Timing Decisions in Organizations: Communication and Authority in a Dynamic Environment[†]

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We consider a problem where an uninformed principal makes a timing decision interacting with an informed but biased agent. Because time is irreversible, the direction of the bias crucially affects the agent's ability to credibly communicate information. When the agent favors late decision making, full information revelation often occurs. In this case, centralized decision making, where the principal retains authority and communicates with the agent, implements the optimal decision-making rule. When the agent favors early decision making, communication is partial, and the optimal decision-making rule is not implemented. Delegation adds value when the bias is for early decision making, but not for late decision making. (*JEL* D21, D23, D82, D83)

Many decisions in organizations deal with the optimal timing of taking a certain action. Because information in organizations is dispersed, the decision maker needs to rely on the information of her better-informed subordinates who, however, may have conflicting preferences. Consider the following two examples of such settings: (i) in a typical hierarchical firm, top executives may be less informed than the product manager about the optimal timing of the launch of a new product. It would not be surprising for an empire-building product manager to be biased in favor of an earlier launch; (ii) the CEO of a multinational corporation is contemplating when to shut down a plant in a struggling economic region. While the local plant manager is better informed about the prospects of the plant, he may be biased toward a later shutdown due to personal costs of relocation.

These examples share a common theme. An uninformed principal faces an optimal stopping-time problem—when to exercise a real option. An agent is better informed than the principal but is biased toward earlier or later exercise. In this paper, we study how organizations make timing decisions in such a setting. We examine the

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effectiveness of centralized decision making, where the principal retains authority and gets information by repeatedly communicating with the agent, and develop implications for the optimal allocation of authority. We show that the economics underlying this problem are quite different from those when the decision is static rather than dynamic, and the decision variable is scale of the action rather than a stopping time, which has been the focus of most of the existing literature. In particular, for timing decisions, the key determinant of the effectiveness of communication and the value of delegating authority is the direction of the agent's bias.

Our setting combines the framework of real option exercise problems with the framework of cheap talk communication between the principal and the agent (Crawford and Sobel 1982). The principal must decide when to exercise an option whose payoff depends on an unknown parameter. The agent knows the parameter, but the agent's payoff from exercise differs from the principal's due to a bias. As a benchmark, we start by analyzing the optimal mechanism if the principal could commit to any decision rule but cannot make monetary transfers to the agent, and show that it takes the form of "interval delegation," similar to static problems.[1] If the agent favors later exercise than the principal, the optimal mechanism features the agent's most desired timing if it is early enough, and pools all types whose most desired timing is too late. If the agent favors earlier exercise than the principal, the optimal mechanism is the opposite: it features the agent's most desired timing if it is late enough, and pools all types whose most desired timing is too early.

We next examine under what conditions the principal is able to implement this optimal decision rule even if she lacks commitment power. In particular, we consider centralized decision making, where the principal has no commitment power and only relies on informal "cheap talk" communication with the agent while retaining authority over the decision. At any point in time, the agent sends a message to the principal about whether or not to exercise the option. Conditional on the received message and the history of the game, the principal chooses whether to exercise or wait. In equilibrium, the agent's communication strategy and the principal's exercise decisions are mutually optimal, and the principal rationally updates her beliefs about the agent's private information.

Our main result is that centralized decision making implements the full-commitment optimal mechanism if the agent is biased toward late exercise, but not if the agent is biased toward early exercise. The intuition for this result lies in the asymmetric nature of time as a decision variable: while the principal always has the choice to exercise at a point later than the present, she cannot do the reverse, i.e., exercise at a point earlier than the present. If the agent is biased toward late exercise, he can withhold information and reveal it later, exactly at the point where he finds it optimal to exercise the option. When the agent with a late exercise bias recommends exercise at his preferred time, the principal learns that it is too late to do so and is tempted to go back in time and exercise the option in the past. This, however, is not feasible, and hence the principal finds it optimal to follow the agent's recommendation. Knowing that, the agent communicates honestly, although communication occurs with delay relative to the principal's optimal

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[1] For example, Melumad and Shibano (1991); Alonso and Matouschek (2008); and Amador and Bagwell (2013). See the end of this section for the discussion of the related literature.

timing. Thus, the inability to go back in time commits the principal to follow the agent's recommendation, which leads to effective communication and allows the principal to implement the full-commitment optimal mechanism despite having no commitment power.

In contrast, if the agent is biased toward early exercise and recommends exercise at his most preferred time, the principal is tempted to delay the decision. Unlike changing past actions, changing future actions is possible, and hence time does not allow the principal to commit to follow the agent's recommendation. Expecting that the principal would not follow his recommendation if he recommends exercising at his most desired time, the agent deviates from this strategy. Therefore, only partial information revelation is possible and the optimal mechanism cannot be implemented if the principal has no commitment power. Moreover, unlike in the late exercise bias case, dynamic communication in the early exercise bias case is no more efficient than static communication: the equilibrium that is most informative and most preferred by the principal in the dynamic communication game also exists in the static communication game, where the players communicate only once at the beginning of the game.

These results have implications for the value of delegating decision-making authority to the agent. Because centralized decision making with communication implements the optimal decision rule when the agent favors late exercise, delegation has no additional value in this case. In contrast, when the agent favors early exercise, delegation may add additional value. In particular, simple once-and-for-all delegation dominates centralized decision making if the agent's bias is sufficiently small, which is similar to the result of Dessein (2002) for static decisions. In addition, delegation that can be timed strategically implements the optimal commitment mechanism and hence always dominates centralized decision making.

The paper proceeds as follows. The remainder of this section discusses the related literature. Section I describes the setup, and Section II solves for the optimal commitment mechanism. Section III analyzes dynamic communication and examines when it implements the optimal mechanism. Section IV compares static and dynamic communication. Section V discusses the value of delegating authority, and Section VI considers two extensions of the basic model. Finally, Section VII concludes. The Appendix presents the proofs of the main propositions and also gives a very simple example, analogous to the quadratic-uniform example in Crawford and Sobel (1982), which illustrates the intuition and findings of the paper. The online Appendix contains additional results, supplementary proofs, and the analysis of alternative versions of the model.

Related Literature.—Our paper is related to the literature that analyzes decision making in the presence of an informed but biased expert. The seminal paper in this literature is Crawford and Sobel (1982), who consider a cheap talk setting, where the expert sends a message to the decision maker and the decision maker cannot commit to the way she reacts to the message. Our paper differs from Crawford and Sobel (1982) in that communication between the expert and the decision maker is dynamic and concerns the timing of option exercise, rather than a static decision such as choosing the scale of a project. To our knowledge, ours is the first paper that studies the problem of optimal timing in a cheap talk setting. Surprisingly, even

though there is no flow of additional private information to the agent, equilibria differ substantially from those in Crawford and Sobel (1982).

Our paper also contributes to the literature on authority in organizations (e.g., Holmström 1984; Aghion and Tirole 1997; Dessein 2002; Alonso and Matouschek 2008). Gibbons, Matouschek, and Roberts (2013); Bolton and Dewatripont (2013); and Garicano and Rayo (2016) provide comprehensive reviews of this literature. Unlike Crawford and Sobel (1982), where the principal has no commitment power, the papers in this literature allow the principal to have some degree of commitment, although most of them rule out contingent transfers to the agent. Dessein (2002) assumes that the principal can commit to delegating full decision-making authority to the agent and shows that delegation dominates centralized decision making with communication if the agent's bias is not too large. Relatedly, Harris and Raviv (2005, 2008) and Chakraborty and Yilmaz (2015) analyze the optimality of delegation in settings with two-sided private information. Alonso, Dessein, and Matouschek (2008, 2014) and Rantakari (2008) compare centralized and decentralized decision making in a multidivisional organization that faces a trade-off between adapting divisions' decisions to local conditions and coordinating decisions across divisions.[2] Our paper contributes to this literature by studying the value of delegation for timing decisions and showing that it crucially depends on the direction of the agent's bias.

Other papers in this literature assume that the principal can commit to a decision rule and thus focus on a partial form of delegation: the principal offers the agent a set of decisions from which the agent can choose his preferred one. These papers include Holmström (1984); Melumad and Shibano (1991); Alonso and Matouschek (2008); Goltsman et al. (2009); Amador and Bagwell (2013); and Frankel (2014). In Baker, Gibbons, and Murphy (1999) and Alonso and Matouschek (2007), the principal's commitment power arises endogenously through relational contracts. Guo (2016) studies the optimal mechanism without transfers in an experimentation setting where the agent prefers to experiment longer than the principal. The optimal contract in her paper is time-consistent but becomes time-inconsistent if the agent prefers to experiment less than the principal, which is related to the asymmetry of our results in the direction of the agent's bias.[3] Our paper differs from this literature because it focuses on the situation where the principal has no commitment power and communicates with the agent.

Several papers analyze dynamic extensions of Crawford and Sobel (1982). In Sobel (1985); Benabou and Laroque (1992); and Morris (2001), the advisor's preferences are unknown and his messages in prior periods affect his reputation with the decision maker.[4] Aumann and Hart (2003); Krishna and Morgan (2004); Goltsman et al. (2009); and Golosov et al. (2014) consider settings with persistent private information where the principal actively participates in communication by

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[2] See also Dessein, Garicano, and Gertner (2010) and Friebel and Raith (2010). Dessein and Santos (2006) study the benefits of specialization in the context of a similar trade-off, but do not analyze strategic communication.

[3] Halac, Kartik, and Liu (2016) also analyze optimal dynamic contracts in an experimentation problem, but in a different setting and allowing for transfers.

[4] Ottaviani and Sorensen (2006a, b) study a single-period reputational cheap talk setting, where the expert is concerned about appearing well informed. Boot, Milbourn, and Thakor (2005) compare delegation and centralization when the agent's reputational concerns can distort her recommendations on whether to accept the project.

either sending messages herself or taking an action following each message of the advisor.[5] Our paper differs from this literature because of the dynamic nature of the decision problem: the decision variable is the timing of option exercise, rather than a static variable. The inability to go back in time creates an implicit commitment device for the principal to follow the advisor's recommendations and thereby improves communication, a feature not present in prior literature.

Finally, our paper is related to the literature on option exercise in the presence of agency problems. Grenadier and Wang (2005); Gryglewicz and Hartman-Glaser (2015); and Kruse and Strack (2015) study such settings but assume that the principal can commit to contracts and make contingent transfers to the agent, which makes the problem conceptually different from ours. Several papers study signaling through option exercise.[6] They assume that the decision maker is informed, while in our setting the decision maker is uninformed.

I. Model Setup

A firm (or an organization, more generally) has a project and needs to decide on the optimal time to implement it. There are two players, the uninformed party (principal, P) and the informed party (agent, A). Both parties are risk-neutral and have the same discount rate $r > 0$. Time is continuous and indexed by $t \in [0, \infty)$. The persistent type θ is drawn and learned by the agent at the initial date $t = 0$. The principal does not know θ. It is common knowledge that θ is a random draw from the uniform distribution over $\Theta = [\underline{\theta}, \overline{\theta}]$, where $0 \leq \underline{\theta} < \overline{\theta}$. Without loss of generality, we normalize $\overline{\theta} = 1$. In Section VI, we generalize our analysis to nonuniform distributions.

We focus on the case of a call option. We will refer to it as the option to invest, but it can capture any perpetual American call option, such as the option to do an initial public offering (IPO) or to launch a new product. We also extend the analysis to a put option (e.g., if the decision is about shutting down a plant) and show that the main results continue to hold (see online Appendix D).

The exercise at time t generates the payoff to the principal of $\theta X(t) - I$, where $I > 0$ is the exercise price (the investment cost), and $X(t)$ follows geometric Brownian motion with drift α and volatility σ,[7]

$$(1) \qquad\qquad dX(t) = \alpha X(t) \, dt + \sigma X(t) \, dW(t),$$

where $\sigma > 0$, $r > \alpha$, and $dW(t)$ is the increment of a standard Wiener process.[8] We assume that the starting point $X(0)$ is low enough.[9] Process $X(t)$, $t \geq 0$ is

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[5] Ely (2015) analyzes a setting with stochastically changing private information, where the informed party can commit to an information policy that shapes the beliefs of the uninformed party.

