Advising the Management*

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Abstract

We study the optimal size and composition of an advisory committee when information is dispersed and informed parties differ in preferences and beliefs. If agents have similar objectives but disagree due to different beliefs, communication by committee members exhibits positive externalities, and the manager’s expertise enhances the committee’s advisory role. Conversely, if agents have conflicting preferences, communication externalities are negative, and managerial expertise impedes the advising effectiveness. The optimal advisory body includes all informed agents under heterogeneous beliefs, but a strict subset of informed agents under heterogeneous preferences. Thus, advisory shareholder voting (board) is optimal in the former (latter) case.

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1 Introduction

Information relevant to corporate decisions is dispersed among many partially informed parties, such as the firm’s managers, employees, shareholders, customers, and industry participants. No manager, even the most experienced and talented one, is fully informed about the optimal course of action, so managers regularly seek advice from other informed agents. In modern firms, there exists a large heterogeneity in advisory structures. On many decisions, advice is provided by a relatively small group of people, such as the board of directors. In fact, advising the management is considered one of the most important functions of the board (e.g., Business Roundtable, 1990). In many “new age” firms, like Facebook, Snap, and Spotify, the advisory role is de facto the only role of the board, as their management has virtually full decision-making authority by holding superior-voting shares. For small companies, including startups, that do not have a formal board of directors, having an advisory board is considered a critical element of the company’s success.¹ On other decisions, advice to decision-makers is provided by a large group of people. For example, shareholders provide advice through a non-binding vote on corporate governance proposals, such as the firm’s anti-takeover defenses, corporate social responsibility policies, and executive compensation (“say on pay” in the U.S.). Although these votes are purely advisory, firms often learn from and respond to their results (Ferri, 2012). Other examples include employee and customer surveys, which are both regularly conducted by companies.²

Why is there such a heterogeneity in the means of providing advice? If advisory voting is indeed informative, why are not shareholders consulted on a greater variety of corporate decisions? When is it optimal to seek advice from a large group of people (e.g., shareholders through a non-binding vote) vs. a small group of people (e.g., the board)? And is an advisory committee more or less efficient at providing advice if the decision-maker is more informed?

The goal of this paper is to tackle these questions by studying the optimal size and composition of the advisory body. We propose a simple and tractable model that captures the two key features of the advisory process. First, relevant information is dispersed among


²For example, according to Watson Wyatt’s 2001 survey of 500 publicly traded companies, 79% regularly surveyed their employees.
multiple agents, which creates value from advising. Second, the agents’ information may not be perfectly communicated to the manager because of communication frictions. A committee is efficient in its advisory role if its members communicate their information to the manager despite these communication frictions.

Specifically, the firm needs to make a decision, whose value is determined by the unknown state of the world. Multiple agents get private signals about the state, with some agents’ signals being potentially more informative than others. The firm designs an advisory committee, which can be any subset of the agents. The committee members communicate with the partially informed manager by sending non-verifiable messages (“cheap talk”), and the manager then decides on which action to take. There is an infinitesimal cost of including a committee member, so the firm does not include agents who are not expected to communicate any information.

Generally, frictions in communication arise if, given the same information, the adviser wants to take a different action than the manager. Indeed, the adviser may then have incentives to misreport his information to tilt the manager towards his preferred action. There are two reasons why optimal actions may differ given the same information — conflicting preferences and differences in beliefs. For example, suppose a firm is deciding on the scale of production in a new market. Consider an agent maximizing shareholder value and the manager. Given the same information, the manager may prefer a larger scale of production for two reasons: he may get private benefits from running bigger operations (different preferences) or he may have “more bullish” priors about the value of increasing the scale (different beliefs). Our model features heterogeneity in both preferences and beliefs. Consistent with the above intuition, differences in preferences and differences in beliefs have a similar effect: the stronger these differences are, the stronger are the advisor’s incentives to misreport his information to the manager, so the lower is the quality of advice. Nevertheless, we show that these communication frictions have drastically different implications for optimal advisory structures.

We start by fixing an advisory body and analyzing the incentives of its members to truthfully communicate their information to the manager. We show that these incentives are strongly affected by whether other members of the advisory body reveal their informa-
tion to the manager. However, the nature of these communication externalities is drastically different depending on the source of communication frictions. If agents have the same preferences but disagree due to different prior beliefs about the distribution of the state, there are positive externalities in communication: When more agents truthfully communicate their information to the manager, the remaining agents have stronger incentives to also communicate their information truthfully. In contrast, if agents have the same prior beliefs but different preferences, there are negative communication externalities: When more agents reveal their information to the manager, the incentives of other agents to truthfully communicate their information decline.

The intuition for the positive externalities effect is that differences in beliefs become less relevant when more agents reveal their information to the manager. Simply speaking, heterogeneous prior beliefs generate disagreement only over the information that is unknown, so there is less to disagree about when more information about the optimal decision is learned. Thus, if the impedient to communication is heterogeneous beliefs, the manager’s and advisers’ optimal actions become more congruent as more agents reveal their information to the manager, which improves communication further. The reason for the negative externalities effect is that the manager’s action responds less to each agent’s message when more agents reveal their information to the manager. Intuitively, when lying to the manager, the agent faces a trade-off: he wants to move the manager’s decision closer to his own ideal decision, but is afraid to make too big of an impact and move the manager’s decision too much, away even from his own ideal decision. This fear encourages truthful communication when the manager strongly reacts to the agent’s message, but is not sufficient to constrain lying when the manager’s reaction to the agent’s message is small. The latter is the case when the manager receives information from many other agents.

These different externalities imply that the size of the advisory body is crucial for the effectiveness of its advisory role, but that the effect of size depends on the nature of communication frictions. We show that if communication is hampered by heterogeneous beliefs, the advisory body is effective (defined as truthful communication of its members’ private information) only if its size is sufficiently large. In contrast, if communication is hampered by heterogeneous preferences, the advisory body is effective only if its size is sufficiently small.
This logic leads to the following result: The optimal advisory body is the set of all informed agents under heterogeneous beliefs, but a strict subset of all informed agents under heterogeneous preferences. Practically, including all informed agents in the advisory body can be interpreted as holding an advisory shareholder vote, combined with asking the opinion of other informed stakeholders such as employees and business partners. Our analysis suggests that such an advisory vote will be efficient in its advisory role if shareholders and the manager have common interests but different beliefs, but will be inefficient if shareholders’ preferences are sufficiently different from the manager. Importantly, this does not imply that the advisory vote alone is optimal: rather, it implies that the vote is a necessary part of the optimal advisory process, together with seeking advice from other informed parties. On the other hand, including a strict subset of all informed agents in the advisory body can be interpreted as appointing an advisory board. Our model implies that in contrast to advisory voting, such an advisory board will be efficient in its advisory role if board members and the manager have the same beliefs but different preferences, but will be inefficient if they disagree due to different beliefs. This suggests, for example, that it is beneficial to seek advice via an advisory shareholder vote on issues such as corporate social responsibility proposals, where there is arguably a lot of heterogeneity in beliefs of different agents. In contrast, on issues such as increasing the scale of production, where conflicts of interest are likely to be more important, a relatively small advisory board will be more efficient at providing advice.\(^3\)

What is the optimal composition of the advisory board in such situations, when conflicts of interest are relatively more important? It is natural to conjecture that, all else equal, the optimal advisory board should include agents with more important information and with preferences that are more aligned to the manager. We show that while the second part of this conjecture is true (because more aligned preferences make advice more credible), the first part is not always true: The negative externalities in communication can make it optimal to choose the less informed director out of two potential candidates, even if they have the same preferences.

In practice, boards of directors provide advice on many decisions, even on those where a

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\(^3\)Of course, in practice, there can be other relevant factors, besides aggregation of dispersed private information, that are relevant for the choice of composition of the advisory body. Since our goal is to isolate one specific channel, we abstract from other potential channels (see Section 4.1 for a discussion).
different advisory structure may be optimal. We thus explore more generally when a given board will be effective at providing advice, and in particular, ask how the board’s advisory role for the manager depends on the manager’s expertise. Are the two complements or substitutes? We show that a more informed manager enhances the board’s advising effectiveness if heterogeneity in beliefs is relatively more important than heterogeneity in preferences, but impedes its advisory role otherwise. Intuitively, when agents disagree due to different beliefs, the manager’s information makes him more congruent with board members, motivating them to report their information truthfully. In contrast, under conflicting preferences, higher managerial expertise makes it less costly for board members to misreport their information because they expect a more informed manager to react less to their advice. Interestingly, this implies that if conflicts of interest are substantial, a more informed manager does not always make more informed decisions. Although a more informed manager makes better decisions due to his own better information, he also finds it more difficult to get informed advice from the board. We show that this negative effect can dominate, so that it can be Pareto improving to appoint a less informed manager, even if he has the same preferences as the more informed manager and any possible advisory board can be appointed.

Our results have implications for the empirical literature on the board’s advisory role. For example, they suggest that CEO expertise is important in explaining the board’s advising effectiveness, but whether its effect is positive or negative systematically differs across decisions and companies. They also suggest that when conflicts of interest are relatively more important than differences in beliefs, then small advisory boards should be observed if there is a strong misalignment in preferences between the manager and potential board members, if the manager is sufficiently informed, or if information is sufficiently asymmetrically distributed among potential directors (cf. Section 6).

Our paper contributes to the literature that studies communication from the board to the manager (Adams and Ferreira, 2007; Harris and Raviv, 2008; Baldenius et al., 2014; Chakraborty and Yilmaz, 2017; Levit, 2018). Differently from these papers, in which the board communicates as a single agent, we analyze communication from multiple informed

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4E.g., Coles et al. (2008), Harford and Schonlau (2013), Dass et al. (2014), Field and Mkrtchyan (2016).
heterogeneous agents and emphasize the externalities in information transmission.\textsuperscript{5,6} Hence, the focus of our paper is on the effects of size and composition of the advisory body and in particular, the optimality of advisory voting vis-à-vis advisory board — a question that has not been studied in the literature before. In contrast, the above papers mostly study the optimal allocation of authority and the role of the board’s independence and alignment with the manager. The exception is Harris and Raviv (2008), who also derive implications for board size, but focus on how large board size impedes information acquisition due to free-riding.\textsuperscript{7} Our other contribution to this literature is to analyze two sources of communication frictions — conflicts of interest and different beliefs, and show that they have starkly different implications for the effectiveness and optimal structure of the advisory body.

