Energy-Based Models: The Cure Against Bayesian Fundamentalism

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See: [LeCun et al. 2006]: "A Tutorial on Energy-Based Learning"

[Ranzato et al. AI-Stats 2007], [Ranzato et al. NIPS 2006]

http://yann.lecun.com/exdb/publis/

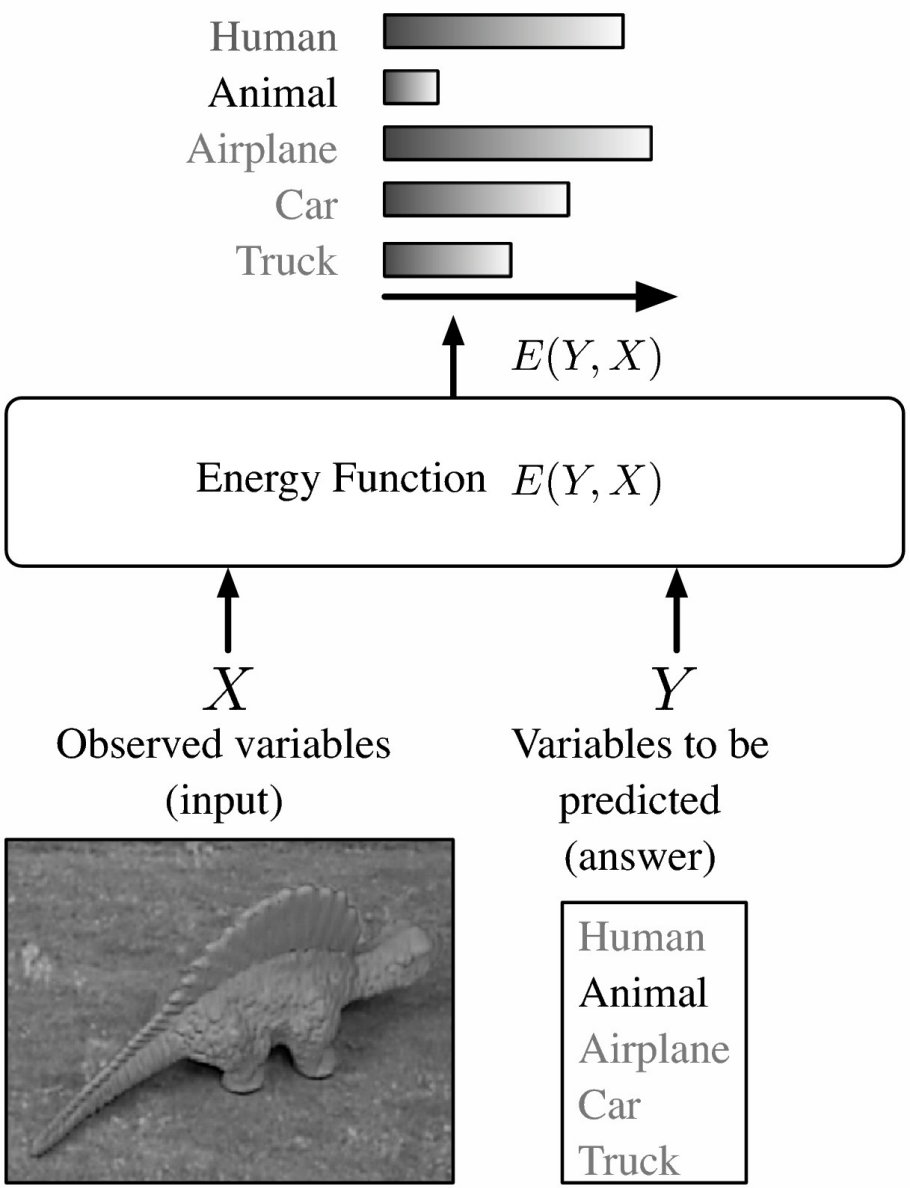
Two Big Problems in Machine Learning

- **1. The "Deep Learning Problem"**
  - "Deep" architectures are necessary to solve the invariance problem in vision (and perception in general)
- **2. The "Partition Function Problem"**
  - Give high probability (or low energy) to good answers
  - Give low probability (or high energy) to bad answers
  - There are too many bad answers!
- **This talk discusses problem #2 first and #1 second.**
  - The partition function problem arises with probabilistic approaches
  - Non-probabilistic approaches may allow us to get around it.
- **Energy-Based Learning provides a framework in which to describe probabilistic and non-probabilistic approaches to learning**
- **Paper: LeCun et al. : "A tutorial on energy-based learning"**
  - http://yann.lecun.com/exdb/publis
  - http://www.cs.nyu.edu/~yann/research/ebm

Plan of the Talks

- **Introduction to Energy-Based Models**
  - Energy-Based inference
  - Examples of architectures and applications, structured outputs
- **Training Energy-Based Models**
  - Designing a loss function. Examples of loss functions.
  - Getting around the partition function problem with EB learning
- **Architectures for structured outputs and sequence labeling**
  - Energy-Based Graphical Models (non-probabilistic factor graphs)
  - Latent variable models
  - Conditional Random Fields, Maximum Margin Markov Nets, Graph Transformer Networks
- **Applications in vision**
  - Hierarchical models of vision and object recognition
  - Unsupervised learning of invariant feature hierarchies.

Energy-Based Model for Decision-Making

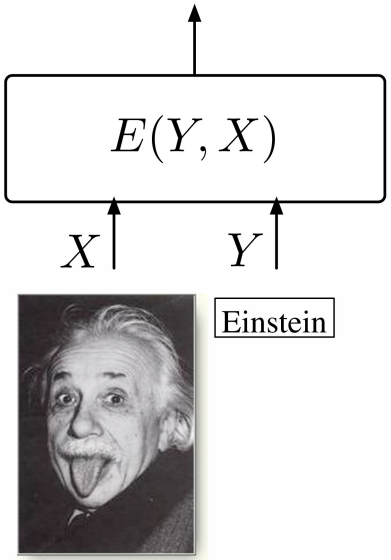


- **Model:** Measures the compatibility between an observed variable X and a variable to be predicted Y through an energy function E(Y,X).

$$Y^* = \text{argmin}_{Y \in \mathcal{Y}} E(Y, X).$$

- **Inference:** Search for the Y that minimizes the energy within a set $\mathcal{Y}$
- If the set has low cardinality, we can use exhaustive search.

Complex Tasks: Inference is non-trivial



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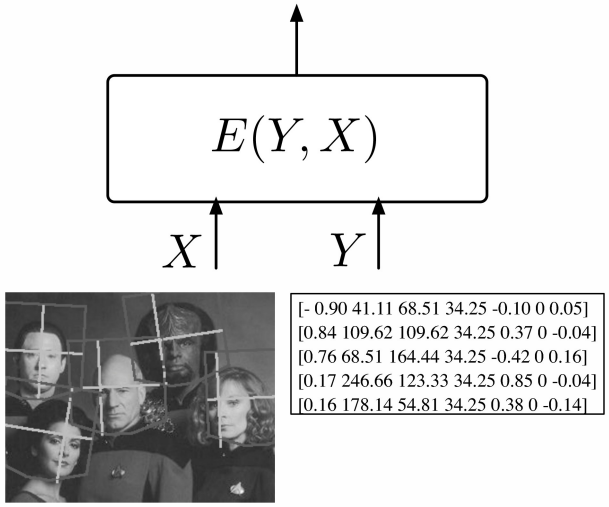

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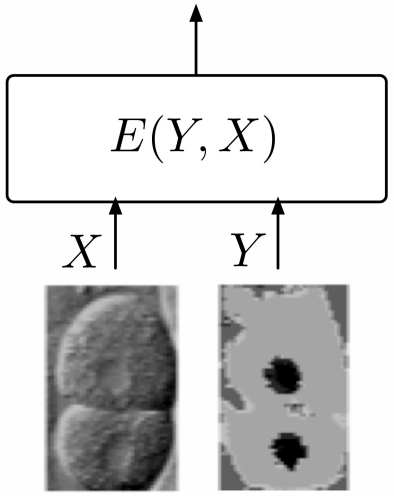

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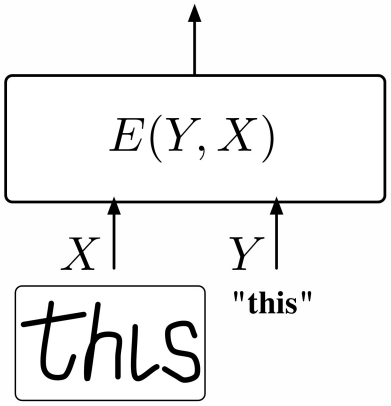

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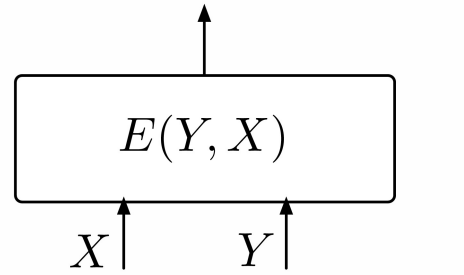

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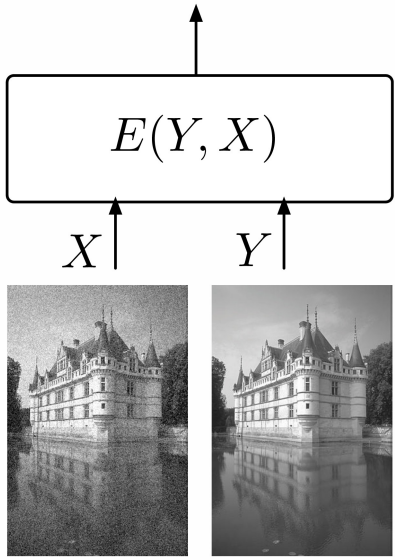

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- When the cardinality or dimension of Y is large, exhaustive search is impractical.
- We need to use a "smart" inference procedure: min-sum, Viterbi, ...

What Questions Can a Model Answer?

- **1. Classification & Decision Making:**
  - "which value of Y is most compatible with X?"
  - Applications: Robot navigation,.....
  - Training: give the lowest energy to the correct answer
- **2. Ranking:**
  - "Is Y1 or Y2 more compatible with X?"
  - Applications: Data-mining....
  - Training: produce energies that rank the answers correctly
- **3. Detection:**
  - "Is this value of Y compatible with X"?
  - Application: face detection....
  - Training: energies that increase as the image looks less like a face.
- **4. Conditional Density Estimation:**
  - "What is the conditional distribution P(Y|X)?"
  - Application: feeding a decision-making system
  - Training: differences of energies must be just so.

Decision-Making versus Probabilistic Modeling

- **Energies are uncalibrated**
  - The energies of two separately-trained systems cannot be combined
  - The energies are uncalibrated (measured in arbitrary units)
- **How do we calibrate energies?**
  - We turn them into probabilities (positive numbers that sum to 1).
  - Simplest way: Gibbs distribution
  - Other ways can be reduced to Gibbs by a suitable redefinition of the energy.

$$P(Y|X) = \frac{e^{-\beta E(Y,X)}}{\int_{y \in \mathcal{Y}} e^{-\beta E(y,X)}},$$

Partition function

Inverse temperature

Architecture and Loss Function

- **Family of energy functions** $\mathcal{E} = \{E(W, Y, X) : \quad W \in \mathcal{W}\}.$
- **Training set** $\mathcal{S} = \{(X^i, Y^i) : i = 1 \ldots P\}$
- **Loss functional / Loss function** $\mathcal{L}(E, \mathcal{S}) \qquad \mathcal{L}(W, \mathcal{S})$
  - Measures the quality of an energy function
- **Training** $W^* = \min_{W \in \mathcal{W}} \mathcal{L}(W, \mathcal{S}).$
- **Form of the loss functional**
  - invariant under permutations and repetitions of the samples

$$\mathcal{L}(E, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} L(Y^i, E(W, \mathcal{Y}, X^i)) + R(W).$$

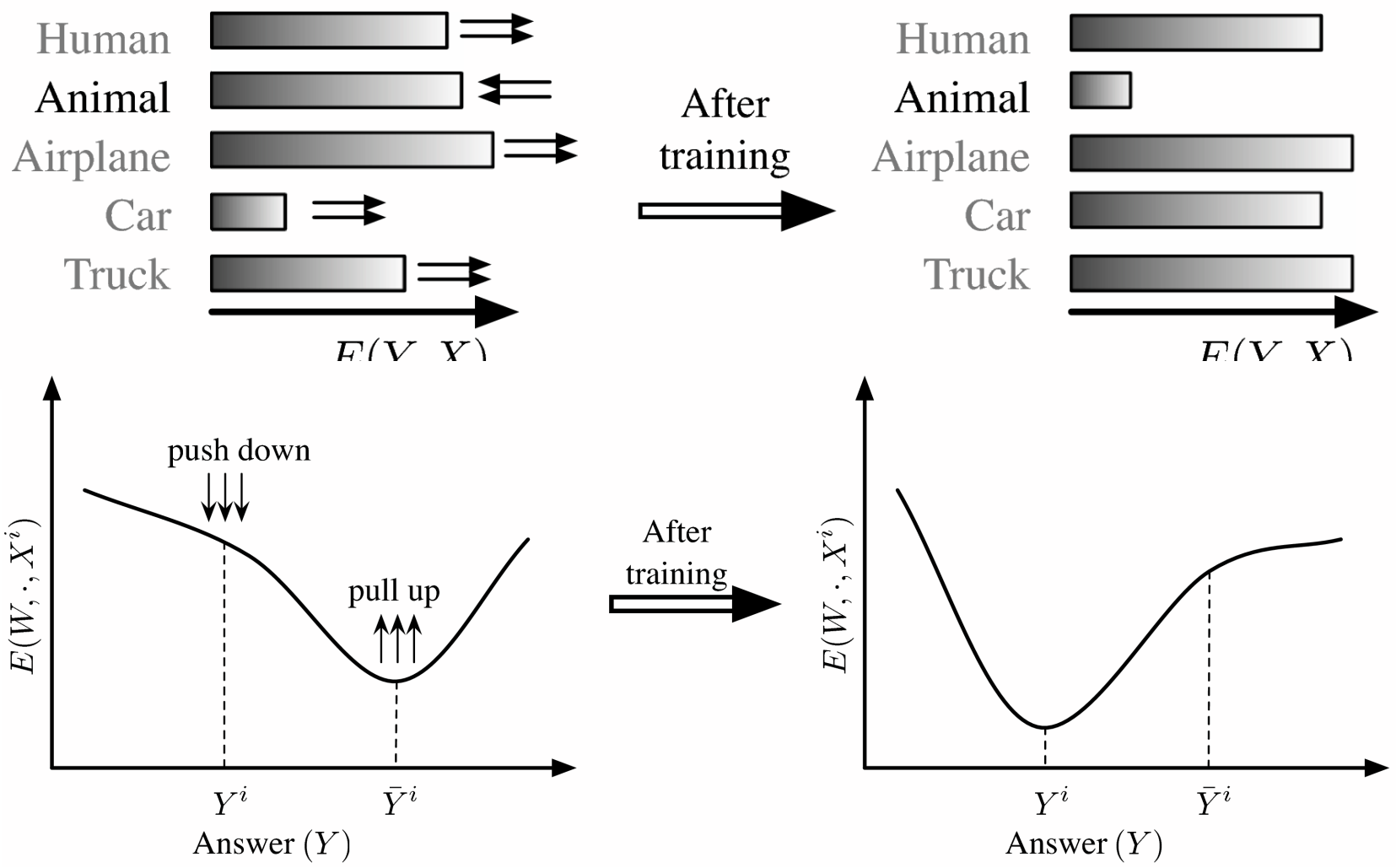
Per-sample loss

Desired answer

Energy surface for a given Xi as Y varies

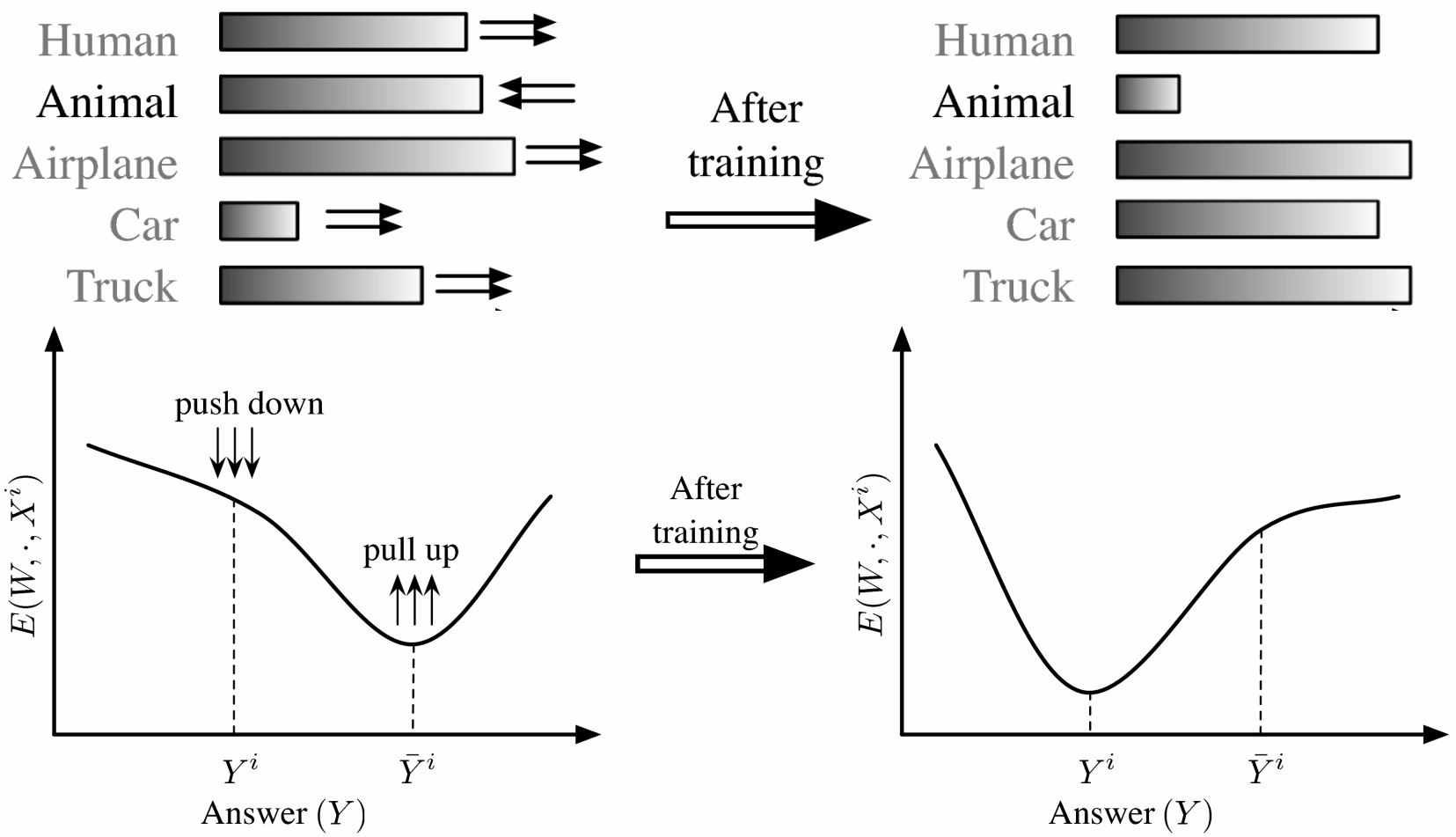
Regularizer

Designing a Loss Functional



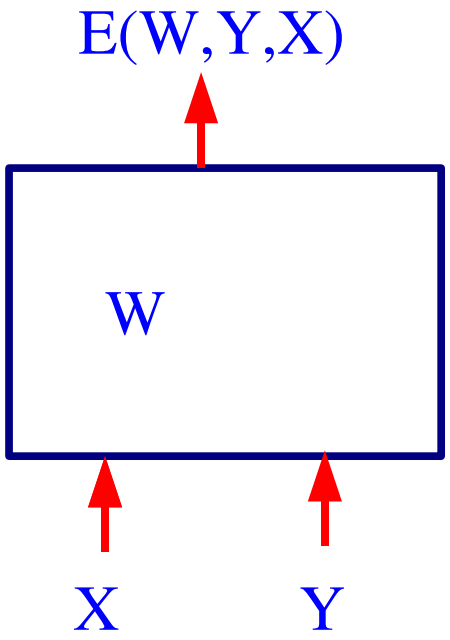
- **Correct answer has the lowest energy -> LOW LOSS**
- **Lowest energy is not for the correct answer -> HIGH LOSS**

Designing a Loss Functional



- **Push down on the energy of the correct answer**
- **Pull up on the energies of the incorrect answers, particularly if they are smaller than the correct one**

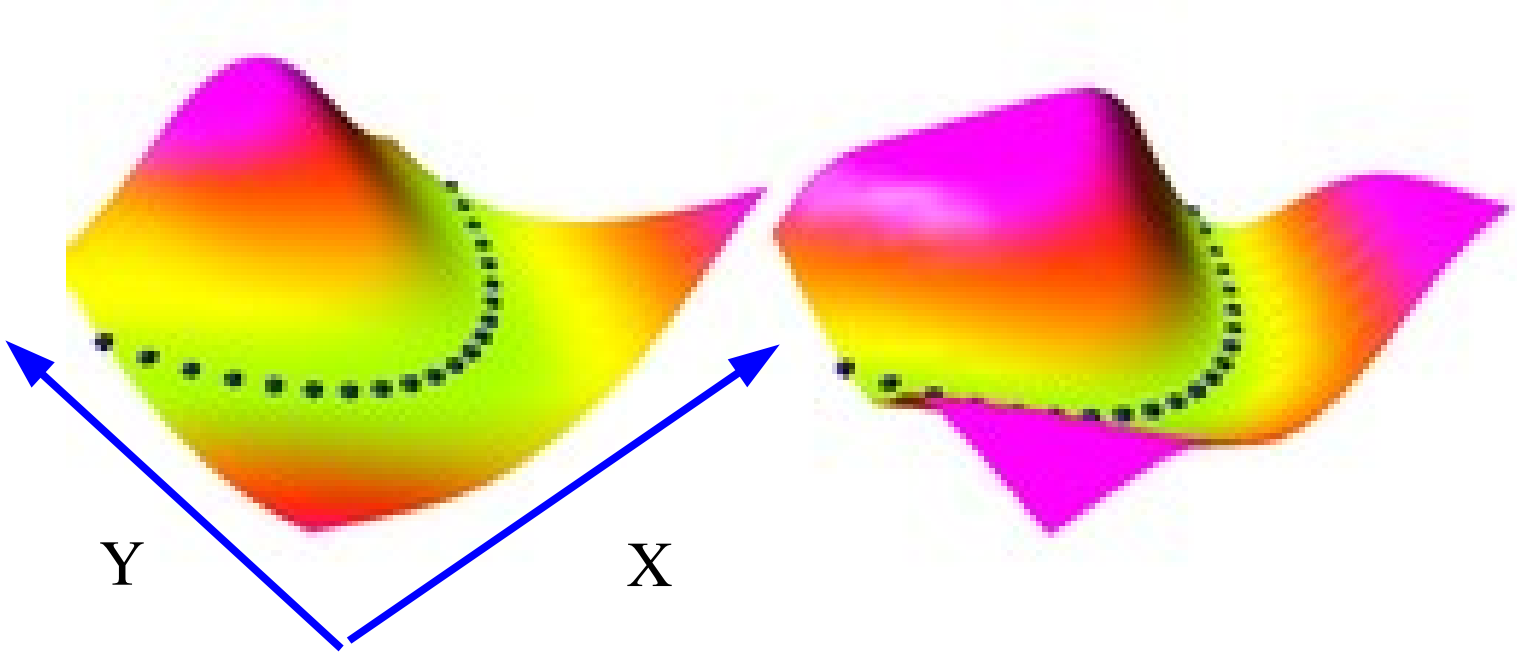
Architecture + Inference Algo + Loss Function = Model



- **1. Design an architecture:** a particular form for E(W,Y,X).
- **2. Pick an inference algorithm for Y:** MAP or conditional distribution, belief prop, min cut, variational methods, gradient descent, MCMC, HMC.....
- **3. Pick a loss function:** in such a way that minimizing it with respect to W over a training set will make the inference algorithm find the correct Y for a given X.
- **4. Pick an optimization method.**