[6] See Grenadier and Malenko (2011); Morellec and Schürhoff (2011); Bustamante (2012); and Grenadier, Malenko, and Strebulaev (2014).

[7] To illustrate the intuition behind our results, we also analyze a very simple example without any stochastic structure in Appendix A. This example is similar to the quadratic-uniform setting in Crawford and Sobel (1982).

[8] Our results also hold if $\sigma = 0$ and $\alpha > 0$, i.e., when the state increases deterministically with time. If $\sigma > 0$, the sign of the drift is not important for the qualitative results.

[9] Specifically, $X(0) < \min\left(X_P^*(1), X_A^*(1)\right)$, where $X_P^*(\theta)$ and $X_A^*(\theta)$ are, respectively, the optimal exercise thresholds of the principal and the agent defined below. This assumption guarantees that there is disagreement between the two parties over the timing of exercise and that immediate exercise does not happen.

observable by both the principal and the agent. As an example, consider an oil-producing firm that owns an oil well and needs to choose the optimal time to begin drilling. The publicly observable oil price process is represented by $X(t)$. The top management of the firm has authority over the decision to drill. The regional manager has private information about how much oil the well contains (θ), which stems from his local knowledge and prior experience with neighboring wells.

While the agent knows θ, he is biased. Specifically, upon exercise, the agent receives the payoff $\theta X(t) - I + b$, where $b \neq 0$ is his commonly known bias. Positive bias $b > 0$ means that the agent is biased toward early exercise: his personal exercise price $(I - b)$ is lower than the principal's (I), so his most preferred timing of exercise is earlier than the principal's for any θ. Similarly, negative bias $b < 0$ means that the agent favors late exercise. These preferences can be viewed as reduced-form implications of an existing revenue-sharing agreement.[10] An alternative way to model the conflict of interest is to assume that $b = 0$ but the players discount the future using different discount rates. An early exercise bias corresponds to the agent being more impatient than the principal and vice versa. We have analyzed this setting and shown that the results are identical to those in the bias setting (see online Appendix D).

Following most of the literature on delegation, we do not allow the principal to make contingent transfers to the agent. In practice, decision making inside firms mostly occurs via the allocation of control rights and informal communication, and hence it is important to study such settings. A plausible rationale for this is that the allocation of control rights is a simple solution to the problem of complexity of contracts with contingent transfers. Indeed, agents in organizations usually make many decisions, and writing complex contracts that specify transfers for all decisions and all possible outcomes of each decision is prohibitively costly. Furthermore, in some organizational settings, such as in government, transfers are explicitly ruled out by law.[11]

A. Optimal Exercise Policy for the Principal and Agent

Before presenting the main analysis, we consider two simple settings: one in which the principal knows θ and the other in which the agent has formal authority to exercise the option.

Optimal Exercise Policy for the Principal.—Suppose that the principal knows θ, so communication with the agent is irrelevant. In the online Appendix, we show

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[10] For example, suppose that the principal supplies financial capital $\hat{I}$, the agent supplies human capital ("effort") valued at $\hat{e}$, and the principal and the agent hold fractions α_P and α_A of equity of the realized value from the project. Then, at exercise, the principal's (agent's) expected payoff is $\alpha_P \theta X(t) - \hat{I} \left(\alpha_A \theta X(t) - \hat{e} \right)$. This is analogous to the specification in the model with $I = \frac{\hat{I}}{\alpha_P}$ and $b = \frac{\hat{I}}{\alpha_P} - \frac{\hat{e}}{\alpha_A}$.

[11] In online Appendix D, we allow the principal to write simple compensation contracts, such as offering the agent a payment upon exercise (for the late exercise bias case) or a flow of payments until exercise (for the early exercise case). We show that the optimal compensation scheme of this type never eliminates the conflict between the agent and the principal, and hence the setting and implications of our paper are robust to allowing for simple compensation contracts.

that following the standard arguments (e.g., Dixit and Pindyck 1994), the optimal strategy for type θ is to exercise the option when $X(t)$ reaches threshold $X_P^*(\theta)$, where

$$(2) \qquad\qquad X_P^*(\theta) = \frac{\beta}{\beta - 1} \frac{I}{\theta}$$

and $\beta > 1$ is the positive root of the quadratic equation $\frac{1}{2} \sigma^2 \beta(\beta - 1) + \alpha\beta - r = 0$.

Optimal Exercise Policy for the Agent.—Suppose that the agent has formal authority over when to exercise the option. If $b < I$, then substituting $I - b$ for I in (2), the agent's optimal exercise strategy is to exercise the option at the first moment when $X(t)$ exceeds the threshold

$$(3) \qquad\qquad X_A^*(\theta) = \frac{\beta}{\beta - 1} \frac{I - b}{\theta}.$$

If $b \geq I$, the optimal exercise strategy for the agent is to exercise the option immediately.

II. Optimal Mechanism with Commitment

In this section, we solve for the optimal decision-making rule if the principal has commitment power. We characterize the optimal mechanism in the class of threshold-exercise policies. A policy is called threshold-exercise if for every type $\theta \in \Theta$, there exists a threshold $\hat{X}(\theta)$, such that the option is exercised when $X(t)$ reaches $\hat{X}(\theta)$ for the first time.[12]

By the revelation principle, we can restrict attention to direct revelation mechanisms, i.e., those in which the message space is $\Theta = [\underline{\theta}, 1]$ and that provide the agent with incentives to report his type θ truthfully. Hence, we consider mechanisms of the form $\{\hat{X}(\theta) \geq X(0), \theta \in \Theta\}$: if the agent reports θ, the principal exercises when $X(t)$ first passes threshold $\hat{X}(\theta)$. Let $\hat{U}_A(\hat{X}, \theta)$ and $\hat{U}_P(\hat{X}, \theta)$ denote the time-zero expected payoffs of the agent and the principal, respectively, when type is θ and exercise occurs at threshold $\hat{X}$. The optimal mechanism maximizes the principal's expected payoff subject to the agent's incentive compatibility (IC) constraint:

$$(4) \qquad\qquad \max_{\{\hat{X}(\theta), \theta \in \Theta\}} \int_{\underline{\theta}}^1 \hat{U}_P\big(\hat{X}(\theta), \theta\big) \frac{1}{1 - \underline{\theta}} \, d\theta$$

$$(5) \qquad\qquad \text{s.t.} \quad \hat{U}_A\big(\hat{X}(\theta), \theta\big) \geq \hat{U}_A\big(\hat{X}(\hat{\theta}), \theta\big) \quad \forall \, \theta, \hat{\theta} \in \Theta.$$

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[12] We restrict attention to mechanisms with threshold-exercise because the goal of this analysis is to provide a benchmark to analyze the effectiveness of centralized decision making, which features threshold-exercise. The solution for the optimal mechanism in a more general class of mechanisms, in particular, those that allow for randomization, is beyond the scope of the paper.

The next result characterizes the optimal threshold-exercise decision-making rule.

LEMMA 1: *The optimal incentive-compatible threshold schedule* $\hat{X}(\theta)$, $\theta \in \Theta$, *is given by the following:*

(i) *If* $b \in \left(-\infty, -\frac{1-\underline{\theta}}{1+\underline{\theta}} I\right] \cup \left[\frac{1-\underline{\theta}}{1+\underline{\theta}} I, \infty\right)$, *then* $\hat{X}(\theta) = \frac{\beta}{\beta - 1} \frac{2I}{\theta + 1}$ *for any* $\theta \in \Theta$.

(ii) *If* $b \in \left(-\frac{1-\underline{\theta}}{1+\underline{\theta}} I, 0\right]$, *then* $\hat{X}(\theta) = \begin{cases} \frac{\beta}{\beta - 1} \frac{I + b}{\theta}, & \text{if} \quad \theta < \frac{I - b}{I + b} \underline{\theta}; \\ \frac{\beta}{\beta - 1} \frac{I - b}{\theta}, & \text{if} \quad \theta \geq \frac{I - b}{I + b} \underline{\theta}. \end{cases}$

(iii) *If* $b \in \left[0, \frac{1-\underline{\theta}}{1+\underline{\theta}} I\right)$, *then* $\hat{X}(\theta) = \begin{cases} \frac{\beta}{\beta - 1} \frac{I - b}{\theta}, & \text{if} \quad \theta < \frac{I - b}{I + b}; \\ \frac{\beta}{\beta - 1} (I + b), & \text{if} \quad \theta \geq \frac{I - b}{I + b}. \end{cases}$

The lemma shows that the optimal threshold-exercise mechanism is interval delegation: the principal lets the agent choose any exercise threshold within a certain interval. This mechanism features perfect separation of types up to a cutoff and pooling beyond the cutoff. The reasoning behind this result is similar to the reasoning of why the optimal decision rule is interval delegation in many static problems (Melumad and Shibano 1991; Alonso and Matouschek 2008; Amador and Bagwell 2013). Intuitively, because the agent does not receive additional private information over time and the optimal stopping rule can be summarized by a threshold, the optimal dynamic contract is similar to the optimal contract in a static game with equivalent payoff functions.

Having derived the optimal full-commitment decision rule, we next analyze under what circumstances the principal can implement this optimal decision rule without commitment power.

III. Centralized Decision Making with Communication

Consider centralized decision making, where the principal has no commitment power and retains formal authority over the decision, while engaging in cheap talk communication with the agent. This problem is the option exercise analogue of Crawford and Sobel's (1982) static cheap talk model.

A. *Timing and Equilibrium Notion*

The timing is as follows. At each time t, knowing his type $\theta \in \Theta$ and the history of the game $\mathcal{H}_t$, the agent decides on a message $m(t) \in M$ to send to the principal, where M is a set of messages. At each t, the principal decides whether to exercise the option or not, given $\mathcal{H}_t$ and the current message $m(t)$. That is, the agent's and the principal's strategies are, respectively, $m_t : \Theta \times \mathcal{H}_t \rightarrow M$ and $e_t : \mathcal{H}_t \times M \rightarrow \{0, 1\}$, where $e_t = 1$ stands for "exercise" and $e_t = 0$ stands for

"wait." Let $\tau(e) \equiv \inf\{t : e_t = 1\}$ denote the stopping time implied by strategy e of the principal. Finally, let $\mu(\theta \mid \mathcal{H}_t)$ and $\mu(\theta \mid \mathcal{H}_t, m(t))$ denote the updated probability that the principal assigns to the type of the agent being θ given the history $\mathcal{H}_t$ before and after getting message $m(t)$, respectively.

We focus on equilibria in pure strategies. The equilibrium concept is perfect Bayesian equilibrium in Markov strategies (PBEM), which requires that the agent's and the principal's strategies are sequentially optimal, beliefs are updated according to Bayes' rule whenever possible, and the equilibrium strategies are Markov. In particular, the Markov property requires that the players' strategies are only functions of the payoff-relevant information at any time t, i.e., the type of the agent, the current value of the state process $X(t)$, and the principal's beliefs about the agent's type. The formal definition of the PBEM is presented in online Appendix A.

Bayes' rule does not apply to messages that are not sent by any type in equilibrium. To restrict beliefs following such off-equilibrium messages, we make the following assumption:

ASSUMPTION 1: *If at any t, the principal's belief $\mu(\cdot \mid \mathcal{H}_t)$ and the observed message $m(t)$ are such that no type that could exist $\big($according to the belief $\mu(\cdot \mid \mathcal{H}_t)\big)$ could send $m(t)$, then the belief is unchanged.*

This assumption is related to a frequently imposed restriction in models with two types that if, at any point, the posterior assigns probability 1 to a given type, then this belief persists no matter what happens (e.g., Rubinstein 1985; Halac 2012). Because our model features a continuum of types, an action that no one was supposed to take may occur off equilibrium even if the belief is not degenerate. As a consequence, we impose a stronger restriction.

