Design of Macro-prudential Stress Tests*

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Abstract

We study the design of macro-prudential stress tests and capital requirements. The tests provide information about correlation in banks portfolios. The regulator chooses contingent capital requirements that create a liquidity buffer in case of a fire sale. The optimal stress test discloses information partially: when systemic risk is low, capital requirements reflect full information; when systemic risk is high, the regulator pools information and requires all banks to hold precautionary liquidity. With heterogeneous banks, weak banks determine level of transparency and strong banks are often required to hold excess capital when systemic risk is high. Moreover, dynamic disclosure and capital adjustments can improve welfare.

Keywords: stress tests, capital requirements, systemic risk, macro-prudential regulation

1 Introduction

Systemically important financial institutions (SIFIs) are at the core of the financial system and are integral to macroeconomic stability. Due to their size and interconnectedness, a fault with any one of these banks imposes large externalities on the rest of the economy and they are often considered "Too Big to Fail". Bank regulators are forward looking, they identify sources of potential instability of the financial system and supervise financial institutions, including the SIFIs, to prevent crisis.

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Stress tests are the main forward-looking regulatory tool to monitor SIFIs and maintain stability of the financial system.

In this paper, we model macro-prudential supervision as a combination of stress tests designed to discover systemic risk (i.e. correlation of bank portfolios) and regulation in the form of capital requirements. We provide a normative analysis of how stress tests should be conducted (i.e. what constitutes passing and failing), under what conditions their results should be disclosed, and what capital requirements should be imposed on the banks upon learning the stress test outcomes. The regulator forces the banks who fail the test to raise (liquid) capital in a precautionary way as to strengthen the financial system and avoid the need for bailouts. The optimal supervision solves the trade-off between allocative efficiency of risky assets within the system and the ability of financial institutions to raise sufficient capital to remain safe.

In our model banks are exposed to correlated cash flow shocks. They can sell long-term illiquid assets to cover their obligations/meet their liquidity need. However, since banks are marginal investors in long-term assets, this correlation generates market risk and forces asset prices to co-move with the aggregate bank liquidity: if a lot of banks try to sell their assets in response to the same shock, the price of the asset can collapse, resulting in a "fire sale." The regulator wants the banks to be well-capitalized in the event of a fire sale. Therefore, the regulator requires that banks sell some of their long-term assets *before* the potential crisis. To identify how much liquid capital needs to be raised, the regulator conducts a stress test. This is the link between stress tests and capital regulation. From an informativeness perspective a very soft scenario passed by all banks, and a very adverse scenario which fails all banks, are equivalent. The difference is that in the first case banks are left alone, while in the second they are forced to raise additional capital.

Asset prices depend on how well banks are capitalized. The regulator takes this into account when determining sufficient capital requirements and informational transparency levels. In particular, when the market expects low fire-sale prices in the future (due to high correlation in bank portfolios), asset prices fall today in anticipation. This hurts the banks' ability to raise capital and creates benefits from pooling information about the level of correlation. However, pooling information implies that the capital requirements cannot be fine-tuned to the state of the system and that prevents the banks from holding the efficient amount of the long-term assets. The optimal macro-prudential stress test solves the trade-off between allocative efficiency or risky assets and the ability of financial institutions to remain liquid.

Our main result is that optimal information disclosure is partial and asymmetric. For low to medium levels of risk the regulator is transparent. For high levels of risk, disclosure takes the form of a pooling message. Capital requirements are contingent on the information disclosed. If systemic risk is low, banks lever up or down depending on whether the aggregate correlation was above or below its expectation. If the systemic risk is high, capital raising ability of the banks becomes a concern, and banks are required to sell a significant fraction of their assets in order to create a liquidity buffer. Pooling information for high levels of systemic risk is important to ensure that the assets prices are high enough in the first period as to allow the banks to recapitalize. The banks are penalized for holding large quantities of risk and are forced to liquidate a large portion of the balance sheet.

Our second result illustrates spillover effects between weak and strong banks in the system. When there is little systemic risk, heterogeneity in bank capitalization does not create additional inefficiencies. For intermediate and high levels of risk, strong banks raise extra capital in order to support asset prices and, by extension, weaker banks. This creates an inefficient transfer from strong to weak banks. Optimal level of transparency is pinned down by the weak banks. Both stringer capital requirements and less information disclosure show that the well-being of the financial system depends on its weakest link.

Finally, we show that the regulator can improve by dynamically disclosing stress test results and adjusting capital requirements over time. If weak banks raise capital first, they, and by extension the whole system, become more stable. This allows for additional information revelation followed by less strict capital requirements for strong banks. Improvement in allocative efficiency under dynamic disclosure comes from two sources. The first source of improvement is the added flexibility provided by dynamic disclosure: coarse information structure needed to keep the weak banks safe can be refined when strong banks issue capital. Second, if banks can transact in the interbank market before stress test results are reported, sequential information disclosure and trading lead to improved risk sharing between banks.

1.1 Related Literature

The main results of our paper speak to optimal information disclosure of stress test results to the market. Goldstein and Sapra (2014) and Leitner (2014) present overviews of the costs and benefits of such disclosures. Our focus is on the optimal disclosure of information related to systemic risk.

When the financial system is relatively safe, we trust the markets to act as a representative agent who benefits from additional information. As the regulator sets capital requirements, banks cannot abuse information in our model. In this way we differ from Alvarez and Barlevy (2015) who argue that information should not be disclosed in good times as it does not lead to improvements in allocative efficiency.

Faria-e Castro, Martinez, and Philippon (2015) study a related model of disclosure in which there is uncertainty about the fraction of good and bad banks in the system. The regulator faces a trade-off in conducting individual bank tests as it exposes her to aggregate uncertainty and, consequently, bailouts. The paper only considers information structures which reveal the aggregate state, but allows for differing precisions of signals about individual banks. The main contribution of Faria-e Castro, Martinez, and Philippon (2015) is the rich analysis of possible fiscal interventions by the regulator as a function of the outcome of the test. We complement it by analyzing unrestricted information policies that can be used by the regulator, while allowing her to commit to contingent capital requirements imposed on the banks as to keep the system default-free. We limit the channels through which banks issue capital to asset sales, but the argument follows through if we let banks issue new equity at some cost. For an in depth analysis of optimal recapitalization see Philippon and Schnabl (2013).

Goldstein and Leitner (2015) study an information design problem by the regulator who wishes to facilitate banks of heterogeneous quality to raise funds in markets plagued by adverse selection. Under some regularity assumptions the optimal test reveals a fraction of bad quality banks who cannot raise funds, while allowing the remaining banks to raise capital at a pooling price. Like in our model, banks raise funds by selling their tradable asset. The information design in question concerns idiosyncratic asset quality which is quite different from disclosure of aggregate uncertainty as the regulator is exposed to aggregate risk in the latter case. Williams (2017) extends the analysis of Goldstein and Leitner (2015) to an endogenous portfolio choice of the banks showing, surprisingly, that banks hold less liquidity under the optimal stress test. In our model capital requirements are designed jointly with stress tests such that in equilibrium no bank defaults. In our opinion, this makes our paper more relevant to regulation of SIFIs, while Goldstein and Leitner (2015) and Williams (2017) are better suited at explaining the optimal tests conducted by FDIC on smaller banks which can be unwound or sold without creating market panic and contagion.[1]

[1] For failed banks entering into FDIC receivership see: https://www.fdic.gov/bank/individual/failed/banklist.html.

Gick and Pausch (2013) and Shapiro and Skeie (2015) consider settings where the regulator balances incentives of investors to withdraw funds from the banking system and risk taking by the banks. The first paper focuses on information design, while the latter on reputation formed by the regulator during two consecutive bank interventions. Both papers result in partially informative scenarios which limit the volatility arising from their actions. The mechanism in our paper differs since we see the market acting as a representative agent when the economy is doing well which results in information being valuable for decision making in good states of nature.

To demonstrate our mechanism we write down a stylized asset pricing model in which aggregate risk faced by the banks matters for asset pricing in both periods. The style of the model is similar to Geanakoplos (2010) and Harrison and Kreps (1978) in which the fluctuating wealth of the marginal investor (bank) creates volatility in asset markets. While we do not model asset choice directly, the inefficiencies arise from illiquidity of the bank's long-term assets when their balance sheets are hit by an aggregate shock. The latter arises from a maturity mismatch between bank's assets and liabilities, a friction studied in Allen and Gale (2000) and Diamond and Dybvig (1983).

Our model features incomplete asset markets. This serves two purposes. First, it does not allow SIFIs to buy insurance against systemic risk. This friction was pointed out in Krishnamurthy (2003) and Allen and Gale (2004) as one driving many of the intermediary asset pricing models. For an excellent overview of intermediary asset pricing literature see Benoit, Colliard, Hurlin, and Pérignon (2016). Second, market incompleteness prevents markets from learning the amount of aggregate risk through prices. This puts the regulator in an advantageous position of being able to aggregate individual bank signals. Marin and Rahi (2000) provide an analysis of the costs and benefits of information structures generated by incomplete markets. Flannery, Hirtle, and Kovner (2017), Petrella and Resti (2013), and Peristiani, Morgan, and Savino (2010) confirm empirically that stress test disclosures in the U.S. in 2009 and Europe in 2011 contained new information manifested through abnormal returns and trading volume.

There are several empirical measures of systemic risk. Acharya, Pedersen, Philippon, and Richardson (2017) and Adrian and Brunnermeier (2016) propose market based measures of systemic risk termed Systemic Expected Shortfall (SES) and Conditional Value at Risk (CoVaR) respectively. We show that there is an intuitive mapping between these measures and what we refer to as systemic risk on our model. As the regulator has superior access to bank balance sheet information, she is in a better position to construct forward looking estimates of these risk measures. An interesting distinction is that while papers above use SES and CoVaR to tax institutions based on their contributions to systemic risk (and thus align individual bank's incentives with the incentives of the regulator), we use these measures to set capital requirements which can be viewed as both an incentive tool and a stability tool. See Brunnermeier, Gorton, and Krishnamurthy (2012) and Duffie (2013) for suggestions over what relevant information the regulator should collect from the banks in order to identify sources of systemic risk.

We impose a requirement on policies that banks do not default even after a systemic shock. It is based on an implicit assumption that the shadow cost of government funds is significantly large[2] and that the banks are too big and interconnected to fail. The latter is supported by arguments of financial contagion Allen and Gale (2000), historical examples Wiggins and Metrick (2015), as well as current proposals on managing failing institutions summarized in "Too Big to Fail: The Path to a Solution" (2013) highlighting the need to keep SIFIs operating even under distress. In our model banks raise capital in a precautionary way by selling illiquid assets in order to be solvent in case a systemic shock arrives.

