Incremental Learning of Robot Dynamics using Random Features

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Cogsys
Cognitive Systems

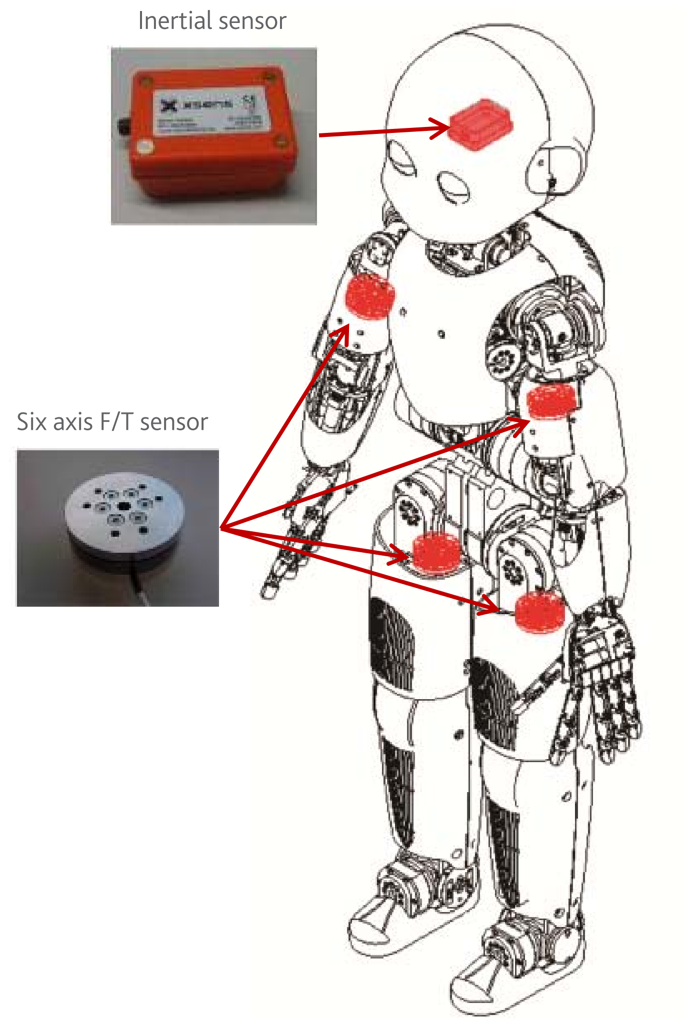
RobotCub

general setting

- learning incrementally
  - because the world is non-stationary (concept drift)
- learn efficiently
  - real-time (hard) constraints
- we'd like to learn
  - accurately (guarantees that learning learns)
  - autonomously (little prior programming)

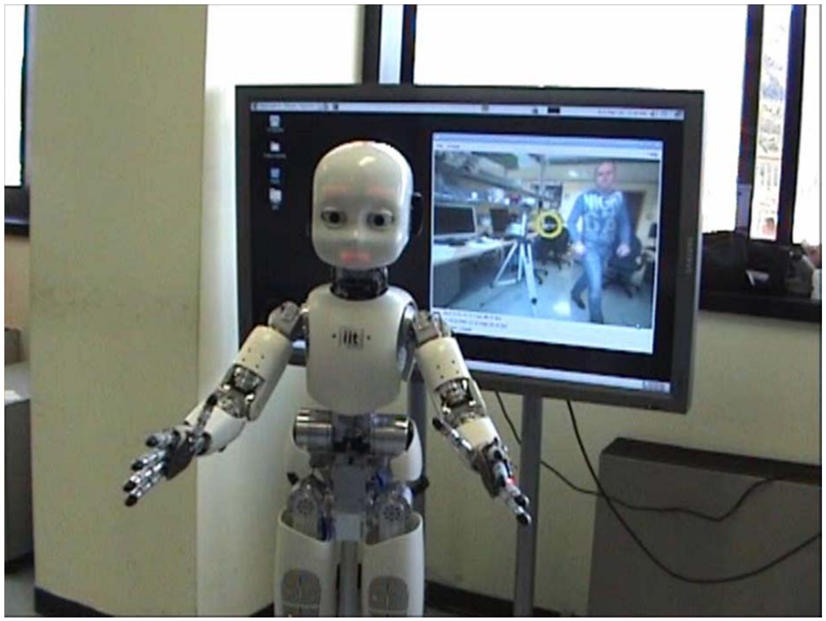
specific setting

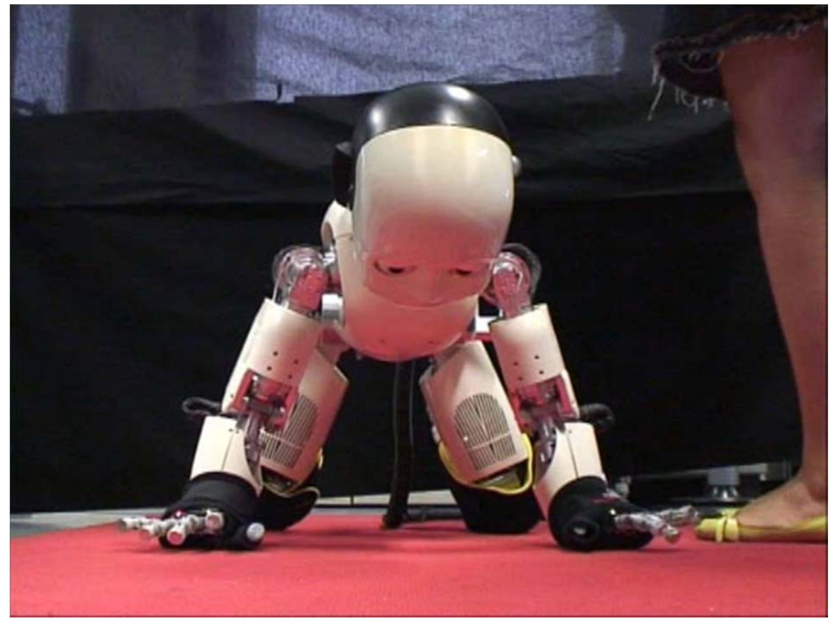
- learning body dynamics
  - compute external forces
  - implement compliant control