Let stopping time $\tau^*(\theta)$ denote the equilibrium exercise time if the type is θ. In almost all standard option exercise models, the optimal exercise strategy for a perpetual American call option is a threshold: it is optimal to exercise at the first instant the state process $X(t)$ exceeds some critical level. It is thus natural to look for equilibria that exhibit a similar property, formally defined as

DEFINITION 1: *An equilibrium is a threshold-exercise PBEM if for all $\theta \in \Theta$, $\tau^*(\theta) = \inf\{t \geq 0 \mid X(t) \geq \overline{X}(\theta)\}$ for some $\overline{X}(\theta)$ (possibly infinite).*

For any threshold-exercise equilibrium, we denote the set of equilibrium exercise thresholds by $\mathcal{X} \equiv \big\{X : \exists \theta \in \Theta \text{ such that } \overline{X}(\theta) = X\big\}$. In the online Appendix, we show that any threshold-exercise equilibrium has the following two properties.

First, the option is exercised weakly later if the agent has less favorable information: $\overline{X}(\theta_1) \geq \overline{X}(\theta_2)$ whenever $\theta_2 \geq \theta_1$. Intuitively, because talk is "cheap," the agent of type θ_1 can adopt the message strategy of type $\theta_2 > \theta_1$, and vice versa. Thus, when choosing between communication strategies that induce exercise at thresholds $\overline{X}(\theta_1)$ and $\overline{X}(\theta_2)$, type θ_1 must prefer the former, and type θ_2 must prefer the latter. This is simultaneously possible only if $\overline{X}(\theta_1) \geq \overline{X}(\theta_2)$.

Second, it is without loss of generality to reduce the message space to binary messages. Intuitively, at each time the principal faces a binary decision: to exercise or to wait. Because the agent's information is important only for the timing of

exercise, one can achieve the same efficiency by choosing the timing of communicating a binary message as through the richness of the message space. Therefore, message spaces that are richer than binary cannot improve efficiency. Specifically, we show that for any threshold-exercise equilibrium, there exists an equilibrium with a binary message space $M = \{0, 1\}$ that implements the same exercise times and hence features the same payoffs of both players and takes the following simple form. At any time t, the agent can send one of two messages, 1 (exercise) or 0 (wait). The agent recommends exercise if and only if $X(t) \geq \overline{X}(\theta)$. The principal also plays a threshold strategy: if she believes that $\theta \in [\check{\theta}_t, \hat{\theta}_t]$, she exercises the option if and only if $X(t) \geq \check{X}(\check{\theta}_t, \hat{\theta}_t)$. As a consequence of the agent's strategy, there is a set $\mathcal{T}$ of "informative" times, when the agent's message has information content, i.e., it affects the belief of the principal and, in turn, her exercise decision. These are instances when $X(t)$ first passes a new threshold from the set $\mathcal{X}$. At all other times, the agent's message has no information content. In equilibrium, type θ recommends exercise at the first time when $X(t)$ passes $\overline{X}(\theta)$ for the first time, and the principal responds by exercising immediately. In what follows, we focus on threshold-exercise PBEM of this form and refer to them as simply equilibria.

B. *When Does Centralization Implement the Optimal Mechanism?*

We now examine under what conditions the optimal full-commitment mechanism can be implemented with no commitment power of the principal. In other words, when does the communication game described above have an equilibrium that features the exercise policy from Lemma 1? We show that the answer crucially depends on whether the agent is biased toward late or early exercise, specifically:

PROPOSITION 1:

(*i*) *If $b < 0$, there always exists an equilibrium of the communication game that implements the optimal mechanism from Lemma* 1*. This equilibrium is as follows:*

If $b \leq -\frac{1 - \underline{\theta}}{1 + \underline{\theta}} I$, the equilibrium is babbling and the principal exercises at the uninformed threshold $\frac{\beta}{\beta - 1} \frac{2I}{\underline{\theta} + 1}$. If $b \in \left(-\frac{1 - \underline{\theta}}{1 + \underline{\theta}} I, 0\right]$, there exists a cutoff X^*, potentially infinite, such that the principal's strategy is: (1) to wait if the agent sends message $m = 0$ and to exercise at the first time t at which the agent sends $m = 1$, provided that $X(t) \in [X_A^*(1), X^*]$ and $X(t) = \max_{s \leq t} X(s)$; (2) to exercise at the first time t at which $X(t) \geq X^*$, regardless of the agent's message. The agent's strategy is to send $m = 1$ at the first moment when $X(t)$ crosses $\min\{X_A^*(\theta), X^*\}$, and to send $m = 0$ before that. Threshold X^* is given by

$$X^* = \begin{cases} \frac{\beta}{\beta - 1} \frac{I + b}{\underline{\theta}} = X_A^*(\hat{\theta}^*) & \text{if} \quad \underline{\theta} > 0, \\ \infty & \text{if} \quad \underline{\theta} = 0, \end{cases}$$

where $\hat{\theta}^* \equiv \frac{I - b}{I + b} \underline{\theta} < 1$.

(ii) *If* $b \in \left(0, \frac{1-\underline{\theta}}{1+\underline{\theta}} I\right)$, *there is no equilibrium that implements the optimal mechanism with commitment. If* $b \geq \frac{1-\underline{\theta}}{1+\underline{\theta}} I$, *the babbling equilibrium where the principal exercises at the uninformed threshold* $\frac{\beta}{\beta-1} \frac{2I}{\underline{\theta}+1}$ *implements the optimal mechanism.*

First, consider the case of an agent biased toward late exercise. Similar to the optimal mechanism in Lemma 1, the equilibrium in Proposition 1 features full separation of types above $\hat{\theta}^*$ (with exercise at the agent's preferred threshold) and pooling of types below $\hat{\theta}^*$. The intuition is as follows. When the agent with a late exercise bias recommends exercise at his most preferred threshold, the principal learns that it is too late to exercise. Because going back in time and exercising in the past is not feasible, the principal finds it optimal to exercise immediately, i.e., to follow the agent's recommendation. Knowing that, the agent communicates honestly, but communication and exercise occur with delay relative to the principal's optimal timing. At any time before receiving the recommendation to exercise, the principal faces the following trade-off. On the one hand, she can wait and see what the agent will recommend in the future. This leads to more informative exercise because the agent communicates his information, but has a drawback in that exercise will be delayed. On the other hand, the principal can disregard the agent's future recommendations and exercise immediately. This results in less informative exercise, but not in excessive delay. Thus, the trade-off is between the value of information and the cost of delay. When the agent's bias is very large, $b \leq -\frac{1-\underline{\theta}}{1+\underline{\theta}} I$, the cost of delay is too high and induces the principal to exercise at her uninformed threshold without waiting for the agent's recommendation. When the agent's bias is moderate, $b > -\frac{1-\underline{\theta}}{1+\underline{\theta}} I$, the cost of delay is not too high and some communication occurs. As time goes by and the agent continues recommending against exercise, the principal learns that θ is not too high (below some cutoff $\hat{\theta}_t$ at time t), and the interval $[\underline{\theta}, \hat{\theta}_t]$, which captures the principal's posterior belief, shrinks over time. For any $\underline{\theta} > 0$, the shrinkage of this interval implies that the remaining value of the agent's information declines over time. Once the interval shrinks to $[\underline{\theta}, \hat{\theta}^*]$, which happens at threshold X^*, the remaining value of the agent's information becomes sufficiently small, so the principal finds it optimal to exercise immediately. The comparative statics of the cutoff type $\hat{\theta}^*$ are intuitive: as b decreases (i.e., the conflict of interest gets bigger), $\hat{\theta}^*$ increases and X^* decreases, implying that the principal waits less for the agent's recommendation. The solid red line in Figure 1 illustrates the exercise threshold in this equilibrium for parameters $\underline{\theta} = 0.15$, $r = 0.15$,[13] $\alpha = 0.05$, $\sigma = 0.2, I = 1$, and $b = -0.25$.

In contrast, if the agent is biased toward early exercise, the optimal commitment mechanism generally cannot be implemented through centralized decision making. This asymmetry occurs because of the asymmetric nature of time: the set of choices that the principal has (when to exercise) shrinks over time. When the agent favors late exercise, then even without formal commitment power, as time passes,

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[13] The discount rate 0.15 can be interpreted as the sum of the risk-free interest rate 0.05 and the intensity 0.1 with which the investment opportunity disappears.
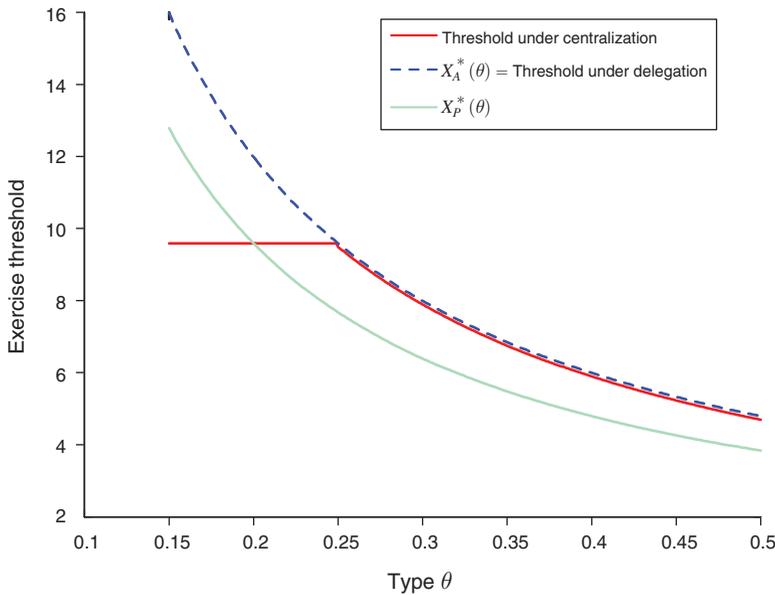
FIGURE 1. EQUILIBRIUM FOR THE CASE $\underline{\theta} > 0, b < 0$

Note: The figure presents the equilibrium with continuous exercise up to a cutoff for parameters $\underline{\theta} = 0.15$, $r = 0.15, \alpha = 0.05, \sigma = 0.2, I = 1$, and $b = -0.25$.

the principal effectively commits to follow the agent's recommendation. In contrast, no such commitment power exists in the case of an early exercise bias: if the agent recommends exercise at his preferred threshold $X_A^*(\theta)$, the principal infers the agent's type perfectly and prefers to delay exercise. Knowing this, the agent is tempted to change his recommendation strategy, mimicking a higher type. Thus, no equilibrium that features separation of types exists in this case. Because the optimal mechanism for any $b < \frac{1 - \underline{\theta}}{1 + \underline{\theta}} I$ features separation of types over some interval, it cannot be implemented.

Note also that a special case of the equilibrium in Proposition 1 is $\underline{\theta} = 0$. As long as the agent's bias is not very high, $b > -I$, there is full information revelation, but communication and exercise are inefficiently (from the principal's perspective) delayed. Using the terminology of Aghion and Tirole (1997), the equilibrium features unlimited real authority of the agent, even though the principal has unlimited formal authority. The reason why this equilibrium is equivalent to full delegation of authority to the agent, rather than delegation up to a finite cutoff, is that when $\underline{\theta} = 0$, the problem exhibits stationarity in the following sense. Because the prior distribution of types is uniform over $[0, 1]$ and the payoff structure is multiplicative, a time-t subgame in which the principal's posterior belief is uniform over $[0, \hat{\theta}]$ is equivalent to the game where the belief is that θ is uniform over $[0, 1]$, the true type is $\frac{\theta}{\hat{\theta}}$, and the modified state process is $\hat{\theta}X(t)$. Because of stationarity, the trade-off between the value of information and the cost of delay persists over time even though the principal updates her belief: as long as the agent's bias is not too high ($b > -I$), the principal finds it optimal to wait for the agent's recommendation for any current belief.

IV. Dynamic versus Static Communication

Proposition 1 shows that in the late exercise bias case, the ability to communicate dynamically is extremely beneficial in that it allows one to implement the optimal mechanism, but that it is not as beneficial in the early exercise bias case. In this section, we highlight this asymmetry even further by showing that when the agent favors early exercise, dynamic communication not only does not help implement the optimal mechanism, but it is actually no more efficient than static communication. To show this, we first characterize the equilibria of the dynamic communication game and then compare them with the equilibria of the static communication game.

