



Investment busts, reputation, and the temptation to blend in with the crowd[☆]



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ABSTRACT

We provide a real-options model of an industry in which agents time abandonment of their projects in an effort to protect their reputations. Agents delay abandonment attempting to signal quality. When a public common shock forces abandonment of a small fraction of projects irrespective of agents' quality, many agents abandon their projects strategically even if they are unaffected by the shock. Such "blending in with the crowd" effect creates an additional incentive to delay abandonment ahead of the shock, leading to accumulation of "living dead" projects, which further amplifies the shock. The potential for moderate public common shocks often improves agents' values.

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1. Introduction

None love the messenger who brings bad news. Love can be even harder to find if the bad news speaks ill of the messenger's abilities. When decision makers face the unappealing task of revealing unsuccessful outcomes that impact their reputations, delay may be their first instinct.¹ Delay becomes even more enticing if they can wait for an industry-wide common shock to hide individual failings and instead "blend in with the crowd" by abandoning their projects strategically when some high-quality projects have to be terminated. In this paper, we argue that real investment decisions can be substantially affected by such a blend-in-with-the-crowd mechanism. More importantly, this may have far-reaching repercussions

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¹ See, e.g., Miller (2002) and Kothari, Shu, and Wysocki (2009).

for the dynamics of whole industries and provide an explanation for industry-wide investment busts following small and not necessarily negative common shocks.

The economic mechanism works as follows. When an agent operates a risky activity, such as a research and development (R&D) project or a start-up investment, a key decision is when to abandon it if it has not yet paid off. If higher-ability agents run better projects, abandonment is perceived as a negative signal about ability. As a consequence, the agent has incentives to delay abandonment. If, in addition, informativeness of abandonment varies over time, the agent has incentives to time abandonment when its informativeness is the lowest. This is the case when the industry is hit by a common shock so that at least some high-ability agents are forced to abandon their projects. In an attempt to fool outsiders into believing that their projects also got hit by the shock, agents managing bad enough projects “blend in with the crowd” and abandon their projects even if they are unaffected by the shock. In a dynamic environment, expectations of a common shock in the future create incentives for agents of lower quality to further delay abandonment in hope of blending in with forced abandonments at the time of the shock. This also leads higher-quality agents to delay their abandonment to separate themselves from lower types. This delay creates “living dead” projects outstanding when the common shock arrives, thereby amplifying its effect even more. As a result, even small common shocks can lead to massive abandonments. These shocks need not be negative on aggregate: the key requirement is that they have a negative effect on a fraction of projects run by high-ability agents, forcing their termination.

To aid in the intuition in the model, consider the following three examples of abandonment options in the context of the temptation to blend in with the crowd.

Example 1. Strategic home mortgage defaults in the presence of systematic shocks to incomes and home prices.

Consider a homeowner contemplating a “strategic default” on a mortgage, which is the decision to walk away from a negative-equity mortgage even when one can afford to pay it. As discussed in [Guiso, Sapienza, and Zingales \(2013\)](#), whether or not a homeowner strategically defaults in the face of significant negative equity shocks depends on personal factors such as moral and social considerations, which are likely difficult to accurately discern by outsiders. Due to the social stigma attached to strategic defaults, reputation is harmed by early exercise of the default option. Thus, strategic default may be postponed in hope of blending in with the crowd at times of market-wide home price or income shocks. As stated in [Guiso, Sapienza, and Zingales \(2013\)](#), “While we do observe defaults, we cannot observe whether a default is strategic. Strategic defaulters have all the incentives to disguise themselves as people who cannot afford to pay...”

Example 2. Business closures in the presence of raw material price shocks.

The option to shut down a business represents a classic real option to abandon. If managerial ability is correlated

with business success and is not perfectly observable, managers will delay shutting down businesses. However, this delay is magnified if common shocks provide an opportunity to pool with higher-ability managers. An example of such a shock can be drawn from the US solar energy industry. In 2011, the price of polysilicon, the primary raw material in many solar panels, dropped to less than \$40 a kilogram from an all-time high of \$475 in 2008, following a large expansion of polysilicon production in China. This shock drove out of business many producers (both high-quality and low-quality) that pursued production technologies that depended on non-polysilicon raw materials.² However, whether or not a firm's demise was due to the raw material price shock was not perfectly observable. For many projects, especially early-stage, the propensity to fail following such a shock depends on subtle variation in technology that is difficult for outsiders to decipher, such as the extent to which the technology can be adjusted.

Example 3. Timing of liquidation of toxic assets.

In a spirit similar to [Rajan \(1994\)](#), consider the timing of financial institutions' liquidations of toxic financial assets. Since reputations matter a great deal, there is an incentive to delay recognition of large declines in asset values. While individual institutions can and do liquidate deteriorated assets at various points in time, there is a strong incentive to cluster such actions at moments in which it is difficult for the market to distinguish between individual characteristics and common shocks. For example, a negative shock to an asset class as a whole (such as the bursting of a bubble or a shock to market sentiment) can provide a protective cover to institutions whose assets were problematic long before the shock. [Rajan \(1994\)](#) discusses how New England banks seemingly under-reported loan loss/earnings announcements during much of 1988 and 1989, while clustering their charge-offs at times of negative exogenous shocks. By liquidating toxic assets at the same moment that high-quality firms are forced to do the same, seemingly small systematic shocks can lead to disproportionately large market outcomes.

To explore this mechanism in more detail, we build a dynamic signaling model in the real-options framework. The starting point is a cross-section of projects that are up and running. Each project is either successful (with a random arrival of payoff) or unsuccessful (payoff never arrives). Neither the agent nor outsiders initially know whether the project can succeed. As they observe whether the project pays off or not, they update their beliefs about the success of the project in a Bayesian manner. However, each agent has private information about the potential payoff from her project, which is correlated with her intrinsic ability. Outsiders are initially aware only about the distribution of ability in the total pool of projects and subsequently either observe ability at the time of the project's successful payoff or learn about it by observing

² Two famous examples are Solyndra and Evergreen Solar. Solyndra manufactured panels based on copper indium gallium selenide (CIGS) cells. Evergreen Solar's technology was based on polysilicon, but it used considerably less polysilicon in its manufacturing process than competitors.

abandonment or lack thereof. To continue investment, the agent must spend a flow cost. The decision that the agent faces is when to abandon the project following a persistent lack of payoff. As higher-ability agents run better projects, they take more time before abandonment. As a consequence, when outsiders observe that a project is abandoned, they lower their estimate of the agent's ability.

We add to this setting a public common shock that arrives randomly and differentially affects outstanding projects. When it arrives, a fraction of outstanding projects is forced to be terminated, irrespective of the agents' abilities. The two requirements on the shock are that it affects some projects run by high-ability agents and that the exposure to the shock is private information of the agent. In this case, because the shock forces abandonment of some projects run by high-ability agents, abandonment at the time of the shock is not as informative about the agent's ability as abandonment outside of the shock. Although outsiders observe abandonment, they are unable to distinguish if it occurs because the agent's project is exposed to the shock or because the agent voluntarily abandons the project that is not exposed to the shock.

In a world with symmetric information about project quality, agents have no incentives to delay abandonment in order to signal higher quality nor to pretend to be exposed to a common shock. In the baseline model of symmetric information, agents optimally time abandonment by trading off the benefits of waiting for a potential project payoff with the costs of continuing to incur the running costs of operation. Given that the agent is indifferent between abandoning the project and continuing it for another instant, arrival of the common shock has no effect on the agent's payoff.

When we move to the case of asymmetric information, agents now delay abandonment for two reasons. First, since outsiders perceive abandonment as a signal of the poor ability of the agent, higher-ability agents delay abandonment of their projects to the point that lower-ability agents have no incentives to mimic them. Second, agents with lower-quality projects have an additional incentive to delay abandonment until the occurrence of the common shock in an effort to pool with projects that were truly exposed to the shock. Hence, when the shock arrives, agents with bad enough projects abandon them even if they are not exposed to the common shock. This effect can be significant. In a simple numerical example, a shock that forces termination of only 1% of outstanding projects leads to an abandonment of 10% of projects at the time of the shock. A variant of the model, albeit with different common shocks, can be applied to explaining an analogous “blending in” phenomenon for waves of investments.³

Although benefits from “blending in” are concentrated among low-ability agents, the possibility of a common shock delays abandonment of all agents. Because outsiders expect low types to delay abandonment, early abandonment becomes an even worse signal of ability than in the model with no common shocks. Consequently, higher-ability agents delay abandonment too as a means of separating from lower-ability agents. Such suboptimal investment in loss-making situations is consistent with substantial psychological research, a trait called escalation of commitment or sunk cost fallacy (Arkes and Blumer, 1985; Staw and Hoang, 1995). Here, escalation of commitment occurs rationally as agents find it optimal to over-invest hoping that a common shock will arrive. This implies that by the time the shock hits, outstanding negative net present value (NPV) projects can be the norm rather than an exception, especially if the shock followed a long period of stable times. For a common shock to trigger an investment bust, it need not be negative on aggregate: As long as it is bad enough news for some projects of high-ability agents, it can trigger strategic abandonments even if other projects are affected positively by the shock. This can rationalize why industry-specific investment busts sometimes do not seem to follow negative fundamental news.

Although expectations of a common shock in the future delay abandonment, once the shock arrives, some agents have incentives to speed up abandonment, occasionally terminating sound projects that would be continued if there were no information frictions. Intuitively, “blending in” can be so attractive to the agent that she prefers receiving a moderate reputation payoff at the time of the shock to continuing the project and facing the risk of abandoning it in a normal time. Both this effect and the delay outside of the shock amplify the observed effect of the shock.

Interestingly, although the presence of common shocks leads to a delay and clustering of abandonments, they are not necessarily detrimental to an industry. Without a common shock, abandonments are excessively delayed. The effect of a common shock is thus twofold. On one hand, a common shock's arrival in the future provides an additional incentive for the agent to delay abandonment today. On the other hand, at the time of the shock some agents, who would otherwise excessively continue investment, abandon their projects. The former effect decreases economic efficiency and the latter can increase it. Although the combined effect is ambiguous, for a reasonable range of parameters, negative common shocks may be value-increasing as they play a “cleansing” role.

1.1. Related literature

This paper extends the literature that uses real-options models to analyze the timing of abandonment of investment projects. Most models do not account for the signaling role of abandonment (e.g., Brennan and Schwartz, 1985; Lambrecht, 2001; Lambrecht and Myers, 2007). Grenadier and Malenko (2011), Morellec and Schürhoff (2011), and Bustamante (2012) study the signaling role of exercise timing, as we do in this paper. Our focus on

³ Consider agents choosing when to invest in irreversible projects, whose payoffs are positively related to ability, although not fully observable to outsiders. Suppose a dramatic common shock hits a small subset of firms (where the exposure of a particular firm is unobservable to outsiders) causing the project's payoff to jump high enough to ensure immediate investment. If this exposure is a strong enough signal of ability, then some agents whose projects are not exposed to the shock will invest to blend-in with the high-ability managers that were exposed to the shock. Thus, a favorable shock that impacts only a few projects can result in a larger wave of investments.

separating equilibria and the solution approach are borrowed from Grenadier and Malenko (2011). However, in all these models there are no common shocks, which is our focus here.⁴ Clustering can also arise in pooling equilibria even without common shocks, as in Bustamante (2012). However, the economic reasoning behind clustering in our model is different. In pooling equilibria, clustering occurs because high types find it too costly to separate from low types by pursuing a different strategy. In contrast, here clustering occurs because low types strategically choose to abandon projects when a common shock hits. It is the active role of low types and the relation of clustering to public common shocks (not necessarily negative) that distinguishes clustering in our model from the prior real-options literature.

Several papers study inefficient delay and/or clustering of decisions due to various information effects. Among them, the most closely related models are Rajan (1994) and Acharya, DeMarzo, and Kremer (2011). Rajan (1994) studies coordination between two banks in the recognition of bad loans when managers care about their reputations. As reputation is less important in the adverse state of the economy, in equilibrium bank managers recognize bad loans if the state is adverse, but not if the state is normal.⁵ While both Rajan's and our model have clustering due to reputation concerns, there are two important differences. First, Rajan's model is effectively a static model in that bank managers can make a decision at one point in time. In contrast, decision-making is dynamic in our model, as abandonment can be undertaken at any time, including before, during, and after any common shock appears. This leads to dynamic effects: For example, expectations of the arrival of the common shock in the future feed back into the decision today, delaying abandonment and adding up to the accumulation of the "living dead" projects. Second, a common shock in our model has a differential effect on projects. In contrast, the adverse state in Rajan's model is the same for all bank managers. Thus, models have different implications for what shocks trigger abandonment. For example, our model implies that industry-wide investment busts can follow common shocks that are neutral or even positive on aggregate.

Strategic pooling of low-ability managers unaffected by the shock with high-ability managers affected by the shock has similarities with pooling in models of voluntary disclosure (Dye, 1985; Jung and Kwon, 1988), in which managers with bad news pool with managers that have no news by not disclosing information. In a dynamic disclosure model, Acharya, DeMarzo, and Kremer (2011) show that bad (but not good) market news triggers immediate disclosure by firms. While their model also features delay and clustering, the economic mechanism is different,

because disclosure is assumed to be truthful, as in other models of voluntary disclosure. Clustering is also present in the equity issuance model of Korajczyk, Lucas, and McDonald (1992), which is based on time-varying firm-level adverse selection, and in models of herding due to limited information of each agent (e.g., Banerjee, 1992).⁶ The decision when to abandon a project under private information has similarities to the decision when to sell an asset in the presence of adverse selection. Thus, the paper is also related to a recent literature on repeated lemons markets (e.g., Daley and Green, 2012; Fuchs and Skrzypacz, 2013; Kurlat, 2013).

The remainder of the paper is organized as follows. Section 2 describes the setup of the model and solves the benchmark case of symmetric information. Section 3 provides the solution for the main case of asymmetric information. Section 4 discusses implications of the model. Section 5 examines pooling equilibria, in which a set of agents abandons their projects simultaneously even outside of the common shock, and shows that many of the insights are not specific to the separating equilibrium. Finally, Section 6 concludes.

2. Model setup

A risk-neutral agent operates an investment project and decides when to abandon it. We model investment as an experimentation problem, which is a common approach to model entrepreneurial and R&D activity.⁷ Specifically, the project can be successful or unsuccessful. If the project is successful, it provides a single payoff of $\theta > 0$ to the agent at a Poisson event that arrives with intensity $\lambda > 0$. If the project is unsuccessful, it never pays off. At any point in time t , prior to the payoff arrival, both the agent and outsiders assess the likelihood that the project is successful by $p(t)$, which is updated in a Bayesian fashion as shown below. The initial prior is $p(0) = p_0 \in (0, 1)$. We define the potential project payoff θ to be the quality of the project and assume that it is the private information of the agent. Outsiders do not know θ except for its prior distribution, which is common knowledge. It is given by the probability density function (p.d.f.) $f(\theta)$ and the cumulative distribution function (c.d.f.) $F(\theta)$ with full support on $[\underline{\theta}, \bar{\theta}]$, $\bar{\theta} > \underline{\theta} > 0$. In practice, the quality of the project is likely to be positively but imperfectly correlated with the ability of the agent. For simplicity, we assume it coincides with θ . To keep the project afloat, the agent must pay cost c per unit time. Thus, the project ends either when it pays off or when it is abandoned by the agent. The discount rate is $r > 0$.⁸

⁴ See also Kanodia, Bushman, and Dickhaut (1989) and Boot (1992) for models in which reputation concerns may cause managers to not stop existing projects. This effect is similar to the delay of abandonment in normal times in our paper. However, common shocks are not studied by Kanodia, Bushman, and Dickhaut (1989) and Boot (1992).

⁵ Agarwal and Koley (2012) present a model similar to Rajan's (1994) to study clustering of corporate layoffs. They also provide empirical evidence of such clustering.

⁶ Models with delay due to information reasons also include Gennotte and Trueman (1996), in which management delays disclosure of negative news until the end of trading hours, and Milbradt (2012), in which an institution delays realizing losses due to fair value accounting and the balance sheet constraint.

⁷ E.g., see Bergemann and Hege (1998, 2005), Miao and Wang (2007), Manso (2011), and Bouvard (forthcoming).