- so far we did it starting from e.g. the cad models
  - but we'd like to avoid it

...so





some incremental learning methods

- LWPR [Vijayakumar et al., 2005]
- Kernel Recursive Least Squares [Engel et al., 2004]
- Local Gaussian Processes [Nguyen-Tuong et al., 2009]
- Sparse Online GPR [Csató and Opper, 2002]

typical problems (not everywhere):

- high per-sample complexity (slow learning)
- increasing or unpredictable computational requirements
- limited theoretical support and understanding

our method

- linear ridge regression as base algorithm
  - efficient, elegant, effective
  - theoretically well-studied

- possible extensions for non-linear regression and incremental updates

$$f(x) = w^T x$$

$$\min_w J = \frac{\lambda}{2}\|w\|^2 + \frac{1}{2}\|y - Xw\|^2$$

$$w = \left(\lambda I + X^T X\right)^{-1} X^T y$$

our method in 3 easy steps

- kernel trick

$$f(x) = \sum_{i=1}^{m} c_i k(x, x_i)$$

$$c = (K + \lambda I)^{-1} y$$

- approximate kernel

Rahimi, A. & Recht, B. (2008)

$$k(x_i, x_j) = E\left[\frac{1}{D}\sum_{d=1}^{D} z_{w_d}(x_i)^T z_{w_d}(x_j)\right]$$

$$z_w(x) = \left[\cos(w^T x), \sin(w^T x)\right]$$

- make it incremental

$$w = (\lambda I + \Phi^T \Phi)^{-1} \Phi^T y$$

\+ Cholesky rank-1 update

features

- $O(1)$ update complexity w.r.t. # training samples
- exact batch solution after each update
- dimensionality of feature mapping trades computation for approximation accuracy
- $O(n^2)$ time and space complexity per update w.r.t. dimensionality of feature mapping
- easy to understand/implement (few lines of code)
- not exclusively for dynamics/robotics learning!

Algorithm 4.1 Incremental RFRLS with isotropic RBF kernel

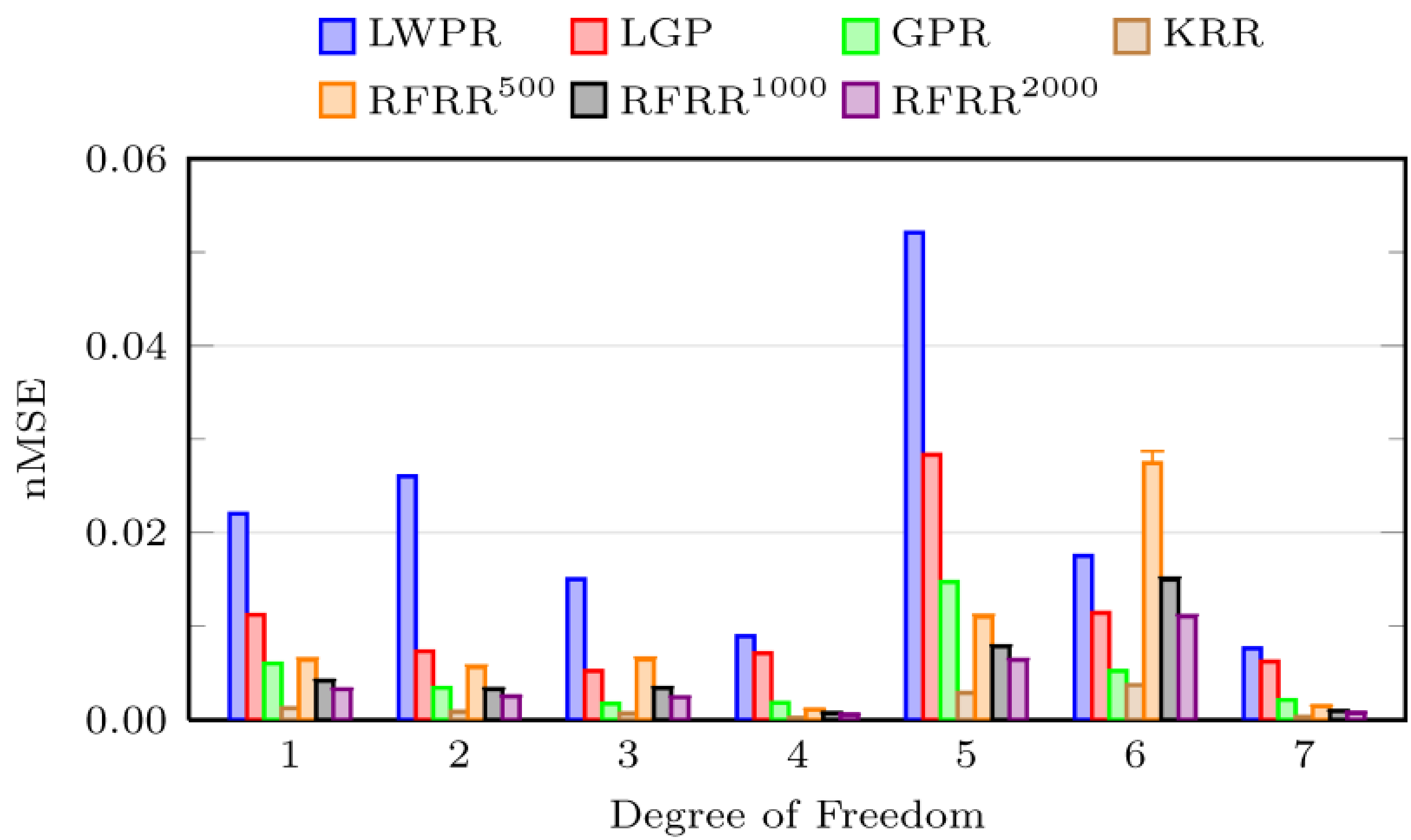
Require: $\lambda > 0, \gamma > 0, n > 0, D > 0$

