Demand Disagreement*

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Abstract

Classical asset pricing models fail to account for the low correlation between macroeconomic fundamentals and (i) stock market returns and (ii) trading volume observed in the data. We develop an overlapping generations model with log utility investors who have heterogeneous time preferences and disagree about investors' future time preferences and, thus, their future demands. There is speculative trade because investors perceive demand shocks differently and, thus, even in the absence of Merton' type hedging demands or early resolution of uncertainty, these demand shocks, which are independent of output shocks, are priced in equilibrium. Our demand disagreement model can reconcile time-varying risk-free rates, excess stock market volatility, and the predictability of stock market returns by the price-dividend ratio, with a low correlation between macroeconomic fundamentals and both asset prices and trading volume.

Keywords: Demand shocks, heterogeneous beliefs, asset prices, trading volume

JEL Classification: D51, G10, G11, G12

1 Introduction

Stock market fluctuations appear to be "excessively volatile" relative to dividends and are only weakly correlated with fundamentals. As the stock market is the discounted value of future dividends, a possible resolution to the "excess volatility" and the "correlation" puzzles is large variability in discount rates driven by other shocks than shocks to fundamentals.

The large variation in discount rates driven by seemingly "non-fundamental" shocks poses challenges for most models of stock markets as typically the stochastic discount factor is functionally related to fundamentals through the marginal utilities of the agents in the economy. Put differently, to be consistent with the excess volatility and the low correlation between returns and fundamentals, the preferences of the agents in the economy must vary over time and be driven by different shocks than that of fundamentals such as consumption and output. If this is the case, a natural question is what the underlying economic forces behind these shocks to demand for risky assets are.

Given that stock market fluctuations appear to be largely driven by non-fundamental shocks, investors must not only understand and forecast future dividends, but also the future demand for risky assets. As there is uncertainty about what the underlying forces behind the shocks to demand are, it is also likely that different investors have opposing views about future demand for risky assets. Hence, there might be "demand disagreement" in which investors with different views about the aggregate demand for risky assets in the future engage in speculative trade based on their views.

In this paper, we propose a tractable model of demand disagreement. Specifically, we consider an overlapping generations model with two types of agents with different preferences and beliefs. Every period, the fraction of each agent type entering the market is random. This randomness effectively works as demand shock as it changes the composition of agents with different preferences in the economy. Agents observe the current mix of agent types, but differ in their view about the future composition. We show that the model with demand disagreement generates excess volatility and low correlation between returns and fundamentals while simultaneously generating empirical patterns such as stochastic volatility and return predictability from the price-dividend ratio.

In our model, all agents have log utility but differ in their time discount rate that can either be high or low, i.e., agents have heterogeneous but constant preferences parameters. The fraction of each type of agents entering the market every period is governed by an exogenous mean reverting process uncorrelated with shocks to aggregate output. This can be interpreted as generational changes in the population due to other reasons than the macroeconomy.

The mix of types entering the market is observable, but the long-run mean of the process is not. Hence, agents in the economy face incomplete information. We assume that agents overestimate the long-run fraction of people with the same preferences as themselves. This implies that the agents in the economy have a false consensus bias (Ross et al. (1977)) as they believe that their preference are more typical than what they in fact are.[1]

Since agents differ in their beliefs about the long-run mean of the distribution of preferences in the economy, agents also disagree about the demand shocks. Importantly, this implies that agents disagree about the patience of the marginal investor in the future and consequently the future discount rate. We show that once the agents in the economy perceive different demand shocks, they also perceive different market prices of risk to these shocks and as a consequence trade on their beliefs.

The trading based on the demand shocks has important implications for equilibrium asset prices. In contrast to an economy with no disagreement, the economy with disagreement feature unpredictable variations in the consumption and wealth shares of agents as they engage in speculative trade. The speculative trade in turn causes the stock market to load on the demand shocks, lowering the correlation between returns and fundamentals. This is due to two reasons. First, shocks to the relative wealth distribution of the agents cause the

[1]For most of our results the assumption that the agents overestimate how typical their preferences are is not crucial. For instance, we could instead have a reverse false consensus bias in which agents believe their preferences are more unique than they actually are.

effective time discount factor in the economy to change, which in turn creates variations in the discount rate through the risk free rate. Second, speculative trade causes the market price of risk for demand shocks to vary as this depends on the relative wealth distribution among the investors with different beliefs.

Although there is a continuum of agents in the economy, we show that the equilibrium real short rate and market prices of risk can be fully described by the fraction of newly born agents of each type and the aggregate consumption share (across all cohorts) of the most patient agent type. In contrast to a similar economy with infinitely lived agents, our economy is stationary as the aggregate consumption share of each agent type has a stationary distribution. This is important for the asset pricing implications of the model since in an infinitely lived economy with equally biased agents only the most patient agent would survive in the long run and prices would fully reflect the preferences and beliefs of this agent type. In fact, we show that if the disagreement is sufficiently high, the fraction of aggregate consumption consumed by the least patient agents might be higher than that of the most patient, making the price impact larger than that of the more patient agents. This overturns the results for infinitely lived economies and shows that the overlapping generations structure can have a non-trivial impact on the unconditional distribution of asset prices in equilibrium.

In the economy with demand disagreement, the market price of risk for demand shocks depends on the consumption weighted average beliefs about the long run mean, which we call the "market view" relative to the true long run mean. When the market view is higher than the true long run mean, then assets that are positively correlated with demand shocks seem expensive and, therefore, from the point of view of an econometrician with the correct belief the market price of risk is negative. When the average consumption share of the most patient agents is above one half, then positive shocks to the fraction of new born agents with high patience carries a negative risk premium as the average market view is relatively optimistic.

We show that the stock market is positively correlated with the demand shocks. This is due a lower discount rate in a response to a demand shock and happens for two reasons. First, the positive demand shock moves wealth towards the more patient agent and therefore the real short decreases. Second, as the more patient agents are also the most optimistic agents with respect to the demand shocks, a larger wealth share decreases the risk premium and, hence, the discount rate is lowered even further. Taken together with the fact that there is no impact on the cash flows from the demand shock, the stock price increases in response to a positive demand shock. Even though the market price of risk for demand shocks might be on average negative and the stock market is loading positively on these shocks, the unconditional risk premium due to the demand shocks might still be positive. The reason for this is that the market price of demand shocks and the stock market loading on these shocks are positively correlated. Hence, in times when the risk premium for demand shocks is strongly positive, the stock market exposure to these shocks is also high.

As the stock market loads on demand shocks when agents disagree, the correlation between stock returns and dividends is lowered. Hence, the model is consistent with excess volatility and low correlation between returns and fundamentals as observed in the data. In addition, we show that the rebalancing of agents, or trading volume, is only driven by demand shocks in the model. The trading generated by the model sets it apart from models based on preference shocks to a representative agent.

The paper relates to several strands of literature. First, our paper relates to the literature on heterogeneous beliefs. Our model has similarities to earlier work such as Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), and Basak (2000).[2] However, our paper has two important differences with the literature above. First, with the exception of Basak (2000) who considers disagreement about extraneous risk, the above papers consider disagreement about macroeconomic quantities such as consumption and inflation

[2]Models with disagreement include, among many others, Basak (2005), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), Cvitanic and Malamud (2011), Cvitanic, Jouini, Malamud, and Napp (2012b), Bhamra and Uppal (2014), and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2017a).

or dividends. Our paper differs as we consider disagreement about the preferences of the marginal investor in the future. Hence, while the previous literature has focused on the disagreement on the "supply side," we focus on disagreement on the demand side. As we show later, demand disagreement has distinct implications for the equilibrium and dynamics of the economy and helps addressing puzzles such as the "correlation puzzle" as well as being consistent with several other asset pricing facts. Second, the disagreement literature has mostly focused on economies with infinitely lived agents.[3] These models are simpler to solve, but often comes at the cost of the model being non-stationary.[4] As we consider an overlapping generations model, the economy is stationary and we can, therefore, study unconditional moments. We show that the stationary consumption share distribution can be very different than that of an economy with infinitely lived agents as it is possible that the aggregate consumption by the most patient agents can be less than that of the less patient agents. This contrasts an economy with infinitely lived agent, where only the most patient agent survives.

Second, our paper relates to the literature on heterogeneous preferences. Papers such as Dumas (1989), Wang (1996), Chan and Kogan (2002), Gomes and Michaelides (2008), Bhamra and Uppal (2009), Weinbaum (2009), Zapatero and Xiouros (2010), Gollier and Zeckhauser (2005), Cvitanic et al. (2012a), Longstaff and Wang (2013), Bhamra and Uppal (2014), and Ehling and Heyerdahl-Larsen (2017) all study the role heterogeneous preferences. Gârleanu and Panageas (2015) consider an overlapping generations model with two types of agents that have heterogeneous recursive preferences. Our setup has the same overlapping generations structure as in Gârleanu and Panageas (2015) with two major differences – 1) the fraction of each agent type being born every period is stochastic and 2) agents disagree about the dynamics of the process governing the fraction of each type being born.

[3] Collin-Dufresne, Johannes, and Lochstoer (2017) and Ehling, Graniero, and Heyerdahl-Larsen (2017b) consider overlapping generations models with heterogeneous beliefs.

[4] A notable expectation is Borovička (2015). He shows that in an economy with Epstein-Zin preferences, the wealth distribution might be stationary even when agents disagree. These restrictions have been used in Baker, Hollifield, and Osambela (2016).

Third, our paper relates to the literature focusing on the role of preference shocks. Garber and King (1983) and Campbell (1993) are early examples economies with preference shocks and the implication for asset prices. In international economics and finance, preferences shocks have been used in papers such as Stockman and Tesar (1995), Pavlova and Rigobon (2007), Pavlova and Rigobon (2008), and Gabaix and Maggiori (2015). Preferences shocks have also been used frequently in the macro literature in works such as Eggertsson and Woodford (2003), Eggertsson (2004), Hall (2014), and Bai et al. (2014). Albuquerque et al. (2016) and Maurer (2012) use preference shocks to the time discount factor to rationalize the correlation puzzle. In their work, recursive preferences are important for the preference shock to be priced in equilibrium. Our paper differs from the above papers as all agents in the economy have constant preference parameters, but the composition changes stochastically over time due to birth and death. Moreover, agents in our economy have incomplete information and disagree about the process determining the evolution of the mix of agents in the economy. As a consequence, agent trade on demand risk due to differences in beliefs. In our economy, shocks to the composition of the newborn agents are priced even without recursive preferences as long as the agents have different time discount factors and beliefs.

Finally, our paper relates to the literature on demand discovery, which is based on the early work of Grossman (1988) and Kraus and Smith (1996). This literature studies how private information about future demand is transmitted through asset prices and trade.[5] In contrast to this literature, investors agree to disagree about future demand and thus there is nothing to learn from prices or trade.

2 Motivation—Empirical Evidence

Before presenting the model of demand disagreement, we discuss the empirical limitations and challenges of existing asset pricing models. In order to do that, consider the stock

[5]See also Jacklin, Kleidon, and Pfleiderer (1992), Leach and Madhavan (1992), Saar (2001), Gallmeyer, Hollifield, and Seppi (2005), and Gallmeyer, Hollifield, and Seppi (2017).

market, defined as a claim on the future aggregate dividend stream. Specifically,

$$S_t = E_t \left[\sum_{\tau=1}^{\infty} \frac{\xi_{t+\tau}}{\xi_t} D_{t+\tau} \right], \quad (1)$$

where ξ_t is the stochastic discount factor and D_t is the dividend. The stochastic discount factor is a function of the fundamentals of the economy, that is, $\xi_t = f\,(\text{fundamentals})$. In classical asset pricing models, all shocks to fundamentals are shocks to the supply side of the economy, e.g. output shocks in Lucas (1978) endowment economies or productivity shocks in Jermann (1998) production economies. Hence, in these models stock market returns are highly correlated with measures of output growth, which is in stark contrast to the weak short and long term correlations between stock returns and output growth, in particular consumption and GDP growth, observed in the data. Cochrane and Hansen (1992), Campbell and Cochrane (1999), and Cochrane (2005) refer to this finding as the "correlation puzzle." For instance, Table 1 shows that the correlation between stock market returns and consumption at the one-, five-, and ten-year horizon does not exceed 30%. While the correlation between stock market returns and dividends is much higher and increasing with the horizon, it is with 65% still significantly lower than predicted by classical asset pricing models.