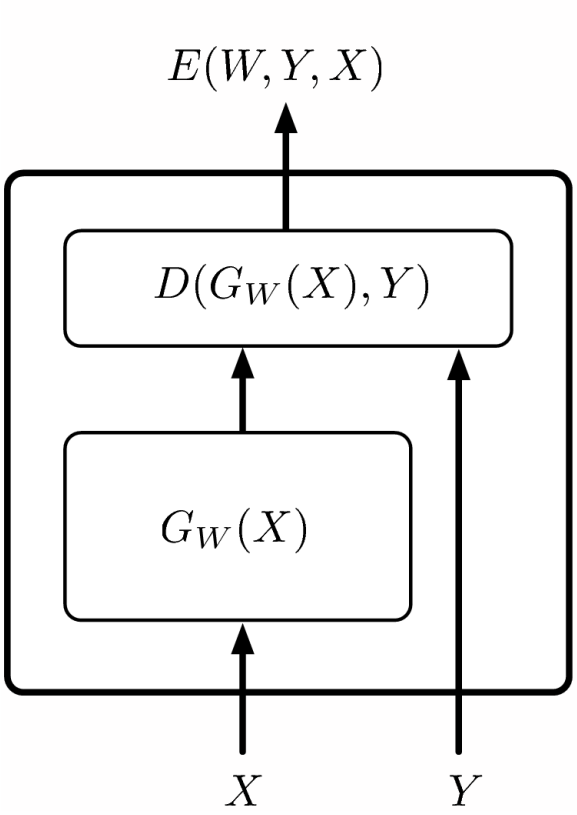
- **PROBLEM: What loss functions will make the machine approach the desired behavior?**

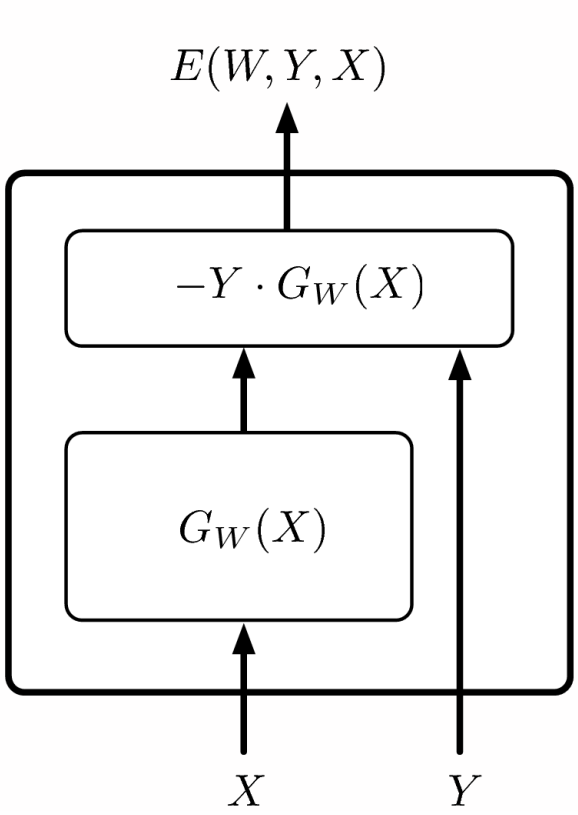
Several Energy Surfaces can give the same answers

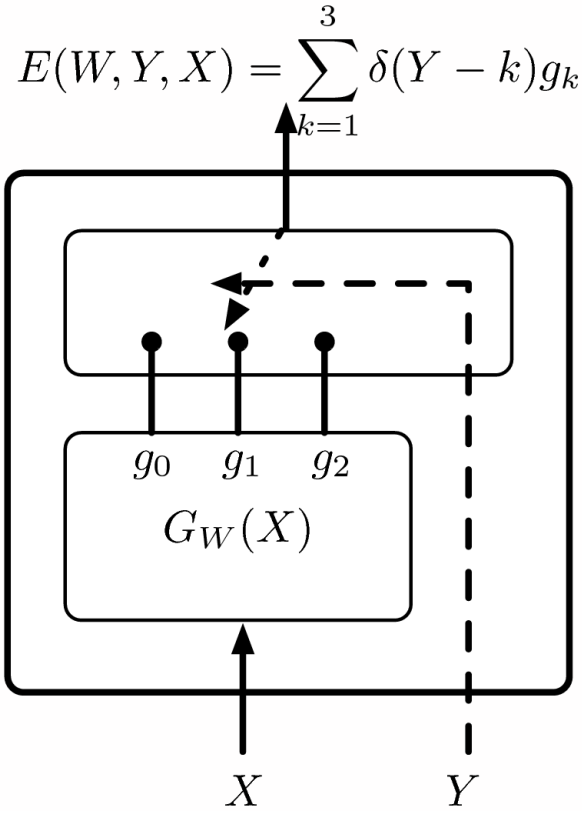


- Both surfaces compute Y=X^2
- MINy E(Y,X) = X^2
- Minimum-energy inference gives us the same answer

Simple Architectures







- **Regression**

$$E(W,Y,X) = \frac{1}{2}||G_W(X) - Y||^2.$$

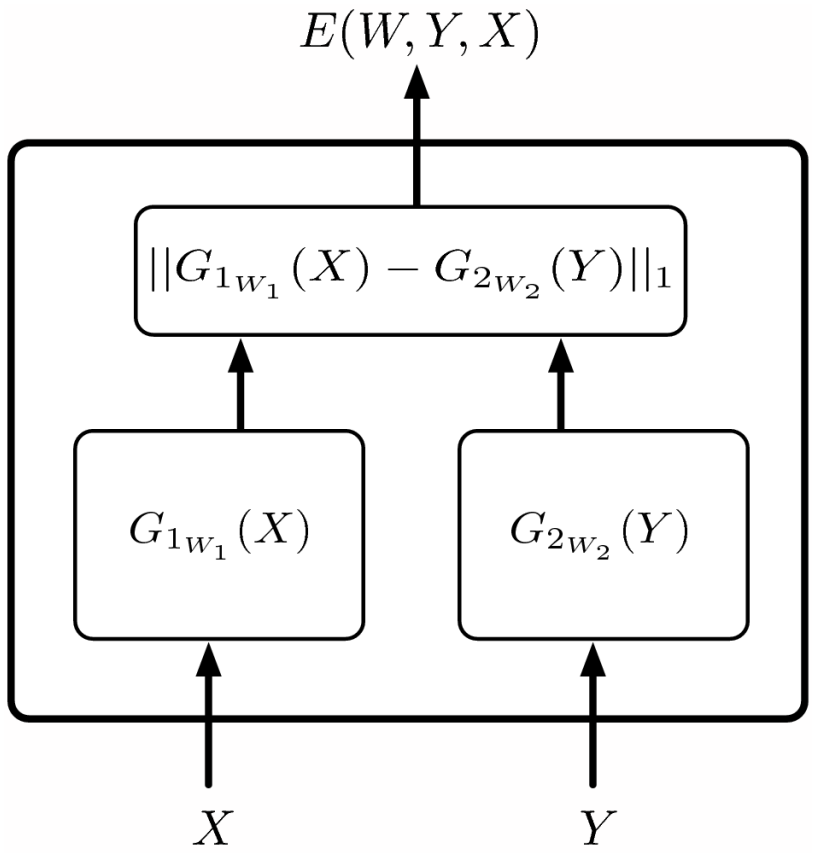
- **Binary Classification**

$$E(W,Y,X) = -YG_W(X),$$

- **Multi-class Classification**

Simple Architecture: Implicit Regression

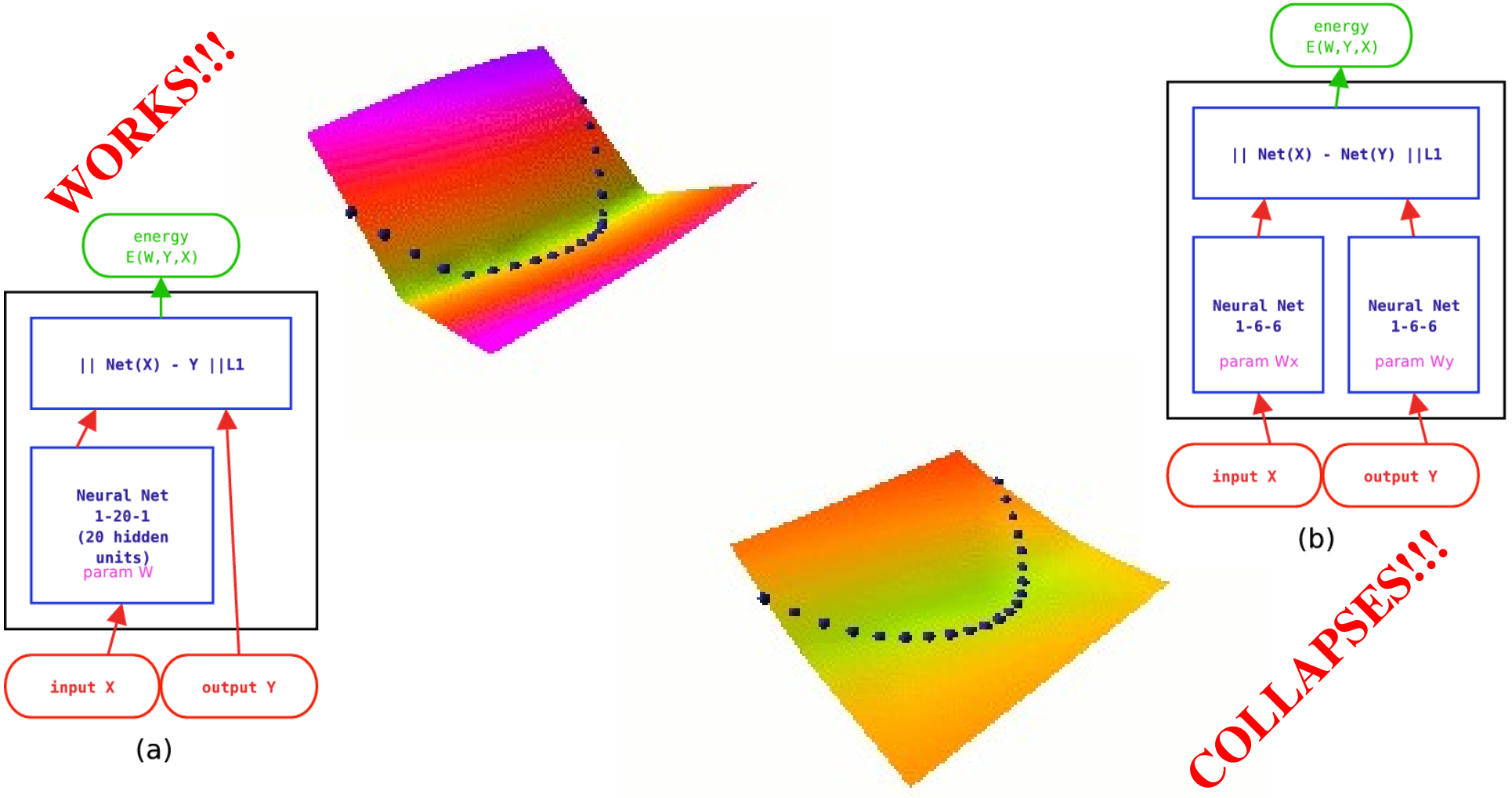
$$E(W, X, Y) = ||G_{1_{W_1}}(X) - G_{2_{W_2}}(Y)||_1,$$



- **The Implicit Regression architecture**
  - allows multiple answers to have low energy.
  - Encodes a constraint between X and Y rather than an explicit functional relationship
  - This is useful for many applications
  - Example: sentence completion: "The cat ate the {mouse,bird,homework,...}"
  - [Bengio et al. 2003]
  - But, inference may be difficult.

Examples of Loss Functions: Energy Loss

- **Energy Loss** $L_{energy}(Y^i, E(W, \mathcal{Y}, X^i)) = E(W, Y^i, X^i).$
  - Simply pushes down on the energy of the correct answer



Examples of Loss Functions:Perceptron Loss

$$L_{perceptron}(Y^i, E(W, \mathcal{Y}, X^i)) = E(W, Y^i, X^i) - \min_{Y \in \mathcal{Y}} E(W, Y, X^i).$$

- **Perceptron Loss [LeCun et al. 1998], [Collins 2002]**
  - Pushes down on the energy of the correct answer
  - Pulls up on the energy of the machine's answer
  - Always positive. Zero when answer is correct
  - No "margin": technically does not prevent the energy surface from being almost flat.
  - Works pretty well in practice, particularly if the energy parameterization does not allow flat surfaces.

Perceptron Loss for Binary Classification

$$L_{perceptron}(Y^i, E(W, \mathcal{Y}, X^i)) = E(W, Y^i, X^i) - \min_{Y \in \mathcal{Y}} E(W, Y, X^i).$$

- **Energy:** $E(W, Y, X) = -Y G_W(X),$
- **Inference:** $Y^* = \text{argmin}_{Y \in \{-1,1\}} - Y G_W(X) = \text{sign}(G_W(X)).$
- **Loss:** $\mathcal{L}_{\text{perceptron}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \left(\text{sign}(G_W(X^i)) - Y^i\right) G_W(X^i).$
- **Learning Rule:** $W \leftarrow W + \eta \left(Y^i - \text{sign}(G_W(X^i))\right) \frac{\partial G_W(X^i)}{\partial W},$
- **If Gw(X) is linear in W:** $E(W, Y, X) = -Y W^T \Phi(X)$

$$W \leftarrow W + \eta \left(Y^i - \text{sign}(W^T \Phi(X^i))\right) \Phi(X^i)$$

Examples of Loss Functions: Generalized Margin Losses

- **First, we need to define the Most Offending Incorrect Answer**

- **Most Offending Incorrect Answer: discrete case**

Definition 1 *Let Y be a discrete variable. Then for a training sample (X^i, Y^i), the* ***most offending incorrect answer*** *$\bar{Y}^i$ is the answer that has the lowest energy among all answers that are incorrect:*

$$\bar{Y}^i = \text{argmin}_{Y \in \mathcal{Y} and Y \neq Y^i} E(W, Y, X^i). \quad (8)$$

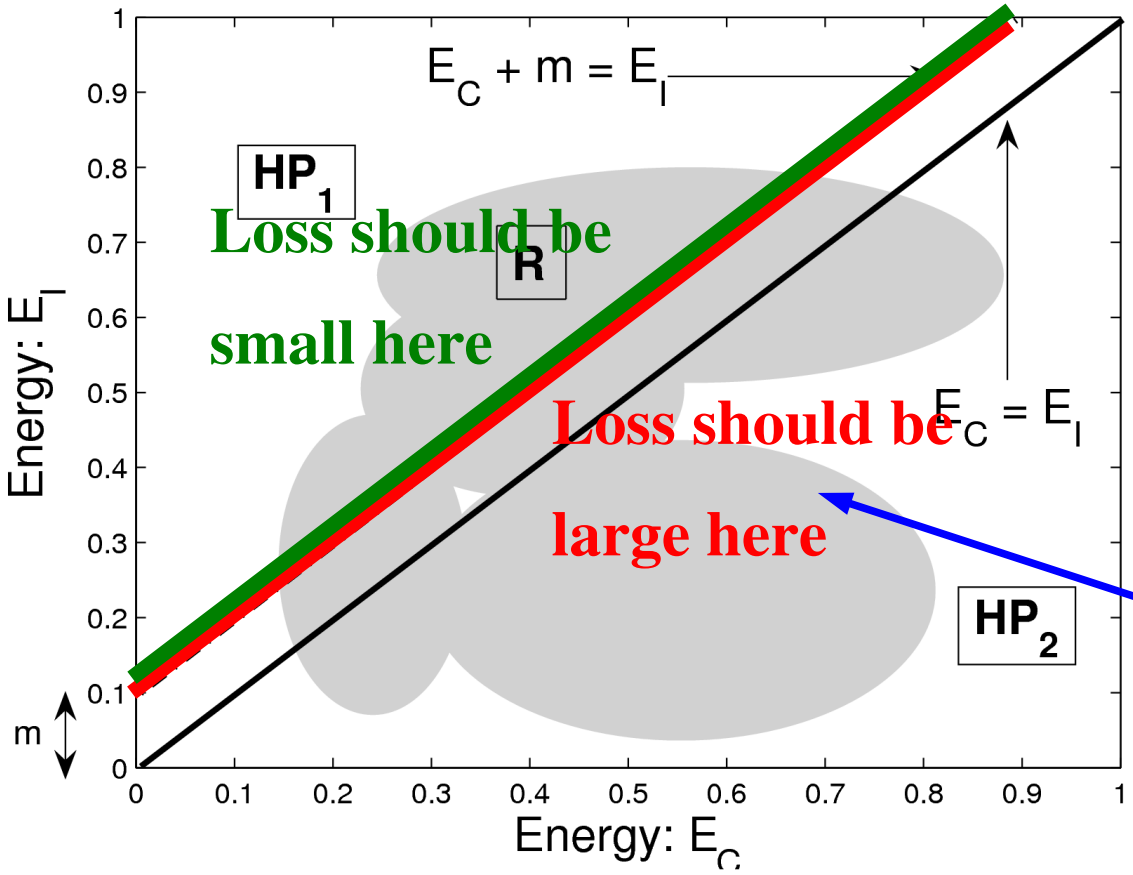
- **Most Offending Incorrect Answer: continuous case**

Definition 2 *Let Y be a continuous variable. Then for a training sample (X^i, Y^i), the* ***most offending incorrect answer*** *$\bar{Y}^i$ is the answer that has the lowest energy among all answers that are at least ϵ away from the correct answer:*

$$\bar{Y}^i = \text{argmin}_{Y \in \mathcal{Y}, \|Y - Y^i\| > \epsilon} E(W, Y, X^i). \quad (9)$$

Examples of Loss Functions: Generalized Margin Losses

$$L_{\text{margin}}(W, Y^i, X^i) = Q_m\left(E(W, Y^i, X^i), E(W, \bar{Y}^i, X^i)\right).$$

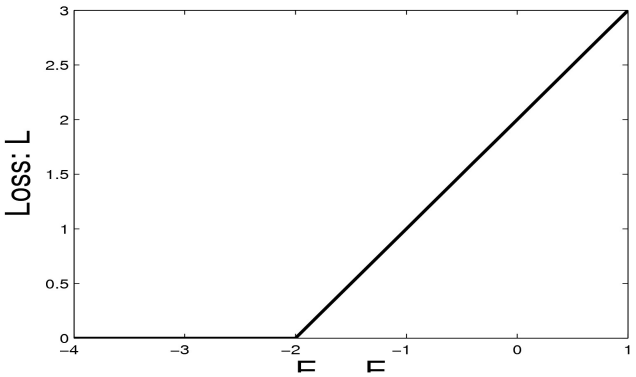


- **Generalized Margin Loss**
  - Qm increases with the energy of the correct answer
  - Qm decreases with the energy of the most offending incorrect answer
  - whenever it is less than the energy of the correct answer plus a margin m.

Examples of Generalized Margin Losses

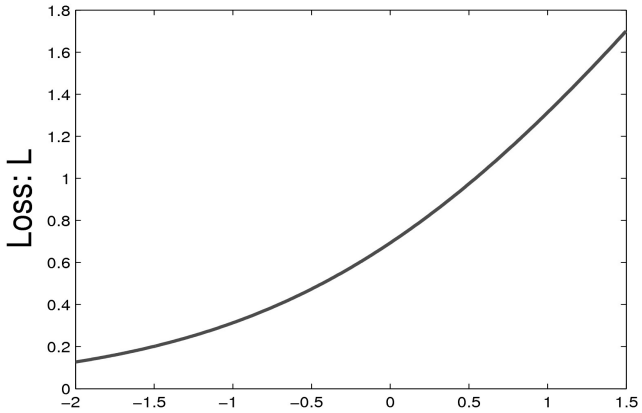
$$L_{\text{hinge}}(W, Y^i, X^i) = \max\left(0, m + E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right),$$

- **Hinge Loss**
  - [Altun et al. 2003], [Taskar et al. 2003]
  - With the linearly-parameterized binary classifier architecture, we get linear SVM



$$L_{\log}(W, Y^i, X^i) = \log\left(1 + e^{E(W,Y^i,X^i) - E(W,\bar{Y}^i,X^i)}\right).$$

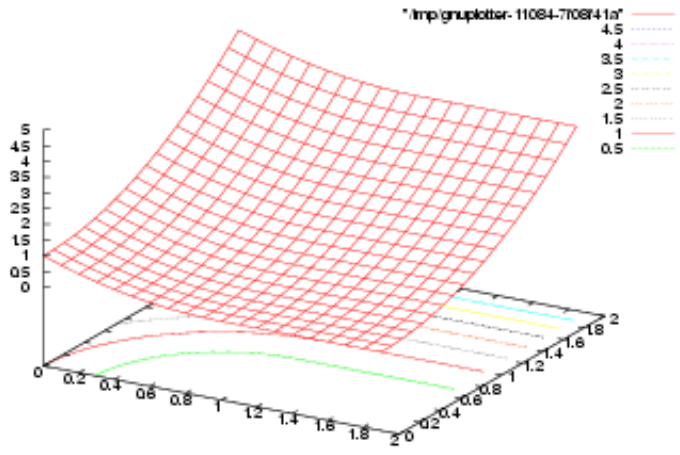
- **Log Loss**
  - "soft hinge" loss
  - With the linearly-parameterized binary classifier architecture, we get linear Logistic Regression

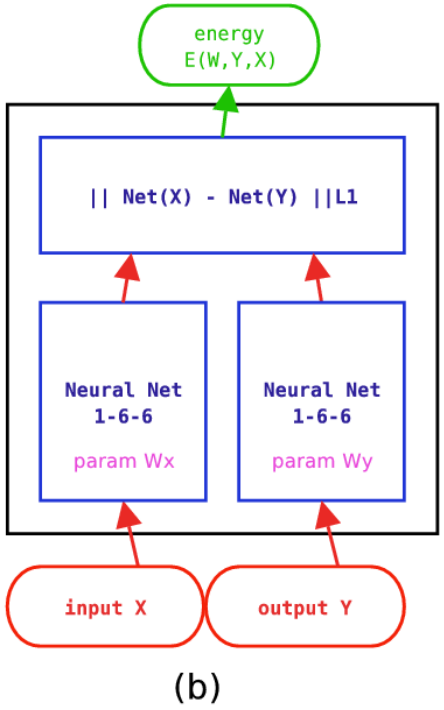


Examples of Margin Losses: Square-Square Loss

$$L_{\text{sq-sq}}(W, Y^i, X^i) = E(W, Y^i, X^i)^2 + \left(\max(0, m - E(W, \bar{Y}^i, X^i))\right)^2 .$$

- **Square-Square Loss**
  - [LeCun-Huang 2005]
  - Appropriate for positive energy functions





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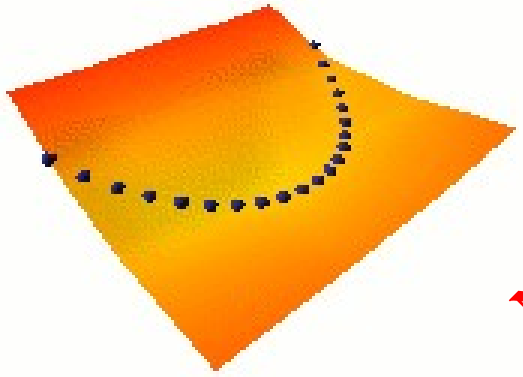
Learning Y = X^2



NO COLLAPSE!!!