1: $\boldsymbol{R} \leftarrow \sqrt{\lambda}\boldsymbol{I}_{D\times D}$
2: $\boldsymbol{w} \leftarrow \boldsymbol{0}_{D\times 1}$
3: $\boldsymbol{b} \leftarrow \boldsymbol{0}_{D\times 1}$
4: $\boldsymbol{\Omega} \sim \mathcal{N}\left(0, (2\gamma)^2\right)_{D\times n}$
5: $\boldsymbol{\beta} \sim \mathcal{U}(-\pi, \pi)_{D\times 1}$
6: **for all** $(\boldsymbol{x}, y)$ **do**
7: $\quad \boldsymbol{\phi} \leftarrow \sqrt{\frac{2}{D}} \cos\left(\boldsymbol{\Omega x} + \boldsymbol{\beta}\right)$ // *Equation* (4.4)
8: $\quad \hat{y} \leftarrow \langle \boldsymbol{w}, \boldsymbol{\phi} \rangle$
9: $\quad \boldsymbol{b} \leftarrow \boldsymbol{b} + \boldsymbol{\phi} y$
10: $\quad \boldsymbol{R} \leftarrow \textsc{CholeskyUpdate}(\boldsymbol{R}, \boldsymbol{\phi})$
11: $\quad \boldsymbol{w} \leftarrow \boldsymbol{R} \backslash \left(\boldsymbol{R}^{\mathrm{T}} \backslash \boldsymbol{b}\right)$
12: $\quad$ **yield** $\hat{y}$
13: **end for**

