow-up work. The end of this section discusses related literature in detail.

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Berg *et al*., 2016). Furthermore, there will be other loan applications that will need to be decided upon in the future.

It is not clear how insights from the existing static studies extend to a setting when the division manager and headquarters interact repeatedly. In particular, the following questions are both important and unanswered: (1) What form does a financing restriction take in a dynamic setting? Is it better to have a long-term budget or a sequence of short-term budgets? (2) When should a project be audited? Should headquarters wait until the division manager spends her budget completely and audit all projects after that, or should they audit projects even if the budget has enough resources to cover the investment? (3) How should the projects be financed: out of the agent's budget, by headquarters, or co-financed by both parties? In the one-shot interaction, the budget is always used up completely: there is no reason to preserve the budget as there will be no more projects.

To examine these questions, I incorporate the static capital allocation framework of Harris and Raviv (1996) into a dynamic environment and study the optimal mechanism design problem. I consider a continuous-time setting in which a risk-neutral principal (headquarters) employs a risk-neutral agent (the division manager) under limited liability and no savings. The firm has access to a sequence of heterogeneous investment opportunities that arrive stochastically over time and whose arrival and type, driving the optimal size of investment, are observed only by the division manager. Headquarters operate in the interest of shareholders, while the division manager enjoys utility from monetary compensation, but also gets an "empire-building" private benefit from each dollar of investment. Thus, headquarters are concerned that if the manager wants to invest a lot, it can be due to private benefits rather than project quality. At any time, headquarters can use two tools to incentivize the division manager. First, they can punish her for high spending today by being tougher and restricting investment in the future. Secondly, they can audit the manager at a cost and find out the quality of the current project. The goal is to find a mechanism that maximizes firm value subject to guaranteeing the division manager her reservation utility. The basic model considers a single, perfect, and deterministic audit technology and assumes that realized project values are not observable. I then relax these assumptions one by one.

The basic model gives rise to the optimality of a simple mechanism, which I call a budgeting mechanism with threshold separation of financing. In this mechanism, headquarters allocate a dynamic spending account to the division manager at the initial date and replenish it over time at a certain rate. At any time, the manager is allowed to draw on this account to finance projects at her discretion. In addition, headquarters specify a threshold on the size of individual projects, such that at any time the manager has an option to pass the project to headquarters claiming that it deserves investment above the threshold. If the division manager passes the project, it gets audited, and if audit confirms that the project indeed deserves an investment above the threshold, it gets financed fully by headquarters. Otherwise, the manager is punished. In equilibrium, the manager passes a project to headquarters if and only if it indeed deserves investment above the threshold. Thus, the mechanism completely separates financing between the parties: all small investment decisions are made at the division level and are financed out of the division manager's spending account; by contrast, all large projects are passed to headquarters and are financed fully by headquarters.

The optimality of this mechanism follows from the following three economic intuitions:

1. To provide incentives not to overspend, headquarters must either audit the project or punish the division manager in the future for high spending today. The latter can be implemented via a spending account. Intuitively, when the division manager draws on the spending account to invest in a project, the account balance goes down, which reduces

the division manager's ability to invest in the future. Setting the replenishment rate of the account balance to reflect the division manager's discount rate aligns the division manager's incentives with those of headquarters: given the budget constraint, the allocation of funds between current and future investment that maximizes headquarters' value also maximizes the division manager's value. This intuition explains why a dynamic spending account is a part of the optimal mechanism. The optimal initial balance on the spending account maximizes the difference between the total value of the relationship and the division manager's rents. The latter are increasing linearly in the account balance, and so a finite initial account balance is optimal.

2. If headquarters audit the project, they do not need to punish the division manager with a reduction in investment in the future. The audit itself already ensures that the division manager does not inflate the type of each project: headquarters find out the true quality of the project in the audit. Because fluctuations in the spending account balance are costly for headquarters, it is optimal to keep the spending account balance unchanged whenever the project is audited. This intuition explains why separation of financing is a part of the optimal mechanism: unaudited projects are financed from the division manager's account balance, while audited projects are separately and fully financed by headquarters.
3. The audit decision is based on the trade-off between the cost of audit and the cost from the increase in the investment distortion due to a reduction in the division manager's spending account balance, which headquarters bear if the project is not audited. While the cost of audit is constant, the latter cost increases with the size of the investment in the project. Therefore, it is optimal to audit the project if and only if its type or, equivalently, the amount of investment is sufficiently high. This intuition explains why small projects are not audited and get financed from the spending account, while large projects are audited and financed by headquarters separately.

Similar mechanisms, but with additional peculiarities, are also optimal in the extensions of the model. In one extension, I show that in a setting with multiple audit technologies, the optimal mechanism features multiple project thresholds, where larger projects are audited using more efficient but costlier audit technologies, and medium projects are co-financed. In another extension, I consider a model with observable and verifiable project values. In this setting, the optimal mechanism can again be implemented with a spending account that gets replenished over time, but the new feature of this mechanism is that the replenishment of the account is sensitive to the division manager's performance.

Together, these analyses allow me to provide answers to the three questions posed above: (1) The optimal financing restriction takes the form of a long-term spending account that gets replenished over time. (2) Waiting until the division manager spends her account completely before auditing projects is not optimal. In fact, auditing a large enough project is optimal even if the account balance is more than enough to cover the investment cost. (3) If the audit technology is perfect, complete separation of financing is optimal: large projects are audited and fully financed by headquarters, while small projects are not audited and get fully financed out of the division's spending account.

The article merges the literature on internal capital allocation in corporate finance and organizational economics with the literature on optimal dynamic contracting and mechanism design. The literature on internal capital allocation was started by Harris *et al.* (1982) and Antle and Eppen (1985), and with rare exceptions, it focuses on a one-shot interaction of headquarters with the division manager. Harris and Raviv (1996, 1998) are the closest papers as they consider a similar agency setting. Harris and Raviv (1996) study a one-project, one-shot setting, and show that the optimal procedure features an initial spending allocation that the division manager can increase under the threat of getting audited by headquarters. Harris and Raviv (1998) extend their earlier paper to the case of two projects and show that the same solution applies, with the difference that the initial spending limit is allocated for both projects.[3] To my knowledge, the problem of dynamic capital budgeting has only been studied in section 6 of Harris and Raviv (1998) and Roper and Ruckes (2012), who consider two-period settings based on Harris and Raviv (1996). However, these models feature no audit and costless audit, respectively, so there is no trade-off between providing incentives through audit and restricting future financing, which is at the heart of my analysis.

Another related strand of literature, started by Holmstrom (1984), studies delegation of a decision to an informed but biased expert.[4] The optimal delegation rule sometimes takes a threshold form. These models are static and, to focus on issues of delegation, these papers rule out transfers. Krishna and Morgan (2008) introduce transfers into the "cheap talk" setting of Crawford and Sobel (1982) under the assumption of limited liability. Like Crawford and Sobel, they assume that actions are not contractible, which makes their model different from mine, even in the benchmark case of a one-shot interaction. Frankel (2014) obtains a quota (or budget) contract as a max–min optimal mechanism. His setting features uncertainty over the agent's preferences, but does not include monetary transfers and audit.

The article also builds on optimal dynamic contracting literature that uses recursive techniques to characterize the optimal contract.[5] Within this literature, the article is most closely related to models with risk-neutral parties based on repeated hidden information.[6] The memory device role of the division manager's spending account is similar to that of a credit line in DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007b) or cash reserves in Biais *et al.* (2007). My model contributes to this literature in several ways. First, I study a different agency setting—organization of internal capital allocation as opposed to the problem of how to split cash flows from the firm. Secondly, my model features investment decisions. Thirdly, it extends the literature by incorporating the costly state verification (CSV) framework of Townsend (1979). Relatedly, Piskorski and Westerfield (2016) introduce non-CSV monitoring into this literature. Finally, path-dependent investment decisions make my article related to the literature on agency and investment dynamics.[7]

The article is organized as follows. Section 1 describes the setup. Section 2 solves for the optimal mechanism and shows that it can be implemented by a budgeting mechanism with threshold separation of financing. Section 3 considers several extensions of the basic model, and Section 4 discusses the empirical implications. Finally, Section 5 concludes.

1. THE MODEL

The organization consists of risk-neutral headquarters (the principal) and a risk-neutral division manager (the agent). It gets a sequence of spending opportunities ("investment projects") that

3. Holmstrom and Ricart i Costa (1986), Bernardo *et al.* (2001, 2004), and Garcia (2014) study the interplay between capital allocation and performance-based compensation. Like Harris and Raviv (1996), but unlike these papers, my main focus is on settings in which performance-based compensation is not feasible.

4. See, for example, Dessein (2002), Harris and Raviv (2005), Marino and Matsusaka (2005), and Alonso and Matouschek (2007, 2008).

5. Green (1987), Spear and Srivastava (1987), and Thomas and Worrall (1988) develop these techniques for discrete-time models. Sannikov (2008) and DeMarzo and Sannikov (2006) extend them for settings in continuous time.

6. See DeMarzo and Sannikov (2006), Biais *et al.* (2007), DeMarzo and Fishman (2007b), Piskorski and Tchistyi (2010), and Tchistyi (2016).

7. See Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007a), Biais *et al.* (2010), DeMarzo *et al.* (2012), Bolton *et al.* (2011), and Gryglewicz and Hartman-Glaser (2015).

arrive randomly over time. Since the focus is on capital budgeting, I assume that headquarters are the only source of capital.[8]

Time is continuous, indexed by $t \geq 0$, and the horizon is infinite. The discount rates of headquarters and the division manager are $r > 0$ and $\rho > r$, respectively. This assumption rules out postponing payments to the division manager forever, which would be strictly optimal for headquarters if the discount rates were equal.[9] Over each infinitesimal period of time $[t, t+dt]$, the firm gets a project with probability λdt. Each project has quality θ, which is an independent and identically distributed (i.i.d.) draw from a distribution with cumulative distribution function (c.d.f.) $F(\theta)$ and finite density $f(\theta) > 0$ defined over $\Theta = [\underline{\theta}, \bar{\theta}]$, $\bar{\theta} > \underline{\theta} > 0$. Formally, the arrival of projects is an independently marked homogeneous point process $((T_n, \theta_n))_{n \geq 1}$, where T_n and θ_n denote the arrival time and the quality of the *nth* investment project. Each project is a take-it-or-leave-it opportunity that generates the net value of $V(k,\theta) - k$ to headquarters, where $k \geq 0$ is the amount of capital spent on the project. Function $V(k,\theta)$ is assumed to satisfy the following restrictions:

Assumption 1. $V(k,\theta)$ has the following properties: (a) $V(0,\theta) = 0$; (b) $V_{kk}(k,\theta) < 0, \lim_{k \to 0} V_k(k,\theta) = \infty$, and $\lim_{k \to \infty} V_k(k,\theta) = 0$; (c) For any $k > 0$ and $\theta \in \Theta$, $V_{k\theta}(k,\theta) > 0$.

These restrictions are natural. Part (a) means that the project generates zero value if there is no investment. Part (b) means that projects exhibit decreasing marginal returns, ranging from infinity for the first dollar invested to zero for the infinite dollar invested. Finally, part (c) means that for the same investment, the marginal return is higher if the quality of the project is higher. Taken together, these restrictions will ensure that investment in each project is positive, finite, and, other things equal, increasing in the quality of the project. If investment occurs when no project is available, it is wasted completely. For convenience, I define $V(k,0) = 0$ to be the value when no project is available.

Let $(dX_t)_{t \geq 0}$ denote the stochastic process describing investment projects. Specifically, $dX_t = 0$ if no project arrives at time t and $dX_t = \theta$ if a project of quality θ arrives at time t. The division manager has informational advantage over headquarters in that she privately observes both the arrival of projects and their qualities (*i.e.* dX_t). This assumption has empirical support: for example, in a recent survey of CFOs by Hoang *et al*. (2017), more than 70% of CFOs believe that divisional managers have superior information about their businesses compared to the information of headquarters. Headquarters can learn about projects from (and only from) two sources. First, they can rely on reports of the division manager. Secondly, at time t headquarters can independently investigate (audit) the division manager and learn dX_t with certainty. As in classical models of CSV (Townsend, 1979, Gale and Hellwig, 1985), when headquarters audit the division manager, they incur a cost $c > 0$, such as the time and effort necessary to find out the true prospects of the project. Let $(dA_t)_{t \geq 0}$ be the stochastic process describing audit decisions of headquarters: $dA_t = 1$ if headquarters audit at time t and $dA_t = 0$ otherwise. The initial values of processes $(dX_t)_{t \geq 0}$ and $(dA_t)_{t \geq 0}$ are normalized to zero.

The basic version of the model assumes that headquarters do not observe (even with noise) realized project values $V(k,\theta)$ ex post. While complete non-observability may seem extreme, many settings feature vastly imperfect observability. Examples include spending activity with

8. The article does not address the question why division does not operate as a stand-alone entity. For studies that examine this issue, see, for example, Gertner *et al*. (1994), Stein (1997), and Scharfstein and Stein (2000). Stein (2003) surveys the literature.

9. I have also studied a finite-horizon analogue of the model, where the assumption $\rho > r$ can be relaxed. The analysis is available upon request.

non-monetary goals, such as corporate social responsibility investment, investment in projects with externalities on other divisions, such as advertising campaigns, when an increase in sales of the advertised product may not reflect the total realized value of the advertising campaign for the firm, and projects with long-term horizons.[10] The assumption of complete unobservability of realized project values is thus a useful benchmark. I consider an extension where project values are observable with noise in Section 3.2.

In addition to investment and audit decisions, I allow for monetary compensation of the division manager. The division manager is moneyless and consumes transfers immediately, so the transfers from headquarters to the manager must be non-negative.[11] Thus, the utility of headquarters from audit decisions $(dA_t)_{t\geq 0}$, non-negative streams of investment $(dK_t)_{t\geq 0}$, and non-negative monetary transfers to the division manager $(dC_t)_{t\geq 0}$ is

$$\int_0^{\infty} e^{-rt}\left(V(dK_t, dX_t)dN_t - dK_t - dC_t - c\,dA_t\right), \tag{1}$$

where $dN_t = 1$ if $t = T_n$ and $dN_t = 0$ otherwise. The agency problem stems from the division manager's desire to overspend. Specifically, the division manager derives utility both from monetary compensation $(dC_t)_{t\geq 0}$ and from spending resources on projects $(dK_t)_{t\geq 0}$:

$$\int_0^{\infty} e^{-\rho t}\left(\gamma\, dK_t + dC_t\right), \tag{2}$$

where $\gamma \in (0,1)$ captures the relative importance of "empire-building" to the division manager.[12] The preference for higher spending may reflect perquisite consumption associated with running larger projects as well as an intrinsic preference for empire-building. In line with this assumption, survey evidence shows that divisional managers prefer running large divisions (Hoang *et al.*, 2017). I assume that the division manager consumes monetary transfers immediately rather than saving them for the future, as can be seen from equation (2). In the Online Appendix, I show that this assumption is without loss of generality: the mechanism that is optimal in this model will also be optimal in the model that allows for savings that yield return r, irrespectively of whether savings are contractible or hidden.[13]

The basic model assumes that the parties can commit to any long-term mechanism subject to the constraint that auditing strategies cannot be random (I allow for random audit in the Online Appendix). Headquarters have all bargaining power subject to delivering the division manager the time-0 utility of at least R. In particular, if the division manager accepts the mechanism at time 0, she cannot quit the relationship.