Our work is also related to the analysis of advisory voting in Levit and Malenko (2011), but it is different in two key aspects. The first is our analysis of conflicting beliefs in addition to conflicting preferences, and the contrast between these two frictions. The second is a different economic mechanism due to the different way we view the role of advisory votes. In their setting, the vote only matters in specific cases – when it changes the manager’s decision from “not implement” the proposal to “implement”, once the vote tally exceeds a certain threshold. In contrast, we view the vote tally as affecting decisions even away from the threshold – e.g., because it affects the extent to which the proposal is implemented.\textsuperscript{8} Accordingly, the mechanism in Levit and Malenko (2011) works through shareholders conditioning their decisions on being pivotal; as a result, if the manager is sufficiently conflicted, the vote does not aggregate information regardless of the number of shareholders. In contrast, in our setting, information aggregation crucially depends on the number of shareholders.

\textsuperscript{5}Harris and Raviv (2008) consider multiple outside directors, but these directors are perfectly aligned and obtain perfectly correlated signals if they become informed, so they communicate as a single agent.

\textsuperscript{6}Levit (2017) studies communication from the board to the shareholders in a tender offer context. Song and Thakor (2006) consider a setting where the manager controls the quality of information available to the board under career concerns. These papers also treat the board as a single entity.

\textsuperscript{7}Warther (1998), Baranchuk and Dybvig (2009), Malenko (2014), Levit and Malenko (2016), Chemmanur and Fedaseyeu (2017), and Donaldson et al. (2018) study interactions between multiple directors within the board, but do not study the board’s advisory role and do not feature the mechanisms that arise in our paper. See Adams et al. (2010) for a comprehensive survey of other papers in the board literature.

\textsuperscript{8}In practice, both roles are important: the literature shows both a monotonic increase in the probability and extent of proposal implementation as a function of the vote tally (e.g., Ertimur et al., 2013) and a discrete jump in the probability of implementation around certain cutoffs (e.g., Cuñat et al., 2012). Technically, this difference is manifested through a binary manager’s action in Levit and Malenko (2011) and a continuous manager’s action in our paper.
More generally, our paper is related to the large literature on cheap talk, which studies transmission of non-verifiable information under conflicting preferences (Crawford and Sobel, 1982). The closest papers in this literature are those analyzing communication by multiple imperfectly informed senders (Austen-Smith, 1993; Battaglini, 2004; Morgan and Stocken, 2008; Galeotti et al., 2013). Our analysis of the case of conflicting preferences is related to Morgan and Stocken (2008) and their result that full information revelation is an equilibrium in a poll with a small sample, but not with a large one. We contribute to this literature in several ways. Most importantly, we show that the results under heterogeneous preferences (which the literature focuses on) are the opposite of those under heterogeneous beliefs. We also show how both communication frictions can be simultaneously captured in a simple tractable model with closed-form solutions. Finally, differently from the above papers, our setting incorporates agents with heterogeneous relevance of private information, allowing us to study the optimal composition of the advisory body, which those papers do not study.

Our paper also relates to the literature on heterogeneous priors. Morris (1995) provides an overview of the heterogeneous prior assumption, discusses why it is consistent with rationality, and concludes that “there are some cases in which differences in prior beliefs are essential to understanding economic phenomena.” Our model also features rational agents: although they have potentially different priors, they are not dogmatic and rationally update their beliefs in a Bayesian way after receiving new information. Overall, there is growing empirical evidence suggesting that heterogeneous priors are important to explain corporate finance decisions and the dynamics of asset prices and volume.\(^9\) Accordingly, there is a large theoretical literature studying the implications of heterogeneous priors.\(^{10}\) The closest papers are Garlappi et al. (2017), who study group decision-making under heterogeneous priors but without private information and communication, and Che and Kartik (2009), Van den Steen (2010), and Alonso and Camara (2016), who study communication under heterogeneous priors but with only one sender and not via cheap talk, and thus do not feature the forces highlighted in our paper.


\(^{10}\)Examples in the finance literature include Harris and Raviv (1993), Kandel and Pearson (1995), Boot, Gopalan, and Thakor (2006), Banerjee, Kaniel, and Kremer (2009), and Banerjee and Kremer (2010), among others.
The paper proceeds as follows. Section 2 describes the setup. Section 3 shows how the externalities in communication depend on the nature of communication frictions. Section 4 studies the optimal composition of the advisory body and provides implications for the use of advisory voting vs. boards. Section 5 examines the role of the manager’s expertise. Section 6 discusses the empirical predictions, and Section 7 concludes.

2 Model

In this section, we present a simple model, which captures heterogeneous preferences, heterogeneous beliefs, and dispersed private information, and has tractable and intuitive solutions.

The firm needs to make a decision, denoted by \( a \). The value of this decision depends on the unknown state of the world \( Z \). The decision affects the payoffs of the firm’s shareholders, managers, employees, customers, industry participants, and other stakeholders of the firm. Specifically, there is a set of \( N \) agents indexed by \( i, i \in \{1,...,N\} \), characterized by their preferences \( b_i \) such that the payoff of agent \( i \) from action \( a \) given state \( Z \) is

\[
U_i(a, Z, b_i) = -(a - Z - b_i)^2. \tag{1}
\]

The information structure is as follows. The state of the world is equal to the weighted sum of \( N \) signals:

\[
Z = \sum_{i=1}^{N} c_i \theta_i, \tag{2}
\]

where \( c_i > 0 \) and \( \theta_i \) are independent and identically distributed. Signals \( \theta_i \) can be thought of as different factors relevant to the decision. Information about these relevant factors is dispersed among the stakeholders of the company: agent \( i \) privately observes signal \( \theta_i \) and is uncertain about other signals. An agent with a higher \( c_i \) can be interpreted as being relatively more informed. Such additive information structure is common in the literature (e.g., Harris and Raviv, 2005 and 2008; Chakraborty and Yilmaz, 2017).

For example, in the context of M&A decisions, \( a \) could be the choice of how much to bid for a potential target, and signals \( \theta_i \) could capture the synergies from the merger, the intrinsic value of the target, the number of potential competing bidders and their bids, the costs of
integrating the two companies, and other relevant factors. Different stakeholders of the firm have expertise about different aspects of the decision, with some being more important than others. We develop this M&A example further below, as we explain the intuition behind the results (see Section 3).

The setup so far captures heterogeneous preferences and information dispersed among the agents. In addition, to capture the possibility that agents may have heterogeneous beliefs, we assume that agents have different priors about the distribution of signals \( \theta_i \). Intuitively, some agents can be ex-ante more bullish about the prospects of the acquisition and the value of the target, while others can be more bearish. To capture this feature in a tractable way and derive closed form solutions, we make the following distributional assumption. We assume that \( \theta_i \) is a binary signal equal to 1 with probability \( \varphi \) and 0 with probability \( 1 - \varphi \), and agents may potentially disagree about \( \varphi \): agent \( i \)'s prior of \( \varphi \) is characterized by the Beta distribution with parameters \( (\rho_i, \tau - \rho_i) \).\(^{11}\) The case of \( \rho_i = \rho \) captures the case of common priors: for example, if \( \rho_i = 1 \) and \( \tau = 2 \), all agents believe that \( \varphi \) is uniformly distributed on \([0, 1]\). Parameter \( \varphi \) captures the intrinsic value of the decision: when \( \varphi \) is higher, the state is likely to be higher, so the optimal action is higher as well. Parameter \( \rho_i \) captures how optimistic agent \( i \) is about the state: those with a higher \( \rho_i \) are ex-ante more optimistic than those with a lower \( \rho_i \).\(^{12}\) While agents may have different prior beliefs, they update their beliefs rationally (according to Bayes’ rule) when they receive new information.

To summarize, each agent is characterized by his preference parameter \( b_i \) (which reflects his ideal action if the state were known), belief \( \rho_i \) (which captures whether he is ex-ante bullish or bearish about the state), and private signal about the state \( \theta_i \) with relative importance \( c_i \). Parameters \( b_i \), \( \rho_i \), and \( c_i \) are publicly known.

We assume that parameters \( b_i \) and \( \rho_i \) satisfy \( (b_i - b_m)(\rho_i - \rho_m) \geq 0 \) for any \( i \). This assumption is made so that agent \( i \) can be interpreted as biased towards a higher or lower action relative to the manager, where this bias can come from a combination of two sources —

\[^{11}\]That is, given the agent’s belief \( \rho_i \), the density of \( \varphi \) is \( f_i(\varphi) = \frac{\varphi^{\rho_i - 1}(1-\varphi)^{\tau-\rho_i-1}}{\text{Beta}(\rho_i, \tau - \rho_i)} \), where \( \text{Beta}(\rho, \tau - \rho) = \frac{\Gamma(\rho)\Gamma(\tau-\rho)}{\Gamma(\tau)} \) and \( \Gamma(\cdot) \) is the gamma function.

\[^{12}\]Indeed, the expected value of a Beta distribution with parameters \( (\rho_i, \tau - \rho_i) \) is \( \frac{\rho_i}{\tau} \), which increases in \( \rho_i \). Hence, given quadratic preferences, the optimal action of an agent with a higher \( \rho_i \) is higher, as formally shown by (4) below.
different preferences over actions and different prior beliefs about the state.\textsuperscript{13} This assumption is automatically satisfied if only one communication friction is present, i.e., if the agents either have common preferences or same priors.

The timeline is as follows. There is an advisory body (committee) $B$, which is a subset of all agents, $B \subset \{1,...,N\}$, and a “decision-maker” from the set of agents, indexed by $m \in \{1,...,N\}$, $m \notin B$. We will refer to this decision-maker as the manager, but it can be any agent with decision-making authority in the organization. After the agents learn their private signals, all members of the advisory body simultaneously communicate their information to the manager (via cheap talk), and the manager then takes the action that maximizes his payoff.

We look for equilibria in pure strategies. Because signals are binary, it is without loss of generality to consider a binary message space: the strategy of agent $i$ is a mapping from his signal $\theta_i \in \{0,1\}$ into a binary non-verifiable message $m_i \in \{0,1\}$. Thus, in equilibrium, agent $i$ either communicates his information truthfully (i.e., $m_i(\theta_i) = \theta_i$ up to relabeling) or sends an uninformative (babbling) message (i.e., $m_i(0) = m_i(1)$). If, for a given advisory body $B$, there exists an equilibrium in which all members of $B$ truthfully communicate their information to the manager, we assume that this equilibrium is played. This equilibrium selection can be justified by the Pareto dominance criterion: as Lemma 2 below demonstrates, such an equilibrium dominates any other possible equilibrium for every agent in the economy.

3 Externalities in communication

Heterogeneity in preferences ($b_i \neq b_m$) and beliefs ($\rho_i \neq \rho_m$) introduces frictions into the communication process and gives the agents incentives to misrepresent their information when communicating it to the manager. In this section, we derive the necessary and sufficient condition for communication from agent $i$ to manager $m$ to be credible. As we show, whether truthful communication is incentive compatible crucially depends on whether other members of the advisory body reveal their information to the manager and on the nature of

\textsuperscript{13}Without this assumption, such an interpretation would not be possible because the two sources of the bias could offset each other, making the agent effectively unbiased relative to the manager.
communication frictions — whether they come from differences in preferences or beliefs.