⁸ An alternative way of modeling private information is to assume that the quality of the project positively affects the probability of the project being successful. This alternative model has similar economics and results but features a more involved solution. The critical driver is

The agent cares about her reputation in the eyes of outsiders. When a successful project pays off, all parties observe θ . For simplicity, we assume that the reputation payoff upon the project payoff is $\gamma\theta$, where $\gamma > 0$ measures the importance of reputation. However, in the event of abandonment outsiders do not observe θ , because the project payoff is not realized. Instead, at any time t before the project either pays off or is abandoned, outsiders hold the conditional expectation $E[\theta|I_t]$, where I_t is the past history that outsiders know at time t . The reputation payoff upon abandonment at any time τ is therefore $\gamma E[\theta|I_\tau]$.

The decision to abandon is publicly observed and irreversible. The project may be abandoned at any time for either of two reasons. First, it may be abandoned because the agent chooses to do so. Abandoning the project allows the agent to stop paying out the flow of c at the cost of forgoing the potential payoff θ . Second, the project may be abandoned exogenously because it gets exposed to a common shock that impacts many projects at once. In such an event, the agent has no choice but to abandon. The common shock arrives as a Poisson event with intensity $\mu > 0$. For tractability, we assume that there can be only one shock. A project is exposed to the common shock with probability q . We assume that the arrival of the common shock is publicly observable, but whether the project is exposed to it is learned privately by the agent upon the arrival of the shock. For simplicity, we assume that if the project is not exposed to the shock, its potential payoff is unaffected. However, this assumption is not necessary for our results: They hold even if the shock positively affects the potential payoff of some projects, implying that sector-specific investment busts need not follow bad news. The key requirement is that the shock forces abandonment of some projects indiscriminately of the agents' quality.

We assume that it is optimal to begin operating all projects. Specifically, even the worst project should be operated for at least an instant. A necessary and sufficient condition for this is

$$\lambda > \frac{1}{p_0} \left(\frac{c}{\theta} + r\gamma \right). \quad (1)$$

2.1. Learning about success of the project

As prospects of the project are uncertain, an important state variable is the belief that the project is successful. Let $p(t)$ denote the posterior probability that the project is successful given that the project has not paid off for time t since inception. Over a short period between t and $t+dt$, the project pays off with probability λdt , if it is successful, and zero, otherwise. Hence, the posterior probability $p(t+dt)$ conditional on not getting a payoff during $[t, t+dt]$ is equal to

$$p(t+dt) = \frac{(1-\lambda dt)p(t)}{(1-\lambda dt)p(t) + 1-p(t)}. \quad (2)$$

(footnote continued)

that abandonment reveals negative information about the ability of the agent to outsiders.

Rewriting (2) and taking the limit $dt \rightarrow 0$, we obtain the following differential equation:

$$dp(t) = -\lambda p(t)(1-p(t)) dt, \quad (3)$$

with the boundary condition

$$p(0) = p_0. \quad (4)$$

The solution to (3)–(4) is

$$p(t) = \frac{p_0}{(1-p_0)e^{\lambda t} + p_0}. \quad (5)$$

The dynamics of the belief process are intuitive. As long as the project does not pay off, the belief that it is successful continuously decreases over time. The speed of learning is proportional to λ , the intensity with which the successful project pays off. Once the project pays off, the belief that it is successful jumps up to one. Note that because the agent's private information does not enter the belief evolution process (5), the agent and outsiders always share the same belief about the likelihood of success.

Eq. (5) implies that there is a one-to-one mapping between $p(t)$ and the time passed since inception of the project t . We can invert (5) to get the time since inception of the project given posterior p :

$$t(p) = \frac{1}{\lambda} \ln \left(\frac{p_0}{1-p_0} \frac{1-p}{p} \right). \quad (6)$$

Thus, either the belief about the success probability of the project p or the time passed since inception of the project t can be used as a state variable. In what follows, we use t as a more intuitive state variable, but all valuation equations can be equivalently rewritten in terms of p .

2.2. Benchmark case

As a benchmark, consider the case of symmetric information in which outsiders also know θ . We break up the value function into two regions depending on whether or not the common shock has yet occurred. Let $V_b^*(t, \theta)$ and $V_a^*(t, \theta)$ denote the value to the agent before and after the arrival of the common shock, respectively, conditional on the project not paying off for time t since inception. Working backwards in time, consider first the problem after the common shock has occurred. By Itô's lemma for jump processes (see, e.g., Shreve, 2004), in the range prior to abandonment, the evolution of $V_a^*(t)$ is given by

$$dV_a^*(t, \theta) = \frac{\partial V_a^*(t, \theta)}{\partial t} dt + (\theta + \gamma\theta - V_a^*(t, \theta)) dN(t), \quad (7)$$

where $N(t)$ is a point process describing the arrival of the project payoff. In (7), the first term reflects the change in the expected value due to learning about the success of the project, and the last term reflects the change in the expected value if the project pays off (the agent receives the project payoff θ , the reputation payoff $\gamma\theta$, and forgoes $V_a^*(t, \theta)$). Because the agent believes that the project is successful with probability $p(t)$, $E_t[dN(t)] = p(t)\lambda dt$. Dividing (7) by dt and taking the expectation:

$$\mathbb{E}_t \left[\frac{dV_a^*(t, \theta)}{dt} \right] = \frac{\partial V_a^*(t, \theta)}{\partial t} + (\theta + \gamma\theta - V_a^*(t, \theta))\lambda p(t). \quad (8)$$

As keeping the project afloat requires a cash outflow of c per unit time, $E_t[dV_a^*(t, \theta)] - c dt$ must equal $rV_a^*(t, \theta) dt$. This yields the following differential equation:

$$(r + \lambda p(t))V_a^*(t, \theta) = -c + \frac{\partial V_a^*(t, \theta)}{\partial t} + \lambda(1 + \gamma)\theta p(t). \quad (9)$$

Let $\tau_a^*(\theta)$ denote the time since inception of the project at which the agent optimally abandons her project, if it has not paid off yet. Eq. (9) is solved subject to the following boundary conditions:

$$V_a^*(\tau_a^*(\theta), \theta) = \gamma\theta, \quad (10)$$

$$\frac{\partial V_a^*(\tau_a^*(\theta), \theta)}{\partial t} = 0. \quad (11)$$

The first equation is the value-matching condition. It reflects the fact that upon exercise of the abandonment option, the value to the agent equals her reputation payoff. The second equation is the smooth-pasting condition. It ensures that the exercise trigger maximizes the agent's value (see, e.g., Dumas, 1991).

Evaluating (9) at $\tau_a^*(\theta)$ yields

$$r\gamma\theta + c = \frac{\lambda\theta p_0}{(1 - p_0)e^{\lambda\tau_a^*(\theta)} + p_0}. \quad (12)$$

The left-hand side of (12) denotes the costs of delaying abandonment for an additional instant: the cost of keeping the project afloat and the delay in getting the reputation payoff. The right-hand side of (12) denotes the benefit of delaying abandonment for an additional instant: the potential payoff from the project. Continuation of investment is optimal as long as the latter value is higher than the former, and condition (1) ensures that it is always the case at the onset of the project. The agent abandons the project when the costs and the benefit of delaying abandonment for an additional instant coincide. Eq. (12) implies the following abandonment threshold:

$$\tau_a^*(\theta) = \frac{1}{\lambda} \ln \left(\frac{p_0}{1 - p_0} \left(\frac{\lambda\theta}{c + r\gamma\theta} - 1 \right) \right). \quad (13)$$

The comparative statics of the abandonment threshold are intuitive: abandonment occurs earlier if the waiting cost c is higher, the success intensity λ of the project is lower, the discount rate r is higher, and reputation γ is more important.

Next, consider the problem of optimal abandonment before the arrival of the common shock. The evolution of $V_b^*(t, \theta)$ is given by

$$dV_b^*(t, \theta) = \frac{\partial V_b^*(t, \theta)}{\partial t} dt + (\theta + \gamma\theta - V_b^*(t, \theta)) dN(t) + [q\gamma\theta - V_b^*(t, \theta) + (1 - q)(V_a^*(t, \theta) - V_b^*(t, \theta))] dM(t), \quad (14)$$

where $M(t)$ is a point process describing the arrival of the common shock. Compared to (7), Eq. (14) includes an additional term reflecting the effect of a potential arrival of the common shock. Upon its arrival, one of the two scenarios takes place. With probability q , the project is exposed to the shock, in which case forced abandonment takes place and the agent gets the reputation payoff $\gamma\theta$. With probability $1 - q$, the project is not exposed to the shock, in which case the agent gets the after-shock

expected value $V_a^*(t, \theta)$. Dividing (14) by dt , taking the expectation, and equating it with $c + rV_b^*(t, \theta)$, we obtain the following differential equation:

$$(r + \lambda p(t) + \mu)V_b^*(t, \theta) = -c + \frac{\partial V_b^*(t, \theta)}{\partial t} + \lambda(1 + \gamma)\theta p(t) + \mu(q\gamma\theta + (1 - q)V_a^*(t, \theta)). \quad (15)$$

Analogous to $\tau_a^*(\theta)$, let $\tau_b^*(\theta)$ denote the time since inception at which the agent optimally abandons her project, if it has not paid off yet. By analogy with (10) and (11), Eq. (15) is solved subject to boundary conditions $V_b^*(\tau_b^*(\theta), \theta) = \gamma\theta$ and $\partial V_b^*(\tau_b^*(\theta), \theta)/\partial t = 0$. Evaluating (15) at $\tau_b^*(\theta)$ and rearranging the terms gives us

$$r\gamma\theta + c = \frac{\lambda\theta p_0}{(1 - p_0)e^{\lambda\tau_b^*(\theta)} + p_0} + \mu(1 - q)(V_a^*(\tau_b^*(\theta), \theta) - \gamma\theta). \quad (16)$$

It is easy to see that $\tau_b^*(\theta) = \tau_a^*(\theta)$ satisfies (16). It is also the unique solution, because the right-hand side of (16) is decreasing in $\tau_b^*(\theta)$. Thus, the abandonment policy of the agent is the same both before and after the arrival of the common shock. This may seem surprising: casual intuition may suggest that a possibility of the common shock in the future adds to the cost of delaying abandonment and leads to earlier abandonment prior to the shock. However, this intuition is incorrect. The reason is that the abandonment threshold is determined by the indifference condition of the agent. When the agent is indifferent between abandoning the project and continuing it for another instant, arrival of the common shock has no effect on the agent's payoff: it simply forces her to accept one option of the two that she is indifferent between. Hence, the optimal abandonment policy of the agent is unaffected by the potential arrival of the common shock. We denote this full-information trigger by $\tau^*(\theta) = \tau_a^*(\theta) = \tau_b^*(\theta)$. In the absence of information frictions between the agent and outsiders, the only consequence of the common shock is to introduce noise in abandonment by forcing the agent to abandon the project prematurely if it gets exposed to the shock. Although the abandonment thresholds are the same before and after the arrival of the shock, the value functions are not. Because the common shock may trigger abandonment at a suboptimal time, $V_b^*(t, \theta) < V_a^*(t, \theta)$ for any $t < \tau^*(\theta)$.

The optimal abandonment strategy in the benchmark case is summarized in Proposition 1:

Proposition 1. Suppose that θ is known to both the agent and outsiders. Then, the optimal strategy of the agent is to abandon the project if it has not paid out for time $\tau^*(\theta)$. When the common shock arrives, the project is abandoned if and only if it is exposed to the shock.

Let us see how $\tau^*(\theta)$ depends on θ :

$$\frac{d}{d\theta} \tau^*(\theta) = \frac{1}{\left(\frac{\lambda\theta}{r\gamma\theta + c} - 1 \right) (r\gamma\theta + c)^2} > 0, \quad (17)$$

because $\tau^*(\theta) > 0$. Abandonment in the benchmark case satisfies three properties. First, the equilibrium abandonment strategy is optimal in that there is no other abandonment strategy that yields a higher expected payoff to the agent. Second, better projects are abandoned later.

Intuitively, if the potential payoff from the project is higher, then the agent optimally waits longer before scrapping it. Finally, the common shock affects abandonment if and only if the project is exposed to the shock.

3. Private information case

In this section, we provide the solution for the main case of asymmetric information between the agent and outsiders. Even though there is no revelation of θ when the project is abandoned, outsiders try to infer θ from observing the timing of abandonment. In turn, the agent attempts to manipulate her abandonment decision to shape the belief of outsiders. If there were no common shock, this would lead to a separating equilibrium, as in Grenadier and Malenko (2011): each type delays abandonment in the hope of fooling outsiders into believing that her type is higher, but in equilibrium outsiders learn θ with certainty. However, with a common shock, the equilibrium is more interesting: the agent may choose to time abandonment to the arrival of the common shock. By doing this, she blends in with agents of higher types that are forced to abandon due to their exposure to the shock.

Because we assume only one common shock, we solve the model by backward induction. First, we consider the timing of abandonment after the shock has already arrived at time s in the past. In this case, no future shock arrival is possible, so the solution is standard. Second, we consider the problem of abandoning the project at the time of the shock. Finally, we consider the problem of abandoning the project prior to the arrival of the shock.

In the first and third stages of the problem, our focus is on the separating Bayesian Nash equilibrium. There are also pooling or semi-pooling equilibria. In these equilibria, several types abandon projects at the same time even without the common shock due to self-fulfilling beliefs. There are two reasons for our focus on the separating equilibrium. First, because we are interested in how common shocks trigger waves of strategic abandonments, it is natural to abstract from possible waves of abandonments in other periods. Second, pooling and semi-pooling equilibria typically do not survive the D1 equilibrium refinement, which intuitively requires outsiders to place zero posterior belief on type θ for any off-equilibrium action whenever there is another type θ' with a stronger incentive to deviate.⁹ In the Appendix we provide intuitive off-the-equilibrium-path beliefs that satisfy the D1 refinement. Such beliefs satisfy an intuitive property that the outsiders' best guess of the agent's type evolves continuously.¹⁰ In contrast, a pooling equilibrium requires outsiders' belief to change discontinuously over time if

no common shock arrives. Because of this, we find the separating equilibrium in the first and third stages of the problem as more empirically plausible, so we relate our implications to it. However, we extend the analysis by considering pooling in the first and third stages of the problem in Section 5. It shows that a common shock triggers strategic abandonments by some unaffected types in this case too. We provide a heuristic analysis in this section and offer formal proofs in the Appendix.

3.1. Abandonment after the arrival of the shock

Consider the decision of type θ to abandon the project after the common shock has already occurred at time s in the past. Conjecture that the distribution of types right after the common shock is truncated at some type $\hat{\theta}$ from below. This conjecture is verified later. Then, right after the common shock at time s , outsiders believe that θ is distributed over $[\hat{\theta}, \bar{\theta}]$ with p.d.f. $f(\theta)/(1-F(\hat{\theta}))$. In the special case of $s=0$ and $\hat{\theta}=\underline{\theta}$, this case corresponds to the overall game if there were no common shocks.