I start with a general communication game with arbitrary message spaces and complete history dependence. The sequence of events over any infinitesimal time interval $[t, t+dt]$ is as follows.

10. For example, I could assume that the division manager leaves the firm at a random Poisson event and all project values are realized after she leaves. This model would be equivalent to the present setup.

11. This assumption is important for the analysis: it will imply that the optimal contract will not feature first-best investment.

12. The assumption that utility from spending accrues at the time of spending is without loss of generality. Equivalently, $\gamma\, dK_t$ can denote the present value of all future private benefits from spending dK_t at time t. For example, if spending dK_t gives the manager a flow of private benefits of $\tilde{\gamma}\, dK_t$, $s \geq t$, then the time-t present value of private benefits is $(\tilde{\gamma}/\rho)\, dK_t$, which is equivalent to equation (2) with $\gamma = \tilde{\gamma}/\rho$.

13. Intuitively, the solution to the problem with hidden savings is the same, because the division manager's discount rate exceeds the return that it can earn via private savings. Thus, saving privately is not optimal. The solution to the problem with contractible savings is the same, because contractible savings are equivalent to the principal saving for the agent.

At the beginning of each period, the division manager learns dX_t: whether the project arrives and if it does, its quality. She sends a message m_t from message space M_t. Given m_t and the prior history, the mechanism prescribes headquarters to audit the report or not. Finally, given m_t, the audit result (if there was audit), and the prior history, the mechanism prescribes investment dK_t and compensation dC_t.

By the revelation principle, any outcome that can be implemented with a general mechanism can also be implemented with a truth-telling direct mechanism.[14] Thus, it is sufficient to focus only on mechanisms in which at any time t the division manager sends a report $d\hat{X}_t \in \{0\} \cup \Theta$, that is, if the project is available and, if it is available, what its quality is, and in which the manager finds it optimal to send truthful reports $d\hat{X}_t = dX_t$. Given this, the analysis proceeds as follows. First, I optimize over the set of truth-telling direct mechanisms. Then, I show that the budgeting mechanism with threshold separation of financing implements the same policies as the optimal truth-telling direct mechanism.

The problem of optimal design of a truth-telling direct mechanism is formalized in the following way. The reporting strategy $\hat{X} = \{d\hat{X}_t \in \{0\} \cup \Theta\}_{t \geq 0}$ is an $\mathcal{F}$-adapted stochastic process, where $\mathcal{F} = \{\mathcal{F}_t\}_{t \geq 0}$ is the filtration generated by $((T_n, \theta_n))_{n \geq 1}$. The direct mechanism Γ is described by a triple (A, K, C) of stochastic processes such that the audit process $A = \{dA_t \in \{0,1\}\}_{t \geq 0}$ is measurable with respect to $\{d\hat{X}_s, s \in [0,t]$ and $dX_u, u \in [0,t)$ for which $dA_u = 1\}_{t \geq 0}$, and the investment and compensation processes, $K = \{dK_t \geq 0\}_{t \geq 0}$ and $C = \{dC_t \geq 0\}_{t \geq 0}$, are measurable with respect to $\{d\hat{X}_s, s \in [0,t]$ and $dX_u, u \in [0,t]$ for which $dA_u = 1\}_{t \geq 0}$.[15] Given Γ and $\hat{X}$, the expected discounted utilities of the division manager and headquarters are

$$\mathbb{E}^{\hat{X}}\left[\int_0^\infty e^{-\rho t}(\gamma dK_t + dC_t)\right], \tag{3}$$

$$\mathbb{E}^{\hat{X}}\left[\int_0^\infty e^{-rt}(V(dK_t, dX_t)dN_t - dK_t - dC_t - c dA_t)\right]. \tag{4}$$

Reporting strategy $\hat{X}$ of the division manager is incentive compatible if and only if it maximizes her expected discounted utility (3) given Γ. A direct mechanism Γ is truth-telling if the truth-telling reporting strategy $\hat{X} = X$ is incentive compatible. The goal is to find a truth-telling direct mechanism $\Gamma = (A, K, C)$ that maximizes the expected discounted utility of headquarters (4) subject to delivering the division manager the initial expected discounted utility of at least R (the division manager's outside option). Throughout the article, I assume that a solution to this problem exists.[16]

1.1. *Static benchmark*

As a benchmark, consider the problem in which headquarters and the division manager interact over one project only. The timing is as follows. Headquarters propose a direct mechanism $\Gamma = \left\{a(\hat{\theta}), k_n(\hat{\theta}), k_a(\theta, \hat{\theta}), c_n(\hat{\theta}), c_a(\theta, \hat{\theta}), \hat{\theta} \in \Theta\right\}$, which the division manager with reservation utility

14. Myerson (1986) provides a general revelation principle for dynamic communication games.

15. Note that dK_t and dC_t can be contingent on dX_t if $dA_t = 1$: If the report at time t is audited, headquarters learn dX_t and thus can make investment and monetary transfer decisions contingent on it. In contrast, dA_t cannot be contingent on dX_t, because audit reveals dX_t only after the audit decision is made.

16. In other words, given existence, I characterize the optimal direct mechanism and show how it can be implemented via capital budgeting processes. However, I do not prove existence of a solution to the Hamilton-Jacobi-Bellman (HJB) equation describing the optimal mechanism. In the special case of $c \to \infty$, it implies an integro-differential equation that is a special case of equation (1.1.1) in Lakshmikantham and Rao (1995) and their existence results can be applied.

R accepts or rejects. Here, $a(\hat{\theta}) \in \{0,1\}$ is the binary audit decision, $k_n(\hat{\theta}) \geq 0$ and $k_a(\theta,\hat{\theta}) \geq 0$ are the investment levels, and $c_n(\hat{\theta}) \geq 0$ and $c_a(\theta,\hat{\theta}) \geq 0$ are the transfers to the division manager in the absence and presence of audit, respectively. If the division manager accepts the mechanism, she learns the project's quality θ, chooses report $\hat{\theta} \in \Theta$ to send, and headquarters behave as prescribed by the mechanism. The optimal mechanism maximizes headquarters' expected value subject to the division manager's truth-telling and participation constraints (the proof of Proposition 1 formulates the problem). It is characterized in the next proposition:

Proposition 1. Let $k_x^*(\theta) \equiv \arg\max_k \{V(k,\theta) - xk\}$ and $u > 0$ and $\mu \geq 0$ be constants defined in the appendix. The optimal mechanism in the one-shot version of the problem is as follows. The audit strategy is $a(\hat{\theta}) = 1$ if $\hat{\theta} \geq \hat{\theta}^*$ and $a(\hat{\theta}) = 0$ if $\hat{\theta} < \hat{\theta}^*$. In the audit region ($\hat{\theta} \geq \hat{\theta}^*$), the investment and transfers are $k_a(\hat{\theta},\hat{\theta}) = k^*_{1-\mu\gamma}(\hat{\theta})\ \forall \hat{\theta}$, $k_a(\theta,\hat{\theta}) = 0$ for $\theta \neq \hat{\theta}$, and $c_a(\theta,\hat{\theta}) = 0\ \forall \theta,\hat{\theta}$. In the no-audit region ($\hat{\theta} < \hat{\theta}^*$), the investment and transfers are $k_n(\hat{\theta}) = \min\left\{k^*_{1-\gamma}(\hat{\theta}), \frac{u}{\gamma}\right\}$ and $c_n(\hat{\theta}) = u - \gamma k_n(\hat{\theta})$.

The intuition is as follows. The division manager must get the same utility u for all reports that are not audited, or she would not have incentives to send truthful reports. If the reported quality $\hat{\theta}$ is low enough, it is optimal to undertake the efficient investment $k^*_{1-\gamma}(\hat{\theta})$ and pay the difference $u - \gamma k^*_{1-\gamma}(\hat{\theta})$ as a transfer. If the reported quality is high, undertaking the efficient investment and providing utility u to the division manager requires that she pays a transfer to headquarters, which is not feasible since the division manager is moneyless. In this case, investment is distorted from $k^*_{1-\gamma}(\hat{\theta})$ to $\frac{u}{\gamma}$ and the transfer equals zero. As the project's quality increases, this underinvestment becomes costlier. Thus, there exists threshold $\hat{\theta}^*$, such that it is optimal to audit all reports above it. The investment in audited projects is $k^*_{1-\mu\gamma}(\theta)$, where μ is the Lagrange multiplier in the headquarters maximization problem for the participation constraint of the division manager. The threshold $\hat{\theta}^*$ is the type θ at which headquarters are indifferent between paying the audit cost c and investing $k^*_{1-\mu\gamma}(\theta)$ and not paying the audit cost but investing $\frac{u}{\gamma}$.[17]

As we shall see, the dynamic model shares with the static benchmark the intuition that the optimal audit policy is given by a threshold rule because the investment distortion caused by lack of audit is increasing in the quality of the project. However, it augments it with two new economic effects that lead to optimality of the spending account and separability of financing, where large projects are financed by headquarters without drawing on the division manager's account.

2. SOLUTION OF THE MODEL

This section solves for the optimal direct mechanism using martingale techniques similar to those in DeMarzo and Sannikov (2006). The idea is to summarize all relevant history using a single state variable and show that its evolution represents the manager's incentives. Then, I show how the optimal policies can be implemented with the budgeting mechanism with threshold separation of financing.

2.1. *Incentive compatibility*

By standard arguments, it is optimal to punish the division manager as much as possible if the audit reveals that her report is not truthful. Intuitively, because lying never occurs in equilibrium,

17. This condition is represented by equation (19) in the Appendix holding as equality.

there is no cost of imposing the maximum punishment. Hence, $dK_t = 0$ and $dC_t = 0$ for any t following a non-truthful report revealed by the audit. Given this, in what follows, I focus only on histories in which audit always confirms reports of the division manager. Then, the past history can be summarized using only the report process $(d\hat{X}_t)_{t\geq 0}$. When deciding what report $d\hat{X}_t$ to send, the division manager evaluates how it will affect her expected utility. Let $W_t(\hat{X})$ denote the expected future utility of the division manager at time t after history $\{d\hat{X}_s, s \leq t\}$, conditional on reporting truthfully in the future:

$$W_t(\hat{X}) = \mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(s-t)}(\gamma dK_s + dC_s)\right]. \tag{5}$$

In other words, $W_t(\hat{X})$ is the expected future utility that the mechanism "promises" to the division manager at time t following history $\hat{X}$, that is, the promised utility. The evolution of $(W_t)_{t\geq 0}$ can be represented in the following way. By definition, the lifetime discounted expected utility of the division manager from the relationship[18] is a martingale. By the martingale representation theorem, there exists an $\mathcal{F}$-predictable function $H_t(d\hat{X}_t)$ that captures the difference between the change in the division manager's total time-t expected utility from sending report $d\hat{X}_t$ and from sending report $d\hat{X}_t = 0$. Using this function, we can represent the evolution of $(W_t)_{t\geq 0}$ as follows:

Lemma 1. At any time $t \geq 0$, the evolution of the division manager's promised utility W_t following report $d\hat{X}_t \in \{0\} \cup \Theta$ is

$$dW_t = \rho W_{t-}dt - \gamma dK_t - dC_t + H_t(d\hat{X}_t) - \left(\lambda \int_{\underline{\theta}}^{\bar{\theta}} H_t(\theta) dF(\theta)\right) dt, \tag{6}$$

where $W_{t-} \equiv \lim_{s\uparrow t} W_s$ denotes the left-hand limit of equation (5) and $H_t(d\hat{X}_t)$ denotes the sensitivity of the division manager's time-t expected utility, which includes promised utility W_t as well as immediate utility $\gamma dK_t + dC_t$, to her report. Function $H_t(d\hat{X}_t)$ satisfies: (1) $H_t(0) = 0$; (2) for any fixed $\theta \in \{0\} \cup \Theta$, $H_t(\theta)$ is $\mathcal{F}$-predictable.