Other Margin-Like Losses

- **LVQ2 Loss** [Kohonen, Oja], [Driancourt-Bottou 1991] <- speech recognition

$$L_{\text{lvq2}}(W, Y^i, X^i) = \min\left(1, \max\left(0, \frac{E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)}{\delta E(W, \bar{Y}^i, X^i)}\right)\right),$$

- **Minimum Classification Error Loss** [Juang, Chou, Lee 1997] <- speech r.

$$L_{\text{mce}}(W, Y^i, X^i) = \sigma\left(E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right),$$

$$\sigma(x) = (1 + e^{-x})^{-1}$$

- **Square-Exponential Loss** [Osadchy, Miller, LeCun 2004] <- face detection

$$L_{\text{sq-exp}}(W, Y^i, X^i) = E(W, Y^i, X^i)^2 + \gamma e^{-E(W, \bar{Y}^i, X^i)},$$

Negative Log-Likelihood Loss

- **Conditional probability of the samples (assuming independence)**

$$P(Y^1, \ldots, Y^P | X^1, \ldots, X^P, W) = \prod_{i=1}^{P} P(Y^i | X^i, W).$$

$$-\log \prod_{i=1}^{P} P(Y^i | X^i, W) = \sum_{i=1}^{P} -\log P(Y^i | X^i, W).$$

- **Gibbs distribution:** $$P(Y | X^i, W) = \frac{e^{-\beta E(W, Y, X^i)}}{\int_{y \in \mathcal{Y}} e^{-\beta E(W, y, X^i)}}.$$

$$-\log \prod_{i=1}^{P} P(Y^i | X^i, W) = \sum_{i=1}^{P} \beta E(W, Y^i, X^i) + \log \int_{y \in \mathcal{Y}} e^{-\beta E(W, y, X^i)}.$$

- **We get the NLL loss by dividing by P and Beta:**

$$\mathcal{L}_{\text{nll}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \left(E(W, Y^i, X^i) + \frac{1}{\beta} \log \int_{y \in \mathcal{Y}} e^{-\beta E(W, y, X^i)} \right).$$

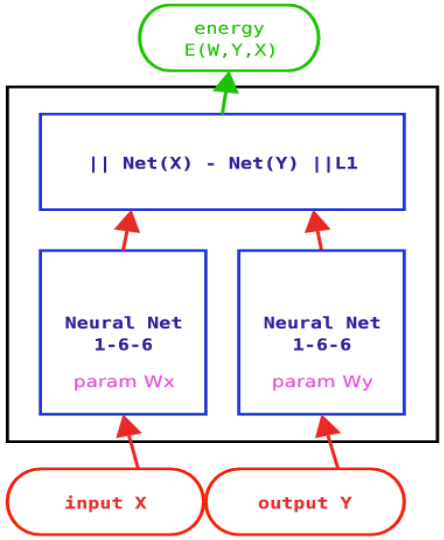
- **Reduces to the perceptron loss when Beta->infinity**

Negative Log-Likelihood Loss

- Pushes down on the energy of the correct answer
- Pulls up on the energies of all answers in proportion to their probability

$$\mathcal{L}_{\text{nll}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \left(E(W, Y^i, X^i) + \frac{1}{\beta} \log \int_{y \in \mathcal{Y}} e^{-\beta E(W, y, X^i)} \right).$$

$$\frac{\partial L_{\text{nll}}(W, Y^i, X^i)}{\partial W} = \frac{\partial E(W, Y^i, X^i)}{\partial W} - \int_{Y \in \mathcal{Y}} \frac{\partial E(W, Y, X^i)}{\partial W} P(Y | X^i, W),$$



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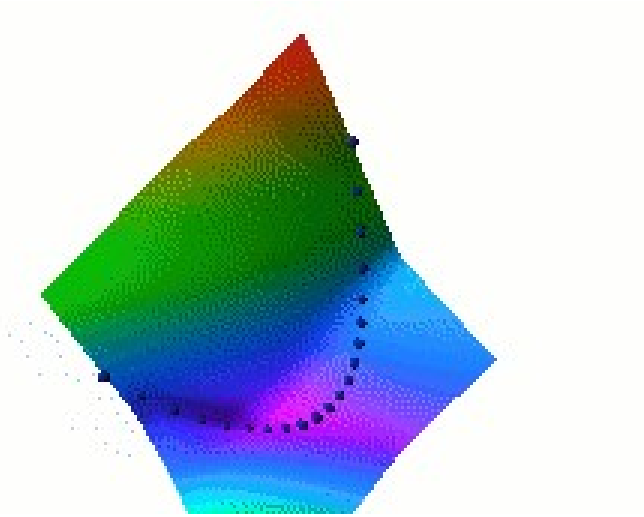


Negative Log-Likelihood Loss: Binary Classification

- **Binary Classifier Architecture:**

$$\mathcal{L}_{\text{nll}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \left[-Y^i G_W(X^i) + \log \left(e^{Y^i G_W(X^i)} + e^{-Y^i G_W(X^i)} \right) \right].$$

$$\mathcal{L}_{\text{nll}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \log \left(1 + e^{-2Y^i G_W(X^i)} \right),$$

- **Linear Binary Classifier Architecture:**

$$\mathcal{L}_{\text{nll}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \log \left(1 + e^{-2Y^i W^T \Phi(X^i)} \right).$$

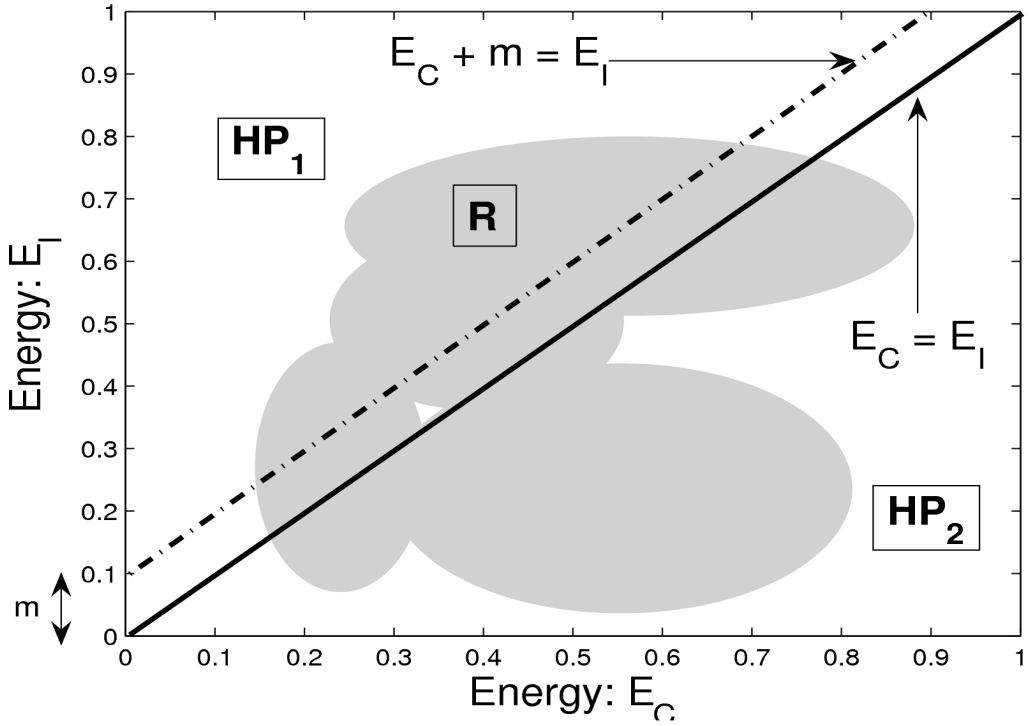
- **Learning Rule in the linear case: logistic regression**
- **NLL is used by lots of speech recognition systems (they call it Maximum Mutual Information), lots of handwriting recognition systems (e.g. Bengio, LeCun 94] [LeCun et al. 98]), CRF [Lafferty et al 2001]**

Negative Log-Likelihood Loss

- **Negative Log Likelihood Loss has been used for a long time in many communities for discriminative learning with structured outputs**
  - Speech recognition: many papers going back to the early 90's [Bengio 92], [Bourlard 94]. They call "Maximum Mutual Information"
  - Handwriting recognition [Bengio LeCun 94], [LeCun et al. 98]
  - Bio-informatics [Haussler]
  - Conditional Random Fields [Lafferty et al. 2001]
  - Lots more.....
  - In all the above cases, it was used with non-linearly parameterized energies.

What Makes a "Good" Loss Function

- **Good loss functions make the machine produce the correct answer**
  - Avoid collapses and flat energy surfaces



Sufficient Condition on the Loss

Let (X^i, Y^i) be the i^{th} training example and m be a positive margin. Minimizing the loss function L will cause the machine to satisfy $E(W, Y^i, X^i) < E(W, Y, X^i) - m$ for all $Y \neq Y^i$, if there exists at least one point (e_1, e_2) with $e_1 + m < e_2$ such that for all points (e'_1, e'_2) with $e'_1 + m \geq e'_2$, we have

$$Q_{[E_y]}(e_1, e_2) < Q_{[E_y]}(e'_1, e'_2),$$

where $Q_{[E_y]}$ is given by

$$L(W, Y^i, X^i) = Q_{[E_y]}(E(W, Y^i, X^i), E(W, \bar{Y}^i, X^i)).$$

What Make a "Good" Loss Function

- **Good and bad loss functions**

Loss (equation #)	Formula	Margin
energy loss	$E(W, Y^i, X^i)$	none
perceptron	$E(W, Y^i, X^i) - \min_{Y \in \mathcal{Y}} E(W, Y, X^i)$	0
hinge	$\max\left(0, m + E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right)$	m
log	$\log\left(1 + e^{E(W,Y^i,X^i) - E(W,\bar{Y}^i,X^i)}\right)$	> 0
LVQ2	$\min\left(M, \max(0, E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right)$	0
MCE	$\left(1 + e^{-\left(E(W,Y^i,X^i) - E(W,\bar{Y}^i,X^i)\right)}\right)^{-1}$	> 0
square-square	$E(W, Y^i, X^i)^2 - \left(\max(0, m - E(W, \bar{Y}^i, X^i))\right)^2$	m
square-exp	$E(W, Y^i, X^i)^2 + \beta e^{-E(W,\bar{Y}^i,X^i)}$	> 0
NLL/MMI	$E(W, Y^i, X^i) + \frac{1}{\beta} \log \int_{y \in \mathcal{Y}} e^{-\beta E(W,y,X^i)}$	> 0
MEE	$1 - e^{-\beta E(W,Y^i,X^i)} / \int_{y \in \mathcal{Y}} e^{-\beta E(W,y,X^i)}$	> 0

Advantages/Disadvantages of various losses

- **Loss functions differ in how they pick the point(s) whose energy is pulled up, and how much they pull them up**
- **Losses with a log partition function in the contrastive term pull up all the bad answers simultaneously.**
  - This may be good if the gradient of the contrastive term can be computed efficiently
  - This may be bad if it cannot, in which case we might as well use a loss with a single point in the contrastive term
- **Variational methods pull up many points, but not as many as with the full log partition function.**
- **Efficiency of a loss/architecture: how many energies are pulled up for a given amount of computation?**
  - The theory for this is does not exist. It needs to be developed

Latent Variable Models

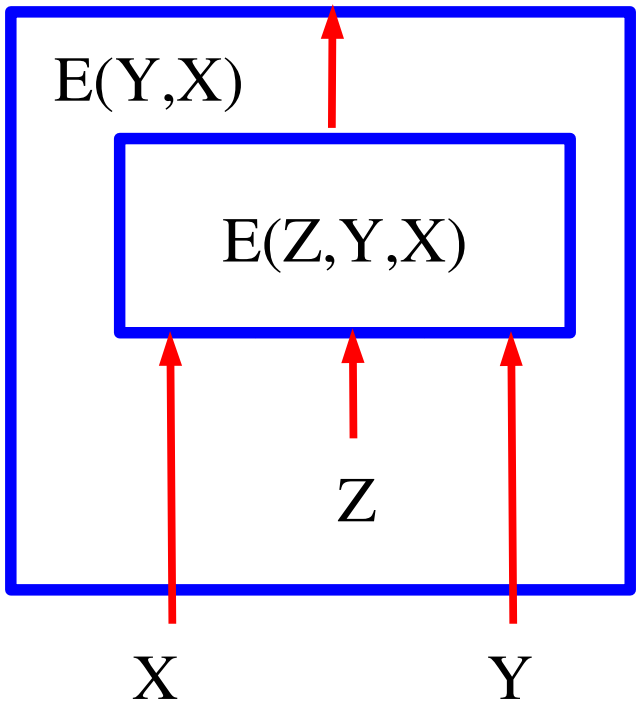
- **The energy includes "hidden" variables Z whose value is never given to us**
  - We can minimize the energy over those latent variables
  - We can also "marginalize" the energy over the latent

Minimization over latent variables:

$$E(Y,X) = \min_{Z \in \mathcal{Z}} E(Z,Y,X).$$

Marginalization over latent variables:

$$E(X,Y) = -\frac{1}{\beta} \log \int_{z \in \mathcal{Z}} e^{-\beta E(z,Y,X)}$$



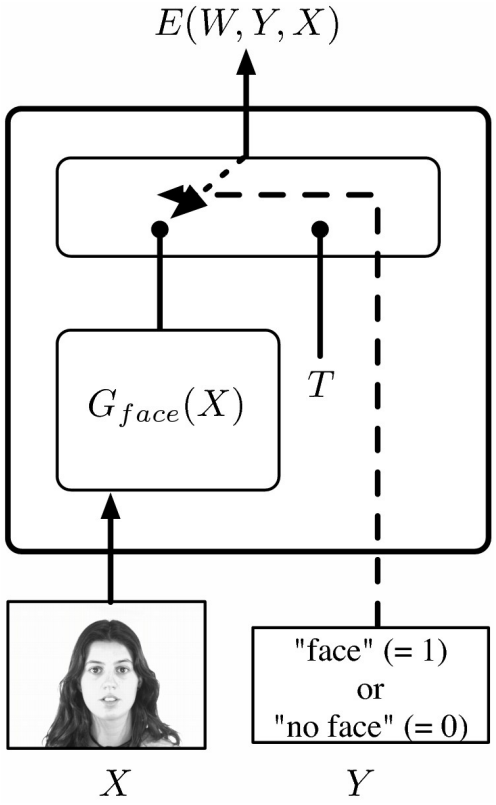
Estimation this integral may require some approximations (sampling, variational methods,....)

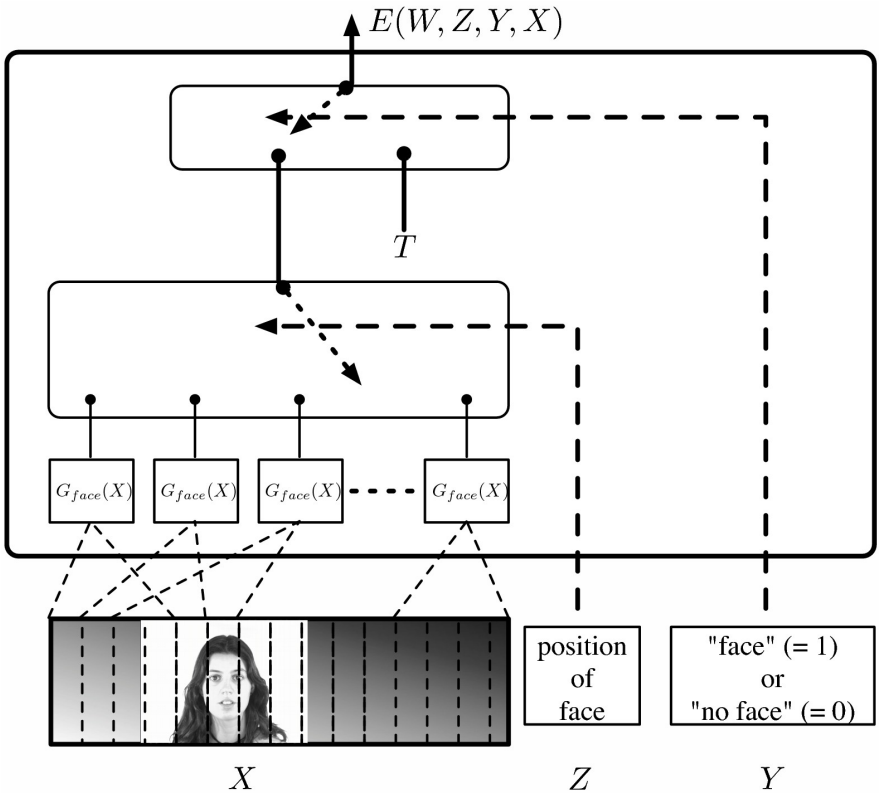
Latent Variable Models

- **The energy includes "hidden" variables Z whose value is never given to us**

$$E(Y, X) = \min_{Z \in \mathcal{Z}} E(Z, Y, X).$$

$$Y^* = \text{argmin}_{Y \in \mathcal{Y}, Z \in \mathcal{Z}} E(Z, Y, X).$$





What can the latent variables represent?

- **Variables that would make the task easier if they were known:**
  - **Face recognition**: the gender of the person, the orientation of the face.
  - **Object recognition**: the pose parameters of the object (location, orientation, scale), the lighting conditions.
  - **Parts of Speech Tagging**: the segmentation of the sentence into syntactic units, the parse tree.
  - **Speech Recognition**: the segmentation of the sentence into phonemes or phones.
  - **Handwriting Recognition**: the segmentation of the line into characters.
- **In general, we will search for the value of the latent variable that allows us to get an answer (Y) of smallest energy.**

Probabilistic Latent Variable Models

- Marginalizing over latent variables instead of minimizing.

$$P(Z,Y|X) = \frac{e^{-\beta E(Z,Y,X)}}{\int_{y\in\mathcal{Y},\ z\in\mathcal{Z}} e^{-\beta E(y,z,X)}}.$$

$$P(Y|X) = \frac{\int_{z\in\mathcal{Z}} e^{-\beta E(Z,Y,X)}}{\int_{y\in\mathcal{Y},\ z\in\mathcal{Z}} e^{-\beta E(y,z,X)}}.$$

- Equivalent to traditional energy-based inference with a redefined energy function:

$$Y^* = \text{argmin}_{Y\in\mathcal{Y}} -\frac{1}{\beta}\log\int_{z\in\mathcal{Z}} e^{-\beta E(z,Y,X)}.$$

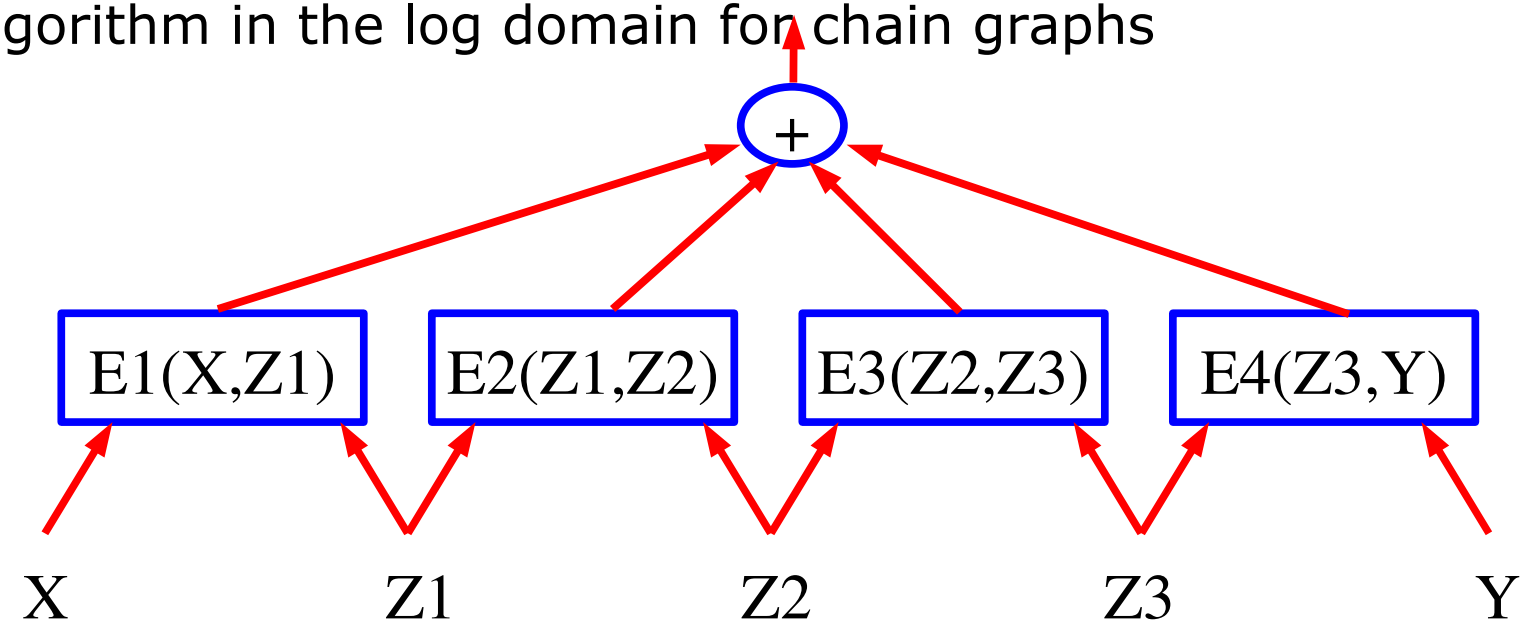
- Reduces to minimization when Beta->infinity

Efficient Inference: Energy-Based Factor Graphs

- **Graphical models have given us efficient inference algorithms, such as belief propagation and its numerous variations.**
- **Traditionally, graphical models are viewed as probabilistic models**
- **At first glance, is seems difficult to dissociate graphical models from the probabilistic view (think "Bayesian networks").**
- **Energy-Based Factor Graphs are an extension of graphical models to non-probabilistic settings.**
- **An EBFG is an energy function that can be written as a sum of "factor" functions that take different subsets of variables as inputs.**
- **Basically, most algorithms for probabilistic factor graphs (such as belief prop) have a counterpart for EBFG:**
  - Operations are performed in the log domain
  - The normalization steps are left out.