Table 1: **Correlation between returns, volume and interest rates and fundamentals** The table shows the correlation between dividend and consumption growth and (i) stock market returns, (ii) stock market trading volume, and (iii) the realized real yield. Stock market, dividend, consumption, and yield data are from Shiller's webpage and stock market trading volume is from the NYSE webpage. We consider annual frequency over the period beginning 1891 until end of 2009.

	Returns		$\Delta_{\%}Volume$		ΔY_1	
	Dividends	Consumption	Dividends	Consumption	Dividends	Consumption
1 year	0.08	0.00	0.21	0.29	0.26	-0.16
5 year	0.46	0.28	0.35	0.38	0.20	0.04
10 year	0.65	0.11	0.31	0.40	0.24	0.06

We show that there is similar disconnect between trading volume and supply side fundamentals. There is no trade in representative agent models such as Campbell and Cochrane

(1999) or Bansal and Yaron (2004), but there is trade in asset pricing models in which agents have different preferences or beliefs. However, these models imply a high correlation between trading volume and measures of output growth, which is inconsistent with the weak short and long term correlations between stock market trading volume and dividend, as well as consumption growth reported in Table 1. Moreover, existing heterogenous beliefs models imply a high correlation between disagreement about future interest rates and disagreement about output or inflation, which is inconsistent with the data as shown in Table 2. Specifically, the R^2 when regressing levels and changes of disagreement about the future nominal one-year yield onto levels and changes of disagreement about one-year GDP growth and inflation rates (and higher order terms to account for possible nonlinearities as in Feldhütter, Heyerdahl-Larsen, and Illeditsch (2016)) are significantly below one.

Table 2: **Yield disagreement versus fundamental disagreement** The table show the regression results from regressing the disagreement about yields on inflation and GDP disagreement. Disagreement is based on Survey of Professional Forecasters (SPD) and is measured as the interquartile range. Yield disagreement is based on the one year forecasts of the 3 month treasury bill rate. We consider quarterly frequency and the time period covers 1981 Q3 – 2017 Q2, i.e. 144 quarters.

	Constant	DIS_{GDP}	DIS_{INFL}	DIS^2_{GDP}	DIS^2_{INFL}	$DIS_{GDP}DIS_{INFL}$	R^2_{adj}
Levels							
	-0.003	0.538	0.355				0.483
	(-1.676)	(3.600)	(2.327)				
	0.007	-0.595	-0.760	19.931	3.370	87.823	0.600
	(2.952)	(-1.167)	(-1.503)	(0.731)	(0.093)	(2.836)	
% changes							
	-0.000	0.065	0.312				0.073
	(-0.804)	(0.493)	(1.790)				
	-0.000	0.085	0.363	35.124	-35.407	32.046	0.090
	(-0.717)	(0.620)	(1.968)	(2.121)	(-1.131)	(0.839)	

To conclude, there is a disconnect between stock market returns and macroeconomic fundamentals, as well as, trading volume in many existing asset pricing models. Similarly, current heterogenous beliefs models focus on disagreement about macroeconomic fundamentals, but this disagreement explains only a part of disagreement about future asset returns.

In the next section, we present a novel asset pricing model that focuses on disagreement about demand shocks, that we micro found by considering heterogenous time preferences and disagreement about the distribution of future time preferences. In Section 4, we show that this model can reconcile the low correlation between output growth and (i) stock market returns and (ii) stock market trading volume. Moreover, disagreement about future risk premia and risk-free rates are unrelated to disagreement about the supply side of the economy.

3 Model

We consider a continuous-time overlapping generations economy in the tradition of Blanchard (1985) and, more recently, Gârleanu and Panageas (2015). Every agent faces an uncertain lifespan where the hazard rate of death is $\nu > 0$. Hence, in each period a fraction ν of the population dies. A new cohort of mass ν is born every period and, therefore, the total population size remains constant. All agents alive at time t receive an endowment Y_t with dynamics given by

$$dY_t = Y_t \left(\mu_Y dt + \sigma_Y dz_{Y,t}\right), \tag{2}$$

and, therefore, aggregate endowment at time t is $\int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y_t ds = Y_t$. There are two types of agents; type a with a time discount rate of ρ_a and type b with a time discount rate of ρ_b, and we assume without loss of generality that $\rho^a < \rho^b$. The fraction of newborns being type a is $\alpha_t = \alpha(l_t) = \frac{1}{1+e^{-l_t}} \in (0,1)$, where l_t is a stochastic process with dynamics

$$dl_t = \kappa \left(\bar{l} - l_t\right) dt + \sigma_l dz_{\alpha,t}, \tag{3}$$

where $z_{\alpha,t}$ is independent of the shocks to the endowment, $z_{Y,t}$. Note that α_t and l_t are positively correlated and that $\lim_{l \to \infty} \alpha(l) = 1$ and $\lim_{l \to -\infty} \alpha_t = 0$.

3.1 Information and Disagreement

Agents observe l_t and know how α relates to l_t, but disagree about the dynamics. Specifically, agents of type i believe that l_t follows

$$dl_t = \kappa\left(\bar{l}^i - l_t\right)dt + \sigma_l dz^i_{\alpha,t}, \tag{4}$$

with $\bar{l}^b < \bar{l} < \bar{l}^a$. Hence, everyone in the economy believes that the long-run mean of people with similar preferences is higher than it actually is. Therefore there is a "false consensus bias" where people tend to overestimate the likelihood that other people share the same preferences, beliefs, opinions etc.[6] We assume that both agents are equally biased about the long-run mean of l and, therefore, the beliefs can be described by a bias parameter d such that $\bar{l}^a = \bar{l} + d$ and $\bar{l}^b = \bar{l} - d$. As agents have biased believes about the long-run mean of l_t they perceive different shocks. The true shock to l_t and how it is perceived by an agent of type i is related through:

$$dz^i_{\alpha,t} = dz_{\alpha,t} - \Delta^i_\alpha dt, \tag{5}$$

where $\Delta^i_\alpha = \frac{\kappa}{\sigma_l}\left(\bar{l}^i - \bar{l}\right)$.

3.2 Security Markets

Agents trade in an instantaneously risk-free asset, which is in zero net supply, with dynamics

$$dB_t = r_t B_t dt, \tag{6}$$

where r_t denotes the equilibrium real short rate.

For tractability, we assume that agents can trade two infinitely lived risky assets in zero

[6]See Ross et al. (1977).

net supply,[7] which evolve according to

$$\frac{dS_t^Y}{S_t^Y} = \left(\mu_{S,t}^Y dt + \sigma_S^Y dz_{Y,t}\right), \tag{7}$$

$$\frac{dS^\alpha}{S_t^\alpha} = \left(\mu_{S,t}^\alpha dt + \sigma_S^\alpha dz_{\alpha,t}\right) = \left(\hat{\mu}_{S,t}^{i,\alpha} dt + \sigma_S^\alpha dz_{\alpha,t}^i\right). \tag{8}$$

In Equation (7) and (8), the diffusion coefficients, σ_S^Y and σ_S^α are exogenous, but the expected returns, $\mu_{S,t}^Y$ and $\mu_{S,t}^\alpha$, are determined in equilibrium. In addition to the two zero net supply risky assets, we also calculate the value of the total endowment of all agents currently alive. We refer to this as the stock market and denote the price as S_t^M. The corresponding return process, R_t^M, is

$$dR_t^M = \mu_{M,t} dt + \sigma_{M,t}^Y dz_{Y,t} + \sigma_{M,t}^\alpha dz_{\alpha,t}, \tag{9}$$

where $\mu_{M,t}$, $\sigma_{M,t}^Y$ and $\sigma_{M,t}^\alpha$ are determined in equilibrium.

Finally, annuity contracts complete the set of available securities as in Yaari (1965). They entitle to an income stream of $\nu W_{s,t}$ per unit of time. In return, the competitive insurance industry receives all financial wealth when the agent dies. Entering such a financial contract is optimal for all agents.

It is convenient to summarize the price system in terms of investor-specific stochastic discount factors that capture the investor-specific beliefs, but common Arrow-Debreu prices across investors. Investor i's stochastic discount factor has dynamics

$$d\xi_t^i = -r_t \xi_t^i - \theta_{Y,t} \xi_t^i dz_{Y,t} - \theta_{\alpha,t}^i \xi_t^i dz_{\alpha,t}^i, \tag{10}$$

where $\theta_{Y,t}$ denotes the market price of risk of the shock to endowment and $\theta_{\alpha,t}^i$ denotes the perceived market price of risk of the shocks to α. Under the true probability measure, the market price of risk of the shocks to α, i.e., the demand shocks, is $\theta_{\alpha,t}$ and is related to the perceived market price of risk through $\theta_{\alpha,t}^i = \theta_{\alpha,t} + \Delta_\alpha^i$. We define the disagreement process,

[7]The risky assets can be interpreted as continuously resettled contracts (e.g., futures contracts). The same asset structure is used, for example, in Basak (2000) and Karatzas, Lehoczky, and Shreve (1994).

η_t^i, through the relation between the stochastic discount factor under the true probability measure and the belief of an agent of type i, i.e, $\xi_t^i = \frac{\xi_t}{\eta_t^i}$, where ξ_t denotes the discount factor under the true measure with dynamics

$$d\xi_t = -r_t\xi_t - \theta_{Y,t}\xi_t dz_{Y,t} - \theta_{\alpha,t}\xi_t dz_{\alpha,t}. \tag{11}$$

The disagreement process, η_t^i, is a Radon Nikodym derivative that allows one to move between the probability measure of an agent of type i to the true probability measure and vice versa. The dynamics of the disagreement processes of the agent of type i is

$$d\eta_t^i = \Delta_{\alpha,t}^i \eta_t^i dz_{\alpha,t}. \tag{12}$$

3.3 Preferences and the Consumption-Portfolio Choice Problem

An agent of type i born at time s maximize lifetime utility given by

$$E_s^i \left[\int_s^\tau e^{-\rho_i(t-s)} log\left(c_{s,t}^i\right) dt \right], \tag{13}$$

where τ is the stochastic time of death and the superscript in the expectation operator denotes that the expectation is taken under the belief of the agent. The random time of death, τ, is exponentially distributed and independent of all other shocks and, therefore, we integrate it out to write the expected lifetime utility as

$$E_s^i \left[\int_s^\infty e^{-(\rho_i+\nu)(t-s)} log\left(c_{s,t}^i\right) dt \right]. \tag{14}$$

The dynamics of financial wealth, $W_{s,t}$, of an agent born at time s follows

$$dW_{s,t}^i = \left(r_t W_{s,t}^i + \pi_{s,t}^i \left(\mu_{S,t}^Y - r_t\right) + \varphi_{s,t}^i \left(\hat{\mu}_{S,t}^{i,\alpha} - r_t\right) + \nu W_{s,t}^i + Y_t - c_{s,t}^i\right) dt + \pi_{s,t}^i \sigma_{S,t}^Y dz_{Y,t} + \varphi_{s,t}^i \sigma_{S,t}^\alpha dz_{\alpha,t}^i, \tag{15}$$

where $\pi^i_{s,t}$ and $\varphi^i_{s,t}$ denote the dollar amounts held in the risky asset loading on $z_{Y,t}$ and $z^i_{\alpha,t}$, respectively. Since agents are born without any financial wealth, we have that $W^i_{s,s} = 0$.

Agents maximize their lifetime utility, Equation (14), subject to the wealth dynamics in equation (15).