Representation (6) underlies the subsequent derivation of the optimal mechanism, so it deserves a detailed explanation. To see the intuition better, rewrite equation (6) as

$$dW_t + \gamma dK_t + dC_t - \rho W_{t-}dt = H_t(d\hat{X}_t) - \left(\lambda \int_{\underline{\theta}}^{\bar{\theta}} H_t(\theta) dF(\theta)\right) dt. \tag{7}$$

When the division manager reports $d\hat{X}_t$, she gets an immediate utility of $\gamma dK_t + dC_t$ and a change in the promised future utility of dW_t, where all of them potentially depend on the report. The martingale condition states that the sum of these terms less the discounting adjustment $\rho W_{t-}dt$, shown on the left-hand side of equation (7), is zero in expectation. Function $H_t(d\hat{X}_t)$ represents the total jump in the division manager's utility from report $d\hat{X}_t$ relative to $d\hat{X}_t = 0$. Because the expectation of the left-hand side of equation (7) is zero, the right-hand subtracts from $H_t(d\hat{X}_t)$ the expected jump in the division manager's utility, which equals the expected jump in utility conditional on project arrival $(\int_{\underline{\theta}}^{\bar{\theta}} H_t(\theta) f(\theta) d\theta)$ times the probability of project arrival over a

18. That is, $\int_0^t e^{-\rho s}(\gamma dK_s + dC_s) + e^{-\rho t} W_t(\hat{X})$.

short period of time (λdt). Condition $H_t(0)=0$ states that if the division manager reports that no project is available, then the evolution of her lifetime expected utility is continuous.[19]

In the optimal mechanism, the division manager finds it optimal to send a truthful report: $d\hat{X}_t = dX_t$. When she decides what report to send, she cares about the effect of the report on her total utility ($dW_t + \gamma dK_t + dC_t$). Depending on report $d\hat{X}_t$, headquarters either audit it or not. Let $D_t^A = \{d\hat{X}_t : dA_t = 1\}$ and $D_t^N = \{d\hat{X}_t : dA_t = 0\}$ be the "audit" and "no audit" regions of reports at time t, respectively. Because any non-truthful report $d\hat{X}_t \in D_t^A$ is revealed in the audit and leads to maximum punishment of the division manager, she never finds it optimal to report $d\hat{X}_t \in D_t^A$ unless $d\hat{X}_t = dX_t$. Therefore, it is sufficient to examine the incentive to report dX_t over sending a different report $d\hat{X}_t$ from the "no audit" region D_t^N. These incentives can be written in terms of restrictions on function $H_t(d\hat{X}_t)$. Because audit is costly, it is never optimal to audit if the division manager reports that there is no project: $\{0\} \in D_t^N$. Consider incentives to send a truthful report when dX_t is in the "no audit" region D_t^N. To have incentives not to send a non-truthful report from D_t^N, any report from this region must have the same effect on the division manager's utility. Otherwise, the division manager would report $d\hat{X}_t$ with the best effect on her utility whenever any $dX_t \in D_t^N$ is realized. Since the division manager can claim that no project has arrived and that report is not audited, it must be that $H_t(d\hat{X}_t)=0$ for any report from the "no audit" region.[20]

Next, consider incentives to send a truthful report when dX_t is in the "audit" region D_t^A. To have incentives not to send a report from D_t^N, $H_t(d\hat{X}_t)$ must weakly exceed the effect of sending the report from the "no audit" region, that is, zero. Intuitively, the division manager must be weakly positively rewarded for sending a report that gets audited, and $H_t(d\hat{X}_t)$ measures the size of this "reward". If this condition were violated, the division manager would be better off sending a report that does not get audited. Since headquarters learn the state when the report is audited, the division manager cannot claim reward $H_t(d\hat{X}_t)$, unless $dX_t = d\hat{X}_t$. Thus, truth-telling does not impose any other restrictions on the size of rewards for audited reports. Lemma 2 summarized these conclusions:

Lemma 2. At any time $t \geq 0$, truth-telling is incentive compatible if and only if: (1) $\forall d\hat{X}_t \in D_t^N$: $H_t(d\hat{X}_t)=0$; and (2) $\forall d\hat{X}_t \in D_t^A : H_t(d\hat{X}_t) \geq 0$.

2.2. *Solution of the optimization problem*

Given the incentive compatibility conditions, I use the dynamic programming approach to solve for the optimal policies subject to delivering the division manager any expected utility W and insuring that truth-telling is incentive compatible. The problem is solved in two steps. First, I fix the audit region and solve for optimal investment and compensation. Then, I optimize over audit strategies. I present a heuristic argument here and verify it in the proof of Proposition 2 in the Appendix.

Let $P(W)$ denote the value that headquarters obtain in the optimal mechanism that delivers expected utility W to the division manager. Because a mechanism can specify randomization between any two levels of the division manager's promised utility, $P(W)$ must be (weakly) concave. For simplicity, my description here is presented based on the assumption that the optimal

19. As mentioned above, equation (6) describes the evolution of W_t only along histories in which audit confirms reports of the division manager. If the audit at time t reveals that the manager's report is not truthful, her expected future utility jumps down to zero.

20. In my setup, I assume that headquarters do not observe whether a project has arrived, which seems a more plausible assumption than the alternative of publicly observed project arrival. If the arrival of projects were observed, the argument would be identical if $\lim_{\theta \downarrow \underline{\theta}} V_k(0,\theta)=0$, that is, if very low-quality projects led to an infinitesimal investment. Otherwise, the truth-telling restriction would be less strict than Lemma 2.

mechanism does not involve randomization, but this assumption is without loss of generality: As I show in the proof of Proposition 2, the optimal mechanism does not feature randomization and $P(W)$ is strictly concave (for $W<W^c$, defined in the next paragraph).

Let W^c be the lowest W at which $P'(W)=-1$.[21] Because headquarters can compensate the division manager with immediate payments, it must be that $P'(W)\geq -1$ for any W. By concavity of $P(W)$, the optimal compensation policy follows the threshold rule: headquarters make payments to the division manager if and only if her promised utility is at least W^c. Thus, the optimal compensation $\{C_t : t>0\}$ is the unique process such that W_t is at most equal to W^c for all $t>0$ and that $dC_t>0$ only when $W_t \geq W^c$. This property is standard to other optimal long-term contracting models with risk-neutral parties.[22] It states that it is cheaper to compensate the division manager with promises when her promised utility is low and with monetary payments when her promised utility is high. Because of this compensation policy, W_t never exceeds W^c, except for the starting point if $W_0>W^c$.

Consider region $W<W^c$. The expected instantaneous change in headquarters' value function is $rP(W_{t-})dt$. It must be equal to the sum of the expected flow of value over the next instant and the expected change in $P(W)$ due to the evolution of W. Since zero investment is optimal if the manager reports that no project arrives, the expected flow of value over the next instant is

$$\lambda dt \int_{\underline{\theta}}^{\bar{\theta}} (V(dK_t,\theta)-dK_t-cdA_t)dF(\theta). \tag{8}$$

To evaluate the expected instantaneous change in $P(W)$, I apply Itô's lemma (*e.g.* Shreve, 2004) and use equation (6):

$$\begin{aligned}\mathbb{E}\left[dP(W_{t-})\right]=&\left[\rho W_{t-}dt-\left(\lambda\int_{\underline{\theta}}^{\bar{\theta}} H_t(\theta)dF(\theta)\right)dt\right]P'(W_{t-})\\ &+\lambda dt\int_{\underline{\theta}}^{\bar{\theta}}\left[P(W_{t-}+H_t(\theta)-\gamma dK_t)-P(W_{t-})\right]dF(\theta).\end{aligned} \tag{9}$$

The first term in equation (9) corresponds to the drift of W, reflecting a change in $P(W)$ if no project arrives. The second term in equation (9) corresponds to the jump in $P(W)$ due to a potential arrival of a project. In particular, when a project of quality θ arrives, the division manager's promised utility changes from W_{t-} to $W_{t-}+H_t(\theta)-\gamma dK_t$, corresponding to the integrand in the second term in equation (9). Note that because there are no monetary transfers in region $W<W^c$, dC_t is absent from equation (9). Combining equations (9) with (8), equating their sum to $rP(W_{t-})dt$, and omitting the time subscript yields the following HJB equation on headquarters' value function $P(W)$ for any $W<W^c$:

$$\begin{aligned}rP(W)=\max_{\{k_\theta,a_\theta,h_\theta\}_{\theta\in\Theta}}\Bigg\{&\lambda\int_{\underline{\theta}}^{\bar{\theta}}(V(k_\theta,\theta)-k_\theta-ca_\theta)dF(\theta)\\ &+\left[\rho W-\left(\lambda\int_{\underline{\theta}}^{\bar{\theta}} h_\theta dF(\theta)\right)\right]P'(W)\\ &+\lambda\int_{\underline{\theta}}^{\bar{\theta}}\left[P(W+h_\theta-\gamma k_\theta)-P(W)\right]dF(\theta),\end{aligned} \tag{10}$$

21. $W^c=\infty$ if $P'(W)>-1$ for all W.

22. For example, Clementi and Hopenhayn (2006), DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007a,b), Piskorski and Tchistyi (2010), Tchistyi (2016), and others.

where the maximization is subject to the constraints $k_\theta \geq 0, a_\theta \in \{0,1\}$, and the incentive compatibility constraints $h_\theta \geq 0$ if $a_\theta = 1$ and $h_\theta = 0$ if $a_\theta = 0$. As W is a parameter in this optimization problem, the optimal policies are functions of W and θ: $k^*(\theta, W)$, $a^*(\theta, W)$, and $h^*(\theta, W)$. They imply investment dK_t, audit dA_t, and reward $H_t(\cdot)$ for each state variable W_{t-} and report $\hat{\theta}$: $dK_t = k^*(\hat{\theta}, W_{t-}), dA_t = a^*(\hat{\theta}, W_{t-})$, and $H_t\left(\hat{\theta}\right) = h^*(\hat{\theta}, W_{t-})$.

Next, I derive the properties of $k^*(\theta, W), a^*(\theta, W)$, and $h^*(\theta, W)$, and the implied evolution of W under the optimal mechanism. First, consider report θ that is not audited. In this case, as we see from equation (6) and Lemma 2, $h^*(\theta, W) = 0$ and thus $dW_t = -\gamma k^*(\theta, W)$:[23]

Property 1. *Under the optimal mechanism, if the division manager's report of project θ is not audited, then the division manager's promised utility W_t declines by the amount of private benefits he gets from the investment: $dW_t = -\gamma k^*(\theta, W)$.*

This property reflects the first intuition from the introduction. Intuitively, if the report is not audited, punishing the manager in the future is the only way to provide incentives not to overstate the prospects of an investment project. Since $dW_t = -\gamma k^*(\theta, W)$, the appropriate punishment lowers future promises to the division manager exactly by the amount of private benefits she gets from investment today. As the next section will show, a dynamic spending account will implement precisely this reduction of future promises to the division manager.

Secondly, consider report θ that is audited, $a_\theta = 1$. The derivative of equation (10) with respect to h_θ is proportional to $-P'(W) + P'(W + h_\theta - \gamma k_\theta)$. This reflects two effects of a higher h_θ. First, it increases the promised utility of the division manager after the investment in the project of type θ. Secondly, it decreases the drift of W. It follows that $h^*(\theta, W) = \gamma k^*(\theta, W)$ and thus $dW_t = 0$, which is summarized in the following property:

Property 2. *Under the optimal mechanism, if the division manager's report of project θ is audited and found to be truthful, then the division manager's promised utility W_t stays unchanged: $dW_t = 0$. If the report is found to be non-truthful, then $dW_t = -W_{t-}$.*

This property reflects the second intuition from the introduction. Lowering the division manager's promised utility by the amount of private benefits is necessary to provide incentives to report the project's type truthfully when the report is not audited. However, if the report is audited, then there is no need to lower the promised utility, because audit already ensures that the division manager does not misreport the project's quality. Given this, it is optimal to keep the spending account balance unchanged whenever the division manager's report is audited and confirmed to be truthful.[24] Note that concavity of headquarters' value function is key for this result. Strict concavity of the value function comes from the assumption that the marginal return on an additional dollar of investment in each product is decreasing in the investment size. Intuitively, higher promised utility of the division manager increases investment in future projects, but this additional investment generates a lower marginal return, which leads to the concavity of headquarters' value function.

Properties 1 and 2 allow to derive optimal investment $k^*(\theta, W)$ for each project type θ and promised utility W in the "audit" and "no audit" regions. Differentiating equation (10) with respect

23. As we see from equation (6), the third and forth terms equal zero, because $dC_t = 0$ and $H_t\left(d\hat{X}_t\right) = h^*(\theta, W_t) = 0$. Since, in addition, the first and last terms are infinitesimal and $\gamma dK_t = \gamma k^*(\theta, W_t)$ is lumpy, we have $dW_t = -\gamma k^*(\theta, W_t)$.

24. The unconstrained optimal choice of h_θ is $h_\theta = \gamma k_\theta$, which implies $dW_t = \gamma k^*(\theta, W_{t-}) - \gamma k^*(\theta, W_{t-}) = 0$. This unconstrained optimal choice always satisfies the constraint $h_\theta \geq 0$.

to k_θ yields

$$\frac{\partial V(k_\theta,\theta)}{\partial k}=1+\gamma P'(W-\gamma k_\theta),\text{ if } a_\theta=0, \tag{11}$$

$$\frac{\partial V(k_\theta,\theta)}{\partial k}=1+\gamma P'(W),\text{ if } a_\theta=1. \tag{12}$$

According to equations (11) and (12), the optimal investment policy maximizes firm value subject to the financing constraint implied by the agency problem. The left-hand sides of (11) and (12) capture the headquarters' marginal value from investing a dollar in the project. The right-hand sides capture headquarters' shadow cost of investing a marginal dollar in the project. It is determined by the division manager's promised utility after the investment: W, if the project is audited, and $W-\gamma k_\theta$, if not. For example, if the project is audited and $W=W^c$, then headquarters' shadow cost of investing a dollar is $1-\gamma$. It is below one by the division manager's private benefit γ, because investing a dollar allows headquarters to reduce the division manager's compensation by γ. Otherwise, headquarters' shadow cost of investing a marginal dollar is above $1-\gamma$.

Thus, the optimal investment policy is: $k^*(\theta,W)=k^n(\theta,W)$ if $a^*(\theta,W)=0$, and $k^*(\theta,W)=k^a(\theta,W)$ if $a^*(\theta,W)=1$, where $k^n(\theta,W)$ and $k^a(\theta,W)$ solve equations (11) and (12), respectively.[25] It satisfies three natural properties. First, $k^n(\theta,W)$ and $k^a(\theta,W)$ are increasing in θ. Secondly, $k^n(\theta,W)$ and $k^a(\theta,W)$ are increasing in W. Headquarters' shadow cost of investing a marginal dollar is determined by the slope of the value function at the post-investment promised utility, as shown on the right-hand sides of equations (11) and (12). It is infinite when W approaches zero, because the marginal return on the first dollar of investment in each project is infinite.[26] As a consequence, $\gamma k^n(\theta,W)<W$, that is, the post-investment promised utility is always positive. Finally, $k^a(\theta,W)>k^n(\theta,W)$: other things equal, investment is higher if the project is audited. The intuition can be seen from equations (11) and (12). If the project is not audited, truth-telling requires that the division manager's post-investment promised utility declines by the amount of private benefits consumed from investment. In contrast, post-investment promised utility is kept constant if the project is audited. Lower promised utility implies higher effective marginal cost of investment for headquarters: for any W and $k_\theta>0$, the right-hand side of equation (11) is higher than that of equation (12). Thus, $k^a(\theta,W)>k^n(\theta,W)$.

An example of headquarters' value function $P(W)$ is shown in Figure 1(a). It has an inverted U-shaped form. When the division manager's promised utility W is low, investment is low because headquarters' shadow cost of investment is high. When the division manager's promised utility is low enough, a marginal increase in it increases headquarters' value. Point W^* denotes the promised utility at which headquarters' value is maximized. When $W>W^c$, it is optimal to compensate the division manager with payments. Hence, the slope of headquarters' value function at these points is -1.

It remains to solve for the optimal audit policy. From equation (10), $a^*(\theta,W)=1$ if and only if

$$\begin{aligned}&V\big(k^a(\theta,W),\theta\big)-k^a(\theta,W)-\gamma k^a(\theta,W)P'(W)\\&-\big[V\big(k^n(\theta,W),\theta\big)-k^n(\theta,W)+P\big(W-\gamma k^n(\theta,W)\big)-P(W)\big]\geq c.\end{aligned} \tag{13}$$

25. Concavity of $V(k_\theta,\theta)$ implies that the left-hand sides of equations (11) and (12) are strictly decreasing in k_θ. Concavity of $P(W)$ implies that their right-hand sides are, respectively, strictly increasing and constant in k_θ. Thus, each equation has at most one solution. Assumptions $\lim_{k\to 0}V_k(k,\theta)=\infty$ and $\lim_{k\to\infty}V_k(k,\theta)=0$ ensure that the solution to each equation exists and is positive.