We start by characterizing the action taken by the manager for a given outcome of the communication stage. Suppose that after communicating with the advisory body, the manager knows the subset \( R \subseteq \{1, \ldots, N\} \) of signals ("revealed" signals) and does not know all the other signals, \(-R \equiv \{1, \ldots, N\}\setminus R\). We use \( R \) and \(-R\) to denote signal indices and \( \theta_R \equiv \{\theta_i, i \in R\} \) and \( \theta_{-R} \equiv \{\theta_i, i \in -R\} \) to denote the corresponding subsets of signal realizations. The subset \( \theta_R \) includes the manager’s own signal \( \theta_m \) and the signals of those members of the advisory body who communicate their information truthfully.

Given the quadratic payoff function, the optimal action of the manager is the sum of \( b_m \) and his expectation of the state given the signals he learned \((\theta_i, i \in R)\) and his prior \( \rho_m \):

\[
a_m(\theta_R) = b_m + \mathbb{E}_m (Z) | \theta_i, i \in R) = b_m + \sum_{i \in R} c_i \theta_i + \sum_{j \in -R} c_j \mathbb{E}_m [\theta_j] | \theta_i, i \in R].
\]

(3)

The subscript \( m \) in the expectation operator \( \mathbb{E}_m \) highlights that the manager uses his own prior \( \rho_m \) to update his beliefs about the unknown signals \( \theta_j, j \in -R \), from his knowledge of signals \( \theta_i, i \in R \). In the appendix, using the properties of the Beta distribution, we derive a simple expression for the manager’s posterior beliefs and obtain the following result:

**Lemma 1 (Optimal action of the manager)** Suppose that after the communication stage, the manager knows the subset \( R \) of signals. Then his optimal action is

\[
a_m(\theta_R) = b_m + \sum_{i \in R} c_i \theta_i + \frac{\rho_m + \sum_{i \in R} \theta_i}{\tau + |R|} \sum_{j \in -R} c_j,
\]

(4)

where \( |R| \) is the number of signals in \( R \).

The higher are the revealed signals \( \theta_i, i \in R \), the higher is the manager’s posterior belief about the state, and hence the higher is his optimal action (e.g., offer price for a target) given this information. Expression (4) also shows the effect of heterogeneous preferences and heterogeneous beliefs on the manager’s action. A higher \( b_m \) induces the manager to take a higher action given the same information. Likewise, a higher \( \rho_m \), capturing more optimistic ex-ante beliefs, also induces the manager to take a higher action. Note, however, that unlike
heterogeneity in preferences, whose effect does not depend on the manager’s information, heterogeneity in beliefs becomes less important as the manager becomes more informed and updates his beliefs. In particular, as the set $R$ expands, the term $\frac{1}{\tau+|R|} \sum_{j \in -R} c_j$, and hence the effect of $\rho_m$ on the manager’s action, decreases. In the extreme case, if $b_m = b_j$ and $R = \{1, ..., N\}$, the manager’s optimal action coincides with the optimal action from the perspective of any other agent.

Using Lemma 1, we next examine when a given committee member will truthfully reveal his information to the manager. Consider any agent $i$ and suppose that the manager knows the subset $R \subset \{1, ..., N\}$ of signals, where $R$ includes the manager’s own signal $\theta_m$ but not agent $i$’s signal $\theta_i$. The manager does not know all the other signals, i.e., agent $i$’s signal $\theta_i$ and all signals in the subset $-R \setminus \{i\}$, where as before, $-R \equiv \{1, ..., N\} \setminus R$. Suppose the manager believes the agent’s message and uses it to update his belief about the state according to (4). If agent $i$ reveals his signal truthfully, (4) implies that the manager’s action is

$$a_m(\theta_R, \theta_i) = b_m + c_i \theta_i + \sum_{k \in R} c_k \theta_k + \frac{\rho_m + \theta_i + \sum_{k \in R} \theta_k}{\tau + 1 + |R|} \sum_{j \in -R \setminus \{i\}} c_j.$$  

(5)

In contrast, if agent $i$ misreports and says that his signal is $1 - \theta_i$, the manager’s action is

$$a_m(\theta_R, 1 - \theta_i) = b_m + c_i (1 - \theta_i) + \sum_{k \in R} c_k \theta_k + \frac{\rho_m + (1 - \theta_i) + \sum_{k \in R} \theta_k}{\tau + 1 + |R|} \sum_{j \in -R \setminus \{i\}} c_j.$$  

(6)

Because agent $i$ does not know the realization of other agents’ signals when he communicates with the manager, he compares his expected payoff from actions $a_m(\theta_R, \theta_i)$ and $a_m(\theta_R, 1 - \theta_i)$ given his signal $\theta_i$ and his own prior beliefs about the distribution of those signals, and reports his signal truthfully if and only if:

$$\sum_{\theta_{-i} \in (0,1)^{N-1}} \left[ (a_m(\theta_R, \theta_i) - Z - b_i)^2 - (a_m(\theta_R, 1 - \theta_i) - Z - b_i)^2 \right] P_i(\theta_{-i}|\theta_i) \leq 0,$$  

(7)

where $\theta_{-i}$ is the set of all signals but $\theta_i$ and $P_i(\theta_{-i}|\theta_i)$ is agent $i$’s belief given his signal $\theta_i$ and his own prior.

The next proposition characterizes the necessary and sufficient conditions for (7) to be satisfied.
Proposition 1 (IC constraint for truthful reporting) Suppose that the manager learns the subset $R$ of signals (which includes his own signal $\theta_m$ but not $\theta_i$) and does not know all the other signals, $-R$. Then agent $i$ reports his signal truthfully if and only if

$$\left| (b_m - b_i) + \frac{\sum_{j \in R \setminus \{i\}} c_j}{\tau + |R| + 1} (\rho_m - \rho_i) \right| \leq \frac{1}{2} \left[ c_i + \frac{\sum_{j \in R \setminus \{i\}} c_j}{\tau + |R| + 1} \right].$$

(8)

Intuitively, the left-hand side of (8) illustrates the agent’s benefit from misreporting the signal, while the right-hand side illustrates his cost of misreporting. The logic is as follows. First, as (8) shows, communication between the agent and the manager can be inhibited by two key frictions — the difference in their preferences, captured by $b_m - b_i$, and the difference in their beliefs, captured by $\rho_m - \rho_i$. If both frictions are absent, $b_m = b_i$ and $\rho_m = \rho_i$, the agent always reveals his information truthfully because he knows that the manager will use this information in the way that is optimal for him. Whenever any of these frictions is present, the agent may have incentive to misreport. In particular, differences in preferences create incentives for misreporting as is standard in cheap talk games: the agent wants to tilt the manager’s action in the direction of his preference, $b_i$. Similarly, if the agent and manager have different priors — for example, if the manager is ex-ante more optimistic about the state than the agent, the agent will want to correct this “bias in beliefs” by reporting a more negative signal. Overall, these results imply that, other things equal, an advisory committee will be less effective in its advisory role if the manager is sufficiently different from the committee members in either preferences of prior beliefs about the optimal decision.

Second, regardless of the source of communication frictions, agent $i$ is more likely to report his signal truthfully if his information is more important: the IC constraint (8) is relaxed when $c_i$ increases. Intuitively, the agent faces a trade-off: while he wants to tilt the manager in the direction of his optimal action (the benefit of misreporting), he is also afraid to tilt it too much, away even from the optimal action from the agent’s perspective (the cost of misreporting). As the agent’s information becomes more important and hence the manager is expected to react more strongly to the agent’s message, this fear makes the agent more reluctant to misreport.
Despite the above similarities between the two sources of communication frictions, we next show that they have very different implications for the optimal advisory structures. Our first observation, which is the key implication of (8), is that the agent’s incentive to report his information truthfully depends on whether other members of the advisory body reveal their information to the manager (i.e., on $R$), but whether this dependence is positive or negative crucially depends on whether communication is hampered by heterogeneity in preferences or heterogeneity in beliefs. In particular, how much information the manager obtains from other committee members is captured by the term \[ \frac{\sum_{j \in -R \setminus \{i\}} c_j}{\tau + |R| + 1} \] on both sides of (8): this term decreases as the set $R$ expands, i.e., as more committee members reveal their information to the manager. As (8) shows, increasing this term relaxes the agent’s IC constraint when communication is inhibited by heterogeneity in preferences ($\rho_i = \rho_m$), but tightens the agent’s IC constraint when communication is inhibited by heterogeneity in beliefs ($b_i = b_m$).

To explain the intuition and the implications more clearly, we rewrite (8) for each of these two cases separately and discuss the intuition behind each.

1. **Heterogeneity in beliefs**: $\rho_i \neq \rho_m, b_i = b_m$

   In this case, agent $i$ reports his signal truthfully if and only if\(^{14}\)

   \[ |\rho_m - \rho_i| \leq \frac{1}{2} \left[ 1 + c_i \frac{\tau + |R| + 1}{\sum_{j \in -R \setminus \{i\}} c_j} \right]. \tag{9} \]

Hence, the more information the manager gets from other members of the advisory body (i.e., the higher is $|R|$ and the lower is $\sum_{j \in -R \setminus \{i\}} c_j$), the more likely it is that agent $i$ will also truthfully communicate his information to him. We refer to this effect as the *positive externality effect* of information transmission, because more information revealed to the manager by some agents has a positive effect on further information aggregation by encouraging other agents to report their information truthfully. The reason is that as the manager learns more information from others, he becomes more congruent with the agent, i.e., the manager’s optimal action becomes closer to agent $i$’s preferred action. This happens for two related and

\(^{14}\)Here if $-R \setminus \{i\}$ is an empty set, the right-hand side of (9) is equal to infinity, i.e., (9) is always satisfied.
complementary reasons. First, heterogeneous prior beliefs generate disagreement only over
the information that is still unknown — once a certain piece of information gets revealed,
there is nothing to disagree about. Hence, the more information gets known to the manager,
the more agreement there is between the manager and the agent about the optimal decision.
The second effect, which is related and acts in the same direction, is that once a signal about
the state is revealed, agents update their posteriors about the distribution of the state based
on the realization of the signal. Hence, even if their prior beliefs were initially very different,
they become closer to each other following the revelation of new information.