3.4 Equilibrium

Definition 1. *Given preferences, endowments, and beliefs, an equilibrium is a collection of allocations* $\left(c^i_{s,t}, \pi^i_{s,t}, \varphi^i_{s,t}\right)$ *and a price system* $\left(r_t, \mu^Y_{S,t}, \hat{\mu}^{i,\alpha}_{S,t}\right)$ *such that the processes* $\left(c^i_{s,t}, \pi^i_{s,t}.\varphi^i_{s,t}\right)$ *maximize utility given in Equation (14) subject to the dynamic budget constraint given in Equation (15) for* $i = a, b$ *and markets clear:*

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s c^a_{s,t} + (1-\alpha_s) c^b_{s,t}\right) ds = Y_t, \tag{16}$$

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s \pi^a_{s,t} + (1-\alpha_s) \pi^b_{s,t}\right) ds = 0, \tag{17}$$

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s \varphi^a_{s,t} + (1-\alpha_s) \varphi^b_{s,t}\right) ds = 0, \tag{18}$$

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s \left(W^a_{s,t} - \pi^a_{s,t} - \varphi^a_{s,t}\right) + (1-\alpha_s)\left(W^b_{s,t} - \pi^b_{s,t} - \varphi^b_{s,t}\right)\right) ds = 0. \tag{19}$$

In the model, the market is not complete as unborn agents cannot trade to hedge market conditions at the time they are born. However, once born, they face complete markets as there are four securities and three sources of risk ($z_{Y,t}$, $z_{\alpha,t}$, and mortality risk).[8] Hence, we can solve the maximization problem by using martingale methods as in Cox and Huang (1989). Consider the static optimization problem for an agent of type i born at time s,

$$\max_{c_s} \quad E^i_s \left[\int_s^{\infty} e^{-(\rho_i+\nu)(t-s)} log\left(c^i_{s,t}\right) dt\right]$$

$$s.t \;\; E^i_s \left[\int_s^{\infty} e^{-\nu(t-s)} \xi^i_t c^i_{s,t} dt\right] = E^i_s \left[\int_s^{\infty} e^{-\nu(t-s)} \xi^i_t Y_t dt\right]. \tag{20}$$

[8] As the risky securities loading on the output shock and the shocks to α are exogenous, we can ensure that the variance-covariance matrix is non-degenerate for all times and states.

The first order conditions (FOCs) of the above problem are

$$\frac{e^{-(\rho_i+\nu)(t-s)}}{c^i_{s,t}} = \kappa_s e^{-\nu(t-s)}\xi^i_t, \tag{21}$$

where κ_s denotes the Lagrange multiplier of the static budget constraint given in Equation (20). Using the FOCs, we can show that the optimal consumption of an agent of type i born at time s at time t is

$$c^i_{s,t} = \beta^i_s Y_s e^{-\rho_i(t-s)}\left(\frac{\eta^i_t}{\eta^i_s}\right)\left(\frac{\xi_s}{\xi_t}\right), \tag{22}$$

where $\beta^i_s = \frac{c^i_{s,s}}{Y_s}$. Let $H_s = E^i_s\left[\int_s^\infty e^{-\nu(t-s)}\xi^i_t Y_t dt\right]$, then using Equation (22) and the static budget condition, (20), it follows that $c^i_{s,s} = (\rho^i+\nu)\,H_s$. Here, the total wealth of a newborn agent is the same as that of the rest of the economy as the endowment is the same for every agent. Consequently, we have that $\beta^i_s = (\rho^i+\nu)\,\phi_s$, where $\phi_s = \frac{S^M_s}{Y_s}$ is the price-dividend ratio of the total wealth in the economy. Hence, the initial consumption of a newborn depends on the prevailing market conditions as measured by the price-dividend ratio, ϕ_t. Inserting the optimal consumption in Equation (22) into the market clearing in the consumption good in Equation (16), we can solve for the stochastic discount factor.

Proposition 1. *In equilibrium, the stochastic discount factor is*

$$\xi_t = \frac{X_t}{Y_t}, \tag{23}$$

where X_t solves the integral equation

$$X_t = \int_{-\infty}^t \nu e^{-\nu(t-s)}\left(\alpha_s\beta^a_s e^{-\rho^a(t-s)}\frac{\eta^a_t}{\eta^a_s} + (1-\alpha_s)\,\beta^b_s e^{-\rho^a(t-s)}\frac{\eta^b_t}{\eta^b_s}\right)X_s ds. \tag{24}$$

The stochastic discount factor in Proposition 1 depends on two parts; (1) the aggregate endowment, Y_t, which enters similarly to a standard infinitely lived representative agent economy with log utility, and (2) the process X_t which capture time discounting and the fact that agents are heterogenous with respect to both their discount factors and their beliefs. It is useful to decompose the process into two parts as $X_t = X_t^a + X_t^b$ where

$$X_t^a = \int_{-\infty}^{t} \nu e^{-(\rho^a+\nu)(t-s)} \alpha_s \beta_s^a \frac{\eta_t^a}{\eta_s^a} X_s ds, \tag{25}$$

$$X_t^b = \int_{-\infty}^{t} \nu e^{-\left(\rho^b+\nu\right)(t-s)} \left(1-\alpha_s\right) \beta_s^b \frac{\eta_t^b}{\eta_s^b} X_s ds. \tag{26}$$

As we will show later, it is not necessary to know the individual consumption share of every agent in the economy to fully characterize asset prices. Instead, it is sufficient to know the total consumption of each agent type. Hence, we define the fraction of consumption consumed by agents of type a at time t as f_t, and it follows that $f_t = \frac{X_t^a}{X_t}$. An application of Ito's lemma to $\frac{X_t}{Y_t}$ and matching the drift and the diffusion coefficients of the stochastic discount factor in Equation (11) yields the equilibrium real short rate and market prices of risk.

Proposition 2. *In equilibrium, the real short rate is*

$$r_t = f_t \rho^a + \left(1-f_t\right) \rho^b + \mu_Y - \sigma_Y^2 + \nu \left(1 - \alpha_t \beta_t^a - \left(1-\alpha_t\right) \beta_t^b\right), \tag{27}$$

and the market prices of risk are

$$\theta_{Y,t} = \sigma_Y, \tag{28}$$

$$\theta_{\alpha,t} = -\bar{\Delta}_{\alpha,t}, \tag{29}$$

where $\bar{\Delta}_{\alpha,t} = f_t \Delta_\alpha^a + \left(1-f_t\right) \Delta_\alpha^b$ *is the consumption share weighted average estimation error.*

The real short rate in Equation (27) can be decomposed into three parts. First, we see that the effective time discounting is the consumption weighted average time discount factor in the economy. When f_t is high, the agents of type a consume a larger fraction of the total consumption and, therefore, their views are more important in determining the interest rate. Second, the intertemporal smoothing and precautionary saving motives are captured by the term, $\mu_Y - \sigma_Y^2$, and are the same as in a standard infinitely lived representative agent economy with log utility. The last term, $\nu\left(1 - \alpha_t \beta_t^a - (1 - \alpha_t)\, \beta_t^b\right)$, is due to the overlapping generation structure. In the OLG economy, the expected growth in aggregate consumption and expected growth of the consumption of agents currently alive might differ. As the current interest rate is only determined by agents currently alive, a displacement effect must be taken into account. Specifically, if agents who are currently alive are expected to consume less on average than agents that are born next period, then the effective consumption growth is lower and, therefore, the interest rate is also lower. Note that the disagreement about the process, α_t, does not enter directly in the real short rate.[9] Yet, as we will show later, disagreement is important in determining the distribution of the real rate. Specifically, the consumption share, f_t, and the relative consumption of newborns to average consumption, β^i, have distributions that depend heavily on the disagreement in the economy.

Turning to the market prices of risk, we see that the price of endowment shocks, $\theta_{Y,t}$, is the same as in a standard log utility economy. Importantly, the market price of shocks to composition of agent types, α, are in generally priced. The price of these demand shocks depend on the consumption weighted average estimation error. Hence, the only time the price of risk is zero is when the consumption share weighted average estimation error is zero. However, the price of risk can be both positive and negative. For instance, if the consumption weighted average estimation error is positive, which means that relative to the true measure the agents believe growth in α to be higher, then the price of demand shocks are negative. To see this, consider an asset that is perfectly positively correlated with α shocks. Under the true measure, this asset will appear to be expensive as the agents in the economy who price the asset believe the growth to be higher or put differently the risk premium is low. Note that when the agents have the correct belief the shocks to α are never priced even though it impact the distribution of preferences. As Proposition 2 illustrates, the consumption share

[9] If we allow for correlation between endowment and α, then the real short rate would directly depend on the disagreement. However, as we are focusing on pure demand disagreement we do not consider this case.

is an important state variable and influence the distribution of the real rate and the market prices of risk. The next Proposition derives the dynamics of the consumption share, f_t.

Proposition 3. *The dynamics of the consumption share of agents of type a is*

$$df_t = \mu_{f,t}dt + \sigma^f_{\alpha,t}dz_{\alpha,t}, \tag{30}$$

where

$$\mu_{f,t} = \nu\left(\alpha_t\beta^a_t\left(1-f_t\right) - \left(1-\alpha_t\right)\beta^b_t f_t\right) + \left(\rho^b-\rho^a\right)f_t\left(1-f_t\right) + f_t\left(1-f_t\right)\bar{\Delta}_{\alpha,t}\left(\Delta^b_\alpha - \Delta^a_\alpha\right), \tag{31}$$

and

$$\sigma^f_{\alpha,t} = f_t\left(\Delta^a_\alpha - \bar{\Delta}_\alpha\right). \tag{32}$$

The diffusion coefficient of the consumption share, $\sigma^f_{\alpha,t}$, is zero whenever the agents agree or when the consumption share approaches the boundaries of zero or one. However, the consumption share is stochastic even without disagreement due to the process α_t that influence the expected growth rate. The drift of the consumption share, $\mu_{f,t}$, has three terms. The first is due to the overlapping generation structure, the second is due to difference in discount factors and third is due to differences in beliefs. It is useful to consider what happens to the expected change in the consumption share when it approaches the boundaries. We have that $\lim_{f_t\to 0}\mu_{f,t} = \nu\alpha_t\beta^a_t > 0$ and $\lim_{f_t\to 1}\mu_{f,t} = -\nu\left(1-\alpha_t\right)\beta^b_t < 0$; hence the consumption share is pushed back into the interior whenever it approaches the boundaries due to the birth of new agents. The second term in the drift, $\left(\rho^b-\rho^a\right)f_t\left(1-f_t\right)$, is positive whenever $\rho^b > \rho^a$. This reflects the market selection force that favours the more patient agent. In a non-OLG economy, the consumption share of the agent with the lowest discount factor approaches one in the limit. The third term, $f_t\left(1-f_t\right)\bar{\Delta}_{\alpha,t}\left(\Delta^b_\alpha - \Delta^a_\alpha\right)$, captures the expected change due to differences in beliefs. Just as for the second term, the effect from disagreement vanishes when the consumption share approaches the boundaries.

The price-dividend ratio, ϕ_t, is important for both the interest rate and the dynamics of the consumption shares through the initial consumption of the new born agents, β_t^i. Proposition 4 shows the equilibrium price-dividend ratio.

Proposition 4. *In equilibrium, the price-dividend ratio, ϕ_t is*

$$\phi_t = \frac{f_t}{\rho^a + \nu} + \frac{1 - f_t}{\rho^b + \nu}. \tag{33}$$

Proposition 4 shows that the price-dividend ratio is a consumption weighted average of the price-dividend ratio that would prevaile in the homogeneous preference economies with either agent a or b as the only agents. Hence, the price dividend ratio is bounded between $\frac{1}{\rho^b+\nu}$ and $\frac{1}{\rho^a+\nu}$. It is easy to show that in the overlapping generation economy with constant α (and no disagreement) the steady state consumption share is constant even though agents have different time discount factors. Hence, the price-dividend ratio in such an economy is also constant. When α is stochastic, but agents agree on its dynamics then the price-dividend ratio is locally deterministic.

Remark 1. *It is interesting to contrast our overlapping generations economy with demand disagreement to a similar economy with homogeneous preferences but disagreement about the endowment process. In the latter economy, the price-dividend ratio is constant just as in an infinitely lived representative agent economy with log utility.*

The next Proposition characterizes the return dynamics of the aggregate wealth, P_t^M.