We first solve for the agent's abandonment strategy for a given belief function of outsiders, and then apply the rational expectations condition that the abandonment strategy and the belief function of outsiders must be consistent with each other. Specifically, suppose that outsiders believe that type θ abandons the project once it has not paid off for time $\tau_a(\theta)$, which is a monotonic and differentiable function of θ . Thus, if the agent abandons the project at time τ , outsiders infer that the type is $\hat{\theta}(\tau) = \tau_a^{-1}(\tau)$. Let $V_a(t, \hat{\theta}, \theta, \tau)$ denote the value to the agent, conditional on the project not paying off for time t , for a fixed belief of outsiders upon abandonment $\hat{\theta}$, where $t \in [s, \tau]$ is the time since inception of the project, θ is the true type, and τ is the abandonment time. In the region prior to abandonment, $V_a(t, \hat{\theta}, \theta, \tau)$ satisfies a differential equation identical to (9):

$$(r + \lambda p(t))V_a = -c + \frac{\partial V_a}{\partial t} + \lambda(1 + \gamma)\theta p(t). \quad (18)$$

Upon abandonment of the project, the agent gets the reputation payoff of $\gamma\hat{\theta}$. It implies the following boundary condition:

$$V_a(\tau, \hat{\theta}, \theta, \tau) = \gamma\hat{\theta}. \quad (19)$$

Solving (18) subject to boundary (19) yields the value to the agent for a given abandonment threshold and the belief of outsiders:

$$V_a(t, \hat{\theta}, \theta, \tau) = e^{(r+\lambda)t} p(t) U(\hat{\theta}, \theta, \tau) - \frac{c}{r} + \frac{\lambda}{r+\lambda} \left(\frac{c}{r} + \theta + \gamma\theta \right) p(t), \quad (20)$$

where:

$$U(\hat{\theta}, \theta, \tau) = \frac{1-p_0}{p_0} e^{-r\tau} \left(\frac{c}{r} + \gamma\hat{\theta} \right) + e^{-(r+\lambda)\tau} \left(\frac{c-\lambda(1+\gamma)\theta}{r+\lambda} + \gamma\hat{\theta} \right). \quad (21)$$

Given the hypothesized inference function $\tau_a(\theta)$, we can substitute $\hat{\theta}(\tau) = \tau_a^{-1}(\tau)$ for $\hat{\theta}$ in (20)–(21) and take the first-order condition with respect to τ :

$$-c - r\gamma\hat{\theta}(\tau) + \lambda p(\tau)(\theta + \gamma(\theta - \hat{\theta}(\tau))) + \gamma\hat{\theta}'(\tau) = 0. \quad (22)$$

⁹ See Cho and Sobel (1990) and Ramey (1996) for the proof of this result for signaling models with discrete types and a continuum of types, respectively. In the Appendix, we also provide the off-the-equilibrium-path beliefs that support the equilibrium and satisfy the D1 refinement.

¹⁰ In addition, the beliefs are such that outsiders identify the most extreme actions with the most extreme types. Any agent that abandons even earlier than the lowest type does in equilibrium is perceived to be the worst type, and any agent that delays abandonment beyond the highest type is perceived to be the highest type.

Eq. (22) reflects the costs and benefits of postponing the abandonment. The first two terms reflect the costs of waiting an additional instant dt : paying a cost $c dt$ and delaying the reputation payoff. The third term reflects the benefits of waiting an additional instant: the possibility that the project will pay off. The probability of the project paying off over the next instant is $\lambda p(\tau) dt$, and the payoff exceeds the abandonment payoff by $\theta + \gamma(\theta - \tilde{\theta}(\tau))$. Finally, the last term reflects the effect of waiting an instant on the outsiders' inference of the agent's type: abandoning the project an instant later changes the reputation payoff by $\gamma \tilde{\theta}'(\tau) dt$.

Applying the rational expectations condition that in equilibrium outsiders' beliefs must be consistent with the agent's abandonment strategy ($\tilde{\theta}(\tau_a(\theta)) = \theta$), we get the equilibrium differential equation:

$$\frac{d\tau_a}{d\theta} = \frac{\gamma}{c + \gamma\theta - \lambda\theta p(\tau_a(\theta))}, \quad \theta \in [\hat{\theta}, \bar{\theta}]. \quad (23)$$

This equation is solved subject to the standard initial value condition that the lowest type in the post-shock history, $\hat{\theta}$, abandons at the symmetric information threshold:

$$\tau_a(\hat{\theta}) = \max\{\tau^*(\hat{\theta}), s\}, \quad (24)$$

where $\tau^*(\cdot)$ is given by (13). Intuitively, if the lowest type did not abandon at the symmetric information threshold, she would find it optimal to deviate to it. This deviation not only would improve the direct payoff from exercise, but also could improve the reputation payoff, since the current belief is already as bad as possible. If $\tau^*(\hat{\theta}) \geq s$, then abandonment at $\tau^*(\hat{\theta})$ is feasible. If $\tau^*(\hat{\theta}) < s$, then abandonment at $\tau^*(\hat{\theta})$ is not feasible. In this case, the most preferred abandonment timing of type $\hat{\theta}$ is immediately after the common shock.

By verifying that $U(\hat{\theta}, \theta, \tau)$ satisfies the regularity conditions in Mailath (1987), Proposition 2 shows that this argument indeed yields a unique (up to the off-equilibrium beliefs) separating equilibrium threshold $\tau_a(\theta)$.

Proposition 2. Let $\tau_a(\theta)$ be the increasing function that solves Eq. (23) subject to the initial value condition (24). Then, $\tau_a(\theta)$ is the abandonment threshold of type θ in the unique (up to the off-equilibrium beliefs) separating equilibrium.

It immediately follows that the equilibrium abandonment threshold after the arrival of the shock depends on the lowest type in this region, $\hat{\theta}$, and the arrival time of the shock s . Thus, we denote the after-shock equilibrium abandonment threshold by $\tau_a(\theta, \hat{\theta}, s)$. We denote the equilibrium value function, $V_a(t, \theta, \theta, \tau_a(\theta, \hat{\theta}, s))$, by $\bar{V}_a(t, \theta, \hat{\theta}, s)$.

Note that the special case of this problem, $s=0$ and $\hat{\theta} = \theta$, corresponds to the whole abandonment problem under asymmetric information, if there were no common shocks. Thus, the equilibrium threshold in the model without common shocks is $\tau^a(\theta, \theta, 0)$. By Proposition 2, it solves (23) subject to $\tau^a(\theta, \theta, 0) = \tau^*(\theta)$. Compared to the case of symmetric information, the agent excessively delays abandonment: $\tau^a(\theta, \theta, 0) > \tau^*(\theta)$ for any $\theta > \underline{\theta}$. We refer to this special case in Section 4.2, in which we decompose the delay into two effects: delay caused by signaling incentives of the agent and delay caused by "blending in" with others at the common shock.

3.2. Abandonment upon the arrival of the shock

Consider the decision of type θ to abandon the project upon the arrival of the common shock. Analogously to the previous section, conjecture that the agent's timing of abandonment before the arrival of the common shock is given by threshold $\tau_b(\theta)$, which is increasing in θ . Conjecture also that $\tau_b(\theta) \geq \tau^*(\theta)$ for any θ , i.e., the agent delays abandonment of her project compared to the benchmark case. Both conjectures are verified in Section 3.3. Given these conjectures, the posterior belief of outsiders upon the arrival of the common shock at time s is that θ is distributed over $[\hat{\theta}, \bar{\theta}]$ with p.d.f. $f(\theta)/(1-F(\hat{\theta}))$, where $\hat{\theta} = \tau_b^{-1}(s)$.

When the common shock arrives, the project may be exposed to it (with probability q) or not exposed to it (with probability $1-q$). In the former case, the agent has no choice but to abandon the project. In the latter case, the agent may abandon the project strategically together with the exposed projects or postpone the abandonment. If the agent abandons the project immediately, she gets a reputation payoff that is independent of her true type. If the agent postpones the abandonment, her expected payoff is increasing in her true type. Hence, there exists type $\hat{\theta} \in [\hat{\theta}, \bar{\theta}]$ such that the non-exposed project is abandoned if $\theta < \hat{\theta}$ and not abandoned if $\theta > \hat{\theta}$. Provided that $\hat{\theta} < \bar{\theta}$, there can be two scenarios depending on the abandonment strategy of the type $\hat{\theta} + \varepsilon$ that is marginally higher than $\hat{\theta}$:

Scenario 1: Type $\hat{\theta} + \varepsilon$ abandons the project immediately after the common shock. This happens if and only if

$$s \geq \tau^*(\hat{\theta}), \quad (25)$$

$$\gamma \int_{\hat{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta + q \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta = \gamma \hat{\theta} \frac{F(\hat{\theta}) - F(\hat{\theta}) + q(1 - F(\hat{\theta}))}{F(\hat{\theta}) - F(\hat{\theta}) + q(1 - F(\hat{\theta}))}. \quad (26)$$

Condition (25) states that the optimal abandonment time of type $\hat{\theta}$ is upon the arrival of the common shock or immediately after. Condition (26) states that type $\hat{\theta}$ is exactly indifferent between blending in and abandoning at the time of the common shock (the payoff of this strategy is the left-hand side of (26)) and separating and abandoning a second after the common shock.

Scenario 2: Type $\hat{\theta} + \varepsilon$ abandons at the symmetric information threshold $\tau^*(\hat{\theta} + \varepsilon)$. This happens if and only if

$$s < \tau^*(\hat{\theta}), \quad (27)$$

$$\gamma \int_{\hat{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta + q \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta = V_a^*(s, \hat{\theta}), \quad (28)$$

where $V_a^*(s, \hat{\theta}) = \bar{V}_a(s, \hat{\theta}, \hat{\theta}, s)$ is the value to type θ in the symmetric information framework, when time s has passed since the inception of the project. Note that $V_a^*(s, \hat{\theta})$ is the value in the world of full information, and thus its solution is simple to calculate. Condition (27) states that the optimal abandonment time of type $\hat{\theta}$ is given by threshold $\tau^*(\hat{\theta})$. Condition (28) states that type $\hat{\theta}$ is indifferent between abandoning at the time of the common shock and postponing abandonment until her optimal separating threshold.

Note that because $V_a^*(t, \hat{\theta}) = \gamma \hat{\theta}$ for all $t \geq \tau^*(\hat{\theta})$, condition (26) is a special case of condition (28).¹¹

Scenarios 1 and 2 imply that the equilibrium can have one of two forms. In the first scenario, abandonment happens continuously with positive probability, while in the second scenario a wave of abandonments triggered by the common shock is followed by a period of no abandonment. The next proposition shows that there exists a unique indifferent type $\hat{\theta}$ in equilibrium, and it is always strictly between $\check{\theta}$ and $\bar{\theta}$:

Proposition 3. *At the moment the common shock hits, types $\theta < \hat{\theta}$ that are not impacted by the shock voluntarily abandon to pool with those that are impacted by the shock. $\hat{\theta} \in (\check{\theta}, \bar{\theta})$ is unique, and computed in the following way. Let $\hat{\theta}_m$ denote the unique solution to (26). If $s \geq \tau^*(\hat{\theta}_m)$, then $\hat{\theta} = \hat{\theta}_m$. If $s < \tau^*(\hat{\theta}_m)$, then $\hat{\theta}$ is determined as the unique solution to (28). Since $\hat{\theta}$ depends on s and $\check{\theta}$, we denote the equilibrium indifference type by $\hat{\theta}(\check{\theta}, s)$.*

The equilibrium structure is illustrated in Fig. 1. Prior to the arrival of the shock, as time goes by, low types abandon their projects: type θ abandons her project at time $\tau_b(\theta)$, where $\tau_b(\theta)$ is to be determined in the next section. This truncates the distribution of existing types from below. When the common shock arrives at date s , the posterior distribution of types is $[\check{\theta}(s), \bar{\theta}]$, where $\check{\theta}(s) = \tau_b^{-1}(s)$. Upon arrival of the shock, fraction q of all types $[\check{\theta}(s), \bar{\theta}]$ is hit by the shock and forced to abandon their projects. On top of this, there exists an intermediate type $\hat{\theta}(\check{\theta}(s), s)$, such that each type in $[\check{\theta}(s), \hat{\theta}(\check{\theta}(s), s)]$ abandons her project even if it is not exposed to the shock.

The results of this section have two interesting implications. First, because $\hat{\theta} > \check{\theta}$, the common shock leads to abandonment of more projects than are affected by the shock directly. The difference between $\hat{\theta}$ and $\check{\theta}$ measures the extent of strategic blending in by low-ability agents. We explore the economic implications of this finding in Section 4.

Second, depending on whether the equilibrium belongs to scenario 1 or scenario 2, there can be different time-series properties of abandonments. In the former case, the common shock triggers a wave of abandonments, but after the shock is over the abandonment rate is comparable to normal times. By contrast, in the latter case, the common shock triggers a wave of abandonments, which is followed by a quiet period of no abandonments. We also discuss these implications in more detail in Section 4.

Let $\theta_{blend}(s, \check{\theta})$ denote the equilibrium expected type conditional on blending in and let $\bar{V}_{cs}(s, \theta, \check{\theta})$ denote the value to type θ when the common shock arrives and before the agent learns if her project is exposed to the shock. Then,

$$\bar{V}_{cs}(s, \theta, \check{\theta}) = q\theta_{blend}(s, \check{\theta}) + (1-q)\max\{\gamma\theta_{blend}(s, \check{\theta}), \bar{V}_a(s, \theta, \hat{\theta}(\check{\theta}, s), s)\}. \quad (29)$$

¹¹ Conditions (28) and (26) implicitly assume that $\check{\theta} < \hat{\theta} < \bar{\theta}$. If $\hat{\theta} = \check{\theta}$ or $\hat{\theta} = \bar{\theta}$, then (28) and (26) may hold as strict inequalities. In the proof of Proposition 3, we establish that $\hat{\theta} \in (\check{\theta}, \bar{\theta})$ indeed.

3.3. Abandonment before the arrival of the shock

To complete the solution of the model, consider the decision of type θ to abandon the project before the arrival of the common shock. Similar to Section 3.1, we solve for the separating equilibrium in two steps. First, we solve for the agent's abandonment strategy for a given inference function of outsiders. Second, we apply the rational expectations condition that the inference function of outsiders must be consistent with the abandonment strategy of the agent.

Suppose that outsiders believe that type θ abandons the project if it has not paid for time $\tau_b(\theta)$ since inception, where $\tau_b(\theta)$ is an increasing and differentiable function of θ . Hence, if the agent abandons the project at time τ , outsiders update their belief about the agent's type to $\tilde{\theta}(\tau) = \tau_b^{-1}(\tau)$. Let $V_b(t, \tilde{\theta}, \theta, \tau)$ denote the value to the agent in the range prior to the arrival of the shock, conditional on the project not paying off for time t , for a fixed belief of outsiders $\tilde{\theta}$, where $t \leq \tau$ is the time since inception of the project, θ is the true type, and τ is the abandonment timing. Using the standard argument, in the region prior to abandonment, $V_b(t, \tilde{\theta}, \theta, \tau)$ satisfies the following differential equation:

$$(r + \lambda p(t) + \mu)V_b = -c + \frac{\partial V_b}{\partial t} + \lambda(1 + \gamma)\theta p(t) + \mu\bar{V}_{cs}(t, \theta, \tilde{\theta}(t)). \quad (30)$$

Compared to (18), Eq. (30) includes an additional term that reflects the possibility that a common shock arrives over the next instant. In this case, the value function jumps to $\bar{V}_{cs}(t, \theta, \tilde{\theta}(t))$, given by (29).

Upon abandonment of the project at τ , the payoff to the agent is $\gamma\tilde{\theta}$, implying the boundary condition:

$$V_b(\tau, \tilde{\theta}, \theta, \tau) = \gamma\tilde{\theta}. \quad (31)$$

If we set $\tilde{\theta} = \tilde{\theta}(\tau)$ in (31), Eqs. (30)–(31) yield the value to the agent for a given abandonment timing τ and outsiders' inference function $\tilde{\theta}(\tau)$. For a given outsiders' inference function, the optimal abandonment timing τ must be such that the agent's payoff at the abandonment point satisfies the smooth-pasting condition:

$$\frac{\partial V_b}{\partial t}\bigg|_{t=\tau} = \gamma\tilde{\theta}'(\tau). \quad (32)$$

Plugging (31) and (32) into (30) yields the following equation for the agent's abandonment timing for a given outsiders' inference function:

$$-c - r\gamma\tilde{\theta}(\tau) + \lambda p(\tau)(\theta + \gamma(\theta - \tilde{\theta}(\tau))) + \gamma\tilde{\theta}'(\tau) + \mu(\bar{V}_{cs}(\tau, \theta, \tilde{\theta}(\tau)) - \gamma\tilde{\theta}(\tau)) = 0. \quad (33)$$

Eq. (33) is similar to the analogous Eq. (22) in Section 3.1. It reflects the costs and benefits of postponing the abandonment for an additional instant. The first two terms reflect the costs and the next two terms reflect the benefits. In addition, Eq. (33) includes an additional term that reflects potential arrival of the common shock. If the agent delays abandonment for an additional instant dt , the common shock will arrive over this instant with probability μdt . Upon the arrival of the common shock, the value of the project to the agent jumps to $\bar{V}_{cs}(\tau, \theta, \tilde{\theta}(\tau))$.