Finally, our paper is related to a broad and growing literature on information design pioneered by Kamenica and Gentzkow (2011). In our setting the stress test is a signal design problem conducted by the regulator in a general equilibrium economy. We provide conditions under which the optimal information structures are monotone partitions of the unknown state. Our analysis is related to Dworczak and Martini (2017) who provide necessary and sufficient conditions for a monotone signal partition in a related class of models.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 provides the analysis for the homogeneous bank model deriving the main information design result of the paper. Section 4 analyzes spillover effects when there are both weak and strong banks in the system.. In Section 5 we show how a dynamic information policy and capital requirements can increase overall welfare. Section 6 concludes.

2 Setup

We start with a brief summary of the model setup. The economy consists of banks, investors, and regulator. Time is discrete and has three periods: $t = 0, 1, 2$. At $t = 0$ the regulator collects

[2] Bailouts both require large sums of funds and they tend to be politically unpopular. We acknowledge that profits for the taxpayers can be made to be the lender of last resort, but the latter is not part of an explicit mandate of the Federal Reserve Banks or other government agencies.

information about portfolios of the banks and learns the aggregate exposure of the banking sector to specific risks. Based on this knowledge, the regulator sets capital/liquidity requirements for the banks and reveals some information to the market. Banks sell long-term assets (that pay only at $t = 2$) in the market in order to meet the new capital requirements. In the next period, $t = 1$, some banks are hit with shocks and sell additional assets at fire sale prices in order to meet their liabilities. When the aggregate exposure of the banking sector is high many banks are hit by the shock at the same time, this reduces the fire sale price forcing some banks to fail. The regulator jointly designs disclosure policy and capital requirements in order to minimize the amount of long term asset sold at $t = 0$ subject to no bank failing at $t = 1$.[3] We proceed to define the model formally.

Banks. There is a continuum of identical[4] banks indexed by $i \in [0, 1]$. Banks are risk-neutral and do not discount future payoffs. Banks portfolio consists of three types of assets: $m > 0$ units of liquid assets (cash), $n > 0$ units of long-term asset, and 1 unit of short-term non-tradable asset. The long-term asset pays $b > 0$ at $t = 2$ and can be traded in both periods. Bank's i non-tradable asset pays X_i at $t = 1$. It is useful to rank these assets with respect to their liquidity – cash is perfectly liquid, X_i is perfectly illiquid. Liquidity of the non-tradable asset depends on the amount of systemic risk in the system. The risk banks face is, in part, coming from a maturity mismatch between their assets and liabilities.

From the individual bank's perspective, the illiquid asset pays either 0, or $-l$ with probabilities $1 - \lambda$ and λ:

$$X_i = \begin{cases} 0 & \text{with probability} \quad 1-\lambda, \\ -l & \text{with probability} \quad \lambda. \end{cases}$$

One can think of X_i as asset returns net of debt liabilities with λ being the tail risk. In the event $X_i = -l$ the bank might not be able to meet its liabilities using only liquid assets and would have to sell some of the long-term assets as well. We set $X_i = 0$ in the good state without loss, since any positive certain payoff can be added to the initial cash position m of the bank.

The risk contained in X_i can be idiosyncratic to bank i, or can be similar to those held by other banks in the system. We model it by introducing a random variable $Z \sim G[\underline{z}, \overline{z}]$ such that a fraction Z of the banks have $X_i = X_j = X$, while a fraction $1 - Z$ hold risks independent of X. A simple

[3] We implicitly assume that every bank is Too Big to Fail and the regulator does not wish to resort to bailouts.

[4] In Section 4 we relax this assumption and consider heterogeneous banks.

and rigorous way to capture this joint distribution is by defining a family of iid random variables $\{\xi, \xi_i\}$ that take values $\{0, -l\}$ with probabilities $(1-\lambda, \lambda)$ respectively such that

$$X_i = \begin{cases} \xi & \text{with probability} \quad Z, \\ \xi_i & \text{with probability} \quad 1-Z. \end{cases}$$

At time $t = 0$ random variable Z is realized but not observed either by banks, or by market participants. Even though every bank observes perfectly its own risk X_i, i.e. the payoff $X_i(\omega)$ for every underlying state $\omega \in \Omega$, this information is insufficient to determine whether $X_i = \xi$ or $X_i = \xi_i$.[5] A greater realized Z implies that banks are more likely to suffer a correlated negative shock. As pointed out in Benoit, Colliard, Hurlin, and Pérignon (2016) correlated exposures of financial institutions are the dominant source of systemic risk and amplification. For the purpose of our main analysis we remain agnostic about the determinants of Z. This approach is valid whenever there is any residual uncertainty about the amount of systemic risk.

Regulator. In our model financial markets are incomplete. Non-tradability of Arrow-Debreu securities on individual states ω ensure that market participants are unable to infer Z from prices. In practice the market might not be able to infer Z from asset prices either, since this would require the knowledge of specific kinds of risks that each bank is exposed to and the magnitudes of such exposures, which is difficult to observe due to opaqueness of banks' portfolios.

The regulator has an informational advantage over the market participants since she can collect information about individual banks' portfolios $\{X_i\}_{i\in[0,1]}$. Knowing how bank portfolios respond to a given state ω, the regulator can aggregate them to compute Z:

$$\int_0^1 \int_0^1 \mathrm{corr}(X_i, X_j)\, di, dj = \mathrm{E}\left[\mathrm{corr}(X_i, X_j) \mid Z\right] = Z^2.$$

Hence, in our model the level of aggregate risk Z can be inferred by the regulator from the average correlation between banks' portfolios. Another way the regulator could calculate Z is through the measures of CoVar introduced by Adrian and Brunnermeier (2016) and Systemic Expected Shortfall by Acharya, Pedersen, Philippon, and Richardson (2017). Since in our model losses are

[5] This is very similar to cash flows being distributed according to $X_i = f(Z\xi + (1-Z)\xi_i)$ where $\{\xi, \xi_i\}$ are normal and iid. In this case it is also the case that the marginal distribution of cash flows of each bank is identical regardless of Z and thus the bank cannot infer Z from its own holdings. We adopt the binary specification for analytical tractability.

binary it is appropriate to look at the probability, given loss size, rather than quantiles. Subject to such a small modification, the regulator can compute CoVaR as

$$\mathrm{P}(\xi = -l | X_i = -l) = \frac{\mathrm{P}(\xi = -l,\, X_i = -l)}{\mathrm{P}(X_i = -l)} = \frac{Z\lambda + (1-z)\lambda^2}{\lambda} = \lambda + Z(1-\lambda).$$

The calculation of Systemic Expected Shortfall is even simpler in the model:

$$\mathrm{SES} = -\mathrm{E}\left[X_i \,|\xi = -l\right] = zl + (1-z)\lambda l = l(\lambda + z(1-\lambda)).$$

Any positive level of aggregate risk Z implies that expected losses (in case of SES) and the probability of aggregate loss (in case of CoVaR) are higher than their unconditional expected values.

A stress test is a combination of an information structure and contingent capital requirements. The information is represented by a signal S which is informative about Z. Capital requirements are a function of the stress test outcome S. Capital requirements reflect the amount of risk in the system in an efficient way. The regulator has authority to set these requirements on the banks even if it violates the incentive compatibility of the banks.[6] Capital requirements are a regulatory cost of doing business for the banks. The regulator, thus, faces a joint design problem – she chooses how much information about the systemic risk to disclose, i.e. which stress test scenarios to run, and what the contingent capital requirements will be following such a test.

Definition. *Regulatory policy is a pair* $(S, (a_i(S))_{i\in[0,1]})$ *consisting of a stress-test* S *and capital requirements* $a_i(S)$ *where*

1. S *is a random variable correlated with* Z *and,*
2. $a_i(s)$ *is the maximum quantity of the long term asset that the bank* i *can hold given the stress test outcome* $S = s$.

The goal of the regulator is to design a policy that imposes minimum capital requirements (maximizes a_i) subject to no bank defaulting at $t = 1$. As evidenced by the 2007-2009 financial crisis, default costs of a systemically important financial institution are incredibly high[7] and the Too Big to Fail problem has been excessively studied by academics and policymakers.[8] In line with this

[6] This is an important departure from Faria-e Castro, Martinez, and Philippon (2015) and Goldstein and Leitner (2015) as it alleviates the lemons problem when banks have discretion over what assets to sell.

[7] For an analysis of the Lehman Bankruptcy see Wiggins and Metrick (2015).

[8] See “Too Big to Fail: The Path to a Solution for details” (2013) by the Bipartisan Policy Center and Duffie (2010).

literature we assume that the regulator optimizes only within the space of default free policies.

Regulatory policy $(S, (a_i(S))_{i\in[0,1]})$ is *default free* if banks are able to meet their liquidity requirements at $t = 1$ with certainty, i.e.

$$m + (n - a_i(s))p_0(s) + a_i(s)p_L(s, z) \geq l \tag{1}$$

for all $i \in [0, 1]$, $s \in \text{supp}(S)$, $z \in \text{supp}(Z|S = s)$, where $p_0(s)$ is the price of the long-term assets at $t = 0$ and $p_L(s, z)$ is the price of the long-term asset at $t = 1$ conditional on realization of common shock $\xi = -l$. Inequality (1) ensures that ex-post no bank defaults regardless of the realization of the correlation of banks' assets Z, information revealed to the market S, or the realization of the systemic shock ξ. In other words, it ensures that all banks survive even in the worst case scenario.

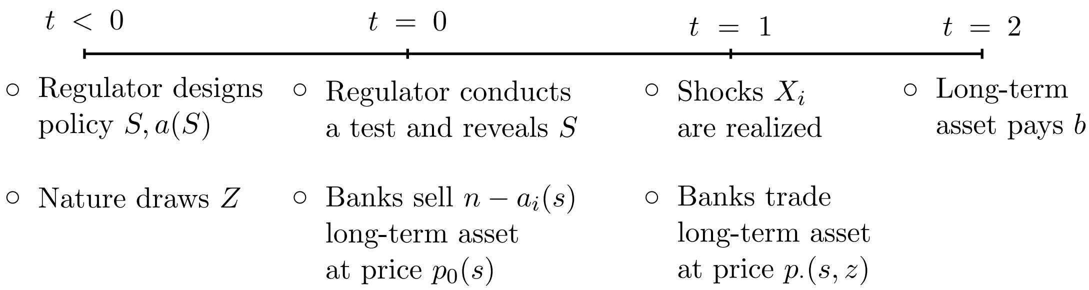
High capital requirements are not free – raising capital in excess of what is needed for bank's solvency is expensive. We assume this manifests itself as the social cost of the banks selling assets to outside investors. This cost stems from several sources. First, banks may be better at holding the risks associated with long-term assets and thus their opportunity cost of holding them is lower. Banks also serve as broker dealers in OTC markets. Thus, every unit of asset held on the balance sheet of the banks generates a higher social gain, than compared to the same asset being held by an average investor in the capital market. Finally, capital markets may have their own investment opportunities, and every unit of capital committed to holding the long-term asset ends up being taken away other investments. This creates a higher opportunity cost for the capital markets of holding the assets. Given the discussion above, the regulator minimizes the amount of assets sold to the outside markets:

$$\min_{(S,a_i)} \mathrm{E}\left[\int_0^1 (n - a_i(S))di\right] \tag{2}$$

over all default free policies $(S, (a_i(S))_{i\in[0,1]})$.

