# Advising the Management*

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**Abstract**

We study the optimal size and composition of an advisory committee when shareholders differ in preferences and beliefs and strategically acquire and communicate information. If shareholders and management have similar objectives but disagree due to different beliefs, and information is cheap, the optimal advisory body includes all shareholders. Conversely, if agents have conflicting preferences or information is sufficiently costly, the optimal advisory body is a strict subset of shareholders. Thus, advisory voting (board) is optimal in the former (latter) case. If ownership is endogenous and shareholders differ in beliefs, combining advisory shareholder voting with a diverse board can be optimal.

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# 1 Introduction

Information relevant to corporate decisions is dispersed among many partially informed parties, such as the firm's managers, employees, shareholders, customers, and industry participants. No manager, even the most experienced and talented one, is fully informed about the optimal course of action, so managers regularly seek advice from other informed agents. In modern firms, there exists a large heterogeneity in advisory structures. On many decisions, advice is provided by a relatively small group of people, such as the board of directors. In fact, advising the management is considered one of the most important functions of the board (e.g., Business Roundtable, 1990). In many "new age" firms, like Facebook, Snap, and Spotify, the advisory role is de facto the only role of the board, as their management has virtually full decision-making authority by holding superior-voting shares. For small companies, including startups, that do not have a formal board of directors, having an advisory board is considered a critical element of the company's success.[1] On other decisions, advice to decision-makers is provided by a large group of people. For example, shareholders provide advice through a non-binding vote on corporate governance proposals, such as the firm's anti-takeover defenses, corporate social responsibility policies, and executive compensation ("say on pay" in the U.S.). Although these votes are purely advisory, firms often learn from and respond to their results (Ferri, 2012). Other examples include employee and customer surveys, which are both regularly conducted by companies.[2]

Why is there such a heterogeneity in the means of providing advice? If advisory voting is indeed informative, why are not shareholders consulted on a greater variety of corporate decisions? When is it optimal to seek advice from a large group of people (e.g., shareholders through a non-binding vote) vs. a small group of people (e.g., the board)? And how do firms' decisions about advisory committees, such as whom to appoint to the board or whether to hold an advisory vote, depend on ownership structure and, in particular, which informed agents become shareholders in the first place?

The goal of this paper is to tackle these questions by studying the optimal size and composition of the advisory body. We propose a simple and tractable model that captures the three key features of the advisory process. First, relevant information is dispersed among

[1]See, e.g. "Who advises the entrepreneur?" (Harvard Business Review, October 22, 2013) and the 2014 BDC study "Advisory boards: An untapped resource for businesses".

[2]According to Watson Wyatt's 2001 survey of 500 publicly traded companies, 79% regularly surveyed their employees.

multiple agents, which creates value from advising. Second, agents' information may not be perfectly communicated to the manager because of communication frictions. Finally, acquiring information is personally costly for the agents.

Specifically, the firm needs to make a decision, whose value is determined by the unknown state of the world. Multiple agents – the firm's shareholders and other stakeholders – are potentially informed about the state. The firm designs an advisory committee, which can be any subset of the agents. For example, a committee that includes all shareholders corresponds to advisory voting, while a committee composed of selected shareholder representatives corresponds to the board of directors. Committee members decide whether to acquire private signals and then communicate with the partially informed manager by sending non-verifiable messages ("cheap talk"). The manager then chooses which action to take.

Generally, frictions in communication arise if, given the same information, the advisor wants to take a different action than the manager: the advisor may then have incentives to misreport his signal to tilt the manager towards his preferred action. This may happen for two reasons – heterogeneous preferences and heterogeneous beliefs. For example, suppose a firm is deciding on the scale of production in a new market. Given the same information, the manager may prefer a larger scale of production than a shareholder for two reasons: he may get private benefits from running bigger operations (different preferences) or he may have "more bullish" priors about the value of increasing the scale (different beliefs). Our model features heterogeneity in both preferences and beliefs. Both differences in preferences and differences in beliefs have a similar effect: the stronger they are, the stronger are the advisor's incentives to misrepresent his information, so the lower is the quality of advice. Nevertheless, we show that these communication frictions have drastically different implications for optimal advisory structures.

We start by analyzing the incentives of an informed advisor to truthfully reveal his information to the manager and show that they are strongly affected by whether other advisors reveal their information. However, the nature of these communication externalities depends on the source of communication frictions. If agents have the same preferences but differ in prior beliefs, communication externalities are positive: when more committee members reveal their information to the manager, others have stronger incentives to communicate truthfully as well. In contrast, if agents have different preferences, communication externalities are negative: as more agents share their information with the manager, other committee members' incentives to communicate truthfully decline. Therefore, the size of the advisory body is crucial for whether it is effective in its advisory role (in the sense of its members' providing truthful advice to the manager). Under heterogeneous beliefs, an advisory body can only be effective if its size is sufficiently large, while under heterogeneous preferences, it can only be effective if its size is sufficiently small.

The intuition for positive externalities is that heterogeneous priors become less relevant as the manager becomes more informed. Simply speaking, differences in priors generate disagreement only over the information that is unknown, so there is less disagreement when more information is learned. Thus, when more other agents share their information with the manager, the manager's and advisor's preferred actions become more congruent, which improves communication between them. The reason for negative externalities is that as the manager becomes more informed, he reacts less to the advisor's messages. Intuitively, when misrepresenting his signal, the advisor faces a trade-off: he wants to move the manager closer to his own preferred decision, but is afraid to make too big of an impact and move the manager's decision too much, away even from his own preferred decision. This concern encourages truthful communication when the manager strongly reacts to the agent's messages, but is not sufficient to constrain misreporting when the manager's reaction to the agent's advice is small, i.e., when he receives advice from many other agents.

This logic leads to the following results about the optimal composition of the advisory body.[3] First, if the cost of information acquisition is sufficiently low, the optimal advisory body is the set of all potentially informed agents under heterogeneous beliefs, but is a strict subset of potentially informed agents under heterogeneous preferences. Second, if the cost of acquiring information is substantial, committee size cannot be too large; otherwise, its members would lack incentives to become informed in the first place. In this case, the optimal advisory body is always a strict subset of potentially informed agents. Practically, including all informed agents in the advisory body can be interpreted as holding an advisory shareholder vote. Our analysis implies that such a vote will be efficient in its advisory role if shareholders and the manager have common interests but different beliefs, and acquiring information is not too costly.[4] On the other hand, including a strict subset of all informed agents in the advisory body can be interpreted as appointing an advisory board. Our results

[3] We assume that there is an infinitesimal cost of including each committee member, and so the optimal committee does not include agents who do not provide any advice.

[4] If there are informed non-shareholder stakeholders such as employees, an advisory shareholder vote alone is not optimal. However, it is still a necessary part of the optimal advisory process, together with seeking advice from other informed stakeholders.

suggest that in contrast to shareholder voting, such a board will be efficient in its advisory role for issues where conflicts of interests are particularly important, or for issues that require extensive analysis. In Section 5, we discuss the implications of these results for different types of corporate decisions.

We next extend the model to endogenize the firm's ownership structure. Intuitively, some potentially informed agents may choose not to become shareholders in the first place, if their prior beliefs or preferences are sufficiently different from those of the management. We show that if agents have the same preferences but differ in prior beliefs, endogeneity of ownership leads to multiple equilibria. First, there exists an efficient equilibrium, in which all agents become shareholders and communicate their information via an advisory vote, so the manager makes the efficient decision and the share price is high. In addition, there can exist an inefficient equilibrium, in which only a few informed agents become shareholders, so the manager makes a suboptimal decision and the share price is low. The source of equilibrium multiplicity are positive externalities in communication: if an investor expects the firm to have few shareholders and the manager's decision to be therefore based on little information, he expects to disagree with the manager's decision ex-post, and hence does not invest in the firm in the first place.

This result suggests two policy implications. First, it can be optimal to combine advisory shareholder voting with forming a diverse advisory board, i.e., a board whose members have strong ex-ante disagreements with management. Second, advisory voting can be more effective in the presence of passive (index-based) investors, whose stake in the firm does not depend on whether they agree or disagree with management. The advice received by management from both diverse advisory directors and informed passive investors can eliminate the inefficient equilibrium and thereby improve decision-making. In addition, our results have new implications for the empirical literature on shareholder voting and boards of directors, which we discuss in detail in Section 5.

The paper proceeds as follows. The remainder of this section reviews the related literature. Section 2 describes the setup. Section 3 shows how the externalities in communication depend on the nature of communication frictions and examines the optimal composition of the advisory committee. Section 4 endogenizes the firm's ownership structure and derives the implications for the advisory role of shareholder voting and the board. Section 5 discusses the empirical predictions, and Section 6 concludes. The appendix contains the proofs of the main results and analyzes an extension that studies the role of the manager's expertise.

## Related literature

Our paper contributes to the literature on the board's advisory role and the board's communication with the manager (Adams and Ferreira, 2007; Harris and Raviv, 2008; Baldenius et al., 2014; Chakraborty and Yilmaz, 2017; Levit, 2018). Differently from these papers, in which the board communicates as a single agent, we analyze communication from multiple informed heterogeneous agents and emphasize the externalities in information transmission.[5,6] Hence, the focus of our paper is on the effects of size and composition of the advisory body and in particular, the optimality of advisory voting vis-à-vis advisory board — a question that has not been studied in the literature before. In contrast, the above papers mostly study the role of the board's independence and the optimal allocation of authority. The exception is Harris and Raviv (2008), who also examine board size, but focus on how large board size impedes information acquisition by directors. While this effect is also present in our paper, our main focus is on the impact of board size on communication.[7] Our other contribution to this literature is to contrast the effects of conflicting preferences vs. beliefs.

Our focus on how shareholders' information is aggregated through the vote relates our paper to the literature on strategic voting (e.g., Feddersen and Pesendorfer, 1997; Maug, 1999; Maug and Yilmaz, 2002; and Persico, 2004; among others). These papers study how the aggregation of voters' information is influenced by factors such as the number of voters, conflicts of interests between them, and the voting rule. Unlike the majority of this literature, where voting is binding, shareholder voting in our paper is non-binding and plays a purely advisory role (as is the case for all shareholder proposals and say-on-pay), which leads to very different mechanisms from those in this literature. In this sense, the closest paper on strategic voting is Levit and Malenko (2011), who also analyze non-binding voting. However, our paper focuses on conflicting beliefs in addition to conflicting preferences, and also features a different economic mechanism leading to different results (see Section 5.2 for details).

More generally, our paper is related to the literature on cheap talk, which studies trans-

[5] Harris and Raviv (2008) consider multiple outside directors, but these directors are perfectly aligned and obtain perfectly correlated signals if they become informed, so they communicate as a single agent.

[6] Levit (2017) studies communication from the board to the shareholders in a tender offer context. Song and Thakor (2006) consider a setting where the manager controls the quality of information available to the board under career concerns. These papers also treat the board as a single entity.

[7] Warther (1998), Baranchuk and Dybvig (2009), Malenko (2014), Levit and Malenko (2016), Chemmanur and Fedaseyeu (2017), and Donaldson et al. (2018) study interactions between multiple directors within the board, but do not study the board's advisory role and do not feature the mechanisms that arise in our paper. See Adams et al. (2010) for a comprehensive survey of other papers in the board literature.

mission of non-verifiable information under conflicting preferences (Crawford and Sobel, 1982). The closest papers in this literature are those analyzing communication by multiple imperfectly informed senders (Austen-Smith, 1993; Battaglini, 2004; Morgan and Stocken, 2008; Galeotti et al., 2013). Our analysis of the case of conflicting preferences is related to Morgan and Stocken (2008) and their result that full information revelation is an equilibrium in a poll with a small sample, but not with a large one. We contribute to this literature in several ways. Most importantly, we show that the results under heterogeneous preferences (which the literature focuses on) are the opposite of those under heterogeneous beliefs. We also show how both communication frictions can be simultaneously captured in a simple tractable model with closed-form solutions. Finally, we highlight how in the corporate governance setting, ownership structure is endogenously determined by investors' preferences and beliefs, and how this, in turn, affects communication between shareholders and management.

Our paper also contributes to the literature on heterogeneous priors. Morris (1995) provides an overview of the heterogeneous prior assumption and discusses why it is consistent with rationality. Our model also features rational agents: although they have different priors, they are not dogmatic and rationally update their beliefs in a Bayesian way after receiving new information. Overall, there is growing empirical evidence suggesting that heterogeneous priors are important to explain corporate finance decisions and the dynamics of asset prices and volume.[8] Accordingly, there is a large theoretical literature studying the implications of heterogeneous priors.[9] The closest papers are Garlappi et al. (2017, 2019), who study group decision-making under heterogeneous beliefs but without private information and communication, and Che and Kartik (2009), Van den Steen (2010), and Alonso and Camara (2016), who study communication under heterogeneous beliefs but with only one sender and not via cheap talk, and thus do not feature the forces highlighted in our paper.

Finally, our analysis of trading relates our paper to the literature on feedback effects from market prices to the real economy, which studies how financial markets affects real decisions when decision-makers can learn from prices. This literature is comprehensively surveyed in Bond et al. (2012).[10] In both this literature and our paper, trading affects how much

[8]E.g., Kandel and Pearson (1995), Diether et al. (2002), Malmendier and Tate (2005), Dittmar and Thakor (2007), and Thakor and Whited (2011), among others.

[9]Examples in the finance literature include Harris and Raviv (1993), Kandel and Pearson (1995), Boot, Gopalan, and Thakor (2006), Banerjee, Kaniel, and Kremer (2009), and Banerjee and Kremer (2010), among many others.

[10]It includes Dow and Gorton (1997), who focus on traders' incentives to produce information relevant

information is incorporated into corporate decisions and thereby affects fundamental firm value. But differently from this literature, where the effect of trading works through prices, in our paper, trading affects information aggregation through its impact on the firm's ownership structure, which, in turn, affects decisions through shareholder voting (communication).

# 2 Setup

In this section, we present a simple model, which captures heterogeneous preferences, heterogeneous beliefs, and dispersed private information, and has tractable and intuitive solutions.

The firm needs to make a decision, denoted by $a$. The value of this decision depends on the unknown state of the world $Z$. There is a set of $N$ shareholders indexed by $i$, $i \in \{1, ..., N\}$, characterized by their preferences $b_i$ such that the payoff of shareholder $i$ from action $a$ given state $Z$ is

$$U_i(a, Z, b_i) = u_0 - (a - Z - b_i)^2, \tag{1}$$

where $u_0 > 0$ is a constant. One of these agents, indexed by $m \in \{1, ..., N\}$, is the manager, who decides on the action $a$. More generally, this could be any agent with decision-making authority in the organization.

The information structure is as follows. The state of the world is equal to the weighted sum of $N$ signals:

$$Z = \sum_{i=1}^{N} c_i \theta_i, \tag{2}$$

where $c_i > 0$ and $\theta_i$ are independent and identically distributed. Signals $\theta_i$ can be thought of as different factors relevant to the decision. Information about these relevant factors is dispersed among the firm's shareholders: shareholder $i \neq m$ can incur cost $\kappa \geq 0$ to privately observe signal $\theta_i$, and is uncertain about other signals. For simplicity, we assume that the manager is endowed with his signal.[11] An agent with a higher $c_i$ can be interpreted as being relatively more informed. Such additive information structure is common in the literature (e.g., Harris and Raviv, 2005 and 2008; Chakraborty and Yilmaz, 2017).

for decision-makers; Subrahmanyam and Titman (1999), who study how the feedback effect affects firms' choice between public and private financing; Bond et al. (2010), who highlight that prices can become less revealing when agents use them to take corrective actions; and many others.