Next, we can apply the rational expectations condition that the agent's abandonment strategy must be consistent

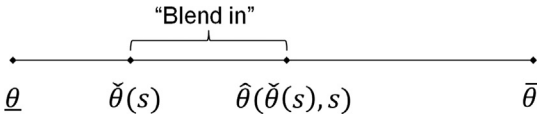


Fig. 1. Equilibrium structure upon the arrival of the shock. The figure illustrates the equilibrium abandonment dynamics when the common shock arrives at time s . Right before the arrival of the shock, types above $\hat{\theta}(s)$ have not abandoned their projects yet. When the shock arrives, types between $\hat{\theta}(s)$ and $\hat{\theta}(\hat{\theta}(s), s)$ abandon their projects even if they are not exposed to the shock and types above $\hat{\theta}(\hat{\theta}(s), s)$ abandon their projects only if they are exposed to the shock.

with outsiders' inference function. This condition implies that at the agent's chosen abandonment timing τ , $\hat{\theta}(\tau) = \theta$. Hence, we can substitute $\tau = \tau_b(\theta)$, $\hat{\theta}(\tau) = \theta$, and $\hat{\theta}(\tau) = 1/\tau_b'(\theta)$ in (33) and get the equilibrium equation for the abandonment timing $\tau_b(\theta)$:

$$-c - r\gamma\theta + \lambda\theta p(\tau_b(\theta)) + \frac{\gamma}{\tau_b(\theta)} + \mu(\bar{V}_{cs}(\tau_b(\theta), \theta, \theta) - \gamma\theta) = 0. \quad (34)$$

Note that $\bar{V}_{cs}(\tau_b(\theta), \theta, \theta)$ is the value that type θ gets at the arrival of the common shock, if she is the lowest type remaining at the time of the shock. Because $\hat{\theta}(\hat{\theta}, \tau_b(\hat{\theta})) > \hat{\theta}$, as established in Section 3.2, type θ always chooses to abandon the project in this case, even if the project is not exposed to the shock. Therefore, $\bar{V}_{cs}(\tau_b(\theta), \theta, \theta) = \gamma\theta_{blend}(\theta, \tau_b(\theta))$. Consequently, Eq. (34) reduces to

$$\frac{d\tau_b}{d\theta} = \frac{\gamma}{c + r\gamma\theta - \lambda\theta p(\tau_b(\theta)) - \mu\gamma(\theta_{blend}(\theta, \tau_b(\theta)) - \theta)}, \quad \theta \in [\underline{\theta}, \bar{\theta}]. \quad (35)$$

Compared to (23), the denominator of (35) includes an additional term $-\mu\gamma(\theta_{blend}(\theta, \tau_b(\theta)) - \theta)$. It reflects an additional incentive to delay abandonment due to the potential arrival of the common shock, which allows the agent to blend in with higher types.

Eq. (35) is solved subject to the appropriate initial value condition. Analogous to Section 3.1, the appropriate initial value condition is that the lowest type $\underline{\theta}$ abandons the project as if her abandonment decision did not reveal information to outsiders:

$$\tau_b(\underline{\theta}) = \frac{1}{\lambda} \ln \left(\frac{p_0}{1 - p_0} \left(\frac{\lambda \underline{\theta}}{c + r\gamma\underline{\theta} - \mu\gamma(\theta_{blend}(\underline{\theta}, \tau_b(\underline{\theta})) - \underline{\theta})} - 1 \right) \right). \quad (36)$$

Note that the right-hand side of (36) is the most preferred threshold of type $\underline{\theta}$ in the symmetric information framework, taking payoff upon arrival of the common shock as given. The intuition behind (36) is simple. If the lowest type did not abandon at this threshold, she would find it optimal to deviate to it. Such deviation not only would increase the direct payoff from abandonment but also could increase the reputation payoff, because the current outsiders' inference is already as low as possible. Eqs. (35)–(36) yield the separating equilibrium abandonment timing $\tau_b(\theta)$. We denote the corresponding equilibrium value function by $\bar{V}_b(t, \theta)$.

By verifying that the solution to (30)–(31) satisfies the regularity conditions in Mailath (1987), Proposition 4 shows that the argument above indeed yields the unique

separating equilibrium threshold $\tau_b(\theta)$. The proof appears in the Appendix.

Proposition 4. Let $\tau_b(\theta)$ be the increasing function that solves differential equation (35) subject to the initial value condition (36). Then, $\tau_b(\theta)$ is the abandonment threshold of type θ in the unique (up to the off-equilibrium beliefs) separating equilibrium in the range prior to the arrival of the common shock.

Taken together, Propositions 2–4 imply that there exists a unique equilibrium in which different types separate via different abandonment strategies. At most times the common shock does not arrive, so different types abandon projects at different instances: worse (lower θ) projects are abandoned earlier, and better (higher θ) projects are abandoned later. However, on rare occasions when a common shock triggers abandonment of some projects for an exogenous reason, a wave of abandonments occurs. Importantly, the wave exceeds the fraction of projects exposed to the shock: agents with bad enough projects choose to blend in with the crowd and abandon their projects together with projects exposed to the shock, even though their projects are not affected by the shock directly.

4. Model implications

In this section we analyze the economic implications of the model. We first show that even relatively small shocks cause many agents to time their abandonment decisions to periods of common shocks. Then, we show that expectations of a common shock arrival in the future feed back to the agents' abandonment strategies today, leading them to delay abandonment. Interestingly, although only low types benefit from blending in, common shocks considerably delay abandonment decisions of all types through the separation effect. This implies that industries can accumulate over time a significant number of the “living dead” projects. We quantify the delay caused by common shocks and by signaling and show that the two sources of delay affect mostly different projects. Finally, we show that small common shocks can actually improve the aggregate present value of all projects in an industry, even though they create additional frictions in abandonment.

4.1. How significant is “blending in”?

First, we quantify the magnitude of the “blending in” effect. Suppose that a fraction q of projects is exposed to the shock. Out of firms that are not exposed to the shock, fraction $(F(\hat{\theta}) - F(\check{\theta})) / (1 - F(\check{\theta}))$ abandons voluntarily, where the lowest-quality project that did not abandon by the time of the shock, $\check{\theta}$, and the higher-quality project that strategically blends in, $\hat{\theta}$, are defined in Section 3.2. To study the importance of strategic abandonment, we introduce two measures:

1. The fraction of projects that blend in out of those not exposed to the shock:

$$\phi(s) = \frac{F(\hat{\theta}(\tau_b^{-1}(s), s)) - F(\tau_b^{-1}(s))}{1 - F(\tau_b^{-1}(s))}. \quad (37)$$

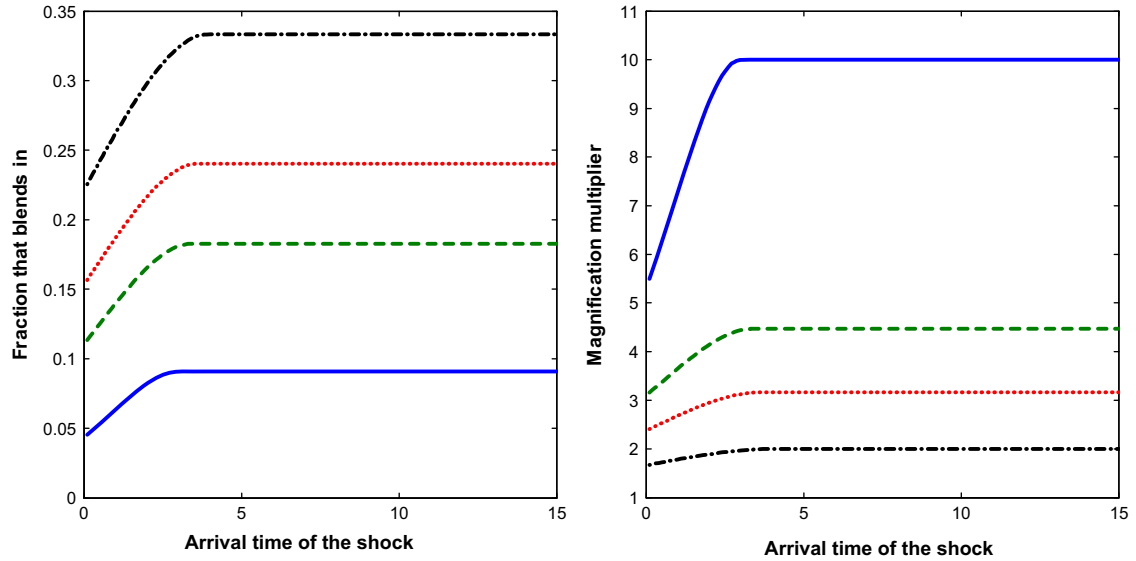


Fig. 2. The “blending in” effect. The panel plots the fraction of projects that blend in out of those not exposed to the shock, $\phi(s)$, and the magnification multiplier, $M(s)$, as a function of the arrival time of the shock. Both $\phi(s)$ and $M(s)$ are plotted for four different values of q : 0.01 (the solid line), 0.05 (the dashed line), 0.1 (the dotted line), and 0.25 (the dash-dotted line). The distribution of θ is uniform on $[1, 2]$. The other parameters are $r = c = 0.05$, $\lambda = \mu = 0.2$, $\gamma = p_0 = 0.5$.

By construction, $\phi \in [0, 1]$. If the “blending in” effect is absent, then $\phi = 0$ as only projects that are exposed to the shock are abandoned upon its arrival. If the “blending in” effect is present, then $\phi > 0$.

2. The magnification multiplier, defined as the ratio of all projects abandoned at the time of the common shock to projects that are abandoned because they are exposed to the shock:

$$M(s) = \frac{q(1 - F(\tau_b^{-1}(s))) + (1 - q)(F(\hat{\theta}(\tau_b^{-1}(s), s)) - F(\tau_b^{-1}(s)))}{q(1 - F(\tau_b^{-1}(s)))} \\ = 1 + \frac{1 - q}{q} \phi(s). \quad (38)$$

By construction, $M \geq 1$. If the “blending in” effect is absent, then the magnification multiplier is equal to one: only projects that are exposed to the common shock are abandoned upon its arrival. If the “blending in” effect is present, then the magnification multiplier exceeds one.

Fig. 2 quantifies the “blending in” effect for four different values of the fraction of firms exposed to the shock, q , for the case of uniform distribution of types. The left panel plots the fraction of projects that blend in out of those not exposed to the shock, $\phi(s)$. The right panel plots the corresponding magnification multiplier, $M(s)$. The “blending in” effect appears to be quite significant. For example, when the common shock affects 5% of outstanding projects, 17.5% of projects that are not exposed to the shock abandon strategically at the same time. This implies the magnification multiplier of 4.5: every project that got hit by the shock leads to 4.5 projects being abandoned. As the common shock becomes bigger (q is higher), the fraction of projects that are abandoned strategically also

increases. For example, a large shock affecting every fourth project triggers abandonment of almost 60% of outstanding projects. At the same time, larger shocks are associated with lower magnification multipliers. Interestingly, even very small shocks can lead to large waves of abandonments. For example, if the common shock hits the project with only 1% probability, the magnification multiplier is 10, meaning that 10% of the projects are abandoned at the shock: 1% are forced and the other 9% do so strategically.

Importantly, large waves of abandonments need not follow bad industry-wide shocks. A neutral or even positive common shock that affects projects differentially may trigger a wave of abandonments, provided that it forces some high types to abandon their projects. Recall the example in the introduction about the impact of the decline in the price of polysilicon on the US solar energy industry. Firms that relied heavily on polysilicon as an input viewed this as a positive shock, while firms whose production technologies depended on non-polysilicon raw materials viewed this as a negative shock. Some of the firms that were forced to liquidate due to their dependence on non-polysilicon raw materials could very well have been of high quality. Provided that it was difficult for outsiders to decipher if project abandonment was caused by a shock or not, such common shocks can lead to massive liquidations, far exceeding the number of projects negatively affected by the shock. This implication contrasts with most other models of clustering, in which information frictions amplify bad news, so clustering always follows a negative aggregate shock.¹²

¹² One exception is a recent model by Nanda and Rhodes-Kropf (2012). They examine “sunspot” equilibria, in which industry-wide investment busts arise without any news, because financiers collectively switch from one equilibrium to the other.

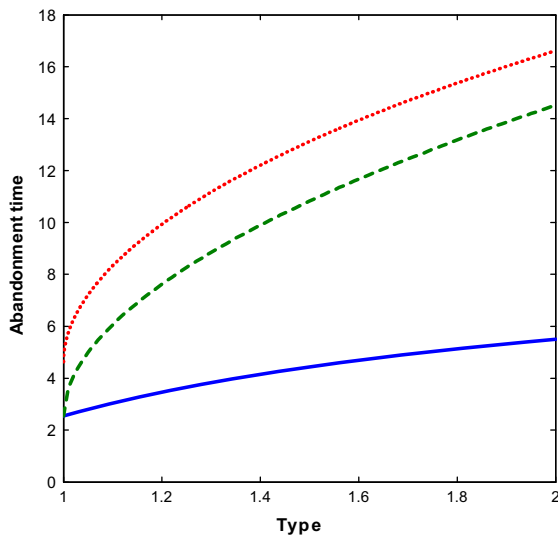


Fig. 3. Comparison of abandonment policies. The figure plots three abandonment thresholds: $\tau^*(\theta)$, abandonment threshold in the frictionless economy (the solid line); $\tau_a(\theta, \theta, 0)$, abandonment threshold under asymmetric information in the economy without common shocks (the dashed line); $\tau_b(\theta)$, abandonment threshold under asymmetric information in the economy with common shocks (the dotted line). The baseline case has $q=0.05$. The distribution of θ is uniform on $[1, 2]$. The other parameters are $r = c = 0.05$, $\lambda = \mu = 0.2$, $\gamma = p_0 = 0.5$.

4.2. Delay due to “blending in” and the “living dead”

Possible arrival of the common shock in the future creates incentives for the agent to delay abandonment today. This is evident from Section 3.3: the equilibrium abandonment timing $\tau_b(\theta)$ increases in the shock arrival intensity, μ . Intuitively, bad types have incentives to delay abandonment in an effort to pool with better types when the common shock hits. Interestingly, abandonments of all projects are delayed, even projects with very high quality θ . This might seem counter-intuitive, because agents managing projects with very high θ have little incentive to wait for the common shock and pool. Intuitively, these agents delay abandonment of their projects because the timing of abandonment is a signaling device about quality. Because lower types delay their abandonments, higher types are forced to delay their abandonments even more to separate from lower types. Hence, higher μ delays abandonment of all types.

Fig. 3 plots three abandonment thresholds: the threshold in the case of symmetric information, $\tau^*(\theta)$, the threshold in the case of asymmetric information when there are no common shocks, $\tau^a(\theta, \theta, 0)$, and the threshold in the case of asymmetric information when there is a possibility of a common shock in the future, $\tau^b(\theta)$. Fig. 3 illustrates two sources of delay compared to the case of symmetric information. The first source of delay is costly signaling: higher types delay abandonment of their projects to separate from earlier abandonments by lower types and thereby get a higher reputation payoff. This effect drives the change of the abandonment trigger from $\tau^*(\theta)$ to $\tau^a(\theta, \theta, 0)$. The second source of delay is “blending in.” Lower types have incentives to wait for the common shock

in expectation of pooling with higher types exposed to the shock. This allows lower types to get a higher reputation payoff at the expense of higher types and leads to an additional delay compared to $\tau^a(\theta, \theta, 0)$.

These effects are illustrated in more detail in Fig. 4. It shows the extent of delay caused by each of the two mechanisms. Signaling incentives lead to a delay in abandonment ranging from no delay (for the lowest type) to 160% (for the highest type) relative to the first-best abandonment timing. The additional effect of “blending in” is more uniform for all types. For the case of $q=0.01$, “blending in” leads to a delay in abandonment timing ranging from 40% (for the lowest type) to 20% (for the highest type). As the magnitude of the shock increases, the extent of delay caused by “blending in” increases as well. For example, if $q=0.05$, a typical project is delayed by 60% due to “blending in.”