Together, these two effects imply that the manager’s and agent’s optimal actions be-
come more congruent as the manager learns more information from his other advisors, in-
creasing the agent’s incentives for truthful communication. To see this most starkly, con-
sider the extreme case where the manager knows all the signals except agent i’s signal:
\[ R = \{1, \ldots, N\} \setminus \{i\} \] and so \(-R \setminus \{i\}\) is an empty set. In this case, truthfully reporting the
last remaining signal \(\theta_i\) results in the manager taking the action that is optimal from the
agent’s perspective, and hence is always incentive compatible.

To illustrate this intuition in the context of our M&A example, suppose that the ac-
quision target is a pharmaceutical company with a pipeline of drugs it is developing. The
optimal offer price then crucially depends on the success probability of each drug, but given
that such drugs have not been developed before, different members of the advisory body
may have very different priors about the likelihood of success of each drug. If they think
the manager is too optimistic about the likelihood of success, they may try to counteract
this optimism by more negative (non-truthful) reports about the aspects of the merger they
are knowledgeable about. If, however, the advisory body includes an expert who knows the
state of research on a particular disease, he can inform the manager about whether a given
drug will be a failure or a success. Once this information is revealed to the manager, the
uncertainty in target value coming from the success of this drug disappears, increasing the
manager’s congruence with other advisors (the first effect described above). Moreover, other
advisors also realize that following the expert’s report, the manager will adjust his prior
belief about the likelihood of success of \(other\) drugs towards its correct value. This makes
the manager even more congruent with other advisors and further decreases their incentives.
to misreport (the second effect described above).

2. **Heterogeneity in preferences**: $\rho_i = \rho_m, b_i \neq b_m$

   In this case, agent $i$ reports his signal truthfully if and only if

   $$|b_m - b_i| \leq \frac{1}{2} \left[ c_i + \sum_{j \in -R \setminus \{i\}} c_j \right].$$

   In contrast to the case of belief heterogeneity, the more information the manager gets from other members of the advisory body (i.e., the higher is $|R|$ and the lower is $\sum_{j \in -R \setminus \{i\}} c_j$), the less likely it is that agent $i$ will truthfully communicate his information to him. We refer to this effect as the *negative externality effect* of information transmission, because more information obtained by the manager from some agents discourages further information aggregation by harming the credibility of other agents. Intuitively, as the manager learns more information from others, the effect of the agent’s message on his actions decreases. Hence, the agent is less worried that misreporting the signal may tilt the manager’s action too far away from his own optimal action and has stronger incentives to misreport (this intuition is similar to the intuition above of why a higher $c_i$ relaxes the agent’s IC constraint).\(^{15}\)

   These externalities in information transmission imply that the size of the advisory body affects whether it is effective in its advisory role, but that the effect of size depends on whether communication is hampered by differences in preferences or differences in beliefs. To formalize this intuition, we define advisory body $B$ to be efficient at providing advice to the manager if truthful communication by all members of $B$ to the manager is an equilibrium. The next result shows how the size $|B|$ of the advisory body affects whether it is efficient in its advisory role.

**Proposition 2 (Committee size and advising effectiveness)** *If agents have heterogeneous beliefs ($b_i = b_m, \rho_i \neq \rho_m$), there is a cutoff on committee size $N_{\text{min}}$, such that committee*

\(^{15}\)This effect is also present in the case of heterogeneous beliefs, but as we show, it is dominated by the positive externalities effects.
$B$ cannot be efficient at providing advice if $|B| < N_{\min}$. If agents have heterogeneous preferences ($b_i \neq b_m$, $\rho_i = \rho_m$), there is a cutoff on committee size $N_{\max}$, such that committee $B$ cannot be efficient at providing advice if $|B| > N_{\max}$.

This result emphasizes that whether the manager should seek advice from a large vs. a small group of people crucially depends on the source of communication frictions. In particular, the first statement of the proposition shows that when communication is hampered by heterogeneous beliefs, truthful communication by all members of the advisory body to the manager is only possible if the size of the body is sufficiently large. Intuitively, unless the advisory body is large enough, the positive externalities in communication are not strong enough to overcome advisers’ incentives to misreport due to differences in beliefs. The opposite result obtains when communication is hampered by heterogeneous preferences: as the second statement of the proposition shows, in this case, truthful communication by all advisory body members is only possible if the size of the body is small enough. This second statement is similar to the result of Morgan and Stocken (2008) about information revelation in polls when constituents have heterogeneous preferences. Differently from Morgan and Stocken (2008), where all agents’ private signals are equally relevant and the decision-maker is uninformed, our setting features agents with heterogeneous relevance of private information and an informed decision-maker (manager). This allows us to examine the optimal composition of the advisory body and the role of the manager’s expertise, which Morgan and Stocken (2008) do not study.

4 Optimal advisory committee

We now use the results in the previous section to analyze the optimal composition of the advisory committee. We start this analysis by showing that each agent is ex-ante better off if the manager is ex-post more informed. Formally:

**Lemma 2 (Ex-ante payoffs)** Suppose that in equilibrium, the manager learns subset $R$ of the signals and does not learn all the other signals, $-R \equiv \{1, ..., N\} \setminus R$. Then the ex-ante
equilibrium payoff of agent $i$ is given by

$$
E_i[U_i | R] = -(b_m - b_i)^2 - A_{im}(R) - B_i(R) - C_{im}(R),
$$

where

$$
A_{im}(R) = \frac{2(b_m - b_i)(\rho_m - \rho_i)}{\tau + |R|} \sum_{j \in -R} c_j,
$$

$$
B_i(R) = \frac{\rho_i(\tau - \rho_i)}{\tau(\tau + 1)} \left( \sum_{j \in -R} c_j^2 + \frac{\left( \sum_{j \in -R} c_j \right)^2}{\tau + |R|} \right),
$$

$$
C_{im}(R) = \left[ \frac{\rho_m - \rho_i}{\tau + |R|} \sum_{j \in -R} c_j \right]^2.
$$

In particular, the ex-ante equilibrium payoff of agent $i$ is increasing in $|R|$ and decreasing in any $c_k$, $k \in -R$.

Intuitively, agent $i$’s utility from the manager’s action is determined by how much information the manager’s action reflects and by how different the manager’s action is from the agent’s optimal action given this information (due to their different preferences and beliefs). Term $B_i(\cdot)$ reflects the former — the loss in the agent’s expected utility due to residual variance in the state, i.e., the fact that the manager’s action does not reflect signals in $-R$. All the other terms capture the latter: Term $(b_m - b_i)^2$ is the loss in agent $i$’s utility due to the fact that the manager’s action reflects his rather than agent $i$’s preference, while terms $A_{im}(\cdot)$ and $C_{im}(\cdot)$ reflect the additional effects due to the ex-post belief divergence between agent $i$ and the manager.

Lemma 2 shows that for any agent in the economy, his ex-ante utility (before learning his private signal) is higher when more information is known to the manager ex-post, i.e., when the set $R$ is larger and $c_j, j \in -R$, is lower. For example, when all agents’ information has the same importance, i.e., $c_i = c$ for all $i$, then for any $\rho_i$ and $b_i$, each agent simply wants to maximize $|R|$, the number of signals that the manager learns from the advisory body.

Lemma 2 is important because it allows us to derive Proposition 3 about the optimal composition of the committee for any possible objective function that weakly increases in the utility of each agent in the economy and strictly increases in the utility of at least one of them, without specifying this objective function. An example of such an objective function is the weighted average of all agents’ expected payoffs with any possible non-negative (and at
least one strictly positive) weights. The only assumption we make for Proposition 3 is that there is an infinitesimally small positive cost of including each agent in the advisory body. This assumption ensures that it is optimal to search for the optimal advisory committee among the set of committees in which all members truthfully communicate their information to the manager.

The next proposition shows that the optimal advisory committee includes all informed agents (other than the manager) when communication is hampered by conflicting beliefs, but is a strict subset of all informed agents when communication is hampered by conflicting preferences.

**Proposition 3 (Optimal advisory committee)** Consider any objective function that weakly increases in the expected utility of each agent and strictly increases in the expected utility of at least one agent.

1. If agents have heterogeneous beliefs ($b_i = b_j$, $\rho_i \neq \rho_j$), the optimal advisory body is the entire set of agents: $B^* = \{1, ..., N\}\{m\}$.

2. If agents have heterogeneous preferences ($b_i \neq b_j$, $\rho_i = \rho_j$) and for at least one agent the preference misalignment with the manager is sufficiently large, $|b_m - b_i| > \frac{1}{2}c_i$, the optimal advisory body is a strict subset of all agents: $B^* \subset \{1, ..., N\}\{m\}$.

Intuitively, if communication is hampered by heterogeneous beliefs, the positive externality effect implies that the more agents are included in the advisory body and reveal their information to the manager, the more likely it is that other agents will truthfully report their information to the manager as well. In fact, if the advisory body is the set of all informed agents, there is an equilibrium in which each agent reports his information truthfully. Because this maximizes the amount of information available to the manager and thus, given Lemma 2, maximizes the ex-ante payoff of all agents, this advisory body is optimal.

In contrast, if communication is hampered by heterogeneous preferences, the negative externality effect and Proposition 2 in particular, imply that if the advisory body is too large (e.g., if it consists of all informed agents) and the misalignment in preferences is significant,\(^{16}\)

\(^{16}\)This is captured by the assumption $|b_m - b_i| > \frac{1}{2}c_i$ for at least some $i$ in the statement of the proposition.
there is no equilibrium in which all members report their information truthfully. Thus, any committee in which all its members communicate truthfully must be a strict subset of all informed agents. As long as there is an infinitesimal cost of including agents who do not contribute any information, the optimal advisory body will be a strict subset of all agents.

Our setup assumes that if all agents in the economy combine their information together, the state is known with certainty. In practice, there could be some residual information that none of the agents knows. The model can be easily extended to capture this feature, and Proposition 3 will continue to hold as long as the amount of this residual unknown information is not too large. If it is very large and an agent’s prior beliefs are very different from the manager’s, then even if all other agents reveal their signals to the manager, it may not be sufficient to align their very different priors, giving the agent incentives to misreport. Importantly, the positive and negative externalities channels continue to hold regardless of the amount of residual unknown information.

4.1 Advisory voting vs. advisory board

Proposition 3 has implications for the use of advisory shareholder voting vs. advisory boards. Under advisory voting, each shareholder of the firm is asked to convey his views to the management via the vote. If, in addition, the management gets the opinion of other informed stakeholders, such as employees, business partners, and external consultants, the resulting decision-making protocol resembles our model with $B = \{1, ..., N\} \setminus \{m\}$. One interpretation of Proposition 3 is that such an advisory vote is optimal if agents have aligned preferences, but disagree due to differences in beliefs. Importantly, our paper does not imply that the advisory vote alone is optimal: for many decisions, the set of informed agents includes not only shareholders, but other informed parties as well. Instead, our paper implies that under heterogeneous beliefs, holding an advisory vote is a necessary part of the optimal advisory process, and that the opinion of shareholders needs to be combined with the opinion of other informed parties.