The timing of the game is summarized by Figure 1. Prior to $t = 0$, without knowing the correlation Z, the regulator commits to a testing and capital requirements policy $(S, a(S))$. At $t = 0$ the regulator conducts the test by learning and disclosing the test outcome $S = s$ conditional on the realized $Z = z$. Banks comply with capital requirements by adjusting their portfolios through a sale of $n - a_i(s)$ units of long-term asset. At $t = 1$ shocks are realized and the banks that are hit sell more of the long-term asset at a price $p_H(s, z)$ when the systemic state is high ($\xi = 0$) or at a fire sale price $p_L(s, z)$ when the systemic state is low ($\xi = -l$). At time $t = 2$ long-term asset pays

Figure 1: Timing of the Model



b if it is held by a bank and nothing otherwise.[9]

Capital Markets. We model financial markets in reduced form by describing pricing at both $t = 0$ and $t = 1$. Intuitively, one can think of the market being comprised of a continuum of competitive unconstrained investors who are risk neutral and do not discount future cash flows. Investors can hold either cash or the long-term tradable asset. They derive no value from holding the long-term asset and price it according to the expected resale value.[10] Given the stress test outcome $S = s$ the price at $t = 0$ is denoted by $p_0(s)$ and is given by

$$p_0(s) = (1 - \lambda)\mathrm{E}\left[p_H(s, Z) \,|\, S = s\right] + \lambda \mathrm{E}\left[p_L(s, Z) \,|\, S = s\right], \tag{3}$$

where $p_H(s, z)$ and $p_L(s, z)$ are the time $t = 1$ price conditional on systemic $\xi = 0$ and $\xi = -l$ respectively.

The price at $t = 1$ is pinned down by the liquidity of banks. When $\xi = 0$ only the banks with idiosyncratic X_i suffer a negative liquidity shock and, thus, the total measure of banks that needs to sell the long-term assets, $\lambda(1 - z)$. This quantity is increasing in z as a bigger fraction of the banks got "lucky" with the systemic shock. Price $p_H(s, z)$ is pinned down by the market clearing condition

$$p_H(s, z) = \min\left[\int_0^1 \Big(m + (n - a_i(s))p_0(s) + a_i(s)p_H(s, z) - l\lambda(1 - z)\Big)\, di,\, b\right]. \tag{4}$$

Note that the price depends on both the realized level of systemic risk z, and the stress test outcome s through the level of capitalization of banks at $t = 1$. We focus on parametric cases when

[9] This assumption is a way to capture market segmentation and specify the need for intermediation.
[10] A similar mechanism to Harrison and Kreps (1978).

$p_H(s, z) = b$.

When the systemic shock $\xi = -l$ is realized the total measure of banks that needs to sell the long-term assets in order to meet their liquidity needs is $z + \lambda(1 - z)$; this puts a downward pressure on the asset price and $p_L(s, z)$ decreasing it below the fundamental value b. Market clearing condition for the price in the low state takes the form

$$p_L(s, z) = \min\left[\int_0^1 \Big(m + (n - a_i(s))p_0(s) + a_i(s)p_L(s, z) - l(z + \lambda(1 - z))\Big)\, di,\, b\right], \tag{5}$$

and we focus on parametric cases where (5) amounts to cash in the market pricing.

3 Optimal Macro-Prudential Stress Test

In this section we characterize the optimal stress test and capital requirements policy in the homogeneous banks setting. First, we examine the consequences of a fully transparent stress test that reveals the level of correlation Z to the market. We show that under such information structure if Z turns out to be large, then banks are unable to raise enough liquidity at $t = 0$ due to anticipated liquidity problems and a positive fraction of them defaults at $t = 1$ under a systemic shock. We proceed by solving for the optimal policy of a relaxed problem in which the banks must be solvent only on average and then verify that the solvency condition is satisfied state by state. We show that the optimal policy imposes weak capital requirements and reveals correlation to the market only when it is sufficiently low and combines strict capital requirements with *pooling* of information when Z is high.

3.1 Optimal Capital Requirements under Full Transparency

In order to highlight the main tension between capital requirements and information design we first solve for the optimal capital requirements under a fully transparent stress test, i.e. $S = Z$. If market participants know the correlation level $Z = z$ then at $t = 0$ investors know the exact quantity of systemic risk and price is determined by the expectation over the realization of the systemic shock; the pricing equation (3) simplifies to

$$p_0 = (1 - \lambda)p_H(z) + \lambda p_L(z).$$

In order for systemic risk to play a role within asset prices in our model, we restrict the parameters of the model to be such that when a low systemic shock arrives, the price is below the fundamental, while if the systemic shock is positive, the price is equal to the fundamental. A higher z implies both a lower fire sale price at $t = 1$ if the systemic shock hits, and a lower expected price at $t = 0$.

Assumption 1. *The model parameters are such that* $p_L(z) < b = p_H(z)$ *for all* $z \in [\underline{z}, \overline{z}]$.[11]

For the disclosed level of correlation Z the regulator sets capital requirements. She can, in principle, set asymmetric capital requirements where bank i is forced to raise more capital than bank j. In the case of identical banks, however, it is easy to see that a regulator's payoff under an asymmetric policy can be replicated by one where all banks are subject to the same requirements. It is thus without loss to assume that $a_i(z) \equiv a(z)$ for all $i \in [0, 1]$. For a given z the regulator is maximizing the quantity of assets $a(z)$ retained by the banks subject to the banks being solvent

$$m + (n - a(z))p_0(z) + a(z)p_L(z) - l \geq 0 \qquad \text{(solvency constraint)} \tag{6}$$

where asset prices are pinned down by their expected price at $t = 0$ and cash in the market pricing at $t = 1$:

$$p_0(z) = (1 - \lambda)b + \lambda p_L(z), \tag{7}$$

$$p_L(z) = m + (n - a(z))p_0(z) + a(z)p_L(z) - l(z + \lambda(1 - z)). \tag{8}$$

Since the fire sale price $p_L(z)$ is lower than the ex-ante price $p_0(z)$, selling long-term asset at $t = 0$ can be used to build-up a liquidity buffer to survive the liquidity shock at $t = 1$. However once a bank has raised enough liquidity at $t = 0$ to satisfy (6), additional asset sales at $t = 0$ are inefficient. Hence, intuitively, the optimal capital requirements should be such that liquidity constraint (6) binds. Formally, if (6) is slack, then the regulator can increase $a(z)$ by $\varepsilon > 0$ small enough such that the liquidity constraint remains slack despite the prices $p_0(z)$ and $p_L(z)$ adjusting. Thus, under the optimal capital requirement the liquidity constraint (6) is binding:

$$m + (n - a(z))p_0(z) + a(z)p_L(z) - l = 0.$$

Taking into account (7) and (8) we obtain our first formal result.

[11] In the Appendix we provide sufficient conditions on the model parameters for the Assumption 1 to hold.

Lemma 1. *Under a fully revealing stress test the optimal capital requirements are given by:*

$$a^I(z) = \frac{m + n[(1-\lambda)b + \lambda p_L(z)] - l}{(1-\lambda)(b - p_L(z))}.$$

The corresponding fire sale price is $p_L(z) = l(1-\lambda)(1-z)$.

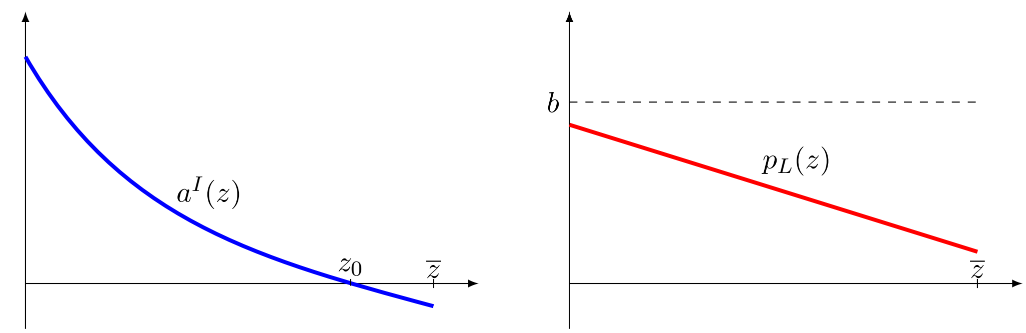


Figure 2: Capital requirements $a^I(z)$ and fire sale prices $p_L(z)$ under a fully revealing stress test.

The macro-prudential nature of optimal capital requirements stems from two sources: the regulator takes into account asset pricing implications of systemic risk Z and internalizes that as banks raise more capital at $t = 0$, they decrease exposure to Z. In other words, the regulator understands how asset prices behave through (8). Higher Z implies that a larger number of banks are hit by the systemic shock and have to sell additional assets at $t = 1$ in order to cover their liabilities. As a result, the price at $p_L(z)$ is decreasing in z. In order to counter a lower $t = 1$ fire sale price, the regulator sets a higher capital requirement at $t = 0$ forcing banks to sell more of their assets (lower $a(z)$). As banks sell more of their asset, this dampens the effect of correlated shocks on the fire sale price. Under the optimal policy, the fire sale price is a linear function of z as illustrated in Figure 2.

Optimal amount of capital retained by the banks $a^I(z)$ described in Lemma 1 is a convex function of z. This property stems from the interaction between the $t = 0$ and $t = 1$ prices. A higher price in the low systemic state implies both a lower capital requirement for the bank, as well as higher price p at which the bank raises capital. A decrease in the level of aggregate correlation z increases $p_L(z)$ directly through the last term in (8). Higher p_L and p_0 imply that the liquidity constraint is slack and that the capital requirements can be relaxed ($a(z)$ goes up). At the same time, higher z implies that the liquidity problem at $t = 1$ is less severe, hence, the difference between ex-ante and ex-post prices $p_0(z) - p_L(z)$ goes down as well, which allows to increase $a(z)$ even further.

If the capital requirement under binding liquidity constraint is always feasible, i.e. $a^I(z) \in [0, n]$ for all $z \in [\underline{z}, \overline{z}]$, then one can show that fully revealing stress test is optimal. However, as shown in Figure 2 there may exist a z_0 such that $a(z) < 0$ for $z > z_0$. When the liquidity problem for high levels of aggregate correlation z is severe, then fully revealing policy fails to be default free. Severe liquidity problems for high z imply that $p_L(z)$ becomes very low. In anticipation of low $t = 1$ prices the $t = 0$ price goes down as well. Low p_0 hurts the banks' ability to raise enough capital to ensure themselves against liquidity problems at $t = 1$. In the remainder of the paper we focus on parameters, such that when $Z = \overline{z}$ the banks are unable to avoid default under the fully transparent stress test.