An interesting consequence of delaying behavior is that many projects are kept alive even though in the first-best world they would have been abandoned a long time ago. To study this phenomenon, we measure the fraction of these firms and refer to them as the “living dead.” In the first-best setting, the agent managing a type θ project will choose to abandon it if it has not paid off for time $\tau^*(\theta)$, determined in Section 2. Let $\theta^*(\tau)$ equal the inverse of $\tau^*(\theta)$. Then, all projects whose types are less than $\theta^*(t)$ will be abandoned by time t . Similarly, in the private information setting prior to the arrival of the common shock, the agent managing a type θ project will choose to abandon it if it has not paid off for time $\tau_b(\theta)$. Let $\theta_b(\tau)$ equal the inverse of $\tau_b(\theta)$. All projects whose types are less than $\theta_b(t)$ will be abandoned by time t . Let $LD(t)$ denote the fraction of outstanding projects at time t that are among the living dead. We can write this as

$$LD(t) = \frac{F[\theta^*(t)] - F[\theta_b(t)]}{1 - F[\theta_b(t)]}. \quad (39)$$

In Fig. 5 we see that for the first 2.5 years there are no living dead because the likelihood of success is high enough to make abandonment suboptimal even in the first-best case. However, beyond that point, agents managing bad enough projects begin abandoning them in the first-best case, while a substantially smaller number of projects are abandoned in the private information case. After about five years, given the small posterior likelihood of success, all types will have abandoned in the first-best case. This implies that all of the non-abandoned projects in the private information setting are among the living dead.

This result implies that in many industries, negative NPV projects are likely to be the norm rather than an exception, especially after a long streak of stable times, during which the industry has experienced few shocks and accumulated “bad cholesterol” projects. It also explains at an intuitive level why small shocks can lead to the avalanche of industry abandonments. By the time the shock hits it is not unlikely that most outstanding projects have negative NPV. Thus, when we observe what appears to be a major collapse in which large numbers of firms exit an industry, we may actually be seeing the public exit of firms that were essentially dead for some time; they were simply waiting for an opportune time to leave in unison.

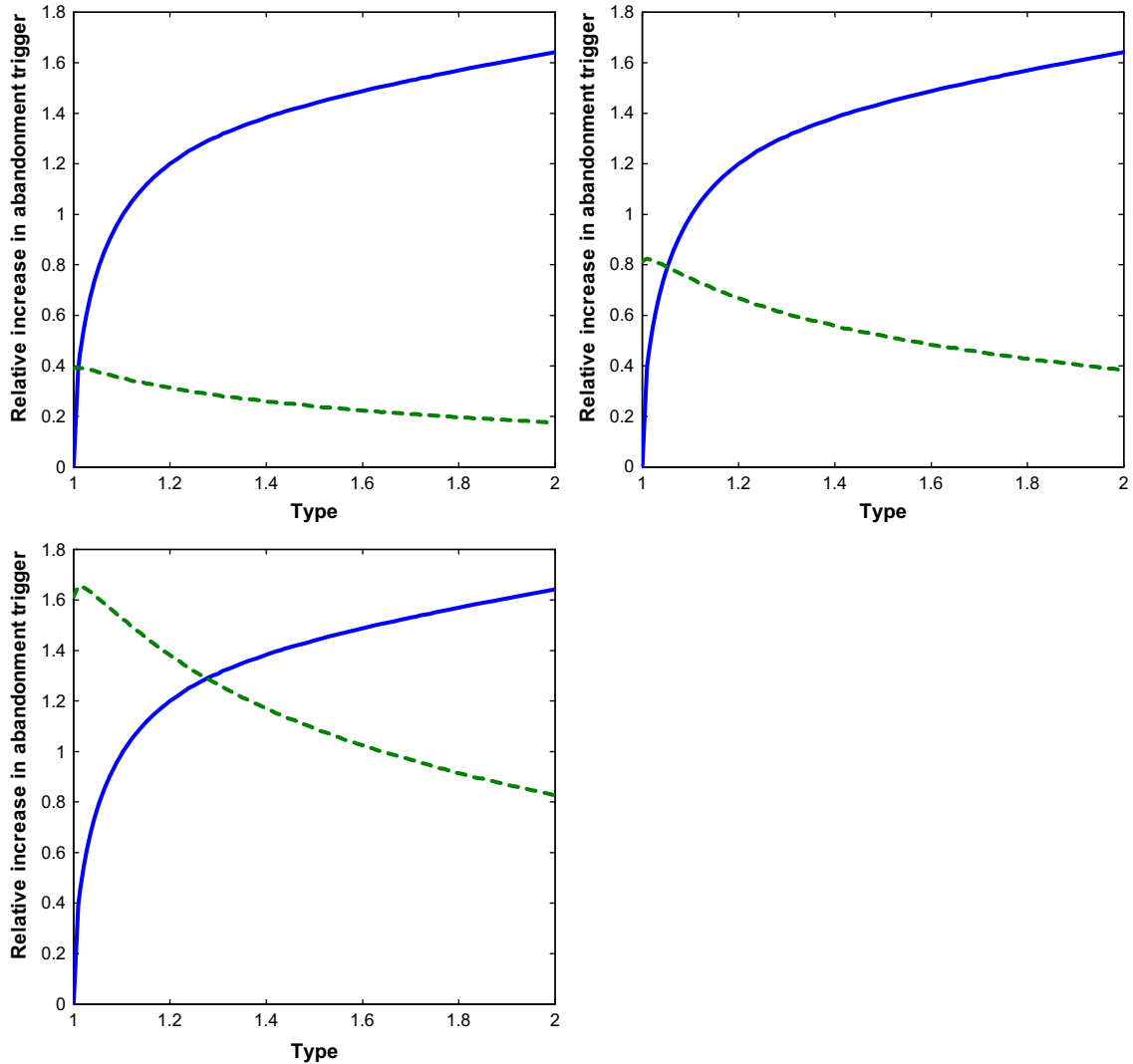


Fig. 4. Delay caused by signaling vs. delay caused by “blending in.” The figure plots the relative delay caused by signaling (the solid line), $(\tau^a(\theta, \theta, 0) - \tau^*(\theta)) / \tau^*(\theta)$, and caused by “blending in” (the dashed line), $(\tau^b(\theta) - \tau^a(\theta, \theta, 0)) / \tau^*(\theta)$, for different θ . The top-left panel plots the case of $q=0.01$. The top-right panel plots the case of $q=0.05$. The bottom panel plots the case of $q=0.25$. The distribution of θ is uniform on $[1, 2]$. The other parameters are $r=c=0.05$, $\lambda=\mu=0.2$, $\gamma=p_0=0.5$.

4.3. The value of common shocks

Although one might think that agents of all types would find the possibility of being hit by a common shock harmful, we find that virtually all types can benefit from the existence of such shocks. Consider the competing impacts of market shocks for different quality types. For a lower-quality agent, having the possibility of a common shock provides the real opportunity for blending in, and thus avoiding recognition of her low type. On the other hand, if the likelihood of a common shock becomes too high, then the agent faces the detrimental event of having a potentially good project exogenously eliminated. We should thus expect lower-quality types to prefer moderate likelihoods of common shocks, at the point at which these trade-offs are equated. For a higher-quality agent, there is

still some moderate benefit from having a possibility of a common shock. While the ability to blend in is rather unimportant for a high type, the possibility of the shock reducing the number of firms means that the cost of signaling its high type is lower, and it can abandon at closer to the first-best time. This moderate benefit is traded off against the cost of being hit by the shock and losing a potentially valuable project. We should thus expect higher-quality types to prefer low, but positive likelihoods of common shocks.

Fig. 6 illustrates this argument. The left panel shows how the expected value of the average agent changes with the intensity of the shock arrival, μ . It suggests that the average agent benefits the most from negative common shocks of moderate size. Under this set of parameter values, she would prefer that the shock have an expected

arrival time of around two years ($\mu = 0.51$). The figure showing the average expected value of agents as a function of the intensity of the shock arrival is similar. The right panel of Fig. 6 shows the value-maximizing intensity of the common shock for several different types, ranging from the lowest to the highest. For the lowest type, the optimal expected arrival time of a shock is the quickest, at around 1.5 years ($\mu = 0.68$), given that they benefit most from the

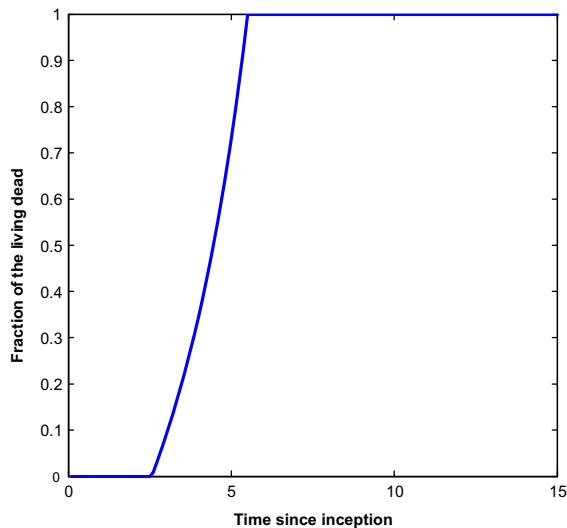


Fig. 5. The “living dead.” The figure plots the fraction $LD(t)$ of outstanding projects at time t that are among the living dead, meaning that they would have already been abandoned if information were symmetric. The distribution of θ is uniform on $[1, 2]$. The other parameters are $r = c = 0.05$, $\lambda = \mu = 0.2$, $\gamma = p_0 = 0.5$.

opportunity to blend in. Higher types usually prefer less frequent common shocks. In the extreme case, we see that for the highest type, the costs of the shock outweigh any of the benefits, and they prefer no possibility of a shock occurring.

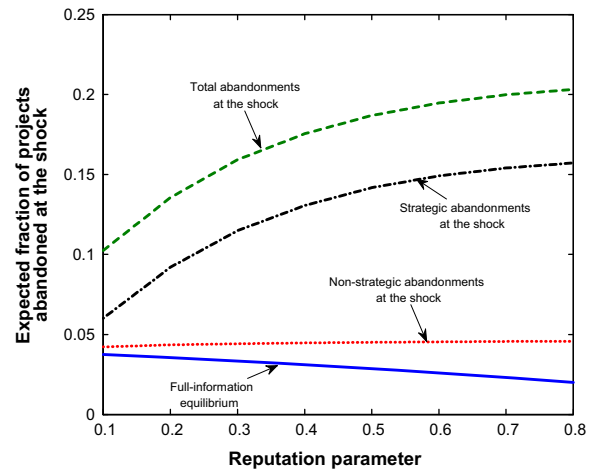


Fig. 7. The effect of reputation on abandonments at the common shock. The figure plots the expected fraction of projects abandoned at the arrival of the shock, as of date 0, as a function of the reputation parameter γ . The solid line shows this fraction under the full-information equilibrium of Section 2.2. The dashed line shows this fraction under the asymmetric-information equilibrium of Section 3. It equals the sum of the expected fraction of projects abandoned at the arrival of the shock because they are exposed to the shock (the dotted line) and because they are not exposed to the shock but choose to blend in (the dash-dotted line). The distribution of θ is uniform on $[1, 2]$. The other parameters are $r = c = 0.05$, $\lambda = \mu = 0.2$, $\gamma = p_0 = 0.5$.

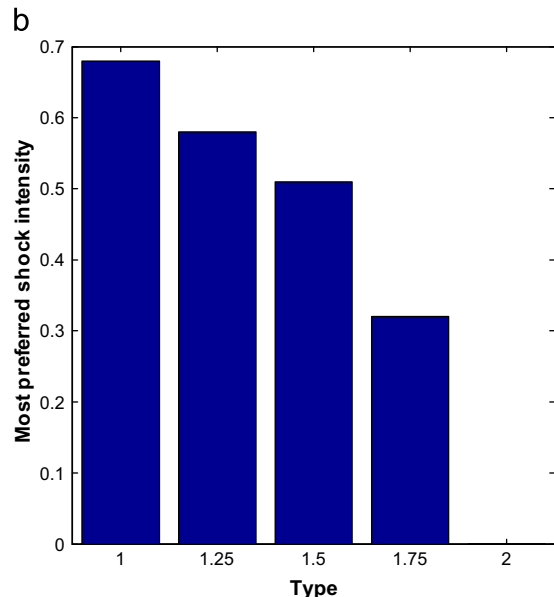
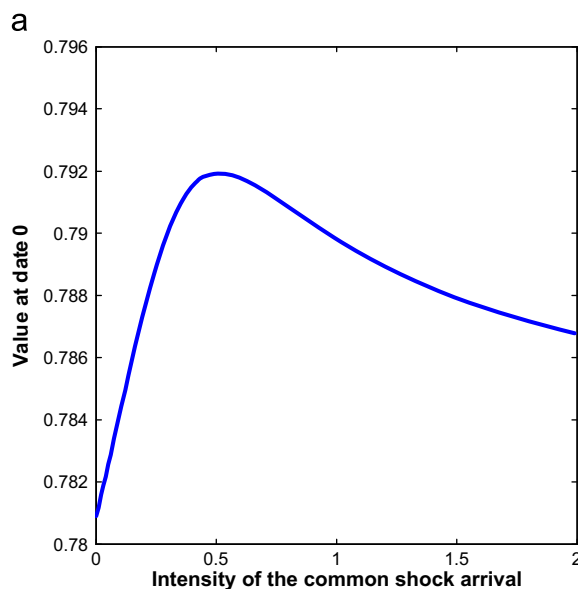


Fig. 6. The effect of the common shock on the agents' expected values. The left panel plots the expected value at date 0, $\bar{V}_b(0, \theta)$, for the average type $\theta = 1.5$, as a function of intensity of the common shock arrival, μ . The right panel plots the intensity of the common shock that maximizes the expected value at date 0 for five different types: 1, 1.25, 1.5, 1.75, and 2. The distribution of θ is uniform on $[1, 2]$. The other parameters are $r = c = 0.05$, $\lambda = \mu = 0.2$, $\gamma = p_0 = 0.5$.

4.4. The role of reputation

An important driver of the incentive to blend in is the magnitude of the reputation component. Higher levels of γ correspond to greater importance of reputation. As γ increases, agents of all types have greater incentives to delay abandonment: low-quality types have added incentives to protect their reputations by blending in, and high-quality types have added incentives to signal their quality. This effect of higher γ leading to greater delay in abandonments implies that higher γ leads to more projects being abandoned at the time of the shock. As the reputation component increases, the act of delaying abandonment leads to more projects being alive at the time of the shock. Thus, there are more projects alive that get hit by the shock, as well as a greater proportion of surviving projects that are voluntarily abandoned.

Fig. 7 displays the impact of reputation on the expected proportion of projects that are abandoned upon the arrival of the shock. In this figure, 5% of projects alive at the time of the shock experience forced abandonment. As a baseline, we see that for the case of full information, a small percentage of projects are expected to be abandoned at the time of the shock. This is depicted in the solid line. Since with full information higher γ leads to earlier abandonment, we see that increasing γ leads to a smaller fraction of projects remaining alive at the time a shock hits. For $\gamma=0.1$, under full information, 3.8% of projects are expected to be abandoned at the time of the shock, while the corresponding value for $\gamma=0.8$ is less than 2%. Thus, with full information and no incentive to signal or pool, higher reputation concerns lead to diminishing chances of witnessing clusters of abandonments.

The dotted and dash-dotted curves on Fig. 7 show the expected fraction of projects that are hit by the shock and are abandoned voluntarily, respectively, in the equilibrium with private information. Their sum, the dashed curve, shows the total expected fraction of projects abandoned at the time of the shock. Now, we find that the strength of the reputation effect greatly impacts the fraction of projects that are expected to be abandoned at the moment of a shock. For example, if $\gamma=0.10$, as of time zero we expect 10% of the industry's projects to be abandoned at the time of the shock. This means that for a low level of the reputation effect, the shock results in roughly the same number of projects that are voluntarily abandoned as the number of projects that are actually hit by the shock. More precisely, 4.2% of projects are abandoned because they are hit by the shock, while 5.8% blend in. As we increase γ to 0.6, we find that we expect around 20% of the industry to abandon at the time of the shock. This means that roughly three times as many projects are voluntarily abandoned as are actually hit by the shock. More specifically, 4.5% of projects will be hit by the shock, while 15% will blend in.