Energy-Based Factor Graphs

- **When the energy is a sum of partial energy functions (or when the probability is a product of factors):**
  - An EBM can be seen as an unnormalized factor graph in the log domain
  - Our favorite efficient inference algorithms can be used for inference (without the normalization step).
  - Min-sum algorithm (instead of max-product), Viterbi for chain graphs
  - (Log/sum/exp)-sum algorithm (instead of sum-product), Forward algorithm in the log domain for chain graphs

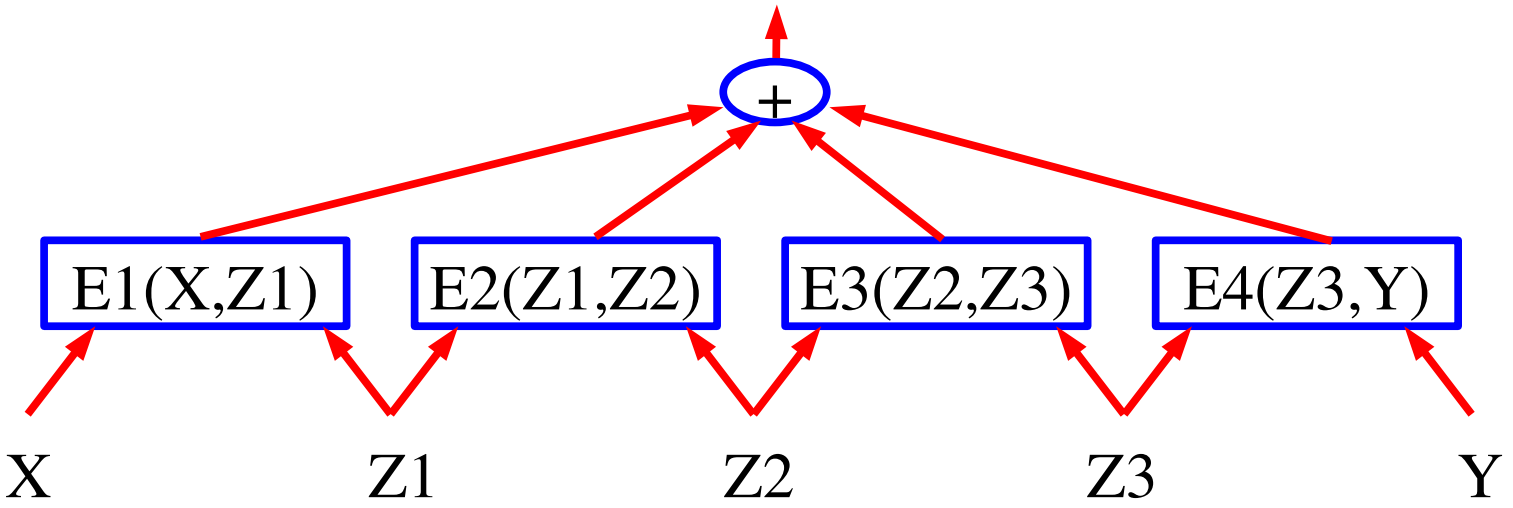


EBFG for Structured Outputs: Sequences, Graphs, Images

- **Structured outputs**
  - When Y is a complex object with components that must satisfy certain constraints.
- **Typically, structured outputs are sequences of symbols that must satisfy "grammatical" constraints**
  - spoken/handwritten word recognition
  - spoken/written sentence recognition
  - DNA sequence analysis
  - Parts of Speech tagging
  - Automatic Machine Translation
- **In General, structured outputs are collections of variables in which subsets of variables must satisfy constraints**
  - Pixels in an image for image restoration
  - Labels of regions for image segmentations
- **We represent the constraints using an Energy-Based Factor Graph.**

Energy-Based Factor Graphs: Three Inference Problems

- X: input, Y: output, Z: latent variables, Energy: E(Z,Y,X)
- Minimization over Y and Z
  - $E(Y,X) = \min_{Z \in \mathcal{Z}} E(Z,Y,X). \qquad Y^* = \text{argmin}_{Y \in \mathcal{Y}} E(Y,X).$
- Min over Y, marginalization over Z (E(X,Y) is a "free energy")
  - $E(X,Y) = -\frac{1}{\beta} \log \int_{z \in \mathcal{Z}} e^{-\beta E(z,Y,X)} \quad Y^* = \text{argmin}_{Y \in \mathcal{Y}} E(Y,X).$
- Marginal Distribution over Y
  - $$P(Y|X) = \frac{e^{-\beta E(Y,X)}}{\int_{y \in \mathcal{Y}} e^{-\beta E(y,X)}},$$

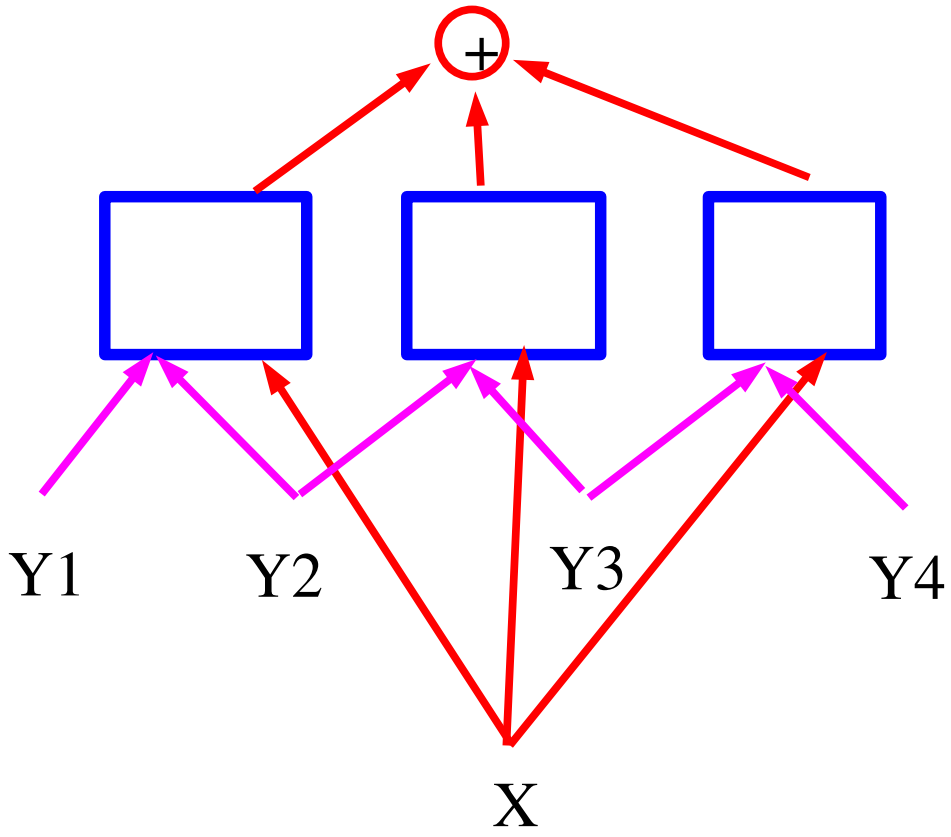


Energy-Based Factor Graphs: simple graphs

- **Sequence Labeling**

$$Y^* = \text{argmin}_{Y \in \mathcal{Y}, Z \in \mathcal{Z}} E(Z, Y, X).$$

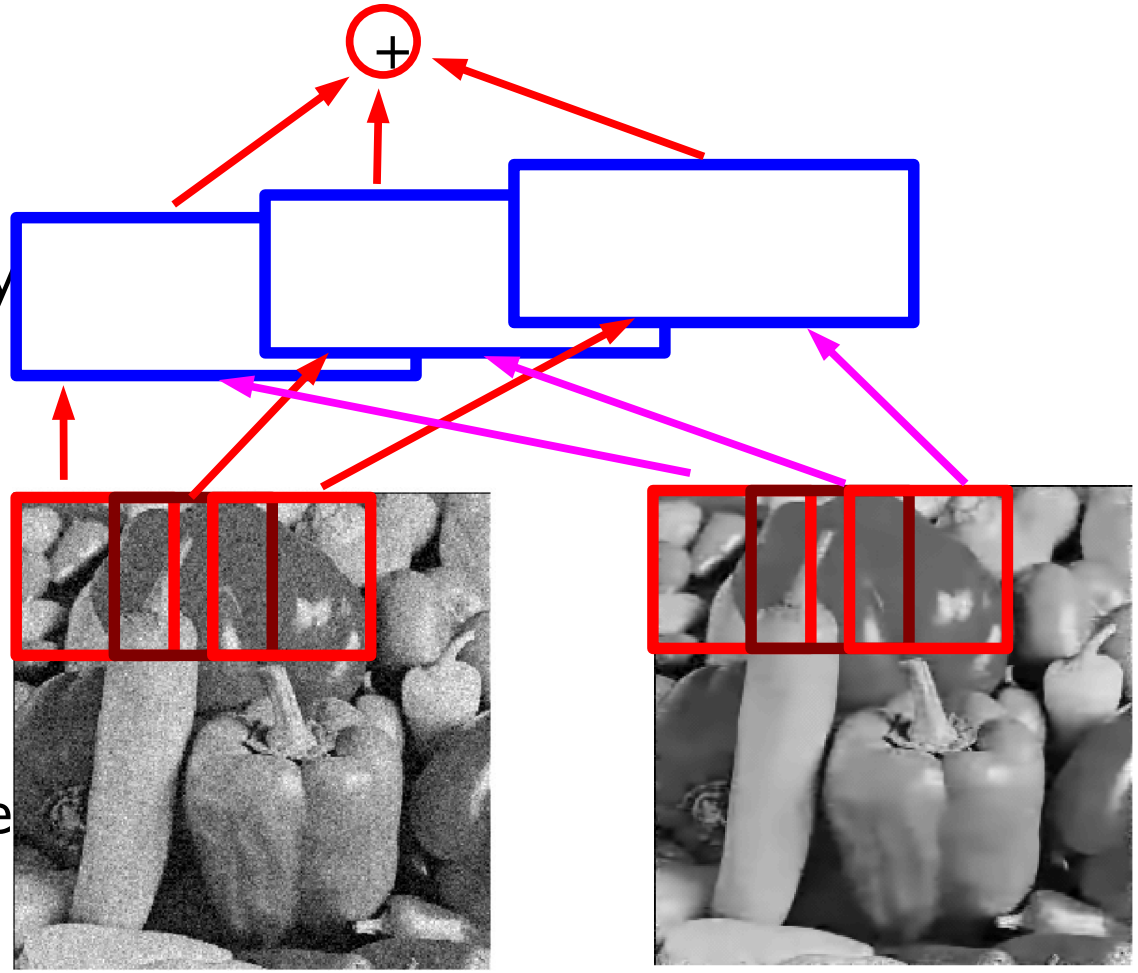
  - Output is a sequence Y1,Y2,Y3,Y4.....
  - NLP parsing, MT, speech/handwriting recognition, biological sequence analysis
  - The factors ensure grammatical consistency
  - They give low energy to consistent sub-sequences of output symbols
  - The graph is generally simple (chain or tree)/
  - Inference is easy (dynamic programming)



Energy-Based Factor Graphs: complex/loopy graphs

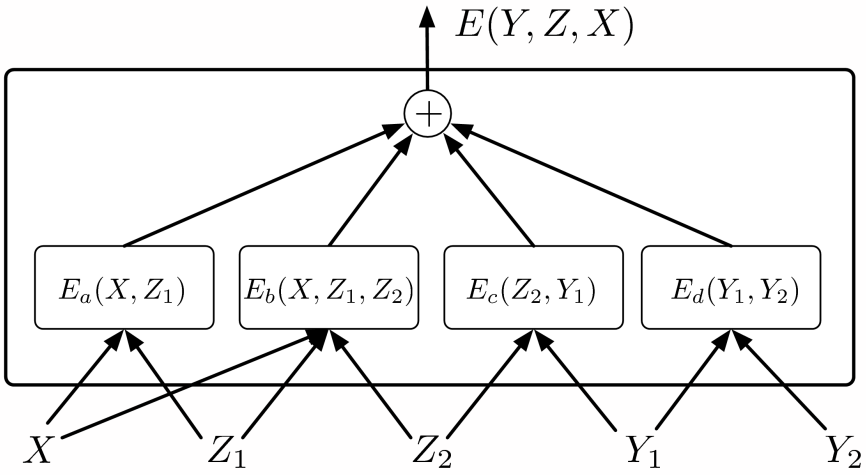
- **Image restoration**
  - The factors ensure local consistency on small overlapping patches
  - They give low energy to "clean" patches, given the noisy versions
  - The graph is loopy when the patches overlap.
  - Inference is difficult, particularly when the patches are large,and when the number of greyscale

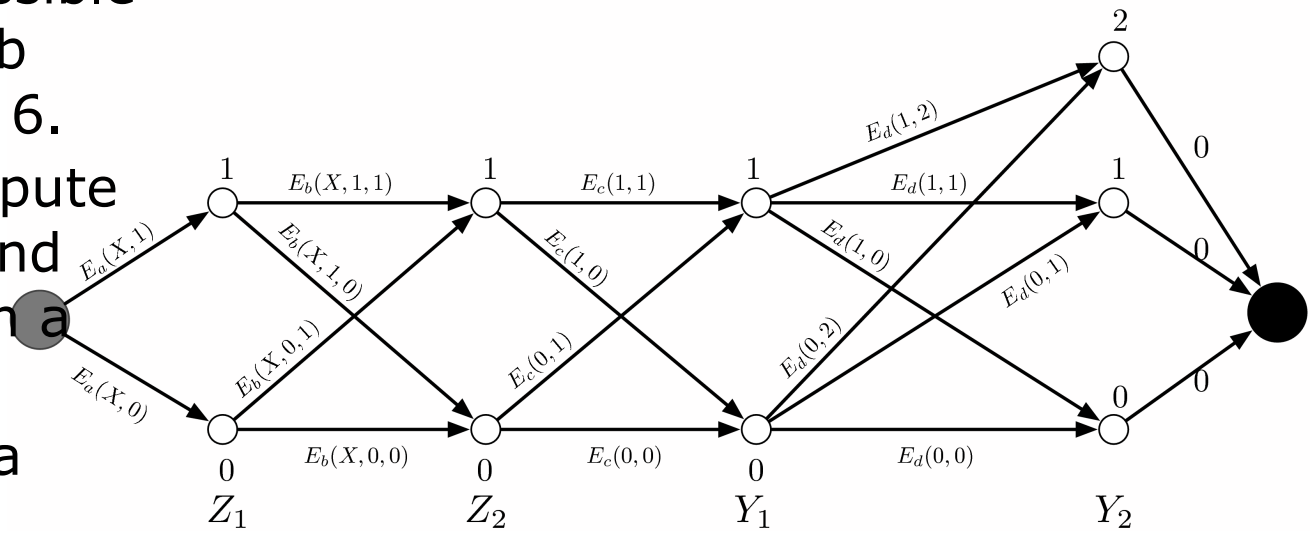
$$Y^* = \text{argmin}_{Y \in \mathcal{Y}} E(Y, X).$$



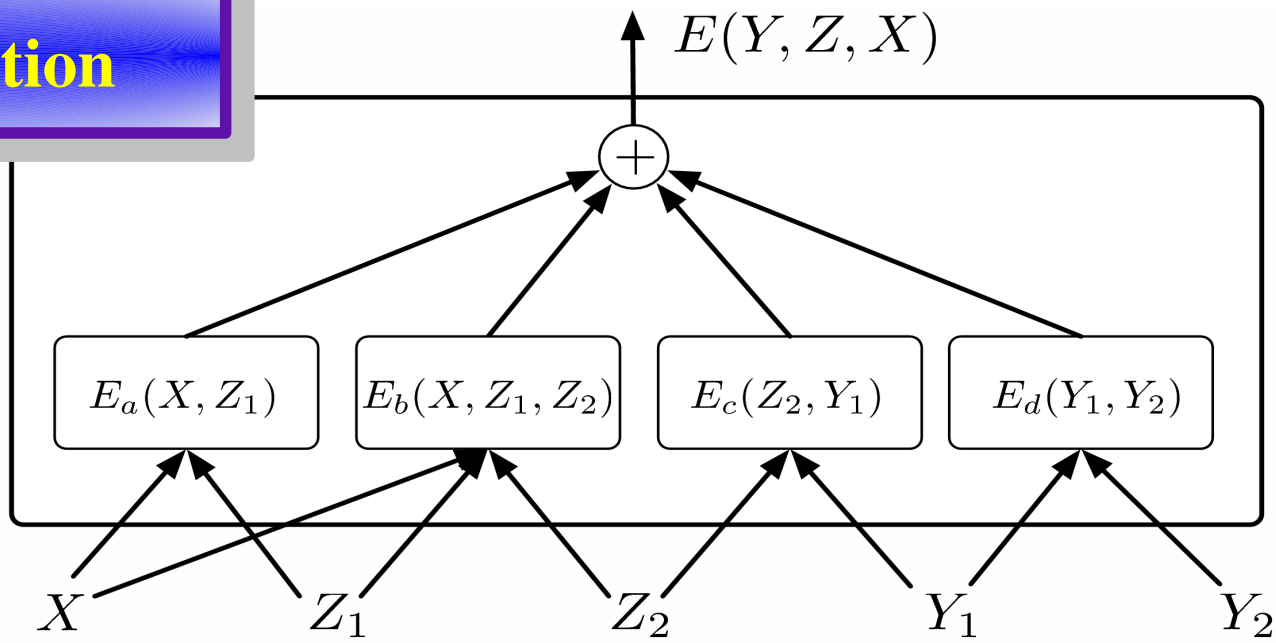
Efficient Inference in simple EBFG

- **The energy is a sum of "factor" functions, the graph is a chain**
- **Example:**
  - Z1, Z2, Y1 are binary
  - Z2 is ternary
  - A naïve exhaustive inference would require 2x2x2x3 energy evaluations (= 96 factor evaluations)
  - BUT: Ea only has 2 possible input configurations, Eb and Ec have 4, and Ed 6.
  - Hence, we can precompute the 16 factor values, and put them on the arcs in a graph.
  - A path in the graph is a config of variable



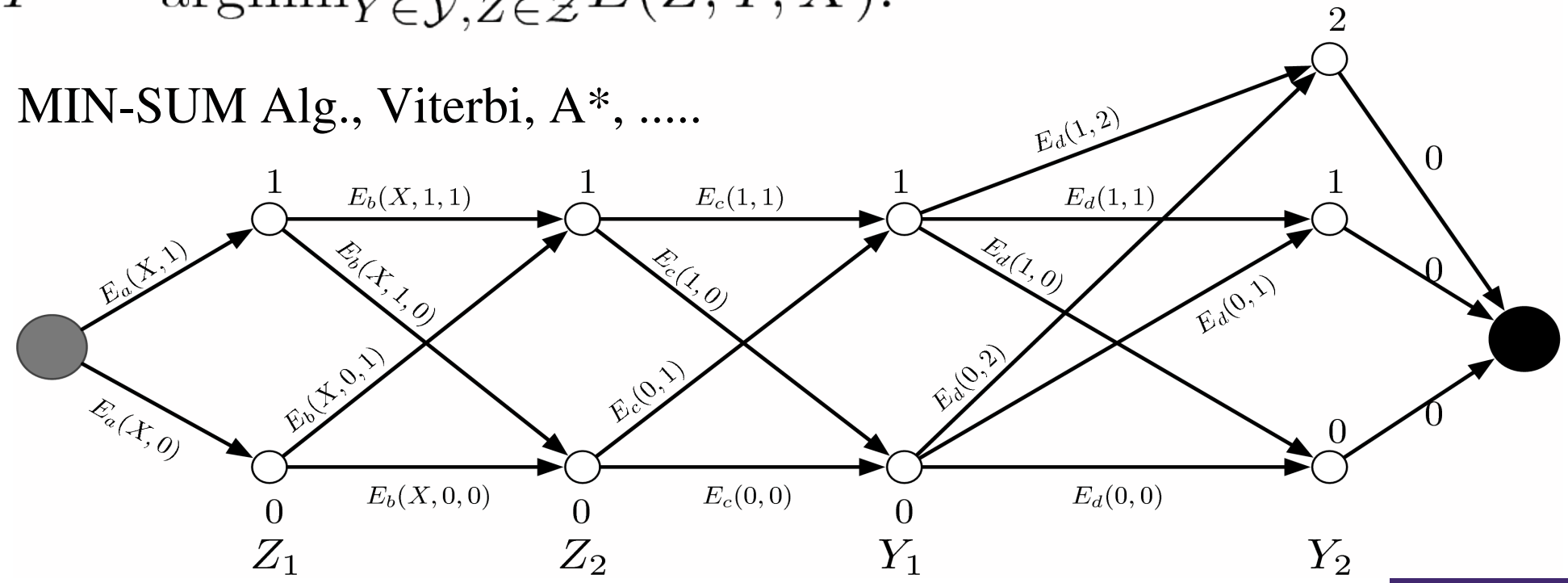


Minimization



$$Y^* = \text{argmin}_{Y \in \mathcal{Y}, Z \in \mathcal{Z}} E(Z, Y, X).$$

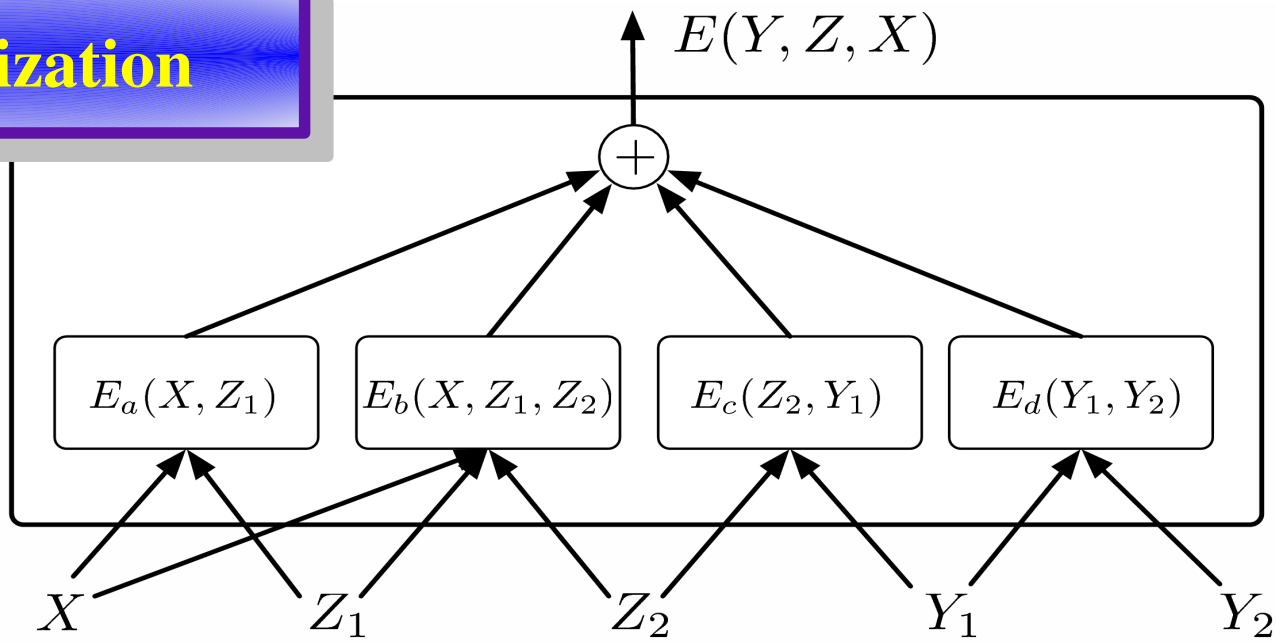
MIN-SUM Alg., Viterbi, A*, ...