Proposition 5. *In equilibrium, the expected return on the aggregate wealth is*

$$\mu_{M,t} = r_t + \theta_{Y,t}\sigma_{M,t}^Y + \theta_{\alpha,t}\sigma_{M,t}^\alpha, \tag{34}$$

and the diffusion coefficients are

$$\sigma^Y_{M,t} = \sigma_Y, \tag{35}$$

$$\sigma^\alpha_{M,t} = \left(\frac{\frac{1}{\rho^a+\nu} - \frac{1}{\rho^b+\nu}}{\frac{f_t}{\rho^a+\nu} + \frac{1-f_t}{\rho^b+\nu}}\right) f_t\left(1-f_t\right)\left(\Delta^a_\alpha - \Delta^b_\alpha\right). \tag{36}$$

The loading on the stock market onto the shock to output, $\sigma^Y_{M,t}$, equals the volatility of output and, hence, it is the same as in a standard log-utility economy without overlapping generations or disagreement. However, the stock market also loads onto shocks to α, i.e., the demand shocks. From Equation (36), it is evident that it is necessary with both heterogeneity in discount rates and beliefs for $\sigma^\alpha_{M,t}$ to be non-zero. Moreover, under the parameter restriction $\rho^a < \rho^b$ and $\Delta^a_\alpha - \Delta^b_\alpha > 0$ (false consensus bias), then the diffusion coefficient, $\sigma^\alpha_{M,t}$, is positive.

As the agents have different preferences and beliefs, the optimal portfolios are also heterogeneous. Proposition 6 below characterizes the optimal investment in the risky securities.

Proposition 6. *The optimal portfolio of an agent of type i born at time $s < t$ is*

$$\pi^i_{s,t} = \frac{\sigma_Y}{\sigma^Y_{S,t}} W^i_{s,t}, \tag{37}$$

$$\varphi^i_{s,t} = \frac{1}{\sigma^\alpha_{S,t}}\left(\hat{W}_{s,t}\left(\Delta^i_\alpha - \bar{\Delta}_{\alpha,t}\right) - S^M_t \sigma^\alpha_{M,t}\right). \tag{38}$$

As each of the risky securities only loads on one shock, the optimal portfolios in Proposition 6 can be interpreted as the desired excess exposure relative to the exposure embedded in the endowment stream. In Proposition 6, it is useful to consider the case in which $\sigma^Y_{S,t} = \sigma_Y$ and $\sigma^\alpha_{S,t} = 1$.[10] In this case, the optimal investment (dollar amount) in the security depending on the output shock is simply the total financial wealth. The reason for this is that agents agree on output shocks and have the same risk preferences; therefore, they fully share the output risk. As a benchmark comparison, if agents agree about demand shocks, then

[10]The value for the exogenous diffusion coefficients of the risky securities does not alter the equilibrium in anyway. Hence, we choose values that makes the interpretation easy.

every agent simply invests his entire financial wealth in the security depending on the output shock. The optimal investment in the risky security depending on the demand shock, $\varphi^i_{s,t}$, depends on the relative disagreement, $\Delta^i_\alpha - \bar{\Delta}_{\alpha,t}$. This implies that keeping everything else constant, if an agent is optimistic relative to the market view, $\bar{\Delta}_{\alpha,t}$, then the agent invest more in the risky security depending on the demand shock. Moreover, the term $S^M_t \sigma^\alpha_{M,t}$ captures the fact that an agent has some intrinsic exposure to the demand shock via the endowment and, therefore, the optimal investment in the security depends on the optimal excess exposure relative to the exposure from the endowment.

4 Numerical Illustrations

4.1 Parameters

The model has a total of nine parameters $\left(\mu_Y, \sigma_Y, \nu, \rho^a, \rho^b, \kappa, \bar{l}, d, \sigma_l\right)$. We set the parameters of the base case in the following way. For the output process, we set the expected output growth, μ_Y, and volatility σ_Y to 2% and 3.3%, respectively. This is similar to the long sample in Campbell and Cochrane (1999). The birth and death intensity, ν, is set to 2% which implies an expected life of 50 years from the start of trading, as in Gârleanu and Panageas (2015). We set the time preference parameter of type a, ρ_a, to 0.001, which is the same as in Gârleanu and Panageas (2015). Time preference parameters in Bansal and Yaron (2004), Chan and Kogan (2002), and Campbell and Cochrane (1999) are with 2.4%, 5.2%, and 11.6% higher and, thus, we set the value of type b, ρ^b, to 0.05. For the dynamics of the process driving the share of the two types, l_t, we set the long-run mean under the true measure, $\bar{l}$, to zero. This implies that α_t evaluated at the steady state is 0.5.[11] For the local volatility, σ_l, we choose a value of 0.1 and we set the speed of mean reversion, κ, to 0.01 in order to capture a slow moving change in the composition of types.

[11] The expected value is not exactly 0.5 due to a Jensen's term. However, for our parameter choice this effect is negligable.

For the specification of the beliefs, we consider three different cases; no disagreement ($d = 0$), medium disagreement ($d = 2$), and high disagreement ($d = 4$), and, hence, relative disagreement, Δ, is 0, 0.4, and 0.8, respectively. Figure 1 shows the mean of $\alpha_{t+\tau}$ conditional on $\alpha_t = 0.5$ under all three beliefs as a function of the horizon τ for the case with medium ($d = 2$) and high ($d = 4$) disagreement. The conditional mean under both agents' belief is within the confidence interval around the truth for medium disagreement and leaves the confidence bound only for horizons above 12 years with high disagreement. To provide further support for the plausibility of our disagreement parameters, we link our model to disagreement about output growth. Specifically, a relative disagreement, Δ, of 0.4 and 0.8 in our demand disagreement model corresponds to disagreement about expected output growth of 1.32% and 2.64%. These numbers are comparable to the time-series average over the interquartile range of one-year ahead GDP growth forecast of 1.35% which based on the Survey of Professional Forecasters (SPF).[12]

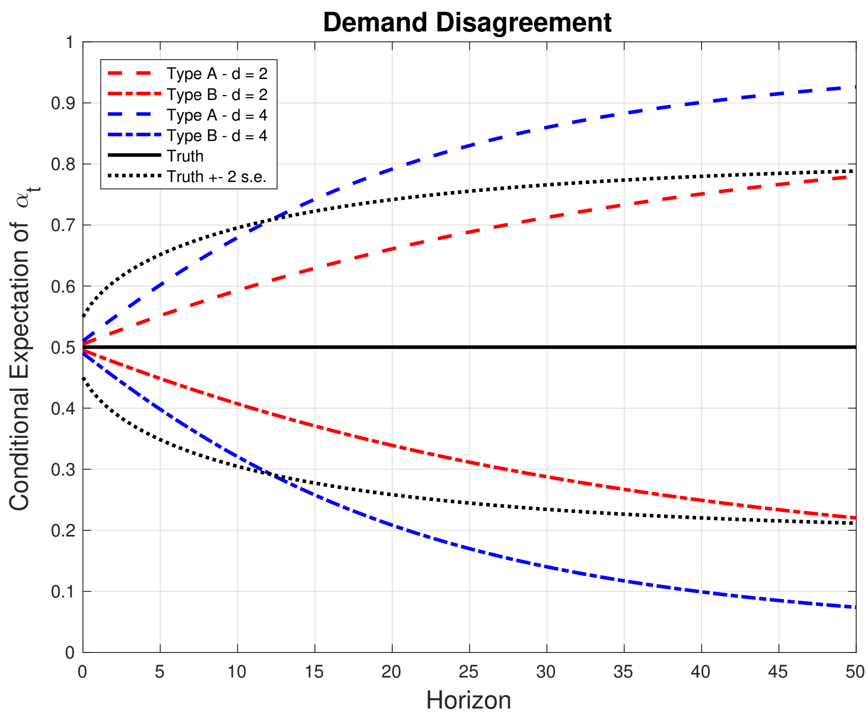
4.2 Asset Prices and the Correlation Puzzle

Correlation puzzle. As documented by Cochrane and Hansen (1992), Campbell and Cochrane (1999), Cochrane (2005), and Albuquerque, Eichenbaum, Luo, and Rebelo (2016), the correlation between stock returns and macroeconomic fundamentals is low. Figure 2 shows the correlation between stock market returns and aggregate consumption for the one, five, and ten year horizon. The figure shows that without disagreement about demand shocks, the correlations are close to one. Hence, demand shocks alone are not sufficient to solve the correlation puzzle. On the other hand, when disagreement increases, then the correlation decreases at an increasing rate, and thus for reasonable disagreement, we get correlations comparable to the one we see in the data.

What is the economic intuition for this solution of the correlation puzzle? First, suppose there are no demand shocks, then the price dividend ratio is constant and stock market

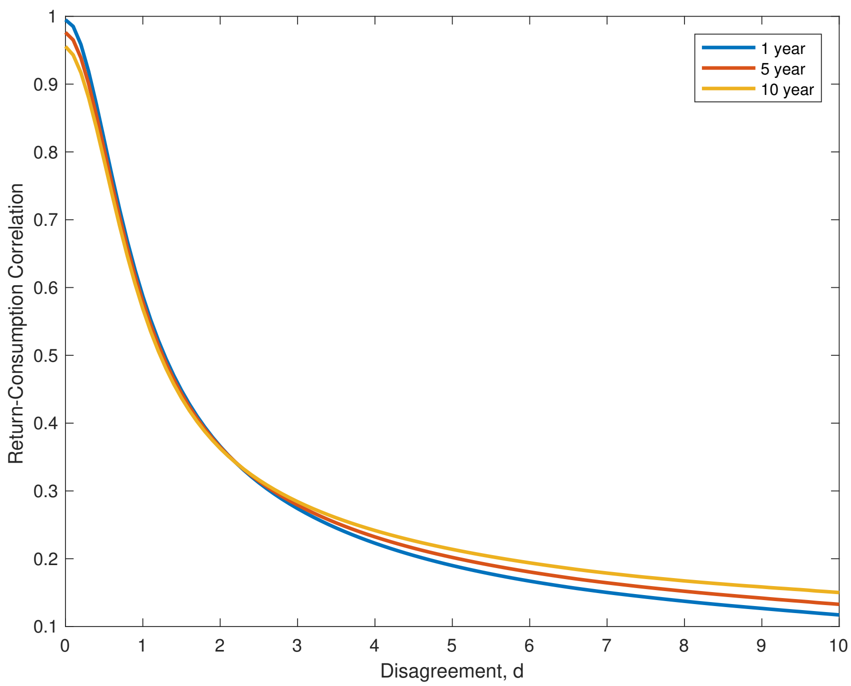
[12]When we convert disagreement Δ into disagreement about GDP growth rates, we multiply by output growth volatility, $\sigma_Y = 3.3\%$.

Figure 1: *Disagreement about the Distribution of* α_t. The figure shows the mean of $\alpha_{t+\tau}$ conditional on $\alpha_t = 0.5$ under the belief of type A, type B, and the true data generating measure as a function of the horizon τ. The red lines show the case of medium disagreement ($d = 2$) and the blue lines show the case of high disagreement ($d = 4$).