Consider these effects in the context of strategic defaults on home mortgages. As evidenced in Guiso, Sapienza, and Zingales (2013), the reputation impact of voluntary defaults on home mortgages depends on the moral and social stigma perceived by mortgagors. Thus, in an environment in which such stigma is considerable, the model predicts that a sizable proportion of defaults that occur at the time of a dramatic

shock to home prices or incomes will be voluntary rather than forced. To the extent that the social stigma of strategic default may have now declined, we should anticipate a smaller “blending in with the crowd” effect.

The model implies that what on the surface appears to have been a large negative common shock could be a relatively small (and not necessarily negative) shock combined with voluntary blending in. Thus, it would be important to see how one could empirically distinguish between a standard large industry shock (large q), and a small industry shock (small q) combined with blending in. The fact that it is likely that projects differ in their reputation incentives (i.e., γ), may allow to identify the true size of q . For example, suppose that there are two groups of projects. Group 1 contains firms with $\gamma=0.50$. Group 2 contains firms with $\gamma=0$. Thus, no firm in Group 2 has an incentive to blend in, so that by looking at what fraction of Group 2 abandoned in a cluster, we can identify q . Then, from observing what fraction of Group 1 abandoned in the cluster, one can identify the amplification multiple. Similarly, if firms (or industries) differ in the likelihoods of shock arrivals (i.e., μ), one could compare how different the firms are in delaying abandonment (i.e., $\tau_b(\theta)$) to estimate the degree of “blending in.”

Empirical estimates of cross-sectional variation in reputation concerns may be helpful in this regard. Chung, Sensoy, Stern, and Weisbach (2012) provide an interesting empirical approach to measuring the strength of reputation concerns in the private equity industry. While private equity agreements typically provide direct compensation in the form of a management fee and earned interest, there is also a reputation component of compensation. General partner lifetime incomes depend on their ability to raise capital in the future. Chung et al. find that the indirect pay for performance from future fund-raising is of the same order of magnitude as direct pay for performance. Importantly, they find that this reputation measure is heterogeneous across the industry. In particular, it is stronger for buyout funds than for venture capital funds. In the study of corporate layoffs, Agarwal and Koley (2012) identify the tenure of the chief executive officer (CEO) in the firm and the proportion of CEO compensation obtained through equity-linked instruments as measures of reputation concerns.¹³ Continuing the example from the preceding paragraph, one might obtain an estimate of q from the layoff decisions of long-tenured CEOs with low pay-for-performance compensation. Then, one could estimate the magnification multiple due to blending in from the layoff decisions of short-tenured CEOs with high pay-for-performance compensation.

5. Pooling equilibria

So far our paper focused on the separating equilibrium outside of the common shock. As a consequence, clustering

¹³ They find that CEOs with shorter tenures and greater equity-linked compensation schemes are more likely to engage in layoffs during recession months.

of abandonments could only be triggered by the arrival of the common shock. However, there may also exist pooling and semi-pooling equilibria, in which abandonments are clustered at times different from the common shock simply because agents believe that only low-quality agents abandon projects at different times.¹⁴ Here, we construct an example of a pooling equilibrium, in which all types abandon projects at time τ_{pool} outside of the common shock. Importantly, the “blending in with the crowd” effect exists in this case too: If a common shock arrives at time $s < \tau_{pool}$, a low enough type will abandon her project even if unaffected by the shock. This suggests that the focus on the separating equilibrium is not critical.

Let τ_{pool} and s be the conjectured abandonment threshold outside of the common shock and the stochastic time at which the common shock arrives, respectively. To allow for the largest set of equilibrium thresholds, we impose the harshest off-equilibrium beliefs. Specifically, if outsiders observe that the agent abandons the project at any time $\tau \notin \{\tau_{pool}, s\}$, then they believe that the agent's type is θ . The following proposition states a sufficient condition for τ_{pool} to be the pooling equilibrium threshold and shows that the arrival of the common shock can trigger abandonment of projects not exposed to the shock:

Proposition 5. Suppose that τ_{pool} satisfies

$$Y(\theta, \tau_d(\theta), \gamma\theta) \leq Y(\theta, \tau_{pool}, \gamma\mathbb{E}[\theta]) \quad \forall \theta \in [\underline{\theta}, \bar{\theta}], \quad (40)$$

$$\hat{Y}(\theta, \hat{\tau}_d(\theta), \gamma\theta, \tau_{pool}) \leq \hat{Y}(\theta, \tau_{pool}, \gamma\mathbb{E}[\theta], \tau_{pool}) \quad \forall \theta \in [\underline{\theta}, \bar{\theta}], \quad (41)$$

where $Y(\theta, \tau, R)$ and $\hat{Y}(\theta, \tau, R, \tau_{pool})$ are given by (70) and (77) in the Appendix, respectively, and $\tau_d(\theta) = \arg \max_{\tau \geq 0} Y(\theta, \tau, \gamma\theta)$ and $\hat{\tau}_d(\theta) = \arg \max_{\tau \geq 0} \hat{Y}(\theta, \tau, \gamma\theta, \tau_{pool})$. Then, there exists an equilibrium in which:

- outside of the common shock (both before and after), each type θ abandons the project that has not paid off yet at time τ_{pool} ;
- if the common shock arrives at time $s < \tau_{pool}$, types $\theta < \hat{\theta}(s)$, where $\hat{\theta}(s)$ is defined in the Appendix, abandon their projects even if they are not impacted by the shock and types $\theta \geq \hat{\theta}(s)$ abandon their projects only if they are impacted by the shock.

The intuition behind Proposition 5 is as follows. For τ_{pool} to be the pooling equilibrium threshold, no type can benefit from deviating from it. Condition (40) ensures that no type wants to deviate from abandoning the project at τ_{pool} in the period after the arrival of the common shock. Similarly, condition (41) ensures that no type wants to deviate in the period prior to the arrival of the common shock. For every type, the decision to deviate entails the trade-off between a better choice of the time to abandon the project and a lower reputation payoff upon abandonment. Conditions (40)–(41) impose bounds on τ_{pool} , ensuring that the cost of deviation always exceeds the benefit.

When a common shock hits, the agent gets an attractive opportunity to abandon her project without harming reputation and having to wait until τ_{pool} . Lower types find it costlier to wait until τ_{pool} to abandon their projects, so they choose to abandon their projects at the common shock strategically, blending in with projects impacted by the shock.

For the parameter set used to illustrate the equilibrium in Sections 3 and 4,¹⁵ conditions (40)–(41) reduce to τ_{pool} being between 2.6 years and 9.6 years. Fig. 8 illustrates the degree of strategic abandonment upon the arrival of the common shock for the case of $\tau_{pool} = 8$. Each project impacted by the shock triggers additional abandonments of up to 1.8 projects that are not impacted by the shock. Interestingly, the degree of “blending in” has an inverted U-shape relationship with the timing of the arrival of the shock. Intuitively, abandoning at the common shock very close to the inception of the project is not very attractive, since the project has a high chance of paying off later. Abandoning at the common shock very late is also not very attractive, since the agent can wait slightly longer until τ_{pool} and get a higher reputation payoff from abandoning the project then. It is the middle interval when “blending in” at the common shock is the most attractive.

6. Conclusion

In this paper, we provide a model of an industry in which agents (corporate managers or entrepreneurs) strategically decide when to stop investing in their projects. We show that when a public common shock forces a small subset of agents to abandon their projects irrespective of their quality, many agents choose to abandon their projects strategically, even if they are unaffected by the shock. Importantly, this “blending in with the crowd” effect feeds back to the agents' investment strategy even outside the shock. Because low-ability agents know that a shock may arrive in the future and offer them an opportunity to hide their private information, they choose to delay abandonment today, before the shock arrives. Delaying abandonment by low-ability agents, in turn, forces high-ability agents to delay abandonment even more to separate from low-ability agents.

These effects lead to the accumulation of “living dead” projects, implying that in many industries negative bad active projects may be the norm rather than the exception, especially during long calm periods. In such times, a small common shock that forces some high-ability agents to abandon their projects will trigger a large wave of abandonments. Such a common shock may even be neutral or positive on aggregate, rationalizing investment busts following neutral or even good news. The key requirement is that a shock negatively affects at least some high-ability agents forcing them to abandon their projects and providing a shelter for other agents to hide their private information by “blending in.”

¹⁴ Bustamante (2012) and Morellec and Schürhoff (2011) derive both separating and pooling equilibria in a real-options setting.

¹⁵ θ is uniformly distributed over $[1, 2]$, $r = 0.05$, $c = 0.05$, $\lambda = 0.2$, $\mu = 0.2$, $\gamma = 0.5$, and $p_0 = 0.5$.

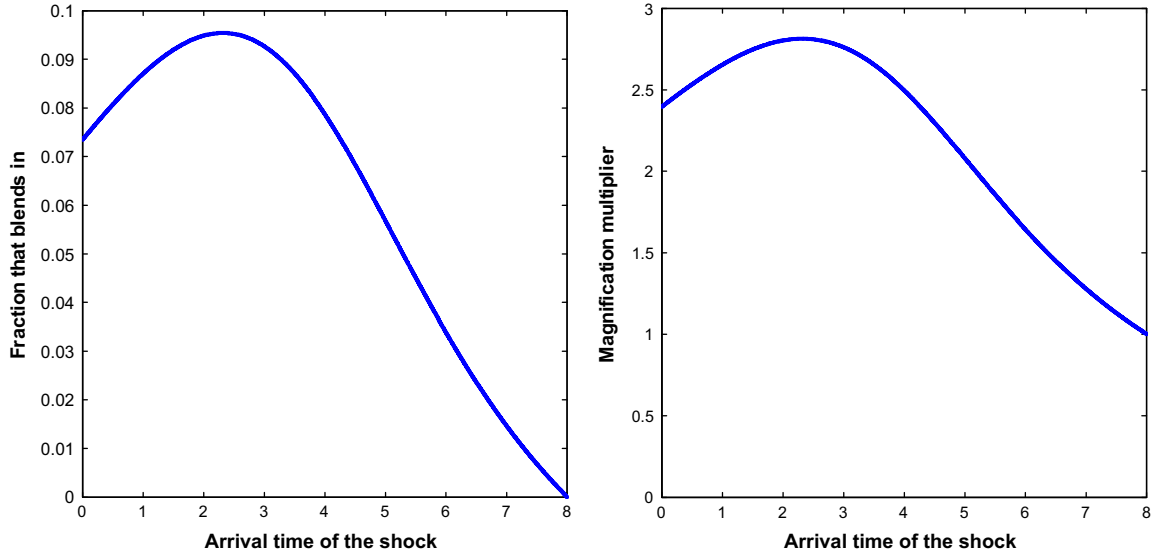


Fig. 8. The “blending in” effect in the pooling equilibrium. The panel plots the fraction of projects that blend in out of those not exposed to the shock, $\phi(s)$, and the magnification multiplier, $M(s)$, as a function of the arrival time of the shock in the pooling equilibrium. The pooling equilibrium abandonment threshold is $\tau_{pool} = 8$. The distribution of θ is uniform on $[1, 2]$. The other parameters are $r = c = 0.05$, $\lambda = \mu = 0.2$, $\gamma = p_0 = 0.5$.

We also briefly explore the possibility of “blending in with the crowd” in investment decisions. Although we suggest that a common shock needs to be very different to trigger an investment wave than an abandonment wave, a formal analysis of this issue could prove useful. Other potentially interesting avenues would be incorporating multiple common shocks, multiple projects, and fire sale discounts to the setting.

Appendix A. Proofs

Proof of Proposition 2. We prove the proposition by applying Theorems 1–3 from Mailath (1987). To do this, we show that $U(\tilde{\theta}, \theta, \tau)$ satisfies regularity conditions from Mailath (1987).

Condition 1 (Smoothness). $U(\tilde{\theta}, \theta, \tau)$ is C^2 on $[\hat{\theta}, \bar{\theta}]^2 \times [s, \infty)$. It is straightforward to see that this condition is satisfied.

Condition 2 (Belief monotonicity). $U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ never equals zero, and so is either positive or negative. Differentiating (21) with respect to $\tilde{\theta}$,

$$U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau) = \frac{1-p_0}{p_0} e^{-r\tau} \gamma + e^{-(r+\lambda)\tau} \gamma > 0. \quad (42)$$

Hence, the belief monotonicity condition is satisfied.

Condition 3 (Type monotonicity). $U_{\theta}(\tilde{\theta}, \theta, \tau)$ never equals zero, and so is either positive or negative. Differentiating (21) with respect to θ ,

$$U_{\theta}(\tilde{\theta}, \theta, \tau) = -e^{-(r+\lambda)\tau} \frac{\lambda(1+\gamma)}{r+\lambda}. \quad (43)$$

Differentiating (43) with respect to τ ,

$$U_{\theta\tau}(\tilde{\theta}, \theta, \tau) = \lambda(1+\gamma)e^{-(r+\lambda)\tau} > 0. \quad (44)$$

Hence, the type monotonicity condition is satisfied.

Condition 4 (“Strict” quasiconcavity). $U_{\tau}(\tilde{\theta}, \theta, \tau) = 0$ has a unique solution in τ , which maximizes $U(\tilde{\theta}, \theta, \tau)$, and $U_{\tau\tau}(\tilde{\theta}, \theta, \hat{\tau}) < 0$ at this solution. Consider the derivative of (21) with respect to τ when $\tilde{\theta} = \theta$:

$$U_{\tau}(\theta, \theta, \tau) = -\frac{1-p_0}{p_0} e^{-r\tau} (c+r\gamma\theta) - e^{-(r+\lambda)\tau} (c+r\gamma\theta - \lambda\theta). \quad (45)$$

Equation $U_{\tau}(\theta, \theta, \tau) = 0$ has a unique solution in τ , given by $\tau^*(\theta)$, determined in Section 2.2. The second derivative is

$$U_{\tau\tau}(\theta, \theta, \tau^*(\theta)) = -\frac{1-p_0}{p_0} e^{-r\tau^*(\theta)} \lambda(c+r\gamma\theta) < 0. \quad (46)$$

Hence, the “strict” quasiconcavity condition is satisfied.

Condition 5 (Boundedness). There exists $\delta > 0$ such that for all $(\theta, \tau) \in [\hat{\theta}, \bar{\theta}] \times [s, \infty)$ $U_{\tau\tau}(\theta, \theta, \tau) \geq 0$ implies $|U_{\tau}(\theta, \theta, \tau)| > \delta$. To ensure that the boundedness condition is satisfied, we restrict the set of potential abandonment times to be bounded by k from above, where k can be arbitrarily large. We will later show that extending the set of times to $\tau \in [s, \infty)$ neither destroys the separating equilibrium nor creates additional separating equilibria. Differentiating $U_{\tau}(\theta, \theta, \tau)$ with respect to τ ,

$$U_{\tau\tau}(\theta, \theta, \tau) = \frac{1-p_0}{p_0} e^{-r\tau} r(c+r\gamma\theta) + e^{-(r+\lambda)\tau} (r+\lambda)(c+r\gamma\theta - \lambda\theta). \quad (47)$$

Condition $U_{\tau\tau}(\theta, \theta, \tau) \geq 0$ is equivalent to $\tau \geq \tau^*(\theta) + (1/\lambda) \ln(1+\lambda/r)$. Hence, for any $(\tau, \theta) \in [\hat{\theta}, \bar{\theta}] \times [s, \infty)$,

$$\begin{aligned} |U_{\tau}(\theta, \theta, \tau)| &= e^{-r\tau} \left| \frac{1-p_0}{p_0} (c+r\gamma\theta) - e^{-\lambda\tau} (\lambda\theta - c - r\gamma\theta) \right| \\ &\geq e^{-r\tau} \left| \frac{1-p_0}{p_0} (c+r\gamma\theta) - \frac{r}{r+\lambda} e^{-\lambda\tau^*(\theta)} (\lambda\theta - c - r\gamma\theta) \right| \\ &= e^{-r\tau} \frac{1-p_0}{p_0} \frac{\lambda(c+r\gamma\theta)}{r+\lambda} \geq e^{-rk} \frac{1-p_0}{p_0} \frac{\lambda(c+r\gamma\theta)}{r+\lambda} > 0, \end{aligned} \quad (48)$$

for any arbitrarily large k . Thus, the boundedness condition is satisfied.