26. See the proof of this result in the Appendix. Intuitively, $\lim_{W\to 0}P'(W)=\infty$, because headquarters earn an infinite return on an infinitesimal investment that will arrive as a Poisson event.

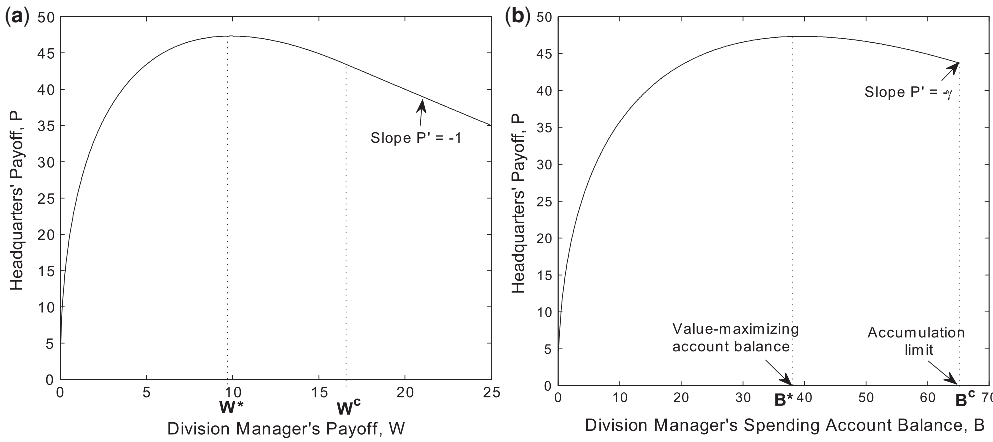

FIGURE 1

Headquarters' value as a function of the manager's promised utility and spending account balance. (a) plots headquarters' payoff P as a function of W, the division manager's promised utility, under the optimal direct mechanism. (b) plots headquarters' payoff P as a function of the division manager's spending account outstanding balance under the optimal direct mechanism. The parameters are $\gamma=0.25, r=0.1, \rho=0.12, \lambda=4, c=1, V(k,\theta)=10\theta\sqrt{k}, F(\theta)=\theta^{\frac{1}{10}}$ on $[0,1]$.

This inequality reflects the third intuition from the introduction. The decision to audit a project reflects the trade-off between the cost of audit (the right-hand side of equation (13)) and the cost from the increased investment distortion, which headquarters bear if the project is not audited. To see how it maps into the left-hand side of equation (13), it is convenient to introduce $\tilde{V}(k,\theta,W)\equiv V(k,\theta)-k\left(1+\gamma P'(W)\right)$ and think about $1+\gamma P'(W)$ as headquarters' current shadow cost of investing a dollar. Then, equation (13) can be decomposed into two parts:

- $\max_k \tilde{V}(k,\theta,W)$ - $\tilde{V}(k^n(\theta,W),\theta,W)$—the loss of value to headquarters due to underinvestment ($k^n(\theta,W)<k^a(\theta,W)$) in the current project, if it is not audited;
- $\int_{-\gamma k^n(\theta,W)}^{0}\left(P'(W+x)-P'(W)\right)dx$—the loss in value due to a higher shadow cost of investing if the current project is not audited.

The first part is similar to the cost of not auditing a project (underinvestment) in the static model of Section 1.1. The second part is new. It captures the increase in the investment distortion in future projects due to the decision of headquarters to not audit the current project. In the Appendix, I show that the cost of not auditing a project is strictly increasing in θ. Intuitively, if the quality of the project is higher, the investment is higher. In turn, higher investment leads to a higher shadow cost of investing a marginal dollar ($1+\gamma P'(W-\gamma k)$ is increasing in k), which determines the size of investment distortion. Since the cost of audit is constant and the cost of not auditing a project is strictly increasing in θ, it is optimal to audit the project if and only if its type or, equivalently, the amount of investment is sufficiently high. This intuition explains why small projects are not audited and get financed from the spending account, while large projects are audited and financed by headquarters directly. The result is summarized as:

Property 3. *Let $\theta^*(W)\in\Theta$ be a point at which (13) holds as equality, if it exists. If (13) holds strictly for all $\theta\in\Theta$, let $\theta^*(W)=\underline{\theta}$. If (13) does not hold for any $\theta\in\Theta$, let $\theta^*(W)$ be any value* *above $\bar{\theta}$. Then, the optimal audit strategy is $a^*(\theta,W)=0$ if $\theta<\theta^*(W)$, and $a^*(\theta,W)=1$ if $\theta\geq\theta^*(W)$.*

The above argument implies the following policies. If the manager reports that no project arrives, her promised utility accumulates continuously. If her promised utility is at threshold W^c, it does not accumulate and the manager gets paid a flow of constant bonus payments such that her promised utility is reflected at W^c. If a project with quality $\theta<\theta^*(W)$ is reported, then the report is not audited, the firm invests $k^n(\theta,W)$, and the promised utility falls by the amount of private benefits consumed from investment. Finally, if a project with quality $\theta\geq\theta^*(W)$ is reported, then the report is audited and, provided that the audit confirms the report, the firm invests $k^a(\theta,W)$ and does not change the manager's promised utility.

Plugging the optimal reward $H_t(\theta)=0$ if $\theta<\theta^*(W_{t-})$ and $H_t(\theta)=\gamma k^a(\theta,W_{t-})$ otherwise, and investment $dK_t=k^*(\theta,W_{t-})$ into equation (6) describing the evolution of the promised utility, we obtain the dynamics of the promised utility when no project arrives: $dW_t=g(W_{t-})W_{t-}dt$, where

$$g(W)=\rho-\lambda\int_{\theta^*(W)}^{\bar{\theta}}\frac{\gamma k^a(\theta,W)}{W}dF(\theta). \tag{14}$$

Intuitively, for the division manager to have correct intertemporal incentives, the flow of expected utility over the next instant must reflect her discount rate and be ρW_{t-}. Part of this flow comes from the growth in promised utility, and part comes from the fact that the division manager expects that a high-quality ($\theta\geq\theta^*(W_{t-})$) project may arrive over the next instant, in which case she will get private benefits $\gamma k^a(\theta,W_{t-})$ without a drop of promised utility. This term is subtracted from ρ to give the rate of growth in promised utility, which leads to equation (14).[27] Since the compensation policy is given by reflective barrier W^c by Property 1, it is optimal to not grow the promised utility when $W_{t-}=W^c$ and give an equivalent utility via monetary compensation, $dC_t=g(W^c)W^c dt$.

Finally, we need to verify that headquarters are better off under this mechanism than not starting the relationship at all and getting the payoff of zero. The maximum joint value that headquarters and the division manager can attain in the mechanism is $W^c+P(W^c)$.[28] Hence, headquarters are better off starting the relationship if and only if $R\leq W^c+P(W^c)$. The following proposition summarizes these findings:

Proposition 2. The following mechanism is optimal. If $R\leq W^c$, then the initial value W_0 is $\max\{R,W^*\}$, where $W^*=\arg\max P(W)$ and R is the outside option of the division manager. If $R\in(W^c,W^c+P(W^c)]$, then an immediate payment of $R-W^c$ is made to the division manager and $W_0=W^c$. At any t, the division manager sends a report $d\hat{X}_t$ from message space $\{0\}\cup\Theta$.

1. If $d\hat{X}_t=0$, then $dK_t=0$ and $dA_t=0$. If $W_{t-}<W^c$, then $dW_t=g(W_{t-})W_{t-}dt$ and $dC_t=0$. If $W_{t-}=W^c$, then $dW_t=0$ and $dC_t=g(W^c)W^c dt$.
2. If $d\hat{X}_t\in[\underline{\theta},\theta^*(W_{t-}))$, then $dK_t=k^n(d\hat{X}_t,W_{t-}), dA_t=0$, and $dW_t=-\gamma dK_t$.
3. If $d\hat{X}_t\in[\theta^*(W_{t-}),\bar{\theta}]$, then $dA_t=1$. If the audit confirms the report, $dK_t=k^a(d\hat{X}_t,W_{t-})$ and $dW_t=0$. If the audit does not confirm the report, $dK_t=0$ and $dW_t=-W_{t-}$.

Headquarters' value function $P(W)$ is strictly concave in the range $W\in(0,W^c)$.

27. Since $P(W)$ is concave, it is optimal to grow the promised utility continuously as opposed to in probabilistic lumps.

28. By strict concavity of $P(W)$, $W+P(W)<W^c+P(W^c)$ for any $W<W^c$, and $W+P(W)=W^c+P(W^c)$ for any $W>W^c$ because of the immediate monetary transfer of $W-W^c$ from headquarters to the division manager.

2.3. *Implementation*

The optimal mechanism from Proposition 2 has little resemblance to real-world capital allocation processes. Here, I show that a simpler mechanism, the budgeting mechanism with threshold separation of financing, is equivalent to the mechanism from Proposition 2 in the sense of implementing the same investment, audit, and compensation policies. I begin by defining such a mechanism:

Definition 1 (budgeting mechanism with threshold separation of financing) Headquarters allocate a spending account B_0 to the division manager at the initial date. The division manager can use the account at her discretion to invest in projects. At time $t \geq 0$ the spending account is replenished at rate $g_t : dB_t = g_t B_t dt$. In addition, there is a threshold on the size of individual investment projects, k_t^*, such that at any time t the division manager can pass the project to headquarters claiming $k^a(\theta, \gamma B_t) \geq k_t^*$, where θ is the quality of the current project. If the division manager passes the project, it gets audited. If the audit confirms that $k^a(\theta, \gamma B_t) \geq k_t^*$, then headquarters invest $k^a(\theta, \gamma B_t)$ and do not alter the account balance. If the audit reveals that $k^a(\theta, \gamma B_t) < k_t^*$, then headquarters punish the division manager by reducing her spending account balance to zero.

This mechanism has three features. First, all investment decisions are delegated to the division manager: she has full discretion to invest any amount in any project provided that she stays within the limit of her account ("budget"). Any investment reduces the remaining balance by the amount of investment. Secondly, the spending account is rigid, meaning that the division manager cannot get extra financing even if it leads to passing by profitable investment opportunities. Third, the mechanism gives the division manager an option to pass the project to headquarters claiming that it deserves investment above some pre-specified threshold. Upon the receipt of the project, headquarters audit it and, provided that audit confirms that the project deserves investment above the threshold, finance the project separately, that is, without changing the manager's account balance. Thus, this mechanism separates financing decisions by a threshold on the size of individual projects: if the division manager exercises the option to pass the project to headquarters if and only if it deserves investment above the threshold, then under this mechanism, investment and financing of all projects below the threshold is delegated to the division manager, while investment and financing of all projects above the threshold is centralized at the level of headquarters. The mechanism has three parameters: initial size B_0, replenishment rate g_t, and project size threshold k_t^*.

The following proposition establishes optimality of this mechanism:

Proposition 3. Consider a budgeting mechanism with threshold separation of financing with the following parameters: (1) the project size threshold of $k_t^* = k^a\left(\theta^*(\gamma B_t), \gamma B_t\right)$; (2) the replenishment rate of $g_t = g(\gamma B_t)$, if $B_t < B^c$, and $g_t = 0$, if $B_t = B^c$, where $B^c = W^c/\gamma$ and function $g(\cdot)$ is the drift of the division manager's promised utility, given by equation (14) in the previous section. Suppose that the monetary compensation of the division manager is $dC_t = 0$, if $B_t < B^c$, and $dC_t = g(\gamma B^c)\gamma B^c dt$, if $B_t = B^c$. Then, γB_t is equal to the division manager's promised utility W_t, and the division manager finds it optimal to (i) allocate the spending account between current and future investment opportunities in the way that maximizes headquarters' value, $V(dK_t, \theta) + P(\gamma(B_t - dK_t))$, where θ is the quality of the project at time t; (ii) pass a project to headquarters if and only if $k^a(\theta, \gamma B_t) \geq k_t^*$. If, in addition, the size of the initial spending account is $B_0 = W_0/\gamma$ and the immediate payment to the division manager is the same as in Proposition 2, then this mechanism is equivalent to the one in Proposition 2.

The intuition behind Proposition 3 is as follows. To provide incentives to invest appropriately, headquarters must either audit the report of the division manager or punish her by reducing her promised utility by the amount of private benefits that she obtains from current investment. If the project's quality is low, the latter tool is optimal and can be implemented using a dynamic spending account. Because investing from the account reduces its balance by the amount of investment, the spending account punishes the division manager in the future for high investment today. Moreover, the decrease in the division manager's promised utility is exactly equal to the amount of private benefits consumed from the current investment. As a consequence, the division manager is indifferent between all ways of allocating her account between the current and future investment projects. In particular, she has incentives to invest in the way that maximizes headquarters' value.

This spending account of the division manager is related to the firm's financial slack in models of investment and contracting based on dynamic agency. DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007) formalize financial slack as a line of credit, while Biais *et al.* (2007) and DeMarzo *et al.* (2012) formalize it as the firm's cash reserves. In these papers, outside investors restrict financial slack of the firm, and at each time it serves as a memory device of the past value-relevant information. In my article, unconstrained headquarters imposes financial constraints on the division manager via a spending account, and its balance plays the memory device role recording value-relevant information from past actions.

The incentive role of a spending account comes at a cost: higher current investment decreases the remaining budget for the future, which constrains future investment of the division. If the current investment is high enough, the increase in the financing constraint leads to a cost above the cost of audit. Hence, any project whose size exceeds a certain threshold is audited, even if the account balance exceeds investment. By concavity of the value function, additional distortions in the spending account balance cannot be beneficial ex-ante. Hence, if the project is audited, the spending account of the division manager remains unaffected—the project is financed without the use of the division manager's account at all. This outcome is implemented through giving the division manager an option to pass the project to headquarters claiming that the optimal investment exceeds the threshold. Because the division manager keeps the same spending account and obtains additional financing, she finds it optimal to pass the project to headquarters if it deserves investment above the threshold. At the same time, because all projects passed to headquarters are audited, the division manager has no incentives to pass the project to headquarters if the optimal investment is below the threshold. The optimal threshold is such that the audit policy implied by this mechanism coincides with the audit policy in Proposition 2.