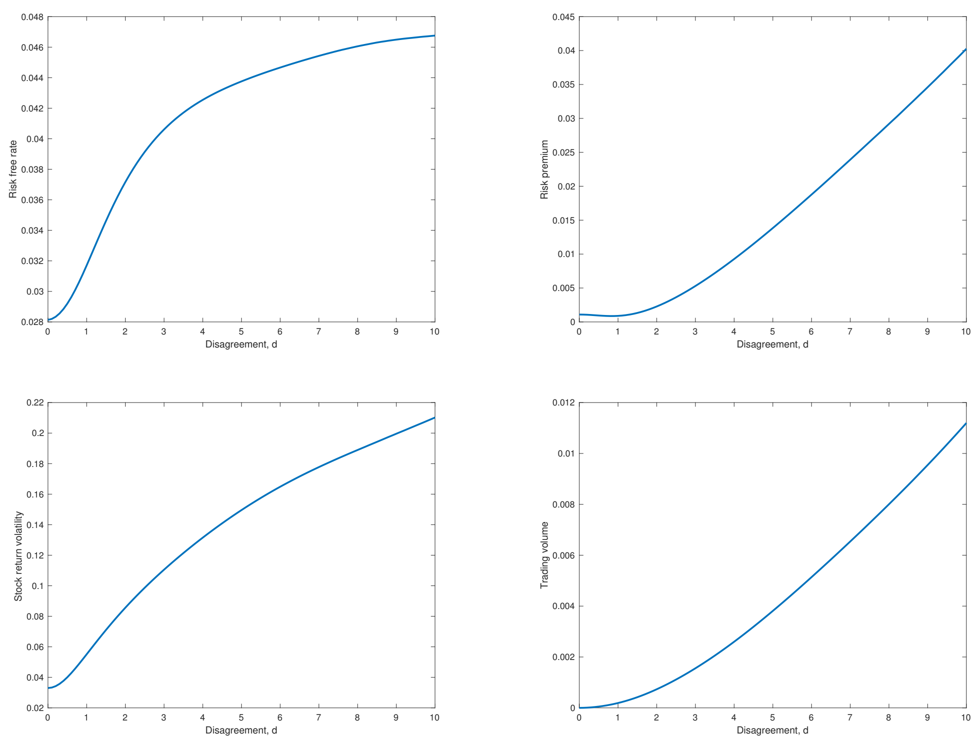
returns are perfectly correlated with output shocks. When there are commonly perceived demands shocks, then the price dividend ratio is stochastic but its dynamics are locally deterministic; hence short term correlations between stock market returns and consumption are close to one. The correlation is 0.95 when measured over ten years and thus the indirect effect of the demand shock through the drift of the consumption share, as discussed in more detail in the next section, is quantitatively small. The indirect effect is small because heterogenous time preferences, while leading to different consumption-savings rates, have no effect on the composition of the risky portfolio. In contrast, when agents have different beliefs about demand shocks, they engage in speculative trade, thus changing there consumption-saving rates and portfolio compositions (the implications for trade are explored in Section 4.6). Therefore demand shocks, which are independent to output shocks, lead to shocks to the price dividend ratio, the risk free rate, and the volatility and risk premium of the stock market. Hence, demand disagreement breaks the tight link between shocks to output growth and stock market returns (and trading volume as discussed in Section 4.6) and solves the correlation puzzle.

Figure 2: *Correlation Puzzle–Demand Disagreement.* The figure shows the correlation between returns and aggregate consumption for one, five and ten years overlapping observations when changing the amount of disagreement measured by d. For each value of disagreement, the correlations are based on 1,000,000 years of monthly observations



Unconditional asset pricing moments. Figure 3 plots the risk free rate, the stock market volatility, the stock market risk premium, and stock market trading volume as a function of disagreement.[13] All four quantities are increasing in disagreement. Moreover, the model can generate a reasonable risk-free rate (with low sensitive to changes in disagreement), a positive risk premium for demand shocks, excess volatility, and trading volume driven by demand shocks. The unconditional stock market risk premium is higher with disagreement even though the average belief of agents coincides with the truth.

Figure 3: *Unconditional asset pricing moments–Demand Disagreement.* The figure shows the risk free rate (top-left), stock market risk premium (top-left), the stock market standard deviation (bottom-left) and the trading volume (bottom-right) as a function of disagreement.For each value of disagreement, the values are averages based on 1,000,000 years of monthly observations

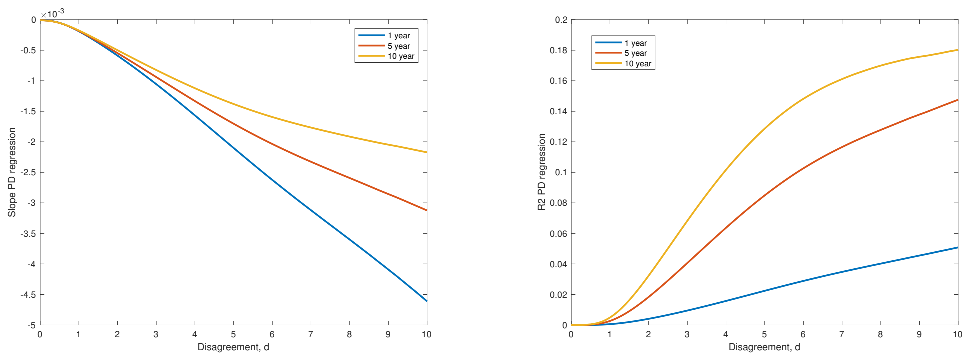


Stock Market Predictability. Figure 4 plots the slope coefficient and R^2 when regressing future realized excess returns onto the price-dividend ratio. The slope coefficient is negative and the R^2 is increasing with the horizon, which is consistent with the data. Hence, a low price-dividend ratio predicts higher future excess returns and by construction does not

[13] We define trading volume in Section 4.6.

predict dividend growth.

Figure 4: *Stock Market Predictability–Demand disagreement.* The figure shows the slope (left) and the R^2 (right) for the price-divided regressions $Rx_{t,t+\tau} = a + b\phi_t + \epsilon_{t+\tau}$, where $Rx_{t,t+\tau}$ is the excess return from t to $t+\tau$. For each value of disagreement, the values are averages based on 1,000,000 years of monthly observations



Stock market trading volume. As the above discussion highlights, the model can replicate the low correlation between stock market returns and output growth because heterogenous time-preferences lead to different consumption-saving rates and speculative trade due to different beliefs lead to different composition of risky portfolios. The latter breaking the strong link between stock market returns and macro economic fundamentals. Disagreement about the stock market risk premium and the resulting trade in the stock market is purely driven by demand shocks and, hence, the correlation between trading volume and macroeconomic fundamentals, which is also low in the data, is by construction zero.[14] The last graph in Figure 3 verifies that trading volume is increasing in demand disagreement.

We now discuss the economic mechanism of our demand disagreement model.

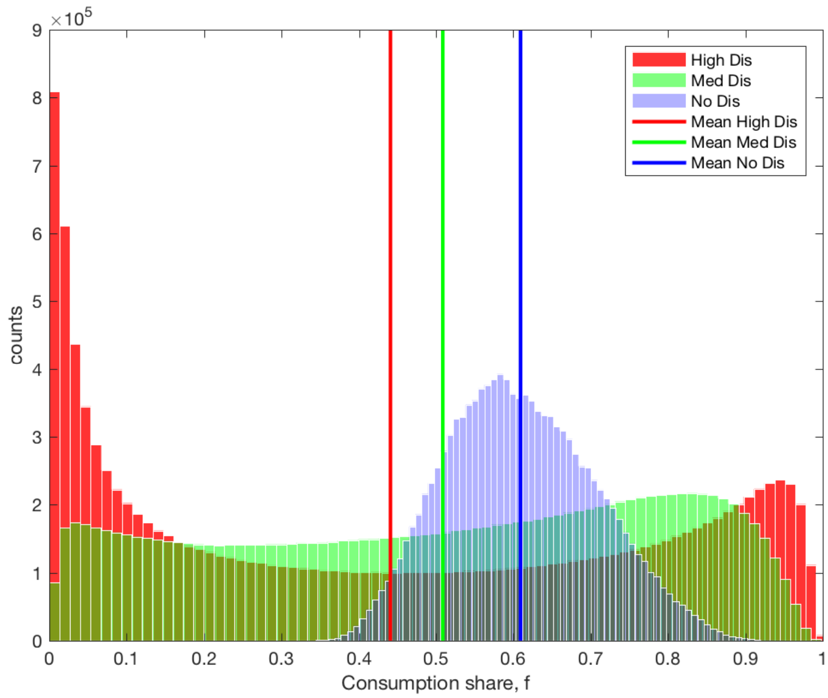
4.3 The Equilibrium Consumption Share

In the model there are two state variables; α_t (or equivalently l_t) which is exogenous and the endogenous consumption share of agents of type a, f_t. In contrast to most models with infinitely lived agents with heterogeneous preferences and beliefs our overlapping generations

[14] It is straightforward to increases this correlation by adding disagreement about output growth to the model.

models is stationary. Hence, we can study the unconditional distribution of the consumption share of agents of type a and b. Figure 5 shows distribution of the consumption share. The three vertical lines indicate the unconditional averages in the three different economies. For the case without disagreement, (No Dis), the average consumption share is 0.61. Note that the unconditional average α is 0.5, and hence in the case without disagreement, the patient agents consume more on average than the less patient. This is consistent with the market selection literature the shows that in an economy with heterogeneous time preferences the most patient agent's consumption share approaches one in the limit. In our economy, agents die and therefore no agent type dominates in the long run, yet there is a shift toward the more patient agent. Interestingly, as disagreement increases the consumption share of the most patient agents decreases, even to a level below 0.5 which is unconditional fraction of agents of type a in the economy. To understand this consider the behavior if the consumption share close to the boundaries. As the consumption share approaches zero, the drift of the consumption share approaches $\nu\alpha_t\frac{\rho^a+\nu}{\rho^b+\nu}$. Similarly, when the consumption share approaches one the drift approaches $-\nu\alpha_t\frac{\rho^b+\nu}{\rho^a+\nu}$. Hence, the pull away from the boundary when the consumption share approaches one is much stronger than when it approaches zero as $\rho^a < \rho^b$. The volatility of the consumption share is higher with more disagreement and therefore more mass is at the boundaries. This is clearly evident in Figure 5 for the high disagreement case. Here we see that much more mass is at the boundaries, but it is not symmetric. This is because the pull back from a high consumption share is stronger, and therefore the consumption share spends less time there than when the consumption share is low where the pull back is less strong. Economically, the reason for the different behavior on the two boundaries is because the newborn agents with a high discount factor consumes more out of their wealth than the newborn agents with a low discount factor. Put differently, while the patient agents slowly increases their consumption share at their extinction boundary due to a high savings rate, the less patient agents quickly increases their consumption share at their extinction boundary due to a low savings rate.

Figure 5: *Distribution of the consumption share.* The figure shows the distribution of the consumption share of agent of type a, f_t, for three different levels of disagreement (No Dis, Med Dis and High Dis) with d set to 0, 2 and 4, respectively. The histograms are based on 1,000,000 years of monthly observations.



In Figure 6 we show the average consumption share when we change the death and birth intensity (hazard rate). When ν approaches zero, the economy approaches that of an infinitely lived agent economy and therefore the most patient agent consumes everything, regardless of the disagreement, as in Yan (2008). However, once the hazard rate increases then the equilibrium consumption share declines and eventually converges to the unconditional value of α_t. While the consumption share without disagreement monotonically decreases, this is not necessarily the case when there is disagreement as illustrated in the high disagreement case. Here the consumption share drops quickly and then slowly increases towards the unconditionally value of α. The reason for this is that as the hazard rate increases, the lifespan of agents is shorter and therefore there is less time for market selection to work.

Figure 6: *Unconditional average consumption share.* The figure shows the unconditional average consumption share of agent of type a, f_t, as a function of the hazard rate ν. The figure shows three different levels of disagreement (No Dis, Med Dis and High Dis) with d set to 0, 2 and 4, respectively. The unconditional averages are based on 1,000,000 years of monthly observations.