By Theorems 1 and 2 from Mailath (1987), any separating equilibrium abandonment threshold $\tau_a(\theta)$ is continuous, differentiable, satisfies Eq. (23), and $d\tau_a(\theta)/d\theta$ has the same sign as $U_{\theta\tau}$. Because $U_{\theta\tau} > 0$, the abandonment threshold $\tau_a(\theta)$ is increasing in θ . To ensure that the increasing solution to Eq. (23) subject to the initial value condition (24) is indeed the unique separating equilibrium, we check the single-crossing condition:

Single-crossing condition: $U_\tau(\tilde{\theta}, \theta, \tau)/U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ is a strictly monotonic function of θ . The ratio of the two derivatives is equal to

$$\frac{U_\tau(\tilde{\theta}, \theta, \tau)}{U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)} = \frac{c + r\gamma\tilde{\theta}}{\gamma} - \frac{p_0 e^{-\lambda\tau} \lambda(\gamma\tilde{\theta} - \theta - \gamma\theta)}{\gamma(p_0 e^{-\lambda\tau} + 1 - p_0)}. \quad (49)$$

Consider the derivative of $U_\tau(\tilde{\theta}, \theta, \tau)/U_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ with respect to θ :

$$\frac{p_0 e^{-\lambda\tau} \lambda(1 + \gamma)}{\gamma(p_0 e^{-\lambda\tau} + 1 - p_0)} > 0. \quad (50)$$

Hence, the single-crossing condition is verified. By Theorem 3 in Mailath (1987), the unique solution to (23)–(24), $\tau_a(\theta)$, is indeed the separating equilibrium.

Finally, it remains to prove that considering the set of abandonment times bounded by k is not restrictive. First, let us prove that $\tau_a(\theta)$ is a separating equilibrium in a problem with $\tau \in [s, \infty)$. Note that the single-crossing condition holds for all $\tau \in [s, \infty)$. Thus, local incentive compatibility guarantees global incentive compatibility for all $\tau \in [s, \infty)$. Hence, $\tau_a(\theta)$ is also a separating equilibrium in a problem with $\tau \in [s, \infty)$. Second, let us prove that no other separating equilibrium exists in a problem with $\tau \in [s, \infty)$. By contradiction, suppose that there is an additional separating equilibrium $\tau_{a2}(\theta)$, other than $\tau_a(\theta)$. Then, $\tau_{a2}(\theta)$ must be infinite for some θ ; otherwise, it would be a separating equilibrium in a problem with $\tau \in [s, k]$ for high enough k . However, an infinite abandonment threshold cannot be optimal for any $\theta \in [\tilde{\theta}, \bar{\theta}]$: over infinite time, $p(t)$ approaches zero, so it would be optimal for this type to deviate to any finite and high enough threshold at which no type abandons. Thus, there is no other separating equilibrium in a problem with $\tau \in [s, \infty)$. \square

Proof of Proposition 3. Let $\phi(\hat{\theta})$ denote the left-hand side of (26) and (28). Note that $\phi(\hat{\theta})$, $[\hat{\theta}, \bar{\theta}]$ is a continuous function that takes values in $[\tilde{\theta}, \bar{\theta}]$. Hence, by Brouwer's fixed point theorem, it has a fixed point. Taking the derivative of $\phi(\hat{\theta})$,

$$\phi'(\hat{\theta}) = \frac{(1-q)f(\hat{\theta})(\hat{\theta}(q+(1-q)F(\hat{\theta})-F(\tilde{\theta})) - \int_{\tilde{\theta}}^{\hat{\theta}} f(\theta) d\theta - q \int_{\tilde{\theta}}^{\bar{\theta}} f(\theta) d\theta)}{(q+(1-q)F(\hat{\theta})-F(\tilde{\theta}))^2}. \quad (51)$$

Hence, $\phi'(\hat{\theta})$ has the same sign as $\hat{\theta} - \phi(\hat{\theta})$. Thus, at any fixed point, $\phi'(\hat{\theta}) = 0$. The fixed point is unique, because any point at which $\phi'(\hat{\theta}) = 0$ must be a local minimum. To see this, note that according to (51) we can write $\phi'(\hat{\theta}) = \psi(\hat{\theta})(\hat{\theta} - \phi(\hat{\theta}))$ for some strictly positive and differentiable function $\psi(\hat{\theta})$. Hence,

$$\phi''(\hat{\theta}) = \psi'(\hat{\theta})(\hat{\theta} - \phi(\hat{\theta})) + \psi(\hat{\theta})(1 - \phi'(\hat{\theta})). \quad (52)$$

Around point $\hat{\theta} : \phi'(\hat{\theta}) = 0$, $\phi''(\hat{\theta}) = \psi(\hat{\theta}) > 0$. Hence, any point $\phi'(\hat{\theta}) = 0$ must be a local minimum. Therefore, the fixed point is unique. In particular, this result implies that Eq. (25) has a unique solution in $\hat{\theta}$, which is also a point at which $\phi(\hat{\theta})$ reaches its minimum.

Next, consider any p . It must be the case that $(1/\gamma)V_a(p, \hat{\theta}) \geq \hat{\theta}$ with the strict equality if and only if $p \leq p^*(\hat{\theta})$. To the left of $\hat{\theta}_m : \phi(\hat{\theta}_m) = 0$, $\phi(\hat{\theta})$ is a decreasing function of $\hat{\theta}$. Because $V_a(p, \hat{\theta})$ is a strictly increasing function in θ for any p taking values from $V_a(p, \hat{\theta})$ to $V_a(p, \hat{\theta}_m) \geq \hat{\theta}_m$, Eq. (28) has a unique solution on $\hat{\theta} \in [\tilde{\theta}, \hat{\theta}_m]$, provided that $V_a(p, \tilde{\theta}) \leq E_{\tilde{\theta}}[\theta]$. If $p > p^*(\hat{\theta}_m)$, this solution is $\hat{\theta} < \hat{\theta}_m$. If $p \leq p^*(\hat{\theta}_m)$, this solution is $\hat{\theta} = \hat{\theta}_m$. The case $V_a(p, \hat{\theta}) > E_{\tilde{\theta}}[\theta]$ corresponds to the case when no type wants to blend in with the crowd, because the option to wait is too valuable to abandon immediately even for the most optimistic belief possible.

Finally, it remains to show that Eq. (28) has no root in the range $\hat{\theta} \in (\hat{\theta}_m, \bar{\theta}]$. Note that because $\phi'(\hat{\theta}) > 0$ for any $\hat{\theta} > \hat{\theta}_m$, $\hat{\theta} > \phi(\hat{\theta})$ for any $\hat{\theta} > \hat{\theta}_m$. However, $(1/\gamma)V_a(p, \hat{\theta}) \geq \hat{\theta}$ by definition of the option. Hence, Eq. (28) has no root in the range $\hat{\theta} \in (\hat{\theta}_m, \bar{\theta}]$. \square

Proof of Proposition 4. The general solution to (30) is

$$V_b(t, \tilde{\theta}, \theta, \tau) = C \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{\lambda t} + p_0} - \frac{c}{r+\mu} + \frac{\lambda p(t)}{r+\lambda+\mu} \left(\frac{c}{r+\mu} + \theta + \gamma\theta \right) - \mu \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{\lambda t} + p_0} \int_0^t \frac{(1-p_0)e^{\lambda x} + p_0 \bar{V}_{cs}(x, \theta, \tilde{\theta}(x))}{e^{(r+\mu+\lambda)x}} dx. \quad (53)$$

To determine constant C , we use boundary (31). This gives us the following value to the agent for a given abandonment timing and outsiders' inference:

$$V_b(t, \tilde{\theta}, \theta, \tau) = \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{\lambda t} + p_0} \hat{U}(\tilde{\theta}, \theta, \tau) - \frac{c}{r+\mu} + \frac{\lambda p(t)}{r+\lambda+\mu} \left(\frac{c}{r+\mu} + \theta + \gamma\theta \right) - \mu \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{\lambda t} + p_0} \int_0^t \frac{(1-p_0)e^{\lambda x} + p_0 \bar{V}_{cs}(x, \theta, \tilde{\theta}(x))}{e^{(r+\mu+\lambda)x}} dx, \quad (54)$$

where

$$\hat{U}(\tilde{\theta}, \theta, \tau) = (1-p_0)e^{-(r+\mu)\tau} \left(\frac{c}{r+\mu} + \gamma\tilde{\theta} \right) + p_0 e^{-(r+\mu+\lambda)\tau} \left(\frac{c - \lambda(1+\gamma)\theta}{r+\lambda+\mu} + \gamma\tilde{\theta} \right) + \mu \int_0^\tau \frac{(1-p_0)e^{\lambda x} + p_0 \bar{V}_{cs}(x, \theta, \tilde{\theta}(x))}{e^{(r+\mu+\lambda)x}} dx. \quad (55)$$

Function $\hat{U}(\tilde{\theta}, \theta, \tau)$ is analogous to function $U(\tilde{\theta}, \theta, \tau)$ in the post-shock case, but includes an additional term. In (55), $\tilde{\theta}(x)$ is the inference function of outsiders and is taken as given, while $\tilde{\theta}$ is a particular point in $[\underline{\theta}, \bar{\theta}]$, which is given by the argument of the function.

Differentiating $\hat{U}(\tilde{\theta}, \theta, \tau)$ with respect to $\tilde{\theta}$:

$$\hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau) = (1-p_0)e^{-(r+\mu)\tau} \gamma + p_0 e^{-(r+\mu+\lambda)\tau} \gamma > 0. \quad (56)$$

Hence, $|\hat{U}_{\tilde{\theta}}(\theta, \theta, \tau)| \leq \gamma$, i.e., $|\hat{U}_{\tilde{\theta}}(\theta, \theta, \tau)|$ is bounded above. By Proposition 5 in Mailath (1987), the restricted initial value problem (35), (36), and $d\tau_b/d\theta > 0$ has a unique solution

on $[\underline{\theta}, \bar{\theta}]$. Let $\tau_b(\theta)$ denote this unique solution. Below we verify that it corresponds to the unique separating equilibrium threshold.

To verify this, we apply Theorems 1–3 from Mailath (1987). We do this in two steps. First, we restrict the space of admissible abandonment times τ from $[0, \infty)$ to $[0, k]$, where k is any finite number above $\tau_b(\bar{\theta})$.¹⁶ We show that in this case $\hat{U}(\tilde{\theta}, \theta, \tau)$ satisfies regularity conditions from Mailath (1987). Hence, we can apply Theorems 1–3 from Mailath (1987) to this problem and conclude that $\tau_b(\theta)$ is the unique separating equilibrium if τ is restricted to be in $[0, k]$. Second, we extend this to show that $\tau_b(\theta)$ is the unique separating equilibrium in the unrestricted problem, where $\tau \in [0, \infty)$.

First, we restrict $\tau \in [0, k]$ and check that $\hat{U}(\tilde{\theta}, \theta, \tau)$ satisfies regularity conditions from Mailath (1987).

Condition 1 (Smoothness). $\hat{U}(\tilde{\theta}, \theta, \tau)$ is C^2 on $[\underline{\theta}, \bar{\theta}]^2 \times [0, \infty)$. The condition is satisfied provided that the last term of (55) is smooth.

Condition 2 (Belief monotonicity). $\hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ never equals zero, and so is either positive or negative. This condition is a direct consequence of (56).

Condition 3 (Type monotonicity). $\hat{U}_{\theta}(\tilde{\theta}, \theta, \tau)$ never equals zero, and so is either positive or negative. Differentiating (55) with respect to θ ,

$$\hat{U}_{\theta}(\tilde{\theta}, \theta, \tau) = -p_0 e^{-(r+\mu+\lambda)\tau} \frac{\lambda(1+\gamma)}{r+\lambda+\mu} + \mu \int_0^{\tau} \frac{(1-p_0)e^{\lambda x} + p_0}{e^{(r+\mu+\lambda)x}} \bar{V}_{cs\theta}(x, \theta, \tilde{\theta}(x)) dx. \quad (57)$$

Differentiating (57) with respect to τ and noting that $\tilde{\theta}(\tau) = \tilde{\theta}$ by construction,

$$\hat{U}_{\theta\tau}(\tilde{\theta}, \theta, \tau) = e^{-(r+\lambda+\mu)\tau} [p_0 \lambda(1+\gamma) + \mu((1-p_0)e^{\lambda\tau} + p_0) \bar{V}_{cs\theta}(\tau, \theta, \tilde{\theta})]. \quad (58)$$

Because $\bar{V}_{cs}(\tau, \theta, \tilde{\theta})$ is non-decreasing in θ , $\hat{U}_{\theta\tau}(\tilde{\theta}, \theta, \tau) > 0$. Hence, the type monotonicity condition is satisfied.

Condition 4 (“Strict” quasiconcavity). $\hat{U}_{\tau}(\theta, \theta, \tau) = 0$ has a unique solution in τ , which maximizes $\hat{U}(\theta, \theta, \tau)$, and $\hat{U}_{\tau\tau}(\theta, \theta, \tau) < 0$ at this solution. Consider the derivative of (55) with respect to τ when $\tilde{\theta} = \theta$:

$$\hat{U}_{\tau}(\theta, \theta, \tau) = -((1-p_0)e^{-(r+\mu)\tau} + p_0 e^{-(r+\lambda+\mu)\tau}) \times (c + r\gamma\theta - \mu(\bar{V}_{cs}(\tau, \theta, \theta) - \gamma\theta) - \lambda\theta p(\tau)). \quad (59)$$

Note that $\bar{V}_{cs}(\tau, \theta, \theta)$ is weakly decreasing in τ . Therefore, the term on the lower line of (59) is strictly increasing in τ . Hence, equation $U_{\tau}(\theta, \theta, \tau) = 0$ has the unique solution in τ . Because $\bar{V}_{cs}(\tau, \theta, \theta) - \gamma\theta \geq 0$, the solution of $U_{\tau}(\theta, \theta, \tau) = 0$ must weakly exceed $\tau^*(\theta)$. However, at any point (τ, θ, θ) such that $\tau \geq \tau^*(\theta)$: $\bar{V}_{cs}(\tau, \theta, \theta) = \gamma\hat{\theta}(\theta)$, where $\hat{\theta}(\theta)$ is the solution of (26). Thus, the unique solution to $U_{\tau}(\theta, \theta, \tau) = 0$ in τ , defined by $\hat{\tau}^*(\theta)$, satisfies

$$c + r\gamma\theta - \mu\gamma(\hat{\theta}(\theta) - \theta) - \lambda\theta p(\hat{\tau}^*(\theta)) = 0, \quad (60)$$

which gives

$$\hat{\tau}^*(\theta) = \frac{1}{\lambda} \ln \left(\frac{p_0}{1-p_0} \left(\frac{\lambda\theta}{c + r\gamma\theta - \mu\gamma(\hat{\theta}(\theta) - \theta)} - 1 \right) \right). \quad (61)$$

The second derivative of $\hat{U}(\theta, \theta, \tau)$ with respect to τ at $\hat{\tau}^*(\theta)$ is given by

$$\hat{U}_{\tau\tau}(\theta, \theta, \hat{\tau}^*(\theta)) = - \left((1-p_0)e^{-(r+\mu)\hat{\tau}^*(\theta)} + p_0 e^{-(r+\lambda+\mu)\hat{\tau}^*(\theta)} \right) \times \left(-\mu \bar{V}_{cs\tau}(\hat{\tau}^*(\theta), \theta, \theta) + \frac{\lambda^2 \theta p_0 (1-p_0) e^{\lambda \hat{\tau}^*(\theta)}}{((1-p_0)e^{\lambda \hat{\tau}^*(\theta)} + p_0)^2} \right) < 0, \quad (62)$$

because $\bar{V}_{cs}(\tau, \theta, \theta)$ is weakly decreasing in τ . Hence, the “strict” quasiconcavity condition is satisfied.