An example of headquarters' value function as a function of the division manager's spending account balance is shown in Figure 1(b). Comparison of Figure 1(a) and 1(b) illustrates the one-to-one correspondence between the manager's spending account balance B and her promised utility W. Under the mechanism in this section, headquarters give the initial spending account to the manager. If the manager's initial required payoff R is below W^*, the size of the initial spending account is $B^* = W^*/\gamma$, the level at which headquarters' value is maximized. If R is above W^* but below W^c, the size of the initial spending account is R/γ. Finally, if R is above W^c, the initial account is $B^c = W^c/\gamma$ and the manager gets an upfront monetary payment of $R - W^c$. As time goes by, the spending account is replenished at the rate of $g(\gamma B_t)$. As investment projects below the threshold arrive, the account is replenished. If the account balance is at the accumulation limit B^c, it is no longer replenished, and the manager receives a flow of constant monetary payments instead.

2.4. *Properties of the optimal mechanism*

First, it is instructive to examine how investment under the optimal mechanism differs from the first-best investment, which maximizes the joint payoff $V(k,\theta)-(1-\gamma)k$. To see these results better, consider the implementation version of equations (11) and (12) that determine investment in the optimal mechanism:

$$\frac{\partial V\left(k_t^*,\theta\right)}{\partial k}=1+\gamma P'\left(\gamma\left(B_t-k_t^*\right)\right),\text{ if }\theta<\theta^*\left(\gamma B_t\right),\tag{15}$$

$$\frac{\partial V\left(k_t^*,\theta\right)}{\partial k}=1+\gamma P'\left(\gamma B_t\right),\text{ if }\theta>\theta^*\left(\gamma B_t\right).\tag{16}$$

As we see, optimal investment equates the marginal return from another dollar of investment in the project (the left-hand side) with the headquarters' shadow cost of investing another dollar in the project, which is determined by the post-investment balance of the spending account: the pre-investment account balance less the investment cost, if the project is not audited, and the pre-investment account balance, if the project is not audited. Figure 2(a) plots headquarters' shadow cost of investing a marginal dollar as a function of the post-investment division manager's spending account balance.[29]

Figure 2(b) plots the investment size as a function of project quality θ both under the first-best and under the optimal mechanism for different levels of the state variable B. The upper and lower solid lines plot investment size that maximizes the joint payoff and the payoff to headquarters only, respectively, and the other lines plot the constrained investment sizes. The discontinuity occurs at the cutoff θ^* above which projects are audited. The figure shows that there is underinvestment relative to the first-best level, unless the account balance is at the limit B^c and the project is audited, in which case investment is at the first-best level. Formally, underinvestment follows from equations (11) and (12): the right-hand side strictly exceeds $1-\gamma$, unless $B_t=B^c$ and $\theta>\theta^*(\gamma B^c)$. The degree of underinvestment is driven by headquarters' shadow cost of investing a marginal dollar, which is determined by the division manager's post-investment account balance. In particular, if $B<B^c$, there is underinvestment relative to the first-best level even in audited projects: it happens because the headquarters shadow cost of investing a marginal dollar in this case $(1+\gamma P'(\gamma B))$ exceeds the social cost of investing a marginal dollar $(1-\gamma)$.

It is also interesting to compare investment to another benchmark, the net present value (NPV) of the project excluding private benefits to the manager, $V(k,\theta)-k$. Compared to this benchmark, there is underinvestment if the post-investment account balance is below B^* (the range $P'(\cdot)>0$) but overinvestment if it is above B^* (the range $P'(\cdot)<0$). Figure 2(b) illustrates this result too.

I next examine the comparative statics of constrained optimal policies with respect to audit cost c and the division manager's private benefit γ. While I did not prove these comparative statics formally, they are quite intuitive and I found them across all numerical specifications I have tried. Figure 2(c) shows that the higher the cost of audit, the higher the optimal cutoff on project quality above which audit occurs. As a result, as Figure 2(d) demonstrates, the replenishment rate of the account is increasing in c. For the manager to have correct intertemporal incentives, her flow of expected utility must reflect her discount rate ρ. When no project is audited, the replenishment rate equals ρ. As more projects get audited and financed by headquarters, the replenishment rate decreases because the manager gets additional flow of utility from headquarters' investment.

29. Alternatively, one can add γ to both sides of equations (15) and (16) and write them as $\frac{\partial V(k_t,\theta)}{\partial k}+\gamma=1+\gamma\left(1+P'(\gamma B_{t+})\right)$. Now, the left-hand side is the marginal value from investment to both headquarters and the division manager. The dotted line of Figure 2(a) plots the right-hand side of this equation as a function of B_{t+}.

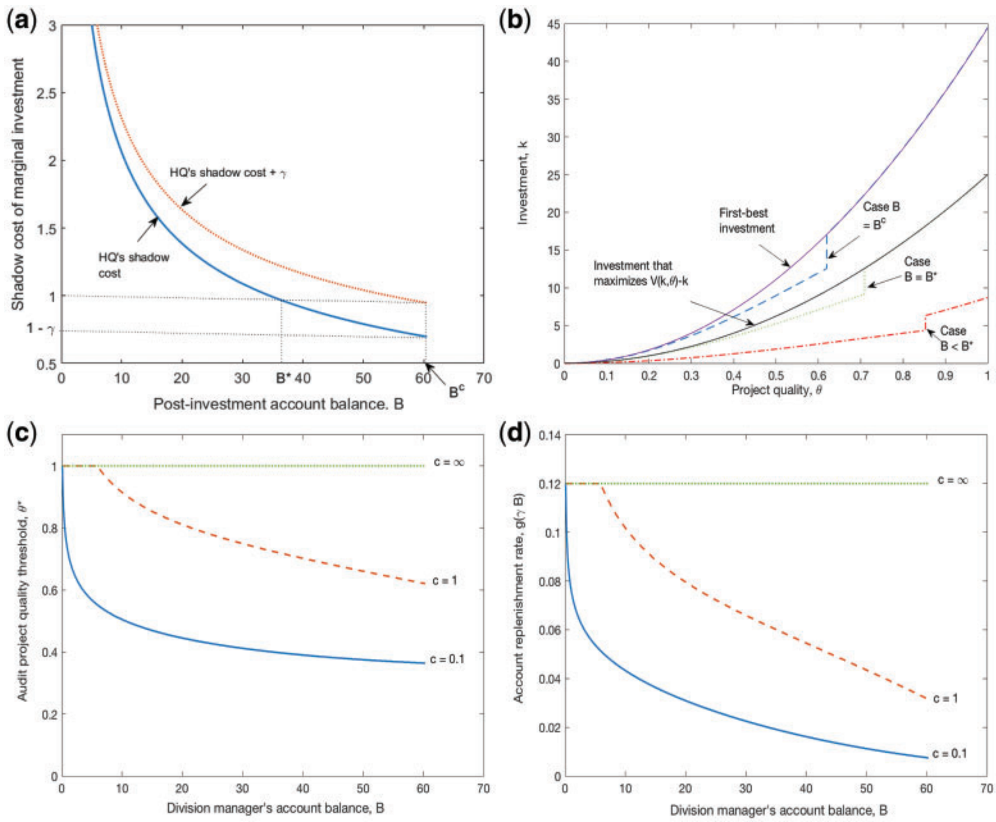

FIGURE 2

Properties of the optimal mechanism. (a) plots headquarters' shadow cost of a marginal dollar of investment (the solid line), $1+\gamma P'(\gamma B)$, and it shifted by γ (the dotted line) as functions of the post-investment account balance of the division manager. (b) plots investment size as a function of project quality θ for first-best investment and investment that maximizes the utility of headquarters (solid lines), and investment under the optimal contract for three cases, $B<B^c$ (dotted line), $B=B^c$, and $B>B^c$. (c) and (d) present comparative statics of the audit cutoff θ^* and the account replenishment rate with respect to the cost of audit c. The parameters are $\gamma=0.25, r=0.1, \rho=0.12, \lambda=4, c=1, V(k,\theta)=10\theta\sqrt{k}, F(\theta)=\theta^{\frac{1}{10}}$ on $[0,1]$.

Finally, the initial account balance, B_0, is weakly increasing with the manager's outside option R.[30] When it is very low, the manager's participation constraint does not bind, so the initial account balance is given by level B^* that maximizes headquarters' value. When R is in the intermediate range, the optimal B_0 is the minimum balance that satisfies the participation constraint, $\frac{R}{\gamma}$. Finally, when R is very high, the optimal B_0 is given by B^c and the manager's participation constraint is satisfied by paying her monetary transfers.

3. EXTENSIONS

The basic model makes a number of assumptions, among which the conceptual are: (1) auditing is non-random and perfect; (2) project values are not observed ex post and thus cannot be contracted

30. I do not depict this relationship as part of Figure 2 for brevity.

on; (3) investment of \$1 generates the same private benefit γ to the division manager regardless of the project; and (4) headquarters have commitment power. In this section, I relax assumptions (1) and (2) and analyse the consequences. In the conclusion, I also discuss how relaxing assumptions (3) and (4) affects the results.[31]

3.1. *Multiple audit technologies and co-financing*

The basic model assumes that the auditing technology is perfect and deterministic. This assumption is important for the result that there is separation of financing between the parties: all small projects are financed out of the division manager's spending account, while all large projects are fully financed separately by headquarters. In this section, I relax this assumption. I first extend the model for multiple deterministic audit technologies. I show that the optimal capital budgeting scheme exhibits co-financing of certain projects. Then, I consider an extension for random audit.

Specifically, consider the following extension to multiple audit technologies, keeping the assumption that random audit is not allowed. There are two audit technologies, 1 and 2. Technology 2 is similar to the one in the basic model: it reveals project type with certainty at cost $c_2 > 0$. Technology 1 costs $c_1 \in (0, c_2)$ but is less efficient: with probability p, it reveals project type with certainty (audit is successful); with probability $1-p$, it does not reveal anything (audit is unsuccessful). The model can similarly be extended to a general number N of audit technologies, such that the *nth* technology implies a higher probability of informative audit but also has a higher cost than the $(n-1)^{th}$ technology.

In Section II of the Online Appendix, I solve for the optimal direct mechanism and show that it admits the following spending account implementation (see Proposition **??**). As in the basic model, headquarters allocate a spending account to the division manager and allow to use it at her discretion. Unlike in the basic model, headquarters specify two thresholds on the size of individual projects, k_t^* and k_t^{**}, that separate the "no audit" region (small projects) from the "audit using technology 1" region (medium projects), and from the "audit using technology 2" region (large projects). As in the basic model, the thresholds are functions of the division manager's current account balance. Section 4 relates this result to the observed capital budgeting practices in corporations (*e.g.*, Ross, 1986).

Figure 3 illustrates how financing of projects depends on the size of the project. If at time t the division manager obtains a project with optimal investment above k_t^{**}, she passes it to headquarters, which then audit it using Technology 2 and finance it fully. If the division manager obtains a project with optimal investment between k_t^* and k_t^{**}, she passes it to headquarters, which then audit it using Technology 1. If audit is successful, headquarters finance the project fully. Interestingly, headquarters provide some financing for the project even if audit is not successful. In this case, the project is co-financed by the two parties or even financed fully by headquarters. The latter happens if the optimal investment level in a project is below a certain threshold k_t^{***}.

The intuition for the optimality of co-financing is as follows. Recall that headquarters have two tools to ensure that the division manager invests in the way that maximizes headquarters' value:

31. Another extension is to allow for persistent observable shocks to investment projects, for example, if the intensity of project arrival follows a two-state Markov regime-switching process. The analysis will be similar to Piskorski and Tchistyi (2010) and DeMarzo *et al*. (2012). The optimal mechanism will likely take the form of a budgeting mechanism in which the replenishment rate and audit threshold depend on the state. When the state switches, headquarters will adjust the manager's account balance to minimize the agency cost. Since investment is more important in the high state, it will be optimal to increase the account balance when the state switches into high and to decrease the balance when the state switches in the opposite direction.

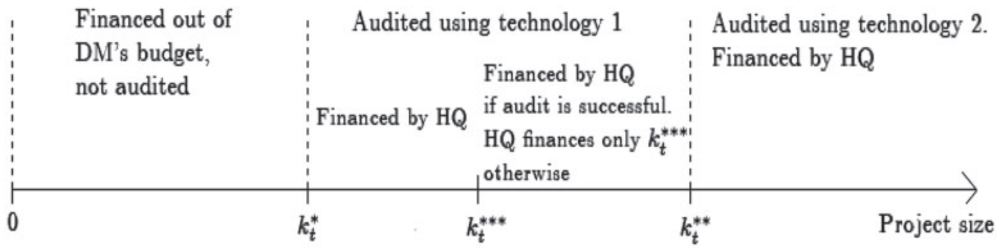

FIGURE 3
Optimal financing in a model with two audit technologies.

the threat of audit followed by the punishment for lying and restricting the manager's ability to invest in future projects. If the project is audited using Technology 2, the former tool of incentive provision has a limited effect, since headquarters could catch lying only with probability p. If the investment is high (between k_t^{***} and k_t^{**} in Figure 3), the potential payoff to the manager from lying and claiming this investment is high. Hence, headquarters must supplement audit with restricting the manager's ability to invest in future projects. This takes the form of requiring the division manager to co-finance the project with headquarters. In contrast, for low investments (between k_t^* and k_t^{***} in Figure 3), the benefits from lying are low and the threat of audit is a sufficient punishment. Hence, it is optimal for headquarters to fully finance these projects.

I analyse the extension to random audit in Section IV of the Online Appendix. Specifically, I solve for the optimal mechanism in the basic model assuming that headquarters can commit to any random audit strategy and show that it can be implemented using a budgeting mechanism that is similar to the one in this section. In particular, there is a threshold on project size, such that projects below this threshold are not audited and get financed from the division manager's spending account. Each project with a size above this threshold is audited with a positive (but below one) probability, which depends on the reported quality and the account balance. If the project is audited and the audit confirms the report, it is financed fully by headquarters and the division manager's account balance is kept constant. If the project is not audited, it is co-financed by headquarters and the division manager.

3.2. *Observable realized values*

The basic model assumes that headquarters do not observe any informative signals about realized project values. In this section, I explore how observability of project values affects the optimal mechanism.