In contrast, when decisions are made with the help of an advisory board, the management does not ask all shareholders for their opinions and gets feedback only from the board members, i.e., from a relatively small subset of people having information. Proposition 3
suggests that this is optimal if agents have heterogeneous preferences but common beliefs.

It is worth pointing out that in this comparison of advisory voting and board, we isolate one channel – how different size and composition of the advisory body affect aggregation of dispersed private information. There are other channels through which the composition of the advisory body could matter, with one notable channel being incentivizing information acquisition. It would be interesting to examine whether heterogeneity in preferences vs. beliefs play a different role in this context as well, but this is beyond the scope of this paper.

While both conflicts of interest and differences in beliefs are likely to play a role in practice, there are certain decisions for which differences in beliefs are arguably more important than conflicts of interest, and vice versa. For example, investments in brand new technologies or CSR activities are likely to be the subject of substantial disagreement. In contrast, if the firm is contemplating to increase the scale of production, then conflicts of interest (e.g., due to managerial empire-building) are likely to play a larger role than differences in beliefs. If this is so, our model implies that management should get input on decisions such as CSR activities from as many informed parties as possible, so using an advisory vote is optimal. Conversely, investment decisions that are subject to significant conflicts of interest should be made using the input from a limited set of agents, such as the board of directors or a committee within the board.

Another relevant example are corporate governance changes, which are likely to combine both strong conflicts of interest and substantial differences in beliefs. Indeed, on the one hand, the decision to allocate more power to shareholders deprives the manager of his private benefits of control, generating a substantial conflict of interest. On the other hand, even shareholders often disagree about which governance arrangement is optimal for any given firm. A recent example is the debate about proxy access, when different shareholders and governance experts disagreed about the optimal terms of proxy access, such as the minimum size and holding period requirements.\textsuperscript{17} Our model suggests that both decision-making protocols — holding an advisory shareholder vote or consulting a relatively small advisory committee (e.g., composed of the firm’s large shareholders) — can be optimal for corporate governance decisions, depending on which friction is more important.

4.2 Optimal board composition

The analysis so far has shown that when agents have heterogeneous priors but common preferences, the optimal advisory body is uniquely determined — it is the set of all informed agents. In contrast, when agents have common priors but heterogeneous preferences, the optimal advisory body is a strict subset of all informed agents, which we will from now on refer to as the board. But which agents should be put on this advisory board? We explore this question in this section.

When agents have common priors, the expected utility of each agent $i$ becomes particularly simple: in equations (11)-(12), $A_{im}(R)$ and $C_{im}(R)$ both equal zero, and the residual variance $B_i(R)$ is the same for every agent. Therefore, board composition that maximizes an agent’s ex-ante expected payoff is the same across all agents — it minimizes the residual variance subject to the constraint that each board member truthfully communicates his information to the manager. In other words, the optimal board $B^*$ solves the following problem:

$$\min_B \sum_{k \in -B_m} c_k^2 + \frac{1}{\tau + |B| + 1} \left[ \sum_{k \in -B_m} c_k \right]^2,$$

subject to $|b_m - b_i| \leq \frac{1}{2} \left[ c_i + \frac{\sum_{k \in -B_m} c_k}{\tau + |B| + 1} \right]$ $\forall i \in B,$

where $-B_m$ is the set all agents other than the board and the manager.\(^{18}\)

Since agents have common priors, they differ in two parameters — the importance of their signals and their preferences. It is natural to conjecture that, all else equal, the optimal advisory board should include agents with more important information (i.e., higher $c_i$) and agents whose preferences are more aligned with the manager (i.e., lower $|b_m - b_i|$). The latter makes advice more credible. The former makes it more relevant. It turns out that while the latter part of the conjecture is true, the former is false.

Specifically, let us first consider two agents, $i$ and $j$, who have the same relevance of information ($c_i = c_j$) but different preferences, and suppose agent $i$ is more aligned with the manager: $|b_m - b_i| < |b_m - b_j|$. Consider any board that includes agent $j$ but not $i$.\(^{18}\) This is because given the infinitesimal positive cost of including a board member, the optimal board does not include agents who do not truthfully communicate their information to the manager. Hence, the set $R$ in (11)-(12) is the board together with the manager, $B_m = B \cup \{m\}$. 

\(^{18}\)This is because given the infinitesimal positive cost of including a board member, the optimal board does not include agents who do not truthfully communicate their information to the manager. Hence, the set $R$ in (11)-(12) is the board together with the manager, $B_m = B \cup \{m\}$. 

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(denote this board $B_j$) and consider a different board, which includes the same members as $B_j$, except that agent $i$ is replaced by $j$ (denote this other board $B_i$). It is easy to see from (13)-(14) that board $B_j$ is weakly dominated by board $B_i$ in the following sense: If board $B_j$ is optimal, then board $B_i$ is optimal as well; however, if board $B_j$ is optimal, then board $B_j$ may be suboptimal. In particular, this could happen if the IC constraint (14) for agent $i$ is satisfied, but it is violated for agent $j$ when we replace agent $i$ by agent $j$. In this sense, all else equal, the optimal advisory board should include agents whose preferences are more aligned with the manager.

In contrast, now consider two agents, $i$ and $j$, with the same preferences ($b_i = b_j$) but different relevance of information, and suppose agent $i$ has information of greater importance: $c_i > c_j$. It is now possible that board $B_j$ (that includes agent $j$ but not $i$) is optimal, while board $B_i$ (i.e., the same board except that agent $j$ is replaced by agent $i$) is not optimal. The reason are negative externalities in communication: replacing agent $j$ by agent $i$ decreases the incentives of other informed board members to communicate their information truthfully, which can be detrimental. Intuitively, a very informed but biased board member needs assurance that his opinion will have a sufficiently high influence to have incentives to communicate his information. If another powerful director joins the board, this influence will be lower, breaking down efficient communication. To see that this negative effect can be significant enough to make board $B_i$ suboptimal, consider the following example.

Example 1 (Adding less informed board members can be optimal)

There is a manager indexed by $N$, and $N - 1$ other informed agents, $i = 1, ..., N - 1$. For simplicity, we normalize $\sum_{i=1}^{N-1} c_i = 1$. Suppose that the set of informed (non-manager) agents includes three agents ($i = 1, 2, 3$) with relatively small preference biases $|b_i - b_m|$ relative to their information, while the remaining agents have very high preference biases relative to their information: $|b_i - b_m| > \frac{c_i}{2}$ for $i > 3$, so that they never truthfully reveal their information. Then, the optimal board will never include any agent $i > 3$. Suppose also that among the first three agents, $c_1 > c_2 > c_3$, but agent 1 has very relevant information compared to agents 2 and 3 ($c_1 >> c_2$), so that the optimal advisory board should include agent 1. Finally, suppose that $b_m = b_2 = b_3 = 0$ but $b_1 = \frac{1}{2} \left[ c_1 + \frac{1-c_1-c_3}{r+3} \right]$. 24
As noted above, the optimal board should include agent 1 and should not include any of the agents $i > 3$. It turns out that the optimal board is $B = \{1, 3\}$ and not $\{1, 2\}$, even though agent 2 has more relevant information than agent 3 and the same preferences. This is because if the board consisted of agents 1 and 2, the IC constraint of agent 1 would be violated because by assumption, $b_1 = \frac{1}{2} \left[ c_1 + \frac{1-c_1-c_3}{\tau+3} \right] > \frac{1}{2} \left[ c_1 + \frac{1-c_1-c_2}{\tau+3} \right]$, so agent 1’s information would not be communicated.

5 Expertise of the manager

In practice, the company’s board of directors provides advice to the management on many different issues, even on those where a different advisory structure would potentially be optimal. Therefore, in this section, rather than asking the question of the optimal composition of the advisory body, we examine the effectiveness of the advisory role of any given board. In particular, an important question is how the advisory function of the board is affected by the manager’s expertise: are they complements or substitutes? Both views have been expressed in the academic literature, but they have not been formally explored in a unified framework. Our analysis shows that whether the two are complements or substitutes strongly depends on the nature of communication frictions.

To study this question, we need to introduce a measure of the manager’s expertise. For this purpose, we consider a small extension of the basic model by assuming that the manager knows a subset $S$ of signals $\{\theta_i, i \neq m\}$ in addition to signal $\theta_m$. If $S = \emptyset$, then the manager only knows his private signal $\theta_m$, as in the basic model. If $S \neq \emptyset$, then the manager also knows some signals of the other agents, in addition to his private signal $\theta_m$. We interpret an expansion/contraction of $S$ (i.e., addition/removal of signals to/from $S$) as an increase/decrease in the manager’s expertise.

Specifically, fix all parameters of the model and consider any board $B$, i.e., a subset of agents $\{1, \ldots, N\}$. As before, we say that board $B$ is efficient at providing advice to the

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19 For example, Armstrong et al. (2010) discuss that managers’ informational advantage may impede the advisory role of outside directors. On the other hand, Sundaramurthy et al. (2014) hypothesize that board members’ experience and expertise will be more impactful when the manager has greater expertise as well.

20 An alternative way to model higher managerial expertise is to increase $c_m$, while normalizing $\sum_{i=1}^{N} c_i = 1$. This model leads to the same result. However, it cannot be used to analyze how firm value changes with the manager’s expertise because a change in $c_m$ changes the distribution of state $Z$ in this formulation.
manager if truthful communication by all members of $B$ to the manager is an equilibrium. Without loss of generality, consider boards in which each member has some information that the manager does not already know: $\{\theta_i, i \in B\} \cap \mathcal{S} = \emptyset$. The next proposition shows how the advising effectiveness of the board varies with the manager’s information:

**Proposition 4 (Manager’s expertise).** Consider any board $B$ with $\{\theta_i, i \in B\} \cap \mathcal{S} = \emptyset$.

1. If $|b_m - b_i| \leq \frac{1}{2}c_i \forall i \in B$ and board $B$ is efficient at providing advice under $\mathcal{S}$, then it is also efficient at providing advice if set $\mathcal{S}$ expands, i.e., as the manager becomes more informed.

2. If $|\rho_m - \rho_i| \leq \frac{1}{2} \forall i \in B$ and board $B$ is efficient at providing advice under $\mathcal{S}$, then it is also efficient at providing advice if set $\mathcal{S}$ contracts, i.e., as the manager becomes less informed.