For tractability, we focus on the case $\underline{\theta} = 0$. The stationary nature of the game for $\underline{\theta} = 0$ allows us to fully characterize the equilibria of the dynamic communication game. It is natural to restrict attention to *stationary* equilibria, where both the message of type $\theta \in [0, \hat{\theta}]$ and the exercise strategy of the principal are the same when the public state is $X(t)$ and the posterior belief is that θ is uniform over $[0, \hat{\theta}]$ as when the state is $\hat{\theta}X(t)$ and the posterior belief is that θ is uniform over $[0, 1]$. The formal definition is provided in online Appendix A.

Stationarity and the property $\overline{X}(\theta_1) \geq \overline{X}(\theta_2)$ for $\theta_2 \geq \theta_1$ imply that any stationary equilibrium must take one of two forms.[14] The first is an equilibrium that features continuous exercise at the agent's optimal threshold $X_A^*(\theta)$, i.e., the equilibrium characterized in the first part of Proposition 1. The second are equilibria that have a partition structure, with the set of types partitioned into intervals and each interval inducing exercise at a given threshold. Moreover, stationarity implies that the set of partitions must be infinite and take the form $[\omega, 1], [\omega^2, \omega], \ldots, [\omega^n, \omega^{n-1}], \ldots,$ $n \in \mathbb{N}$, for some $\omega \in [0, 1)$, where $\mathbb{N}$ is the set of natural numbers. This implies that the set of exercise thresholds $\mathcal{X}$ is given by $\left\{ \overline{X}, \frac{\overline{X}}{\omega}, \frac{\overline{X}}{\omega^2}, \ldots, \frac{\overline{X}}{\omega^n}, \ldots \right\}$, $n \in \mathbb{N}$, for some $\overline{X} > 0$, such that if $\theta \in (\omega^n, \omega^{n-1})$, the option is exercised at threshold $\frac{\overline{X}}{\omega^{n-1}}$. We refer to an equilibrium of this form as a ω-*equilibrium* and illustrate it in Figure 2.

For ω and $\overline{X}$ to constitute an equilibrium, the IC conditions for the principal and the agent must hold. Pair $(\omega, \overline{X})$ satisfies the agent's IC condition only if types above ω have incentives to recommend exercise at threshold $\overline{X}$ rather than to wait, whereas types below ω have incentives to recommend delay. The proof of Proposition 2 shows that the agent's IC condition holds if and only if type ω is exactly indifferent between exercising the option at threshold $\overline{X}$ and at threshold $\frac{\overline{X}}{\omega}$, and this indifference condition reduces to the constraint

$$(6) \qquad \overline{X} = Y(\omega) \equiv \frac{(1 - \omega^\beta)(I - b)}{\omega(1 - \omega^{\beta-1})}.$$

Next, consider the principal's problem. For ω and $\overline{X}$ to constitute an equilibrium, the principal must have incentives: (i) to exercise immediately when the agent sends

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[14] The argument is as follows. If there is separation (pooling) of types between some cutoff $\hat{\theta}$ and 1, there must also be separation (pooling) of types between $\hat{\theta}^2$ and $\hat{\theta}$. Iterating this argument implies that either all types separate or there is a sequence of partitions, each being a multiple of the previous one.
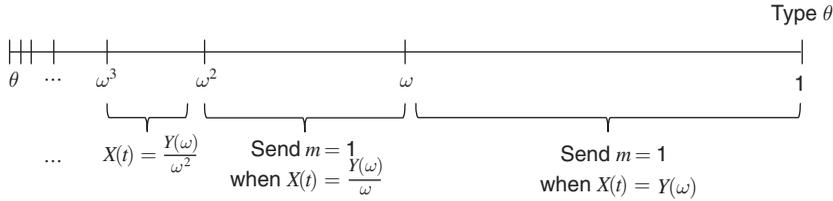
FIGURE 2. PARTITIONS IN A ω-EQUILIBRIUM

message $m = 1$ at a threshold in $\mathcal{X}$; and (ii) not to exercise before getting $m = 1$. We refer to the former (latter) IC condition as the ex post (ex ante) IC constraint. The proof of Proposition 2 shows that when $b < 0$, the principal's ex post IC constraint is satisfied for any $\omega \in (0, 1)$. Intuitively, this is because the agent is biased toward late exercise, and hence the principal does not benefit from further delay. In contrast, when the agent favors early exercise, the ex post IC constraint is satisfied if and only if ω is low enough. The intuition why the ex post IC constraint is violated if ω is large is similar to the standard intuition of why sufficiently efficient information revelation is impossible in cheap talk games: because the agent has an early exercise bias and the principal can wait and exercise later after getting the agent's message to exercise, the agent's message cannot be too informative about his type. Formally, we show that for any $b \in (0, I)$, there exists a unique $\omega^* \in (0, 1)$ for which the ex post IC constraint is satisfied as an equality, and that the principal's ex post IC condition is satisfied if and only if $\omega \leq \omega^*$.

Finally, the principal's ex ante IC condition is satisfied if and only if communication is informative enough, which puts a lower bound on ω, denoted $\underline{\omega} > 0$. The set of equilibria with partitioned exercise is illustrated in Figure B.1 of the online Appendix.

The following proposition summarizes the set of all stationary equilibria:

PROPOSITION 2: *If $b \in [-I, I)$, the set of nonbabbling stationary equilibria is given by:*

(i) *Equilibria with partitioned exercise (ω-equilibria) exist if and only if $b \in (-I, I)$. If $b \in (-I, 0)$, there exists a unique ω-equilibrium for each $\omega \in [\underline{\omega}, 1)$, and if $b \in (0, I)$, there exists a unique ω-equilibrium for each $\omega \in [\underline{\omega}, \omega^*]$, where $0 < \underline{\omega} < \omega^* < 1$, ω^* is the unique solution to $Y(\omega) = \frac{\beta}{\beta - 1} \frac{2I}{\omega + 1}$, and $\underline{\omega}$ is uniquely defined by the condition that the principal's ex ante IC constraint is binding. In the ω-equilibrium, the principal exercises at time t at which $X(t)$ crosses threshold $Y(\omega), \frac{1}{\omega} Y(\omega), \dots$ for the first time, provided that the agent sends message $m = 1$ at that point, where $Y(\omega)$ is given by (6). The principal does not exercise the option at any other time. The agent of type θ sends $m = 1$ at the first moment when $X(t)$ crosses threshold $\frac{1}{\omega^n} Y(\omega)$, where $n \geq 0$ is such that $\theta \in (\omega^{n+1}, \omega^n)$.*

(ii) *Equilibrium with continuous exercise exists if and only if $b \in [-I, 0)$. The principal exercises at the first time t at which the agent sends $m = 1$,*

provided that $X(t) \geq X_A^*(1)$ and $X(t) = \max_{s \leq t} X(s)$. The agent of type θ sends $m = 1$ at the first moment when $X(t)$ crosses his preferred threshold $X_A^*(\theta)$.

If $b < -I$ or $b \geq I$, the only stationary equilibrium is babbling, where the principal exercises the option at her optimal uninformed threshold $\overline{X}_u = \frac{\beta}{\beta - 1} 2I$.

Not all of these equilibria are equally reasonable. In online Appendix B, we show that when $b < 0$, the equilibrium with continuous exercise Pareto dominates all others: both the principal's expected payoff and the agent's expected payoff for each realization of θ are higher than in any other possible equilibrium. Similarly, when $b > 0$, the ω^*-equilibrium dominates other stationary equilibria in the sense of yielding both a higher expected payoff to the principal and a higher ex ante (before θ is realized) expected payoff to the agent.

It is instructive to highlight the role of dynamic communication by comparing the equilibria above to those in the benchmark model, where communication is static and limited to a one-shot interaction at the beginning of the game. Specifically, consider a restricted version of the model, where instead of communicating with the principal continuously, the agent sends a single message at time $t = 0$ and there is no subsequent communication. After receiving the message, the principal updates her beliefs about θ and exercises the option at the optimal threshold given these beliefs. The following proposition examines which stationary equilibria of the dynamic game from Proposition 2 have equivalent equilibria in the static communication game:

PROPOSITION 3: *If $b < 0$, there is no nonbabbling stationary equilibrium of the dynamic communication game that is also an equilibrium of the static communication game. If $b > 0$, the only nonbabbling stationary equilibrium of the dynamic communication game that is also an equilibrium of the static communication game is the ω^*-equilibrium.*

The intuition is as follows. All nonbabbling stationary equilibria of the dynamic communication game for $b < 0$ feature delay relative to what the principal's optimal timing of exercise would have been ex ante, given the information she learns in equilibrium. In a dynamic communication game, this delay is feasible because the principal learns information with delay, after her optimal (conditional on this information) exercise time has passed. In contrast, in a static communication game, this delay cannot be sustained: since the principal learns all the information at time zero, her exercise decision is always optimal given the available information.[15] By the same argument, the only sustainable equilibrium of the dynamic communication game for $b > 0$ is the one that features no delay relative to the principal's optimal threshold, i.e., the ω^*-equilibrium.

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[15] For the same reason, in the nonstationary case, the equilibrium with continuous exercise up to a cutoff, described in Proposition 1, does not exist in the static communication game either.

Proposition 3 further emphasizes that the ability to communicate dynamically is crucial when the agent favors late exercise, but not when he favors early exercise. In particular, when $b > 0$, dynamic communication is not only unable to implement the optimal mechanism, but is also no more efficient than static communication: the ω^*-equilibrium, which is both most informative and most preferred by the principal in the dynamic communication game, also exists in the static communication game.

V. Implications for the Value of Delegation

Our analysis has implications for the value of delegating authority over timing decisions. In particular, the principal can either keep authority and play the communication game analyzed in the previous sections or can delegate authority to exercise the option to the agent. Our results imply that in the context of timing decisions, the direction of the conflict of interest is the key determinant of whether delegating authority adds additional value. The formal comparison of delegation and centralized decision making is presented in online Appendix C, and we briefly summarize the results in this section.

We first compare centralized decision making to once-and-for-all delegation. Because centralized decision making implements the optimal commitment mechanism when the agent favors late exercise, the principal is weakly better off (and strictly better off if $\underline{\theta} > 0$) retaining control and getting advice from the agent rather than delegating the exercise decision once-and-for-all. This is different from the implications for static decisions, where delegation dominates centralized decision making if the agent's bias is sufficiently small (Dessein 2002). In contrast, when the agent favors early exercise, communication is not as efficient and delegation can dominate because it allows for more effective use of the agent's information. We show that the trade-off between information and bias makes delegation superior when the agent's bias is sufficiently small. This result is similar to the result of Dessein (2002) for static decisions, which is expected given that the most efficient equilibrium of the dynamic communication game also exists in the static communication game.

In a dynamic setting, delegation does not have to be once-and-for-all but can be time-contingent: the principal can retain authority for some time and delegate it later or can take authority from the agent after some period of time. We show that there always exists a time-contingent delegation policy that implements the optimal mechanism in Lemma 1. When the agent favors early exercise, this policy involves waiting and not exercising the option until $X(t)$ reaches some cutoff level and delegating authority to the agent after that moment. When the agent favors late exercise, this policy involves delegating authority to the agent at the beginning, but then taking authority away and exercising the option at the first moment when $X(t)$ reaches a certain threshold. Therefore, time-contingent delegation is equivalent to centralized decision making in the case of a late exercise bias, but is superior in the case of an early exercise bias.

To sum up, this analysis suggests that the ability to delegate authority adds additional value if the agent has an early exercise bias, but has no additional value if the bias is toward late exercise.

VI. Extensions

We develop two extensions of the basic model. First, we relax the assumption that the distribution of types is uniform and extend our main results to a large class of distributions. Second, we analyze a setting in which at any point in time, the principal might learn the agent's information with some probability, even without any communication from the agent.