[11] This allows us to analyze the optimal composition of the advisory committee by focusing on its members' information acquisition and communication decisions, while abstracting from the effect of the committee structure on the manager's private information.

For example, in the context of M&A decisions, $a$ could be the choice of how much to bid for a potential target, and signals $\theta_i$ could capture the synergies from the merger, the intrinsic value of the target, the number of potential competing bidders and their bids, the costs of integrating the two companies, and other relevant factors. Different shareholders of the firm have expertise about different aspects of the decision, with some being more important than others. We develop this M&A example further below, as we explain the intuition behind the results (see Section 3.1).

Note also that another interpretation of the $N$ informed players is that they include not only the firm's shareholders, but also its employees, customers, industry participants, and other stakeholders who care about the firm's decision and have relevant expertise. For this reason, we will often refer to these players as simply "agents."

The setup so far captures heterogeneous preferences and information dispersed among the agents. In addition, to capture the possibility that agents may have heterogeneous beliefs, we assume that agents have different priors about the distribution of signals $\theta_i$. Intuitively, some shareholders can be ex-ante more bullish about the prospects of the acquisition and the value of the target, while others can be more bearish. To capture this feature in a tractable way and derive closed form solutions, we make the following distributional assumption. We assume that $\theta_i$ is a binary signal equal to 1 with probability $\varphi$ and 0 with probability $1-\varphi$, and agents may potentially disagree about $\varphi$: agent $i$'s prior of $\varphi$ is characterized by the Beta distribution with parameters $(\rho_i, \tau - \rho_i)$.[12] The case of $\rho_i = \rho$ captures the case of common priors: for example, if $\rho_i = 1$ and $\tau = 2$, all agents believe that $\varphi$ is uniformly distributed on $[0,1]$. Parameter $\varphi$ captures the intrinsic value of the decision: when $\varphi$ is higher, the state is likely to be higher, so the optimal action is higher as well. Parameter $\rho_i$ captures how optimistic agent $i$ is about the state: those with a higher $\rho_i$ are ex-ante more optimistic than those with a lower $\rho_i$.[13] While agents may have different prior beliefs, they update their beliefs rationally (according to Bayes' rule) when they receive new information.

To summarize, each agent is characterized by his preference parameter $b_i$ (which reflects his ideal action if the state were known), belief $\rho_i$ (which captures whether he is ex-ante bullish or bearish about the state), and private signal about the state $\theta_i$ with relative im-

[12] That is, given the agent's belief $\rho_i$, the density of $\varphi$ is $f_i(\varphi) = \frac{\varphi^{\rho_i - 1}(1-\varphi)^{\tau-\rho_i-1}}{Beta(\rho_i, \tau-\rho_i)}$, where $Beta(\rho, \tau-\rho) = \frac{\Gamma(\rho)\Gamma(\tau-\rho)}{\Gamma(\tau)}$ and $\Gamma(\cdot)$ is the gamma function.

[13] Indeed, the expected value of a Beta distribution with parameters $(\rho_i, \tau-\rho_i)$ is $\frac{\rho_i}{\tau}$, which increases in $\rho_i$. Hence, given quadratic preferences, the optimal action of an agent with a higher $\rho_i$ is higher, as formally shown by (4) below.

portance $c_i$. Parameters $b_i$, $\rho_i$, and $c_i$ are publicly known.

We assume that parameters $b_i$ and $\rho_i$ satisfy $(b_i - b_m)(\rho_i - \rho_m) \geq 0$ for any $i$. This assumption is made so that agent $i$ can be interpreted as biased towards a higher or lower action relative to the manager, where this bias can come from a combination of two sources – different preferences over actions and different prior beliefs about the state.[14] This assumption is automatically satisfied if only one communication friction is present, i.e., if the agents either have common preferences or same priors.

The timeline is as follows. There is an advisory body (committee) $B$, which is a subset of all agents excluding the manager, $B \subseteq \{1, ..., N\} \backslash \{m\}$, and the manager $m$, $m \notin B$. First, all agents in the advisory body simultaneously decide whether to incur a private cost to acquire their private signals. For simplicity, we assume that agents' information acquisition decisions are observed, and this happens after the communication stage.[15] Next, all informed members of the advisory body simultaneously communicate their information to the manager (via cheap talk). Finally, the manager takes the action that maximizes his payoff.

We look for equilibria in pure strategies at the information acquisition and communication stages. Because signals are binary, it is without loss of generality to consider a binary message space at the communication stage: the communication strategy of agent $i$ is a mapping from his signal $\theta_i \in \{0, 1\}$ into a binary non-verifiable message $m_i \in \{0, 1\}$. Thus, in equilibrium, an agent who has information either communicates his information truthfully (i.e., $m_i(\theta_i) = \theta_i$ up to relabeling) or sends an uninformative (babbling) message (i.e., $m_i(0) = m_i(1)$).

If, for a given advisory body $B$, there exists an equilibrium in which all members of $B$ acquire information and truthfully communicate it to the manager, we assume that this equilibrium is played. Clearly, no agent would acquire information if he does not plan to communicate it, so the second part of this equilibrium selection is immaterial. The first part of the equilibrium selection is natural, since any alternative equilibrium would imply that

[14] Without this assumption, such an interpretation would not be possible because the two sources of the bias could offset each other, making the agent effectively unbiased relative to the manager.

[15] Without this assumption, an agent who deviates from his equilibrium strategy and does not invest in information, may want to mislead the manager and try to send a signal that he did not in fact acquire. Making the above assumption makes such deviations impossible and hence simplifies the analysis. In addition, assuming that information acquisition is observed after the communication stage rather than before simplifies the incentive compatibility constraint on information acquisition, because it implies that other agents do not change their behavior when one agent deviates to not acquiring information. However, most of the analysis remains unchanged if information acquisition decisions are unobserved: the only difference is an additional incentive compatibility constraint on information acquisition, which does not change the results qualitatively.

advisory body $B$ is equivalent to a smaller advisory body, which excludes members of $B$ that do not acquire information.

# 3 Analysis of the model

## 3.1 Externalities in communication

We start the analysis by considering the communication stage. Since we focus on pure strategy equilibria at the information acquisition stage, the set of agents at the communication stage consists of two subsets: those known to be informed and those known to be uninformed. The manager will ignore any messages sent by the latter group of agents, and hence we can focus on incentives of the former group of agents to communicate truthfully. Heterogeneity in preferences and beliefs introduces frictions into the communication process and gives the informed agents incentives to misrepresent their information when communicating it to the manager. As we show below, whether truthful communication is incentive compatible crucially depends on whether other members of the advisory body reveal their information to the manager and on the nature of communication frictions — whether they come from differences in preferences or beliefs.

We start by characterizing the action taken by the manager for a given outcome of the communication stage. Suppose that after communicating with the advisory body, the manager knows the subset $R \subseteq \{1, ..., N\}$ of signals ("revealed" signals) and does not know all the other signals, $-R \equiv \{1, ..., N\} \backslash R$. We use $R$ and $-R$ to denote signal indices and $\theta_R \equiv \{\theta_i, i \in R\}$ and $\theta_{-R} \equiv \{\theta_i, i \in -R\}$ to denote the corresponding subsets of signal realizations. The subset $\theta_R$ includes the manager's own signal $\theta_m$ and the signals of those members of the advisory body who become informed and communicate their information truthfully.

Given the quadratic payoff function, the optimal action of the manager is the sum of $b_m$ and his expectation of the state given the signals he learned ($\theta_i, i \in R$) and his prior $\rho_m$:

$$a_m(\theta_R) = b_m + \mathbb{E}_m \left( Z \,|\, \theta_i, i \in R \right) = b_m + \sum_{i \in R} c_i \theta_i + \sum_{j \in -R} c_j \; \mathbb{E}_m \left[ \theta_j | \theta_i, i \in R \right]. \tag{3}$$

The subscript $m$ in the expectation operator $\mathbb{E}_m$ highlights that the manager uses his own prior $\rho_m$ to update his beliefs about the unknown signals $\theta_j, j \in -R$, from his knowledge of signals $\theta_i, i \in R$. In the appendix, using the properties of the Beta distribution, we derive a simple expression for the manager's posterior beliefs and obtain the following result:

**Lemma 1 (Optimal action of the manager)** *Suppose that after the communication stage, the manager knows the subset $R$ of signals. Then his optimal action is*

$$a_m(\theta_R) = b_m + \sum_{i \in R} c_i \theta_i + \frac{\rho_m + \sum_{i \in R} \theta_i}{\tau + |R|} \sum_{j \in -R} c_j, \tag{4}$$

*where $|R|$ is the number of signals in $R$.*

The higher are the revealed signals $\theta_i, i \in R$, the higher is the manager's posterior belief about the state, and hence the higher is his optimal action (e.g., offer price for a target) given this information. Expression (4) also shows the effect of heterogeneous preferences and heterogeneous beliefs on the manager's action. A higher $b_m$ induces the manager to take a higher action given the same information. Likewise, a higher $\rho_m$, capturing more optimistic ex-ante beliefs, also induces the manager to take a higher action. Note, however, that unlike heterogeneity in preferences, whose effect does not depend on the manager's information, heterogeneity in beliefs becomes less important as the manager becomes more informed and updates his beliefs. In particular, as the set $R$ expands, the term $\frac{1}{\tau+|R|} \sum_{j \in -R} c_j$, and hence the effect of $\rho_m$ on the manager's action, decreases. In the extreme case, if $b_m = b_j$ and $R = \{1, ..., N\}$, the manager's optimal action coincides with the optimal action from the perspective of any other agent.

Using Lemma 1, we next examine when a given committee member will truthfully reveal his information to the manager. Consider any informed agent $i$ and suppose that the manager knows the subset $R \subset \{1, ..., N\}$ of signals, where $R$ includes the manager's own signal $\theta_m$ but not agent $i$'s signal $\theta_i$. The manager does not know all the other signals, i.e., agent $i$'s signal $\theta_i$ and all signals in the subset $-R\backslash \{i\}$, where as before, $-R \equiv \{1, ..., N\} \backslash R$. Suppose the manager believes the agent's message and uses it to update his belief about the state according to (4). If agent $i$ reveals his signal truthfully, (4) implies that the manager's action is

$$a_m(\theta_R, \theta_i) \equiv b_m + c_i \theta_i + \sum_{k \in R} c_k \theta_k + \frac{\rho_m + \theta_i + \sum_{k \in R} \theta_k}{\tau + 1 + |R|} \sum_{j \in -R \backslash \{i\}} c_j. \tag{5}$$

In contrast, if agent $i$ misreports and says that his signal is $1-\theta_i$, the manager's action is

$$a_m\left(\theta_R, 1-\theta_i\right) \equiv b_m + c_i\left(1-\theta_i\right) + \sum_{k\in R} c_k\theta_k \; + \frac{\rho_m + (1-\theta_i) + \sum_{k\in R}\theta_k}{\tau+1+|R|} \sum_{j\in -R\setminus\{i\}} c_j. \tag{6}$$

Because agent $i$ does not know the realization of other agents' signals when he communicates with the manager, he compares his expected payoff from actions $a_m\left(\theta_R,\theta_i\right)$ and $a_m\left(\theta_R,1-\theta_i\right)$ given his signal $\theta_i$ and his own prior beliefs about the distribution of those signals, and reports his signal truthfully if and only if:

$$\sum_{\theta_{-i}\in\{0,1\}^{N-1}} \left[\left(a_m\left(\theta_R,\theta_i\right)-Z-b_i\right)^2 - \left(a_m\left(\theta_R,1-\theta_i\right)-Z-b_i\right)^2\right] P_i(\theta_{-i}|\theta_i) \le 0, \tag{7}$$

where $\theta_{-i}$ is the set of all signals but $\theta_i$ and $P_i\left(\theta_{-i}|\theta_i\right)$ is agent $i$'s belief given his signal $\theta_i$ and his own prior.

The next proposition characterizes the necessary and sufficient conditions for (7) to be satisfied.

**Proposition 1 (IC constraint for truthful reporting)** *Suppose that the manager learns the subset $R$ of signals (which includes his own signal $\theta_m$ but not $\theta_i$) and does not know all the other signals, $-R$. Then agent $i$ reports his signal truthfully if and only if*

$$\left|(b_m-b_i) + \frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau+|R|+1}(\rho_m-\rho_i)\right| \le \frac{1}{2}\left[c_i + \frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau+|R|+1}\right]. \tag{8}$$

Intuitively, the left-hand side of (8) illustrates the agent's benefit from misreporting the signal, while the right-hand side illustrates his cost of misreporting. The logic is as follows. First, as (8) shows, communication between the agent and the manager can be inhibited by two key frictions — the difference in their preferences, captured by $b_m-b_i$, and the difference in their beliefs, captured by $\rho_m-\rho_i$. If both frictions are absent, $b_m=b_i$ and $\rho_m=\rho_i$, the agent always reveals his information truthfully because he knows that the manager will use this information in the way that is optimal for the agent. Whenever any of these frictions is present, the agent may have incentive to misreport. In particular, differences in preferences create incentives for misreporting as is standard in cheap talk games: the agent wants to tilt the manager's action in the direction of his preference, $b_i$. Similarly, if the agent and manager have different priors — for example, if the manager is ex-ante more optimistic about the state, the agent will want to correct this "bias in beliefs" by reporting a more negative signal. Overall, these results imply that, other things equal, an advisory committee will be less effective in its advisory role if the manager is sufficiently different from the committee members in either preferences of prior beliefs about the optimal decision.

Second, regardless of the source of communication frictions, agent $i$ is more likely to report his signal truthfully if his information is more important: the IC constraint (8) is relaxed when $c_i$ increases. Intuitively, the agent faces a trade-off: while he wants to tilt the manager in the direction of his optimal action (the benefit of misreporting), he is also afraid to tilt it too much, away even from the optimal action from the agent's perspective (the cost of misreporting). As the agent's information becomes more important and hence the manager is expected to react more strongly to the agent's message, this fear makes the agent more reluctant to misreport.

Despite the above similarities between the two sources of communication frictions, we next show that they have very different implications for the optimal advisory structures. Our first observation, which is the key implication of (8), is that the agent's incentive to report his information truthfully depends on whether other members of the advisory body reveal their information to the manager (i.e., on $R$), but whether this dependence is positive or negative crucially depends on whether communication is hampered by heterogeneity in preferences or heterogeneity in beliefs. In particular, how much information the manager obtains from other committee members is captured by the term $\frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau+|R|+1}$ on both sides of (8): this term decreases as the set $R$ expands, i.e., as more committee members reveal their information to the manager. As (8) shows, increasing this term relaxes the agent's IC constraint when communication is inhibited by heterogeneity in preferences ($\rho_i = \rho_m$), but tightens the agent's IC constraint when communication is inhibited by heterogeneity in beliefs ($b_i = b_m$).

To explain the intuition and the implications more clearly, we rewrite (8) for each of these two cases separately and discuss the intuition behind each.