Consider the following extension of the basic model. Suppose that an investment project with type θ and investment k has a binary realized output. With probability θ, it succeeds generates the value of $v(k)$ to headquarters, where $v(k)$ is an increasing and concave function satisfying $v(0)=0, \lim_{k\to 0} v'(k)=\infty$, and $\lim_{k\to\infty} v'(k)=0$. With probability $1-\theta$, the project fails and generates zero value. If the project succeeds, there are two possibilities. With probability $p\in[0,1]$, value $v(k)$ is realized as an immediate verifiable cash flow. With probability $1-p$, value $v(k)$ is realized as non-verifiable long-run value to headquarters, but the immediate verifiable cash flow is zero and thus cannot be distinguished from a project that fails. In this setup, parameter p captures the degree of information about project values.[32] In the limit case of $p=0$, this extension maps

32. Equivalently, I can assume that the project of quality θ succeeds with probability $p\theta$ and fails with probability $1-p\theta$. Both success and failure are observed immediately after the investment. If the project succeeds (fails), it generates the cash flow of $v(k)/p$ (zero).

into the basic model with $V(k,\theta)=\theta v(k)$. To keep the analysis tractable, I assume that audit is infinitely costly, $c=\infty$.

I begin by defining an extension of the budgeting mechanism in the basic model:

Definition 2 (performance-sensitive budgeting mechanism) Headquarters allocate a spending account with balance B_0 to the division manager at the initial date. At time $t\geq 0$ the spending account is replenished at the rate g_t: $dB_t=g_tB_tdt$. All projects are financed out of the allocated account and are at the discretion of the division manager. If the investment done at time t results in immediate verifiable success, the account gets increased by B_t^+, so that $dB_t=B_t^+$. If the investment fails, the account gets reduced by B_t^-, so that $dB_t=-B_t^-$.

The performance-sensitive budgeting mechanism augments the simple budgeting mechanism with the feature that the replenishment of the spending account can be contingent on the realized performance of investment projects. The division manager gets a bonus in the form of an addition to her account if her past investment resulted in the verifiable success. Similarly, she gets fined in the form of a reduction of her account if her past investment did not result in the verifiable success.

The next result establishes optimality of the performance-sensitive budgeting mechanism:

Proposition 4. Consider $p\in(0,1]$ and a performance-sensitive budgeting mechanism with the following parameters: (1) the replenishment rate of $g_t=g_{ov}(\gamma B_t)$, given by (A47) in the Online Appendix, if $B_t<B_{ov}^c$, and $g_t=0$ if $B_t\geq B_{ov}^c$, where $B_{ov}^c=W_{ov}^c/\gamma$, and W_{ov}^c is defined in the Online Appendix; (2) the bonus for verifiable success and fine for the lack of verifiable success are $B_t^+=B^+(k_t,B_t)$ and $B_t^-=B^-(k_t,B_t)$, where k_t is the division manager's investment at time t. Functions $B^+(\cdot),B^-(\cdot)$, and constant B_{ov}^c are defined in (A48)–(A50) in the Online Appendix. Suppose that the monetary compensation of the division manager is $dC_t=0$, if $B_t<B_{ov}^c, dC_t=g\left(\gamma B_{ov}^c\right)\gamma B_{ov}^c dt$, if $B_t=B_{ov}^c$. If B_t increases above limit B_{ov}^c due to a bonus, the division manager gets a one-time monetary payment of $dC_t=\gamma\left(B_t-B_{ov}^c\right)$ and the account balance reduces to B_{ov}^c. If, in addition, the size of the initial spending account is $B_0=\max\left\{R,W_{ov}^*\right\}/\gamma$, where W_{ov}^* is the division manager's promised utility at which the payoff of headquarters is maximized in the optimal direct mechanism, then this mechanism is optimal.

The intuition is as follows. The realized payoff from investment provides a noisy signal of the quality of the investment project. Thus, headquarters can curb the division manager's desire to overinvest by rewarding her for investment successes and punishing for the lack of verifiable successes. As in the basic model, promises of future payoffs can be implemented via the spending account, controlled by the division manager. The optimal reward $B^+(k,B_t)$ and fine $B^-(k,B_t)$ functions trade-off more efficient incentive provision for the current investment project against the cost of greater financial constraints in the future. Thus, the division manager's preferences about the project's quality are derived endogenously through contracting. Functions $B^+(k,B_t)$ and $B^-(k,B_t)$ are determined by solving a dynamic screening problem, outlined in the Online Appendix. When p converges to zero, the optimal performance-sensitive budgeting mechanism converges to the standard budgeting mechanism: Providing incentives via bonuses and fines for (lack) of verifiable success is no longer feasible, so headquarters does it only through constraining future investment activity in the form of a spending account.

4. EMPIRICAL PREDICTIONS

The model shows that the optimal dynamic capital budgeting system has two features. The first feature is a spending account that gets replenished over time and gives the manager the flexibility to draw on it to finance projects. Spending accounts are common in organizations. Examples include R&D accounts of research groups, investment budgets in corporations, budgets of loan officers, and research accounts in academic institutions. The second feature is the separation of decision-making. The model predicts that the optimal separation takes the form of a cutoff on the size of individual investment projects:

Prediction 1. Decisions over projects below a certain size are made by the divisional manager, while decisions over projects above a certain size are made by headquarters. The fraction of audited projects decreases in the costs of audit.

Such separation of authority is common. In a recent survey of capital allocation practices of 115 firms, Hoang *et al*. (2017) show that nearly all (97%) firms in their survey have formal investment thresholds that feature central approval for investments above the threshold. They also find that divisions get considerable discretion over smaller capital expenditures, estimating that 40% of overall capital expenditures are made without central approval. Similarly, in a typical firm in the sample of Ross (1986), a plant manager makes decisions on small projects, but passes larger projects to one of the upper levels of the organizational hierarchy.[33] The analysis in Section 3.1 suggests that when there are two audit technologies, the allocation of authority is given by two thresholds on project size, such that the smallest projects do not get audited, medium projects are audited using the less effective and cheaper technology, and the largest projects are audited using the more effective technology. One interpretation of the two audit technologies is audit by the CEO (more efficient and expensive) and by the investment committee, consistent with Ross (1986).

Connected to Prediction 1, the model predicts that the way projects are financed is tied to whether or not they get audited. It is optimal for headquarters to finance projects they audit either fully (in the basic model) or partially, but not finance projects they do not audit.

Prediction 2. Projects that are not audited are financed out of the division manager's spending account. Projects that are audited are, at least partially, financed by headquarters.

Whether it is optimal for headquarters to fully finance or co-finance large projects depends on the quality of the audit technology: as Section 3.1 demonstrates, full financing is optimal if the audit technology is good enough, and co-financing is optimal if the audit technology is poor.

Prediction 3. Given the same account balance, headquarters replenish the account at a slower rate if the audit cost is lower.

This prediction follows from equation (14) that determines the drift of the continuation utility under the optimal mechanism (the replenishment rate of the account in the capital budgeting implementation). Lower audit cost means that more projects get financed by headquarters. Because a higher fraction of future investment is expected to come from headquarters' direct financing, the division manager's spending account should be replenished at a slower rate to match the same total growth of utility. Note that Prediction 3 relies on the assumption that the division manager consumes private benefits irrespectively from whether the project is audited or not.

33. For other surveys with similar evidence, see Gitman and Forrester (1977), Slagmulder *et al*. (1995), Ryan and Ryan (2002), and Akalu (2003).

To test Predictions 1 and 3, one needs data on how costly audit is and, even better, random variation in the cost of audit. One interesting proxy for the cost of audit is proximity of headquarters to a given plant of the firm. Giroud (2013) uses the introduction of new airline routes as a plausibly exogenous shock to the cost of monitoring and finds that it leads to an increase in investment and productivity. If his research design was applied to the data on the internal resource allocation processes, such as survey evidence in Ross (1986) and Hoang *et al*. (2017), Predictions 1 and 3 would be credibly tested.

We next turn to the properties of investment implied by the model:

Prediction 4. Given similar quality, the project gets a higher investment if it is financed by headquarters than from the division manager's account.

This prediction follows from the comparison of equations (11) with (12): Given borderline quality $\theta^*(\gamma B_t)$ at which the project can be either audited or not, the project gets strictly lower investment if it is not audited and gets financed from the division manager's account. Intuitively, if the project is financed from the spending account, the post-investment account balance will be lower resulting in a higher marginal cost of investment and, consequently, lower investment. The jump in the investment curve on Figure 2(b) also illustrates this effect.

Prediction 5. All else equal, higher investment by the division today is associated with lower future investment of the division if the investment today is financed from the division manager's account but not if it is financed by headquarters.

Prediction 5 implies that the intertemporal correlation of investment levels depends on the source of financing of investment. Like Prediction 4, it follows from equations (11) and (12). Consider a division manager with a certain account balance B and hold all parameters of the model constant. If the division manager gets a project financed from the spending account ($\theta < \theta^*(\gamma B)$), then higher investment today lowers the post-investment balance of the spending account, making the division manager more financially constrained tomorrow and lowering future investment. In contrast, if the division manager's current project is financed by headquarters ($\theta > \theta^*(\gamma B)$), then higher investment today does not affect the post-investment balance of the spending account and hence, future investment.

5. CONCLUSION

The article studies optimal dynamic capital budgeting when the key problem is the agent's desire to overspend. My main result is that the optimal mechanism can be implemented as a combination of a dynamic spending account and a threshold on the size of individual projects that separates financing between the two parties. Headquarters allocate a spending account to the division manager, replenish it over time, and give the manager discretion to spend this account on projects. Headquarters also set a threshold on the size of projects, such that at any time the manager can pass the project to headquarters claiming that investment should exceed the threshold. In this case, headquarters audit it and finance it separately. Thus, the optimal mechanism features threshold separation: all small projects are financed out of the manager's spending account, while all large projects are passed to headquarters and financed independently. The analysis also highlights situations in which co-financing of projects is optimal and in which replenishment of the account is sensitive to past performance.

Several extensions can be worth pursuing. First, it can be interesting to consider a model with persistent private information. Fernandes and Phelan (2000) develop recursive techniques to study principal–agent problems with persistent private information, and the same tools can be applied to the setting in this article, if the base model is simplified by having finitely many, such as two, project qualities that follow the Markov chain.[34] Building on Fernandes and Phelan (2000) and related papers, we can summarize the relevant history in this case by a vector of state variables, which include the most recent reported type and the vector of the manager's continuation utilities, contingent on each report of the next project. Thus, the spending account balance is not a sufficient state variable in the optimal mechanism, leading to two questions. First, can a modified version of the budgeting mechanism with threshold division of financing be optimal? Secondly, do positive shocks to projects lead to the headquarters being more or less aggressive at auditing the projects?

Another extension would be to study a model in which each project is an investment option that can be delayed, which is a realistic feature of many capital budgeting applications. In unreported analysis, I study an investment timing analogue of the single-project model from Section 1.1.[35] The optimal mechanism is similar to the one in my basic model, once we interpret earlier investment as larger investment.

APPENDIX

Proof of Proposition 1 The optimal mechanism maximizes headquarters' expected value

$$\max_{\Gamma}\int_{\theta:a(\theta)=0}(V(k_n(\theta),\theta)-k_n(\theta)-c_n(\theta))\,dF(\theta)$$
$$+\int_{\theta:a(\theta)=1}(V(k_a(\theta,\theta))-k_a(\theta,\theta)-c_a(\theta,\theta)-c)\,dF(\theta)$$

subject to the division manager's truth-telling and participation constraints:

$$(1-a(\theta))(\gamma k_n(\theta)+c_n(\theta))+a(\theta)(\gamma k_a(\theta,\theta)+c_a(\theta,\theta))$$
$$\geq(1-a(\hat\theta))\left(\gamma k_n(\hat\theta)+c_n(\hat\theta)\right)+a(\hat\theta)\gamma k_a(\theta,\hat\theta)+c_a(\theta,\hat\theta)),$$
$$\int_{\theta:a(\theta)=0}(\gamma k_n(\theta)+c_n(\theta))dF(\theta)+\int_{\theta:a(\theta)=1}(\gamma k_a(\theta,\theta)+c_a(\theta,\theta))dF(\theta)\geq R,$$

where the first inequality must hold $\forall\theta,\hat\theta\in\Theta$.

Consider the region of audited reports $D^A=\{\theta:a(\theta)=1\}$. Since lying never occurs in equilibrium, it is optimal to impose maximum punishment for $\theta\neq\hat\theta$, $\hat\theta\in D^A$. Thus, $k_a\left(\theta,\hat\theta\right)=c_a\left(\theta,\hat\theta\right)=0$ in this case. Let $D^N\equiv\Theta\setminus D^A$. No type θ has no incentives to report $\hat\theta\in D^A$, $\hat\theta\neq\theta$, because of the maximal punishment. Type $\theta\in D^N$ has no incentive to misreport $\hat\theta\in D^N$ if and only if

$$\gamma k_n\left(\hat\theta\right)+c_n\left(\hat\theta\right)\leq\gamma k_n(\theta)+c_n(\theta). \tag{17}$$

Because equation (17) must hold for all $\theta,\hat\theta\in D^N$, there must exist u, such that $\gamma k_n(\theta)+c_n(\theta)=u\;\forall\theta\in D^N$. Finally, type $\theta\in D^A$ has no incentive to report $\hat\theta\in D^N$ if and only if $\gamma k_a(\theta,\theta)+c_a(\theta,\theta)\geq u$. Denoting $\tilde k(\theta)$and $\tilde c(\theta)$ to be $k_n(\theta)$ and $c_n(\theta)$, if $a(\theta)=0$, and $k_a(\theta,\theta)$ and $c_a(\theta,\theta)$, if $a(\theta)=1$, we can write the optimization problem of headquarters as:

$$\max\int_{\theta\in D^N}\left(V\left(\frac{u-\tilde c(\theta)}{\gamma},\theta\right)-\frac{u-\tilde c(\theta)}{\gamma}-\tilde c(\theta)\right)dF(\theta)$$
$$+\int_{\theta\in D^A}\left(V\left(\tilde k(\theta),\theta\right)-\tilde k(\theta)-\tilde c(\theta)-c\right)dF(\theta),$$

subject to $\gamma\tilde k(\theta)+\tilde c(\theta)\geq u$, if $a(\theta)=1$, and the participation constraint. Here, the maximization is over $\tilde c(\theta)\geq 0$, $\tilde k(\theta)\geq 0$, and $a(\theta)\in\{0,1\}$.

34. Fu and Krishna (2016) study a dynamic financial contracting model of Clementi and Hopenhayn (2006) but with persistent private information, applying tools that are similar (but not the same) to Fernandes and Phelan (2000).

35. This setup is related to Grenadier and Wang (2005) and Gryglewicz and Hartman-Glaser (2018), who study investment timing in different principal–agent models.