Assumption 2. *The model parameters are such that* $a^I(\underline{z}) < 1$ *and* $a^I(\overline{z}) < 0$*.*

Under assumption 2, capital requirements cannot completely immunize banks against systemic risk if all information is revealed. If it turns out that Z is sufficiently large, then banks with a negative systemic shock default at $t = 1$. Regulation by setting capital requirements reaches its limit. The regulator must resort to the second lever she has in her disposal: strategic management of stress test information. In subsequent sections we analyze the joint problem of choosing a stress test disclosure strategy and contingent capital requirements.

3.2 Optimal Macro-Prudential Test

A stress test is a signal S that the regulator commits to communicating to the market. Before the stress test is run the regulator announces the joint distribution $Law((S, Z))$. The market then forms posteriors on Z following the publicly disclosed realization of S. She also commits to a contingent capital requirement plan that can be summarized by the quantity of assets $a(s)$ retained by the banks at $t = 0$ for any $s \in \text{supp}(S)$.

The signal design problem we consider is not straightforward. The regulator internalizes the general equilibrium effects of both systemic risk Z and her chosen policy. This implies that the set of liquidity constraints (1) that the optimal policy needs to satisfy depend on the stress test itself. We overcome this difficulty by considering a relaxed problem in which the liquidity constraint is satisfied only on average, rather than state by state. We then show that under the optimal policy in the relaxed problem banks remain solvent state by state.

We define the relaxed problem by replacing liquidity constraint (1) with

$$\mathrm{E}\left[m + (n - a(s))p_0(s) + a(s)p_L(s, Z) \,|\, S = s\right] \geq l \quad \forall\, s \in \operatorname{supp}(S). \tag{9}$$

Unlike (1), the *average* constraint (9) has to be binding for the optimal policy that solves the relaxed problem. If it were not to bind for some s, then an increase in $a(s)$ would improve the regulator's objective and not violate the constraint. If (9) is binding for all s, then

$$\mathrm{E}\left[p_L(s, Z) \,|\, S = s\right] = \mathrm{E}\left[l(1-\lambda)(1-Z) \,\middle|\, S = s\right] = l(1-\lambda)(1 - \mathrm{E}\left[Z \,|\, S = s\right]).$$

Thus, conditional on $S = s$ the regulator's objective depends only on the conditional average belief $\mathrm{E}\left[Z \,|\, S = s\right]$ induced by the stress test outcome. In order to avoid severe liquidity problems for high levels of aggregate correlation Z the optimal test cannot be fully informative. It needs to pool some high risk states together with states of lower Z. Whether a particular realization z will be revealed, or pooled with the high systemic risk states depends on the cost-benefit calculation for the regulator. The benefit stems from a decrease in the average belief about aggregate correlation Z and the corresponding increase in the average price $p_L(s, Z)$, which translates into higher $p_0(s)$. The cost of including a lower state into the pool stems from excessive (ex-post) capital requirements and depends on the shape of $a^I(z)$. The next proposition characterizes the optimal policy of the relaxed problem.

Proposition 1 (Relaxed Test)**.** *If, absent additional information, banks are able to issue enough capital to remain solvent ($a^I(\mathrm{E}\left[Z\right]) > 0$), then the optimal default free policy $(S, a^*(S))$ of the relaxed problem is characterized by a single threshold z_d. Optimal stress test and corresponding capital requirements are*

$$S = \begin{cases} Z, & \textit{if } Z < z_d, \\ z_0, & \textit{if } Z \geq z_d, \end{cases} \qquad\qquad a^*(S) = \begin{cases} a^I(S), & \textit{if } S < z_d, \\ 0, & \textit{if } S = z_0, \end{cases}$$

and threshold z_d solves $\mathrm{E}\left[Z \,|\, Z \geq z_d\right] = z_0$ where $a^I(z_0) = 0$. If $a^I(\mathrm{E}\left[Z\right]) < 0$ then no default free policy exists.

Proposition 1 establishes that the optimal policy has only one pooling region $[z_d, \overline{z}]$ which contains the highest levels of aggregate correlation. The intuition behind the above result is as follows.

The regulator understands that she cannot disclose Z fully since for high levels of Z the system would be risky even under full recapitalization. By forcing banks to fully recapitalize in the region $Z \in [z_d, 1]$, outside investors cannot distinguish whether $Z = z_d$ or $Z = 1$. Thus their expectation about the severity of systemic risk can be maintained to be sufficiently low. The cost of such a signal is that conditional on $Z \in [z_d, z_0]$, the banks will be holding excess capital. The benefit is that banks are able to remain safe conditional on $Z \in [z_0, 1]$.

The regulator could force recapitalization of states in which Z is very low. For instance, the regulator could force banks to raise the maximum amount of capital conditional on $Z \in [\underline{z}, \hat{z}] \cup [z_0, 1]$ such that the expected level of risk in the system would be low from the perspective of investors getting this signal. However, if Z is low, the banks could hold a lot of assets and still be safe. Thus, recapitalizing the banks in those states carries a large opportunity cost for the regulator. The optimal pooling region in which banks raise the maximum amount of capital turns out to belong to the interval adjacent to the failing states $[z_0, 1]$.

Formally, the regulator minimizes the opportunity cost associated with banks raising excess liquidity:

$$\min_{\phi(\cdot)} \int_0^1 a^I(z)\phi(z)\,dz$$

where

$$a^I(z) = \frac{m + n((1-\lambda)b + \lambda p_L(z)) - l}{(1-\lambda)(b - p_L(z))}$$

and subject to $\phi(z) \in [0, g(z)]$ for all $z \in [\underline{z}, 1]$ and

$$\phi(z) = g(z) \qquad \text{for} \quad z \geq z_0 \tag{10}$$

$$\frac{1}{\int_0^1 \phi(z)\,dz} \cdot \int_0^1 z\phi(z)\,dz \geq z_0 \tag{11}$$

Constraint (10) ensures that the regulator forces states with very high Z to raise the maximum capital. Constraint (11) simply states that the expected value of z given pooling is sufficiently large that banks can be solvent if they sell all of the asset at $t = 0$. Denote by η the Lagrange multiplier corresponding to (11). The regulator thus minimizes the Lagrangian with respect a density function

ϕ:

$$\min_{\phi(\cdot)} \left[\int_0^1 (a^I(z) - \eta(z - z_0))\phi(z)\, dz \right],$$

subject to constraints on $\phi(\cdot)$. The above expression is minimized if $\phi(\cdot)$ is chosen as large as possible for z's such that

$$\phi(z) = \begin{cases} g(z) & \text{if} \quad z > z_0, \\ g(z) & \text{if} \quad a^I(z) - \eta(z - z_0) < 0, \\ 0 & \text{if} \quad a^I(z) - \eta(z - z_0) > 0. \end{cases} \tag{12}$$

Because $a^I(z)$ is decreasing and convex, the set $\{z : a^I(z) - \eta(z - z_0) < 0\}$ can be respresented as equivalent to $[z_d, 1]$. Thus $\phi(z) = g(z) \cdot \mathbb{1}\{z > z_d\}$ which correspond to a pooling region $[z_d, \bar{z}]$. For a formal proof we refer to the Appendix.

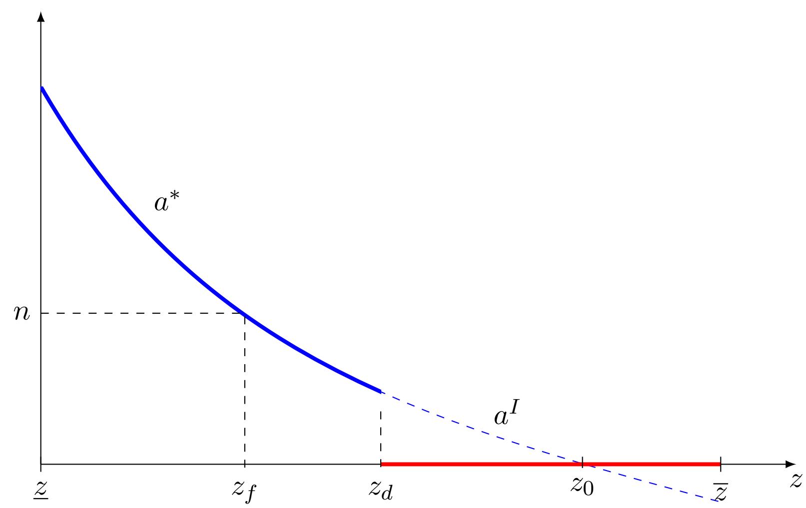


Figure 3: Optimal capital requirements $a^*(z)$. Correlation Z is disclosed conditional on $Z < z_d$. A representative bank passes the test if $Z < z_f$ and fails the test otherwise.

Figure 3 plots the optimal capital requirement $a^*(\cdot)$ as a function of the underlying state Z. For low levels of aggregate correlation $Z < z_d$ the optimal test perfectly reveals Z to the market and implements binding capital requirements $a^*(z) = a^I(z)$ computed in Lemma 1. For levels of systemic risk $Z \geq z_d$ it is optimal to impose very strict capital requirements $a^* = 0$, but only disclose a signal $S = \mathbb{1}\{Z > z_d\}$, but not which particular Z it is. For states $Z \in [z_d, z_0]$ the regulator forces banks to raise the maximum amount of capital available. This might seem counterintuitive as, upon disclosure of Z in that region, banks can still recapitalize at $t = 0$ and be safe at $t = 1$.

However that cannot be optimal ex-ante since then in states $Z > z_0$ banks do not raise enough capital and default with positive probability at $t = 1$.

Notice that if $Z < z_d$, the optimal policy reveals the state to the market, therefore the average constraint (9) is identical to the ex-post constraint (1). If $Z \geq z_d$, the average solvency constraint (9) is weaker than (1). However in this region the optimal policy imposes $a^* = 0$ and thus the banks are not exposed to the residual asset price risk in $t = 1$. The banks thus remain default free even in the worst case scenario $Z = \overline{z}$. This observation brings us to the following conclusion.

Corollary 1. *If $a^I(\mathrm{E}[Z]) > 0$ then the policy $(S, a^*(S))$ defined in Proposition 1 is optimal in the original problem with ex-post liquidity constraint* (1)*.*

The optimal stress test can be thought of as two thresholds: $\{z_f,\, z_d\}$ where z_f solves $a^*(z_f) = n$. If the amount of aggregate risk in the system is sufficiently small, i.e. $z < z_f$, then banks pass the stress test, optimal capital requirements are set based on the publicly disclosed level of systemic risk Z and banks are allowed to expand their portfolio of illiquid assets ($a^*(z) > n$). If the amount of systemic risk exceeds z_f, then the whole system is considered insufficiently capitalized, i.e. the banks fail the stress test and are required to sell some of their illiquid assets in order to increase the liquidity cushion. Two cases are possible if banks fail the stress test. If $z \in (z_f, z_d)$, then aggregate amount of risk in the system is revealed, and the capital requirements are chosen optimally given full information. However, when the aggregate amount of risk in the system is too high, $Z > z_d$, then the regulator imposes very strict capital requirements without providing additional information to the market.