Energy-Based Belief Prop: Minimization over Latent Variables

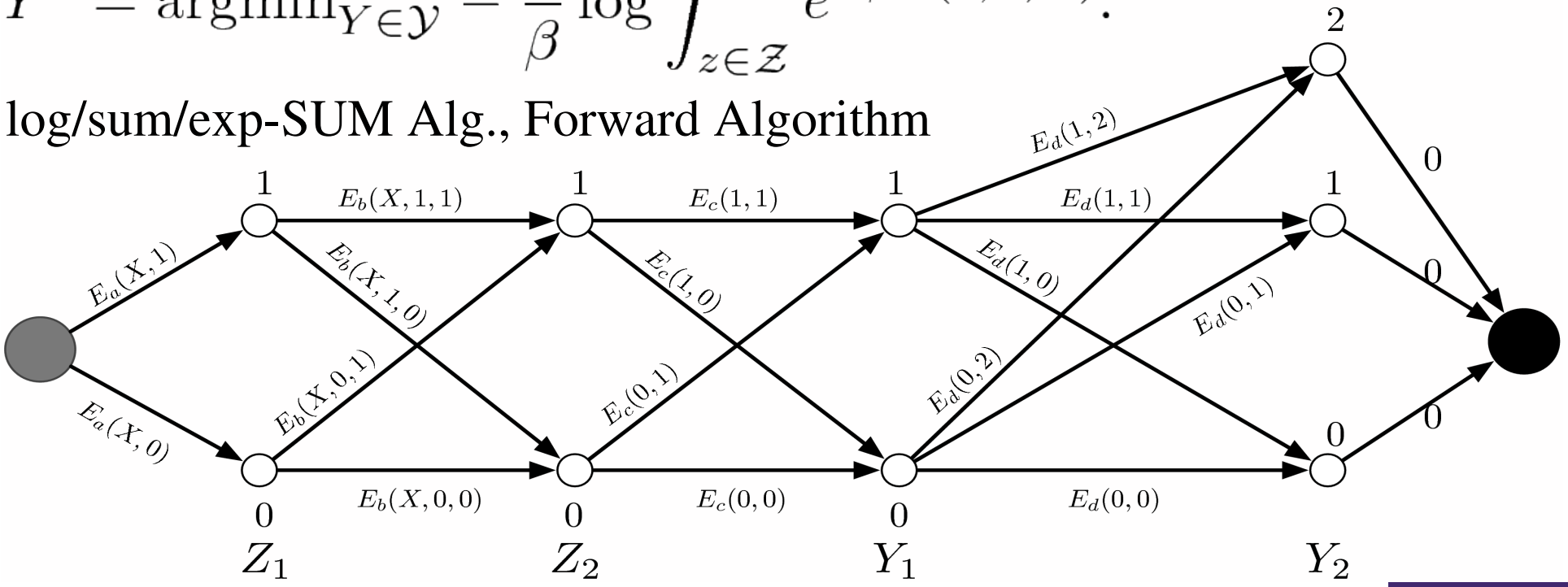
- The previous picture shows a chain graph of factors with 2 inputs.
- The extension of this procedure to trees, with factors that can have more than 2 inputs is the "min-sum" algorithm (a non-probabilistic form of belief propagation)
- Basically, it is the sum-product algorithm with a different semi-ring algebra (min instead of sum, sum instead of product), without the normalization step.
  - [Kschischang, Frey, Loeliger, 2001][McKay's book]

Marginalization



$$Y^* = \text{argmin}_{Y \in \mathcal{Y}} -\frac{1}{\beta} \log \int_{z \in \mathcal{Z}} e^{-\beta E(z,Y,X)}.$$

log/sum/exp-SUM Alg., Forward Algorithm



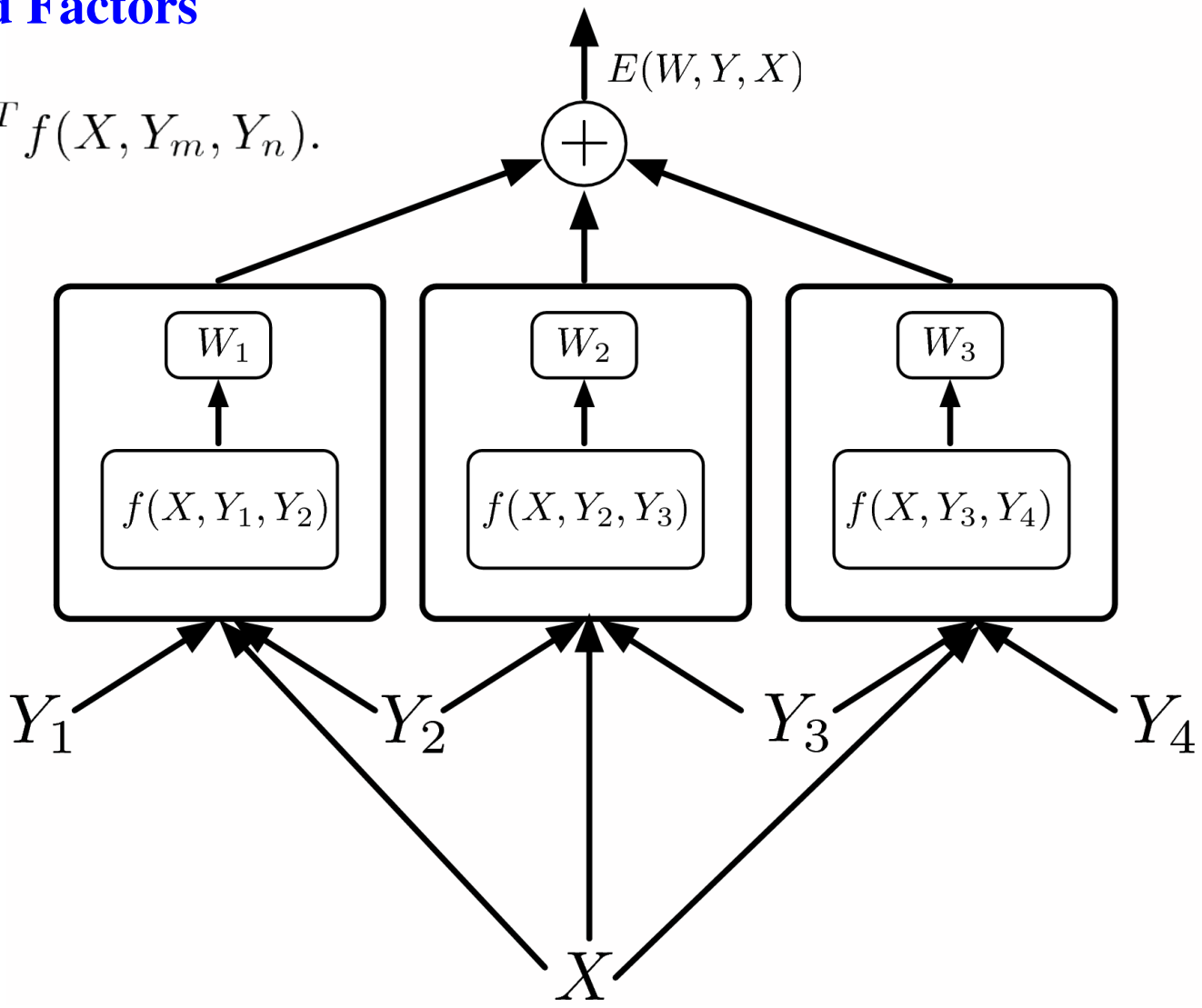
Energy-Based Belief Prop: Marginalization over Latent Variables

- **The previous picture shows a chain graph of factors with 2 inputs.**
  - Going along a path: add up the energies
  - When several paths meet: compute $-\frac{1}{\beta}\log\sum_i e^{-\beta E_{ji}}$
- **The extension of this procedure to trees, with factors that can have more than 2 inputs is the "[log/sum/exp]-sum" algorithm (a non-probabilistic form of belief propagation)**
- **Basically, it is the sum-product algorithm with a different semi-ring algebra (log/sum/exp instead of sum, sum instead of product), and without the normalization step.**
  - [Kschischang, Frey, Loeliger, 2001][McKay's book]

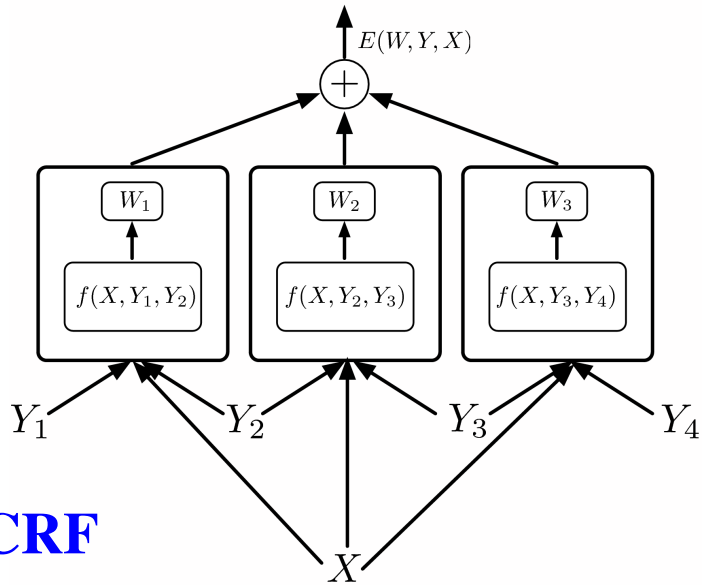
A Simple Case: Linearly Parameterized Factors: CRF, MMMN

Linearly Parameterized Factors

$$E(W, Y, X) = \sum_{(m,n)\in\mathcal{F}} W^T f(X, Y_m, Y_n).$$



Linearly Parameterized Factors + Negative Log Likelihood Loss = Conditional Random Fields



- **Linearly Parameterized Factors + NLL loss = CRF**
  - [Lafferty, McCallum, Pereira, 2001]

$$\mathcal{L}_{\text{nll}}(W) = \frac{1}{P}\sum_{i=1}^{P} W^T F(X^i, Y^i) + \frac{1}{\beta}\log\sum_{y\in\mathcal{Y}} e^{-\beta W^T F(X^i,y)}.$$

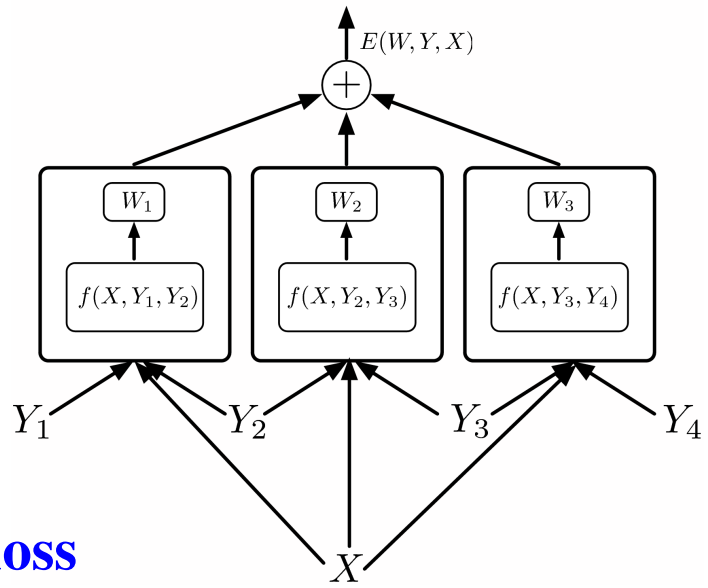
$$\frac{\partial \mathcal{L}_{\text{nll}}(W)}{\partial W} = \frac{1}{P}\sum_{i=1}^{P} F(X^i, Y^i) - \sum_{y\in\mathcal{Y}} F(X^i, y) P(y|X^i, W),$$

$$P(y|X^i, W) = \frac{e^{-\beta W^T F(X^i,y)}}{\sum_{y'\in\mathcal{Y}} e^{-\beta W^T F(X^i,y')}}.$$

simplest/best learning procedure:

stochastic gradient

Linearly Parameterized Factors + Perceptron Loss = Sequence Perceptron



- **Linearly Parameterized Factors + Perceptron loss**
  - [LeCun, Bottou, Bengio, Haffner 1998, Collins 2000, Collins 2001]

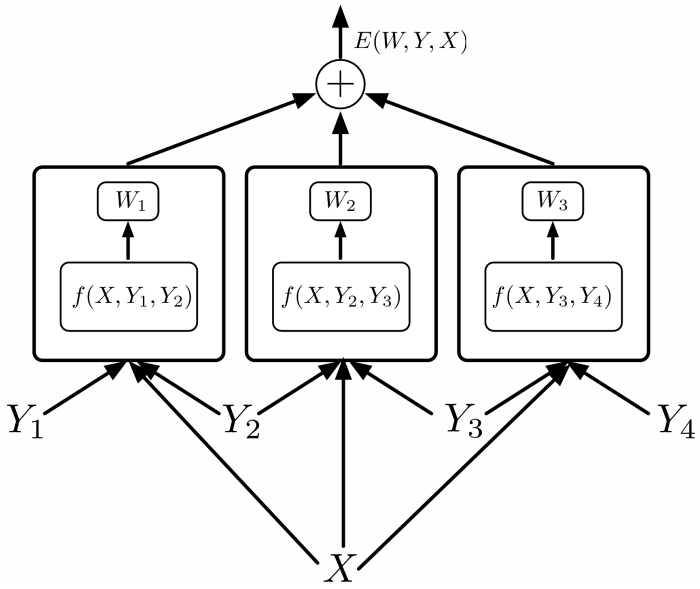
$$\mathcal{L}_{\text{perceptron}}(W) = \frac{1}{P}\sum_{i=1}^{P} E(W, Y^i, X^i) - E(W, Y^{*i}, X^i),$$

$$\mathcal{L}_{\text{perceptron}}(W) = \frac{1}{P}\sum_{i=1}^{P} W^T \left(F(X^i, Y^i) - F(X^i, Y^{*i})\right).$$

$$W \leftarrow W - \eta \left(F(X^i, Y^i) - F(X^i, Y^{*i})\right).$$

(but [LeCun et al. 1998] used non-linear factors)

Linearly Parameterized Factors + Hinge Loss = Max Margin Markov Networks



- **Linearly Parameterized Factor + Hinge loss**
  - [Altun et a. 2003, Taskar et al. 2003]

$$\mathcal{L}_{\text{hinge}}(W) = \frac{1}{P}\sum_{i=1}^{P} \max(0, m + E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)) + \gamma||W||^2.$$

$$\mathcal{L}_{\text{hinge}}(W) = \frac{1}{P}\sum_{i=1}^{P} \max\left(0, m + W^T \Delta F(X^i, Y^i)\right) + \gamma||W||^2,$$

$$\Delta F(X^i, Y^i) = F(X^i, Y^i) - F(X^i, \bar{Y}^i)$$

Simple gradient descent rule:

$$\text{If } \Delta F(X^i, Y^i) > -m \text{ then } W \leftarrow W - \eta \Delta F(X^i, Y^i) - 2\gamma W$$

Can be performed in the dual (like an SVM)

Non-Linear Factors

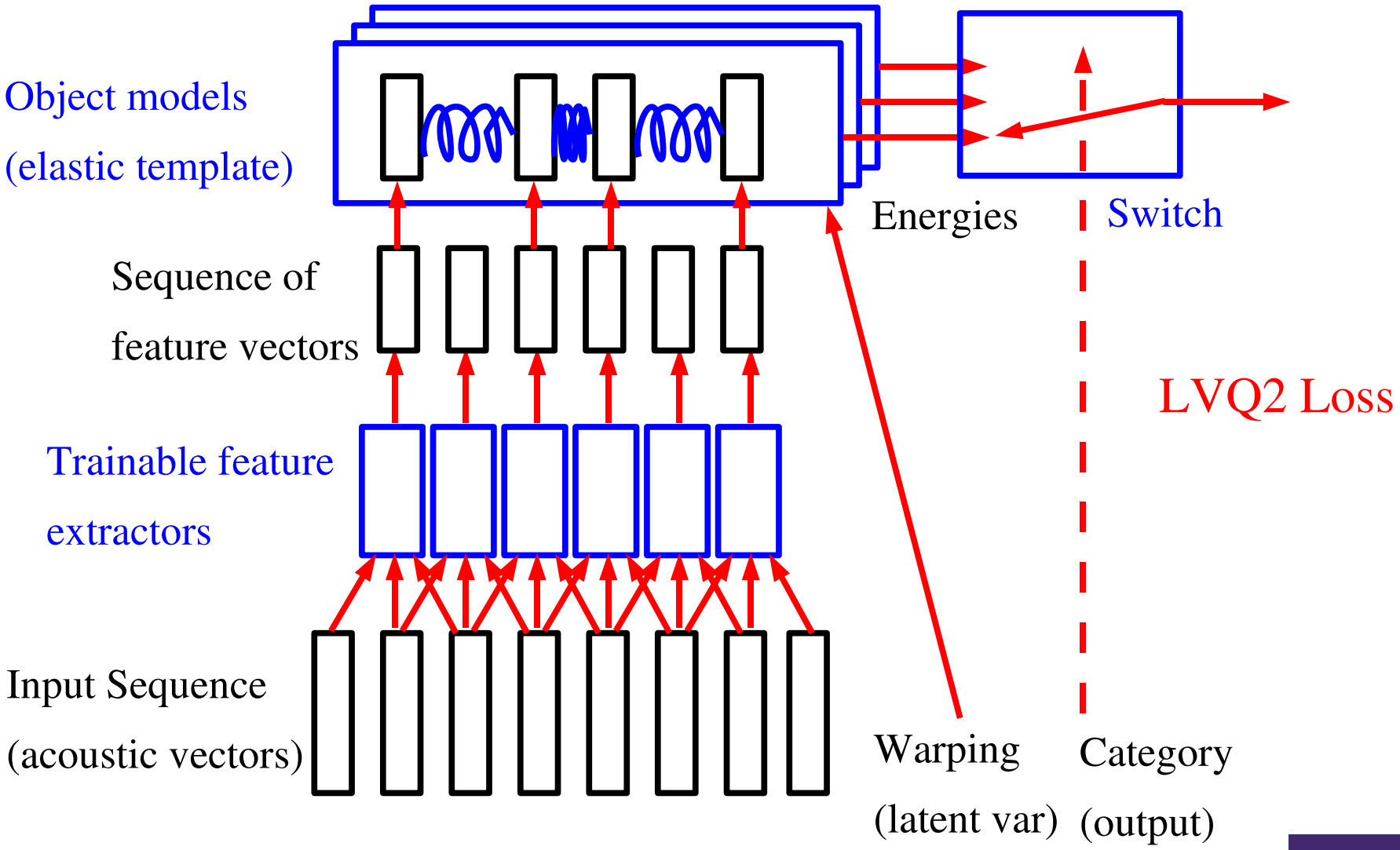
- **Energy-Based sequence labeling systems trained discriminatively have been used since the early 1990's**
- **Almost all of them used non-linear factors, such as multi-layer neural nets or mixtures of Gaussians.**
- **They were used mostly for speech and handwriting recognition**
- **There is a huge literature on the subject that has been somewhat ignored or forgotten by the NIPS and NLP communities.**
- **Why use non linear factors?**
  - :-(the loss function is non-convex
  - :-o You have to use simple gradient-based optimization algorithms, such as stochastic gradient descent (but that's what works best anyway, even in the convex case)
  - :-) linear factors simply don't cut it for speech and handwriting (including SVM-like linear combinations of kernel functions)

Deep Factors / Deep Graph: ASR with TDNN/HMM

- **Discriminative Automatic Speech Recognition system with HMM and various acoustic models**
  - Training the acoustic model (feature extractor) and a (normalized) HMM in an integrated fashion.
- **With Minimum Empirical Error loss**
  - Ljolje and Rabiner (1990)
- **with NLL:**
  - Bengio (1992)
  - Haffner (1993)
  - Bourlard (1994)
- **With MCE**
  - Juang et al. (1997)
- **Late normalization scheme (un-normalized HMM)**
  - Bottou pointed out the **label bias problem** (1991)
  - Denker and Burges proposed a solution (1995)

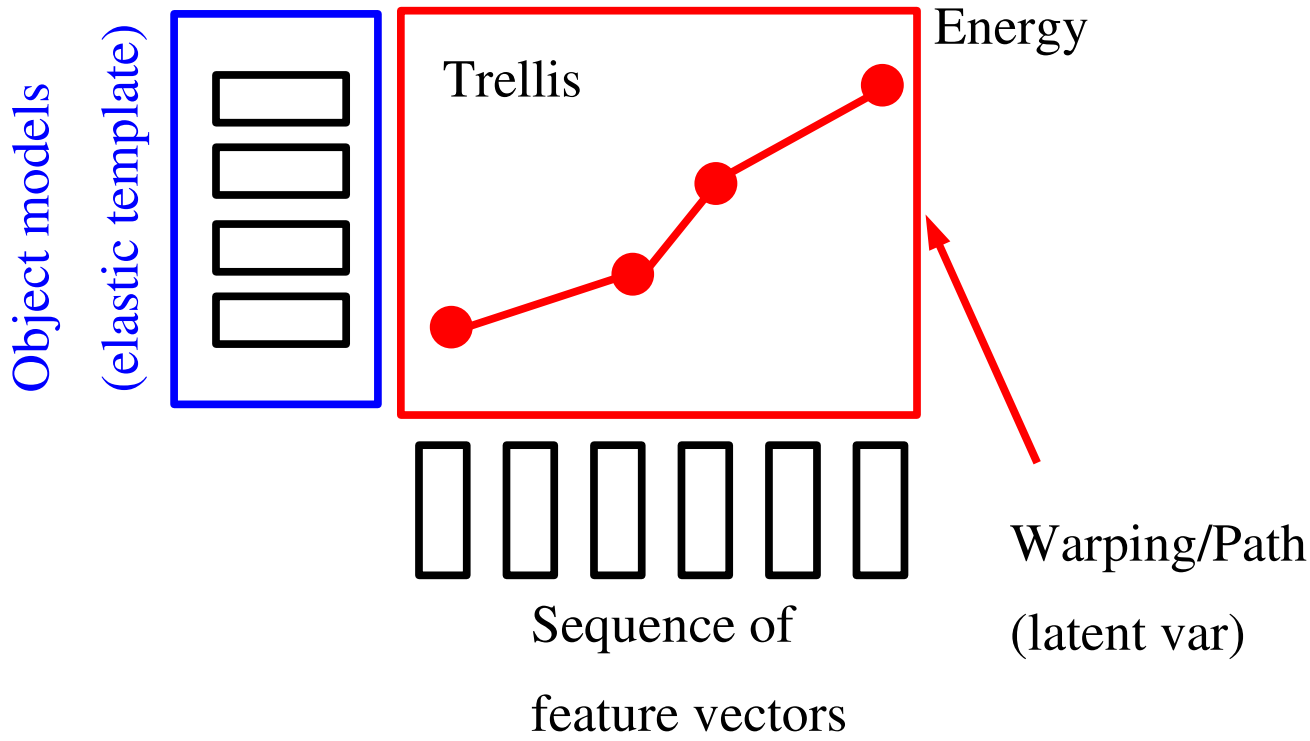
Example 1: Integrated Disc. Training with Sequence Alignment

- **Spoken word recognition with trainable elastic templates and trainable feature extraction [Driancourt&Bottou 1991, Bottou 1991, Driancourt 1994]**



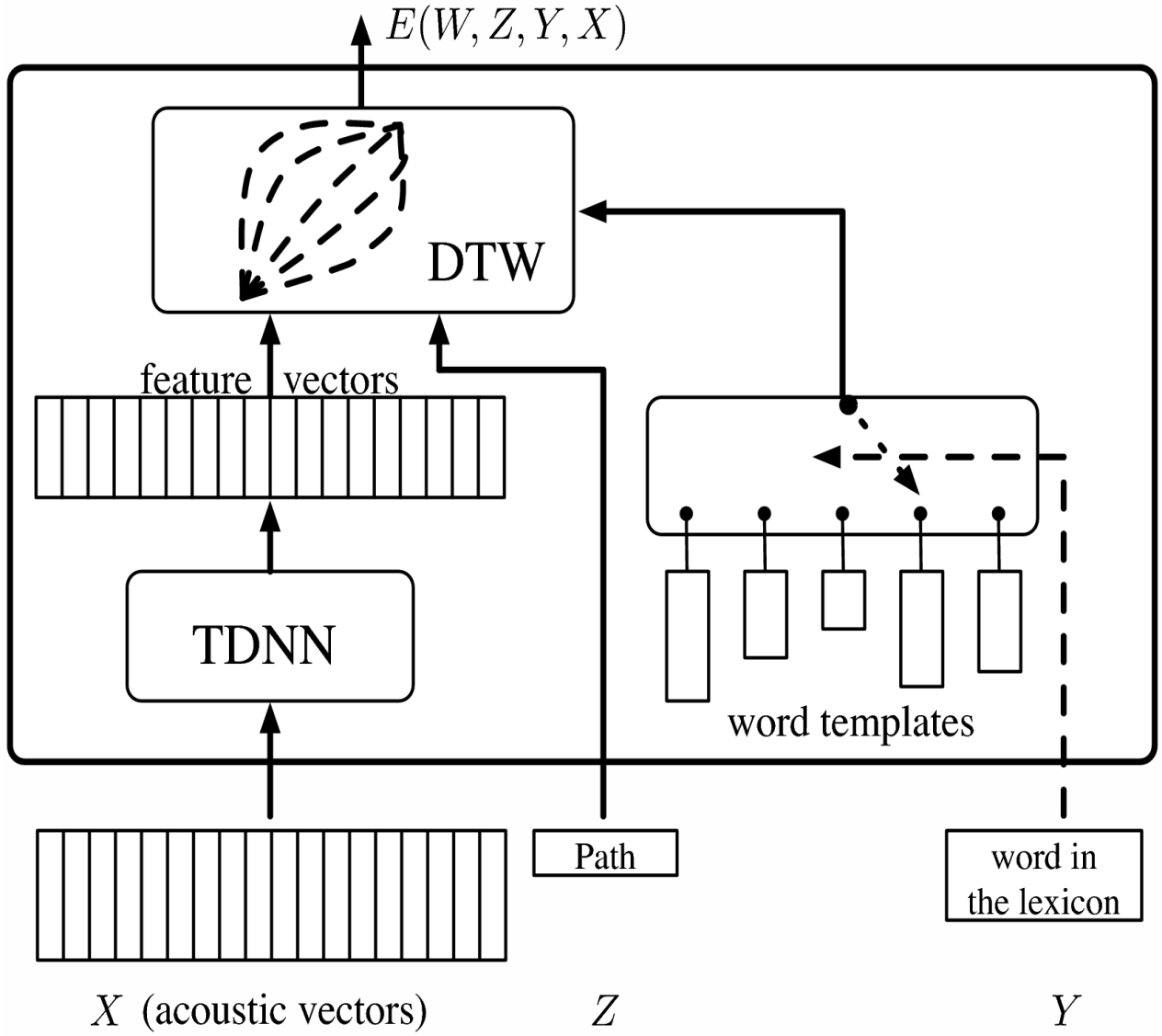
Example: 1-D Constellation Model (a.k.a. Dynamic Time Warping)

- Spoken word recognition with trainable elastic templates and trainable feature extraction [Driancourt&Bottou 1991, Bottou 1991, Driancourt 1994]
- Elastic matching using dynamic time warping (Viterbi algorithm on a trellis).
- The corresponding EBFG is implicit (it changes for every new sample).