Intuitively, as the manager becomes more informed, there is less information relevant for the decision that neither the manager nor the board knows. This has different effects depending on the nature of communication frictions. When heterogeneous beliefs are relatively more important, the key consequence is the increase in the manager’s congruence because there is less information that the manager and board members can disagree about. As a result, board members have stronger incentives to truthfully reveal their information to the manager, explaining the first statement of the proposition. In contrast, when communication is primarily hampered by conflicts of interest, greater managerial expertise decreases directors’ costs of misreporting their information because the manager is expected to react less to each director’s message. This explains the second statement of the proposition.

It is interesting to compare this analysis of the manager’s expertise to the analysis of the optimal advisory committee in Section 4. On the one hand, the intuition behind Proposition 4 is close to the intuition behind the positive and negative externalities in Section 3: it relies on how an increase in the manager’s information set affects any given committee member’s incentives to truthfully reveal his information. On the other hand, the two are also quite

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21 Clearly, there is no benefit of adding an agent to the advisory board if he has no information that the manager does not already possess.
different. In Proposition 4, the manager is more vs. less informed for an exogenous reason, based on his expertise, ability, and experience. In Section 3, whether the manager is more or less informed arises endogenously — it depends on which members of the advisory committee communicate their information truthfully, which is a function of the committee composition. This difference implies that the problem with multiple senders (advisors) does not reduce to the problem with a single sender, and the two problems have different implications. To see this, consider a single sender-receiver (i.e., single advisor-manager) problem. In such a problem, for both sources of communication frictions, it is optimal to have the manager who is perfectly informed about the state — indeed, according to Lemma 2, this maximizes the ex-ante utility of all agents in the economy. If the two problems were equivalent, one could conclude that in the problem with multiple senders, the optimal advisory body should consist of the entire set of informed agents. This, however, is not the case under heterogeneous preferences (as shown in Proposition 3). Another difference relates to the information set of the sender: in games with a single sender, an increase in the sender’s information set typically improves his communication with the receiver. If the problems were equivalent, one could conclude that in the case of multiple senders (advisors), including more informed agents in the advisory body is always beneficial. This, however, is not the case either, as demonstrated in Section 4.2 and Example 1.

Interestingly, the result of Proposition 4 that the manager’s expertise may impede the board’s advising effectiveness implies that higher managerial expertise is not always beneficial. It can be Pareto improving to appoint a less informed manager, even if the less informed manager has the same preferences as the more informed manager, and even if the composition of the advisory board can be chosen optimally. Intuitively, when heterogeneity in beliefs is small while heterogeneity in preferences is substantial, the manager’s expertise has two opposing effects on the quality of decision-making. On the one hand, a less informed manager makes worse decisions due to his own information about the state being worse. On the other hand, a less informed manager improves the advisory role of the board and hence obtains more information from board members, increasing the quality of decision-making. The numerical example below illustrates that this second effect can dominate.
Example 2 (Manager’s expertise can be harmful)

There is a manager and 100 other agents, divided into two groups. The parameters are: $c_m = 0.3$, $b_m = 0.0475$, $b_i = 0 \forall i \neq m$, $\rho_i = \rho = 2$, $\tau = 4$. The first group are the relatively more informed agents: it contains $N = 10$ agents with $c_i = c = 0.05$. The second group are the relatively less informed agents: it contains $N = 90$ agents with $c_i = c = 0.2/90$ (for simplicity, $\sum_{i=1}^{N} c_i$ is normalized to one). Thus, the manager’s signal $\theta_m$ has weight 30% in the state, the sum of all signals of the more informed agents has weight 50% in the state, and the sum of all signals of the less informed agents has weight 20% in the state. If the manager only knows signal $\theta_m$ (i.e., $S = \emptyset$), then the optimal board is comprised of 5 agents from the first group. These agents report their signals to the manager truthfully, and the implied expected payoff of each agent $i \neq m$ is $V = -0.0093$ (the payoff of the manager is higher by $b_m^2$). In contrast, if the manager also knows one of the signals with $c_i = c = 0.2/90$ and the board is comprised of 5 agents from the first group, the IC constraint is violated and truthful revelation by all members of this board is not an equilibrium. Instead, the optimal board is comprised of 4 agents from the first group. The implied expected payoff of each agent $i \neq m$ is $V = -0.0109$, which is lower than if the manager is less informed. Thus, a reduction in the manager’s information improves the values of all agents by promoting more efficient communication.

In contrast, the fact that a more informed manager improves the advisory role of the board when conflicts of interest play a small role relative to differences in beliefs, implies that in this case, both effects act in the same direction. As the manager becomes more informed, he both makes better decisions due to his own information and can also get better advice from other agents.

6 Implications

The literature on the board’s advisory role studies how the presence of directors with a certain type of expertise is related to corporate policies and performance (e.g., Dass et al., 2014; Field and Mkrtchyan, 2016; Harford and Schonlau, 2013; and others). The analysis in
Section 5 suggests that this relationship will be affected by CEO expertise, but whether the link between directors’ expertise and corporate outcomes (which could proxy for the board’s advising effectiveness) is strengthened or weakened by CEO expertise depends on the source of communication frictions. In cases where board members and the CEO have conflicting preferences, CEO expertise will weaken the advisory role of the board. In contrast, in cases where they have aligned preferences but disagree due to different beliefs, CEO expertise will enhance the board’s advising effectiveness.

Whether conflicting preferences or differences in beliefs are more important is likely to differ both across decisions and across companies. For example, there is likely to be strong heterogeneity in prior beliefs about the success of a brand new technology or the development of an innovative drug. Likewise, there is often substantial disagreement about the effect of corporate governance policies, even among parties with very similar interests, such as shareholders with similar portfolios. The literature has proposed several measures of belief heterogeneity that could be used to assess how different agents’ beliefs are in a given situation (e.g., Thakor and Whited, 2011; Diether et al., 2002; Malmendier and Tate, 2005). On the other hand, strong heterogeneity in preferences is likely to arise if the decision involves a clear conflict of interest, such as an investment/acquisition that brings large private benefits to the manager. Across firms, all else equal, heterogeneity in preferences is more likely to arise in firms where the manager’s incentives are not aligned due to ineffective compensation and poor governance controls.

Our model also provides implications about the determinants of the optimal advisory committee size. Consider the case where agents have common priors but heterogeneous preferences, so that the optimal advisory committee consists of a strict subset of all informed agents (Proposition 3), which we refer to as the board. When should we observe smaller versus larger advisory boards? Recall that the optimal advisory board in this case solves problem (13)-(14). As (13)-(14) show, it is generally difficult to make statements about the optimal board size, because the solution to this problem depends a lot on the specifics of heterogeneity among agents. It is, however, possible to make several observations about

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22 The only paper we are aware of that tests the relationship between CEO expertise and directors’ advisory role is Sundaramurthy et al. (2014) in the context of IPOs by biotech firms.

when a small board consisting of only a few members is more likely to be observed.

First, the model implies that a small board is optimal if there is strong misalignment in preferences between the manager and potential board members. As (14) shows, in this case, board members will have incentives to misreport their information unless the board is sufficiently small. 24 Second, a small board is optimal if the manager is sufficiently informed. Indeed, as Proposition 4 shows, when agents have common beliefs but heterogeneous preferences, the manager’s expertise impedes the advisory role of any board. Hence, if the board is large, board members will not truthfully reveal their information to a highly informed manager. Third, a small board is optimal if information is highly asymmetrically distributed among potential directors in the following sense. Suppose that relatively few agents have information that is highly relevant (high $c_i$) and a large number of agents have information of relatively low relevance (very low $c_i$), similarly to Example 2 above. If all agents have sufficiently different preferences from those of the manager, the optimal board will only include members from the first group (as shown in Example 2), so if there are few highly informed agents, the optimal board size will be small. By the same logic, a large advisory board is optimal if potential directors are relatively congruent with the manager, if the manager is relatively uninformed, and if information is relatively uniformly dispersed across potential board members. The second condition is broadly consistent with the evidence in Coles et al. (2008), who study board size in the context of the board’s advisory role. They find that boards of more complex firms whose CEOs require more advice (which they proxy, e.g., by firm diversification and size) have larger boards, and this larger board size is driven by a larger number of outside directors. In the context of our model, the set of outside directors can be interpreted as the firm’s advisory committee (which is also the interpretation of Coles et al., 2008), while inside directors can be interpreted as the decision-maker.

Since board composition is chosen endogenously, our analysis does not generally predict

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24 To see this in the simplest way, suppose all potential board members have equally relevant information. Then, the optimal advisory board is the committee of the highest possible size that allows truthful communication by all its members. As (13)-(14) imply, the solution is to rank all agents by the degree of their preference misalignment with the manager $|b_i - b_m|$, and then gradually expand $B$ by including agents with progressively larger preference misalignments, up to the point when the next included agent’s IC constraint for truth-telling is violated. In this setting, if we now increase the degree of misalignment by scaling up $|b_i - b_m|$ by some constant $C$ for all $i$, then the optimal board size will monotonically decrease in $C$ and will become zero as $C$ becomes large enough.
any specific correlation between board size and performance. It does, however, predict how performance will be affected by exogenous changes in board size. One example is the 1976 law in Germany, which introduced new requirements for supervisory board size. Jenter et al. (2018) study this regulation via regression-discontinuity and difference-in-differences analyses and conclude that forcing firms to have larger boards lowers performance and value. This evidence is consistent with our model if heterogeneity in preferences is strong (differences in preferences are particularly likely in German supervisory boards, which contain shareholder and employee representatives, but no executive directors). Indeed, our model predicts that adding members to the optimal board determined by (13)-(14) decreases value because it does not lead to more communication and better advice but imposes additional costs, for example, the costs of new board members’ compensation.\footnote{These costs are assumed to be infinitesimally positive in our setting, but could be significant in practice.}

Finally, the paper has implications for advisory (non-binding) shareholder voting. The literature has looked at the degree of management’s responsiveness to the advisory vote tally, which can be interpreted as the effectiveness of the vote’s advisory role (e.g., Ertimur et al., 2010; Ferri, 2012). Our analysis predicts that, holding the differences in beliefs between shareholders and management fixed, a greater misalignment in preferences between them decreases the advisory effectiveness of the vote.