A. *General Distribution*

So far, we have assumed that the distribution of types θ is uniform. While this assumption makes the analysis more tractable, it is not critical for the main results. Suppose that type θ is drawn from a continuous distribution Φ with support $\left[\underline{\theta}, \overline{\theta}\right]$, where $0 < \underline{\theta} < \overline{\theta}$, and strictly positive continuous density ϕ. Assume that the distribution satisfies the following assumption:

ASSUMPTION 2: *Distribution Φ is such that: (i) $\Phi(\theta) + \frac{b}{I}\theta\phi(\theta)$ is nondecreasing for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$; (ii) the equation $E\left[\tilde{\theta}\,|\,\tilde{\theta} \leq \theta\right] = \frac{I}{I-b}\theta$ has at most one solution on $\left[\underline{\theta}, \overline{\theta}\right]$ for $b < 0$.*

The next proposition presents an analog of our main results for a general distribution satisfying Assumption 2.

PROPOSITION 4: *Suppose that Assumption 2 holds and $b \in \left(-\frac{\overline{\theta} - E[\theta]}{E[\theta]}\,I,\, \frac{E[\theta] - \underline{\theta}}{E[\theta]}\,I\right)$. For $b \in \left(-\frac{\overline{\theta} - E[\theta]}{E[\theta]}\,I,\, 0\right)$, there is a unique solution to $E\left[\tilde{\theta}\,|\,\tilde{\theta} \leq \theta\right] = \frac{I}{I-b}\theta$; denote it θ_L. For $b \in \left(0,\, \frac{E[\theta] - \underline{\theta}}{E[\theta]}\,I\right)$, there is at least one solution to $E\left[\tilde{\theta}\,|\,\tilde{\theta} \geq \theta\right] = I\frac{\theta}{I-b}$; denote the highest one by θ_H. Then,*

(i) *The optimal incentive-compatible threshold schedule $\hat{X}(\theta), \theta \in \Theta$, is given by*
$$\hat{X}(\theta) = \frac{\beta}{\beta - 1}\frac{I - b}{\max\{\theta, \theta_L\}} \quad \text{if} \quad b \in \left(-\frac{\overline{\theta} - E[\theta]}{E[\theta]}\,I,\, 0\right), \quad \text{and} \quad \hat{X}(\theta) = \frac{\beta}{\beta - 1}$$
$$\times \frac{I - b}{\min\{\theta, \theta_H\}} \text{ if } b \in \left(0,\, \frac{E[\theta] - \underline{\theta}}{E[\theta]}\,I\right).$$

(ii) *If $b \in \left(-\frac{\overline{\theta} - E[\theta]}{E[\theta]}\,I,\, 0\right)$, centralized decision making with communication implements the optimal mechanism from part 1. Specifically, there exists the following equilibrium, which implements $\hat{X}(\theta)$. The principal's strategy is: (1) to wait if the agent sends $m = 0$ and to exercise at the first time t at which the agent sends $m = 1$, provided that $X(t) \in \left[X_A^*(\overline{\theta}), X_A^*(\theta_L)\right]$ and $X(t) = \max_{s \leq t} X(s)$; (2) to exercise at the first time t at which $X(t) \geq X_A^*(\theta_L)$, regardless of the agent's message. The agent's strategy is to send $m = 1$ at the first moment when $X(t)$ crosses $X_A^*(\max\{\theta, \theta_L\})$, and to send $m = 0$ before that.*

(iii) *If* $b \in \left(0, \frac{E[\theta] - \underline{\theta}}{E[\theta]} I\right)$, *there is no equilibrium that implements the optimal mechanism from part 1.*

The argument behind Proposition 4 is the same as in the model with uniformly distributed types, but it can be helpful to highlight the sufficient conditions on the distribution. Restriction (i) of Assumption 2 is identical to the restriction in the static delegation problem of Amador and Bagwell (2013) and guarantees that the optimal contract in part 1 of the proposition is interval delegation.[16] Restriction (ii) of Assumption 2 is an additional restriction that is only needed to prove part 2 of the proposition. It requires that the agent's optimal exercise threshold for the highest outstanding type $\hat{\theta}$ and the principal's optimal exercise threshold given the belief $\theta \in \left[\underline{\theta}, \hat{\theta}\right]$ cross only once at θ_L $\left(\text{if } b \in \left(-\frac{\overline{\theta} - E[\theta]}{E[\theta]} I, 0\right)\right)$ or never $\left(\text{if } b < -\frac{\overline{\theta} - E[\theta]}{E[\theta]} I\right)$. This condition implies that in the proof of part 2, it is sufficient to verify the principal's ex ante IC constraint at first-passage times. In the online Appendix, we show that Assumption 2 is satisfied for the uniform and truncated standard normal distributions, and that the power distribution $\Phi(\theta) = \left(\frac{\theta - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right)^{\alpha}$ satisfies Assumption 2 if and only if $\alpha \leq 1$.

B. *Arrival of News about the Project*

In this section, we show that the structure of the equilibrium in the delay bias case remains the same even if the principal can independently learn θ with some probability. Specifically, we consider the same setup as before, but introduce a Poisson news arrival process that reveals type θ to the principal upon arrival. The arrival rate is $\lambda > 0$. In this setting, the equilibrium under centralized decision making with communication takes the following form:

PROPOSITION 5: *If* $b \in \left(-\frac{1 - \underline{\theta}}{1 + \underline{\theta}} I, 0\right)$, *there exists the following equilibrium. The principal's strategy after the arrival of the news is to exercise the option at the first time t at which $X(t) \geq X_P^*(\theta)$. The principal's strategy prior to the arrival of the news is: (i) to wait if the agent sends $m = 0$ and to exercise at the first time t at which the agent sends $m = 1$, provided that $X(t) \in \left[\tilde{X}_A(1), \check{X}\right]$ and $X(t) = \max_{s \leq t} X(s)$; (ii) to exercise at the first time t at which $X(t) \geq \check{X}$, regardless of the agent's message, where threshold $\check{X}$ and function $\tilde{X}_A(\theta)$ are defined in the online Appendix. The agent's strategy is to send $m = 1$ at the first moment when $X(t)$ crosses $\min\{\tilde{X}_A(\theta), \check{X}\}$, and to send $m = 0$ before that. Furthermore, $X_P^*(\theta) < \tilde{X}_A(\theta) < X_A^*(\theta)$.*

As in the basic model, there is full separation of types up to a certain cutoff, but communication occurs with delay. However, the delay in communication is

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[16] If equation $E[\tilde{\theta} | \tilde{\theta} \leq \theta] = \frac{I}{I - b} \theta$ has multiple solutions, i.e., part (ii) of Assumption 2 is violated, define θ_L as the lowest solution.

smaller than in the basic model: $\tilde{X}_A(\theta) < X_A^*(\theta)$. Intuitively, consider the agent who chooses between recommending exercising now and delaying for a little bit. The value of delaying the recommendation is lower in the extended model than in the basic model: unlike in the basic model, where the principal waits until the agent's recommendation to exercise, there is now a possibility that the principal will learn θ and not wait. Because the agent's equilibrium threshold is determined from his indifference between recommending exercising at that threshold and delaying it for a marginal amount of time and because the value of delay is lower, the agent recommends exercise earlier than in the model without news.

This implies that the possibility of learning θ has two positive effects on the principal's payoff. The direct effect is that the principal might learn θ independently and exercise the option at a better time than when the agent would recommend doing it. The indirect effect is that the possibility of news arrival improves communication, $\tilde{X}_A(\theta) < X_A^*(\theta)$.

VII. Conclusion

This paper studies timing decisions in organizations. We consider a problem in which an uninformed principal is deciding when to exercise an option and has to rely on the information of a better-informed but biased agent. Our results emphasize that the effectiveness of communication between the agent and the principal, as well as the value of delegating authority over timing decisions, crucially depend on whether the agent is biased toward early or late exercise.

We first examine centralized decision making, where the principal retains authority and repeatedly communicates with the agent via cheap talk. When the agent favors late exercise, there is often full information revelation but suboptimal delay in option exercise. Moreover, decision making under centralized decision making implements the optimal full-commitment mechanism without transfers even though the principal has no commitment power. In contrast, when the agent favors early exercise, there is partial revelation of information, exercise is either unbiased or delayed, and the principal is worse off than under the optimal full-commitment mechanism. The reason for these strikingly different results for the two directions of the agent's bias is the asymmetric nature of time: upon getting information, the principal can wait and exercise the option at a later point in time, but cannot go back and exercise the option at an earlier point. When the agent favors late exercise, this inability to go back in time creates an implicit commitment device for the principal to follow the agent's recommendation and thereby makes communication very efficient, but it does not help when the agent favors early exercise.

We next discuss the implications of our analysis for the value of delegating authority over timing decisions. While delegation adds no additional value in the case of a late exercise bias, it can be beneficial in the case of an early exercise bias. In particular, we show that when the agent favors early exercise, delegation dominates centralized decision making if the agent's bias is sufficiently small or if the principal can delay the delegation decision strategically.

Our results also imply that in an alternative setting, where the principal is biased toward early exercise (as in the case of an empire-building top manager), it is

possible to ensure unbiased decision making by having an unbiased agent, even if the principal has formal authority. Thus, as in Landier, Sraer, and Thesmar (2009), divergence of preferences between the principal and her subordinate can enhance decision-making quality, although the mechanism in our paper is very different.

APPENDIX

A. *An Example*

Here, we present a simple example that illustrates how communication over time differs from static communication and why the direction of the agent's bias is the first-order determinant of communication efficiency.

The principal needs to choose the timing of investment, and the optimal timing depends on an unknown to the principal parameter θ. The agent learns θ at the initial date. It is common knowledge that θ is a draw from a uniform distribution over $[1, 2]$. If the principal invests at time t, the principal and the agent obtain the following payoffs at the time of investment:

$$U_P(t, \theta) = C - (t - \theta)^2,$$

$$U_A(t, \theta) = C - (t - \theta + b)^2,$$

where b is the agent's bias and C is a large enough constant. Time moves continuously starting at zero, and there is no discounting. Thus, given θ, the optimal timing is $t = \theta$ from the position of the principal and $t = \theta - b$ from the position of the agent.

First, suppose that communication occurs only at the initial date $t = 0$. In this case, the problem is identical to the quadratic-uniform example in Crawford and Sobel (1982). Consider $b = -\frac{1}{8}$. In addition to the babbling equilibrium, where the agent does not communicate anything and the principal invests at $t = 1\frac{1}{2}$, the only equilibrium that exists has two partitions $[1, 1\frac{3}{4}]$ and $[1\frac{3}{4}, 2]$. All types in partition $[1, 1\frac{3}{4}]$ send the same message, upon which the principal invests at time $t = 1\frac{3}{8}$, and all types in partition $[1\frac{3}{4}, 2]$ send the same message, upon which the principal invests at $t = 1\frac{7}{8}$. Similarly, if $b = \frac{1}{8}$, there is one nonbabbling equilibrium, and it consists of two partitions, $[1, 1\frac{1}{4}]$ and $[1\frac{1}{4}, 2]$.

Now suppose that the agent and the principal communicate dynamically. If $b = -\frac{1}{8}$, the game has the following equilibrium. The agent of type θ plays a threshold strategy of recommending to "wait" ($m = 0$) before his preferred investment time $\theta - b = \theta + \frac{1}{8}$ and recommending to "invest" ($m = 1$) once time reaches $\theta + \frac{1}{8}$. Consider the best response of the principal. If the principal receives a recommendation to "invest" at $t \in [1\frac{1}{8}, 2\frac{1}{8}]$, she infers that the agent's type is $t + b = t - \frac{1}{8}$. Since $U_P(t, \theta)$ is strictly decreasing in t in the range $t \geq \theta$, the best response of the principal is to invest immediately upon receiving the recommendation to invest. If the principal has not received the recommendation to invest from the agent by time t, her optimal strategy is to wait for the agent's recommendation

to invest until time τ and to invest at τ if the agent has not recommended investing yet, where $\tau \leq 2$ maximizes her expected payoff:

$$C - \int_{t+b}^{\tau+b} b^2 \, \frac{d\theta}{2 - t - b} - \int_{\tau+b}^{2} (\tau - \theta)^2 \, \frac{d\theta}{2 - t - b},$$

yielding $\tau = 2 + b = 1\frac{7}{8}$. That is, the best response of the principal is to follow the agent's recommendation up to $\tau = 1\frac{7}{8}$ and to invest then if the agent has not recommended investing yet. Given that, the agent of type $\theta \leq 1\frac{3}{4}$ does not want to deviate from the strategy of recommending investment at time $\theta + \frac{1}{8}$ because by following this strategy, he gets his preferred investment time. Similarly, no type $\theta > 1\frac{3}{4}$ benefits from a deviation since the principal does not delay investment beyond $\tau = 1\frac{7}{8}$. As in the paper, it can be shown that this equilibrium of the cheap talk game implements the optimal commitment mechanism.