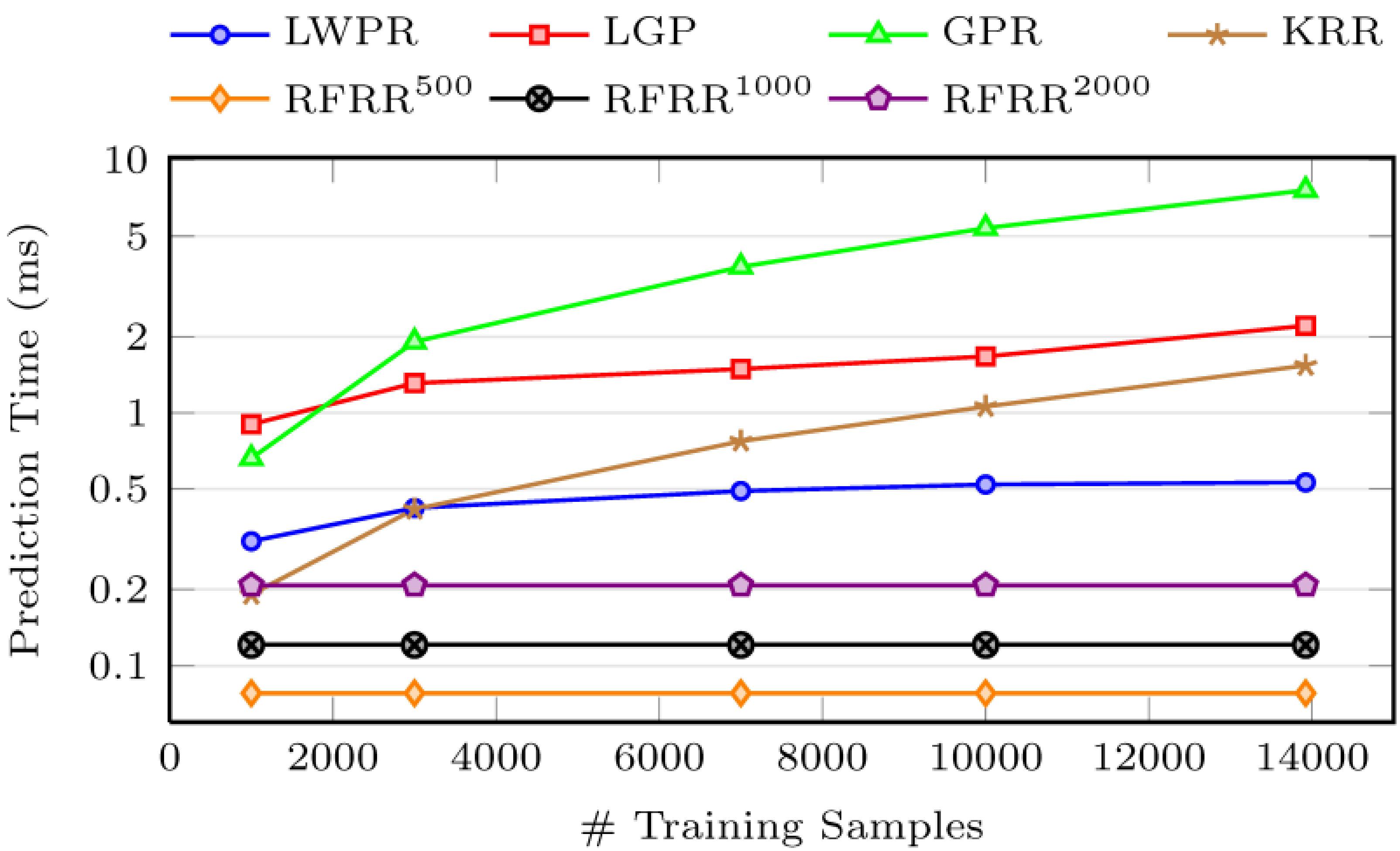
batch experiments

- 3 inverse dynamics datasets: Sarcos, Simulated Sarcos, Barrett [Nguyen-Tuong et al., 2009]
- approximately 15k training and 5k test samples
- comparison with LWPR, GPR, LGP, Kernel RR
- RFRR with 500, 1000, 2000 random features
- hyperparameter optimization by exploiting functional similarity with GPR (log marginal likelihood optimization)