**1. Heterogeneity in beliefs**: $\rho_i \neq \rho_m, b_i = b_m$

In this case, agent $i$ reports his signal truthfully if and only if[16]

$$|\rho_m - \rho_i| \leq \frac{1}{2}\left[1 + c_i \frac{\tau + |R| + 1}{\sum_{j \in -R \backslash \{i\}} c_j}\right]. \tag{9}$$

Hence, the more information the manager gets from other members of the advisory body (i.e., the higher is $|R|$ and the lower is $\sum_{j \in -R \backslash \{i\}} c_j$), the more likely it is that agent $i$ will also truthfully communicate his information to him. We refer to this effect as the *positive externality effect* of information transmission, because more information revealed to the manager by some agents has a positive effect on further information aggregation by encouraging other agents to report their information truthfully. The reason is that as the manager learns more information from others, he becomes more congruent with the agent, i.e., the manager's optimal action becomes closer to agent $i$'s preferred action. This happens for two related and complementary reasons. First, heterogeneous prior beliefs generate disagreement only over the information that is still unknown — once a certain piece of information gets revealed, there is nothing to disagree about. Hence, the more information gets known to the manager, the more agreement there is between the manager and the agent about the optimal decision. The second effect, which is related and acts in the same direction, is that once a signal about the state is revealed, agents update their posteriors about the distribution of the state based on the realization of the signal. Hence, even if their prior beliefs were initially very different, they become closer to each other following the revelation of new information.

Together, these two effects imply that the manager's and agent's optimal actions become more congruent as the manager learns more information from his other advisors, increasing the agent's incentives for truthful communication. To see this most starkly, consider the extreme case where the manager knows all the signals except agent $i$'s signal: $R = \{1, ..., N\} \backslash \{i\}$ and so $-R \backslash \{i\}$ is an empty set. In this case, truthfully reporting the last remaining signal $\theta_i$ results in the manager taking the action that is optimal from the agent's perspective, and hence is always incentive compatible.

To illustrate this intuition in the context of our M&A example, suppose that the acquisition target is a pharmaceutical company with a pipeline of drugs it is developing. The optimal offer price then crucially depends on the success probability of each drug, but given

[16] Here if $-R \backslash \{i\}$ is an empty set, the right-hand side of (9) is equal to infinity, i.e., (9) is always satisfied.

that such drugs have not been developed before, different members of the advisory body may have very different priors about the likelihood of success of each drug. If they think the manager is too optimistic about the likelihood of success, they may try to counteract this optimism by more negative (non-truthful) reports about the aspects of the merger they are knowledgeable about. If, however, the advisory body includes an expert who knows the state of research on a particular disease, he can inform the manager about whether a given drug will be a failure or a success. Once this information is revealed to the manager, the uncertainty in target value coming from the success of this drug disappears, increasing the manager's congruence with other advisors (the first effect described above). Moreover, other advisors also realize that following the expert's report, the manager will adjust his prior belief about the likelihood of success of *other* drugs towards its correct value. This makes the manager even more congruent with other advisors and further decreases their incentives to misreport (the second effect described above).

**2. Heterogeneity in preferences**: $\rho_i = \rho_m, b_i \neq b_m$

In this case, agent $i$ reports his signal truthfully if and only if

$$|b_m - b_i| \leq \frac{1}{2}\left[c_i + \frac{\sum_{j\in -R\setminus\{i\}} c_j}{\tau + |R| + 1}\right]. \tag{10}$$

In contrast to the case of belief heterogeneity, the more information the manager gets from other members of the advisory body (i.e., the higher is $|R|$ and the lower is $\sum_{j\in -R\setminus\{i\}} c_j$), the less likely it is that agent $i$ will truthfully communicate his information to him. We refer to this effect as the *negative externality effect* of information transmission, because more information obtained by the manager from some agents discourages further information aggregation by harming the credibility of other agents. Intuitively, as the manager learns more information from others, the effect of the agent's message on his actions decreases. Hence, the agent is less worried that misreporting the signal may tilt the manager's action too far away from his own optimal action and has stronger incentives to misreport (this intuition is similar to the intuition above of why a higher $c_i$ relaxes the agent's IC constraint).[17]

[17] This effect is also present in the case of heterogeneous beliefs, but as we show, it is dominated by the positive externalities effects.

The externalities in information transmission imply that the size of the advisory body affects whether its members truthfully reveal their information, but that the effect of size depends on whether communication is hampered by differences in preferences or differences in beliefs. The following result formalizes this intuition.

**Proposition 2 (Committee size and communication)** *Consider a committee $B$ of informed agents. If agents have heterogeneous beliefs ($b_i = b_m$, $\rho_i \neq \rho_m$), there is a cutoff on committee size $N_{\min}$, such that an equilibrium where all committee members truthfully communicate to the manager does not exist if $|B| < N_{\min}$. If agents have heterogeneous preferences ($b_i \neq b_m$, $\rho_i = \rho_m$), there is a cutoff on committee size $N_{\max}$, such that an equilibrium where all committee members truthfully communicate to the manager does not exist if $|B| > N_{\max}$.*

Proposition 2 emphasizes that whether the manager should seek advice from a large vs. a small group of people crucially depends on the source of communication frictions. Intuitively, when communication is hampered by heterogeneous beliefs, the positive externalities in communication are not strong enough to overcome advisers' incentives to misreport unless the advisory body is large enough. This leads to the first statement of the proposition. The opposite result obtains when communication is hampered by heterogeneous preferences: due to negative externalities in communication, truthful information revelation by all committee members is only possible if the committee is small enough. This second statement is similar to the result of Morgan and Stocken (2008) about information revelation in polls when constituents have heterogeneous preferences.

## 3.2 Optimal advisory committee

We now use the results in the previous section to analyze the optimal composition of the advisory committee. In order to define the optimal committee, it is useful to derive each agent's ex-ante expected utility as a function of a given committee. The next result shows that each agent is ex-ante better off if the manager is ex-post more informed:

**Lemma 2 (Ex-ante payoffs)** *Suppose that in equilibrium, the manager learns subset $R$ of the signals and does not learn all the other signals, $-R \equiv \{1, ..., N\} \backslash R$. Then the ex-ante* *equilibrium payoff of agent $i$ (excluding his information acquisition costs) is given by*

$$\mathbb{E}_i[U_i|R] = u_0 - (b_m - b_i)^2 - \mathsf{A}_{im}(R) - \mathsf{B}_i(R) - \mathsf{C}_{im}(R), \tag{11}$$

*where*

$$\begin{aligned} \mathsf{A}_{im}(R) &= \tfrac{2(b_m - b_i)(\rho_m - \rho_i)}{\tau + |R|} \textstyle\sum_{j \in -R} c_j, \\ \mathsf{B}_i(R) &= \tfrac{\rho_i(\tau - \rho_i)}{\tau(\tau+1)} \left( \textstyle\sum_{j \in -R} c_j^2 + \frac{\left[\sum_{j \in -R} c_j\right]^2}{\tau + |R|} \right), \\ \mathsf{C}_{im}(R) &= \left[ \tfrac{\rho_m - \rho_i}{\tau + |R|} \textstyle\sum_{j \in -R} c_j \right]^2. \end{aligned} \tag{12}$$

*In particular, $\mathbb{E}_i[U_i|R]$ is increasing in $|R|$ and decreasing in any $c_k,\ k \in -R$.*

Intuitively, agent $i$'s utility from the manager's action is determined by how much information the manager's action reflects and by how different the manager's action is from the agent's optimal action given this information (due to their different preferences and beliefs). Term $\mathsf{B}_i(\cdot)$ reflects the former — the loss in the agent's expected utility due to residual variance in the state, i.e., the fact that the manager's action does not reflect signals in $-R$. All the other terms capture the latter: Term $(b_m - b_i)^2$ is the loss in agent $i$'s utility due to the fact that the manager's action reflects his rather than agent $i$'s preference, while terms $\mathsf{A}_{im}(\cdot)$ and $\mathsf{C}_{im}(\cdot)$ reflect the additional effects due to the ex-post belief divergence between agent $i$ and the manager. Lemma 2 shows that for any agent in the economy, his ex-ante utility before acquiring and learning his private signal is higher when more information is known to the manager ex-post, i.e., when the set $R$ is larger and $c_j, j \in -R$, is lower. For example, when all agents' information has the same importance, i.e., $c_i = c$ for all $i$, then for any $\rho_i$ and $b_i$, each agent simply wants to maximize $|R|$, the number of signals that the manager learns from the advisory body.

We assume that the objective function in choosing the optimal committee is to maximize the sum of all $N$ agents' expected utilities (11), net of their information acquisition costs. If $\kappa = 0$, then our results generalize to any objective function that weakly increases in the expected utility of each agent and strictly increases in the expected utility of at least one agent.[18] We also make an assumption (which is similar to that in Persico, 2004) that there is an infinitesimally small positive cost of including each agent in the advisory body. This assumption ensures that it is optimal to search for the optimal advisory committee among the

[18] This is because if $\kappa = 0$, the optimal committee is the same from any agent $i$'s point of view, as Lemma 2 and the proof of Proposition 3 imply.

set of committees in which all members acquire information and truthfully communicate it to the manager. Thus, committee $B$ is *optimal* if it maximizes the sum of expected utilities of all $N$ agents, net of information acquisition costs of committee members, subject to information acquisition and truthful communication being optimal for all committee members.

We start with the case in which the cost $\kappa$ of acquiring information is sufficiently small, so that the information acquisition friction is not relevant, i.e., committee members find it optimal to acquire information if they truthfully communicate it. This allows us to focus on the results that rely solely on communication frictions (Proposition 3) and demonstrate the intuition behind them in the simplest way. In Proposition 4, we consider general information acquisition costs $\kappa$ and study how communication frictions interact with the information acquisition friction. In both cases, our results highlight that the composition of the optimal committee crucially depends on the nature of communication frictions – conflicting beliefs vs. conflicting preferences.

**Proposition 3 (Optimal advisory committee)** *Suppose $\kappa \leq \kappa_l$, where $\kappa_l > 0$ is defined in the proof.*

1. *If agents have heterogeneous beliefs ($b_i = b_j$, $\rho_i \neq \rho_j$), the optimal advisory body is the entire set of agents: $B^* = \{1, ..., N\} \backslash \{m\}$.*

2. *If agents have heterogeneous preferences ($b_i \neq b_j$, $\rho_i = \rho_j$) and for at least one agent the preference misalignment with the manager is sufficiently large, $|b_m - b_i| > \frac{1}{2} c_i$, the optimal advisory body is a strict subset of all agents: $B^* \subset \{1, ..., N\} \backslash \{m\}$.*

Intuitively, if communication is hampered by heterogeneous beliefs, the positive externality effect implies that the more agents are included in the advisory body and reveal their information to the manager, the more likely it is that other agents will truthfully report their information to the manager as well. In fact, if the advisory body is the set of all informed agents, there is an equilibrium in which each agent reports his information truthfully. Indeed, if the manager learns all the $N$ signals, there is no ex-post disagreement between him and other agents, and this, in turn, implies that truthful communication by all agents is indeed optimal. Because this maximizes the amount of information available to the manager, this advisory body is optimal, leading to the first statement of the proposition. In contrast, if communication is hampered by heterogeneous preferences, the negative externality effect and Proposition 2 in particular, imply that if the advisory body is too large (e.g., if it consists of all informed agents) and the misalignment in preferences is sufficiently large, there is no equilibrium in which all members report their information truthfully. Thus, any committee in which all its members communicate truthfully must be a strict subset of all informed agents. As long as there is an infinitesimal cost of including agents who do not contribute any information, the optimal advisory body will be a strict subset of all agents.

Our setup assumes that if all agents in the economy combine their information together, the state is known with certainty. In practice, there could be some residual information that none of the agents knows. The model can be easily extended to capture this feature, and Proposition 3 will continue to hold as long as the amount of this residual unknown information is not too large. If it is very large and an agent's prior beliefs are very different from the manager's, then even if all other agents reveal their signals to the manager, it may not be sufficient to align their very different priors, giving the agent incentives to misreport. Importantly, the positive and negative externalities channels continue to hold regardless of the amount of residual unknown information.

We next consider the general case, allowing for any possible information acquisition cost $\kappa$. In order to derive concrete implications about the optimal advisory committee, we make the simplifying assumption that all signals are equally informative, $c_i = c$, for the rest of the paper.[19]

**Assumption 1** *All signals are equally informative: $c_i = c$ for all $i$.*

If the cost of information $\kappa$ is sufficiently large, the requirement that all committee members invest in information acquisition imposes an additional restriction on the size of the advisory body: regardless of whether agents differ in their preferences or beliefs, the advisory body cannot be too large. Specifically:

**Lemma 3** *Consider any committee $B$ for which truth-telling conditions (8) are satisfied. Then, all members of committee $B$ find it optimal to acquire information if and only if*

[19] The assumption $c_i = c$ implies that, as long as the incentive compatibility constraints on information acquisition and communication are satisfied, committee size is a sufficient statistic for value $U_i$ of each agent $i$. Indeed, according to Lemma 2, if $c_i = c$, each agent simply wants to maximize the number of signals the manager learns from the committee.

*$|B| \leq N_B(\kappa)$, where $N_B(\kappa)$ decreases in $\kappa$. In particular, $N_B(\kappa) < N-1$ for any $\kappa > \kappa_l$. In the special case of $\rho_i = \rho\ \forall i$, $N_B(\kappa) = N(\kappa)$, i.e., it is independent of $B$.*

Intuitively, the larger is the size of the committee, and hence the larger is the aggregate information that the committee members possess, the lower is the marginal value of any additional signal. Therefore, if the committee is large enough, $N-1 > N_B(\kappa)$, it is not optimal for all of its members to acquire information. This implies, in particular, that when agents have heterogeneous beliefs but information acquisition costs are non-trivial, then, differently from the result in Proposition 3, the optimal advisory body is no longer the set of all agents. Otherwise, some agents would end up not acquiring information in the first place, and hence it would be better to exclude them, given the infinitesimally small cost of including each agent in the advisory body.

Lemma 3 thus imposes an upper bound on the size of the optimal committee. In addition, the requirement that all committee members truthfully communicate their information imposes additional restrictions on the committee size (Proposition 2). Note also that because information acquisition has positive externalities on other agents, there is never overinvestment in it. Therefore, the optimal committee is simply the committee of the maximum size that features information acquisition and truthful communication by all its members. The next proposition characterizes the solution:

**Proposition 4**
*1. If agents have heterogeneous beliefs ($b_i = b_j$, $\rho_i \neq \rho_j$), the optimal advisory body includes all agents with sufficiently low $|\rho_i - \rho_m|$, and its size $N^*$ is:*

- *if $\kappa \leq \kappa_l$, $N^* = N-1$;*
- *if $\kappa \in (\kappa_l, \kappa_h)$, $N^* \in [N_{\min}, N-1)$ and decreases in $\kappa$, where $N_{\min}$ is defined in Proposition 2;*
- *if $\kappa \geq \kappa_h$, $N^* = 0$.*

*2. If agents have heterogeneous preferences ($b_i \neq b_j$, $\rho_i = \rho_j$), the optimal advisory body includes all agents with sufficiently low $|b_i - b_m|$. Its size is $N^* = \min(N(\kappa), N_{\max})$ and decreases in $\kappa$, where $N(\kappa)$ and $N_{\max}$ are defined in Lemma 3 and Proposition 2, respectively.*

In both cases, as information acquisition costs $\kappa$ increase, the size of the optimal committee decreases, in order to maintain its members' incentives to acquire information. When agents have heterogeneous preferences, this smaller committee size improves their communication with the manager due to negative communication externalities, and hence the optimal committee size gradually declines with $\kappa$ until it reaches zero (Figure 1, panel (b)). In contrast, when agents have heterogeneous beliefs, positive communication externalities imply that as the committee becomes smaller, each agent's incentives to misreport his information increase. At some point, when $\kappa = \kappa_h$, communication breaks down completely, and hence, interestingly, *any* advisory committee becomes completely ineffective: the information acquisition and truthful communication constraints (which are both necessary for the committee to be effective) are no longer compatible. As a result, when $\kappa = \kappa_h$, the optimal committee size jumps to zero (Figure 1, panel (a)).