Finally, suppose that the agent and the principal communicate dynamically but $b = \frac{1}{8}$, i.e., the agent has a bias for investing earlier than the principal. As above, suppose that the agent of type θ follows the strategy of recommending to "wait" before his preferred investment time $\theta - b = \theta - \frac{1}{8}$ and recommending to "invest" once time reaches $\theta - \frac{1}{8}$. Now, if the principal receives a recommendation to "invest" at time $t \in \left[\frac{7}{8}, 1\frac{7}{8}\right]$, she infers that the agent's type is $\theta = t + \frac{1}{8}$ and delays investment by $\frac{1}{8}$ until time $t + \frac{1}{8}$. Knowing this, the agent deviates from following the strategy above. As a consequence, the equilibrium where the agent credibly communicates his information up to a cutoff does not exist.

This example illustrates two properties of dynamic communication. First, because the principal cannot go back in time, the set of actions that the principal can take (when to invest) shrinks over time. This gives the principal commitment power not to overrule the agent when the agent is biased toward later investment $\left(b = -\frac{1}{8}\right)$, making communication very efficient. Second, because the principal can always delay the decision, the commitment power is one-sided: if the agent has a bias for earlier investment $\left(b = \frac{1}{8}\right)$, the inability to go back in time does not help the principal to commit to follow the agent's recommendation.

B. *Proofs*

This section contains the main parts of the proofs of Propositions 1–3. The proofs of Lemma 1, Propositions 4 and 5, which describe the extensions of the basic model, and the proofs of some auxiliary results are presented in online Appendix B.

PROOF OF PROPOSITION 1:

Part 1 ($b < 0$)**:** The proof includes the special case $\underline{\theta} = 0$. First, consider $b > -\frac{1 - \bar{\theta}}{1 + \bar{\theta}} I$. This implies $b > -I$, and hence $b > -\frac{1 - \underline{\theta}}{1 + \underline{\theta}} \Leftrightarrow b + I > (I - b)\underline{\theta} \Leftrightarrow \hat{\theta}^* \equiv \frac{I - b}{I + b}\underline{\theta} < 1$. Given that the principal plays the strategy stated in the proposition, it is clear that the strategy of any type θ of the agent is incentive-compatible. Indeed, for any type $\theta \geq \frac{I - b}{I + b}\underline{\theta}$, exercise occurs at his most

preferred time. Therefore, no type $\theta \geq \frac{I-b}{I+b}\,\underline{\theta}$ can benefit from a deviation. Any type $\theta < \frac{I-b}{I+b}\,\underline{\theta}$ cannot benefit from a deviation either: the agent would lose from inducing the principal to exercise earlier because he is biased toward late exercise, and it is not feasible for him to induce the principal to exercise later because the principal exercises at threshold X^* regardless of the recommendation. For the principal's strategy to be optimal, we need to check that the principal has incentives to exercise the option immediately when the agent sends message $m = 1$ (the ex post IC constraint), and not to exercise the option before getting message $m = 1$ (the ex ante IC constraint). We first show that the principal's ex post IC constraint is satisfied. If the agent sends a message to exercise when $X(t) < X^*$, the principal learns the agent's type θ and realizes that it is already too late $\left(X_P^*(\theta) < X_A^*(\theta)\right)$ and thus does not benefit from delaying exercise even further. If the agent sends a message to exercise when $X(t) = X^*$, the principal infers that $\theta \leq \hat{\theta}^*$ and that she will not learn any additional information by waiting more. Given the belief that $\theta \in \left[\underline{\theta}, \hat{\theta}^*\right]$, the optimal exercise threshold for the principal is given by $\dfrac{\beta}{\beta-1}\dfrac{2I}{\underline{\theta}+\hat{\theta}^*} = \dfrac{\beta}{\beta-1}\dfrac{2I}{\underline{\theta}+\left(\frac{I-b}{I+b}\right)\underline{\theta}} = \dfrac{\beta}{\beta-1}\dfrac{I+b}{\underline{\theta}} = X^*$, and hence the ex post IC constraint is satisfied. Finally, in the online Appendix, we show that if the principal's ex ante IC constraint is violated for $b > -\frac{1-\underline{\theta}}{1+\underline{\theta}}\,I$, then the mechanism derived in Lemma 1 cannot be optimal, which is a contradiction. This completes the proof of existence of equilibrium with continuous exercise for $b > -\frac{1-\underline{\theta}}{1+\underline{\theta}}\,I$. Next, consider $b \leq -\frac{1-\underline{\theta}}{1+\underline{\theta}}\,I$. According to Lemma 1, the optimal mechanism is characterized by $\hat{X}(\theta) = \dfrac{\beta}{\beta-1}\dfrac{2I}{\underline{\theta}+1}$. Clearly, the equilibrium implementing this mechanism exists: the principal exercises at her optimal uninformed threshold $\dfrac{\beta}{\beta-1}\dfrac{2I}{\underline{\theta}+1}$ and the agent babbles.

Part 2 $(b > 0)$**:** According to the proof of Lemma 1, for any $b > 0$, there is a unique exercise policy $\hat{X}(\theta)$ that maximizes the principal's expected utility. Hence, if $b \in \left(0, \frac{1-\underline{\theta}}{1+\underline{\theta}}\,I\right)$, an equilibrium implementing the optimal mechanism can exist only if in this equilibrium exercise happens at $X_A^*(\theta)$ for all $\theta < \frac{I-b}{I+b}$. This, however, is not possible because the principal's optimal exercise time is later than the agent's. Indeed, if a type $\theta < \frac{I-b}{I+b}$ follows the strategy of recommending exercise at his most-preferred threshold $X_A^*(\theta)$, the principal infers the agent's type perfectly and prefers delay over immediate exercise upon getting the recommendation to exercise. Knowing this, the agent is tempted to change his recommendation strategy, mimicking a lower type. Thus, no equilibrium with full separation of types over an interval $\theta < \frac{I-b}{I+b}$ can exist. Finally, if $b \geq \frac{1-\underline{\theta}}{1+\underline{\theta}}\,I$, the equilibrium implementing the optimal mechanism $\hat{X}(\theta) = \dfrac{\beta}{\beta-1}\dfrac{2I}{\underline{\theta}+1}$ exists: the principal exercises at her optimal uninformed threshold $\dfrac{\beta}{\beta-1}\dfrac{2I}{\underline{\theta}+1}$ and the agent babbles. ∎

PROOF OF PROPOSITION 2:

Part 1 (Existence of Equilibrium with Continuous Exercise)**:** According to the proof of Proposition 1, this equilibrium does not exist for $b > 0$. We next prove that for $b < 0$, this equilibrium exists if and only if $b \geq -I$. Because the agent's IC constraint and the principal's ex post IC constraint are satisfied, we only need to satisfy the principal's ex ante IC constraint. Let $V_P^c(X, \hat{\theta})$ denote the principal's expected value in this equilibrium, given that the public state is X and the principal's belief is that θ is uniform over $\left[0, \hat{\theta}\right]$. If the agent's type is θ, exercise occurs at threshold $\frac{\beta}{\beta - 1} \frac{I - b}{\theta}$, and the principal's payoff upon exercise is $\frac{\beta}{\beta - 1}(I - b) - I$. Hence,

$$(A1) \qquad V_P^c(X, \hat{\theta}) = \int_0^{\hat{\theta}} \frac{1}{\hat{\theta}} X^\beta \left(\frac{\beta}{\beta - 1} \frac{I - b}{\theta} \right)^{-\beta} \frac{I - \beta b}{\beta - 1} d\theta$$

$$= \frac{(X\hat{\theta})^\beta}{\beta + 1} \left(\frac{\beta}{\beta - 1}(I - b) \right)^{-\beta} \frac{I - \beta b}{\beta - 1}.$$

By stationarity, it is sufficient to verify the principal's ex ante IC constraint for $\hat{\theta} = 1$, which yields

$$(A2) \qquad V_P^c(X, 1) \geq \frac{1}{2} X - I \quad \forall X \leq X_A^*(1).$$

We show that (A2) is satisfied if and only if $b \geq -I$. Using (A1), (A2) is equivalent to

$$(A3) \quad \frac{1}{\beta + 1} \left(\frac{\beta}{\beta - 1}(I - b) \right)^{-\beta} \frac{I - \beta b}{\beta - 1} \geq \max_{X \in (0, X_A^*(1)]} X^{-\beta} \left(\frac{1}{2} X - I \right).$$

The function $X^{-\beta}\left(\frac{1}{2} X - I \right)$ is inverse U-shaped with a maximum at $\overline{X}_u \equiv \frac{\beta}{\beta - 1} 2I$, where $\overline{X}_u > X_A^*(1) \Leftrightarrow b > -I$. First, suppose that $b < -I$, and hence $\overline{X}_u < X_A^*(1)$. Then, (A3) is equivalent to

$$(A4) \qquad \frac{1}{\beta + 1} \left(\frac{\beta}{\beta - 1}(I - b) \right)^{-\beta} \frac{I - \beta b}{\beta - 1} \geq \overline{X}_u^{-\beta} \left(\frac{1}{2} \overline{X}_u - I \right)$$

$$\Leftrightarrow \frac{1}{\beta + 1}(I - b)^{-\beta}(I - \beta b) \geq (2I)^{-\beta} I.$$

Consider $f(b) \equiv (I - b)^{-\beta}(I - \beta b) - (\beta + 1)(2I)^{-\beta} I$. Note that $f(-I) = 0$ and $f'(b) > 0$. Hence, $f(b) \geq 0 \Leftrightarrow b \geq -I$, and hence (A3) is violated when $b < -I$.

Second, suppose that $b \geq -I$, and hence (A4) is satisfied. Since, in this case, $\overline{X}_u \geq X_A^*(1)$, then $\max_{X \in (0, X_A^*(1)]} X^{-\beta} \left(\frac{1}{2} X - I \right) \leq \overline{X}_u^{-\beta} \left(\frac{1}{2} \overline{X}_u - I \right)$, and hence the inequality (A3) follows from the fact that inequality (A4) is satisfied.

Note also that if $b = -I$, the equilibrium with continuous exercise brings the principal the same payoff as the babbling equilibrium with exercise at $\overline{X}_u$.

Part 2 (The Necessary and Sufficient Conditions for a ω-Equilibrium to Exist)**:** For ω and $\overline{X}$ to constitute an equilibrium, the IC conditions for the principal and the agent must hold. Because the problem is stationary, it is sufficient to only consider the IC conditions for the game up to reaching the first threshold $\overline{X}$. First, consider the agent's problem. Pair $(\omega, \overline{X})$ satisfies the agent's IC condition if and only if types above ω have incentives to recommend exercise ($m = 1$) at threshold $\overline{X}$ rather than to wait, whereas types below ω have incentives to recommend delay ($m = 0$). From the agent's point of view, the set of possible exercise thresholds is given by $\mathcal{X}$: the agent can induce exercise at any threshold in $\mathcal{X}$ by sending $m = 1$ at the first instant when $X(t)$ reaches a desired point in $\mathcal{X}$, but cannot induce exercise at any point not in $\mathcal{X}$. This implies that the agent's IC condition holds if and only if type ω is exactly indifferent between exercising the option at threshold $\overline{X}$ and at threshold $\frac{\overline{X}}{\omega}$:

$$(A5) \qquad \left(\frac{X(t)}{\overline{X}}\right)^{\beta}(\omega\overline{X} + b - I) = \left(\frac{X(t)}{\overline{X}/\omega}\right)^{\beta}\left(\omega\,\frac{\overline{X}}{\omega} + b - I\right),$$

which simplifies to $\omega\overline{X} + b - I = \omega^{\beta}(\overline{X} + b - I)$. Indeed, if (A5) holds, then $\left(\frac{X(t)}{\overline{X}}\right)^{\beta}(\theta\overline{X} + b - I) \gtrless \left(\frac{X(t)}{\overline{X}/\omega}\right)^{\beta}\left(\theta\,\frac{\overline{X}}{\omega} + b - I\right)$ if $\theta \gtrless \omega$. Hence, if type ω is indifferent between exercise at threshold $\overline{X}$ and at threshold $\frac{\overline{X}}{\omega}$, then any higher type strictly prefers recommending exercise at $\overline{X}$, while any lower type strictly prefers recommending delay at $\overline{X}$. By stationarity, if (A5) holds, then type ω^2 is indifferent between recommending exercise and recommending delay at threshold $\frac{\overline{X}}{\omega}$, so types in (ω^2, ω) strictly prefer recommending exercise at threshold $\frac{\overline{X}}{\omega}$, and so on. Thus, (A5) is necessary and sufficient for the agent's IC condition to hold. Equation (A5) is equivalent to (6).