Deep Factors / Deep Graph: ASR with TDNN/DTW

- **Trainable Automatic Speech Recognition system with convolutional nets (TDNN) and dynamic time warping (DTW)**
- **Training the feature extractor as part of the whole process.**
- **with the LVQ2 Loss :**
  - Driancourt and Bottou's speech recognizer (1991)
- **with NLL:**
  - Bengio's speech recognizer (1992)
  - Haffner's speech recognizer (1993)

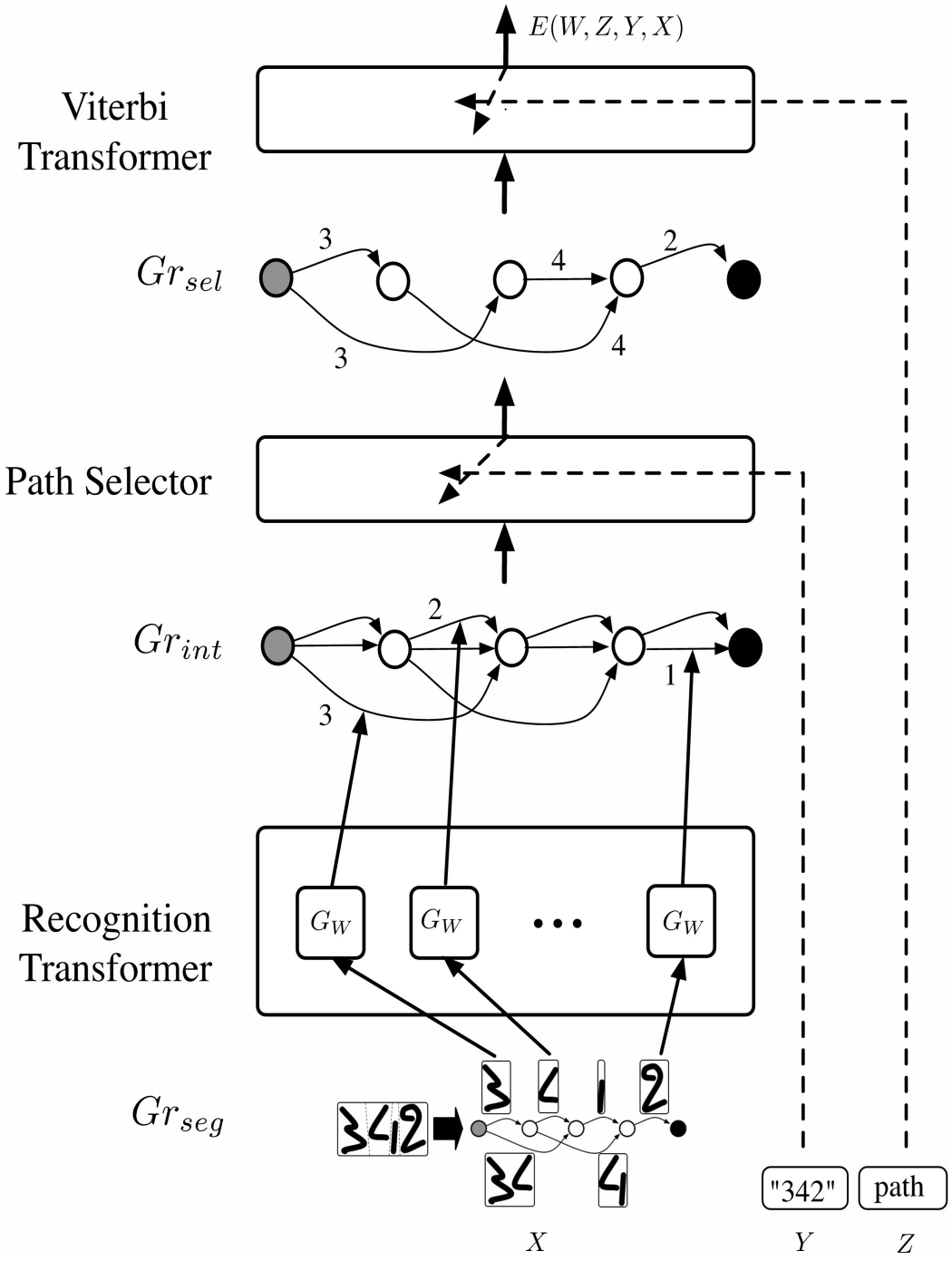


Complex Trellises: procedural representation of trellises

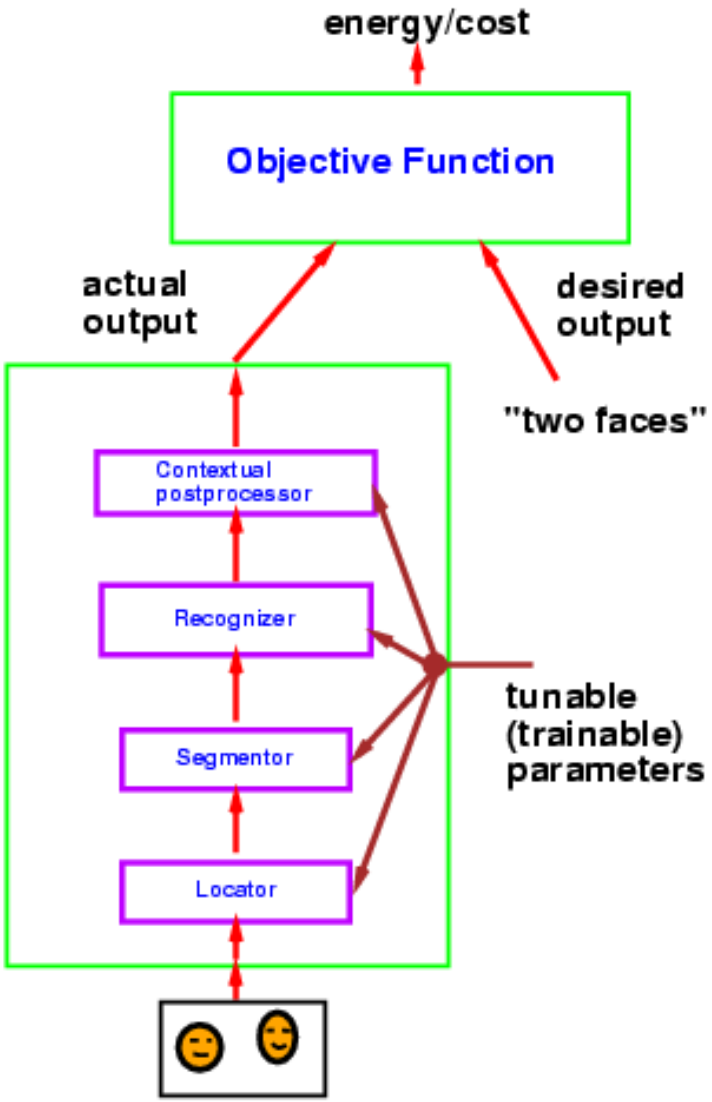
- **When the trellis is too large, we cannot store it in its entirety in memory.**
  - We must represent it procedurally
- **The cleanest way to represent complex graphs proceduraly is through the formalism of finite-state transducer algebra**
  - [Mohri 1997, Pereira et al.]

Really Deep Factors / Really Deep Graph

- **Handwriting Recognition with Graph Transformer Networks**
- **Un-normalized hierarchical HMMs**
  - Trained with Perceptron loss [LeCun, Bottou, Bengio, Haffner 1998]
  - Trained with NLL loss [Bengio, LeCun 1994], [LeCun, Bottou, Bengio, Haffner 1998]
- **Answer = sequence of symbols**
- **Latent variable = segmentation**

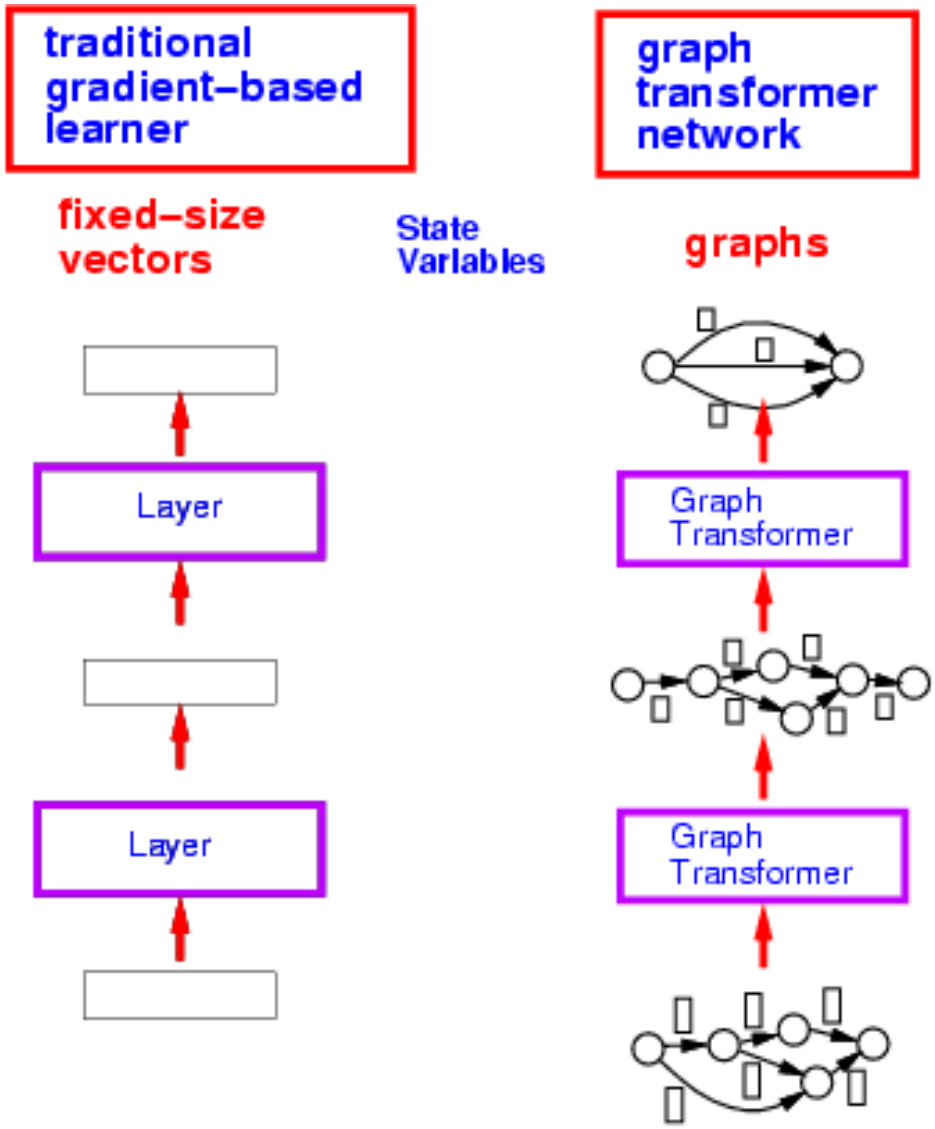


End-to-End Learning.



- Making every single module in the system trainable.
- Every module is trained simultaneously so as to optimize a global loss function.

Using Graphs instead of Vectors.

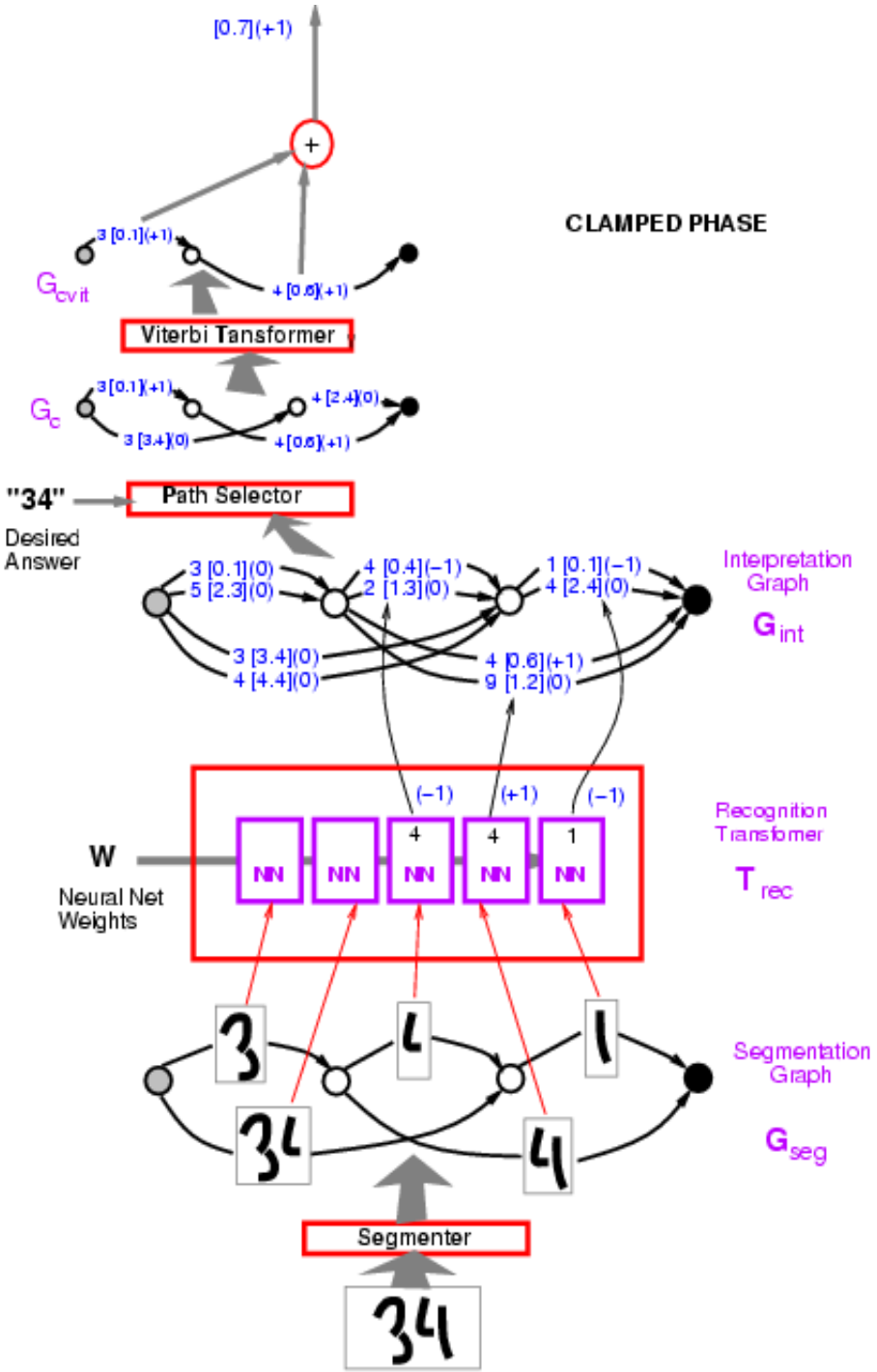


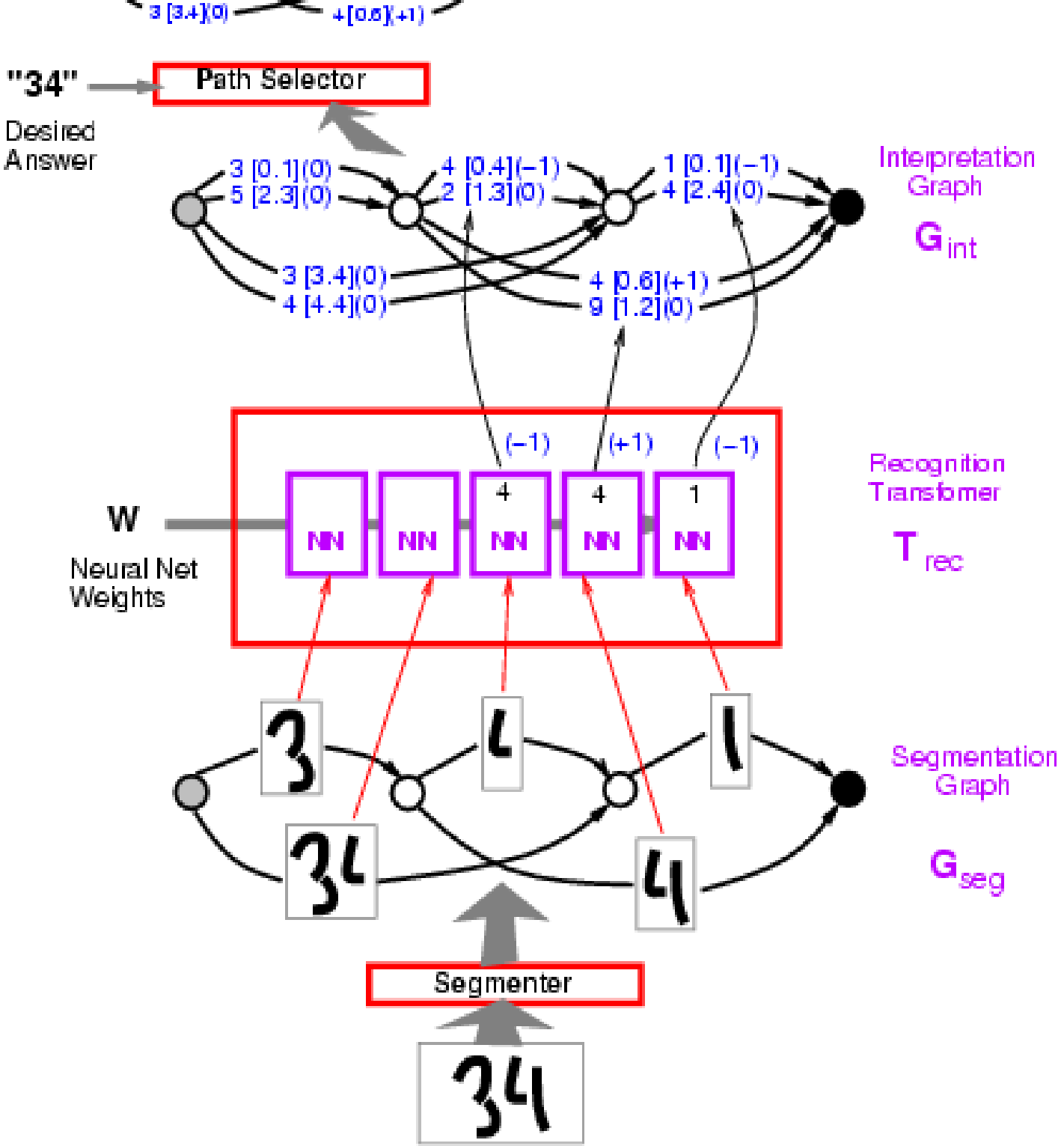
- Whereas traditional learning machines manipulate fixed-size vectors, Graph Transformer Networks manipulate graphs.

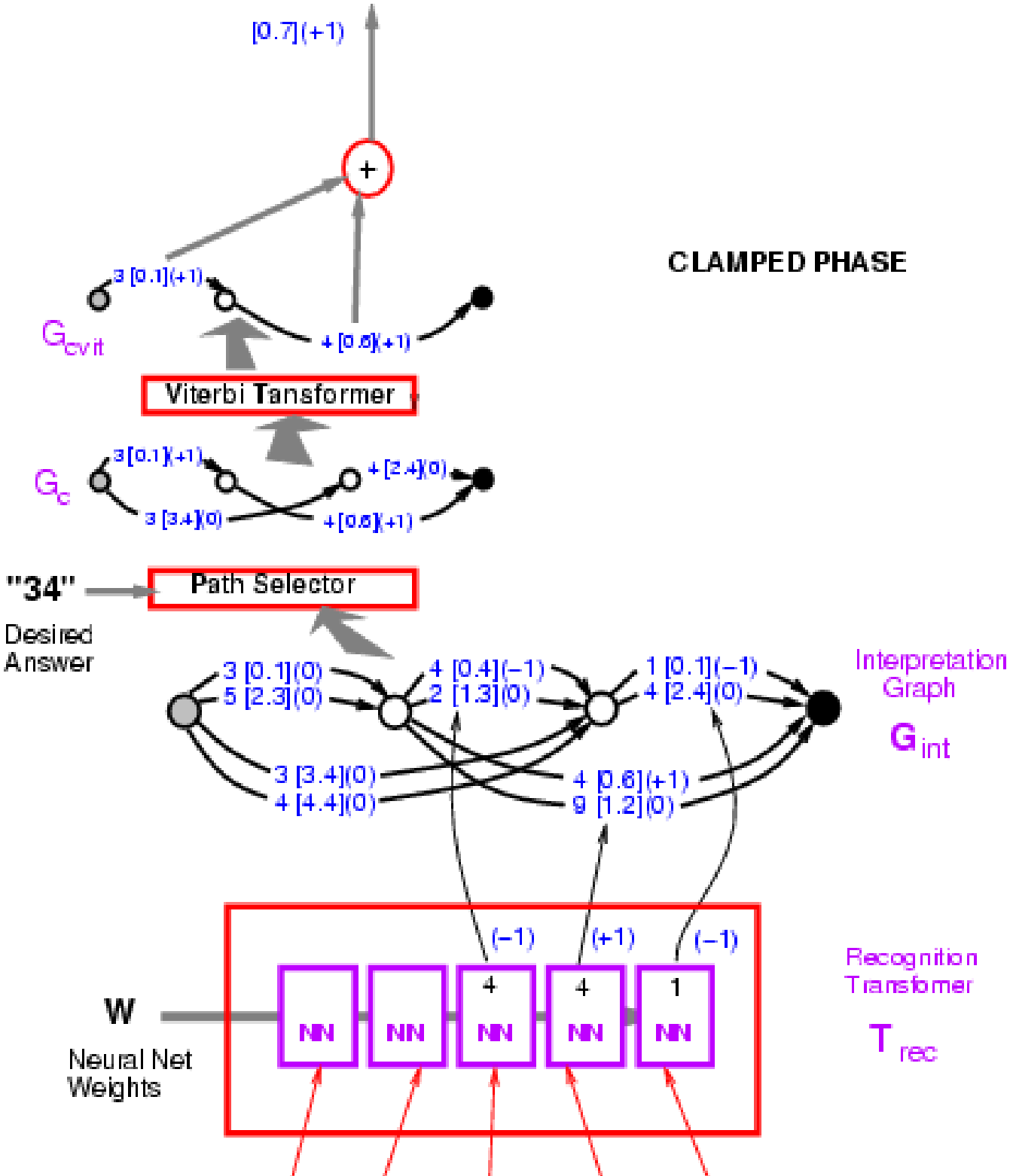
Graph Transformer Networks

- **Variables:**
  - X: input image
  - Z: path in the interpretation graph/segmentation
  - Y: sequence of labels on a path
- **Loss function: computing the energy of the desired answer:**

$$E(W, Y, X)$$




"34"
Desired Answer
Path Selector
3 [0.1](0)
5 [2.3](0)
4 [0.4](−1)
2 [1.3](0)
1 [0.1](−1)
4 [2.4](0)
Interpretation Graph
G int
3 [3.4](0)
4 [4.4](0)
4 [0.6](+1)
9 [1.2](0)
(−1)
(+1)
(−1)
W
Neural Net Weights
NN
NN
4
NN
4
NN
1
NN
Recognition Transformer
T rec
3
4
1
34
4
Segmentation Graph
G seg
Segmenter
34