Constraint $\gamma\tilde{k}(\theta)+\tilde{c}(\theta)\geq u$, if $a(\theta)=1$ cannot bind in the optimal mechanism. If it did for some $\tilde{\theta}\in D^A$, the original mechanism would be dominated by the same mechanism but with $a\left(\tilde{\theta}\right)=0$, $k_n\left(\tilde{\theta}\right)=k_a\left(\tilde{\theta},\tilde{\theta}\right)$, and $c_n\left(\tilde{\theta}\right)=c_a\left(\tilde{\theta},\tilde{\theta}\right)$. This mechanism satisfies all IC constraints whenever the original mechanism does, has the same investment and transfers, but results in a lower cost of auditing. Therefore, I consider the problem that ignores constraint $\gamma\tilde{k}(\theta)+\tilde{c}(\theta)\geq u$, if $a(\theta)=1$. Define the Lagrangean as

$$\begin{aligned}\mathcal{L}\left(\tilde{\mathbf{k}},\tilde{\mathbf{c}},\mathbf{a},u|\mu,\lambda\right)=&\int_{\underline{\theta}}^{\bar{\theta}}\left(\begin{array}{c}a(\theta)\left(V\left(\tilde{k}(\theta),\theta\right)-\tilde{k}(\theta)-\tilde{c}(\theta)-c\right)\\+(1-a(\theta))\left(V(\frac{u-\tilde{c}(\theta)}{\gamma})-\frac{u-\tilde{c}(\theta)}{\gamma}-\tilde{c}(\theta)\right)\end{array}\right)dF(\theta)\\&+\mu\left(u+\int_{\underline{\theta}}^{\bar{\theta}}a(\theta)\left(\gamma\tilde{k}(\theta)+\tilde{c}(\theta)-u\right)dF(\theta)-R\right)+\int_{\underline{\theta}}^{\bar{\theta}}\lambda(\theta)\tilde{c}(\theta)dF(\theta),\end{aligned}\tag{18}$$

where $\mathbf{x}\equiv\{x(\theta),\theta\in\Theta\}$ for $x\in\left\{\tilde{k},\tilde{c},a\right\}$, μ is the Lagrange multiplier associated with the participation constraint of the division manager, and $\lambda(\theta)$ is the Lagrange multiplier associated with the limited liability constraint of type θ.[36] Taking the first-order conditions, we obtain $\tilde{k}(\theta)=k^*_{1-\mu\gamma}(\theta)$ and $\tilde{c}(\theta)=0$, if $a(\theta)=1$, and $\tilde{k}(\theta)=\min\left\{k^*_{1-\gamma}(\theta),\frac{u}{\gamma}\right\}$ and $\tilde{c}(\theta)=u-\gamma\tilde{k}(\theta)$, if $a(\theta)=0$. Next, consider the optimal choice of $a(\theta)$. As shown above, $\gamma\tilde{k}(\theta)+\tilde{c}(\theta)>u$ whenever $a(\theta)=1$. Therefore, $k^*_{1-\mu\gamma}(\theta)>\frac{u}{\gamma}$ whenever $a(\theta)=1$. Because of the possibility of transfers, $\mu\leq1$, which implies $k^*_{1-\gamma}(\theta)\geq k^*_{1-\mu\gamma}(\theta)>\frac{u}{\gamma}$, if $a(\theta)=1$. Therefore, $a(\theta)=0$ in the range $\theta:k^*_{1-\gamma}(\theta)\leq\frac{u}{\gamma}$. In the range $\theta:k^*_{1-\gamma}(\theta)>\frac{u}{\gamma}$, $a(\theta)=1$ if and only if

$$V\left(k^*_{1-\mu\gamma}(\theta),\theta\right)-(1-\mu\gamma)k^*_{1-\mu\gamma}(\theta)-V\left(\frac{u}{\gamma},\theta\right)+(1-\mu\gamma)\frac{u}{\gamma}\geq c.\tag{19}$$

By the envelope theorem, the derivative of the left-hand side is

$$V_\theta\left(k^*_{1-\mu\gamma}(\theta),\theta\right)-V_\theta\left(\frac{u}{\gamma},\theta\right)=\int_{\frac{u}{\gamma}}^{k^*_{1-\mu\gamma}(\theta)}V_{k\theta}(k,\theta)dk.$$

By part 3 of Assumption 1, it is negative if $k^*_{1-\mu\gamma}(\theta)<\frac{u}{\gamma}$, and positive otherwise. Since $a(\theta)=1$ implies $k^*_{1-\mu\gamma}(\theta)>\frac{u}{\gamma}$, we conclude that equation (19) has at most one solution in the range $\left[k^{*-1}_{1-\mu\gamma}\left(\frac{u}{\gamma}\right),\overline{\theta}\right]$. Therefore, $a(\theta)=0$, if and only if $\theta\leq\hat{\theta}^*(u,\mu)$, where $\hat{\theta}^*(u,\mu)$ is such that equation (19) holds as equality, or $\hat{\theta}^*(u,\mu)=\overline{\theta}$, if (19) is negative at $\overline{\theta}$. By the envelope theorem, the derivative of the left-hand side of equation (19) at $\hat{\theta}^*(u,\mu)$ in μ is $\gamma\left(k^*_{1-\mu\gamma}\left(\hat{\theta}^*(u,\mu)\right)-\frac{u}{\gamma}\right)>0$. The derivative of the left-hand side of equation (19) at $\hat{\theta}^*(u,\mu)$ in u is $-\frac{1}{\gamma}\left(V_k\left(\frac{u}{\gamma},\hat{\theta}^*(u,\mu)\right)-(1-\mu\gamma)\right)<0$. Therefore, $\hat{\theta}^*(u,\mu)$ is weakly increasing in u and weakly decreasing in μ, and strictly so in the range $\hat{\theta}^*(u,\mu)<\overline{\theta}$. It is also weakly increasing in c and strictly so in the range $\hat{\theta}^*(u,\mu)<\overline{\theta}$.

If the participation constraint is slack (*i.e.*, if R is low enough), then $\mu=0$ and u is determined from the first-order condition for maximizing $\mathcal{L}(\cdot)$ in u, which yields

$$\mathbb{E}\left[\max\left\{1-\gamma,V_k\left(\frac{u}{\gamma},\theta\right)\right\}|\theta\leq\hat{\theta}^*(u,0)\right]=1.\tag{20}$$

Since the left-hand side is continuous in u, taking values from infinity (for $u\to0$) to $1-\gamma$ (for $u\to\infty$), equation (20) has a solution, denoted u^*. If (20) has multiple solutions, let u^* denote the one that maximizes headquarters' value. Then, if $\theta\leq\hat{\theta}^*(u^*,0)$, then $a(\theta)=0$, $\tilde{k}(\theta)=\min\left\{k^*_{1-\gamma}(\theta),\frac{u^*}{\gamma}\right\}$, and $\tilde{c}(\theta)=u^*-\tilde{k}(\theta)$; if $\theta>\hat{\theta}^*(u^*,0)$, then $\tilde{k}(\theta)=k^*_1(\theta)$ and $\tilde{c}(\theta)=0$. Let R^*denote the impled expected payoff to the division manager:

$$R^*=u^*+\int_{\hat{\theta}^*(u^*,0)}^{\bar{\theta}}\left(\gamma k^*_1(\theta)-u^*\right)dF(\theta).$$

Then, this is the optimal mechanism if and only if $R\leq R^*$.

If the participation constraint binds ($R>R^*$), then $\mu>0$ and u are determined from the first-order condition maximizing $\mathcal{L}(\cdot)$ in u and the participation constraint, which yield:

$$\mathbb{E}\left[\max\left\{1-\gamma,V_k\left(\frac{u}{\gamma},\theta\right)\right\}|\theta<\hat{\theta}^*(u,\mu)\right]=1-\mu\gamma,$$

$$u+\int_{\hat{\theta}^*(u,\mu)}^{\bar{\theta}}\left(\gamma k^*_{1-\mu\gamma}(\theta)-u\right)=R.$$

36. I ignore constraint $\tilde{k}(\theta)\geq0$, since it is trivially satisfied by $\lim_{k\to\infty}\frac{\partial}{\partial k}V(k,\theta)=\infty\forall\theta\in\Theta$.

Denoting the solution by $\tilde{u}(R)$ and $\tilde{\mu}(R)$, the optimal policies in this case are: if $\theta \le \hat{\theta}^*(\tilde{u}(R), \tilde{\mu}(R))$, then $a(\theta)=0$, $\tilde{k}(\theta)=\min\left\{k^*_{1-\gamma}(\theta), \frac{\tilde{u}(R)}{\gamma}\right\}$, and $\tilde{c}(\theta)=\tilde{u}(R)-\tilde{k}(\theta)$; if $\theta > \hat{\theta}^*(\tilde{u}(R), \tilde{\mu}(R))$, then $a(\theta)=1$, $\tilde{k}(\theta)=k^*_{1-\tilde{\mu}(R)\gamma}(\theta)$, and $\tilde{c}(\theta)=0$.

Proof of Lemma 1 Note that $W_t(\hat{X})$ is also the division manager's expected future utility if $\left\{d\hat{X}_s, 0\le s\le t\right\}$ were the realizations that the division manager reported truthfully. Hence, without loss of generality, it is sufficient to prove the lemma for the case of truthful reporting by the division manager. Let $U_t(X)$ denote the lifetime expected utility of the division manager, evaluated conditionally on information available at time t:

$$U_t(X)=\int_0^t e^{-\rho s}(\gamma dK_s+dC_s)+e^{-\rho t}W_t(X). \tag{21}$$

By definition, process $U(X)=\{U_t(X)\}_{t\ge 0}$ is a right-continuous $\mathcal{F}$-martingale. By the martingale representation theorem for marked point processes (Theorem 1.13.2 on pages 25–26 in Last and Brandt (1995)), for any t there exists a function $h_t(\cdot)$, where $h_t(\theta)$ is $\mathcal{F}$-predictable for any fixed $\theta\in\Theta$, such that

$$dU_t=\begin{cases} -\left(\lambda\int_{\underline{\theta}}^{\bar{\theta}} h_t(\theta)f(\theta)d\theta\right)dt, & \text{if } t\neq T_n \text{ for any } n\ge 1,\\ h_t(\theta_n)-\left(\lambda\int_{\underline{\theta}}^{\bar{\theta}} h_t(\theta)f(\theta)d\theta\right)dt, & \text{if } t=T_n \text{ for some } n\ge 1.\end{cases} \tag{22}$$

For convenience, rescale $h_t(\cdot)$ by factor $e^{\rho t}$ and write as a function of $dX_t\in\{0\}\cup\Theta$, defining it to be zero if $dX_t=0$:

$$H_t(dX_t)=\begin{cases} 0, & \text{if } dX_t=0,\\ e^{\rho t}h_t(dX_t), & \text{if } dX_t\in\Theta.\end{cases} \tag{23}$$

Notice that $H_t(dX_t)$ is $\mathcal{F}$-predictable for any fixed $dX_t\in\{0\}\cup\Theta$, because $h_t(\theta)$ is $\mathcal{F}$-predictable for any fixed $\theta\in\Theta$ and $H_t(0)=0$. Then,

$$dU_t=e^{-\rho t}\left(H_t(dX_t)-\left(\lambda\int_{\underline{\theta}}^{\bar{\theta}} H_t(\theta)f(\theta)d\theta\right)dt\right). \tag{24}$$

From (21):

$$dU_t=e^{-\rho t}(\gamma dK_t+dC_t)-\rho e^{-\rho t}W_{t-}(X)+e^{-\rho t}dW_t(X). \tag{25}$$

Equating (25) with (24) and rearranging the terms yields (6).

Proof of Lemma 2 Consider any $dX_t\in D_t^N$. Report dX_t dominates any $d\hat{X}_t\in D_t^A$, because any report $d\hat{X}_t\in D_t^A$ leads to zero utility due to maximum punishment. Any report $d\hat{X}_t\in D_t^N$ instead of dX_t leads to the net gain of $H_t\left(d\hat{X}_t\right)-H_t(dX_t)$. Therefore, truthful reporting is optimal for all $dX_t\in D_t^N$ if and only if $H_t\left(d\hat{X}_t\right)-H_t(dX_t)\le 0\ \forall dX_t, d\hat{X}_t\in D_t^N$. Hence, $H_t(dX_t)$ must be constant $\forall dX_t\in D_t^N$. Because $\{0\}\in D_t^N$ and $H_t(0)=0$, $H_t(dX_t)=0\forall dX_t\in D_t^N$. Next, consider any $dX_t\in D_t^A$. Again, report dX_t dominates any $d\hat{X}_t\in D_t^A$ due to punishment. Report $d\hat{X}_t\in D_t^N$ instead of dX_t leads to the net gain of $H_t\left(d\hat{X}_t\right)-H_t(dX_t)$. Therefore, truthful reporting is optimal for all $dX_t\in D_t^A$ if and only if $H_t\left(d\hat{X}_t\right)-H(dX_t)\le 0\forall dX_t\in D_t^A, d\hat{X}_t\in D_t^N$. Because $H_t\left(d\hat{X}_t\right)=0\forall d\hat{X}_t\in D_t^N$, this condition is equivalent to $H_t(dX_t)\ge 0\forall dX_t\in D_t^A$.

Proof of the result that $\lim_{W\to 0}P'(W)=\infty$. Headquarters' value function must satisfy $P(W)\ge \frac{\lambda}{r+\lambda}\left(\int_{\underline{\theta}}^{\bar{\theta}} V\left(\frac{W}{\gamma},\theta\right)dF(\theta)-\frac{W}{\gamma}\right)-\frac{\rho W}{r+\lambda}$, because one policy that yields expected value of W to the division manager is: Invest $\frac{W}{\gamma}$ in the project (irrespectively of reported quality) when the division manager reports the arrival of a project for the first time and pay the division manager a flow of compensation ρW prior to that; after the division manager reports the arrival of a project for the first time, pay no compensation to him and never invest anything. The expected value from this policy to the division manager is W. The expected value of this policy to headquarters is as follows. First, headquarters get $E\left[V\left(\frac{W}{\gamma},\theta\right)\right]-\frac{W}{\gamma}$ at the arrival of the next investment opportunity, which arrives with intensity λ. The present value of this term is the first term of the right-hand side of the inequality. Second, headquarters pays a flow of ρW to the division manager until the arrival of the investment opportunity. Its expected value is

$$\int_0^\infty\left(\int_0^\tau e^{-rt}\rho W dt\right)\lambda e^{-\lambda\tau}d\tau=\frac{\lambda\rho}{r}W\int_0^\infty\left(1-e^{-r\tau}\right)e^{-\lambda\tau}d\tau=\frac{\rho W}{r+\lambda},$$

which is the second term of the right-hand side of the inequality. Because this is one possible policy, headquarters cannot do worse under the optimal policy than under this policy. Thus, $P(W)$ cannot be lower. We also know that $P(0)=0$, because the only admissible policy that yields zero continuation value to the division manager is to never invest and pay him anything. Hence,

$$P'(0)=\lim_{\varepsilon\to 0}\frac{P(\varepsilon)-P(0)}{\varepsilon}\geq\lim_{\varepsilon\to 0}\frac{\frac{\lambda}{r+\lambda}\left(\int_{\underline{\theta}}^{\bar{\theta}}V\left(\frac{\varepsilon}{\gamma},\theta\right)dF(\theta)-\frac{\varepsilon}{\gamma}\right)-\frac{\rho\varepsilon}{r+\lambda}}{\varepsilon}$$
$$=\frac{\lambda}{(r+\lambda)\gamma}\left(\int_{\underline{\theta}}^{\bar{\theta}}\lim_{\varepsilon\to 0}\frac{V\left(\frac{\varepsilon}{\gamma},\theta\right)}{\varepsilon/\gamma}dF(\theta)\right)-\frac{\frac{\lambda}{\gamma}+\rho}{r+\lambda}.$$

Since $\lim_{k\to 0}V_k(k,\theta)=\infty$, it follows that $\lim_{\varepsilon\to 0}\frac{V\left(\frac{\varepsilon}{\gamma},\theta\right)}{\varepsilon/\gamma}=\infty$ for any θ. Therefore, $P'(0)=\infty$.