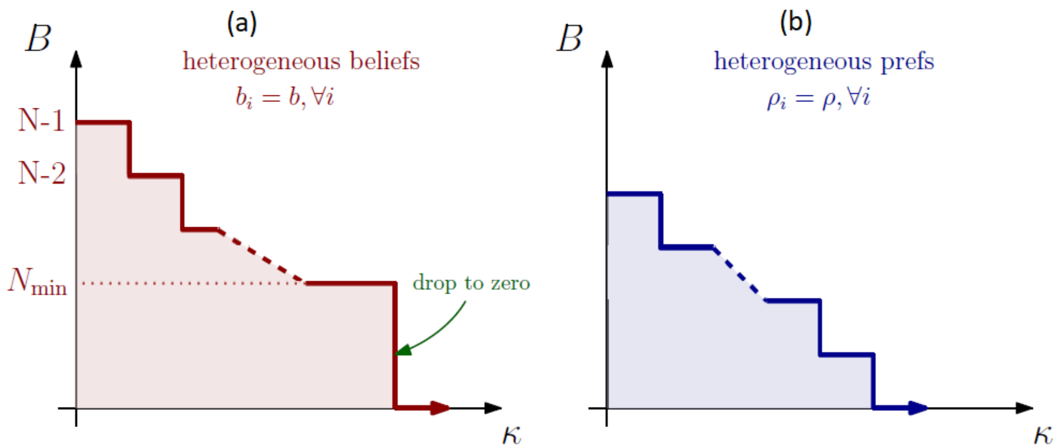


**Figure 1. Optimal advisory committee size.**

# 4 Shareholder voting and the role of the board

In this section, we apply the above results and intuition to analyze the advising effectiveness of shareholder voting and the board of directors, discuss when one is more effective than the other, and when the two should be used in combination. We start by considering the direct implications of the case with exogenous ownership structure, analyzed in the previous section. After that, we endogenize the firm's ownership structure, which allows us to study how market trading affects real efficiency through its effects on shareholder base, and, in turn, on shareholder communication with management.

Propositions 3 and 4 have direct implications for the use of advisory shareholder voting vs. advisory boards. Under advisory voting, all of the firm's shareholders are asked to convey their views to the management via the vote. The resulting decision-making protocol resembles our model with $B = \{1, ..., N\} \backslash \{m\}$. One interpretation of Proposition 3 is that as long as the costs of information acquisition are not too large, such an advisory vote is the optimal decision-making protocol if agents have aligned preferences, but disagree due to differences in beliefs. Under the more general interpretation that the set of $N$ players includes not only the firm's shareholders, but also other informed stakeholders who care about the firm's decisions, Proposition 3 implies that holding an advisory shareholder vote is still a necessary part of the optimal advisory process, but that it can be useful to combine such an advisory vote with the opinion of other informed stakeholders – e.g., by also conducting employee and customer surveys.

In contrast, when decisions are made with the help of a board, the management does not ask all shareholders for their opinions and gets feedback only from the board members, i.e., from a relatively small subset of the firm's shareholders, or stakeholders more generally. Propositions 3 and 4 suggest that this is always optimal if agents have heterogeneous preferences but common beliefs, regardless of their costs of information acquisition: negative externalities in communication limit the size of the optimal advisory committee. In addition, Lemma 3 and Proposition 4 imply that when information is sufficiently costly, then even under heterogeneous beliefs, restricting the size of the advisory body may be optimal to incentivize information acquisition in the first place, and this can make advisory voting no longer optimal. In Section 5, we discuss the implications of these results for different types of corporate decisions.

## 4.1 Endogenous ownership structure

So far, we have assumed that the firm's ownership structure consists of all $N$ agents, and thus all $N$ agents care about the firm's decisions. In reality, potentially informed agents can choose not to become shareholders. In this case, they generally do not care about the firm's decisions and hence have no incentives to communicate their information, as well as to acquire it in the first place. In addition, even if such agents had relevant information and wanted to share it with the manager, they would generally have little opportunity to do so.

In this section, we extend the model to capture this feature and endogenize the decisions of the $N$ potentially informed agents to become shareholders. In particular, we analyze each agent's decision whether and how many shares of the firm to hold, and study the implications of this endogenous ownership structure for the effectiveness of advisory shareholder voting. If agent $i \in \{1, ..., N\}$ does not become a shareholder, he does not acquire and/or communicate his potential information $\theta_i$ to the manager.[20]

Specifically, we extend the setup in Section 2 by adding a trading stage: At the beginning of the game, all $N$ agents participate in the market for the firm's shares, during which the total stock of the firm is sold in a competitive market. After that, the game proceeds as in the basic model (for simplicity, we assume that $\kappa$ is zero, so that information acquisition is not a relevant friction).

Suppose that the stock is in unit supply, so that holding $\alpha_i$ shares is equivalent to holding fraction $\alpha_i$ of the firm. We adopt the specification of Vives (1993) and Manzano and Vives (2019) and assume that agent $i$'s utility from holding stake $\alpha_i$ is given by

$$\alpha_i \left(\mathbb{E}_i[U_i] - p\right) - \frac{\lambda}{2}\alpha_i^2, \tag{13}$$

where $U_i$ is the director's utility from each share and is given by (1), $p$ is the share price, and $\lambda > 0$ captures the holding cost, which can be viewed as a proxy for limited diversification and risk aversion. For example, $\lambda$ is likely to increase with firm size and volatility because holding a given fraction of the firm is costlier when the firm is larger and more risky. We also assume that constant $u_0$ in the payoff specification $U_i$ is sufficiently high, so that the equilibrium share price is positive.[21]

The firm's shareholder base, $S \subseteq \{1, ..., N\}$, is thus endogenously determined and consists of all agents who hold a positive number of shares after the trading stage: $S = \{i : \alpha_i > 0\}$. The firm forms an optimal advisory committee $B$ out of its shareholders, $B \subseteq S$, to maximize shareholder value.[22] For example, $B = S$ corresponds to an advisory vote of all shareholders, and $B \subset S$ corresponds to a board composed of selected shareholder

[20] This essentially implies that the firm's stakeholders who do not hold shares in the firm have little means to communicate with the firm's management or do not have strong enough incentives to become informed in the first place. This assumption is made for simplicity, but the model could be easily extended to allow stakeholders who do not hold shares to also communicate with management.

[21] The only role of this assumption is to make the interpretation more intuitive, but none of the results depends on $u_0$.

[22] Since $c_i = c_j$ and information acquisition costs are zero, maximizing the utility of each shareholder is equivalent to forming a committee of the maximum size that features truthful communication by all its members.

representatives. After that, the game proceeds as in the basic model. First, each shareholder receives his private signal $\theta_i$ about the state. Next, all members of the advisory body $B$ simultaneously communicate their information to the manager. Finally, the manager chooses the action, and the payoffs are realized.

This setup assumes that agents' trading is based on their preferences and prior beliefs regarding the firm's decision, but that agents do not trade again ex-post, after learning their private signals. There are two arguments for this simplifying assumption. One is tractability: the model in which agents both trade on private information and then decide whether to reveal it to the manager is very difficult to analyze. The second argument is more fundamental. As discussed in the literature review, prior research has extensively studied how market trading incorporates agents' private information into real decisions through its impact on *prices*. Our unique contribution is to examine how trading incorporates agents' information into real decisions through a different channel, *communication*: trading determines the firm's shareholder base and thus, determines which agents communicate their information to the manager via voting or being on the board. Assuming that agents to not trade based on private information allows us to abstract from the price channel and focus on the more novel communication channel.

Finally, note that another interpretation of this setup is that $b_i$ and $\rho_i$ capture agents' general preferences and beliefs regarding the firm (e.g., how congruent they are with the overall strategic direction the firm is pursuing), and are not decision-specific. In this interpretation, the firm's shareholder base $S$ captures the firm's long-term shareholders, and hence it is reasonable to assume that such long-term shareholders' ownership stakes are not affected by more transitory, decision-specific private information.

**Analysis.** We solve the model by backward induction. The analysis at the communication stage remains unchanged, and hence we only need to consider the trading stage. Using (13), the optimal ownership stake of agent $i$ is

$$\alpha_i(p) = \max\left\{\frac{\mathbb{E}_i[U_i] - p}{\lambda}, 0\right\}. \tag{14}$$

As expected, a larger holding cost $\lambda$ decreases the agent's demand for shares, while higher expected utility $\mathbb{E}_i[U_i]$ from holding each share increases his demand. In particular, as Lemma 2 and expressions (11)-(12) for $\mathbb{E}_i[U_i]$ demonstrate, the shareholder's expected utility is higher when the manager's action reflects more of other agents' information. This implies that an agent's optimal stake in the firm is affected by the firm' overall ownership structure, which will be important for the results that follow.

Market clearing implies $1 = \sum_{i=1}^{N} \alpha_i(p) = \sum_{i \in S} \frac{\mathbb{E}_i[U_i]-p}{\lambda}$, and hence

$$p^* = \frac{1}{|S|}\left(\sum_{i \in S} \mathbb{E}_i[U_i] - \lambda\right). \tag{15}$$

Similarly to the basic model, the implications under endogenous ownership structure are, as we show next, very different depending on whether agents differ in their preferences or beliefs. We consider each of these cases separately.

#### 4.1.1 Heterogeneity in beliefs: $\rho_i \neq \rho_m$, $b_i = b_m$

Our first observation is that there always exists an equilibrium in which all informed agents become shareholders and communicate their information truthfully. Indeed, according to the proof of Proposition 3, if the shareholder base is $S = \{1, ..., N\}$, and hence the advisory vote includes all informed agents, there exists an equilibrium in which all shareholders report their information truthfully. Intuitively, if the manager learns all the $N$ signals, there is no ex-post disagreement between him and any of the shareholders, which makes truthful communication by all shareholders indeed optimal. Moreover, because of no ex-post disagreement between agents, they all value the firm in the same way and thus acquire the same stakes, $\alpha_i = \frac{1}{N}$, despite their ex-ante differences in beliefs. This equilibrium is efficient in that it maximizes total welfare, defined as the sum of the seller's proceeds and payoffs of all shareholders. Intuitively, it is efficient both because the firm's decision uses the maximum available information, and because agents' total holdings costs are minimized since the asset is evenly divided among them. However, as we show next, there can also exist an inefficient equilibrium, in which only a subset of the agents become shareholders and hence, the manager's decision is not based on all the available information and total holding costs are larger. The share price in this inefficient equilibrium is lower than in the efficient equilibrium. Specifically:

**Proposition 5** *Suppose agents have heterogeneous beliefs ($b_i = b_m = b$, $\rho_i \neq \rho_m$). There always exists an efficient equilibrium in which all $N$ agents become shareholders, acquire equal shares $\alpha_i = \frac{1}{N}$ of the firm, the optimal committee is the set of all agents other than the* *manager, and the efficient action $a = b + Z$ is undertaken. However, there can also exist an inefficient equilibrium, in which a strict subset of agents become shareholders, acquire unequal shares of the firm, and an inefficient action $a \neq b + Z$ is undertaken.*

The source of equilibrium multiplicity are positive externalities in communication. Intuitively, if only a subset of agents become shareholders and communicate their information to the manager, there is still ex-post disagreement between the manager and the shareholders who have different prior beliefs. Realizing this at the trading stage, shareholders who have substantially different beliefs from those of the manager (i.e., have large enough $|\rho_i - \rho_m|$), do not buy shares in the first place. The set of shareholders thus only includes a relatively narrow set of investors whose prior beliefs are sufficiently close to those of the manager, consistent with the initial conjecture.

This analysis demonstrates that the effectiveness of advisory voting crucially depends on the firm's ownership structure. In the basic model, advisory voting always resulted in efficient decision-making (Proposition 3) because we assumed that the shareholder base included all $N$ agents. In contrast, when the shareholder base is endogenously determined, it may no longer be optimal to rely on advisory voting alone: this may lead to an inefficient equilibrium, in which only investors who have very similar prior views to those of the management end up becoming shareholders. What are possible ways for a firm to avoid this inefficiency?

**Implication 1: Advisory voting combined with a diverse board.** One interesting implication of the above results is that it can be optimal to *combine* advisory shareholder voting with a board consisting of relatively diverse directors. To see this, recall that the set of shareholders in the inefficient equilibrium of Proposition 5 consists of all agents whose priors are close enough to those of the management (small $|\rho_i - \rho_m|$). One way to avoid this inefficient equilibrium is to not only conduct the advisory vote, but also to form an advisory board by inviting some "diverse" agents, i.e., some of the agents whose ex-ante views are different from those of the management (large $|\rho_i - \rho_m|$). Of course, if these advisory directors do not hold any stake in the firm, they will not have any incentives to communicate their information and acquire it in the first place, so the firm needs to provide incentives to such advisory directors — e.g., by compensating them with shares of the firm. Such an advisory board may be optimal for the firm despite the costs of compensating its members for advice: the information provided to the manager by the advisory directors reduces ex-post disagreements among all potential shareholders, so an inefficient equilibrium no longer exists.

**Implication 2: The role of index funds.** There is an ongoing active debate about the role of passive (index-based) asset managers in corporate governance. Some market participants have expressed concerns that the presence of passive investors weakens governance and even proposed that they should be restricted from voting, noting that they may lack the adequate incentives to become informed. Others argue that index funds actively engage with their portfolio companies, even more so than investors who can easily exit.[23] An implication of our analysis is that the presence of index funds can make advisory shareholder voting more effective. Indeed, suppose investors expect that a large fraction of the firm's shareholders will consist of index funds, whose stake in the firm will not depend on whether their fund managers agree or disagree with the firm's CEO. In this case, the inefficient equilibrium may again cease to exist due to positive externalities in communication, as long as these passive investors are sufficiently informed and have diverse prior beliefs. This logic highlights a positive effect of passive investors on the voting of *other* shareholders, with a caveat that it relies on passive investors investing in private information.

### 4.1.2 Heterogeneity in preferences: $\rho_i = \rho_m, b_i \neq b_m$

Recall that when agents have heterogeneous preferences, communication externalities are negative. Since positive externalities were the key reason for equilibrium multiplicity under heterogeneous beliefs, it is intuitive to expect that equilibrium under heterogeneous preferences is unique. This is indeed the case, as shown in Proposition 6. In this equilibrium, unlike in the efficient equilibrium under heterogeneous beliefs, the shareholder base is generally restricted and consists of agents whose preferences are sufficiently aligned with those of the manager (small $|b_m - b_i|$). This is because under heterogeneous preferences, there are always ex-post disagreements between the manager and shareholders about the optimal course of action, regardless of the amount of information conveyed.

[23] See "Vanguard, Trian and the problem with 'passive' index funds", *Forbes*, HBS Working Knowledge (Feb 15, 2017) and "The case against passive shareholder voting," *CLS Blue Sky Blog* (August 2, 2017) for negative views on the governance role of passive investors; and "Passive investment, active ownership," *Financial Times* (April 6, 2014) for a positive view.