Condition 5 (Boundedness). There exists $\delta > 0$ such that for all $(\theta, \tau) \in [\underline{\theta}, \bar{\theta}] \times [0, \infty)$ $\hat{U}_{\tau\tau}(\theta, \theta, \tau) \geq 0$ implies $|\hat{U}_{\tau}(\theta, \theta, \tau)| > \delta$. Differentiating (59) with respect to τ :

$$\hat{U}_{\tau\tau}(\theta, \theta, \tau) = ((1-p_0)(r+\mu) + p_0(r+\lambda+\mu)) \times e^{-(r+\mu)\tau} (c + r\gamma\theta - \mu(\bar{V}_{cs}(\tau, \theta, \theta) - \gamma\theta) - \lambda\theta p(\tau)) - (1-p_0 + p_0 e^{-\lambda\tau}) e^{-(r+\mu)\tau} \left(-\mu \bar{V}_{cs\tau}(\tau, \theta, \theta) + \frac{\lambda^2 \theta p_0 (1-p_0) e^{\lambda\tau}}{((1-p_0)e^{\lambda\tau} + p_0)^2} \right). \quad (63)$$

Consider any $\tau \leq \hat{\tau}^*(\theta)$. The top line of (63) is non-positive by monotonicity of the term in the last brackets. The bottom line of (63) is negative, because $\bar{V}_{cs}(\tau, \theta, \theta)$ is non-increasing in τ . Therefore, $\hat{U}_{\tau\tau}(\theta, \theta, \tau) < 0$ for any $\tau \leq \hat{\tau}^*(\theta)$. By continuity of $\hat{U}_{\tau\tau}(\theta, \theta, \tau)$ at $\hat{\tau}^*(\theta)$, $\hat{U}_{\tau\tau}(\theta, \theta, \tau) \geq 0$ is only possible if $\tau > \hat{\tau}^*(\theta) + \varepsilon$ for some $\varepsilon > 0$. Hence, $\forall (\theta, \tau) \in [\underline{\theta}, \bar{\theta}] \times [0, \infty)$: $\hat{U}_{\tau\tau}(\theta, \theta, \tau) \geq 0$:

$$|\hat{U}_{\tau}(\theta, \theta, \tau)| = e^{-(r+\mu)\tau} (1-p_0 + p_0 e^{-\lambda\tau}) (c + r\gamma\theta - \mu(\bar{V}_{cs}(\tau, \theta, \theta) - \gamma\theta) - \lambda\theta p(\tau)) > e^{-(r+\mu)k} (1-p_0) (c + r\gamma\theta - \mu\gamma(\theta_{blend}(\theta, \tau) - \theta) - \lambda\theta p(\hat{\tau}^*(\theta) + \varepsilon)) \geq \delta, \quad (64)$$

where

$$\delta \equiv e^{-(r+\mu)k} (1-p_0) \times \min_{\theta \in [\underline{\theta}, \bar{\theta}]} \{c + r\gamma\theta - \mu\gamma(\theta_{blend}(\theta, \tau) - \theta) - \lambda\theta p(\hat{\tau}^*(\theta) + \varepsilon)\}. \quad (65)$$

Note that $\delta > 0$, because $\varepsilon > 0$ and $\delta = 0$ if $\varepsilon = 0$. Thus, the boundedness condition is satisfied.

By Theorems 1 and 2 from Mailath (1987), any separating equilibrium abandonment threshold $\tau_b(\theta)$ is continuous, differentiable, satisfies Eq. (35), subject to (36), and $d\tau_b(\theta)/d\theta > 0$. Because $\tau_b(\theta)$ is the unique solution to this problem, there is at most one separating equilibrium in the restricted problem. To ensure that $\tau_b(\theta)$ is indeed the unique separating equilibrium in the restricted problem, we check the single-crossing condition:

Single-crossing condition: $\hat{U}_{\tau}(\tilde{\theta}, \theta, \tau)/\hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ is a strictly monotonic function of θ . The ratio of the two derivatives is equal to

$$\frac{\hat{U}_{\tau}(\tilde{\theta}, \theta, \tau)}{\hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)} = - \frac{c + r\gamma\tilde{\theta} - \mu(\bar{V}_{cs}(\tau, \theta, \tilde{\theta}) - \gamma\tilde{\theta})}{\gamma} - \frac{p_0 e^{-\lambda\tau} \lambda(\gamma\tilde{\theta} - \theta - \gamma\theta)}{\gamma(p_0 e^{-\lambda\tau} + 1 - p_0)}. \quad (66)$$

¹⁶ The fact that $\tau_b(\bar{\theta})$ is finite is a direct consequence of Proposition 5 from Mailath (1987).

Consider the derivative of $\hat{U}_\tau(\tilde{\theta}, \theta, \tau)/\hat{U}_{\tilde{\theta}}(\tilde{\theta}, \theta, \tau)$ with respect to θ :

$$\frac{\mu \bar{V}_{cs}(\tau, \theta, \tilde{\theta})}{\gamma} + \frac{p_0 e^{-\lambda \tau} (1 + \gamma)}{\gamma (p_0 e^{-\lambda \tau} + 1 - p_0)} > 0, \quad (67)$$

because $\bar{V}_{cs}(\tau, \theta, \tilde{\theta})$ is non-decreasing in θ . Hence, the single-crossing condition is verified. By Theorem 3 in Mailath (1987), $\tau_b(\theta)$ is indeed the separating equilibrium in the restricted problem.

Finally, it remains to prove that $\tau_b(\theta)$ is also the unique separating equilibrium in the unrestricted problem. The single-crossing condition holds for all $\tau \in [0, \infty)$. Hence, local incentive compatibility guarantees global incentive compatibility for all $\tau \in [0, \infty)$. Consequently, $\tau_b(\theta)$ is also a separating equilibrium in the unrestricted problem. Second, we prove that no other separating equilibrium exists in a problem with $\tau \in [0, \infty)$. By contradiction, suppose that there is another separating equilibrium $\tau_{b2}(\theta) \neq \tau_b(\theta)$. Then, $\tau_{b2}(\theta) = \infty$ for some θ , as otherwise, it would be a separating equilibrium in a problem with $\tau \in [0, k]$ for a high enough k . However, an infinite abandonment threshold cannot be optimal for any $\theta \in [\underline{\theta}, \bar{\theta}]$: over infinite time, $p(t)$ approaches zero, so it would be optimal for this type to deviate to any finite and high enough threshold at which no type abandons. Thus, there is no other separating equilibrium in a problem with $\tau \in [0, \infty)$. \square

Proof of Proposition 5. We solve for the equilibrium by backward induction. First, consider any time $t > s$ and the decision of type θ to abandon the project at time $\tau \geq t$. Let $W_a(t, \theta, \tau, R)$ denote the value to this agent at time t , if she follows the abandonment strategy τ , and gets the reputation payoff of R upon abandonment. By analogy with Section 3.1, it satisfies

$$(r + \lambda p(t))W_a = -c + \frac{\partial W_a}{\partial t} + \lambda p(t)(1 + \gamma)\theta, \quad (68)$$

subject to the boundary condition $W_a(\tau, \theta, \tau, R) = R$. The solution is

$$W_a(t, \theta, \tau, R) = e^{(r+\lambda)t} p(t) Y(\theta, \tau, R) - \frac{c}{r} + \frac{\lambda}{r+\lambda} \left(\frac{c}{r} + \theta + \gamma \theta \right) p(t), \quad (69)$$

where

$$Y(\theta, \tau, R) = \frac{1-p_0}{p_0} e^{-r\tau} \left(\frac{c}{r} + R \right) + e^{-(r+\lambda)\tau} \left(\frac{c-\lambda(1+\gamma)\theta}{r+\lambda} + R \right). \quad (70)$$

The beliefs of outsiders imply

$$R = \begin{cases} \gamma \mathbb{E}[\theta | \theta \geq \hat{\theta}] & \text{if } \tau = \tau_{pool}, \\ \gamma \underline{\theta} & \text{otherwise,} \end{cases} \quad (71)$$

where $\hat{\theta}$ is the lowest possible type on the equilibrium path in the post-shock case. Type θ has no incentives to deviate from τ_{pool} if and only if $Y(\theta, \tau, \gamma \underline{\theta}) \leq Y(\theta, \tau_{pool}, \gamma \mathbb{E}[\theta | \theta \geq \hat{\theta}]) \forall \tau \geq s$. Then, a sufficient condition for incentive compatibility of τ_{pool} is that (40) is satisfied for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

Second, consider time $s < \tau_{pool}$ at which the shock arrives and the decision of type θ to abandon the project. If it is exposed to the shock (with probability q), it gets abandoned irrespectively of its type θ . Otherwise, the agent may either abandon the project strategically or postpone

the abandonment. If the agent abandons the project, she gets the reputation payoff, which is independent of θ . If she postpones, she gets the highest feasible continuation payoff. Because $W_a(s, \theta, \tau, R)$ is strictly increasing in θ for $s < \tau$, the highest feasible continuation payoff is also strictly increasing in θ . Hence, like in Section 3.2, there exists type $\hat{\theta}(s) \geq \underline{\theta}$, such that the non-exposed project is abandoned if and only if $\theta < \hat{\theta}(s)$. If $W_a(s, \underline{\theta}, \tau_{pool}, \gamma \mathbb{E}[\theta]) \geq \gamma \mathbb{E}[\theta]$, i.e., the continuation payoff is higher even for the lowest type $\underline{\theta}$, then $\hat{\theta}(s) = \underline{\theta}$. Otherwise, $\hat{\theta}(s)$ is given as a unique solution to

$$\gamma \frac{\int_{\underline{\theta}}^{\hat{\theta}(s)} \theta dF(\theta) + q \int_{\hat{\theta}(s)}^{\bar{\theta}} \theta dF(\theta)}{F(\hat{\theta}(s)) + q(1 - F(\hat{\theta}(s)))} = W_a(s, \hat{\theta}(s), \tau_{pool}, \gamma \mathbb{E}[\theta | \theta \geq \hat{\theta}(s)]). \quad (72)$$

The argument from the proof of Proposition 3 applies to show that $\hat{\theta}(s) \in (\underline{\theta}, \mathbb{E}[\theta])$. Let $\bar{W}_{cs}(s, \theta, \tau_{pool})$ denote the equilibrium value to type θ at the time of arrival of the common shock but before the agent learns if her project is exposed to the shock:

$$\begin{aligned} \bar{W}_{cs}(s, \theta, \tau_{pool}) &= q \gamma \frac{\int_{\underline{\theta}}^{\hat{\theta}(s)} \theta dF(\theta) + q \int_{\hat{\theta}(s)}^{\bar{\theta}} \theta dF(\theta)}{F(\hat{\theta}(s)) + q(1 - F(\hat{\theta}(s)))} \\ &\quad + (1-q) \max \left\{ \gamma \frac{\int_{\underline{\theta}}^{\hat{\theta}(s)} \theta dF(\theta) + q \int_{\hat{\theta}(s)}^{\bar{\theta}} \theta dF(\theta)}{F(\hat{\theta}(s)) + q(1 - F(\hat{\theta}(s)))}, \right. \\ &\quad \left. W_a(s, \theta, \tau_{pool}, \gamma \mathbb{E}[\theta | \theta \geq \hat{\theta}(s)]) \right\}. \end{aligned} \quad (73)$$

If type θ does not abandon at τ_{pool} and the common shock arrives at time $s > \tau_{pool}$, then she abandons the project at the time of the shock if and only if her project is exposed to the shock, as outsiders' off-equilibrium belief is always $\underline{\theta}$. Let $\hat{W}_{cs}(s, \theta, \tau_{pool})$ denote the highest continuation payoff to type θ at the arrival of the common shock at time s . Then:

$$\hat{W}_{cs}(s, \theta, \tau_{pool}) = \begin{cases} \bar{W}_{cs}(s, \theta, \tau_{pool}) & \text{if } s < \tau_{pool}, \\ q \gamma \underline{\theta} + (1-q) W_a(s, \theta, \max\{\tau, \tau_d(\theta)\}, \gamma \underline{\theta}) & \text{if } s \geq \tau_{pool}. \end{cases} \quad (74)$$

Finally, consider the decision of type θ to abandon the project prior to the arrival of the common shock. Let $W_b(t, \theta, \tau, R, \tau_{pool})$ denote the value to this agent at time t before the common shock arrives, where upon abandonment at time τ , the agent gets a reputation payoff of R , and other agents follow the abandonment policy τ_{pool} . In the region prior to abandonment, the expected payoff to the agent satisfies:

$$(r + \lambda p(t) + \mu) W_b = -c + \frac{\partial W_b}{\partial t} + \lambda p(t)(1 + \gamma)\theta + \mu \hat{W}_{cs}(s, \theta, \tau_{pool}), \quad (75)$$

which is solved subject to the boundary condition $W_b(\tau, \theta, \tau, R, \tau_{pool}) = R$. The solution is

$$\begin{aligned} W_b(t, \theta, \tau, R, \tau_{pool}) &= \hat{Y}(\theta, \tau, R, \tau_{pool}) \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{2t} + p_0} \\ &\quad - \frac{c}{r+\mu} + \frac{\lambda p(t)}{r+\lambda+\mu} \left(\frac{c}{r+\mu} + \theta + \gamma \theta \right) \end{aligned}$$

$$-\mu \frac{e^{(r+\mu+\lambda)t}}{(1-p_0)e^{\lambda t} + p_0} \int_0^t \frac{(1-p_0)e^{\lambda x} + p_0}{e^{(r+\mu+\lambda)x}} \hat{W}_{cs}(x, \theta, \tau_{pool}) dx, \quad (76)$$

where

$$\begin{aligned} \hat{Y}(\theta, \tau, R, \tau_{pool}) = & (1-p_0)e^{-(r+\mu)\tau} \left(\frac{c}{r+\mu} + R \right) \\ & + p_0 e^{-(r+\mu+\lambda)\tau} \left(\frac{c-\lambda(1+\gamma)\theta}{r+\lambda+\mu} + R \right) \\ & + \mu \int_0^\tau \frac{1-p_0+p_0e^{-\lambda s}}{e^{(r+\mu)s}} \hat{W}_{cs}(s, \theta, \tau_{pool}) ds. \end{aligned} \quad (77)$$

Type θ has no incentives to deviate from τ_{pool} if and only if (41) is satisfied for all $\tau \geq 0$.

Therefore, any τ_{pool} such that (40) holds for all $\theta \in [\underline{\theta}, \bar{\theta}]$ and (41) holds for all $\theta \in [\underline{\theta}, \bar{\theta}]$ and $\tau \geq 0$ yields a pooling equilibrium, in which all agents abandon their projects at time τ_{pool} . If the common shock arrives at time $s < \tau_{pool}$, all types $\theta < \hat{\theta}(s)$ abandon their projects, even if they are not exposed to the shock. \square

A.1. Off-the-equilibrium-path beliefs

There are many off-the-equilibrium-path beliefs that support the equilibrium of Section 3. Below, we provide the beliefs that satisfy the D1 refinement. These beliefs are monotone: As time goes by, outsiders become weakly more optimistic about the type of the agent.

- *Prior to the arrival of the shock case:* If abandonment occurs before $\tau_b(\theta)$, then outsiders believe that the agent's type is the worst possible, i.e., $\underline{\theta}$. If abandonment occurs after $\tau_b(\bar{\theta})$, then outsiders believe that the agent's type is $\bar{\theta}$. The differential equation ensures that no type wants to deviate to an abandonment strategy that is marginally different from the equilibrium one. The single-crossing condition further ensures that no type wants to deviate to any threshold in the interval $[\tau_b(\underline{\theta}), \tau_b(\bar{\theta})]$. The off-equilibrium-path beliefs ensure that no type wants to deviate to any threshold that does not belong to $[\tau_b(\underline{\theta}), \tau_b(\bar{\theta})]$.
- *After the arrival of the common shock case:* Suppose the common shock arrives at date s , and the lowest type in the post-shock history (according to outsiders' beliefs) is $\hat{\theta}$. If abandonment occurs before $\tau_a(\hat{\theta}, \hat{\theta}, s)$, then outsiders believe that the agent's type is $\hat{\theta}$, i.e., the worst possible type in the post-shock history. If abandonment occurs after $\tau_a(\bar{\theta}, \hat{\theta}, s)$, then outsiders believe that the agent's type is $\bar{\theta}$. The differential equation and the single-crossing condition ensure that no type wants to deviate to any threshold in the interval $[\tau_a(\hat{\theta}, \hat{\theta}, s), \tau_a(\bar{\theta}, \hat{\theta}, s)]$. The off-equilibrium-path beliefs ensure that no type wants to deviate to any threshold that does not belong to $[\tau_a(\hat{\theta}, \hat{\theta}, s), \tau_a(\bar{\theta}, \hat{\theta}, s)]$.

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