To sum up, the findings of this section are two-fold. First, we show that optimal capital requirements in a macro-prudential stress test are increasing in the amount of systemic risk. The regulator must take into account general equilibrium effects to ensure both that the system remains liquid upon being hit by a systemic shock and that banks do not raise excess liquidity. Second, we show that the for high levels of systemic risk, the regulator cannot just rely on capital requirements to maintain a safe system. We argue that the region where banks fail needs to be split into two parts. For the moderate amount of systemic risk, optimal policy perfectly reveals the aggregate state to the market and requirements are fine-tuned. When the aggregate amount of systemic risk is too high, the regulator combines strict capital requirements with lack of disclosure.

4 Heterogeneous Banks

In this section we characterize the optimal stress test and capital requirements in the heterogeneous banks setting. When banks differ in their risk capacity and the strength of their liquidity position, it is the liquidity constraint of the weakest bank that binds first. We show that the optimal test creates cross subsidization between strong and weak banks. It forces the strong banks to hold significantly more liquid assets than they need to cover their own liabilities in order to increase the strength of the *average* bank. Through this channel the regulator reduces the fire-sale severity at $t = 1$ and consequently increases asset prices in $t = 0$. This allows the weak banks to improve their liquidity position. We characterize conditions under which the optimal test is of a threshold type and show that optimal disclosure cutoff depends on the liquidity needs of weak banks and the liquidity buffer of the strong banks.

We extend the model described in Section 2 by considering two types of banks. There is a measure w of weak and measure $1 - w$ of strong banks characterized by the triplets (m, n, l) and (M, N, L) respectively. Every weak (strong) bank holds a portfolio consisting of m (M) units of liquid assets, n (N) illiquid long-term assents and has one unit of risky non-tradable asset X_i which either pays 0 or $-l$ ($-L$). The strength of the bank is ultimately determined by the liquidity position relative to the losses that the bank is exposed to. In order to ensure that the weak bank's liquidity constraint is more restrictive than the strong bank's liquidity constraint we assume that $(l-m)/n < (L-M)/N$. Whether the bank is strong or weak is uncorrelated with the type of risk (idiosyncratic or systemic) that the bank holds. Thus, the aggregate losses of the banking system conditional on the arrival of systemic shock are

$$\int_0^1 X_i di = -Z(wl + (1-w)L) - (1-Z)\lambda(wl + (1-w)L) = -Z\bar{l} - (1-Z)\lambda\bar{l},$$

where $\bar{l} = wl + (1-w)L$.

We allow the regulator to impose differential capital constraints for the weak and strong banks. Consequently, a symmetric[12] regulatory policy is a triplet $(S, a(\cdot), A(\cdot))$. As before, the random variable S is the stress test, and $a(s)$ and $A(s)$ are the maximal quantity of the long term assets that the weak and strong bank can hold given the test outcome $S = s$.

[12] Similar to Section 3 the regulator would not benefit from using non-symmetric policies, hence, we focus on symmetric policies only in order to simplify notation and exposition.

A default free regulatory policy ensures that banks remain solvent with certainty at $t = 1$:

$$m + (n - a(s))p_0(s) + a(s)p_L(s, z) \geq l, \tag{13}$$

$$M + (N - A(s))p_0(s) + A(s)p_L(s, z) \geq L \tag{14}$$

for all $s \in \text{supp}(S)$, $z \in \text{supp}(Z|S = s)$. The regulator's objective is to minimize the dead-weight losses of recapitalization

$$\min_{S,a,A} \text{E}\left[w \cdot (n - a(S)) + (1 - w) \cdot (N - A(S))\right]$$

over all default free policies $(S, a(S), A(S))$.

The time $t = 0$ price p_0 is again an expectation of $t = 1$ prices $p_H(s, z)$ and $p_L(s, z)$, and the pricing equation (5)-(4) take the form of

$$\begin{aligned}
p_L(s, z) = \min\Bigg[& w\Big[m + (n - a(s))p_0(s) + a(s)p_L(s, z)\Big] + \\
& (1 - w)\Big[M + (N - A(s))p_0(s) + A(s)p_L(s, z)\Big] - \bar{l}(z + \lambda(1 - z)),\, b\Bigg], \\
p_H(s, z) = \min\Bigg[& w\Big[m + (n - a(s))p_0(s) + a(s)p_L(s, z)\Big] + \\
& (1 - w)\Big[M + (N - A(s))p_0(s) + A(s)p_L(s, z)\Big] - \bar{l}\lambda(1 - z),\, b\Bigg].
\end{aligned}$$

To solve for the optimal triplet $(S, a(\cdot), A(\cdot))$ we first analyze what happens under a fully transparent stress test $S = Z$. We show that optimal capital requirements create implicit cross subsidization between the strong and the weak banks. Then we characterize the optimal information structure S chosen by the regulator.

4.1 Cross Subsidization under Full Transparency

Similar to Section 3 we focus on the region of model parameters in which the fire-sale price p_L reflects the illiquidity discount, i.e. $p_L(z) < b$, and conditional on absence of the aggregate shock the price p_H reflects fundamentals, i.e. $p_H = b$. Sufficient condition for that to happen can be formulated in terms of the average bank with $\bar{m} = wm + (1-w)M$ liquid assets, $\bar{n} = wn + (1-w)N$ illiquid assets, and potential losses $\bar{l} = wl + (1-w)L$ analogous to Assumption 1.

Under full transparency, the choice of $a(z)$ and $A(z)$ is equivalent to choosing how slack the corresponding liquidity constraints (13) and (14) are. When both liquidity constraints are tight, the prices $p_L(z)$ and $p_0(z)$ depend only on the aggregate losses of the system, i.e., $\bar{l}(z + \lambda(1 - z))$. The objective of the regulator is a weighted average of the banks' long-term asset holdings can be expressed in terms of the *average* bank as well. To see this, one needs to take a convex combination of binding liquidity constraints (13)-(14) with weights w and $1 - w$ and solve for $wa(z) + (1 - w)A(z)$. The discussion above brings us to the following lemma.

Lemma 2. *If the liquidity constraint of both the weak and the strong banks is binding, then prices and welfare are determined only by the average bank:*

$$wa^I + (1 - w)A^I(z) = \frac{\bar{m} + \bar{n}[(1 - \lambda)b + \lambda p_L(z) - \bar{l}]}{(1 - \lambda)(b - p_L(z))},$$

with $p_L(z) = \bar{l}(1 - \lambda)(1 - z)$.

Lemma 2 shows that when it is possible to satisfy both liquidity constraints at the same time, then the problem of the regulator with heterogeneous banks is equivalent to one with symmetrical average banks, and that two liquidity constraints (13)-(14) can be replaced with one average constraint. This is generically feasible when the aggregate amount of risk in the system z is sufficiently low and the corresponding price $p_L(z)$ is sufficiently high.

As the level of systemic risk z increases, the fire sale price decreases and both types of banks need to sell more of their existing long-term assets in order to increase their liquidity buffer. At some critical level of z_w the weak banks can no longer improve their liquidity position by selling long-term assets since $a^I(z_w) = 0$. However, the liquidity position of the week banks can be further improved by imposing stricter capital requirements on the strong banks

Cross subsidization between strong and weak banks works through the price channel. When strong banks sell more long-term assets than they need to satisfy their liquidity constraint (14) at $t = 0$, they reduce the amount of long-term asset that they need to sell conditional on being hit by the shock at $t = 1$. As a consequence of fewer assets being sold by the banks conditional on the arrival of systemic shock the fire sale is less severe, i.e. the price p_L goes up. Anticipating higher prices at $t = 1$ investors are trade at higher prices at $t = 0$. This allows weak banks to raise more capital at $t = 0$ for the same quantity of assets sold.

Subsidizing weak banks through strong banks is inefficient. To see this, construct the following deviation from the solution of Lemma 2: tighten the capital requirements of the strong banks by ε, i.e. reduce $A(z)$ by ε. This change introduces slack $\varepsilon(p_0 - p_L)$ into the liquidity constraint of the strong banks and consequently increases price p_L by $\varepsilon(p_0 - p_L)(1-w)$ and the price p_0 by $\lambda\varepsilon(p_0 - p_L)(1-w)$. The change in prices relaxes the liquidity constraint of the weak banks and allows the regulator to increase $a(z)$. The overall effect can be written as

$$(1-w)\varepsilon\Big[-1 + w[(1-\lambda)a + n\lambda]\Big] < 0.$$

As the equation above illustrates there are three sources of inefficiency. First, since subsidization works through prices it affects welfare only indirectly through the liquidity constraint of the weak banks. Low w implies that few weak banks are able to enjoy the benefits of higher prices, hence, the positive impact is dampened. Second, higher liquidity buffer of the strong banks affects the prices only in the low systemic state, i.e. only the price p_L. Since the weak banks need to raise capital at $t = 0$ and $\lambda < 1$, the price at which they are able to sell long-term assets only partially reflects the increase in p_L. Finally, positive effect is dampened since the benefit that the weak banks receive comes through the prices of the long-term assets the size of the subsidy depends on the amount of long-term assets held by those banks ($n, a \leq 1$).

When the aggregate amount of risk is above z_w, price $p_L(z)$ no longer equals $\bar{l}(1-\lambda)(1-z)$, instead it is supported at the high enough level to allow the weak banks to stay default free. In order for that to happen, time $t = 0$ price $p_0(z)$ needs to solve

$$m + np_0(z) = l,$$

thus, $p_L(z) = (l - m - \lambda nb)/n(1-\lambda)$. The capital requirements for the strong banks $A(z)$ are then derived from the pricing equation for $p_L(z)$ rather than from the binding liquidity constraint (14). The resulting capital constraints are plotted in Figure 4.