**Proposition 6** *Suppose agents have heterogeneous preferences ($b_i \neq b_m$, $\rho_i = \rho_m = \rho$). In the unique equilibrium, agent $i$ becomes a shareholder if and only if $|b_m - b_i|$ is sufficiently low. The equilibrium number of shareholders is increasing in $\lambda$. There exists cutoff $\lambda^* \in (0, \infty)$, such that:*

- *if $\lambda \leq \lambda^*$, then $B = S \backslash \{m\}$, i.e., the optimal committee includes all non-manager shareholders;*
- *if $\lambda > \lambda^*$, then $B \subset S \backslash \{m\}$, i.e., the optimal committee is a strict subset of non-manager shareholders.*

As Proposition 6 shows, the key factor that determines the advising effectiveness of shareholder votes and the optimal advisory committee is the holding cost $\lambda$. Intuitively, the holding cost affects how concentrated vs. dispersed the firm's ownership structure is, and hence how close shareholders' preferences are to those of the management. If holding costs are small, the equilibrium features concentrated ownership: the firm's shareholders hold relatively large stakes in the firm and have similar preferences to those of the manager. Under such concentrated ownership, shareholder voting is effective in its advisory role in the sense that in equilibrium, all shareholders truthfully convey their information to the manager. Hence, the optimal advisory committee includes all of the firm's shareholders. Note that in this case, the firm can improve efficiency even further, if it both conducts the advisory shareholder vote and, in addition, forms an advisory board consisting of some non-shareholders and gives them incentives to become informed (similarly to Implication 1 in Section 4.1.1). Such an arrangement is consistent with startup firms (which tend to have a concentrated shareholder base) frequently having purely advisory boards with outside directors.

In contrast, if holding costs are large, concentrated ownership becomes too expensive, so the firm's shareholder base is relatively diverse and includes many shareholders whose preferences are misaligned with those of the manager. In this case, advisory voting is no longer effective: there is no equilibrium in which all of the shareholders truthfully convey their views to the manager. Hence, advice is optimally provided by a board consisting of a subset of the firm's shareholders, rather than through an advisory vote.

# 5 Implications

There is a large empirical literature that studies the advisory role of the board and advisory shareholder voting. Our model offers new empirical predictions about the advising effectiveness of the board and shareholder votes, as well as the role of board size.

## 5.1 Advisory role of the board

The literature on the board's advisory role studies how the presence of directors with a certain type of expertise is related to corporate policies and performance. For example, Dass et al. (2014) analyze directors' expertise in related industries, Guner, Malmendier, and Tate (2008) study the role of financial expertise, while Harford and Schonlau (2013) and Field and Mkrtchyan (2016) focus on the role of directors' experience in mergers and acquisitions. This literature typically views the advisory role of a given director in isolation. In contrast, the unique prediction of our model is that the advisory role of a director crucially depends on the expertise of other board members. Specifically, the analysis of communication externalities in Section 3.1 implies the following prediction:

**Prediction 1**: *When directors and manager have conflicting preferences (beliefs), the advisory role of a given director is weakened (enhanced) by the expertise of other directors.*

To test the above prediction, one needs to proxy for heterogeneity in preferences and beliefs, which is likely to differ both across different types of decisions and across companies. For example, there is likely to be strong heterogeneity in prior beliefs about the success of a brand new technology, or the development of an innovative drug. Likewise, there is often substantial disagreement about the effect of corporate governance policies, even among parties with similar interests, such as shareholders with similar portfolios.[24] The literature has proposed several measures of belief heterogeneity that could be used to assess how different agents' beliefs are in a given situation (e.g., Thakor and Whited, 2011; Diether et al., 2002; Malmendier and Tate, 2005). On the other hand, strong heterogeneity in

[24] See, e.g., "A Lack of Consensus on Corporate Governance", *The New York Times* (September 29, 2015), discussing shareholder disagreements on the issue of CEO-chairman separation. Another recent example is the debate about proxy access, when different shareholders and governance experts disagreed about the optimal terms of proxy access, such as the minimum size and holding period requirements. See "The Proxy Access Debate", *The New York Times* (October 9, 2009).

preferences is likely to arise if the decision involves a clear conflict of interest, such as an investment/acquisition or an increase in the scale of production that brings large private benefits to the manager. Across firms, all else equal, heterogeneity in preferences is more likely to arise in firms where the manager's incentives are not aligned due to ineffective compensation and poor governance controls.

Our next prediction connects the advisory role of the board to the expertise of the manager. Are the two complements or substitutes? Both views have been expressed in the academic literature, but they have not been formally explored in a unified framework.[25] Our analysis suggests that whether the two are complements or substitutes strongly depends on the nature of communication frictions. To see this more formally, we analyze an extension in Appendix B, which shows that the effect of the manager's expertise on the board's advising effectiveness is negative when the manager and directors have different preferences, but positive when they differ in beliefs. The intuition directly connects to the discussion of positive and negative externalities in Section 3.1. While Section 3.1 describes how the transmission of some agents' information to the manager imposes externalities on other agents' communication to him, the same logic applies if the manager is more informed for an exogenous reason, such as having greater experience and expertise. This leads to the following prediction.

**Prediction 2**: *When directors and manager have conflicting preferences (beliefs), the advisory role of the board is weakened (enhanced) by the expertise of the manager.*

Interestingly, the fact that the manager's expertise impedes the board's advisory role under conflicting preferences implies that it can be optimal for the firm to appoint a less informed manager. Intuitively, the manager's expertise has two opposing effects on the quality of decision-making. On the one hand, a less informed manager makes worse decisions due to his own information being worse. On the other hand, a less informed manager improves the advisory role of the board and hence obtains more information from board members, increasing the quality of decision-making. As we show in Appendix B, this second effect can dominate.

[25] For example, Armstrong et al. (2010) discuss that managers' informational advantage may impede the advisory role of outside directors. On the other hand, Sundaramurthy et al. (2014), which is the only paper we know that tests the relationship between CEO expertise and directors' advisory role, hypothesizes that board members' experience and expertise are more impactful when the manager has greater expertise as well.

## 5.2 Advisory role of shareholder voting

Our analysis also has implications for the advisory role of shareholder votes. Voting on many types of proposals (such as say-on-pay and proposals sponsored by shareholders) is non-binding: even if a proposal is approved by the majority of the votes, the management is not legally obligated to implement it. In this sense, such non-binding shareholder votes play a purely advisory role, which is why they are also called advisory. The literature has studied the effectiveness of such votes' advisory role by measuring management's responsiveness to the vote tally (e.g., Ertimur et al., 2010; Ferri, 2012; Cuñat et al., 2012; and others).

One immediate prediction of our analysis is that the advising effectiveness of the vote is lower when there is a greater conflict of interest between shareholders and management. This prediction is also present in Levit and Malenko (2011). However, our paper offers additional implications about the advisory role of the votes, which are not present in Levit and Malenko (2011). The first concerns the number of shareholders: in their paper, if the manager is conflicted, the vote does not aggregate information regardless of the number of shareholders, while in our setting, information aggregation crucially depends on the number of shareholders.[26] Specifically:

**Prediction 3**: *When shareholders and manager have conflicting preferences (beliefs), the advising effectiveness of shareholder voting is higher when the firm has a smaller (larger) number of shareholders.*[27]

The second new prediction follows from our analysis of endogenous ownership structure. Recall that the holding cost $\lambda$ proxies for limited diversification and risk aversion, and hence is likely to be higher in larger and riskier firms. Then, the analysis in Section 4.1 implies the

[26] The reason our results differ is due to a different economic mechanism: unlike in our paper, the mechanism in Levit and Malenko (2011) works through shareholders conditioning their decisions on being pivotal. This is due to the different way we view the role of advisory votes. In their paper, the vote only matters in specific cases – when it changes the manager's decision from "not implement" the proposal to "implement", once the vote tally exceeds a certain cutoff. In contrast, we view the vote tally as affecting decisions even away from the cutoff – e.g., because it affects the extent to which the proposal is implemented. In practice, both roles are important: the literature shows both a monotonic increase in the probability and extent of proposal implementation as a function of the vote tally (e.g., Ertimur et al., 2013) and a discrete jump in the probability of implementation around certain cutoffs (e.g., Cuñat et al., 2012).

[27] Formally, this prediction follows from a slight variation of the model in which $N$ is the overall number of signals that determine the state and $S \subseteq N$ is the number of the firm's shareholders, which is also the number of agents communicating to the manager. The comparative statics of the IC constraint for truthful communication with respect to $S$ immediately implies Prediction 3.

following:

**Prediction 4**: *The advising effectiveness of shareholder votes is higher in smaller and less risky firms.*[28]

## 5.3 Board size

A number of empirical studies examine the determinants and effects of board size (e.g., Yermack, 1996; Coles et al., 2008; Jenter et al., 2018; and others). One drawback of large boards that is sometimes discussed in this literature is the free-rider problem, whereby a larger board size discourages each individual investor from exerting effort. A related effect is also present in our framework: as Lemma 3 and Proposition 4 demonstrate, the larger the board, the lower are each director's incentives to acquire private information, leading the optimal board size to decrease in the costs of information acquisition. In addition, our model predicts that a smaller board is optimal if there is stronger misalignment in preferences between the manager and potential directors. Indeed, as (24) shows, when conflicts of interest are substantial, directors will have incentives to misreport their information unless the board is sufficiently small.[29] We summarize these predictions as follows:

**Prediction 5**: *The optimal advisory board size is larger if:*

(*i*) *the manager's and potential directors' preferences are more aligned;*

(*ii*) *directors' costs of acquiring private information are lower.*

In addition, building further on the analysis of the manager's expertise in Appendix B, we expect that under heterogeneous preferences, it is optimal to have a larger advisory board if

[28] For the case of heterogeneous preferences, this immediately follows from Proposition 6 and its proof: when $\lambda$ is large, shareholder voting is not fully effective in its advisory role, in the sense that truthful communication by all shareholders is not an equilibrium; in contrast, when $\lambda$ is small, all shareholders communicate truthfully if an advisory vote is held among them. While this comparative statics with respect to $\lambda$ is only derived under heterogeneous preferences, it also weakly holds under heterogeneous beliefs. In this case, according to Proposition 5, the efficient equilibrium always exists, and in this equilibrium, the advising effectiveness of the vote does not depend on $\lambda$: for any $\lambda$, the set of shareholders includes all informed agents, and they all communicate truthfully.

[29] We formally show this prediction at the end of the proof of Proposition 4.

the manager is less informed. This prediction is broadly consistent with the evidence in Coles et al. (2008), who study board size in the context of the board's advisory role. They find that boards of more complex firms whose CEOs require more advice (which they proxy, e.g., by firm diversification and size) have larger boards, and this larger board size is driven by a larger number of outside directors. In the context of our model, the set of outside directors can be interpreted as the firm's advisory committee (which is also the interpretation of Coles et al., 2008), while inside directors can be interpreted as the manager.

Note also that since we focus on the advisory role of the board and abstract from its monitoring role, we expect the above predictions to be particularly strong in cases where the board's primary role is advising. These include dual-class firms, where the board's monitoring role is less relevant since management controls the majority of the votes, or private firms with purely advisory boards.

**Effect of board size on performance.** Since board composition is chosen endogenously, our analysis does not generally predict any specific correlation between board size and performance. It does, however, predict how performance will be affected by exogenous changes in board size. One example is the 1976 law in Germany, which introduced new requirements for supervisory board size. Jenter et al. (2018) study this regulation via regression-discontinuity and difference-in-differences analyses and conclude that forcing firms to have larger boards lowers performance and value. This evidence is consistent with our model if heterogeneity in preferences is strong: differences in preferences are particularly likely in German supervisory boards, which contain shareholder and employee representatives, but no executive directors. Indeed, our model predicts that adding members to the optimal board characterized in Proposition 4 decreases value because it does not lead to more communication and better advice but imposes additional costs, for example, the costs of new board members' compensation.[30]

# 6 Conclusion

When information about the company is dispersed among multiple shareholders and stakeholders, the manager can benefit from seeking advice of these informed parties. Two key examples of such provision of advice to management are through the company's board of directors and through advisory shareholder voting. The goal of this paper is to study the

[30] These costs are assumed to be infinitesimally positive in our setting, but could be significant in practice.

optimal design of the advisory body and derive implications for the use of advisory shareholder voting vis-à-vis advisory board. We analyze a setting where members of the advisory body decide whether to acquire private information and whether to communicate it to a partially informed manager, who then takes the action optimal for him. In this setting, communication from any committee member to the manager can be inhibited by two key frictions – conflicting preferences and conflicting prior beliefs: both frictions may induce the advisor to misreport his information, in order to tilt the manager's action in the advisor's preferred direction. We study both of these frictions and show that they have very different implications for the optimal composition of the advisory body and the effectiveness of advisory voting vis-à-vis the board of directors.

Specifically, when agents have heterogeneous prior beliefs, communication from committee members to the manager exhibits positive externalities: as more advisors reveal their information to the manager, the incentives of other advisors to truthfully reveal their information increase even further. In contrast, when agents have heterogeneous preferences, communication externalities are negative: as more advisors reveal their information, other advisors have stronger incentives to misreport. Due to these externalities, the optimal advisory committee is the set of all potentially informed agents if agents have heterogeneous beliefs and their costs of acquiring information are low. In contrast, if agents have heterogeneous preferences or collecting information is sufficiently costly, the optimal advisory committee is a strict subset of informed agents. This implies that it is better to seek advice from a small advisory board on issues involving a significant misalignment in preferences, such as the decision to scale up the production. Conversely, advisory shareholder voting, combined with the manager seeking the advice of other informed stakeholders, is more effective for issues involving considerable heterogeneity in beliefs, such as the development of a new product. Finally, we study the implications of market trading and endogenous ownership structure for the effectiveness of advisory voting and thereby real efficiency. When agents have heterogeneous beliefs, endogeneity of ownership can lead to inefficient equilibria, in which the manager gets advice from very few informed parties and makes suboptimal decisions. In this case, combining an advisory shareholder vote with an advisory board can eliminate this inefficiency and improve decision-making. Our analysis offers novel implications concerning the advising effectiveness of shareholder votes and boards of directors, the role of the manager's expertise, and the effects of board size.