7 Conclusion

The goal of this paper is to study the optimal design of the advisory committee when information about the decision is dispersed among multiple stakeholders of the company. We analyze a setting where members of the advisory committee communicate their information to a partially informed decision-maker (the manager), who then takes the optimal action from his perspective. In this setting, communication from any advisory committee member to the manager can be inhibited by two key frictions, conflicting preferences and disagreements due to different prior beliefs: Both frictions may induce the advisor to misreport his information, in order to tilt the manager’s action in the advisor’s preferred direction. We study both of these frictions and show that they have drastically different implications for the advising effectiveness of any committee and for the composition of the optimal committee.
Specifically, when agents have aligned preferences but disagree due to differences in beliefs, communication from committee members to the manager exhibits positive externalities: As more agents truthfully reveal their information to the manager, the incentives of other agents to reveal their information increase, inducing further information aggregation. In contrast, when communication is primarily impeded by heterogeneous preferences, there are negative externalities: As more agents truthfully reveal their information to the manager, other agents have stronger incentives to misreport. We show that due to these externalities, the optimal advisory body is the set of all informed agents under heterogeneous beliefs, but a strict subset of all informed agents under heterogeneous preferences. This implies that advisory shareholder voting, combined with the manager seeking the advice of other informed stakeholders, is effective for issues involving considerable heterogeneity in beliefs, such as the firm’s CSR practices. Conversely, it is better to seek advice from a small advisory board on issues involving a significant misalignment in preferences, such as the decision to scale up the production.

We also study the advising effectiveness of any given board and show that the manager’s expertise enhances the board’s advisory role when heterogeneous beliefs are the key communication friction, but impedes it when heterogeneous preferences are most important. Finally, we highlight that it is not always optimal to seek advice from the most informed agents or to appoint the most informed decision-makers: When heterogeneity in preferences is substantial, picking the less informed board member out of two potential candidates with the same preferences can be Pareto improving. Likewise, it can be Pareto improving to pick the less informed manager out of two candidates with the same preferences.
References


Appendix: Proofs

Proof of Lemma 1

Since \( \theta_i \) is a binary signal equal to 1 with probability \( \varphi \) and 0 with probability \( 1 - \varphi \), the manager’s optimal action (4) can be written as:

\[
a_m(\theta_R) = b_m + \sum_{i \in R} c_i \theta_i + \mathbb{E}_m[\varphi|\theta_i; i \in R] \sum_{j \in -R} c_j .
\]

Let \( 1_R \equiv \sum_{i \in R} \theta_i \) be the number of signals in \( R \) equal to 1. The conditional probability that \( 1_R \) signals out of \( |R| \) are equal to one given \( \varphi \) is \( P(1_R|\varphi) = \left(\frac{|R|}{1_R}\right)\varphi^{1_R}(1 - \varphi)^{|R| - 1_R} \).

Since the prior distribution is Beta and the likelihood function is Binomial, the posterior distribution is also Beta but with different parameters (this is a known property of the Beta distribution). Formally, let \( P_i(1_R) \) be agent \( i \)'s assessed probability that \( 1_R \) signals out of \( |R| \) are equal to 1 (over all possible values of \( \varphi \)). Using Bayes rule, agent \( i \)'s posterior belief of \( \varphi \), \( P_i(\varphi|1_R) \), is

\[
P_i(\varphi|1_R) = \frac{f_i(\varphi)P(1_R|\varphi)}{P_i(1_R)} = \frac{\varphi^{\rho_i-1}(1 - \varphi)^{\tau - \rho_i - 1}}{Beta(\rho_i, \tau - \rho_i)} \frac{1}{P_i(1_R)} \left(\frac{|R|}{1_R}\right)\varphi^{1_R}(1 - \varphi)^{|R| - 1_R} \\
= \frac{1}{Beta(\rho_i, \tau - \rho_i)P_i(1_R)} \left(\frac{|R|}{1_R}\right) \times \varphi^{\rho_i+1_R-1}(1 - \varphi)^{\tau - \rho_i + |R| - 1_R},
\]

which is some constant that does not depend on \( \varphi \) times \( \varphi^{\rho_i+1_R-1}(1 - \varphi)^{\tau - \rho_i + |R| - 1_R} \).

Since the posterior beliefs must integrate to one over possible values of \( \varphi \), this automatically implies that the posterior belief also follows a Beta distribution with parameters \((\rho_i + 1_R, \tau - \rho_i + |R| - 1_R)\) and density

\[
P_i(\varphi|1_R) = \frac{1}{Beta(\rho_i + 1_R, \tau - \rho_i + |R| - 1_R)} \varphi^{\rho_i + 1_R - 1}(1 - \varphi)^{\tau - \rho_i + |R| - 1_R}.
\]

It is known that the mean of a Beta distribution with parameters \((\alpha, \beta)\) is \( \frac{\alpha}{\alpha + \beta} \). Therefore, using these expressions and the above posterior distribution, agent \( i \)'s expected value of \( \varphi \) is \( \mathbb{E}_i(\varphi|1_R) = \frac{\rho_i + 1_R}{\tau + |R|} \), which proves the lemma.

Auxiliary Lemma A.1

Suppose \( \varphi \sim Beta(\rho, \tau - \rho) \) and \( X = \{x_1, x_2, \cdots, x_n\} \), where \( x_i \in \{0, 1\} \) are independent draws with \( x_i = 1 \) with probability \( \varphi \). Let \( 1_X \equiv \sum_{i=1}^n x_i \). Then

\[
\mathbb{E}_X[1_X] = n \frac{\rho}{\tau} \\
\mathbb{E}_X[1_X^2] = n \rho \frac{\tau - \rho + n(\rho + 1)}{\tau(\tau + 1)}.
\]
Proof. It is known that the first two moments of a random variable $X$ distributed according to a Beta distribution with parameters $\alpha$ and $\beta$ are $E[X] = \frac{\alpha}{\alpha + \beta}$ and $E[X^2] = \frac{\alpha(\alpha + 1)}{(\alpha + \beta)(\alpha + \beta + 1)}$. Hence, $E[\varphi] = \frac{\rho}{\tau}$ and $E[\varphi^2] = \frac{\rho(\rho + 1)}{\tau(\tau + 1)}$. Using this, we get

$$E[1_X] = E\left[\sum_{i=1}^{n} x_i\right] = nE[x_i] = nE[\varphi] = \frac{n\rho}{\tau}$$

and

$$E[1_X^2] = E\left[\sum_{i=1}^{n} x_i^2 + \sum_{i\neq j} x_ix_j\right] = E\left(nE[x_i^2|\varphi] + n(n-1)E[x_i|\varphi]^2\right)$$

$$= nE[\varphi] + n(n-1)E[\varphi^2] = \frac{n\rho}{\tau(\tau + 1)}(\tau - \rho + n(\rho + 1)).$$

Proof of Lemma 2

Let $1_R = \sum_{i \in R} \theta_i$ denote the number of signals 1 in $R$. Using Lemma 1, we obtain agent $i$’s ex-ante payoff, $E_i(a_m(\theta_R) - Z - b_i)^2$, as follows:

$$E_i[U_i|R] = -(b_m - b_i)^2 - U_1 - U_2,$$

where

$$U_1 \equiv 2(b_m - b_i)E_i\left[\left(\frac{\rho_m + 1_R}{\tau + |R|} \sum_{k \in -R} c_k - \sum_{k \in -R} c_k \theta_k\right) |R|\right],$$

$$U_2 \equiv E_i\left[\left(\frac{\rho_m + 1_R}{\tau + |R|} \sum_{k \in -R} c_k - \sum_{k \in -R} c_k \theta_k\right)^2 |R|\right].$$

Using independence of $\theta_k$ conditional on $\varphi$, and Auxiliary Lemma A.1, $U_1$ simplifies to

$$U_1 = 2(b_m - b_i)\frac{\rho_m - \rho_i}{\tau + |R|} \left(\sum_{k \in -R} c_k\right) = A_{im}(R).$$

To simplify $U_2$, we use the law of iterated expectations:

$$U_2 = E_i\left[\left(\frac{(\rho_m + 1_R) \sum_{k \in -R} c_k}{\tau + |R|}\right)^2 - \frac{2(\rho_m + 1_R)(\rho_i + 1_R)(\sum_{k \in -R} c_k)^2}{(\tau + |R|)^2} |R|\right]$$

$$+ E_i\left[\left(\sum_{k \in -R} c_k \theta_k\right)^2 |\theta_R, R\right] |R|$$

(17)
where we used $\mathbb{E}_i \left[ \sum_{k \in -R} c_k \theta_k | \theta_R, R \right] = \left( \sum_{k \in -R} c_k \right) \mathbb{E}_i [\varphi | \theta_R, R] = \left( \sum_{k \in -R} c_k \right) \frac{\rho_i + 1_R}{\tau + |R|}$. Consider the last term under the expectation sign:

$$
\mathbb{E}_i \left[ \left( \sum_{k \in -R} c_k \theta_k \right)^2 | \theta_R, R \right] = \mathbb{E}_i \left[ \sum_{k \in -R} c_k^2 \text{Var}_i [\theta_k | \varphi, R] + \varphi^2 \left( \sum_{k \in -R} c_k \right)^2 | \theta_R, R \right]
$$

$$
= \mathbb{E}_i \left[ \sum_{k \in -R} c_k^2 \varphi (1 - \varphi) + \varphi^2 \left( \sum_{k \in -R} c_k \right)^2 | \theta_R, R \right]
$$

$$
= \rho_i + 1_R \frac{1}{\tau + |R|} \left( \sum_{k \in -R} c_k^2 \right) - \rho_i \left( \sum_{k \in -R} c_k \right) \left( \frac{\rho_i + 1_R + 1}{\tau + |R| + 1} \right),
$$

where the second equality is due to $\text{Var}_i [\theta_k | \varphi, R] = \varphi (1 - \varphi)$ and the last equality is due to the fact that the agent $i$’s posterior distribution of $\varphi$ conditional on $\theta_R$ is Beta with parameters $\rho_i + 1_R$ and $\tau + |R| - \rho_i - 1_R$, whose first and second moments are, respectively, $\frac{\rho_i + 1_R}{\tau + |R|}$ and $\left(\frac{\rho_i + 1_R}{\tau + |R|}\right) \left(\frac{\rho_i + 1_R}{\tau + |R|} + 1\right)$. Plugging this expression into (17) and simplifying using Auxiliary Lemma A.1,

$$
U_2 - C_{im}(R) = \mathbb{E}_i \left[ \left( \sum_{k \in -R} c_k^2 \right) \frac{(\rho_i + 1_R)}{\tau + |R|} \right] - \left( \frac{\rho_i + 1_R}{\tau + |R|} \right)^2 |R|
$$

$$
+ \left( \frac{\left( \sum_{k \in -R} c_k \right)^2 - \sum_{k \in -R} c_k^2 \rho_i}{\tau + |R|} \right) \mathbb{E}_i \left[ \frac{(\rho_i + 1_R) (\rho_i + 1_R)}{\tau + |R| + 1} \right]
$$

$$
= \left( \frac{\left( \sum_{k \in -R} c_k \right)^2}{\tau + |R|} + \sum_{k \in -R} c_k^2 \right) \mathbb{E}_i \left[ \frac{(\rho_i + 1_R) (\tau + |R| - \rho_i - 1_R)}{(\tau + |R|)(\tau + |R| + 1)} \right]
$$

$$
= \left( \frac{\left( \sum_{k \in -R} c_k \right)^2}{\tau + |R|} + \sum_{k \in -R} c_k^2 \right) \frac{\rho_i (\tau - \rho_i)}{\tau (\tau + 1)} = B_i(R).
$$