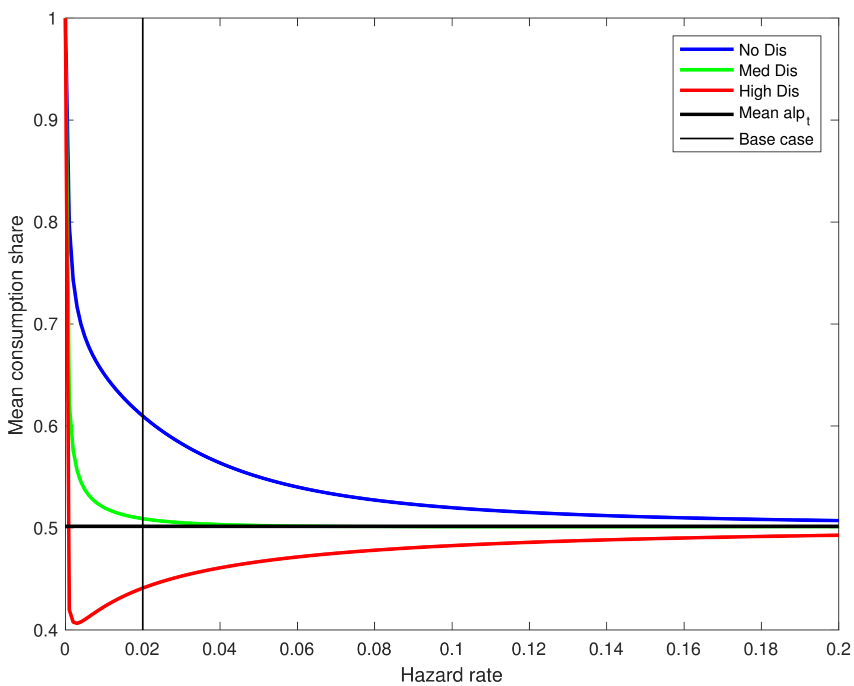
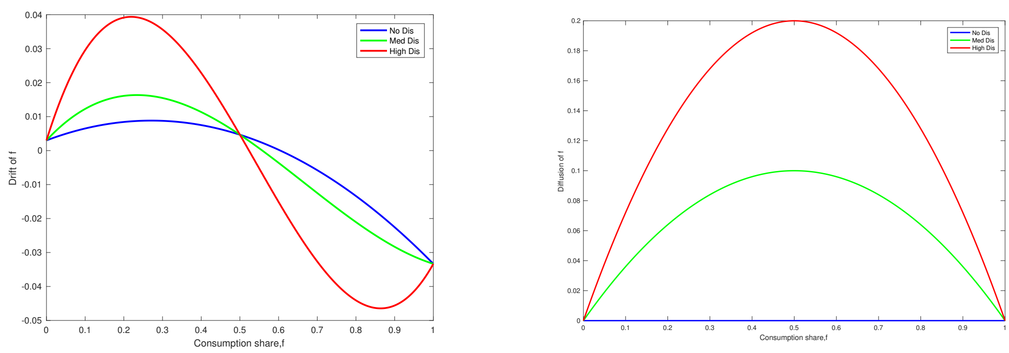


Figure 7 shows the drift (left) and diffusion (right) coefficient of the consumption share as a function of the consumption share itself while keeping $\alpha_t = 0.5$. From the figure one can see that the pull back is stronger when the consumption share approaches one than when it approaches zero. The right hand plot in Figure 7 shows the diffusion coefficient of the consumption share. One can see that the maximum volatility is attained when the consumption share is one half, and this maximum is increasing in the disagreement. The reason for this is that the both type of agents can take large speculative positions when they are of equal size and therefore making the exposure to shocks to α large.

Figure 7: *Drift and diffusion of the consumption share.* The figure shows the drift (left) and diffusion (right) of the consumption share as a function of the consumption share for three levels of disagreement (No Dis, Med Dis and High Dis) with d set to 0, 2 and 4, respectively. The fraction of agents of type a, α_t, is fixed at 0.5.



4.4 The Market Prices of Risk and the Real Short Rate

Proposition 2 characterizes the stochastic discount factor in the economy with demand disagreement, which is pinned down by the market prices of risk for output and demand shocks and the real short rate. As the proposition shows, the price of shocks to output it the same as in an infinitely lived representative agent economy with log utility. Hence, there is no variation in the market price of output shocks. This is not the case for demand shocks. Importantly, disagreement itself does not change the market price of risk, only the relative pessimism/optimism of the consumption weighted average belief matters. That is, the market price of risk is the same in an economy with high disagreement or low disagreement as long as the consumption weighted average beliefs stays the same.[15] Specifically, the unconditional mean of the market price of risk for demand shocks is

$$\begin{aligned} E\left(\theta_{\alpha,t}\right) &= \frac{\kappa}{\sigma_l}\left(\bar{l}-\left(E\left(f_t\right)\bar{l}^a+\left(1-E\left(f_t\right)\right)\bar{l}^b\right)\right) \\ &= \frac{\kappa}{\sigma_l}\left(1-2E\left(f_t\right)\right)d. \end{aligned} \tag{39}$$

From Equation (39) and the fact that $d > 0$, it follows that the unconditional value of the market price of risk for demand shocks is positive (negative) when $E\left(f_t\right) < 0.5$. Economically, when the unconditional value of the consumption share is less than 0.5, then the agents with the pessimistic beliefs consume more on average and consequently their belief has a larger impact on prices. The left hand side plot of Figure of 8 shows the distribution of the market price of demand shocks in economies with no, medium and high disagreement. In the no disagreement case, the market price of demand shocks in trivially zero. Interestingly, the market price of risk in the medium and high disagreement cases have opposite sign. This is due to the fact that in the medium disagreement case, the unconditional average consumption share is greater than 0.5 as the more patient agent accumulate more wealth over time. In contrast, the unconditional average consumption share in the high disagreement case is below 0.5 and this is due to the boundary behavior of the consumption share unique to the overlapping generations framework, as discussed above.

The left hand plot of Figure 9 shows how the unconditional market price of risk changes with the hazard rate in the three economies. Here we see that both the medium and high disagreement has a market price of risk that equals -0.4 when the hazard rate approaches zero. This is because in this case the most patient agent's consumption share approaches one and this agent is also more optimistic than the true beliefs, i.e., $\bar{l} < \bar{l}^a$. However, once the hazard rate increases both the market price of risk in the medium and high disagreement cases increase.

[15] See the discussion in Ehling et al. (2017a) for details on how to compare economies with different levels of disagreement.

The right hand side of Figure 8 shows the distribution of the real short rate. The unconditional average short rate is increasing in the disagreement. This is consistent with the results in Ehling et al. (2017a). However, the mechanism is quite different. In Ehling et al. (2017a), the real rates are higher due to the relative strength of the income and substitution effects and there would be no effect on the real rate with log utility. In our model with log utility, the reason for increase in the short rate when the economy has higher disagreement is to due two reasons. First, as illustrated above, the expected consumption share of the low discount factor agents declines from the no disagreement to high disagreement case, making the consumption weighted average patient lower. This in turn increases the interest rate. Second, a lower expected consumption share decreases the price dividend ratio and consequently the consumption of newborn relative to the rest of the population, $\alpha_t \beta_t^a + (1-\alpha_t)\,\beta_t^b = \left(\alpha_t\,(\rho^a+\nu) + (1-\alpha_t)\left(\rho^b+\nu\right)\right)\phi_t$, which implies that the expected consumption growth of agents currently alive is higher, pushing the interest rate up.

The right hand side of Figure 9 shows the average real short rate when increasing the hazard rate. For the case of no disagreement, the real short rate is monotonically increasing. This follows from the fact that as the hazard rate increases, the market selection due to different preferences has less bite as life expectancy is lower. For the economies with disagreement the relation does not have to be monotonic as the consumption share quickly drops from one to below 0.5, then slowly converging toward the unexpected value of α which is 0.5, as illustrated in the high disagreement case.

4.5 Stock Return Volatility and the Risk Premium

The left plot in Figure 10 shows the stock market's loading on the demand shocks, $\sigma^{\alpha}_{M,t}$, as a function of the consumption share. The maximum loading increases with disagreement and is zero when there is no disagreement. As the next proposition illustrates, the loading onto demands shocks is not symmetric around one half.

Proposition 7. *We have* $\lim_{f\to 0}\sigma^{\alpha}_{M,t} = \lim_{f\to 1}\sigma^{\alpha}_{M,t} = 0$. *Moreover,* $|\sigma^{\alpha}_{M,t}|$ *attains the*

Figure 8: *Distribution of the market price of α shocks and the real short rate.*The figures show the distribution of $\theta_{\alpha,t}$ (left) and the real short rate (right) for three different levels of disagreement (No Dis, Med Dis and High Dis) with d set to 0, 2 and 4, respectively. The histograms are based on 1,000,000 years of monthly observations

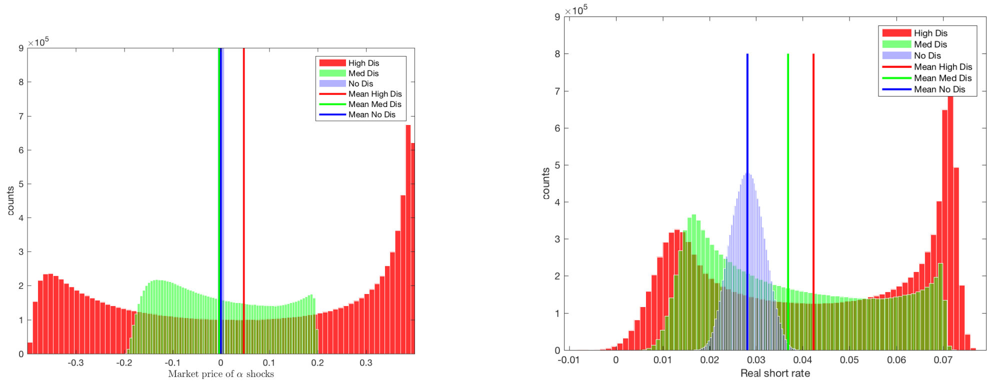
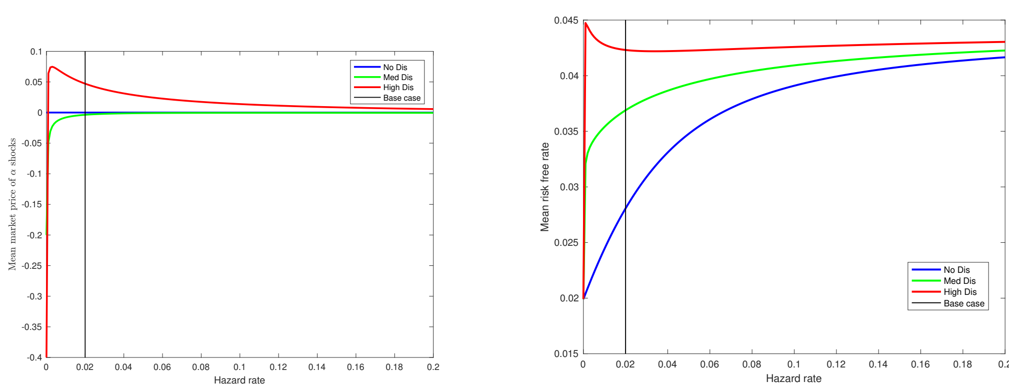


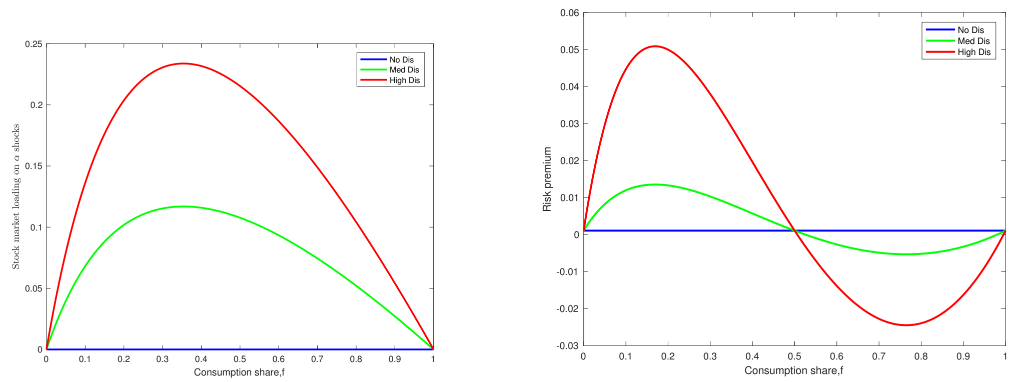
Figure 9: *Unconditional average of the market price of α shocks and the real short rate.* The figures show the unconditional average of $\theta_{\alpha,t}$ (left) and the real short rate (right) as a function of the hazard rate ν. The figures show three different levels of disagreement (No Dis, Med Dis and High Dis) with d set to 0, 2 and 4, respectively. The unconditional averages are based on 1,000,000 years of monthly observations.



maximum at $f^* = \left(\sqrt{\frac{\rho^b+\nu}{\rho^a+\nu}}+1\right)^{-1}$.