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# Appendix A: Proofs

## Proof of Lemma 1

Since $\theta_i$ is a binary signal equal to 1 with probability $\varphi$ and 0 with probability $1-\varphi$, the manager's optimal action (4) can be written as:

$$a_m(\theta_R) = b_m + \sum_{i\in R} c_i\theta_i + \mathbb{E}_m\left[\varphi|\theta_i, i\in R\right]\sum_{j\in -R} c_j\ .$$

Let $\mathbf{1}_R \equiv \sum_{i\in R}\theta_i$ be the number of signals in $R$ equal to 1. The conditional probability that $\mathbf{1}_R$ signals out of $|R|$ are equal to one given $\varphi$ is $P(\mathbf{1}_R|\varphi) = \binom{|R|}{\mathbf{1}_R}\varphi^{\mathbf{1}_R}(1-\varphi)^{|R|-\mathbf{1}_R}$. Since the prior distribution is Beta and the likelihood function is Binomial, the posterior distribution is also Beta but with different parameters (this is a known property of the Beta distribution). Formally, let $P_i(\mathbf{1}_R)$ be agent $i$'s assessed probability that $\mathbf{1}_R$ signals out of $|R|$ are equal to 1 (over all possible values of $\varphi$). Using Bayes rule, agent $i$'s posterior belief of $\varphi$, $P_i(\varphi|\mathbf{1}_R)$, is

$$\begin{aligned} P_i(\varphi|\mathbf{1}_R) &= \frac{f_i(\varphi)P(\mathbf{1}_R|\varphi)}{P_i(\mathbf{1}_R)} = \frac{\varphi^{\rho_i-1}(1-\varphi)^{\tau-\rho_i-1}}{Beta(\rho_i,\tau-\rho_i)}\frac{1}{P_i(\mathbf{1}_R)}\binom{|R|}{\mathbf{1}_R}\varphi^{\mathbf{1}_R}(1-\varphi)^{|R|-\mathbf{1}_R} \\ &= \frac{1}{Beta(\rho_i,\tau-\rho_i)P_i(\mathbf{1}_R)}\binom{|R|}{\mathbf{1}_R}\times\varphi^{\rho_i+\mathbf{1}_R-1}(1-\varphi)^{\tau-\rho_i+|R|-\mathbf{1}_R-1}, \end{aligned}$$

which is some constant that does not depend on $\varphi$ times $\varphi^{\rho_i+\mathbf{1}_R-1}(1-\varphi)^{\tau-\rho_i+|R|-\mathbf{1}_R-1}$. Since the posterior beliefs must integrate to one over possible values of $\varphi$, this automatically implies that the posterior belief also follows a Beta distribution with parameters $(\rho_i+\mathbf{1}_R, \tau-\rho_i+|R|-\mathbf{1}_R)$ and density

$$P_i(\varphi|\mathbf{1}_R) = \frac{1}{Beta(\rho_i+\mathbf{1}_R,\tau-\rho_i+|R|-\mathbf{1}_R)}\varphi^{\rho_i+\mathbf{1}_R-1}(1-\varphi)^{\tau-\rho_i+|R|-\mathbf{1}_R-1}.$$

It is known that the mean of a Beta distribution with parameters $(\alpha,\beta)$ is $\frac{\alpha}{\alpha+\beta}$. Therefore, using these expressions and the above posterior distribution, agent $i$'s expected value of $\varphi$ is $\mathbb{E}_i(\varphi|\mathbf{1}_R) = \frac{\rho_i+\mathbf{1}_R}{\tau+|R|}$, which proves the lemma.

## Auxiliary Lemma A.1

*Suppose $\varphi \sim Beta(\rho,\tau-\rho)$ and $X = \{x_1, x_2, \cdots, x_n\}$, where $x_i \in \{0,1\}$ are independent draws with $x_i = 1$ with probability $\varphi$. Let $\mathbf{1}_X \equiv \sum_{i=1}^n x_i$. Then*

$$\begin{aligned} \mathbb{E}_X[\mathbf{1}_X] &= n\frac{\rho}{\tau} \\ \mathbb{E}_X[\mathbf{1}_X^2] &= n\rho\,\frac{\tau-\rho+n(\rho+1)}{\tau(\tau+1)}. \end{aligned}$$

**Proof.** It is known that the first two moments of a random variable $X$ distributed according to a Beta distribution with parameters $\alpha$ and $\beta$ are $\mathbb{E}[X] = \frac{\alpha}{\alpha+\beta}$ and $\mathbb{E}[X^2] = \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}$. Hence, $\mathbb{E}[\varphi] = \frac{\rho}{\tau}$ and $\mathbb{E}[\varphi^2] = \frac{\rho(\rho+1)}{\tau(\tau+1)}$. Using this, we get

$$\mathbb{E}[1_X] = \mathbb{E}\left[\sum_{i=1}^{n} x_i\right] = n\mathbb{E}[x_i] = n\mathbb{E}[\varphi] = n\frac{\rho}{\tau}$$

and

$$\begin{aligned}
\mathbb{E}\left[1_X^2\right] &= \mathbb{E}\left[\sum_{i=1}^{n} x_i^2 + \sum_{i\neq j} x_i x_j\right] = \mathbb{E}\left(n\mathbb{E}\left[x_i^2|\varphi\right] + n(n-1)\,\mathbb{E}\left[x_i|\varphi\right]^2\right) \\
&= n\mathbb{E}[\varphi] + n(n-1)\,\mathbb{E}\left[\varphi^2\right] = \frac{n\rho}{\tau(\tau+1)}\left(\tau-\rho+n(\rho+1)\right).
\end{aligned}$$

## Proof of Lemma 2

Let $\mathbf{1}_R = \sum_{i\in R}\theta_i$ denote the number of signals 1 in $R$. Using Lemma 1, we obtain agent $i$'s ex-ante payoff, $\mathbb{E}_i(a_m(\theta_R) - Z - b_i)^2$, as follows:

$$\mathbb{E}_i[U_i|R] = u_0 - (b_m - b_i)^2 - U_1 - U_2, \tag{16}$$

where

$$\begin{aligned}
U_1 &\equiv 2(b_m - b_i)\,\mathbb{E}_i\left[\left(\frac{\rho_m + \mathbf{1}_R}{\tau+|R|}\sum_{k\in -R} c_k - \sum_{k\in -R} c_k\theta_k\right)|R\right], \\
U_2 &\equiv \mathbb{E}_i\left[\left(\frac{\rho_m + \mathbf{1}_R}{\tau+|R|}\sum_{k\in -R} c_k - \sum_{k\in -R} c_k\theta_k\right)^2 |R\right].
\end{aligned}$$

Using independence of $\theta_k$ conditional on $\varphi$, and Auxiliary Lemma A.1, $U_1$ simplifies to

$$U_1 = 2(b_m - b_i)\frac{\rho_m - \rho_i}{\tau+|R|}\left(\sum_{k\in -R} c_k\right) = \mathsf{A}_{im}(R). \tag{17}$$

To simplify $U_2$, we use the law of iterated expectations:

$$\begin{aligned}
U_2 &= \mathbb{E}_i\left[\left(\frac{(\rho_m + \mathbf{1}_R)\sum_{k\in -R} c_k}{\tau+|R|}\right)^2 - 2\frac{(\rho_m + \mathbf{1}_R)(\rho_i + 1_R)\left(\sum_{k\in -R} c_k\right)^2}{(\tau+|R|)^2}|R\right] \\
&\quad + \mathbb{E}_i\left[\mathbb{E}_i\left[\left(\sum_{k\in -R} c_k\theta_k\right)^2 |\theta_R, R\right]|R\right],
\end{aligned} \tag{18}$$

where we used $\mathbb{E}_i\left[\sum_{k\in -R} c_k\theta_k|\theta_R, R\right] = \left(\sum_{k\in -R} c_k\right)\mathbb{E}_i\left[\varphi|\theta_R, R\right] = \left(\sum_{k\in -R} c_k\right)\frac{\rho_i+1_R}{\tau+|R|}$. Consider the last term under the expectation sign:

$$\begin{aligned}
\mathbb{E}_i\left[\left(\sum_{k\in -R} c_k\theta_k\right)^2|\theta_R, R\right] &= \mathbb{E}_i\left[\sum_{k\in -R} c_k^2 Var_i\left[\theta_k|\varphi, R\right] + \varphi^2\left(\sum_{k\in -R} c_k\right)^2|\theta_R, R\right] \\
&= \mathbb{E}_i\left[\sum_{k\in -R} c_k^2\varphi\left(1-\varphi\right) + \varphi^2\left(\sum_{k\in -R} c_k\right)^2|\theta_R, R\right] \\
&= \frac{\rho_i+1_R}{\tau+|R|}\left(\sum_{k\in -R} c_k^2 + \left(\left(\sum_{k\in -R} c_k\right)^2 - \sum_{k\in -R} c_k^2\right)\frac{\rho_i+1_R+1}{\tau+|R|+1}\right),
\end{aligned}$$

where the second equality is due to $Var_i\left[\theta_k|\varphi, R\right] = \varphi\left(1-\varphi\right)$ and the last equality is due to the fact that the agent $i$'s posterior distribution of $\varphi$ conditional on $\theta_R$ is Beta with parameters $\rho_i + 1_R$ and $\tau + |R| - \rho_i - 1_R$, whose first and second moments are, respectively, $\frac{\rho_i+1_R}{\tau+|R|}$ and $\frac{(\rho_i+1_R)(\rho_i+1_R+1)}{(\tau+|R|)(\tau+|R|+1)}$. Plugging this expression into (18) and simplifying using Auxiliary Lemma A.1,

$$\begin{aligned}
U_2 - \mathsf{C}_{im}(R) &= \mathbb{E}_i\left[\frac{\left(\sum_{k\in -R} c_k^2\right)\left(\rho_i+1_R\right)}{\tau+|R|} - \left(\frac{\left(\sum_{k\in -R} c_k\right)\left(\rho_i+1_R\right)}{\tau+|R|}\right)^2|R\right] \\
&\quad + \left(\left(\sum_{k\in -R} c_k\right)^2 - \sum_{k\in -R} c_k^2\right)\mathbb{E}_i\left[\frac{\left(\rho_i+1_R+1\right)\left(\rho_i+1_R\right)}{\left(\tau+|R|+1\right)\left(\tau+|R|\right)}|R\right] \\
&= \left(\frac{\left(\sum_{k\in -R} c_k\right)^2}{\tau+|R|} + \sum_{k\in -R} c_k^2\right)\mathbb{E}_i\left[\frac{\left(\rho_i+1_R\right)\left(\tau+|R|-\rho_i-1_R\right)}{\left(\tau+|R|\right)\left(\tau+|R|+1\right)}\right] \\
&= \left(\frac{\left(\sum_{k\in -R} c_k\right)^2}{\tau+|R|} + \sum_{k\in -R} c_k^2\right)\frac{\rho_i\left(\tau-\rho_i\right)}{\tau\left(\tau+1\right)} = \mathsf{B}_i(R).
\end{aligned}$$

Combining with (16) and (17) gives (11)-(12). This immediately shows that the ex-ante payoff of any agent $i$ is increasing in $|R|$ and is decreasing in $c_k$ for any $k \in -R$. In other words, when the manager learns an additional signal, $\mathsf{A}_{im}(\cdot), \mathsf{B}_i(\cdot)$ and $\mathsf{C}_{im}(\cdot)$ are reduced. Indeed, the greater information and the smaller unknown part of the state $Z$ imply that the residual variance $\mathsf{B}_i(\cdot)$ decreases. The first and third terms, i.e., $\mathsf{A}_{im}(\cdot)$ and $\mathsf{C}_{im}(\cdot)$, decrease as well, because agent $i$ expects the additional signal to "persuade" the manager, such that they have a smaller expected divergence in their ex-post beliefs. This intuition holds in the opposite direction when $c_k, k \in -R$ increases.

## Proof of Proposition 1

Plugging (5) and (6) into (7) gives

$$0 \geq \sum_{\theta_{-i} \in \{0,1\}^{N-1}} \left[ c_i(2\theta_i - 1) + (\sum_{j \in -R \setminus \{i\}} c_j) \cdot \frac{2\theta_i - 1}{\tau + |R| + 1} \right]$$

$$\times \left[ 2(b_m - b_i) + c_i(1 - 2\theta_i) - 2 \sum_{j \in -R \setminus \{i\}} c_j \theta_j + \frac{2(\rho_m + \mathbf{1}_R) + 1}{\tau + |R| + 1} \sum_{j \in -R \setminus \{i\}} c_j \right] P_i(\theta_{-i} | \theta_i).$$

Note that the first multiple in each term equals $(2\theta_i - 1)[c_i + \frac{\sum_{j \in -R \setminus \{i\}} c_j}{\tau + |R| + 1}]$, where $c_i + \frac{\sum_{j \in -R \setminus \{i\}} c_j}{\tau + |R| + 1}$ is positive and is constant across all terms in the sum. Thus, the above inequality is equivalent to

$$0 \geq (2\theta_i - 1) \sum_{\theta_{-i}} P_i(\theta_{-i} | \theta_i) \left( 2(b_m - b_i) + c_i(1 - 2\theta_i) - 2 \sum_{j \in -R \setminus \{i\}} c_j \theta_j + \frac{2(\rho_m + \mathbf{1}_R) + 1}{\tau + |R| + 1} \sum_{j \in -R \setminus \{i\}} c_j \right).$$

Since $\sum_{\theta_{-R \setminus \{i\}}} \left( \sum_{j \in -R \setminus \{i\}} c_j \theta_j \right) P_i(\theta_{-R \setminus \{i\}} | \theta_i, \theta_R) = \frac{\rho_i + \mathbf{1}_R + \theta_i}{\tau + |R| + 1} \sum_{j \in -R \setminus \{i\}} c_j$, we can further simplify it to

$$(2\theta_i - 1) \left[ 2(b_m - b_i) + c_i(1 - 2\theta_i) + \frac{2(\rho_m - \rho_i) + 1 - 2\theta_i}{\tau + |R| + 1} \sum_{j \in -R \setminus \{i\}} c_j \right] \leq 0.$$

We consider two separate cases. If $\theta_i = 0$, the above inequality becomes:

$$2(b_m - b_i) + c_i + \frac{2(\rho_m - \rho_i) + 1}{\tau + |R| + 1} \sum_{j \in -R \setminus \{i\}} c_j \geq 0,$$

and if $\theta_i = 1$, it becomes

$$2(b_m - b_i) - c_i + \frac{2(\rho_m - \rho_i) - 1}{\tau + |R| + 1} \sum_{j \in -R \setminus \{i\}} c_j \leq 0,$$

Together we get (8), which completes the proof.

## Proof of Proposition 2

Consider advisory committee $B$, where all members are informed, and define $B_m \equiv B \cup \{m\}$. Full information revelation by all members of $B$ to $m$ is an equilibrium if and only if (8) holds for any agent $i$ in $B$, where $R = B_m \setminus \{i\}$. Since $|R| = |B_m| - 1 = |B|$, (9) for all $i$ in

$B$ becomes

$$|\rho_m - \rho_i| \leq \frac{1}{2}\left[1 + c_i \frac{\tau + |B| + 1}{\sum_{j \in \{1,...,N\} \setminus B_m} c_j}\right] \ \forall i \in B,$$

which is equivalent to

$$|B| \geq \left[\max_{i \in B} \frac{2\,|\rho_m - \rho_i| - 1}{c_i}\right] \sum_{j \in \{1,...,N\} \setminus B_m} c_j - \tau - 1.$$

Hence, truth-telling by all members of the advisory body cannot be an equilibrium if $|B|$ is below

$$\min_{B \subseteq \{1,2,\cdots,N\} \setminus \{m\}} \left[\max_{i \in B} \frac{2\,|\rho_m - \rho_i| - 1}{c_i}\right] \sum_{j \in \{1,...,N\} \setminus B_m} c_j - \tau - 1,$$

which is strictly positive if heterogeneity in beliefs is sufficiently strong.

Likewise, (10) for each $i$ in $B$ becomes

$$|b_m - b_i| \leq \frac{1}{2}\left[c_i + \frac{\sum_{j \in \{1,...,N\} \setminus B_m} c_j}{\tau + |B| + 1}\right] \ \forall i \in B \Leftrightarrow \left[\max_{i \in B} 2\,|b_m - b_i| - c_i\right] \leq \frac{\sum_{j \in \{1,...,N\} \setminus B_m} c_j}{\tau + |B| + 1}.$$

Hence, truth-telling by all members of the advisory body cannot be an equilibrium if $|B|$ is above

$$\max_{B \subseteq \{1,2,\cdots,N\} \setminus \{m\}} \frac{\sum_{j \in \{1,...,N\} \setminus B_m} c_j}{\max_{i \in B} 2\,|b_m - b_i| - c_i} - \tau - 1,$$

which is strictly smaller than $N - 1$ if heterogeneity in preferences is sufficiently strong.