batch error on 7-DOF Sarcos arm



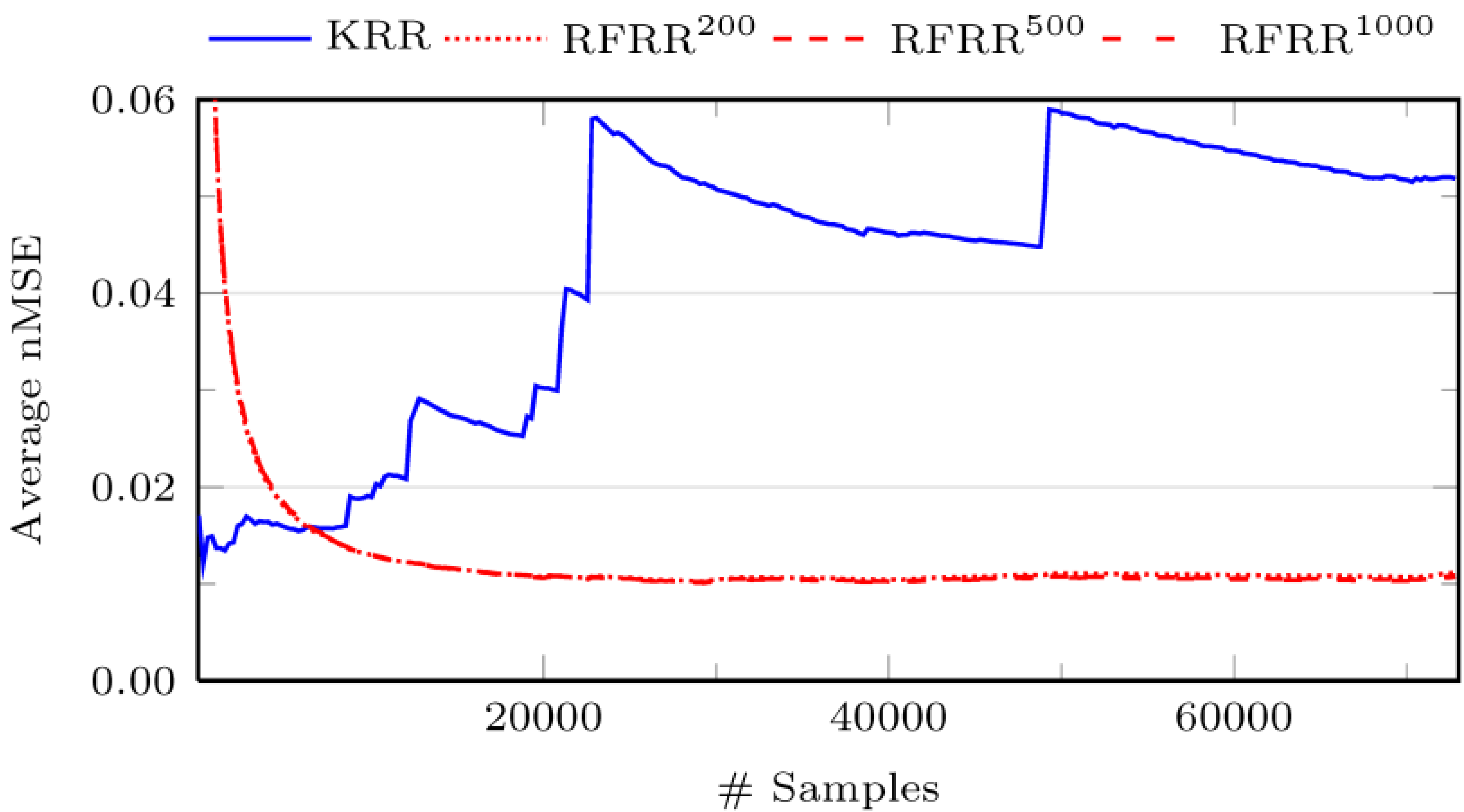
prediction time



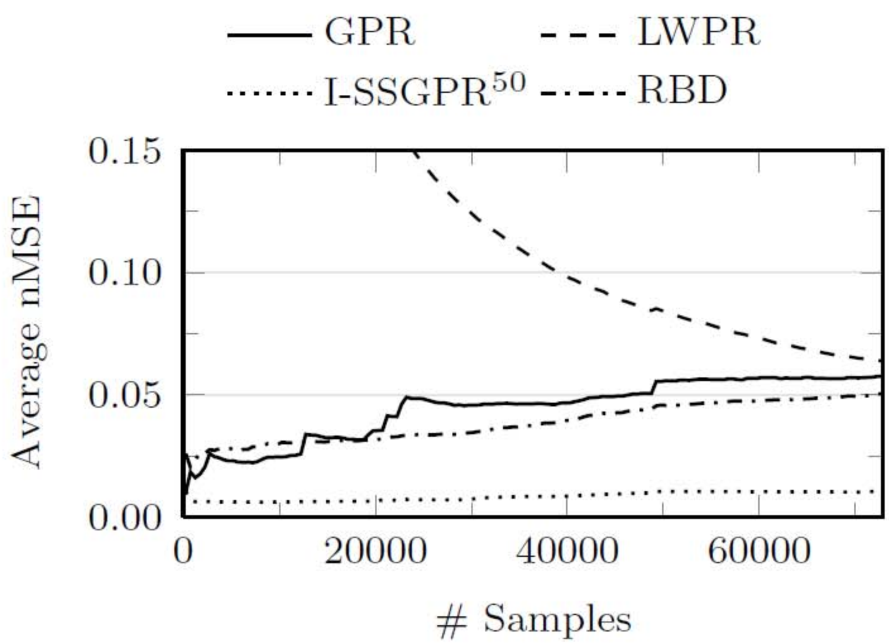
incremental experiments

- two large scale inverse dynamics datasets from "James" and iCub humanoids (4-DOF)
- realistic scenario: initial 15k training and remaining approx. 200k and 80k test samples
- RFRR with 200, 500, 1000 random features
- RFRR uses training samples only for hyperparameter optimization
- comparison with batch Kernel RR (identical hyperparameters)

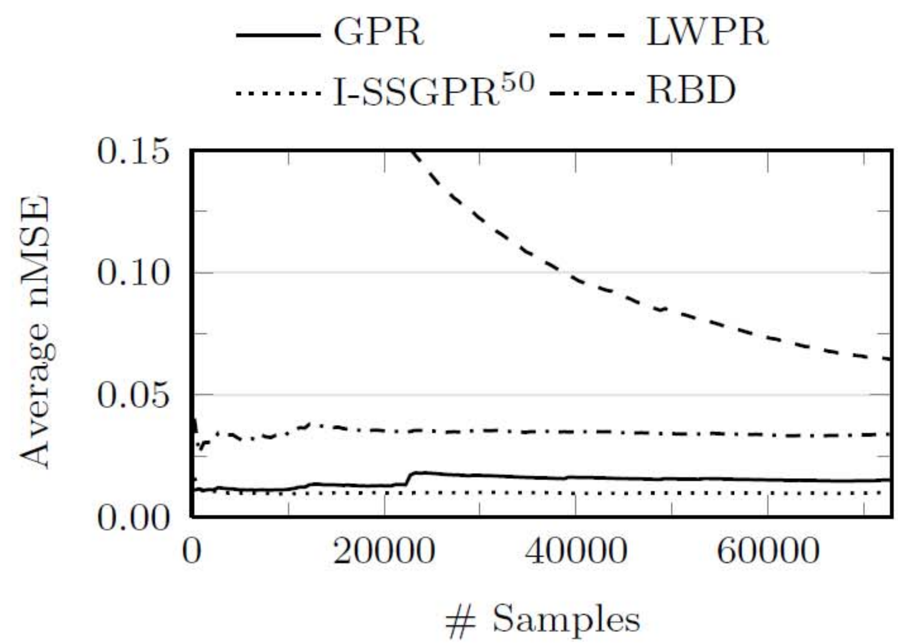
batch vs. incremental



verification (learning dynamics)