Next, consider the principal's problem. For ω and $\overline{X}$ to constitute an equilibrium, the principal must have incentives: (i) to exercise the option immediately when the agent sends message $m = 1$ at a threshold in $\mathcal{X}$ (the ex post IC constraint); and (ii) not to exercise the option before getting message $m = 1$ (the ex ante IC constraint). Suppose that $X(t)$ reaches threshold $\overline{X}$ for the first time, and the principal receives recommendation $m = 1$ at that instant. By Bayes' rule, the principal updates her beliefs to θ being uniform on $[\omega, 1]$. If the principal exercises immediately, her expected payoff is $\frac{\omega+1}{2}\,\overline{X} - I$. If the principal delays, she expects that there will be no further informative communication in the continuation game. Thus, upon receiving message $m = 1$ at threshold $\overline{X}$, the principal faces the standard perpetual call option exercise problem (e.g., Dixit and Pindyck 1994) as if the type of the project were $\frac{\omega+1}{2}$. Immediate exercise is optimal if and only if exercising at threshold $\overline{X}$ dominates waiting until $X(t)$ reaches a higher threshold Z and exercising the option then for any possible $Z > \overline{X}$:

$$(A6) \qquad \overline{X} \in \arg\max_{Z \geq \overline{X}} \left(\frac{\overline{X}}{Z}\right)^{\beta}\left(\frac{\omega + 1}{2}\,Z - I\right).$$

Using $\overline{X} = Y(\omega)$ and the fact that the right-hand side is an inverted U-shaped function of Z with a maximum at $\frac{\beta}{\beta - 1} \frac{2I}{\omega + 1}$, the ex post IC condition for the principal is equivalent to

$$(A7) \qquad\qquad Y(\omega) \geq \frac{\beta}{\beta - 1} \frac{2I}{\omega + 1}.$$

This condition has a clear intuition. Suppose that $X(t)$ reaches threshold $\overline{X} = Y(\omega)$ for the first time, and the principal receives recommendation $m = 1$ at that instant. By Bayes' rule, the principal updates her beliefs to θ being uniform on $[\omega, 1]$. Condition (A7) ensures that the current value of the state process, $Y(\omega)$, exceeds the optimal exercise threshold of the principal given these updated beliefs, $\frac{\beta}{\beta - 1} \frac{2I}{\omega + 1}$, and hence the principal finds it optimal to exercise immediately. In contrast, if (A7) is violated, the principal delays exercise, so the recommendation loses its responsiveness as the principal does not follow it. As with the IC condition of the agent, stationarity implies that if (A7) holds, then a similar condition holds for all higher thresholds in $\mathcal{X}$. The fact that constraint (A7) is an inequality rather than an equality highlights the asymmetric nature of time: When the agent recommends exercise, the principal can either exercise immediately or can delay, but cannot go back in time and exercise in the past, even if it is tempting to do so.

Let $V_P(X(t), \hat{\theta}_t; \omega)$ denote the expected value to the principal in the ω-equilibrium, given that the public state is $X(t)$ and the principal's belief is that θ is uniform over $[0, \hat{\theta}_t]$. In the online Appendix, we solve for the principal's value in closed form and show that if $\hat{\theta}_t = 1$,

$$(A8) \qquad V_P(X, 1; \omega) = \frac{1 - \omega}{1 - \omega^{\beta+1}} \left(\frac{X}{Y(\omega)} \right)^{\beta} \left(\frac{1}{2} (1 + \omega) Y(\omega) - I \right)$$

for any $X \leq Y(\omega)$. By stationarity, (A8) can be generalized to any $\hat{\theta}$:

$$(A9) \;\; V_P(X, \hat{\theta}; \omega) = V_P(\hat{\theta} X, 1; \omega) = \frac{1 - \omega}{1 - \omega^{\beta+1}} \left(\frac{X\hat{\theta}}{Y(\omega)} \right)^{\beta} \left(\frac{1}{2} (1 + \omega) Y(\omega) - I \right).$$

The principal's ex ante IC constraint requires that the principal is better off waiting, rather than exercising immediately, at any time prior to receiving message $m = 1$ at $X(t) \in X$:

$$(A10) \qquad\qquad V_P(X(t), \hat{\theta}_t; \omega) \geq \frac{\hat{\theta}_t}{2} X(t) - I$$

for any $X(t)$ and $\hat{\theta}_t = \sup \left\{ \theta : \overline{X}(\theta) > \max_{s \leq t} X(s) \right\}$. By stationarity, it is sufficient to verify the ex ante IC constraint for $X(t) \leq \overline{X}(1) = Y(\omega)$ and beliefs equal to the prior:

$$(A11) \qquad\qquad V_P(X, 1; \omega) \geq \frac{1}{2} X - I \quad \forall X \leq Y(\omega).$$

This inequality states that at any point up to threshold $Y(\omega)$, the principal is better off waiting than exercising the option. If (A11) does not hold for some $X \leq Y(\omega)$, then the principal is better off exercising the option when $X(t)$ reaches X, rather than waiting for informative recommendations from the agent. If (A11) holds, then the principal does not exercise the option prior to reaching threshold $Y(\omega)$. By stationarity, if (A11) holds, then a similar condition holds for the nth partition for any $n \in N$, which implies that (A11) and (A10) are equivalent.

To summarize, a ω-equilibrium exists if and only if conditions (6), (A7), and (A11) are satisfied.

Part 3 (Existence of ω-Equilibria for $b < 0$)**:** We first show that if $b < 0$, then for any positive $\omega < 1$, the principal's ex post IC is strictly satisfied, i.e., $Y(\omega) > \frac{\beta}{\beta - 1} \frac{2I}{1 + \omega}$. In the online Appendix, we prove that $G(\omega) \equiv \frac{(1 - \omega^{\beta})(I - b)}{\omega(1 - \omega^{\beta - 1})} - \frac{\beta}{\beta - 1} \frac{2(I - b)}{1 + \omega} > 0$ for all $\omega \in [0, 1)$, or equivalently, that $Y(\omega) > \frac{\beta}{\beta - 1} \frac{2(I - b)}{1 + \omega}$. Since $b < 0$, this implies that $Y(\omega) > \frac{\beta}{\beta - 1} \frac{2I}{1 + \omega}$, and hence the ex post IC condition of the principal is satisfied for any $\omega < 1$. Thus, the ω-equilibrium exists if and only if the ex ante IC (A11) is satisfied, where $V_P(X, 1; \omega)$ is given by (A8). Because $X^{-\beta}V_P(X, 1; \omega)$ does not depend on X, we can rewrite (A11) as

$$(\text{A12}) \qquad X^{-\beta}V_P(X, 1; \omega) \geq \max_{X \in (0, Y(\omega)]} X^{-\beta}\left(\frac{1}{2}X - I\right).$$

We pin down the range of ω that satisfies this condition in the following steps, each of which is proved in the online Appendix.

Step 1: If $b < 0$, $V_P(X, 1; \omega)$ is strictly increasing in ω for any $\omega \in (0, 1)$.

Step 2: $\lim_{\omega \to 1} V_P(X, 1; \omega) = V_P^c(X, 1)$.

Step 3: Suppose $-I < b < I$. For ω close enough to zero, the ex ante IC condition (A12) does not hold.

Step 4: Suppose $-I < b < I$. Then (A12) is satisfied for any $\omega \geq \overline{\omega}$, where $\overline{\omega}$ is the unique solution to $Y(\omega) = \overline{X}_u$. For any $\omega < \overline{\omega}$, (A12) is satisfied if and only if $X^{-\beta}V_P(X, 1; \omega) \geq \overline{X}_u^{-\beta}\left(\frac{1}{2}\overline{X}_u - I\right)$.

Combining the four steps above yields the statement of the proposition for $b < 0$. First, if $b \leq -I$, then $I - b \geq 2I$, and hence $\lim_{\omega \to 1} Y(\omega) = \frac{\beta(I - b)}{\beta - 1} \geq \overline{X}_u$. Since $Y(\omega)$ is decreasing, it implies that $Y(\omega) > \overline{X}_u$ for any $\omega < 1$, and hence (A12) is equivalent to $X^{-\beta}V_P(X, 1; \omega) \geq \overline{X}_u^{-\beta}\left(\frac{1}{2}\overline{X}_u - I\right)$. According to Steps 1 and 2, for any $\omega < 1$, $V_P(X, 1; \omega) < \lim_{\omega \to 1} V_P(X, 1; \omega) = V_P^c(X, 1)$. As shown in the proof of the equilibrium with continuous exercise above, $X^{-\beta}V_P^c(X, 1) \leq \overline{X}_u^{-\beta}\left(\frac{1}{2}\overline{X}_u - I\right)$ for $b \leq -I$, and hence (A12) is violated. Hence, there is no ω-equilibrium in this case. Second, if $0 > b > -I$, then according to Step 4,

(A12) is satisfied for any $\omega \geq \overline{\omega}$, and for any $\omega < \overline{\omega}$, (A12) is satisfied if and only if $X^{-\beta}V_P(X, 1; \omega) \geq \overline{X}_u^{-\beta}\left(\frac{1}{2}\overline{X}_u - I\right)$. The left-hand side of this inequality is increasing in ω according to Step 1, while the right-hand side is constant. Hence, if (A12) is satisfied for some $\tilde{\omega}$, it is satisfied for any $\omega \geq \tilde{\omega}$. According to Step 3, for ω close to 0, (A12) does not hold. Together, this implies that there exists a unique $\underline{\omega} \in (0, \overline{\omega})$ such that the principal's ex ante IC (A12) holds if and only if $\omega \geq \underline{\omega}$, and that $X^{-\beta}V_P(X, 1; \underline{\omega}) = \overline{X}_u^{-\beta}\left(\frac{1}{2}\overline{X}_u - I\right)$.

Part 4 (Existence of ω-Equilibria for $b < 0$)**:** Since the agent's IC condition is guaranteed by (6), we only have to ensure that the principal's ex post and ex ante IC conditions are satisfied. First, we check the principal's ex post IC condition (A7). In the online Appendix, we prove that (i) $Y(\omega) = \frac{\beta}{\beta - 1}\frac{2I}{1 + \omega}$ has only one solution $\omega = \omega^*$; and (ii) $Y(\omega)$ is strictly decreasing in ω for $\omega \in (0, 1)$. Since $\lim_{\omega \to 0}Y(\omega) = +\infty$, it follows that the principal's ex post IC condition is equivalent to $\omega \leq \omega^*$. Next, we check the principal's ex ante IC condition (A11), which is equivalent to (A12), where $V_P(X, 1; \omega)$ is given by (A8). We pin down the range of ω that satisfies this condition in the following steps, which are proved in the online Appendix.

Step 5: If $b > 0$, $V_P(X, 1; \omega)$ is strictly increasing in ω for any $\omega \in (0, \omega^*)$.

Step 6: If $0 < b < I$, then the ex ante IC condition (A12) holds as a strict inequality for $\omega = \omega^*$.