## Proof of Proposition 3

We start with the first statement of the proposition, when $b_i = b_j$, $\rho_i \neq \rho_j$. Suppose that the advisory body is $B = \{1, ..., N\} \setminus \{m\}$. We show that there exists $\kappa_l > 0$ such that for any $\kappa \leq \kappa_l$, there exists an equilibrium in which all agents in the advisory body acquire information and truthfully reveal it to the manager. Consider such an equilibrium and any agent $i \in B$. Then $R = \{1, ..., N\} \setminus \{i\}$ and $-R \setminus \{i\} = \emptyset$, so using (8), the IC constraint on communication of agent $i$ is equivalent to $0 \leq c_i$, which is always satisfied. Consider the agent's decision to acquire information in this equilibrium. If agent $i$, $i \neq m$ acquires his signal, his expected utility is $\mathbb{E}_i[U_i|\{1, ..., N\}] - \kappa$, where $\mathbb{E}_i[U_i|R]$ is given by (11). If the agent deviates and does not acquire his signal, his expected utility is $\mathbb{E}_i[U_i|\{1, ..., N\} \setminus \{i\}]$: because information acquisition decisions are not observed at the communication stage, the agent's deviation does not change other agents' incentives to communicate truthfully, but at the decision-making stage, the manager will make his decision knowing that the agent is uninformed. Define:

$$\kappa_l \equiv \min_{i \in \{1,...,N\} \setminus \{m\}} \{\mathbb{E}_i[U_i|\{1, ..., N\}] - \mathbb{E}_i[U_i|\{1, ..., N\} \setminus \{i\}]\}. \tag{19}$$

Then, the IC constraint on information acquisition is satisfied for any $\kappa \leq \kappa_l$. Hence, for $\kappa \leq \kappa_l$, such an equilibrium indeed exists.

Moreover, according to Lemma 2, the sum of expected utilities of all agents in this equilibrium is $Nu_0 - (N-1)\,\kappa$, since $\mathbb{E}_i\left[U_i|\{1,...,N\}\right] = u_0\ \forall i$. For any other advisory body, the sum of expected utilities of all agents is $\sum_{i=1}^{N} \mathbb{E}_i[U_i|R] - \kappa\,(|R|-1)$, where $R$ is the set of signals that the manager knows after the communication stage. Recall from Lemma 2 that $\mathbb{E}_i[U_i|R]$ increases as $R$ expands. Using

$$\begin{aligned} u_0 - \kappa &\geq \mathbb{E}_i[U_i|\{1,...,N\}\backslash\{i\}]\} \geq \mathbb{E}_i[U_i|R]\ \forall \kappa \leq \kappa_l, \\ u_0 &> \mathbb{E}_i[U_i|R], \end{aligned}$$

we obtain $Nu_0 - (N-1)\,\kappa > \sum_{i=1}^{N} \mathbb{E}_i[U_i|R] - \kappa\,(|R|-1)$. Therefore, $B = \{1,...,N\}\backslash\{m\}$ is indeed optimal.

We next prove the second statement, when $b_i \neq b_j$, $\rho_i = \rho_j$. First, notice that when $\rho_i = \rho$ for all $i$, all agents have exactly the same preferences about the manager's ex-post information set: each agent's utility (11) is a constant term minus the following residual variance:

$$\sum_{j \in -B_m} c_j^2 + \frac{1}{\tau + |B| + 1}\left[\sum_{j \in -B_m} c_j\right]^2, \tag{20}$$

where $B_m \equiv B \cup \{m\}$. Notice that the value of agent $i$'s from acquiring a signal declines as committee $B$ expands. Therefore, if $\kappa \leq \kappa_l$, all agents on the committee will acquire information provided that they expect it to be truthfully communicated to the manager. Thus, the optimal committee minimizes

$$N\left(\sum_{j \in -B_m} c_j^2 + \frac{1}{\tau + |B| + 1}\left[\sum_{j \in -B_m} c_j\right]^2\right) + \kappa\,|B|\,,$$

subject to the IC constraints on truthful communication of the committee members. Consider the solution to this minimization problem. Under the conditions in the proposition, it is a strict subset of $\{1,...,N\}\backslash\{m\}$. Indeed, if the advisory body is the set of all agents, $B = \{1,...,N\}\backslash\{m\}$, then $-B_m = \emptyset$, so (8) becomes $|b_m - b_i| \leq \frac{1}{2}c_i$. Since, by assumption, this inequality is violated for at least one of the agents, there is no equilibrium in which all agents truthfully reveal their information to the manager. Hence, $B = \{1,...,N\}\backslash\{m\}$ is not an optimal advisory body, which implies that the optimal advisory body is a strict subset of all agents.

## Proof of Lemma 3

Suppose there is an equilibrium in which all members of the committee acquire information, which, in turn, requires that they communicate it truthfully. Consider the information acquisition decision of any member $i$ of the committee, $i \in B$. Since truth-telling conditions

(8) are satisfied, his expected payoff is $\mathbb{E}_i[U_i|\{m\},B]$. If he deviates from his equilibrium strategy and does not acquire information, his utility is $\mathbb{E}_i[U_i|\{m\},B\backslash\{i\}]$ (similar to the argument in the proof of Proposition 3). Hence, the incentive compatibility condition on information acquisition is equivalent to

$$\mathbb{E}_i[U_i|\{m\},B]-\mathbb{E}_i[U_i|\{m\},B/\{i\}]\geq\kappa. \tag{21}$$

Simplifying (11)-(12) for $c_i=c$ and denoting $\mathcal{G}(|B|)\equiv\frac{N-|B|-1}{\tau+|B|+1}$, we get

$$\begin{array}{c} u_i\left(|B|\right)\equiv\mathbb{E}_i[U_i|\{m\},B]=u_0-(b_m-b_i)^2-2c(b_m-b_i)(\rho_m-\rho_i)\mathcal{G}\left(|B|\right) \\ -c^2\frac{\rho_i(\tau-\rho_i)(N+\tau)}{\tau(\tau+1)}\mathcal{G}(|B|)-c^2\left[\left(\rho_m-\rho_i\right)\mathcal{G}(|B|)\right]^2. \end{array} \tag{22}$$

Note that $\mathcal{G}\left(|B\right)>0$ and that $\mathcal{G}(|B|)$ and hence $\mathcal{G}^2(|B|)$ decrease in $|B|$. In addition, $\mathcal{G}''(|B|)>0$ and hence $\left(\mathcal{G}^2\right)''(|B|)>0$ as well. It follows that the function $u_i\left(|B|\right)$ is increasing and concave in $|B|$, and hence $\mathbb{E}_i[U_i|\{m\},B]-\mathbb{E}_i[U_i|\{m\},B/\{i\}]=u_i\left(|B|\right)-u_i\left(|B|-1\right)$ is decreasing in $|B|$. Let $N_i\left(\kappa\right)$ be the highest value of $|B|$ for which $u_i\left(|B|\right)-u_i\left(|B|-1\right)\geq\kappa$. Then, (21) is equivalent to $|B|\leq N_i\left(\kappa\right)$. Thus, the incentive compatibility condition on information acquisition is satisfied for all committee members $i\in B$ if and only if $|B|\leq N_B\left(\kappa\right)\equiv\min_{i\in B}N_i\left(\kappa\right)$. Since $N_i\left(\kappa\right)$ is weakly decreasing in $\kappa$ for any $i$, $N_B\left(\kappa\right)$ is also weakly decreasing in $\kappa$. Note that according to the proof of Proposition 3, $\kappa_l=u\left(N-1\right)-u\left(N-2\right)$, and hence $N_B\left(\kappa\right)<N-1$ for any $\kappa>\kappa_l$. Finally, in the special case of $\rho_i=\rho_j$,

$$u_i\left(|B|\right)-u_i\left(|B|-1\right)=c^2\frac{\rho(\tau-\rho)\left(N+\tau\right)}{\tau(\tau+1)}\left(\mathcal{G}(|B|-1)-\mathcal{G}(|B|)\right),$$

which is independent of $i$. Therefore, the highest value of $|B|$ for which $u_i\left(|B|\right)-u_i\left(|B|-1\right)\geq\kappa$ is the same for every agent $i$. Denoting it by $N\left(\kappa\right)$, we conclude that $N_B\left(\kappa\right)=N\left(\kappa\right)$ in this case.

## Proof of Proposition 4

Since $c_i=c$, the expected utility of agent $i$ depends on committee $B$ only via its size $|B|$. First, we prove that the problem of choosing the optimal committee reduces to maximizing its size subject to the IC constraints on information acquisition and truth-telling of its members. For contradiction, suppose that such committee $B$ is not optimal. Then, there exists committee $B'$ with $|B'|<|B|$ that yields higher sum of all agents' expected utilities, net of information acquisition costs. Since information acquisition is optimal for all members of committee $B$, then $u_i\left(|B|\right)-u_i\left(|B|-1\right)\geq\kappa\;\forall i\in B$, where $u_i\left(\cdot\right)$ is defined by (22). Since $u_i\left(\cdot\right)$ is increasing in $|B|$, $u_i\left(|B|\right)-u_i\left(|B'|\right)\geq\kappa$ for all $i\in B$ and $u_i\left(|B|\right)-u_i\left(|B'|\right)\geq 0$ for all $i$. It follows that

$$\sum_{i=1}^{N} \left(u_i\left(|B|\right) - u_i\left(|B'|\right)\right) \geq \kappa\,|B| > \left(|B| - |B'|\right)\kappa.$$

Hence, committee $B'$ cannot be optimal, so the problem indeed reduces to maximizing a committee size subject to the IC constraints on information acquisition and truth-telling of its members.

Consider part 1 of the proposition. First, consider the conditions for which the committee consisting of all $N-1$ agents satisfies IC constraints on information acquisition and truthful communication. According to the proof of Lemma 3, information acquisition is incentive compatible for all $N-1$ agents if and only if $u_i\left(N-1\right) - u_i\left(N-2\right) \geq \kappa\ \forall i$. Using $b_i = b_j$ and $\mathcal{G}(N-1) = 0$, this constraint reduces to

$$c^2 \frac{\rho_i(\tau-\rho_i)\left(N+\tau\right)}{\tau(\tau+1)\left(\tau+N-1\right)} + c^2 \left[\frac{(\rho_m-\rho_i)}{\tau+N-1}\right]^2 \geq \kappa\ \forall i. \tag{23}$$

Then, $\kappa_l$ from (19) simplifies to:

$$\kappa_l = c^2 \min_{i\in\{1,\ldots,N\}\backslash m} \left\{ \frac{\rho_i(\tau-\rho_i)\left(N+\tau\right)}{\tau(\tau+1)\left(\tau+N-1\right)} + \left[\frac{(\rho_m-\rho_i)}{\tau+N-1}\right]^2 \right\}.$$

Then, information acquisition is incentive compatible for all $N-1$ agents if and only if $\kappa \leq \kappa_l$. According to Proposition 2, if $b_i = b_j$, the committee consisting of all $N-1$ members satisfies IC conditions for truthful communication of all members. Therefore, the committee consisting of all $N-1$ agents is optimal if $\kappa \leq \kappa_l$.

Consider $\kappa > \kappa_l$. Then, condition (23) is violated for at least one agent $i \in \{1, ..., N\}\backslash m$. Hence, the optimal committee is a subset of all agents, implying $N^* < N-1$. According to Proposition 2, there is a cutoff $N_{\min}$ such that truthful communication by all committee members is not an equilibrium if $|B| < N_{\min}$. Therefore, unless the optimal committee is $B = \emptyset$, it must be the case that $|B| \in [N_{\min}, N-1)$. The problem of choosing the optimal advisory committee is equivalent to maximizing its size subject to the IC constraints on information acquisition and communication. The IC constraints on information acquisition become more stringent as $\kappa$ increases. The IC constraints on truthful communication do not depend on $\kappa$. Therefore, if a certain committee satisfies both sets of constraints for some $\kappa$, it also satisfies them for any lower $\kappa$. Therefore, $N^*$ is (weakly) decreases in $\kappa$. Let $\kappa_h$ denote the lowest $\kappa$ for which there is no committee $B \neq \emptyset$ that satisfies incentive compatibility of information acquisition and truthful communication for all its members. Then, $N^* \in [N_{\min}, N-1)$ for $\kappa \in (\kappa_l, \kappa_h)$ and $N^* = 0$ for $\kappa \geq \kappa_h$.

Consider part 2 of the proposition. Using $c_i = c$, the IC condition for truth-telling of all members of committee $B$ simplifies to

$$|b_m - b_i| \leq \frac{c}{2} \frac{\tau + N}{\tau + |B| + 1} \ \forall i \in B. \tag{24}$$

Reorder the agents in the order of (weakly) increasing $|b_m - b_i|$, with the manager being number 1, and the agent with the highest $|b_m - b_i|$ being number $N$. Then, $N_{\max}$ is given by the lowest integer for which

$$|b_m - b_{N_{\max}+1}| \leq \frac{c}{2} \frac{\tau + N}{\tau + N_{\max} + 1}$$

is satisfied. Then, there exists a committee with $K$ members satisfying the IC conditions for truth-telling of all its members if and only if $K \leq N_{\max}$. According to Lemma 3, any committee with $K$ members, for which IC conditions for truth-telling are satisfied, satisfies the IC constraints on information acquisition if and only if $K \leq N(\kappa)$. Given this and the fact that the problem of choosing the optimal committee is equivalent to maximizing its size, we conclude that $N^* = \min\{N_{\max}, N(\kappa)\}$. This concludes the proof of the proposition.

In addition, the above analysis implies that the optimal size of the committee increases in the degree of preference alignment between the manager and potential directors (Prediction 5 ($i$) in Section 5). Indeed, the optimal committee is one of the largest possible size that allows information acquisition and truthful communication by all its members. As (24) implies, to find the committee of the largest size that allows truth-telling conditional on information acquisition, the solution is to rank all agents by the degree of their preference misalignment with the manager $|b_m - b_i|$, and then gradually expand $B$ by including agents with progressively larger preference misalignments, up to the point when the next included agent's IC constraint for truth-telling is violated. In this setting, if we now increase the degree of misalignment by scaling up $|b_i - b_m|$ by some constant $C$ for all $i$, then the maximum committee size that allows truthful communication will monotonically decrease in $C$ and will become zero as $C$ becomes large enough. According to Lemma 3, any committee with $K$ members, for which IC conditions for truth-telling are satisfied, satisfies the IC constraints on information acquisition if and only if $K \leq N(\kappa)$. Given this, the optimal committee size weakly decreases in $C$.

## Proof of Proposition 5

We prove the existence of the efficient equilibrium by showing that no agent wants to deviate from the strategies described in the proposition. Consider a subgame that happens after all $N$ agents become shareholders. According to Proposition 3 and its proof, there exists an equilibrium in which all $N$ agents communicate their information truthfully, and hence the optimal committee consists of the set of all agents (excluding the manager). Since all agents communicate their signals truthfully, the manager takes action $a_m = b + Z$ (a special case of Lemma 1). Consider the trading game, provided that each agent expects that all agents will become shareholders and the manager will undertake action $a_m = b + Z$. Given this expectation, Lemma 2 implies that the expected per-share payoff of each agent $i$ (excluding the holding cost) is $\mathbb{E}_i[U_i|R] = u_0$. Hence, from (15), the market-clearing price reduces to $p^* = u_0 - \frac{\lambda}{N}$. From (14), at this price, the fraction of the firm purchased by agent $i$ is $\alpha_i = \frac{u_0 - p^*}{\lambda} = \frac{1}{N}$. This equilibrium is efficient in the sense of maximizing the sum of the seller's proceeds and the utilities of all the agents, because it (1) achieves the most efficient action from all agents' point of view ($b+Z$) and (2) minimizes the agents' total holding costs by distributing the asset evenly among all agents ($\alpha_i = \frac{1}{N}$).