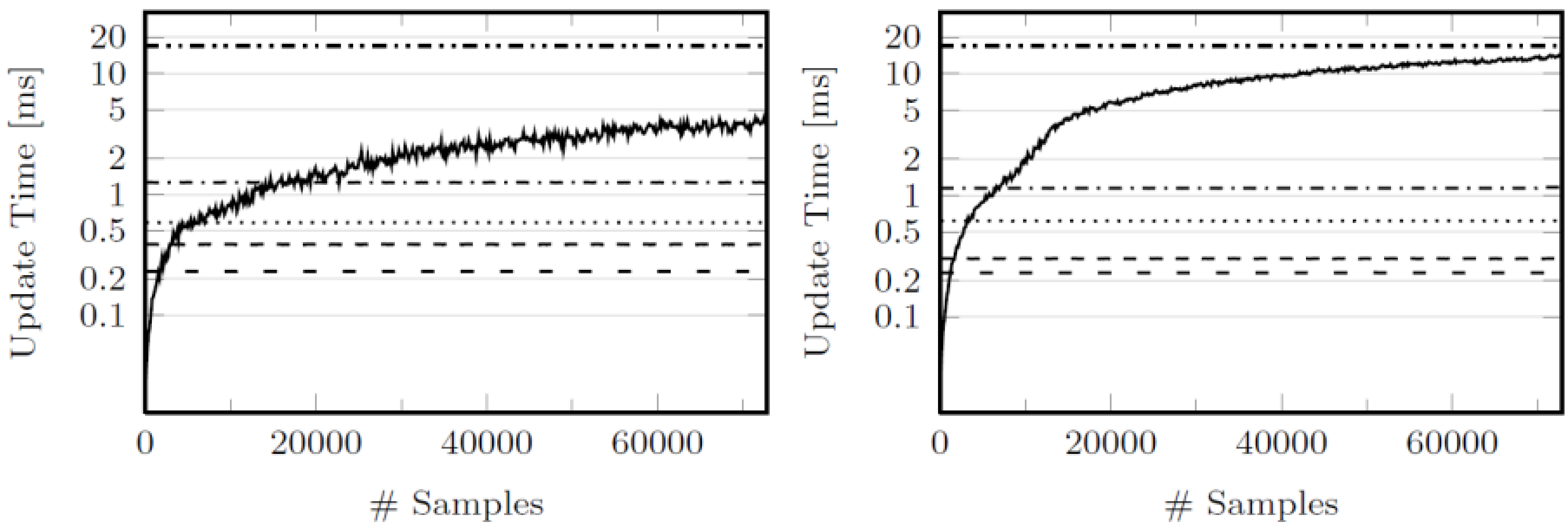


(d) iCub Forces



(e) iCub Torques

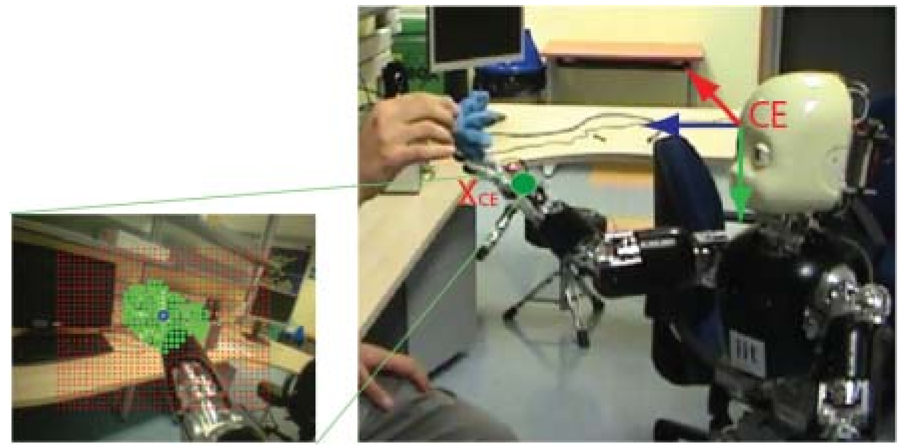
verification: time



(d) iCub Forces

(e) iCub Torques

verification: reaching



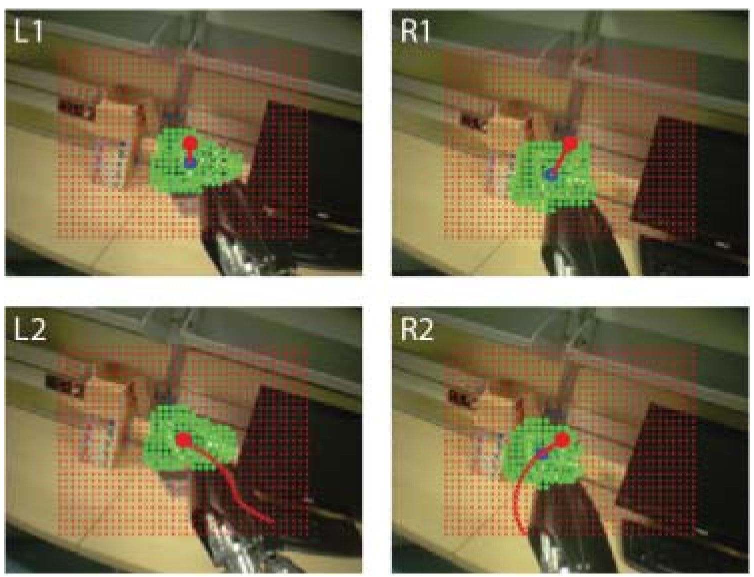
$$(x, y, z)_{CE} = M\left(\underbrace{u_l, v_l, u_r, v_r}_{\text{image}}, T, V_s, V_g\right)$$