When the amount of the aggregate risk Z is sufficiently small Lemma 2 shows that ex-post welfare $wa^I(z) + (1-w)A^I(z)$ coincides with the ex-post welfare of the economy consisting of symmetric average banks $a^I_{avg}(z)$. When the amount of the aggregate risk z is above z_w the two start to differ. Since the liquidity constraint imposed by the weak banks is more restrictive than the average liquidity constraint, the ex-post welfare with heterogeneous banks dips below that of the average bank. Moreover, since in this region $a^I(z) = 0$ and prices $p_L(z)$ and $p_0(z)$ do not depend on z

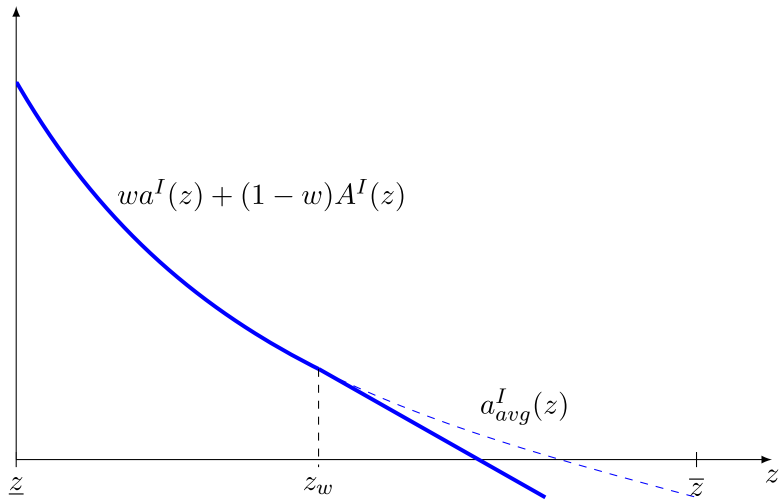


Figure 4: Capital requirements under full transparency of the average banks a^{FI}_{avg} and heterogeneous banks $wa^I(z) + (1-w)A^I(z)$

the feedback loop between capital constraints and prices breaks and the resulting welfare loses convexity.

Under the Assumption 2 for the average bank $(\bar{m}, \bar{n}, \bar{l})$ the fully transparent test fails to ensure that the weak banks remain default free when the amount of aggregate risk is sufficiently high $Z = \bar{z}$. Thus, the optimal stress test has to be only partially revealing. In the next subsection we analyze the trade-off between pooling of information that results in ex-post suboptimal capital requirements and revealing the information that might undermine the ability of the weak bank to stay default-free.

4.2 Optimal Test

In this section we show that under certain parametric conditions the optimal stress test can be characterized by the threshold z_d, such that the information is pooled from $Z \geq z_d$ and perfectly disclosed for $Z < z_d$. The size of the pooling region is determined by jointly the liquidity needs of the weak banks and the ability of stronger banks to support higher prices at $t = 1$.

We solve the optimal signal design problem in a fashion similar to Section 3. First, we relax the state-by-state liquidity constraints (13)-(14) to hold only in expectation, conditional on the realized outcome of the test $S = s$. Second we show that the solution to the relaxed problem satisfies the constraints for every z and, thus, solves the original problem.

The regulator has two levers to improve the liquidity position of the weak banks when $a(z) = 0$

either through information pooling or by tightening capital requirements for the strong banks. Information pooling creates allocative distortion since the capital requirements can not be tailored to the particular level of aggregate risk z, otherwise they would reveal additional information to the market. Tightening capital requirements is also inefficient. The next proposition characterizes the shape of the optimal policy when on the margin it is more costly to pool information than to impose stricter capital requirements for the strong bank.

Proposition 2. *If* $\operatorname{cav}(wa^I + (1-w)A^I)(z_w) > wa^I(z_w) + (1-w)A^I(z_w)$ *and* $\mathrm{E}\left[Z \,|\, Z \geq \hat{z}\right] > z_0$*, where* $\hat{z}$ *is the intersection of the convex part of* $wa^I(z) + (1-w)A^I(z)$ *with the continuation of the linear part. Then the optimal default free policy* $(S, a(S), A(S))$ *can be characterized by a single cut-off* z_d *such that optimal stress test and corresponding capital requirements are*

$$S = \begin{cases} Z, & \text{if } Z < z_d, \\ z_0, & \text{if } Z \geq z_d, \end{cases} \qquad wa^*(S) + (1-w)A^*(S) = \begin{cases} wa^I(S) + (1-w)A^I(S), & \text{if } S < z_d, \\ 0, & \text{if } S = z_0, \end{cases}$$

and threshold z_d *solves* $wa^I(\mathrm{E}\left[Z \,|\, Z \geq z_d\right]) + (1-w)A^I(\mathrm{E}\left[Z \,|\, Z \geq z_d\right]) = 0.$

The optimal policy of Proposition 2 has two distinct regions. When the systemic risk is sufficiently low $Z < z_d$, the regulator optimally discloses it and achieves allocative efficiency by setting capital constraints $(a^*(z), A^*(z))$ such that the liquidity constraint of both types of banks binds. In the pooling region $Z > z_d$ the regulator sets strict capital requirements, $a^*(z) = A^*(z) = 0$ in order to avoid the failure of the weak banks in the event of systemic shock arrival.

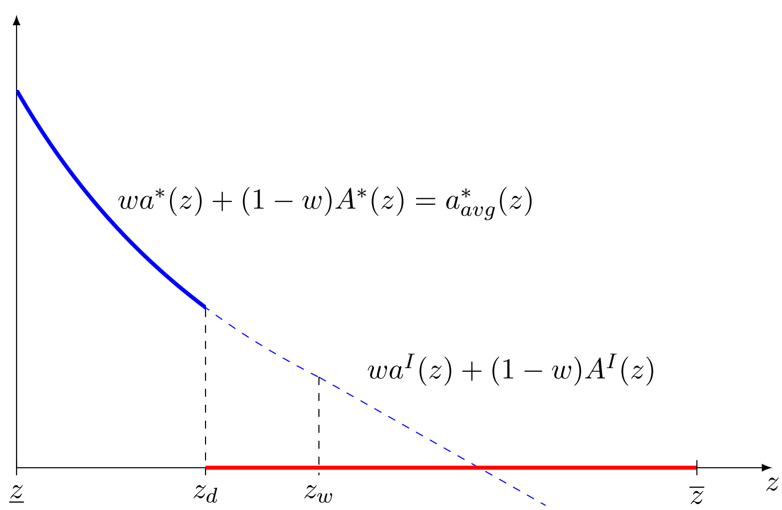


Figure 5: Optimal capital requirements with heterogeneous banks

The size of the pooling region is such that the liquidity constraint of the weak banks binds conditional on $S = z_d$, i.e.

$$m + np_0(z_d) - l = 0.$$

Nevertheless, the threshold z_d is determined not only by the liquidity needs of the weak banks but also by the liquidity buffer of the strong banks. Since $M + Np_0 - L > m + np_0 - l$ for any price $p_0 > 0$, the optimal capital requirements require strong banks to sell more assets than they need to avoid failing. Parametric restriction of the Proposition 2 implies that on the margin pooling information is more costly relative to imposing stricter capital requirements on the strong banks, therefore, the optimal policy exploits the cross-subsidization channel up to its capacity by setting $A^*(z) = 0$ for all $z > z_d$.

For low levels of systemic risk Z the aggregate welfare can be summarized in terms of the *representative* bank $(\bar{m}, \bar{n}, \bar{l})$. It is precisely such aggregation that makes the optimal test revealing for low levels of aggregate risk Z. Intuitively, it is always better to reveal more information in a single agent framework. Despite the aggregation result, the regulator applies differential treatment to weak and strong banks. In the revealing region the regulator optimally imposes capital requirements that allow the strong banks to hold more illiquid assets $A(z)$ as compared to the weak banks $a(z)$.

5 Dynamic Communication

In this section we show how dynamic and private communication can improve welfare when banks are heterogeneous. The optimal policy characterized in Proposition 2 imposes an extra burden on the strong banks by requiring them to hold extra liquidity in order to support the asset prices needed for survival of the weak banks. Moreover, tight capital requirements are combined with the lack of disclosure, thus, ex-post the prices often end up being higher than needed for the strong banks to survive. Such cross subsidization is especially inefficient when there are only a few weak banks. Dynamic communication allows the regulator to improve the liquidity position of the weakest banks first, then when once the whole system is more stable to reveal additional information. The difference between the two implementations that we consider lies in the source of liquidity for the weak banks. In the first case we consider a scenario in which the weak banks raise funds from the market. Then we show that such policy could be further improved if one allows for the weak banks to raise capital from the strong banks to improve risk sharing.

5.1 Weak Banks Raise Funds First

Suppose that the regulator could conduct two rounds of communications, i.e. reveal two pieces S_1 and S_2 of information about the systemic risk Z, and that the weak banks could improve their liquidity position via asset sales before the second piece of information is disclosed.

Introduction of the second round of communication after the optimal policy of Proposition 2 can be welfare improving. If S_1 is fully revealing, then the signal S_2 is not needed. However if S_1 pools information that $Z \geq z_d$ then there is a role for potentially disclosing more information to the market, especially once the weak banks reduce their exposure to the price fluctuation of the long-term assets. The benefit of additional disclosure comes from the ability of the regulator to adjust capital requirements for the strong banks and improving allocative efficiency. However, in this particular case such adjustments are impossible. In order for the weak banks not to fail at $t = 1$, the price at which they need to sell the assets has to be at least $p_0 = (1 - \lambda)b + \lambda p_L^I(z_0)$. Under the optimal test of Proposition 1 the expected price $\mathrm{E}\left[p_L(z)\,|Z > z_d\right] = p_L^I(z_0)$, where $p_L(z)$ is the ex-post price when both types of banks get rid of the long term assets. Since after revealing additional information the regulator can only impose $A(S_2) \geq 0$ the ex-post price is weakly lower then $p_L(z)$, which implies that if $A(S_2)$ differs from 0, then price that investors are willing to pay conditional on $S_1 = \{Z \geq z_d\}$ will not allow the weak banks to stay default free.

Nevertheless, dynamic communication can improve welfare if the pooling region of S_1 is strictly greater then $[z_d, \overline{z}]$. In this case, if the strong banks were to have $A(S_2) \equiv 0$ in this region, the liquidity constraint of the weak banks would be lax. This slack could be used to disclose more information and to increase $A(S_2)$ above 0. This dynamic policy involves a trade-off between the welfare created by the strong and weak banks, and results in a welfare increase if there are not too many weak banks in the system.

Proposition 3. *If the fraction of the weak banks is not too high, then the following dynamic policy is welfare improving relative to Proposition 2:*

$$S_1 = \begin{cases} Z, & \text{if } Z < z_d - \varepsilon, \\ z_d - \varepsilon, & \text{if } Z \geq z_d - \varepsilon, \end{cases} \qquad S_2 = \begin{cases} Z, & \text{if } Z < z_d + \Delta(\varepsilon), \\ z_d + \Delta(\varepsilon), & \text{if } Z \geq z_d + \Delta(\varepsilon), \end{cases}$$

$$a(S_1) = \begin{cases} a^I(S_1), & \textit{if } S_1 < z_d - \varepsilon, \\ 0, & \textit{if } S_1 = z_d - \varepsilon, \end{cases} \qquad A(S_2) = \begin{cases} A^I(S_2), & \textit{if } S_2 < z_d + \Delta(\varepsilon), \\ 0, & \textit{if } S_2 = z_d + \Delta(\varepsilon), \end{cases}$$

where $\varepsilon > 0$ is sufficiently small and $\Delta(\varepsilon)$ solves

$$\begin{aligned} &\mathrm{E}\left[p^I_{1,L}(z)\,|Z \in [z_d - \varepsilon, z_d + \Delta(\varepsilon))\right] \cdot \mathrm{P}(Z < z_d + \Delta(\varepsilon)\,|Z \geq z_d - \varepsilon) + \\ &\mathrm{E}\left[p_L(z)\,|Z \geq z_d + \Delta(\varepsilon)\right] \cdot \mathrm{P}(Z \geq z_d + \Delta(\varepsilon)\,|Z \geq z_d - \varepsilon) = \mathrm{E}\left[p_L(z)\,|Z \geq z_d\right]. \end{aligned}$$

The policy defined in Proposition 3 can be implemented as follows. First, a fairly lax stress test S_1 is conducted. If the level of systemic risk is sufficiently small $Z < z_d - \varepsilon$, then the outcome of the test is disclosed and both weak and strong banks implement full information capital requirements. If the level of systemic risk Z is above the cutoff $z_d - \varepsilon$, then the outcome of the test is not disclosed and the weak banks are required to significantly improve their liquidity position $a(S_1) = 0$. Once the weak banks sell all of their long-term assets to the market, the regulator reveals additional piece of information S_2. If the level of systemic risk is intermediate $Z \in [z_d - \varepsilon, z_d + \Delta(\varepsilon))$, then this information is disclosed to the market, and the strong banks' capital requirements are given by $A^I(z)$. However, if the level of systemic risk is too high, then even the strong banks are subjected to tight capital requirements.