We prove the possibility of equilibrium multiplicity by construction of an example. Suppose there are two groups of agents, $\Gamma_1$ and $\Gamma_2$. Agents in the first group have the same prior beliefs as the manager: $\rho_i = \rho_m \ \forall i \in \Gamma_1$. In contrast, agents in the second group have different beliefs than the manager: $\rho_i \neq \rho_m$ with $|\rho_m - \rho_i| = \delta > 0$. Next, we show that if $\delta$ is sufficiently high, there exists an equilibrium in which only agents from group $\Gamma_1$ become shareholders and an inefficient action $a \neq b+Z$ is implemented. Consider a subgame that happens after trading, in which only agents from group $\Gamma_1$ became shareholders. Since $\rho_i = \rho_m$ for these agents, the IC condition for truthful communication is satisfied for each $i \in \Gamma_1$. Hence, the optimal committee is the set of all $i \in \Gamma_1$ (excluding the manager). Using Lemma 1, the action that the manager undertakes is

$$a_m(\theta_R) = b + c\sum_{i\in\Gamma_1}\theta_i + \frac{\rho_m + \sum_{i\in\Gamma_1}\theta_i}{\tau + |\Gamma_1|} c\left(N - |\Gamma_1|\right). \tag{25}$$

The expected per-share payoff of each agent $i \in \Gamma_1$ (excluding the holding cost) is

$$u_0 - c^2 \frac{\rho_m\left(\tau - \rho_m\right)\left(N + \tau\right)}{\tau(\tau+1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|}.$$

Hence, from (15), the market-clearing price is

$$p^* = u_0 - c^2 \frac{\rho_m\left(\tau - \rho_m\right)\left(N + \tau\right)}{\tau(\tau+1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|} - \frac{\lambda}{|\Gamma_1|}.$$

From (14), at this price, the fraction of firm purchased by agent $i \in \Gamma_1$ is $\alpha_i = \frac{1}{|\Gamma_1|}$. To show that this is an equilibrium, it remains to show that no agent $i \in \Gamma_2$ is better off deviating to buying shares at price $p^*$ if $\delta$ is sufficiently high. Suppose that one agent $j \in \Gamma_2$ deviates to $\alpha_j > 0$. Using Proposition 1, the IC condition on truthful reporting for this agent, (9), is violated if $\delta > \delta_1$, where

$$\delta_1 \equiv \frac{1}{2}\left[1 + \frac{\tau + |\Gamma_1|}{N - |\Gamma_1|}\right].$$

Hence, if agent $j$ deviates to $\alpha_j > 0$, the optimal committee and firm's action will remain the same. Therefore, the expected per-share payoff of this agent (excluding the holding cost) is

$$u_0 - c^2 \frac{\rho_j\left(\tau - \rho_j\right)\left(N + \tau\right)}{\tau(\tau+1)} \frac{N - |\Gamma_1|}{\tau + |\Gamma_1|} - c^2\delta^2\left(\frac{N - |\Gamma_1|}{\tau + |\Gamma_1|}\right)^2.$$

Using (14), a sufficient condition for deviation to $\alpha_j > 0$ to be unprofitable for agent $j$ is that $\delta > \delta_2$, where

$$\delta_2 \equiv \frac{\tau + |\Gamma_1|}{N - |\Gamma_1|}\sqrt{\frac{\lambda}{c^2\,|\Gamma_1|} + \frac{(N+\tau)}{\tau(\tau+1)}\frac{N-|\Gamma_1|}{\tau+|\Gamma_1|}\rho_m\,(\tau-\rho_m)}.$$

Hence, this strategy profile is indeed an equilibrium if $\delta > \max\{\delta_1, \delta_2\}$.

## Proof of Proposition 6

Let $S$ denote the equilibrium set of agents that become shareholders, $B \subseteq S$ be the equilibrium committee chosen out of these shareholders, and $|B|$ denote the equilibrium committee size. Then, the expected per-share payoff of each agent $i \in \Gamma_1$ (excluding the holding cost) is

$$u_0 - (b_m - b_i)^2 - c^2 \frac{\rho(\tau-\rho)\,(N+\tau)}{\tau(\tau+1)}\mathcal{G}(|B|).$$

Plugging this expression into the demand equation (14) and using the market-clearing condition $\sum_{j\in S}\alpha_j^* = 1$, we obtain the equilibrium price:

$$p = u_0 - c^2\frac{\rho(\tau-\rho)\,(N+\tau)}{\tau(\tau+1)}\mathcal{G}(|B|) - \frac{1}{|S|}\left(\sum_{j\in S}(b_m-b_j)^2 + \lambda\right).$$

Plugging this price into the demand equation, we obtain:

$$\alpha_i^* = \max\left\{\frac{1}{|S|} + \frac{1}{\lambda}\left(\frac{1}{|S|}\sum_{j\in S}(b_m-b_j)^2 - (b_m-b_i)^2\right), 0\right\}. \tag{26}$$

Note that this expression does not depend on the composition of the committee. Hence, we can solve for the set of all equilibria in two steps: (1) solve for the equilibrium set of shareholders; (2) find optimal committee given the solution to the first step.

Consider the first step. Reorder the agents in the order of (weakly) increasing $|b_m - b_i|$, with the manager being number 1, and the agent with the highest $|b_m - b_i|$ being number $N$. Then, (26) implies that $|S|$ is given by the last index $K$ at which

$$\lambda + \sum_{j\le K}(b_m-b_j)^2 - K(b_m-b_K)^2 > 0.$$

Note that this expression is monotone decreasing in $K$. To see this, take the difference between $K+1$ and $K$:

$$K\left((b_m-b_K)^2 - (b_m-b_{K+1})^2\right) < 0.$$

Therefore, there is a unique $K$. Notice that the number of agents that become shareholders, $K$, is increasing in $\lambda$. In particular, if $\lambda \to 0$, then only one agent (the manager) is the shareholder. Conversely, if $\lambda$ exceeds $\sum_{j=1}^{N}(b_m - b_j)^2 - N(b_m - b_N)^2$, then all agents become shareholders.

Next, consider the optimal committee given that $K$ agents become shareholders. The IC constraint on truthful reporting for committee member $i$ is

$$(\tau + |B| + 2)\,|b_m - b_i| \leq \frac{\tau + N + 2}{2}. \tag{27}$$

Since agents are ranked in order of increasing $|b_m - b_i|$ and have the same quality of information, it is without loss of generality to consider committees that include all agents of sufficiently low rank (except the manager). Notice that expression $(\tau + K + 2)\,|b_m - b_K|$ is strictly increasing in integer $K$, taking values from zero for $K = 1$ to a value exceeding $\frac{\tau+N+2}{2}$ for $K = N$. Therefore, there is a unique integer $\hat{K}$, such that (27) holds for all agents of committee size $\hat{K} - 1$ or below, but not for committees of size above $\hat{K} - 1$. Therefore, the optimal committee size is equal to $\min\left\{\hat{K}, K\right\} - 1$. Since $K$ is increasing in $\lambda$, taking values from 1 to $N$, there exists a point $\lambda^*$, such that the optimal committee includes all non-manager shareholders if and only if $\lambda \leq \lambda^*$.

# Appendix B: Expertise of the manager

In this section, we analyze the effect of the manager's expertise on the board's advising effectiveness. To study this question, we need to introduce a measure of the manager's expertise. For this purpose, we consider a small extension of the basic model by assuming that the manager knows a subset $\mathcal{S}$ of signals $\{\theta_i, i \neq m\}$ in addition to signal $\theta_m$. If $\mathcal{S} = \emptyset$, then the manager only knows his private signal $\theta_m$, as in the basic model. If $\mathcal{S} \neq \emptyset$, then the manager also knows some signals of the other agents, in addition to his private signal $\theta_m$. We interpret an expansion/contraction of $\mathcal{S}$ (i.e., addition/removal of signals to/from $\mathcal{S}$) as an increase/decrease in the manager's expertise.[31] For simplicity, we assume that the costs of information acquisition are sufficiently small that all directors acquire information and focus on the effects of communication frictions.

Let us fix all parameters of the model and consider any board $B$, i.e., a subset of agents $\{1, ..., N\}$. As before, we say that board $B$ is efficient at providing advice to the manager if truthful communication by all members of $B$ to the manager is an equilibrium. Without loss of generality, consider boards in which each member has some information that the manager does not already know: $\{\theta_i, i \in B\} \cap \mathcal{S} = \emptyset$.[32] The next proposition shows how the advising effectiveness of the board varies with the manager's information:

[31] An alternative way to model higher managerial expertise is to increase $c_m$, while normalizing $\sum_{i=1}^{N} c_i = 1$. This model leads to the same result. However, it cannot be used to analyze how firm value changes with the manager's expertise because a change in $c_m$ changes the distribution of state $Z$ in this formulation.

[32] Clearly, there is no benefit of adding an agent to the advisory board if he has no information that the manager does not already possess.

**Proposition 7 (Manager's expertise).** *Consider any board $B$ with $\{\theta_i, i \in B\} \cap \mathcal{S} = \emptyset$.*

1. *If $|b_m - b_i| \leq \frac{1}{2}c_i$ $\forall i \in B$ and board $B$ is efficient at providing advice under $\mathcal{S}$, then it is also efficient at providing advice if set $\mathcal{S}$ expands, i.e., as the manager becomes more informed.*

2. *If $|\rho_m - \rho_i| \leq \frac{1}{2}$ $\forall i \in B$ and board $B$ is efficient at providing advice under $\mathcal{S}$, then it is also efficient at prodiving advice if set $\mathcal{S}$ contracts, i.e., as the manager becomes less informed.*

The intuition is close to the intuition behind positive and negative externalities in Section 3.1. As the manager becomes more informed, there is less information relevant for the decision that neither the manager nor the board knows. This has different effects depending on the nature of communication frictions. When the friction of heterogeneous beliefs is relatively more important, the key consequence is the increase in the manager's congruence because there is less information that the manager and board members can disagree about. As a result, board members have stronger incentives to truthfully reveal their information to the manager, explaining the first statement of the proposition. In contrast, when communication is primarily hampered by conflicts of interest, greater managerial expertise decreases directors' costs of misreporting their information because the manager is expected to react less to each director's message. This explains the second statement of the proposition.

Since Proposition 7 applies to any committee, both optimal and suboptimal ones, we can conclude that the manager's expertise enhances the advisory role of the board if communication is mainly hampered by disagreement between the manager and board members, but impedes its advisory role if communication is mainly hampered by conflicts of interest.

Interestingly, the latter result implies that it can be Pareto improving to appoint a less informed manager, even if the less informed manager has the same preferences as the more informed manager, and even if the composition of the advisory board can be chosen optimally. Intuitively, when heterogeneity in beliefs is small while heterogeneity in preferences is substantial, the manager's expertise has two opposing effects. First, a less informed manager has worse private information, which impedes decision-making. Second, a less informed manager obtains more information from the board members, which improves decision-making. The numerical example below illustrates that this second effect can dominate.

**Example 2 (Manager's expertise can be harmful)**

There is a manager and 100 other agents, divided into two groups. The parameters are: $c_m = 0.3$, $b_m = 0.0475$, $b_i = 0$ $\forall i \neq m$, $\rho_i = \rho = 2$, $\tau = 4$. The first group are the relatively more informed agents: it contains $\bar{N} = 10$ agents with $c_i = \bar{c} = 0.05$. The second group are the relatively less informed agents: it contains $\underline{N} = 90$ agents with $c_i = \underline{c} = 0.2/90$ (for simplicity, $\sum_{i=1}^{N} c_i$ is normalized to one). Thus, the manager's signal $\theta_m$ has weight 30% in the state, the sum of all signals of the more informed agents has weight 50% in the state, and the sum of all signals of the less informed agents has weight 20% in the state. If the manager only knows signal $\theta_m$ (i.e., $\mathcal{S} = \emptyset$), then the optimal board is comprised of 5 agents from the first group. These agents report their signals to the manager truthfully, and the implied expected payoff of each agent $i \neq m$ is $V = -0.0093$ (the payoff of the manager is higher by $b_m^2$). In contrast, if the manager also knows one of the signals with $c_i = 0.2/90$ and the board is comprised of 5 agents from the first group, the IC constraint is violated and truthful revelation by all members of this board is not an equilibrium. Instead, the optimal board is comprised of 4 agents from the first group. The implied expected payoff of each agent $i \neq m$ is $V = -0.0109$, which is lower than if the manager is less informed. Thus, a reduction in the manager's information improves the values of all agents by promoting more efficient communication.

In contrast, the fact that a more informed manager improves the advisory role of the board when conflicts of interest play a small role relative to differences in beliefs, implies that in this case, both effects act in the same direction. As the manager becomes more informed, he both makes better decisions due to his own information and can also get better advice from other agents.

## Proof of Proposition 7

Rewriting the IC constraint from Proposition 1 and using $(b_m - b_i)(\rho_m - \rho_i) \geq 0$, board $B$ is efficient if and only if $\mathcal{I}_i \geq 0$ for all $i \in B$, where

$$\mathcal{I}_i \equiv \frac{\tau + |B| + |\mathcal{S}| + 2}{\sum_{j \in -B_m} c_j} \left( \frac{1}{2} c_i - |b_m - b_i| \right) + \frac{1}{2} - |\rho_m - \rho_i| \,,$$

where $-B_m$ is a set of all signal indices that are not known to the board or the manager. Consider an expansion of $\mathcal{S}$ by one element. If this element belongs to $\{\theta_i, i \in B\}$, then all statements of the proposition are vacuously true, as the IC constraints are unaffected. Thus, consider the case when this element does not belong to $\{\theta_i, i \in B\}$. In this case, an expansion in $\mathcal{S}$ increases $|\mathcal{S}|$ and decreases $\sum_{j \in -B_m} c_j$. Suppose that $|b_m - b_i| \leq \frac{1}{2} c_i \; \forall i \in B$. Then, an expansion in $\mathcal{S}$ increases $\mathcal{I}_i$ for any $i$. Hence, if $\mathcal{I}_i \geq 0$ for all $i$ for some $\mathcal{S}$, then $\mathcal{I}_i \geq 0 \; \forall i$ for any expansion in set $\mathcal{S}$. This proves the first statement of the proposition.

To prove the second statement, rewrite the IC constraint from Proposition 1 as $\mathcal{J}_i \geq 0$, where

$$\mathcal{J}_i \equiv \frac{\sum_{j \in -B_m} c_j}{\tau + |B| + |\mathcal{S}| + 2} \left( \frac{1}{2} - |\rho_m - \rho_i| \right) + \frac{1}{2} c_i - |b_m - b_i| \,.$$

Again consider an expansion of $\mathcal{S}$ by one element that does not belong to $\{\theta_i, i \in B\}$. Suppose that $|\rho_m - \rho_i| \leq \frac{1}{2} \; \forall i \in B$. Then, an expansion in $\mathcal{S}$ reduces $\mathcal{J}_i$ for any $i$, because it increases $|\mathcal{S}|$ and decreases $\sum_{j \in -B_m} c_j$. Hence, if $\mathcal{J}_i \geq 0$ for all $i$ for some $\mathcal{S}$, then $\mathcal{J}_i \geq 0 \; \forall i$ for any contraction in set $\mathcal{S}$.