fixation point

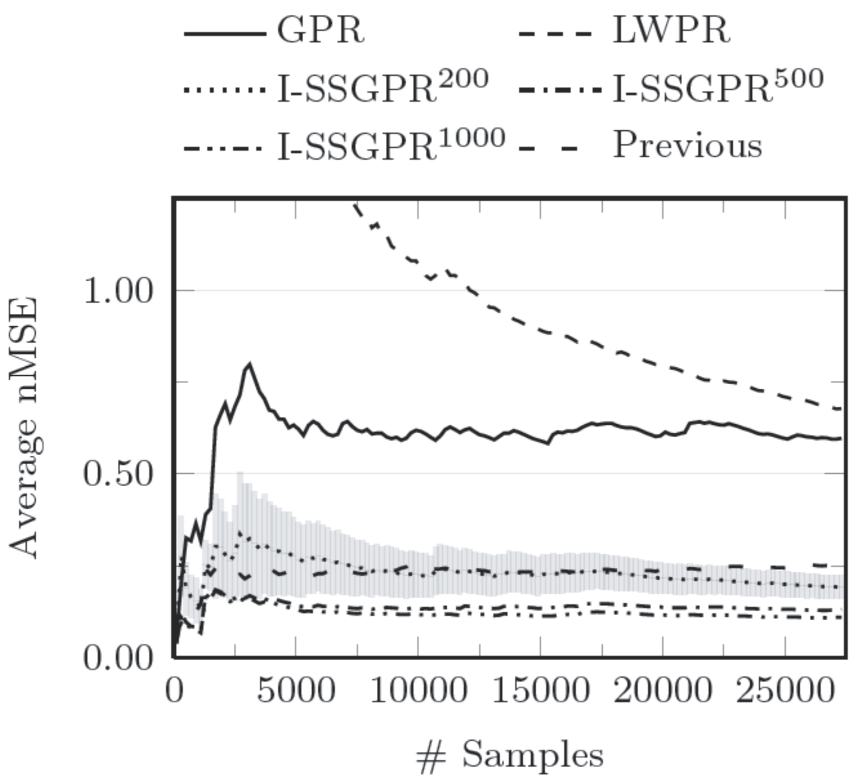
to learn

image

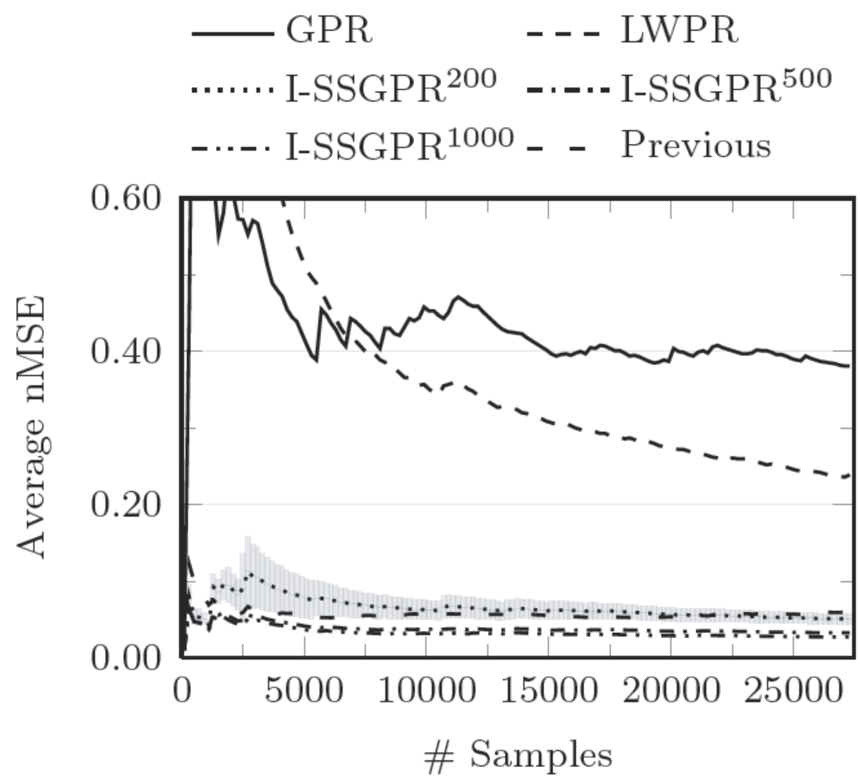
eye configuration



verification

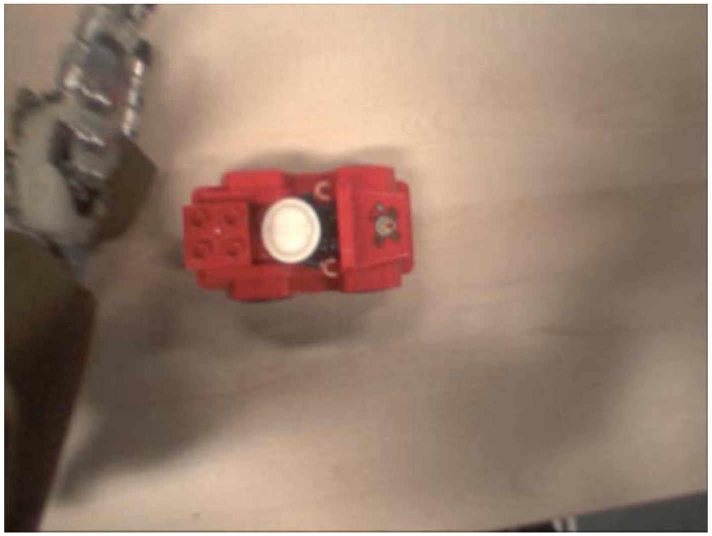


(a) u_{left}

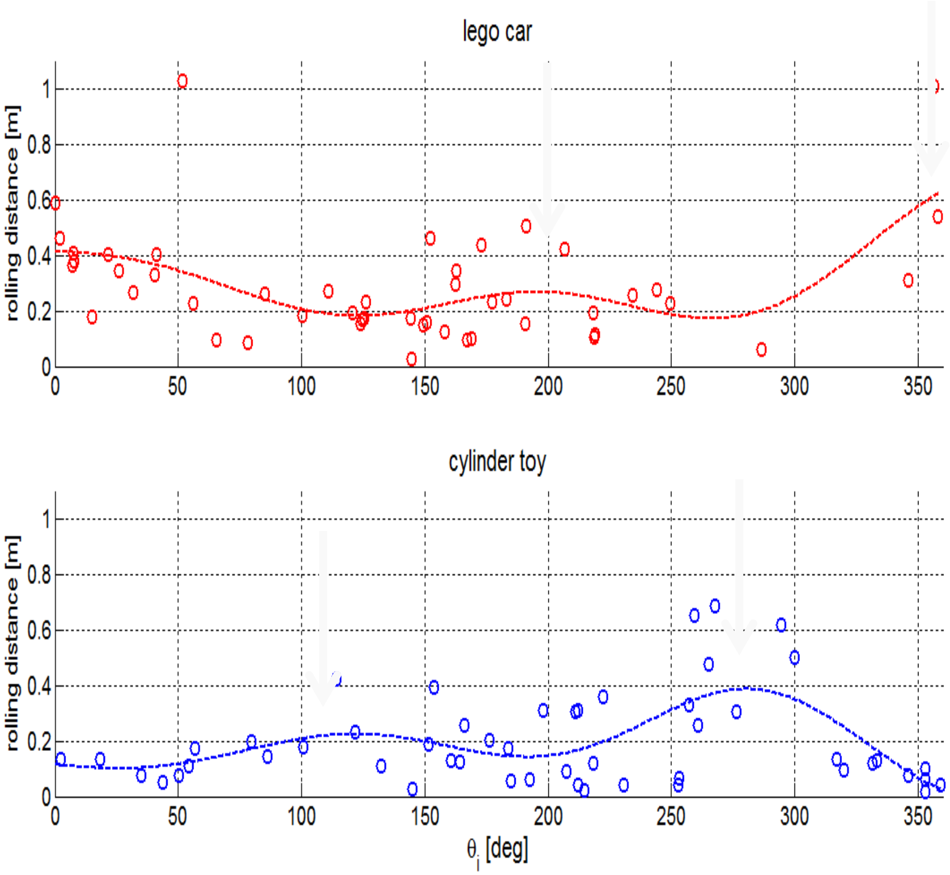


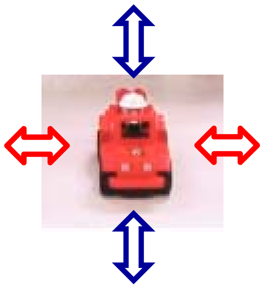
(b) u_{right}

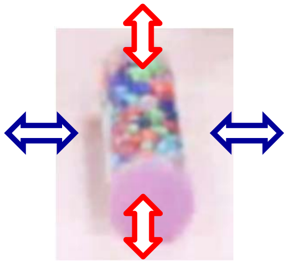
affordances (learning objects)

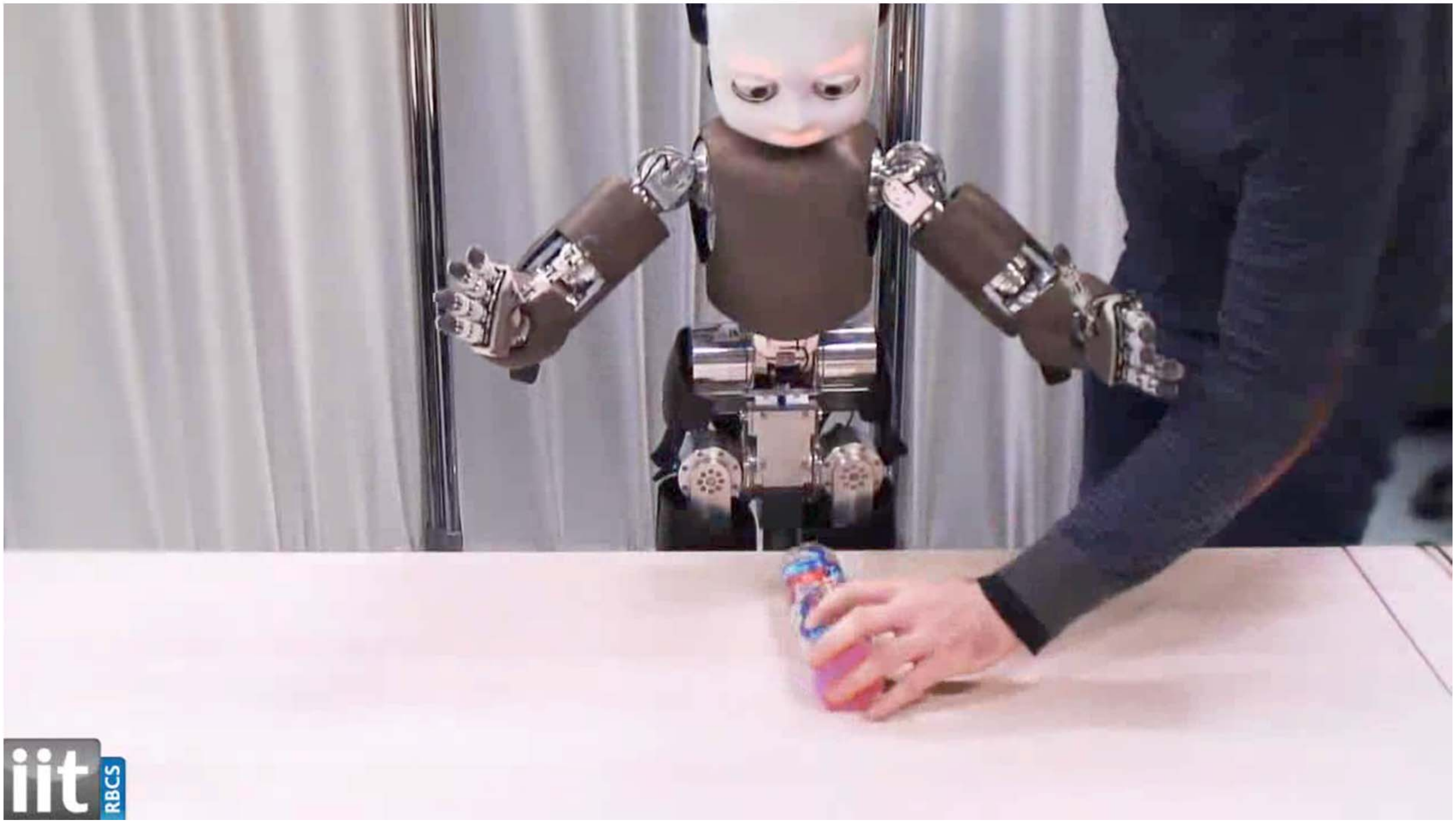


learning object behavior








iit
RBCS

conclusions

- incremental learning is advantageous when models cannot be assumed stationary
- ridge regression with kernel approximation and exact update rule for efficient incremental learning
- RFRR has an *O(1)* time and space complexity per update (suitable for hard real-time)
- number of random features regulates computation vs. accuracy tradeoff

sponsors

- EU Commission projects:
  - RobotCub, grant FP6-004370, http://www.robotcub.org
  - CHRIS, grant FP7-215805, http://www.chrisfp7.eu
  - ITALK, grant FP7-214668, http://italkproject.org
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