In the intermediate region $[z_d - \varepsilon, z_d + \Delta(\varepsilon))$ the regulator is able to have softer capital requirements $A^I(z)$ relative to the policy of Proposition 2. Such capital requirements are optimal (conditional on z) even though the holdings of the banks $a(z) = 0$ and $A(z) = A^I(z)$ differ from the full information ones. The reason behind the optimality of $A^I(z)$ lies in the binding liquidity constraint of the weak banks who sell their assets at expected prices, conditional on S_1. When both liquidity constraints are binding, the ex-post price equals to $p^I_L(z)$, hence, the optimal $A(z)$ coincides with the one with full information.

5.2 Weak Banks Sell To Strong Banks

When the regulator can enforce asset sales in the interbank market at prices that might differ from those at which investors are willing to trade, then welfare could be improved even further. We show, that interbank transfers lead to aggregation result, i.e., the welfare is equivalent to the case with a single *representative* bank $(\bar{n}, \bar{m}, \bar{l})$.

Suppose that the regulator could enforce the following contract: every weak bank sells Δ_n long-term assets at some price p and the strong banks need to buy their pro-rate share. If Δ_n is chosen such that

$$\Delta_n = n - \bar{n}\frac{m - l + \Delta_n p}{\bar{m} - \bar{l}},$$

then regardless of price p the following equality holds

$$\frac{M - \frac{w}{1-w}\Delta_n p - L}{N + \frac{w}{1-w}\Delta_n} = \frac{m + \Delta_n p - l}{n - \Delta_n} = \frac{\bar{m} - \bar{l}}{\bar{n}}.$$

That is, if both strong and weak banks sell all their long-term assets to the market at some price $p_0(S_1)$, then they will fail in $t = 1$ if and only if the representative bank $(\bar{n}, \bar{m}, \bar{l})$ also fails. The next proposition shows that the price p can be chosen such that the aggregation result holds.

Proposition 4. *There exists a transfer price $p(z)$ such that regulator achieves welfare equivalent to the one in economy with a single representative bank.*

The interbank transfers do no impact welfare in the region where the liquidity constraints of both banks can be satisfied both before and after transfers. This follows from the fact, that $p_L = (1-\lambda)(1-z)\bar{l}$ and explicit calculations of $wa^I(z) + (1-w)A^I(z)$ show that the transfers exactly cancel out. In particular, there is no need to use transfers for $z < z_w$. However, the regulator can use such transfers in order to expand the region where the liquidity constraints of both banks can bind simultaneously. For example, for $z > z_w$ the minimum transfer price p that makes both constraints bind solves

$$m + \Delta_n p + (n - \Delta_n)[(1-\lambda)b + \lambda p_L^I(z)] - l = 0,$$

that is after the transfer from the strong banks it is optimal for the weak banks to sell all the remaining assets to the market at a price $p_0(z) = (1-\lambda)b + \lambda p_L^I(z)$. The equation above implies that $p > (1-\lambda)b + \lambda p_L^I(z)$ for $z > z_w$, that is the strong banks are directly subsiding the weak banks. Such direct subsidy is efficient and does no involve losses associated with indirect subsidization methods discussed in Section 4.

6 Conclusion

Bank regulation must be forward looking. It is unlikely that the next financial crisis will arise from junk bonds or housing markets. A stress test is the regulatory tool used to identify new risks in the financial system. The regulator conducts a stress test by subjecting banks to hypothetical adverse scenarios and seeing which banks remain solvent under it. The goal is to identify weak banks and common sources for exposure. In this paper we analyze the optimal design of such tests. The optimal disclosure policy reveals low and intermediate levels of risk, allowing for efficient asset allocation in those states, while pooling states of nature with high levels of correlation. In the latter case the regulator imposes strict capital requirements and systemic banks raise large amounts of liquidity in the public markets. When banks vary in their liquidity positions, whether a bank passes a stress test depends both on its own liquid holdings, but even more importantly on the liquidity and risks held by other banks. Under the optimal test capital requirements are driven by the weakest bank and strong banks are forced to raise excess liquidity to support prices in case of a fire sale. We also show that dynamic disclosure of stress test results and corresponding capital adjustments can improve allocative efficiency of assets in the system relative to a static test.

Appendix

Justification of Assumptions 1.

Put $\int_0^1 a_i(z)di = \bar{a}$ and suppose that $p_L(z) < b$ and $p_H(z) < b$, then

$$\begin{aligned} p_L(z) &= m + (n - \bar{a})p_0(z) + \bar{a}p_L(z) - l(z + \lambda(1 - z)), \\ p_H(z) &= m + (n - \bar{a})p_0(z) + \bar{a}p_H(z) - l\lambda(1 - z). \end{aligned}$$

Since $p_0(z) = (1-\lambda)p_H(z)+\lambda p_L(z)$ we can take a convex combination of the $t = 1$ pricing equations to obtain

$$p_0(z) = m + (n - \bar{a})p_0(z) + \bar{a}p_0(z) - l\lambda,$$

which implies that

$$p_0(z) = \frac{m - \lambda l}{1 - n}.$$

Since $p_L(z) \leq p_0(z) \leq p_H(z)$ it is sufficient to require

$$\frac{m - \lambda l}{1 - n} > b \tag{15}$$

to have $p_H(z) = b$.

Under restriction (15) we have $p_0(z) = (1 - \lambda)b + \lambda p_L(z)$. Suppose that $p_L(z) < b$, then it is given by

$$\begin{aligned} p_L(z) &= m + (n - \bar{a})[(1 - \lambda)b + \lambda p_L(z)] + \bar{a}p_L(z) - l(z + \lambda(1 - z)) \\ &= m + (n - \bar{a})(1 - \lambda)b - l(z + \lambda(1 - z)) + (\lambda(n - \bar{a}\lambda + \bar{a})p_L(z) \\ &\leq m + (n - \bar{a})(1 - \lambda)b - l(z + \lambda(1 - z)) \\ &\leq m + (1 - \lambda)b - \lambda l. \end{aligned}$$

It is sufficient to require that

$$m - \lambda l < \lambda b, \tag{16}$$

to have $p_L(z) < b$.

Constrains (15) and (16) imply that in aggregate banks need to hold a large fraction of the long-term assets

$$\lambda > 1 - n.$$

Justification of Assumption 2.

The threshold $\underline{z}$ should satisfy

$$\begin{aligned}
a^I(\underline{z}) &< 1 \\
m + n[(1-\lambda)b + \lambda p_L(\underline{z})] - l &< (1-\lambda)(b - p_L(z)) \\
[n\lambda + (1-\lambda)]p_L(\underline{z}) &< l - m + (1-\lambda)(1-n)b \\
[n\lambda + (1-\lambda)]l(1-\lambda)(1-\underline{z}) &< l - m + (1-\lambda)(1-n)b \\
1 - \underline{z} &< \frac{l - m + (1-\lambda)(1-n)b}{l(1-\lambda)[n\lambda + (1-\lambda)]}
\end{aligned}$$

The upper threshold $\overline{z}$ should satisfy

$$\begin{aligned}
a^I(\underline{z}) &< 0 \\
m + n[(1-\lambda)b + \lambda p_L(\overline{z})] - l &< 0 \\
n\lambda p_L(\overline{z}) &< l - m - n(1-\lambda)b \\
n\lambda l(1-\lambda)(1-\overline{z}) &< l - m - n(1-\lambda)b \\
1 - \overline{z} &< \frac{l - m - n(1-\lambda)b}{nl\lambda(1-\lambda)}
\end{aligned}$$

There always exists a pair of $0 \leq \underline{z} < \overline{z} \leq 1$ satisfying inequalities above as long as

$$l - m - n(1-\lambda)b > 0,$$

or equivalently

$$m - \lambda l < (1-\lambda)(l - nb). \tag{17}$$

Proof of Proposition 1. Since the liquidity constraint (9) is binding for the optimal policy, we obtain that

$$a(s) = \frac{m + np_0(s) - l}{p_0(s) - \mathrm{E}\left[p_L(s, Z) \mid S = s\right]}$$
$$= \frac{m + n[(1-\lambda)b + \lambda(1-\lambda)l(1 - \mathrm{E}\left[Z|S = s\right]) - l]}{(1-\lambda)[b - l(1-\lambda)(1 - \mathrm{E}\left[Z|S = s\right])]}.$$

Thus, the objective of the regulator depends only on the posterior mean $\mathrm{E}\left[Z|S = s\right]$ induced by the signal S. The payoff of the relaxed problem satisfies conditions of Dworczak and Martini (2017), which implies that the optimal signal induces a monotone partition of the state space $[\underline{z}, \overline{z}]$. In other words, the optimal signal breaks the set $[\underline{z}, \overline{z}]$ into a finite number of intervals $\{[\underline{z}_i, \overline{z}_i]\}_{i=1}^k$ such that inside of each interval $[\underline{z}_i, \overline{z}_i]$ the optimal signal S either fully reveals z, or provides no information apart from $Z \in [\underline{z}_i, \overline{z}_i]$.

For every interval $[\underline{z}_i, \overline{z}_i]$ fully contained in the region $a(z) > 0$ the value function is convex in z, thus, it is strictly optimal to reveal information. No interval $[\underline{z}_i, \overline{z}_i]$ can be fully contained in the region $a(z) < 0$, since this violates the expected liquidity constraint conditional on $Z \in [\underline{z}_i, \overline{z}_i]$. The only remaining possibility is that the optimal test is characterized by a threshold z_d, such that z is perfectly revealed below z_d and is pooled otherwise. Binding expected liquidity constraint conditional on $Z \in [z_d, \overline{z}]$ implies that

$$a(\mathrm{E}\left[Z \mid Z \in [z_d, \overline{z}]\right]) \geq 0,$$

however, due to convexity of $a(z)$ the inequality above can not be strict. $\square$

Proof of Proposition 2. Step 1: There is only one pooling region. First, notice that for every pooling region i, $U(\mathrm{E}\left[Z|Z \in \text{pooling set } i\right]) = 0$. Clearly $U(\mathrm{E}\left[Z|Z \in \text{pooling set } i\right]) \geq 0$ but if the inequality was strict, one could shrink the pooling region by disclosing lower values of z and increase expected welfare.[13] Since the average Z in all the pooling regions is the same, we can combine them without loss of generality into one.