Combining with (15) and (16) gives (11)-(12). This immediately shows that the ex-ante payoff of any agent $i$ is increasing in $|R|$ and is decreasing in $c_k$ for any $k \in -R$. In other words, when the manager learns an additional signal, $A_{im}(\cdot)$, $B_i(\cdot)$ and $C_{im}(\cdot)$ are reduced. Indeed, the greater information and the smaller unknown part of the state $Z$ imply that the residual variance $B_i(\cdot)$ decreases. The first and third terms, i.e., $A_{im}(\cdot)$ and $C_{im}(\cdot)$, decrease as well, because agent $i$ expects the additional signal to “persuade” the manager, such that they have a smaller expected divergence in their ex-post beliefs. This intuition holds in the opposite direction when $c_k, k \in -R$ increases.
Proof of Proposition 1

Plugging (5) and (6) into (7) gives

\[
0 \geq \sum_{\theta_i \in \{0, 1\}^N-1} \left[ c_i(2\theta_i - 1) + \left( \sum_{j \in -R \backslash \{i\}} c_j \right) \cdot \frac{2\theta_i - 1}{\tau + |R| + 1} \right] \times \left[ 2(b_m - b_i) + c_i(1 - 2\theta_i) - 2 \sum_{j \in -R \backslash \{i\}} c_j\theta_j + \frac{2(\rho_m + 1_R) + 1}{\tau + |R| + 1} \sum_{j \in -R \backslash \{i\}} c_j \right] P_i(\theta_i - i\theta_i).
\]

Note that the first multiple in each term equals \((2\theta_i - 1)[c_i + \sum_{j \in -R \backslash \{i\}} c_j]/\tau + |R| + 1\), where \(c_i + \sum_{j \in -R \backslash \{i\}} c_j\) is positive and is constant across all terms in the sum. Thus, the above inequality is equivalent to

\[
0 \geq (2\theta_i - 1) \sum_{\theta_i} P_i(\theta_i - i\theta_i) \left[ 2(b_m - b_i) + c_i(1 - 2\theta_i) - 2 \sum_{j \in -R \backslash \{i\}} c_j\theta_j + \frac{2(\rho_m + 1_R) + 1}{\tau + |R| + 1} \sum_{j \in -R \backslash \{i\}} c_j \right].
\]

Since \(\sum_{\theta_i-R \backslash \{i\}} (\sum_{j \in -R \backslash \{i\}} c_j\theta_j) P_i(\theta_i - R \backslash \{i\}|\theta_i) = \frac{\rho_i + 1_{R+1}}{\tau + |R| + 1} \sum_{j \in -R \backslash \{i\}} c_j\), we can further simplify it to

\[
(2\theta_i - 1) \left[ 2(b_m - b_i) + c_i(1 - 2\theta_i) + \frac{2(\rho_m - \rho_i) + 1 - 2\theta_i}{\tau + |R| + 1} \sum_{j \in -R \backslash \{i\}} c_j \right] \leq 0.
\]

We consider two separate cases. If \(\theta_i = 0\), the above inequality becomes:

\[
2(b_m - b_i) + c_i + \frac{2(\rho_m - \rho_i) + 1}{\tau + |R| + 1} \sum_{j \in -R \backslash \{i\}} c_j \geq 0,
\]

and if \(\theta_i = 1\), it becomes

\[
2(b_m - b_i) - c_i + \frac{2(\rho_m - \rho_i) - 1}{\tau + |R| + 1} \sum_{j \in -R \backslash \{i\}} c_j \leq 0,
\]

Together we get (8), which completes the proof.

Proof of Proposition 2

Consider advisory body \(B\) and define \(B_m \equiv B \cup \{m\}\). Full information revelation by all members of \(B\) to \(m\) is an equilibrium if and only if (8) holds for any agent \(i\) in \(B\), where
$R = B_m \setminus \{i\}$. Since $|R| = |B_m| - 1 = |B|$, (9) for all $i$ in $B$ becomes
\[|\rho_m - \rho_i| \leq \frac{1}{2} \left[ 1 + c_i \sum_{j \in \{1, \ldots, N\} \setminus B_m} \frac{\tau + |B| + 1}{c_j} \right] \quad \forall i \in B,
\]
which is equivalent to
\[|B| \geq \max_{i \in B} \left[ 2 \frac{|\rho_m - \rho_i| - 1}{c_i} \right] \sum_{j \in \{1, \ldots, N\} \setminus B_m} c_j - \tau - 1.
\]
Hence, truth-telling by all members of the advisory body cannot be an equilibrium if $|B|$ is below
\[\min_{B \subseteq \{1, 2, \ldots, N\} \setminus \{m\}} \max_{i \in B} \left[ 2 \frac{|\rho_m - \rho_i| - 1}{c_i} \right] \sum_{j \in \{1, \ldots, N\} \setminus B_m} c_j - \tau - 1,
\]
which is strictly positive if heterogeneity in beliefs is sufficiently strong.

Likewise, (10) for each $i$ in $B$ becomes
\[|b_m - b_i| \leq \frac{1}{2} \left[ c_i + \frac{\sum_{j \in \{1, \ldots, N\} \setminus B_m} c_j}{\tau + |B| + 1} \right] \quad \forall i \in B \iff \max_{i \in B} \left[ 2 |b_m - b_i| - c_i \right] \leq \frac{\sum_{j \in \{1, \ldots, N\} \setminus B_m} c_j}{\tau + |B| + 1}.
\]
Hence, truth-telling by all members of the advisory body cannot be an equilibrium if $|B|$ is above
\[\max_{B \subseteq \{1, 2, \ldots, N\} \setminus \{m\}} \frac{\sum_{j \in \{1, \ldots, N\} \setminus B_m} c_j}{\max_{i \in B} 2 |b_m - b_i| - c_i} - \tau - 1,
\]
which is strictly smaller than $N - 1$ if heterogeneity in preferences is sufficiently strong.

**Proof of Proposition 3**

We start with the first statement of the proposition, when $b_i = b_j$, $\rho_i \neq \rho_j$. Suppose that the advisory body is $B = \{1, \ldots, N\} \setminus \{m\}$. We show that there exists an equilibrium in which all agents in the advisory body truthfully reveal their information to the manager. Consider such an equilibrium and any agent $i \in B$. Then $R = \{1, \ldots, N\} \setminus \{i\}$ and $-R \setminus \{i\} = \emptyset$, so using (8), the IC constraint of agent $i$ is equivalent to $0 \leq c_i$, which is always satisfied. Hence, indeed, there exists an equilibrium in which the manager makes the decision knowing the information of all the agents, $i = \{1, \ldots, N\}$. Moreover, given Lemma 2, any agent $i$ achieves his maximum possible ex-ante utility when $R = \{1, \ldots, N\}$. Therefore, $B = \{1, \ldots, N\} \setminus \{m\}$ is indeed optimal in this case.

We next prove the second statement, when $b_i \neq b_j$, $\rho_i = \rho_j$. First, note that when $\rho_i = \rho$ for all $i$, then all agents have exactly the same preferences about the manager’s ex-post
information set $R$: they would like to minimize

$$\sum_{j \in - B_m} c_j^2 + \frac{1}{\tau + |B| + 1} \left[ \sum_{j \in - B_m} c_j \right]^2,$$

where $B_m \equiv B \cup \{m\}$. Thus, for any objective function that weakly increases in each agent’s ex-ante utility and strictly increases in at least one agent’s ex-ante utility, the optimal advisory body is the set of agents such that 1) all agents truthfully communicate their information to the manager 2) this set minimizes (18). Consider the solution to this minimization problem. We next prove that it is a strict subset of $\{1, ..., N\} \setminus \{m\}$ under the conditions in the proposition. Indeed, if the advisory body is the set of all agents, $B = \{1, ..., N\} \setminus \{m\}$, then $-B_m = \emptyset$, so (8) becomes $|b_m - b_i| \leq \frac{1}{2} c_i$. Since, by assumption, this inequality is violated for at least one of the agents, there is no equilibrium in which all agents truthfully reveal their information to the manager. Hence, $B = \{1, ..., N\} \setminus \{m\}$ is not an optimal advisory body, which implies that the optimal advisory body is a strict subset of all agents.

**Proof of Proposition 4**

Rewriting the IC constraint from Proposition 1 and using $(b_m - b_i) (\rho_m - \rho_i) \geq 0$, board $B$ is efficient if and only if $I_i \geq 0$ for all $i \in B$, where

$$I_i \equiv \frac{\tau + |B| + |S| + 2}{\sum_{j \in - B_m} c_j} \left( \frac{1}{2} c_i - |b_m - b_i| \right) + \frac{1}{2} - |\rho_m - \rho_i|,$$

where $-B_m$ is a set of all signal indices that are not known to the board or the manager. Consider an expansion of $S$ by one element that does not belong to $\{\theta_i, i \in B\}$, then all statements of the proposition are vacuously true, as the IC constraints are unaffected. Thus, consider the case when this element does not belong to $\{\theta_i, i \in B\}$. In this case, an expansion in $S$ increases $|S|$ and decreases $\sum_{j \in - B_m} c_j$. Suppose that $|b_m - b_i| \leq \frac{1}{2} c_i \forall i \in B$. Then, an expansion in $S$ increases $I_i$ for any $i$. Hence, if $I_i \geq 0$ for all $i$ for some $S$, then $I_i \geq 0 \forall i$ for any expansion in set $S$. This proves the first statement of the proposition.

To prove the second statement, rewrite the IC constraint from Proposition 1 as $J_i \geq 0$, where

$$J_i \equiv \frac{\sum_{j \in - B_m} c_j}{\tau + |B| + |S| + 2} \left( \frac{1}{2} - |\rho_m - \rho_i| \right) + \frac{1}{2} c_i - |b_m - b_i|.$$  

Again consider an expansion of $S$ by one element that does not belong to $\{\theta_i, i \in B\}$. Suppose that $|\rho_m - \rho_i| \leq \frac{1}{2} \forall i \in B$. Then, an expansion in $S$ reduces $J_i$ for any $i$, because it increases $|S|$ and decreases $\sum_{j \in - B_m} c_j$. Hence, if $J_i \geq 0$ for all $i$ for some $S$, then $J_i \geq 0 \forall i$ for any contraction in set $S$.  

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