Combining the steps above yields the statement of the proposition for $b > 0$. Suppose $b < I$. As shown above, the ex post IC condition holds if and only if $\omega \leq \omega^*$. Recall that $Y(\omega^*) = \frac{\beta}{\beta - 1}\frac{2I}{\omega^* + 1} < \frac{\beta}{\beta - 1}2I = \overline{X}_u$, and hence $\omega^* > \overline{\omega}$. According to Step 4 from the proof of case $b < 0$ above, the ex ante IC condition (A12) is satisfied for any $\omega \geq \overline{\omega}$, and for any $\omega < \overline{\omega}$, (A12) is satisfied if and only if $X^{-\beta}V_P(X, 1; \omega) \geq \overline{X}_u^{-\beta}\left(\frac{1}{2}\overline{X}_u - I\right)$. The left-hand side of this inequality is increasing in ω for $\omega \leq \omega^*$ according to Step 5, while the right-hand side is constant. Together, this implies that if (A12) is satisfied for some $\tilde{\omega}$, it is satisfied for any $\omega \geq \tilde{\omega}$. According to Step 3 from the proof of case $b < 0$ above, for ω close to 0, (12) does not hold. Hence, there exists a unique $\underline{\omega} \in (0, \overline{\omega})$ such that the principal's ex ante IC (A12) holds if and only if $\omega \geq \underline{\omega}$, and $X^{-\beta}V_P(X, 1; \underline{\omega}) = \overline{X}_u^{-\beta}\left(\frac{1}{2}\overline{X}_u - I\right)$. Because, $\underline{\omega} < \overline{\omega}$ and $\overline{\omega} < \omega^*$ by Step 6, we have $\underline{\omega} < \omega^*$. We conclude that both the ex post and the ex ante IC conditions hold if and only if $\omega \in [\underline{\omega}, \omega^*]$. Finally, consider $b \geq I$. In this case, all types of agents want immediate exercise, which implies that the principal must exercise the option at the optimal uninformed threshold $\overline{X}_u = \frac{\beta}{\beta - 1}2I$. ∎

PROOF OF PROPOSITION 3:

First, consider the case $b < 0$. Proposition 1 shows that in the dynamic communication game, there exists an equilibrium with continuous exercise, where for each type θ, the option is exercised at threshold $X_A^*(\theta)$. No such equilibrium exists in the

static communication game. Indeed, continuous exercise requires that the principal perfectly infers the agent's type. However, since the principal gets this information at time 0, she will exercise the option at $X_P^*(\theta) \neq X_A^*(\theta)$.

We next show that for $b < 0$, no stationary equilibrium with partitioned exercise exists in the static communication game either. To see this, note that for such an equilibrium to exist, the following conditions must hold. First, the boundary type ω must be indifferent between exercise at $\overline{X}$ and at $\frac{\overline{X}}{\omega}$. Repeating the derivations in the proof of Proposition 2, this requires that (6) holds: $\overline{X} = Y(\omega) \equiv \frac{(1 - \omega^\beta)(I - b)}{\omega(1 - \omega^{\beta-1})}$. Second, because the exercise threshold $\overline{X}$ is optimally chosen by the principal given the belief that $\theta \in [\omega, 1]$, it must satisfy $\overline{X} = \frac{\beta}{\beta - 1} \frac{2I}{\omega + 1}$. Combining these two equations, ω must solve $Y(\omega) = \frac{\beta}{\beta - 1} \frac{2I}{\omega + 1}$, which can be rewritten as

$$(A13) \qquad 2\beta I(\omega - \omega^\beta) - (\beta - 1)(I - b)(1 + \omega)(1 - \omega^\beta) = 0.$$

We next show that the left-hand side of $(A13)$ is negative for any $b < 0$ and $\omega < 1$. Since $b < 0$, it is sufficient to prove that $2\beta(\omega - \omega^\beta) < (\beta - 1)(1 + \omega) \times (1 - \omega^\beta)$, or equivalently, that

$$(A14) \qquad s(\omega) \equiv 2\beta(\omega - \omega^\beta) + (\beta - 1)(\omega^{\beta+1} - \omega - 1 + \omega^\beta) < 0.$$

It is easy to show that $s'(1) = 0$ and that $s''(\omega) < 0 \Leftrightarrow \omega < 1$, and hence $s'(\omega) > 0$ for any $\omega < 1$. Since $s(1) = 0$, then, indeed, $s(\omega) < 0$ for all $\omega < 1$.

Next, consider $b > 0$. As argued above, for ω-equilibrium to exist in the static communication game, ω must satisfy $Y(\omega) = \frac{\beta}{\beta - 1} \frac{2I}{\omega + 1}$. According to Proposition 2, for $b > 0$, this equation has a unique solution, denoted by ω^*. Thus, among equilibria with $\omega \in [\underline{\omega}, \omega^*]$, which exist in the dynamic communication game, only equilibrium with $\omega = \omega^*$ is an equilibrium of the static communication game. ∎

PROOF OF PROPOSITIONS 4 AND 5:
    See the online Appendix.

REFERENCES

Aghion, Philippe, and Jean Tirole. 1997. "Formal and Real Authority in Organizations." *Journal of Political Economy* 105 (1): 1–29.

Alonso, Ricardo, Wouter Dessein, and Niko Matouschek. 2008. "When Does Coordination Require Centralization?" *American Economic Review* 98 (1): 145–79.

Alonso, Ricardo, Wouter Dessein, and Niko Matouschek. 2015. "Organizing to Adapt and Compete." *American Economic Journal: Microeconomics* 7 (2): 158–79.

Alonso, Ricardo, and Niko Matouschek. 2007. "Relational Delegation." *RAND Journal of Economics* 38 (4): 1070–89.

Alonso, Ricardo, and Niko Matouschek. 2008. "Optimal Delegation." *Review of Economic Studies* 75 (1): 259–93.

Amador, Manuel, and Kyle Bagwell. 2013. "The Theory of Optimal Delegation with an Application to Tariff Caps." *Econometrica* 81 (4): 1541–99.

Aumann, Robert J., and Sergiu Hart. 2003. "Long Cheap Talk." *Econometrica* 71 (6): 1619–60.

Baker, George, Robert Gibbons, and Kevin J. Murphy. 1999. "Informal Authority in Organizations." *Journal of Law, Economics, and Organization* 15 (1): 56–73.

Benabou, Roland, and Guy Laroque. 1992. "Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility." *Quarterly Journal of Economics* 107 (3): 921–58.

Bolton, Patrick, and Mathias Dewatripont. 2013. "Authority in Organizations." In *Handbook of Organizational Economics*, edited by Robert Gibbons and John Roberts, 342–72. Princeton, NJ: Princeton University Press.

Boot, Arnoud W. A., Todd T. Milbourn, and Anjan V. Thakor. 2005. "Sunflower Management and Capital Budgeting." *Journal of Business* 78 (2): 501–27.

Bustamante, M. Cecilia. 2012. "The Dynamics of Going Public." *Review of Finance* 16 (2): 577–618.

Chakraborty, Archishman, and Bilge Yilmaz. 2015. "Authority, Consensus and Governance." https://sites.google.com/site/archishmanchakrabortyhomepag/home (accessed July 22, 2016).

Crawford, Vincent P., and Joel Sobel. 1982. "Strategic Information Transmission." *Econometrica* 50 (6): 1431–51.

Dessein, Wouter. 2002. "Authority and Communication in Organizations." *Review of Economic Studies* 69 (4): 811–38.

Dessein, Wouter, Luis Garicano, and Robert Gertner. 2010. "Organizing for Synergies." *American Economic Journal: Microeconomics* 2 (4): 77–114.

Dessein, Wouter, and Tano Santos. 2006. "Adaptive Organizations." *Journal of Political Economy* 114 (5): 956–95.

Dixit, Avinash K., and Robert S. Pindyck. 1994. *Investment under Uncertainty.* Princeton, NJ: Princeton University Press.

Ely, Jeffrey C. 2015. "Beeps." http://jeffely.com (accessed July 22, 2016).

Frankel, Alexander. 2014. "Aligned Delegation." *American Economic Review* 104 (1): 66–83.

Friebel, Guido, and Michael Raith. 2010. "Resource Allocation and Organizational Form." *American Economic Journal: Microeconomics* 2 (2): 1–33.

Garicano, Luis, and Luis Rayo. 2016. "Why Organizations Fail: Models and Cases." *Journal of Economic Literature* 54 (1): 137–92.

Gibbons, Robert, Niko Matouschek, and John Roberts. 2013. "Decisions in Organizations." In *Handbook of Organizational Economics*, edited by Robert Gibbons and John Roberts, 373–431. Princeton, NJ: Princeton University Press.

Golosov, Mikhail, Vasiliki Skreta, Aleh Tsyvinski, and Andrea Wilson. 2014. "Dynamic Strategic Information Transmission." *Journal of Economic Theory* 151 (0): 304–41.

Goltsman, Maria, Johannes Horner, Gregory Pavlov, and Francesco Squintani. 2009. "Mediation, Arbitration and Negotiation." *Journal of Economic Theory* 144 (4): 1397–420.

Grenadier, Steven R., and Andrey Malenko. 2011. "Real Options Signaling Games with Applications to Corporate Finance." *Review of Financial Studies* 24 (12): 3993–4036.

Grenadier, Steven R., Andrey Malenko, and Ilya A. Strebulaev. 2014. "Investment Busts, Reputation, and the Temptation to Blend in with the Crowd." *Journal of Financial Economics* 111 (1): 137–57.

Grenadier, Steven R., and Neng Wang. 2005. "Investment Timing, Agency, and Information." *Journal of Financial Economics* 75 (3): 493–533.

Gryglewicz, Sebastian, and Barney Hartman-Glaser. 2015. "Dynamic Agency and Real Options." https://sites.google.com/site/barneyhartmanglaser/ (accessed July 22, 2016).

Guo, Yingi. 2016. "Dynamic Delegation of Experimentation." *American Economic Review* 106 (8): 1969–2008.

Halac, Marina. 2012. "Relational Contracts and the Value of Relationships." *American Economic Review* 102 (2): 75–79.

Halac, Marina, Navin Kartik, and Qingmin Liu. 2016. "Optimal Contracts for Experimentation." *Review of Economic Studies* 83 (3): 1040–91.

Harris, Milton, and Artur Raviv. 2005. "Allocation of Decision-Making Authority." *Review of Finance* 9 (3): 353–83.

Harris, Milton, and Artur Raviv. 2008. "A Theory of Board Control and Size." *Review of Financial Studies* 21 (4): 1797–832.

Holmström, Bengt. 1984. "On the Theory of Delegation." In *Bayesian Models in Economic Theory*, edited by Marcel Boyer and Richard Kihlstrom, 115–141. New York: North-Holland.

Krishna, Vijay, and John Morgan. 2004. "The Art of Conversation: Eliciting Information from Experts through Multi-Stage Communication." *Journal of Economic Theory* 117 (2): 147–79.

Kruse, Thomas, and Philipp Strack. 2015. "Optimal Stopping with Private Information." *Journal of Economic Theory* 159: 702–27.

Landier, Augustin, David Sraer, and David Thesmar. 2009. "Optimal Dissent in Organizations." *Review of Economic Studies* 76 (2): 761–94.

Melumad, Nahum D., and Toshiyuki Shibano. 1991. "Communication in Settings with No Transfers." *RAND Journal of Economics* 22 (2): 173–98.

Morellec, Erwan, and Norman Schürhoff. 2011. "Corporate Investment and Financing under Asymmetric Information." *Journal of Financial Economics* 99 (2): 262–88.

Morris, Stephen. 2001. "Political Correctness." *Journal of Political Economy* 109 (2): 231–65.

Ottaviani, Marco, and Peter Norman Sorensen. 2006a. "Reputational Cheap Talk." *RAND Journal of Economics* 37 (1): 155–75.

Ottaviani, Marco, and Peter Norman Sorensen. 2006b. "The Strategy of Professional Forecasting." *Journal of Financial Economics* 81 (2): 441–66.

Rantakari, Heikki. 2008. "Governing Adaptation." *Review of Economic Studies* 75 (4): 1257–85.

Rubinstein, Ariel. 1985. "A Bargaining Model with Incomplete Information about Time Preferences." *Econometrica* 53 (5): 1151–72.

Sobel, Joel. 1985. "A Theory of Credibility." *Review of Economic Studies* 52 (4): 557–73.