Step 2: Pooling region is an interval $[z_d, \overline{z}]$.

Suppose that the pooling region is not connected, then there exists $z' < z'' < \mathrm{E}\left[Z|Z \in \text{pooling set}\right] = z_0$ such that $z' \in$ pooling set and $z'' \notin$ pooling set. Adjust the distribution of Z conditional on the pooling signal in the following way: remove weight ε from the point z' and add weight $\alpha\varepsilon$ on z''

[13] Here we rely on convexity of $U(z)$ to the left of $\hat{z}$ and the fact that $U'(\hat{z}) \leq U'(z)$ for all $z \geq \hat{z}$.

such that the average does no change. Constant expectation pins down α to be the solution of

$$(z' - z_0)g(z') = \alpha(z'' - z_0)g(z'').$$

The change to the value function is

$$g(z')(U(z') - U(z_0)) - \alpha g(z'')(U(z'') - U(z_0)),$$

where U_0 is the regulator's utility in the pooling region. This change is positive if and only if

$$\frac{U(z') - U(z_0)}{z' - z_0} > \frac{U(z'') - U(z_0)}{z'' - z_0} \tag{18}$$

Since $\mathrm{E}\left[Z \mid Z \geq \hat{z}\right] > z_0$, a continuous pooling region needs to go beyond $\hat{z}$. This is even more true for the if the pooling region is not connected, hence $z' < \hat{z}$. But $U(z)$ is convex to the left of $\hat{z}$ and $U'(\hat{z}) \leq U'(z)$ for all $z \geq \hat{z}$, thus the inequality (18) holds and shifting weight from z' to z'' is an improvement over the pooling region with disconnected support. □

Parametric conditions for Proposition 2.

Proving optimality of the threshold test in Proposition 2 is challenging, since the value function $U(z)$ defined as

$$U(z) = wa^I(z) + (1 - w)A^I(z)$$

is not globally convex. Namely, at the point z_w, where the weak types sell all long-term assets $a^I(z_w) = 0$ the value function has a concave kink. The derivative of $U(z)$ to the right of z_w can be computed as follows: the price $p_L(z_w)$ should be supported by $A(z)$, hence, $A(z)$ solves

$$p_L(z_w) = w[m + np_0(z_w)] + (1 - w)[M + (N - A(z))p_0(z_w) + A(z)p_L(z_w)] - \bar{l}(z + \lambda(1 - z)).$$

But neither $p_L(z_w)$ nor $p_0(z_w)$ depend on z in this region, hence differentiation of the equation above gives

$$\begin{aligned} 0 &= (1-w)A'(z)(p_L(z_w) - p_0(z_w)) - \bar{l}(1-\lambda) \\ U'(z) = (1-w)A'(z) &= -\frac{\bar{l}(1-\lambda)}{p_0(z_w) - p_L(z_w)} \\ &= -\frac{\bar{l}(1-\lambda)}{(1-\lambda)b + \lambda p_L(z_w) - p_L(z_w)} \\ &= -\frac{\bar{l}}{b - p_L(z_w)} \end{aligned}$$

Define $\hat{z} < z_w$ as the intersection of the continuation of the linear part of $U(z)$ with the convex part of $U(z)$ illustrated in Figure 6.

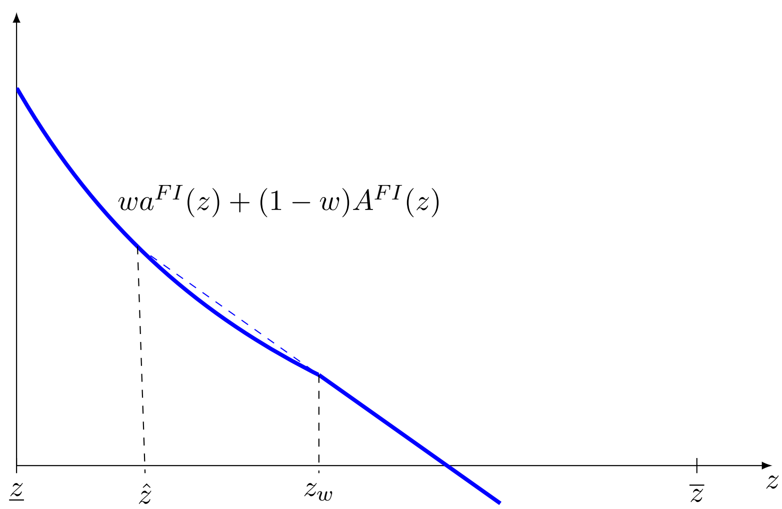


Figure 6: Definition of $\hat{z}$.

We next derive sufficient conditions for the $\hat{z}$ to be well defined. First, notice that the intersection point is more likely to exist if, keeping the parameters of the average bank $(\bar{m}, \bar{n}, \bar{l})$ constant the weak bank needs a very low price $p_L(z_w)$ to survive. The lowest $p_L(z_w)$ occurs when $z_w \to 1$ and $p_L(z_w) \to 0$. For the $\hat{z}$ to be well defined we need to have $U(\underline{z}) > (1-\underline{z})\frac{\bar{l}}{b}$. Notice that this inequality trivially holds when $\underline{z} \to 0$, $\lambda \to 0$ and $n \to 1$:

$$
\begin{aligned}
U(\underline{z}) \to \frac{\bar{m} + b - \bar{l}}{b - \bar{l}} &> \frac{\bar{l}}{b} \\
(\bar{m} + b - \bar{l})b &> \bar{l}(b - \bar{l}) \\
\bar{m}b + b^2 - \bar{l}b &> \bar{l}b - \bar{l}^2 \\
\bar{m}b + (b - \bar{l})^2 &> 0
\end{aligned}
$$

Proof of Proposition 3. Take a derivative w.r.t ε of the equation the defines $\Delta(\varepsilon)$ and put $\varepsilon = 0$:

$$
\frac{[p^I_{1,L}(z_d)g(z_d) + p^I_{1,L}(z_d)g(z_d)\Delta'(0) - p_L(z_d)g(z_d)\Delta'(0)] \int_{z_d}^1 g(z)dz - g(z_d) \int_{z_d}^1 p_L(z)g(z)dz}{\left(\int_{z_d}^1 g(z)dz\right)^2} = 0
$$

$$
p^I_{1,L}(z_d) + (p^I_{1,L}(z_d) - p_L(z_d))\Delta'(0) = \mathrm{E}\left[p_L(z)|Z > z_d\right] = p^I_{1,L}(z_0)
$$

$$
\Delta'(0) = \frac{p^I_{1,L}(z_d) - p^I_{1,L}(z_0)}{p_L(z_d) - p^I_{1,L}(z_d)}
$$

Thus, for ε small enough the first order welfare loss is $\varepsilon w a^I(z_d)$, and the first order welfare gain is $\varepsilon\Delta'(0)(1 - w)A^I(z_d)$. Since $A^I(z_d) > a^I(z_d) > 0$ for the dynamic policy to be welfare improving it is sufficient to have

$$
\frac{w}{1 - w} \leq \Delta'(0).
$$

The denominator of $\Delta'(0)$ can be rewritten as

$$
p_L(z_d) - p^I_{1,L}(z_d) = (1 - w)[M + N((1 - \lambda)b + \lambda p^I_{1,L}(z_0)) - L],
$$

hence the following inequalities are equivalent

$$
\begin{aligned}
\frac{w}{1 - w} &\leq \Delta'(0) \\
\frac{w}{1 - w} &\leq \frac{p^I_{1,L}(z_d) - p^I_{1,L}(z_0)}{(1 - w)[M + N((1 - \lambda)b + \lambda p^I_{1,L}(z_0)) - L]} \\
w[M + N((1 - \lambda)b + \lambda p^I_{1,L}(z_0)) - L] &\leq p^I_{1,L}(z_d) - p^I_{1,L}(z_0).
\end{aligned}
$$

The last inequality clearly holds for w small enough.

□

Proof of Proposition 4. The welfare is achieved with the stress test for the representative bank derived in Proposition 1 together with the following capital requirements and transfer prices.

For $z \leq z_w$ put $\Delta_n = p = 0$ and let

$$a(z) = \frac{m - l + n[(1-\lambda)b + \lambda(1-\lambda)(1-z)\bar{l}]}{(1-\lambda)(b - (1-\lambda)(1-z)\bar{l})}$$
$$A(z) = \frac{M - L + N[(1-\lambda)b + \lambda(1-\lambda)(1-z)\bar{l}]}{(1-\lambda)(b - (1-\lambda)(1-z)\bar{l})}$$

For $z \in (z_w, z_d)$ put

$$a(z) = 0$$
$$A(z) = \frac{M' - L + N'[(1-\lambda)b + \lambda(1-\lambda)(1-z)\bar{l}]}{(1-\lambda)(b - (1-\lambda)(1-z)\bar{l})}$$

where $M' = M - \frac{w}{1-w}\Delta_n p$ and $N' = N + \frac{w}{1-w}\Delta_n$. The transfer price p solves

$$m + n[(1-\lambda)b + \lambda(1-\lambda)(1-z)\bar{l}] - l + \Delta_n(p - [(1-\lambda)b + \lambda(1-\lambda)(1-z)\bar{l}]) = 0$$

and

$$\Delta_n = n \cdot \frac{\bar{m} - \bar{l} + \bar{n}[(1-\lambda)b + \lambda(1-\lambda)(1-z)\bar{l}]}{\bar{m} - \bar{l} + [(1-\lambda)b + \lambda(1-\lambda)(1-z)\bar{l}]}.$$

Finally, for $z \geq z_d$ put $a(z) = A(z) = 0$ and let the transfers solve

$$m + n[(1-\lambda)b + \lambda(1-\lambda)(1-z_0)\bar{l}] - l + \Delta_n(p - [(1-\lambda)b + \lambda(1-\lambda)(1-z_0)\bar{l}]) = 0$$

and

$$\Delta_n = n \cdot \frac{\bar{m} - \bar{l} + \bar{n}[(1-\lambda)b + \lambda(1-\lambda)(1-z_0)\bar{l}]}{\bar{m} - \bar{l} + [(1-\lambda)b + \lambda(1-\lambda)(1-z_0)\bar{l}]} = 0.$$

Notice that in the region $z \geq z_d$ the interbank transfers essentially implement a bailout of the weak banks by the strong banks.

□

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