From Proposition 7 we see that when $\rho^a < \rho^b$, then the diffusion attains the maximum at a value less than $\frac{1}{2}$. As we will show later, this asymmetry is important for the unconditional risk premium for the stock. Moreover, the loading onto the demand shocks approaches zero when the consumption approaches either zero or one. This is due to the fact that at the boundaries the local volatility is effectively the same as in an economy without heterogeneity.

Figure 10: *Stock market loading on α shocks and the stock market risk premium.* The figure shows stock market loading on shocks to α (left) and the stock market risk premium (right) as a function of the consumption share for three levels of disagreement (No Dis, Med Dis and High Dis) with d set to 0, 2 and 4, respectively. The fraction of agents of type a, α_t, is fixed at 0.5.



The left hand side plot in Figure 11 shows the distribution of local volatility of the stock return. The expected local volatility is increasing from the medium disagreement to the high disagreement. Two forces determine the total effect. First, fixing the consumption share, the stock market's loading onto demand shock is weakly increasing in disagreement.[16] Second, the disagreement lowers the unconditional value of the consumption share. Going from the medium to the high disagreement case this effect also increase the loading of the stock market onto demand shocks as the unconditional value of the consumption share is closer to the value that maximizes the local volatility.

The left hand plot in Figure 12 shows that the volatility initially increases with the hazard rate to then decline monotonically. The initial increase is due to the market selection; when

[16] For any f in the interior, the loading on the demand shock is strictly increasing in d.

the hazard rate is zero the unconditional value of the consumption share is one and therefore the loading on the demand shocks is zero. However, once the hazard rate is greater than zero, the unconditional value of the consumption share drops, increasing the local volatility of the stock market.

The right hand plot in Figure 11 shows the distribution of the risk premium. As in the case of the local volatility, the local risk premium is only due to shocks to output and the loading and market price of risk of these shocks are both constant. In this case the risk premium is 11bp. However, once agents disagree, part of the risk premium is due to the demand shocks. The unconditional average is higher for the high disagreement than the medium disagreement case. Interestingly, even though the average price of demand shocks is negative and the average loading is positive in the medium disagreement case, the total stock risk premium is higher than the no disagreement case. This can be seen from the following

$$\text{Local stock risk premium} = E\left(\sigma^{\alpha}_{M,t}\theta_{\alpha,t}\right) = E\left(\sigma^{\alpha}_{M,t}\right) E\left(\theta_{\alpha,t}\right) + cov\left(\sigma^{\alpha}_{M,t}, \theta_{\alpha,t}\right). \qquad (40)$$

In the medium disagreement case, $E\left(\sigma^{\alpha}_{M,t}\right) E\left(\theta_{\alpha,t}\right)$ is negative but the covariance between the market price of risk and the stock market loading on demand shocks is positive and outweigh the first part.

4.6 Exposure to Demand Risk and Trading

As the agents disagree, they optimally have heterogeneous portfolio positions. As illustrate above, the differences in beliefs creates excess volatility and therefore the total exposure to demand shocks are higher than in an otherwise identical economy without disagreement. However, the total exposure of the agents in the economy might be higher than the exposure of total wealth due to speculative trade. The exposure of an agent in the economy is

Figure 11: *Distribution of the stock market volatility and the risk premium.* The figures show the distribution of the stock market volatility (left) and the risk premium(right) for three different levels of disagreement (No Dis, Med Dis and High Dis) with d set to 0, 2 and 4, respectively. The histograms are based on 1,000,000 years of monthly observations

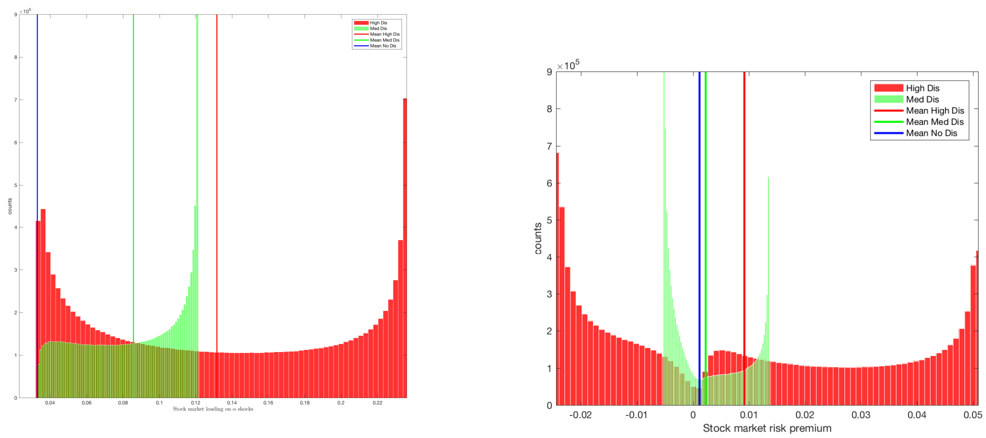
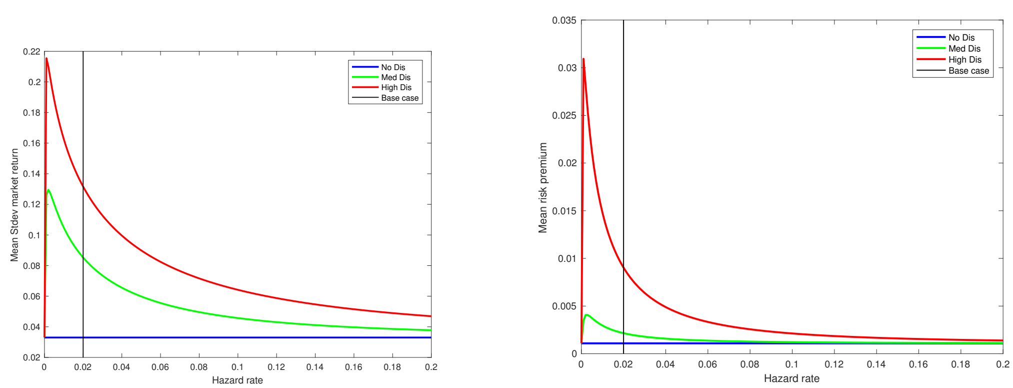


Figure 12: *Unconditional average of the stock market volatility and the risk premium.* The figures show the unconditional average of stock market volatility (left) and the risk premium (right) as a function of the hazard rate ν. The figures show three different levels of disagreement (No Dis, Med Dis and High Dis) with d set to 0, 2 and 4, respectively. The unconditional averages are based on 1,000,000 years of monthly observations.



determined by the diffusion coefficient of total wealth

$$d\hat{W}^i_{s,t} = \ldots dt + \underbrace{\left(\sigma^Y_{S,t}\pi^i_{s,t} + \sigma^Y_{M,t}S^M_t\right)}_{\text{output shock exposure}} dz_{Y,t} + \underbrace{\left(\sigma^\alpha_{S,t}\varphi^i_{s,t} + \sigma^\alpha_{M,t}S^M_t\right)}_{\text{demand shock exposure}} dz_{\alpha,t} \tag{41}$$

Definition 2 introduces the total excess exposure to output shocks and demand shocks in the economy.

Definition 2. *We define the total excess exposure to output shocks, $E_{Y,t}$, as the sum of the absolute value of the output shock exposure of every agent in the economy minus the exposure of aggregate wealth to output shocks, normalized by total wealth in the economy:*

$$E_{Y,t} = \frac{\int_{-\infty}^t \nu e^{\nu(t-s)} \left(\alpha_s|\sigma^Y_{S,t}\pi^a_{s,t} + \sigma^Y_{M,t}S^M_t| + (1-\alpha_s)\,|\sigma^Y_{S,t}\pi^b_{s,t} + \sigma^Y_{M,t}S^M_t|\right) ds}{S^M_t} - |\sigma^Y_{M,t}|. \tag{42}$$

We define the total excess exposure to demand shocks, $E_{\alpha,t}$, as the sum of the absolute value of the demand shock exposure of every agent in the economy minus the exposure of the aggregate wealth to demand shocks, normalized by total wealth in the economy:

$$E_{\alpha,t} = \frac{\int_{-\infty}^t \nu e^{\nu(t-s)} \left(\alpha_s|(\sigma^\alpha_{S,t}\varphi^a_{s,t} + \sigma^\alpha_{M,t}S^M_t| + (1-\alpha_s)\,|(\sigma^\alpha_{S,t}\varphi^b_{s,t} + \sigma^\alpha_{M,t}S^M_t|\right) ds}{S^M_t} - |\sigma^\alpha_{M,t}|. \tag{43}$$

Proposition 8 show the total excess exposure of output and demand shocks in the economy,

Proposition 8. *Let $\sigma^Y_{S,t} = \sigma_{Y,t}$ and $\sigma^\alpha_{s,t} = 1$. The total excess exposure of output shocks is always zero, i.e., $E_{Y,t} = 0$. The total excess exposure to demand shocks is*

$$E_{\alpha,t} = \frac{2|\Delta^a_\alpha - \Delta^b_\alpha|}{\rho^b + \nu} \frac{f_t\,(1-f_t)}{\frac{f_t}{\rho^a+\nu} + \frac{1-f_t}{\rho^b+\nu}}. \tag{44}$$

From Proposition 8, we see that the total excess exposure to output shocks is always zero. This is due to the fact that agents have homogenous beliefs about the output shocks and therefore do not want to trade on these shocks. For demand shocks this is not the case. Here we see that if agents disagree then there is excess exposure to demand shocks. Comparing the expression of the loading on the aggregate wealth on demand shocks, $\sigma^{\alpha}_{M,t}$, with the excess exposure to demand shocks we see that they are proportional, i.e., $E_{\alpha,t} = \bar{E}_{\alpha}|\sigma^{\alpha}_{M,t}|$ where $\bar{E}_{\alpha} = \frac{2}{\rho^b+\nu}|\frac{1}{\rho^a+\nu} - \frac{1}{\rho^b+\nu}|^{-1}$. Hence, the excess exposure peaks at the same time as the stock market volatility peaks. As we have a continuous time model, trading volume is not easily defined. Often the quadratic variation of portfolio policies are used as a measure of the trading intensity.[17] We follow a similar approach by calculating the diffusion of the excess exposure for the demand shock and use the absolute value as a measure of trading intensity. This measure the sensitivity of $E_{\alpha,t}$ to demand shocks itself. If the value if high, this implies that a demand shock will cause large changes in the relative positions of the agents in the economy. If the excess exposure has the dynamics $dE_{\alpha,t} = \mu_{E_{\alpha},t}dt + \sigma_{E_{\alpha},t}dz_{\alpha,t}$, then we use $|\sigma_{E_{\alpha},t}|$ as a measure of the trading volume in the economy. The next Proposition characterize the trading volume.