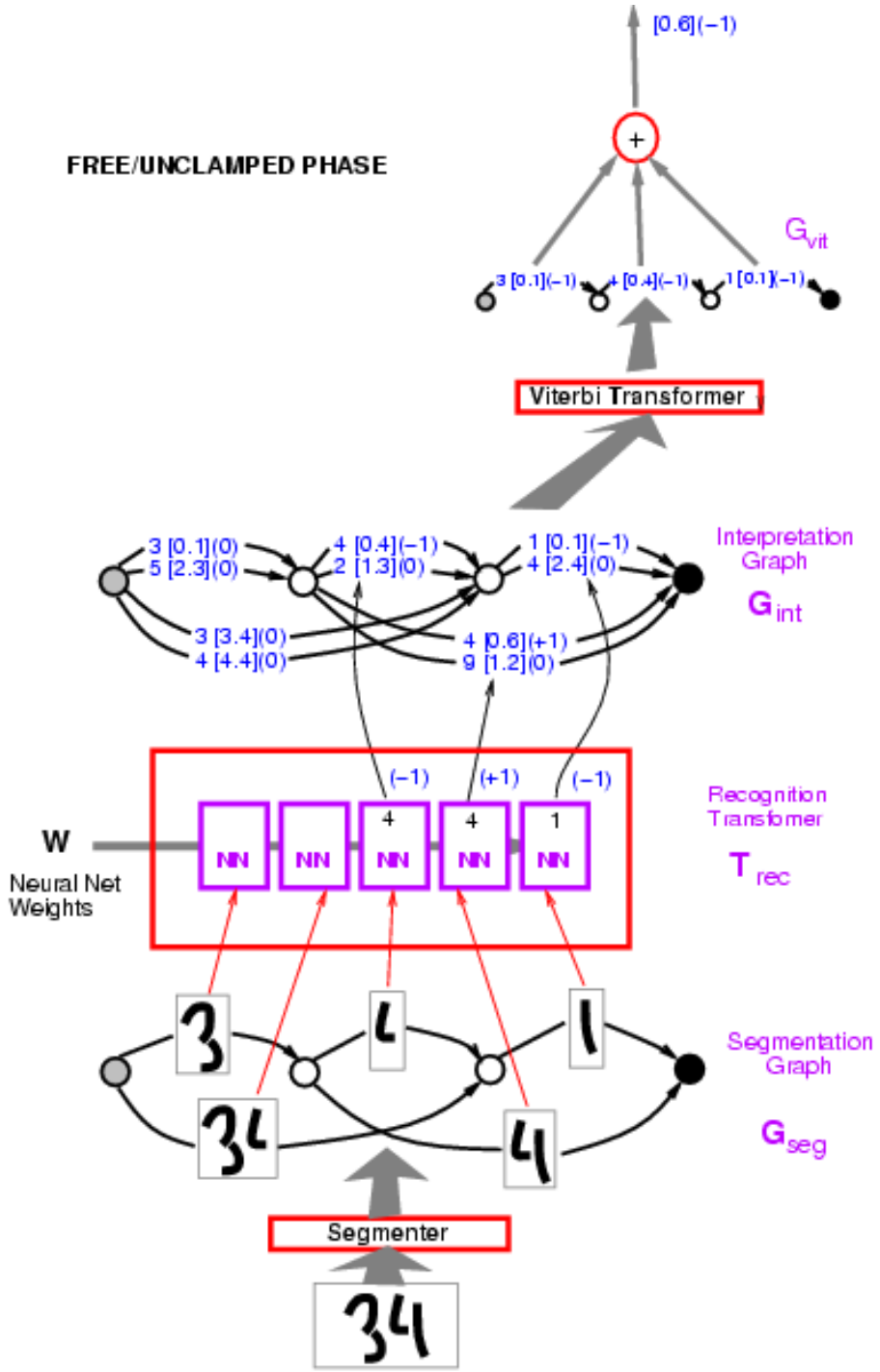


CLAMPED PHASE

Graph Transformer Networks

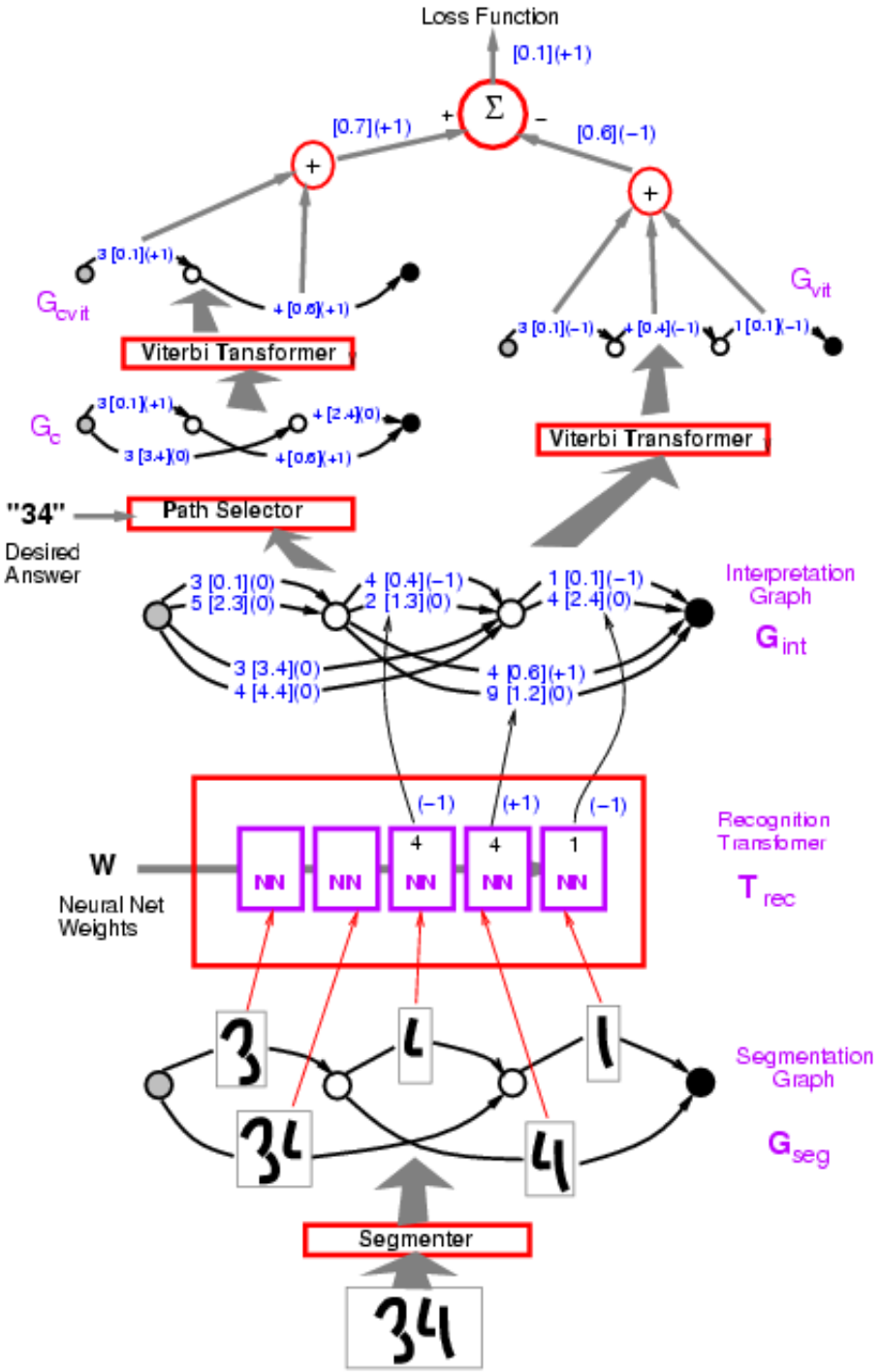
- **Variables:**
  - X: input image
  - Z: path in the interpretation graph/segmentation
  - Y: sequence of labels on a path
- **Loss function: computing the constrastive term:**

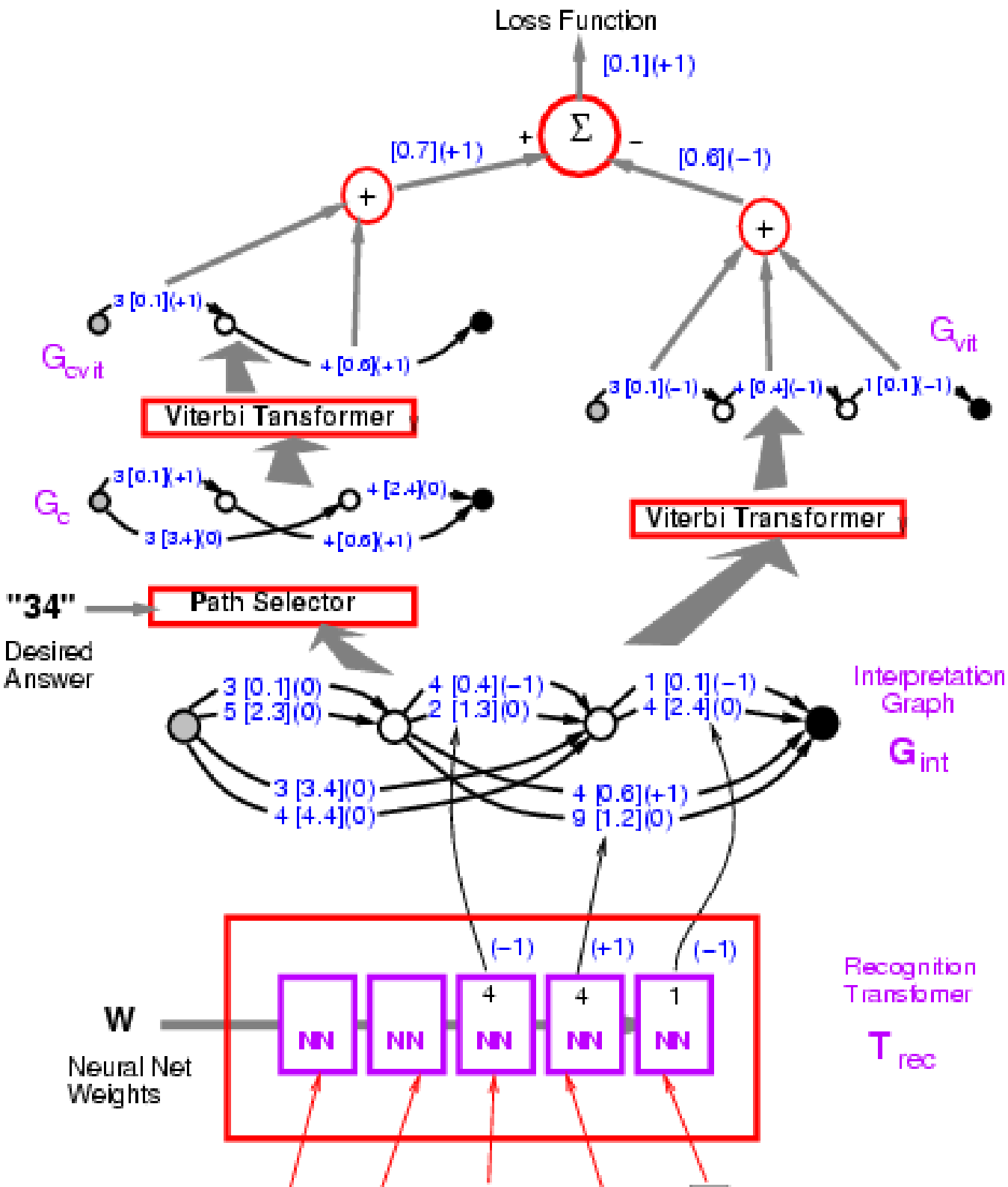
$$E(W, \check{Y}, X)$$



Graph Transformer Networks

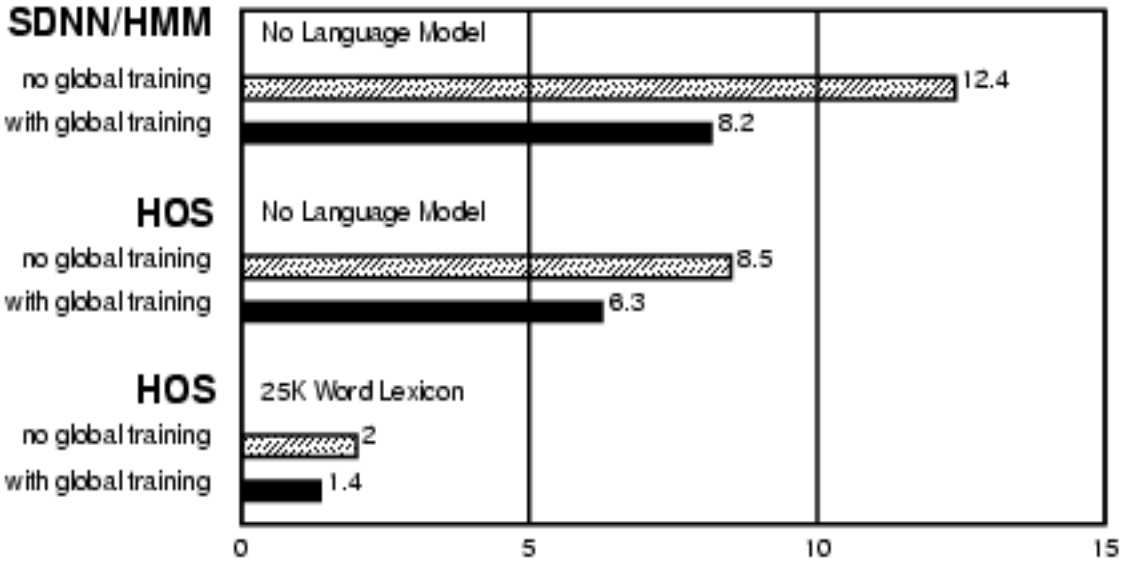
- **Example: Perceptron loss**
- **Loss = Energy of desired answer – Energy of best answer.**
  - (no margin)

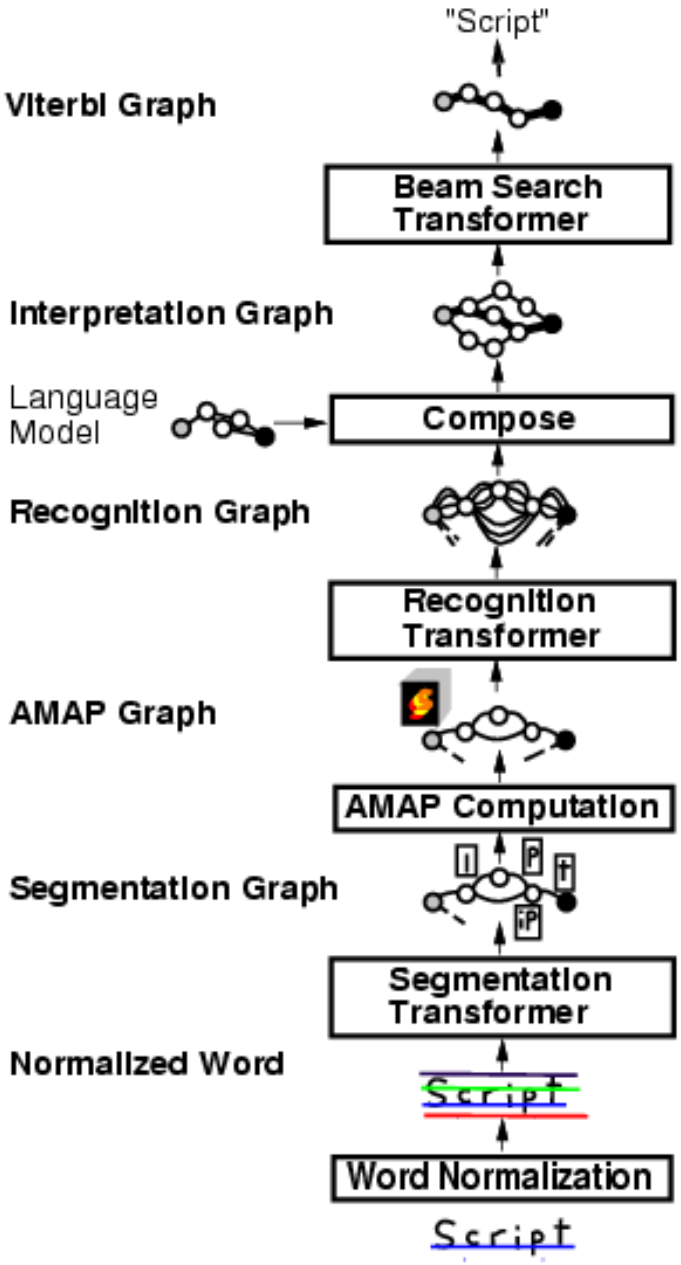



Loss Function
[0.1](+1)
Σ
+
−
[0.7](+1)
[0.6](−1)
+
+
3 [0.1](+1)
4 [0.6](+1)
G cv it
Viterbi Tansformer
3 [0.1](+1)
4 [2.4](0)
3 [3.4](0)
4 [0.6](+1)
G c
G vit
3 [0.1](−1)
4 [0.4](−1)
1 [0.1](−1)
Viterbi Transformer
"34"
Desired Answer
Path Selector
3 [0.1](0)
5 [2.3](0)
4 [0.4](−1)
2 [1.3](0)
1 [0.1](−1)
4 [2.4](0)
3 [3.4](0)
4 [4.4](0)
4 [0.6](+1)
9 [1.2](0)
Interpretation Graph
G int
(−1)
(+1)
(−1)
4
4
1
NN
NN
NN
NN
NN
W
Neural Net Weights
Recognition Transformer
T rec

Global Training Helps

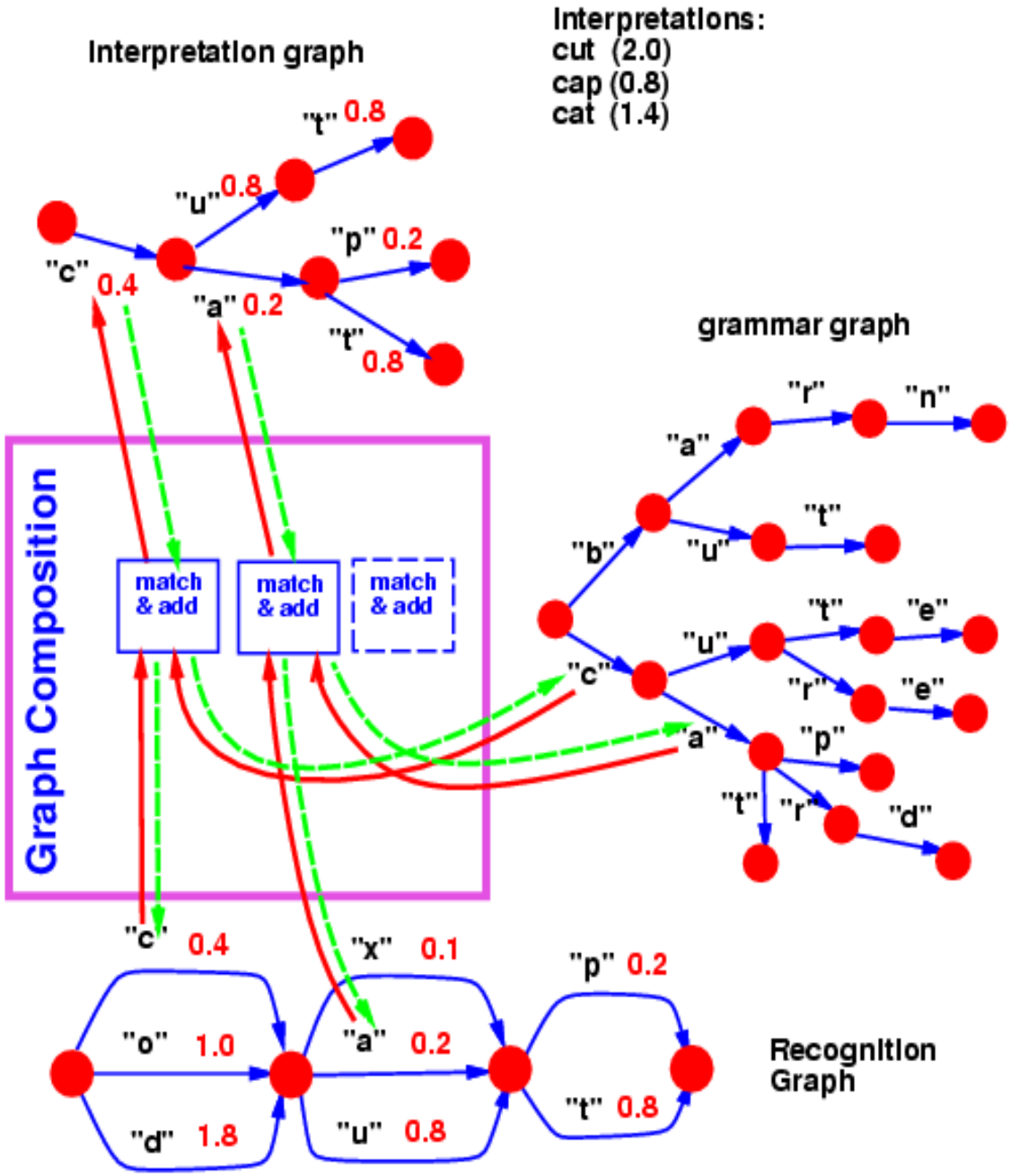
- **Pen-based handwriting recognition (for tablet computer)**
  - [Bengio&LeCun 1995]
  - Trained with NLL loss (aka MMI)





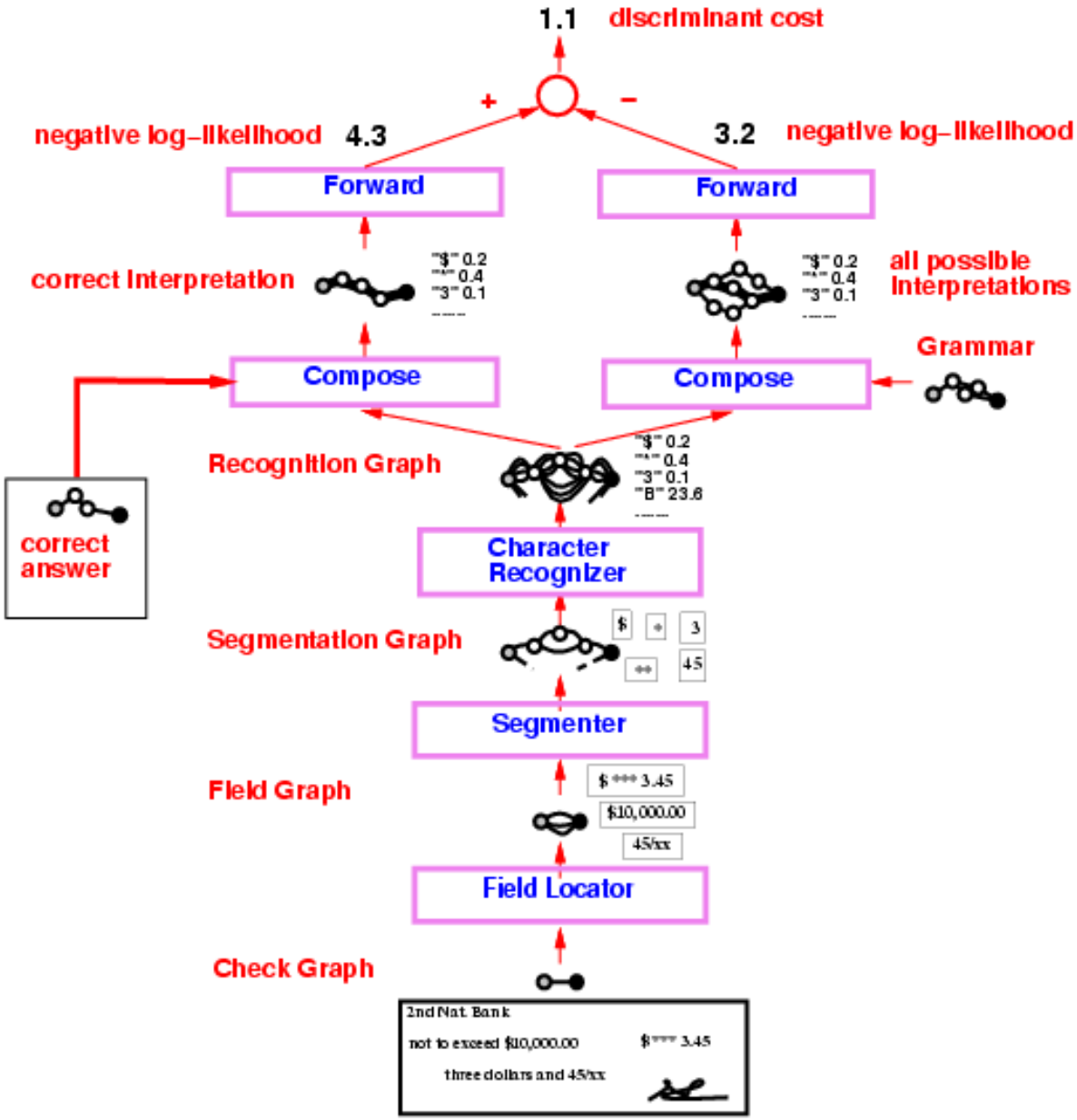
Graph Composition, Transducers.