Proof of Property 3 First, I show that the left-hand side of (13) is a strictly increasing function of θ. It equals $F^a(\theta,W)-F^n(\theta,W)$, where

$$F^a(\theta,W)\equiv\max_{k\in\mathbb{R}_+}\left\{V(k,\theta)-k+P(W)-\gamma kP'(W)\right\},\tag{26}$$

$$F^n(\theta,W)\equiv\max_{k\in\mathbb{R}_+}\{V(k,\theta)-k+P(W-\gamma k)-P(W)\}.\tag{27}$$

Applying the envelope theorem for parameter θ,[37]

$$\frac{d[F^a(\theta,W)-F^n(\theta,W)]}{d\theta}=\frac{\partial V(k^a(\theta,W),\theta)}{\partial\theta}-\frac{\partial V(k^n(\theta,W),\theta)}{\partial\theta}\tag{28}$$
$$=\int_{k^n(\theta,W)}^{k^a(\theta,W)}\frac{\partial^2V(k,\theta)}{\partial k\partial\theta}dk\geq 0,$$

because $k^a(\theta,W)\geq k^n(\theta,W)$, as follows from (11) and (12), and $\partial^2V(k,\theta)/\partial k\partial\theta>0$ by Assumption 1. Therefore, the left-hand side of (13) is a (weakly) increasing function of θ.

Second, I use this result to conclude that Property 3 holds. There are three cases. First, if the left-hand side of (13) is above c for all $\theta\in\Theta$, then it is optimal to audit all investment projects. Hence, $a^*(\theta,W)=1$ for any $\theta\geq\underline{\theta}=\theta^*(W)$. Secondly, if the left-hand side of equation (13) is below c for all $\theta\in\Theta$, then audit is not optimal for any project θ. Hence, $a^*(\theta,W)=0$ for any $\theta\leq\bar{\theta}<\theta^*(W)$. Finally, if the left-hand side of equation (13) is neither above nor below c for all $\theta\in\Theta$, then the result that the left-hand side of equation (13) is an increasing function of θ implies that there is a point $\theta^*(W)\in\Theta$ at which equation (13) holds as equality. In this case, the left-hand side of (13) is not higher than c (hence, $a^*(\theta,W)=0$) for all $\theta<\theta^*(W)$ and not lower than c (hence, $a^*(\theta,W)=1$) for $\theta\geq\theta^*(W)$.

Proof of Proposition 2 First, I prove that headquarters' value funcion implied by the mechanism is strictly concave in the range $W\in(0,W^c)$. Then, I verify that the direct mechanism described in the proposition indeed maximizes headquarters' value.

Differentiating both sides of the HJB equation (10), using the envelope theorem, and rearranging the terms yield

$$P''(W)\int_{\underline{\theta}}^{\bar{\theta}}k^a(\theta,W)1_{\{a^*(\theta,W)=1\}}dF(\theta)$$
$$+\int_{\underline{\theta}}^{\bar{\theta}}\left(P'(W)-P'(W-\gamma k_n(\theta,W))\right)1_{\{a^*(\theta,W)=0\}}dF(\theta)\tag{29}$$
$$=\frac{\rho-r}{\lambda}P'(W)+\frac{\rho}{\lambda}WP''(W).$$

Because $P(\cdot)$ is weakly concave, the left-hand side of (29) is non-positive. Therefore, the right-hand side is non-positive. Hence, $P''(W)<0$ for all $W:P'(W)>0$, *i.e.*, for all $W<W^*$. Thus, there is no randomization of the division manager's promised utility in the range $W\in(0,W^*)$. Next, we show that there is also no randomization in the range $W\in\left[W^*,W^c\right]$. By contradiction, suppose that there is randomization over some interval $[W_l,W_h]$, and consider the lowest such interval. Then, $P(W)=P(W_h)+d(W-W_h)$ for all $W\in[W_l,W_h]$, where $d=\frac{P(W_h)-P(W_l)}{W_h-W_l}\in(-1,0)$. Optimality of randomization implies that $rP(W)=rP(W_h)+rd(W-W_h)$ for any $W\in(W_l,W_h)$ exceeds the right-hand side of the HJB equation (10). Rearranging the terms, we have inequality:

$$l(W)=rP(W_h)+rd(W-W_h)-\rho Wd-\phi(W)>0,\tag{30}$$

where

$$\phi(W)=\lambda\int_{\underline{\theta}}^{\bar{\theta}}\max\left\{\max_{k_\theta}\tilde{\phi}_0(k_\theta,W),\tilde{\phi}_1\right\}dF(\theta),\tag{31}$$

37. The envelope theorem holds, because $V(k,\theta)$ is differentiable in both parameters and $P(W-\gamma k)$ is differentiable in k (*e.g.* Milgrom and Segal (2002)).

$$\tilde{\phi}_0(k_\theta, W) = V(k_\theta, \theta) - k_\theta + P(W - \gamma k_\theta) - P(W),$$

$$\tilde{\phi}_1 = \max_{k_\theta}\{V(k_\theta, \theta) - (1+\gamma d)k_\theta - c\}$$

defined over $W \in (W_l, W_h)$. We know that $\lim_{W\to W_l} l(W) = 0$ and $\lim_{W\to W_h} l(W) = 0$, because $P(W)$ satisfies (10) at the ends of the randomization interval. Since $l(W) > 0$ for $W \in (W_l, W_h)$, we have $\lim_{W\to W_l} l'(W) > 0$ and $\lim_{W\to W_h} l'(W) < 0$. Differentiating (30), $l'(W) = (r-\rho)d - \phi'(W)$, so we have $\lim_{W\to W_h}\phi'(W) > \lim_{W\to W_l}\phi'(W)$. If I show that $\phi(W)$ is concave, it will contradict this inequality. Thus, it is sufficient to establish concavity of $\phi(W)$. For this, let me show that $\tilde{\phi}_0(k_\theta, W)$ is concave. The Hessian of $\tilde{\phi}_0$ is:

$$\mathbf{H}_{\tilde{\phi}_0} = \left(\begin{pmatrix} V_{kk}(k_\theta,\theta) + \gamma^2 P''(W-\gamma k_\theta) & -\gamma P''(W-\gamma k_\theta) \\ -\gamma P''(W-\gamma k_\theta) & P''(W-\gamma k_\theta) \end{pmatrix}\right).$$

Thus,

$$(k_\theta \;\; W)\mathbf{H}_{\tilde{\phi}_0}\begin{pmatrix} k_\theta \\ W \end{pmatrix} = V_{kk}(k_\theta,\theta)k_\theta^2 + P''(W-\gamma k_\theta)(W-\gamma k_\theta)^2 < 0$$

by $V_{kk}(\cdot,\cdot) < 0$ and $P''(\cdot) \le 0$. Therefore, $\mathbf{H}_{\tilde{\phi}_0}$ is negative definite, so $\tilde{\phi}_0$ is concave. Concavity of $\tilde{\phi}_0(k_\theta, W)$ implies that $\max_{k_\theta}\tilde{\phi}_0(k_\theta, W)$ is concave in W. Hence, the integrand of equation (31) is concave in W for any θ. Thus, $\phi(W)$ is indeed concave in W. Hence, there is also no randomization in the range $W \in [W^*, W^c]$.

Next, I verify that the direct mechanism conjectured in the proposition indeed maximizes headquarters' value. The proof follows the logic of standard problems in optimal control theory. First, I show headquarters' value from any incentive compatible mechanism that delivers the initial expected value of W_0 to the manager is at most $P(W_0)$. Secondly, I argue that headquarters' value from the mechanism that satisfies the conditions of the proposition and delivers the initial expected value of W_0 to the division manager is $P(W_0)$.

Let G_t be defined as

$$G_t \equiv \int_0^t e^{-rs}(V(dK_s, dX_s)dN_s - dK_s - dC_s - cdA_s) + e^{-rt}P(W_t). \tag{32}$$

Consider an arbitrary direct mechanism satisfying incentive compatibility of truth-telling. Because any mechanism that wastes resources when there is no investment opportunity cannot be optimal, it is enough to restrict attention to mechanisms with $dK_t = 0$ if $d\hat{X}_t = 0$. The evolution of the division manager's expected future utility implied by the mechanism is given by equation (6). Applying Itô's lemma, multiplying by e^{rt}, and rearranging the terms,

$$\begin{aligned} e^{rt}dG_t = {} & V(dK_t, dX_t)dN_t - dK_t - cdA_t \\ & - \left(\lambda\int_{\underline{\theta}}^{\bar{\theta}}(V(dK_t,\theta) - dK_t - cdA_t)f(\theta)d\theta\right)dt \\ & + \left(\lambda\int_{\underline{\theta}}^{\bar{\theta}}(V(dK_t,\theta) - dK_t - cdA_t)f(\theta)d\theta\right. \\ & + \left[\rho W_{t-} - \left(\lambda\int_{\underline{\theta}}^{\bar{\theta}} H_t(\theta)f(\theta)d\theta\right)\right]P'(W_{t-}) \\ & \left. + \lambda\int_{\underline{\theta}}^{\bar{\theta}}\left[P(W_{t-} + H_t(\theta) - \gamma dK_t) - P(W_{t-})\right]f(\theta)d\theta - rP(W_{t-})\right)dt \\ & + \left(P'(W_{t-}) - 1\right)dC_t. \end{aligned}$$

The expectation of the sum of the terms on the first two lines is zero. From equation (10), the sum of the terms on lines 3–5 is less than or equal to zero. Finally, because $P'(W_{t-}) \ge -1$, the term on line 6 is less than or equal to zero. Therefore, $(dG_t)_{t\ge 0}$ is a supermartingale. Consider headquarters' value at time 0. For any $t < \infty$,

$$\begin{aligned} & \mathbb{E}\left[\int_0^\infty e^{-rs}(V(dK_s, dX_s)dN_s - dK_s - dC_s - cdA_s)\right] \qquad (33) \\ & = \mathbb{E}[G_t] + e^{-rt}\mathbb{E}\left[\mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}(V(dK_s, dX_s)dN_s - dK_s - dC_s - cdA_s)\right]\right. \\ & \qquad \left. - P(W_t)\right] \\ & \le P(W_0) + e^{-rt}\mathbb{E}\left[P^0 - W_t - P(W_t)\right], \end{aligned}$$

where P^0 is the "first-best" value of operations such that investment that maximizes $V(k,\theta)-(1-\gamma)k$ is always made and audit never occurs. The last inequality holds, because $\mathbb{E}[G_t] \leq P(W_0)$ by the supermartingale property of G and the value of operations can never exceed P^0. Letting $t \to \infty$,

$$\mathbb{E}\left[\int_0^\infty e^{-rs}(V(dK_s,dX_s)-dK_s-dC_s-cdA_s)\right] \leq P(W_0). \tag{34}$$

Therefore, headquarters' expected utility from any incentive compatible mechanism that delivers the initial expected value of W_0 to the manager is at most $P(W_0)$.

Suppose that the mechanism satisfies the conditions of the proposition. Then, G_t is a martingale. Therefore, headquarters' initial expected payoff from the mechanism is $G_0 = P(W_0)$. Consequently, this mechanism is optimal, since no other direct incentive compatible mechanism can achieve the initial expected payoff above $P(W_0)$.

Proof of Proposition 3 The mechanism is optimal if and only if at any time t it leads to the same investment, audit, compensation policies, and evolution of the division manager's expected future utility as the mechanism in Proposition 2.

First, I show that the evolution of γB_t is the same as the evolution of W_t in Proposition 2. The starting point is $\gamma B_0 = W_0$ and the evolution of γB_t if $B_t < B^c$ and the division manager does not pass the project to headquarters is

$$d(\gamma B_t) = (g(\gamma B_t)B_t dt - dK_t)\gamma. \tag{35}$$

Hence, the evolutions of γB_t and W_t are the same if the investment policies are the same. Because the change in the division manager's utility, $dW_t + \gamma dK_t = g(W_t)W_t dt$, does not depend on dK_t, allocating the spending account between the current and future investment opportunities in the way that maximizes headquarters' value is incentive compatible. The implied amount of investment solves

$$\max_{k \in \mathbb{R}_+} \{V(\theta,k) + P(\gamma(B_t - k))\}. \tag{36}$$

Investment that solves this problem is exactly $k^n(\theta,\gamma B_t) = k^n(\theta,W_t)$.

Consider the division manager's decision to pass the project to headquarters. If the division manager believes that the audit would confirm that $k^a(\theta,\gamma B_t) \geq k_t^*$, then the division manager finds it optimal to pass the project to headquarters, because her account balance would be unaffected and she would get additional utility from private benefits. By contrast, if the division manager believes that the audit would not confirm that $k^a(\theta,\gamma B_t) \geq k_t^*$, then the division manager does not find it optimal to pass the project to headquarters, because she would get punished.

Next, the audit decisions implied by this mechanism are the same as the audit decisions in the mechanism in Proposition 2. Conditional on getting financed by headquarters, the optimal level of investment in a project is $k^a(\theta,\gamma B_t)$. Because $k^a(\theta,\gamma B_t)$ is an increasing function of θ and $k_t^* = k^a(\theta^*(\gamma B_t),\gamma B_t)$, the division manager will pass the project to headquarters if and only if $\theta \geq \theta^*(\gamma B_t) = \theta^*(W_t)$. Therefore, this mechanism implies the same audit decisions as the mechanism in Proposition 2. Finally, the mechanism implies the same compensation of the division manager as the mechanism in Proposition 2.

Proof of Proposition 4 See Section III of the Online Appendix.

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

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