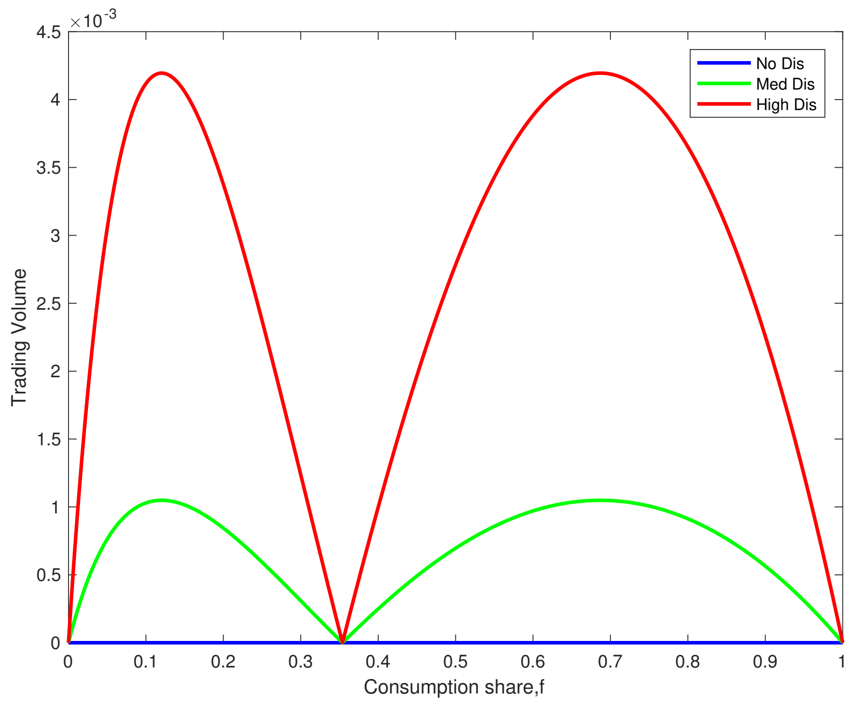
Proposition 9. *Given $\rho^a < \rho^b$ and $\Delta^a_{\alpha} > \Delta^b_{\alpha}$, the trading volume in the economy, $|\sigma_{E_{\alpha},t}|$, is*

$$|\sigma_{E_{\alpha},t}| = \frac{2}{\frac{1}{\rho^a+\nu} - \frac{1}{\rho^b+\nu}}\sigma^{\alpha}_{M,t}|(1-2f_t)\left(\Delta^a_{\alpha} - \Delta^b_{\alpha}\right) - \sigma^{\alpha}_{M,t}| \tag{45}$$

The trading volume in Figure 13 shows that the trading volume has a peak both to the left and right of $f = 0.5$. Given that total excess exposure is maximized when the stock market volatility peaks, the trading volume is zero at that point. Moreover, as the consumption share converges to zero or one the trading volume converges to zero. For the no disagreement case, the trading volume is constant and equal to zero and it is increasing from the medium to high disagreement.

[17]See Grossman and Zhou (1996), Longstaff and Wang (2013) and Ehling and Heyerdahl-Larsen (2017).

Figure 13: *Trading volume as function of the consumption share.* The figure shows the trading volume,$|\sigma_{E_\alpha,t}|$, as a function of the consumption share for three levels of disagreement (No Dis, Med Dis and High Dis) with d set to 0, 2 and 4, respectively. The fraction of agents of type a, α_t, is fixed at 0.5.



5 Conclusion

In classical asset pricing models all shocks to fundamentals are shocks to the supply side of the economy; e.g. output shocks in exchange economies or productivity shocks in production economies. While these models are successful in explaining many empirical stylized facts of asset returns, they fail to account for the low correlation between measure of output growth and stock market returns, as well as, trading volume; a phenomenon known as the correlation puzzle. We develop an overlapping generation model with log utility investors who have heterogenous time preferences and who disagree about investors' future time preferences and, thus, their future demands. There is speculative trade because investors perceive demand shocks differently and, thus, even in the absence of Merton' type hedging demands or early resolution of uncertainty, these demand shocks, which are independent of output shocks, are priced in equilibrium. Moreover, investors make different consumption-saving and portfolio decisions, which leads to a stochastic consumption share that is only exposed to demand shocks. Hence, our demand disagreement model can reconcile time-varying risk-free rates, excess stock market volatility, the predictability of stock market returns by the price-dividend ratio, with a low correlation between macroeconomic fundamentals and (i) stock market returns, (ii) trading volume, and (iii) disagreement about future yields and excess stock market returns.

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A Proofs

Proof of Proposition 1. Plugging optimal consumption of type a and type b agents (see equation (22)) into the resource constraint

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s c_{s,t}^{A} + (1-\alpha_s) c_{s,t}^{B}\right) ds = y_t \tag{46}$$

leads to

$$y_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s \beta_s^a e^{-\rho^a(t-s)} \frac{\eta_t^a}{\eta_s^a} + (1-\alpha_s)\, \beta_s^b e^{-\rho^a(t-s)} \frac{\eta_t^b}{\eta_s^b}\right) y_s \frac{\xi_s}{\xi_t}\, ds. \tag{47}$$

Multiplying both sides of equation (47) with ξ_t and defining $X_t = y_t \xi_t$ completes the proof.

□

Proof of Proposition 2. Applying Itô's lemma to equation (23) and using equation (24) and $\beta_t^i = (\rho^i + \nu)\, \phi_t$ with ϕ_t given in equation (33) leads to the dynamics of the stochastic discount factor ξ_t and, thus, the real short rate and the market price of demand and supply shocks given in equation (27), (28) and (29). □

Proof of Proposition 3. The consumption shares in this economy are $f_t = X_t^a / X_t$ and $(1 - f_t) = X_t^b / X_t$ with X_t, X_a, and X_b given in equations (24), (25), and (26). Applying Itô's lemma to f_t leads to the dynamics of f_t given in equation (3) and thus completes the proof. □

Proof of Proposition 4. The stock market portfolio is defined as a claim on the output stream of all agents currently alive and, thus, the price-dividend ratio is

$$\phi_t = \mathrm{E}_t \left[\int_{t}^{\infty} e^{-\nu(u-t)} \frac{\xi_u}{\xi_t} \frac{y_u}{y_t}\, du\right]. \tag{48}$$

The price-dividend ratio is not a martingale but the process

$$M_t \equiv e^{-\nu t} y_t \xi_t \phi_t + \int_{-\infty}^{t} e^{-\nu u} \xi_u y_u du = e^{-\nu t} X_t \phi_t + \int_{-\infty}^{t} e^{-\nu u} X_u du \tag{49}$$

is. Applying Itô's lemma to the martingale M_t leads to a PDE that the price-dividend ratio given in equation (33) satisfies. □

Proof of Proposition 5. The price-dividend ratio is given in closed form and, thus, applying Itô's Lemma to $S_t^M = \phi_t Y_t$ leads to the exposures of the stock market to supply and demand shocks given in equation (35) and (36), respectively. The drift of the market portfolio can be computed from the fundamental pricing equation

$$\frac{dS_t^M}{S_t^M} - r_t\, dt = -\frac{d\xi_t}{\xi_t}\frac{dS_t^M}{S_t^M}. \tag{50}$$

□

Proof of Proposition 6. We have that $c_{s,t}^i = (\rho^i + \nu)\, \hat{W}_{s,t}^i$. Hence, $\frac{dc_{s,t}^i}{c_{s,t}^i} = \frac{d\hat{W}_{s,t}^i}{\hat{W}_{s,t}^i}$. Applying Ito's lemma to Equation (22) we have

$$\frac{dc_{s,t}^i}{c_{s,t}^i} = \ldots dt + \sigma_Y dz_{Y,t} + \left(\Delta_{\alpha,t}^i - \bar{\Delta}_{\alpha,t}\right) dz_{\alpha,t}^i. \tag{51}$$

Applying Ito's lemma to $\hat{W}_{s,t}^i = H_t + W_t^i$ and noting that $H_t = S_t^M$, we have

$$d\hat{W}_{s,t}^i = \ldots dt + \left(S_t^M \sigma_{M,t}^Y + \phi_{s,t}^i \sigma_{s,t}^Y\right) dz_{Y,t} + \left(S_t^M \sigma_{M,t}^\alpha + \varphi_{s,t}^i \sigma_{s,t}^Y\right) dz_{\alpha,t}^i. \tag{52}$$

Matching diffusions in Equation (51) and (52) we get

$$\sigma_Y \hat{W}_{s,t}^i = S_t^M \sigma_{M,t}^Y + \phi_{s,t}^i \sigma_{s,t}^Y \tag{53}$$

$$\left(\Delta_{\alpha,t}^i - \bar{\Delta}_{\alpha,t}\right) \hat{W}_{s,t}^i = S_t^M \sigma_{M,t}^\alpha + \varphi_{s,t}^i \sigma_{s,t}^Y. \tag{54}$$

Solving Equation (53) and (54) for $\pi^i_{s,t}$ and $\varphi^i_{s,t}$ yields the optimal portfolios.

$\square$

Proof of Proposition 7. Taking the limit of $\sigma^{\alpha}_{M,t}$ given in equation (36) for f going to 0 or 1 leads to 0 in both cases. Taking the first derivative of $\sigma^{\alpha}_{M,t}$ given in equation (36) w.r.t. f and setting it to 0 leads to $f^* = \left(\sqrt{\frac{\rho^b+\nu}{\rho^a+\nu}}+1\right)^{-1}$. Since $\sigma^{\alpha}_{M,t}$ is a concave function of f, it attains its maximum at f^*. $\square$

Proof of Proposition 8. By market clearing we have $\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s \hat{W}^a_{s,t} + (1-\alpha_s)\hat{W}^b_{s,t}\right) ds = S^M_{s,t}$. Note that the exposure of any agent to the shock to output is $\sigma_Y \hat{W}^i_{s,t} > 0$ for all s and i and therefore it follows directly from the above market clearing that the excess exposure is zero. For the excess exposure to demand shocks we have from Equation (54) that this equals $\left(\Delta^i_{\alpha,t} - \bar{\Delta}_{\alpha,t}\right)\hat{W}^i_{s,t}$. Applying Ito's lemma to the market clearing condition above, the loading on the demand shocks is

$$d\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_t \hat{W}^a_{s,t} + (1-\alpha_t)\hat{W}^b_{s,t}\right) ds = \ldots dt + \ldots dz^Y_t + F_t dz_{\alpha,t}, \tag{55}$$

where

$$\begin{aligned} F_t &= \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \alpha_t \hat{W}^a_{s,t} ds |\Delta^a_{\alpha,t} - \bar{\Delta}_{\alpha,t}| + \int_{-\infty}^{t} \nu e^{-\nu(t-s)} (1-\alpha_t)\hat{W}^b_{s,t} ds |\Delta^b_{\alpha,t} - \bar{\Delta}_{\alpha,t}| \\ &= Y_t \frac{f_t}{\rho^a+\nu} |\,(1-f_t)\Delta| + Y_t \frac{(1-f_t)}{\rho^b+\nu} |-f_t \Delta| \\ &= Y_t f_t (1-f_t) |\,\Delta| \left(\frac{1}{\rho^a+\nu} + \frac{1}{\rho^b+\nu}\right). \end{aligned} \tag{56}$$

Normalizing Equation (56) by aggregate wealth we get

$$\frac{f_t(1-f_t)|\,\Delta| \left(\frac{1}{\rho^a+\nu} + \frac{1}{\rho^b+\nu}\right)}{\phi_t}, \tag{57}$$

and therefore the excess exposure to demand shocks is

$$
\begin{aligned}
E_{\alpha,t} &= \frac{f_t\left(1-f_t\right)\left|\,\Delta\right|\left(\frac{1}{\rho^a+\nu}+\frac{1}{\rho^b+\nu}\right)}{\phi_t}-\sigma^{\alpha}_{M,t} \\
&= \frac{f_t\left(1-f_t\right)\left|\,\Delta\right|\left(\frac{1}{\rho^a+\nu}+\frac{1}{\rho^b+\nu}-\frac{1}{\rho^a+\nu}+\frac{1}{\rho^b+\nu}\right)}{\phi_t} \\
&= 2\frac{f_t\left(1-f_t\right)\left|\,\Delta\right|}{\phi_t}\frac{1}{\rho^b+\nu}. \qquad (58)
\end{aligned}
$$

Note that $E_{\alpha,t}=\bar{E}_{\alpha}|\sigma^{\alpha}_{M,t}|$ where $\bar{E}_{\alpha}=\frac{\frac{2}{\rho^b+\nu}}{|\frac{1}{\rho^a+\nu}-\frac{1}{\rho^b+\nu}|}$. □

Proof of Proposition 9. This follows from applying Ito's lemma to Equation (44). □