- The composition of two graphs can be computed, the same way the dot product between two vectors can be computed.
- General theory: semi-ring algebra on weighted finite-state transducers and acceptors.



Check Reader

- Graph transformer network trained to read check amounts.
- Trained globally with Negative-Log-Likelihood loss.
- 50% percent corrent, 49% reject, 1% error (detectable later in the process.
- Fielded in 1996, used in many banks in the US and Europe.
- Processes an estimated 10% of all the checks written in the US.



Learning when the space of Y is huge

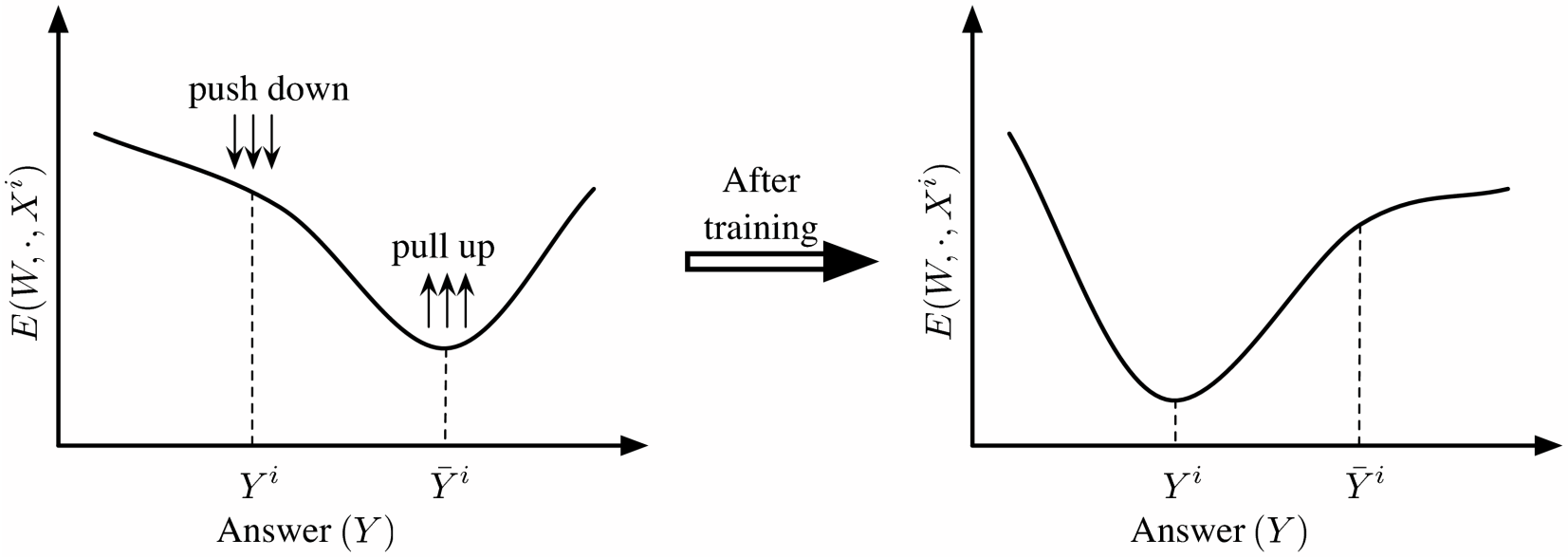
- learning when Y is in a high-dimensional continuous spaces
- Image restoration, Image segmentation
- Unsupervised learning in high-dimensional space

Learning when the space of Y is huge

- **Solutions:**
- **Use an energy function such that contrastive term in the loss is either constant or easy to compute**
  - e.g. Energy is quadratic: convex (inference is easy), integral of exponential is easily computable or constant.
- **Approximate the derivative of the contrastive term in the loss with a variational approximation**
- **Simple sampling approximation:**
  - Pull down on the energy of the training samples
  - Pull up on the energies of other configurations that have low energy (that are threatening)
  - Question: how do we pick those configurations?
  - One idea: **contrastive Divergence [Hinton 2000]**

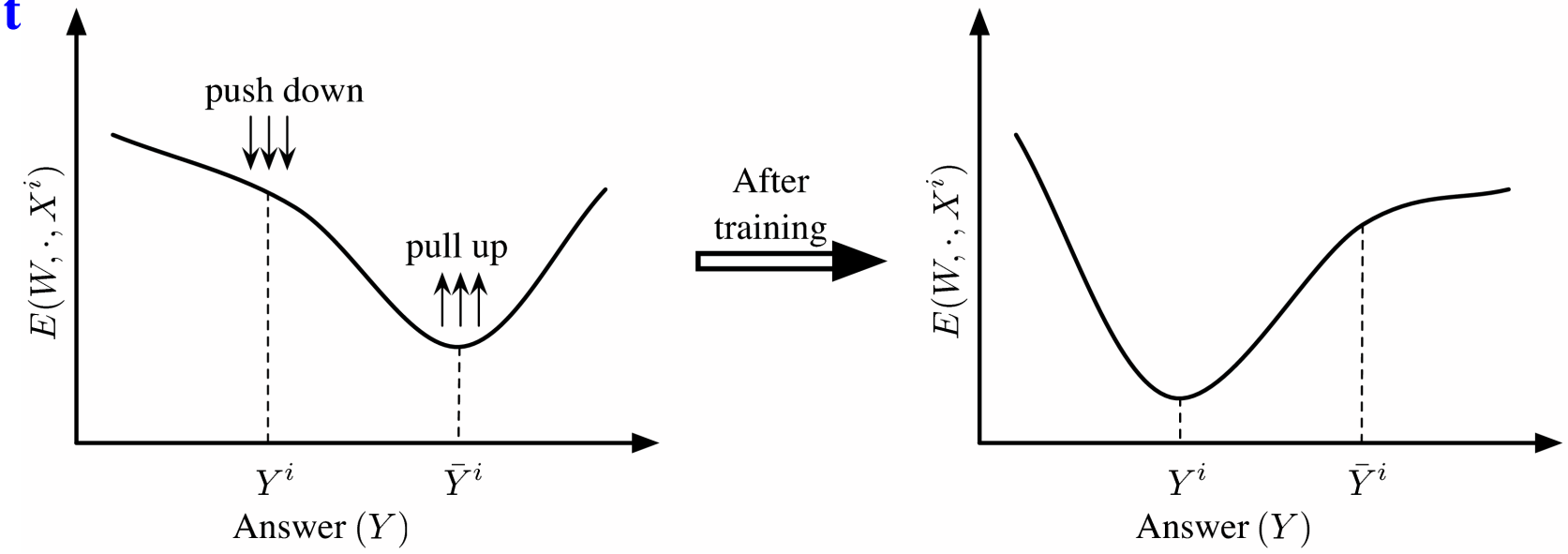
Contrastive Divergence

- To generate the "bad" configurations:
- 1. Start from the correct value of Y
- 2. Pull down the energy of the correct value
- 3. To obtain a "bad" configuration, go down the energy surface with "some noise"
- 4. pull up the energy of the obtained configuration



Contrastive Divergence

- To generate the "bad" configurations:
- Hybrid Monte-Carlo Sampling: simulate a ball rolling down the energy surface in Y space.
- Kick the ball in the a random direction (with a random momentum), and run the simulation for a few iterations.
- The final configuration is quite likely to have lower energy than t

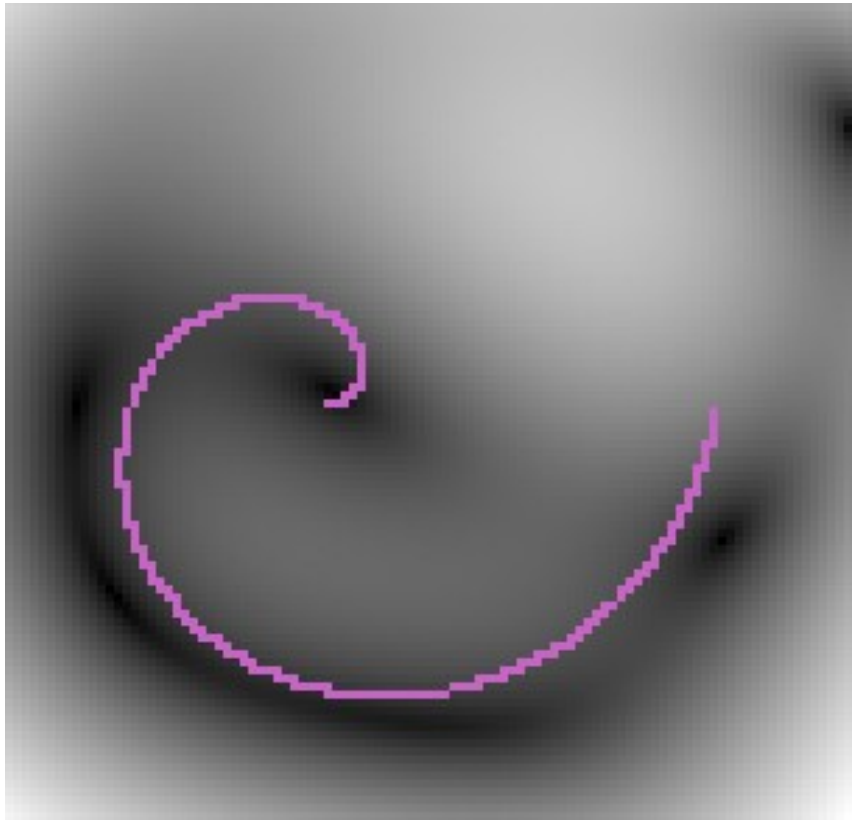


Energy-Based Unsupervised Learning with Margin Loss

- **Example: learning a spiral in 2D**
- **Energy: || Y – F(W,Y)||^2 where F is a 2-layer neural net**

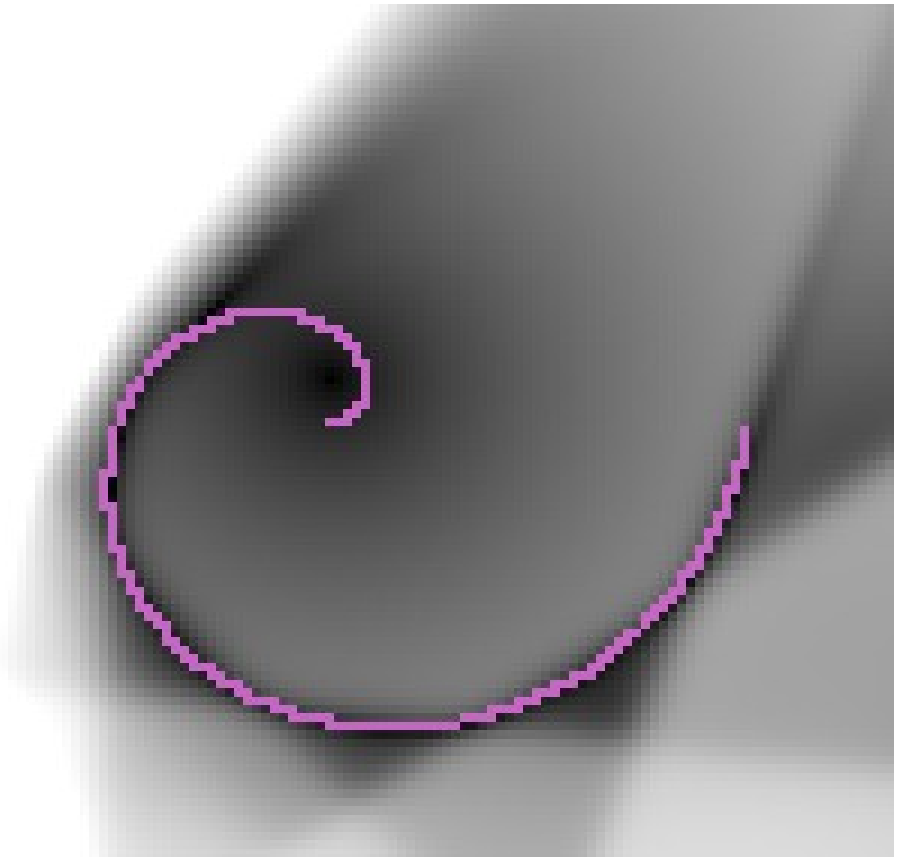
$$L(Y, W) = \alpha E(Y, W) + max(0, m - E(\bar{Y}, W))$$

$$\bar{Y} \leftarrow \bar{Y} - \eta \frac{\delta E(\bar{Y})}{\delta Y} + \epsilon$$



Wide auto-encoder with sparse code

- Sparse Codes
  - Limiting the information content of the code prevents flat energy surfaces, without the need to explicitly push up the bad points
  - Idea is to make the high dimensional code sparse by forcing each variable to be zero most of the time



Challenges of Visual Neuroscience (and Computer Vision)

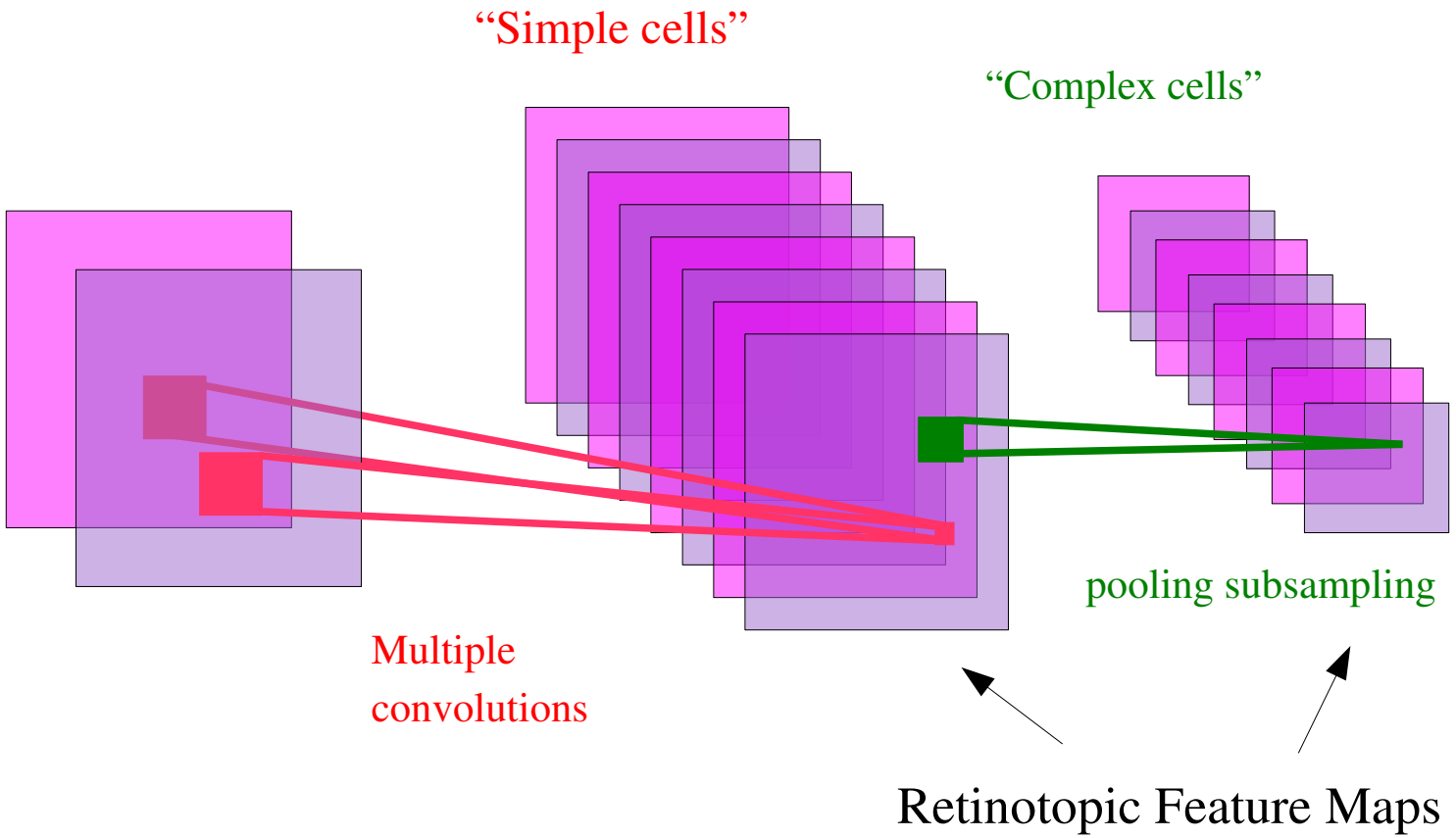
- **The recognition of everyday objects is a very fast process.**
  - Experiments by Simon Thorpe and others have shown that the recognition of common object is essentially "feed forward."
  - Not all of vision is feed forward (what would all those feed-back connection be there for?).
- **How much of the visual system is the result of learning?**
  - How much prior structure must be built into the visual system to enable it to learn to see?
  - Are V1/V2/V4 neurons learned or hard-wired?
- **If the visual system is learned, what is the learning algorithm?**
  - What learning algorithm can train neural network as "deep" as the visual system (10 layers?).
- **Let's try to train an artificial vision system from end to end and see what it can do.**

Questions?

- **Is there a magic bullet for visual learning?**
  - Is there a general principle, or should we just resort to a bunch of tricks?
  - Is there a universal learning algorithm/architecture which, given a small amount of appropriate prior structure, can produce an intelligent vision system?
  - Or do we need to accumulate a large repertoire of "modules" to solve each specific problem an intelligent vision system must solve. How would we assemble those modules?
- **How far can we get by training a vision system end to end**
  - Let us train a complete vision system from raw pixels to object categories, or to robot actions.

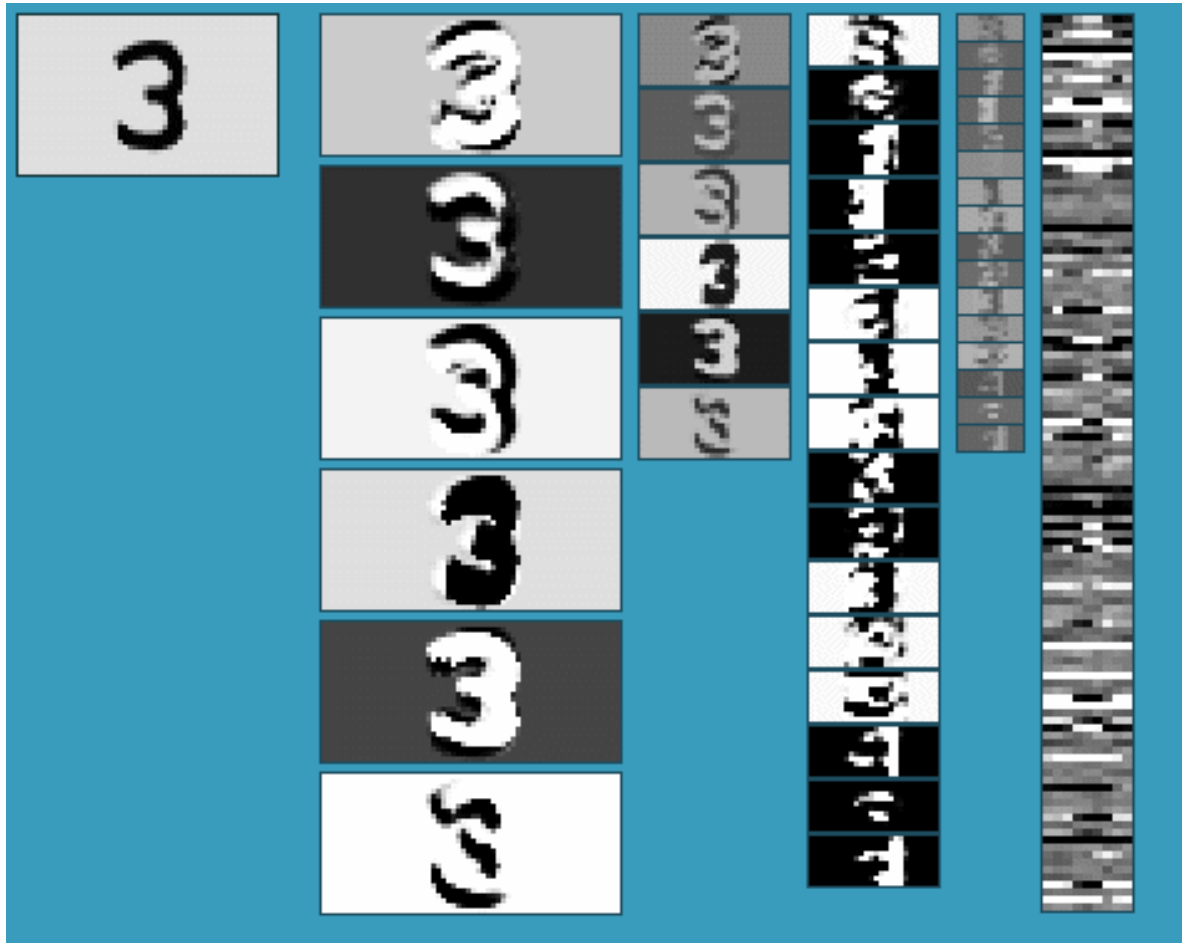
An Old Idea for Local Shift Invariance

- **[Hubel & Wiesel 1962]:**
  - simple cells detect local features
  - complex cells "pool" the outputs of simple cells within a retinotopic neighborhood.



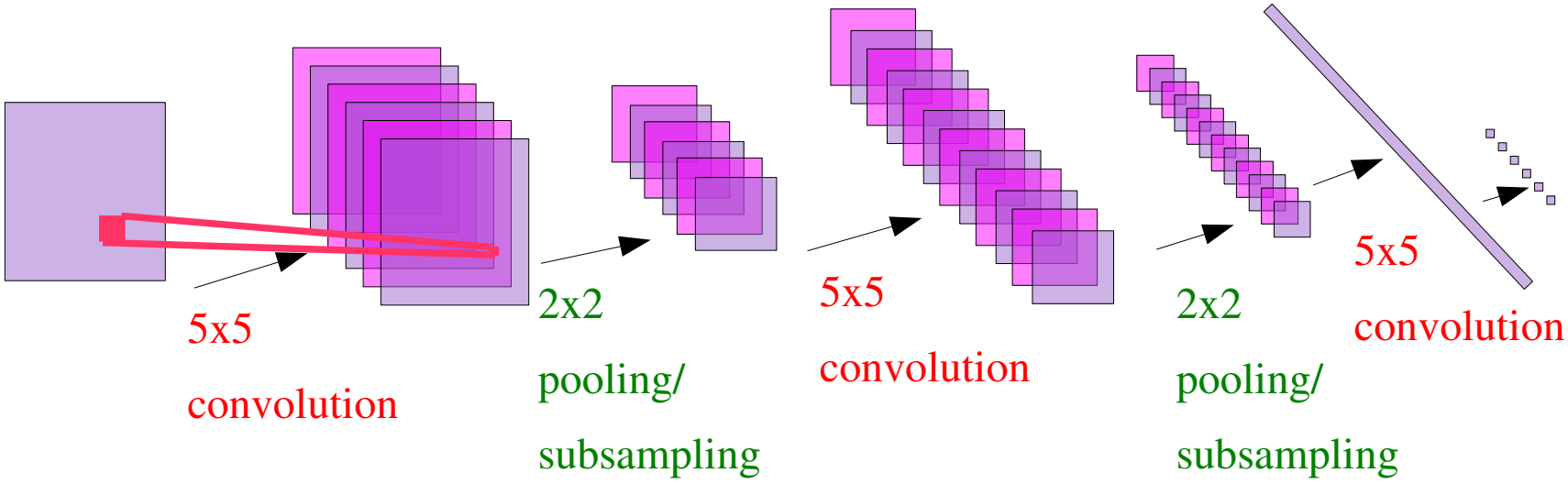
The Multistage Hubel-Wiesel Architecture

- **Building a complete artificial vision system:**
  - Stack multiple stages of simple cells / complex cells layers
  - Higher stages compute more global, more invariant features
  - Stick a classification layer on top
  - [Fukushima 1971-1982]
    - neocognitron
  - [LeCun et al. 1988-2007]
    - convolutional net
  - [Poggio et al. 2002-2006]
    - HMAX
  - [Ullman 2002-2006]
    - fragment hierarchy
  - [Lowe 2006]
    - HMAX



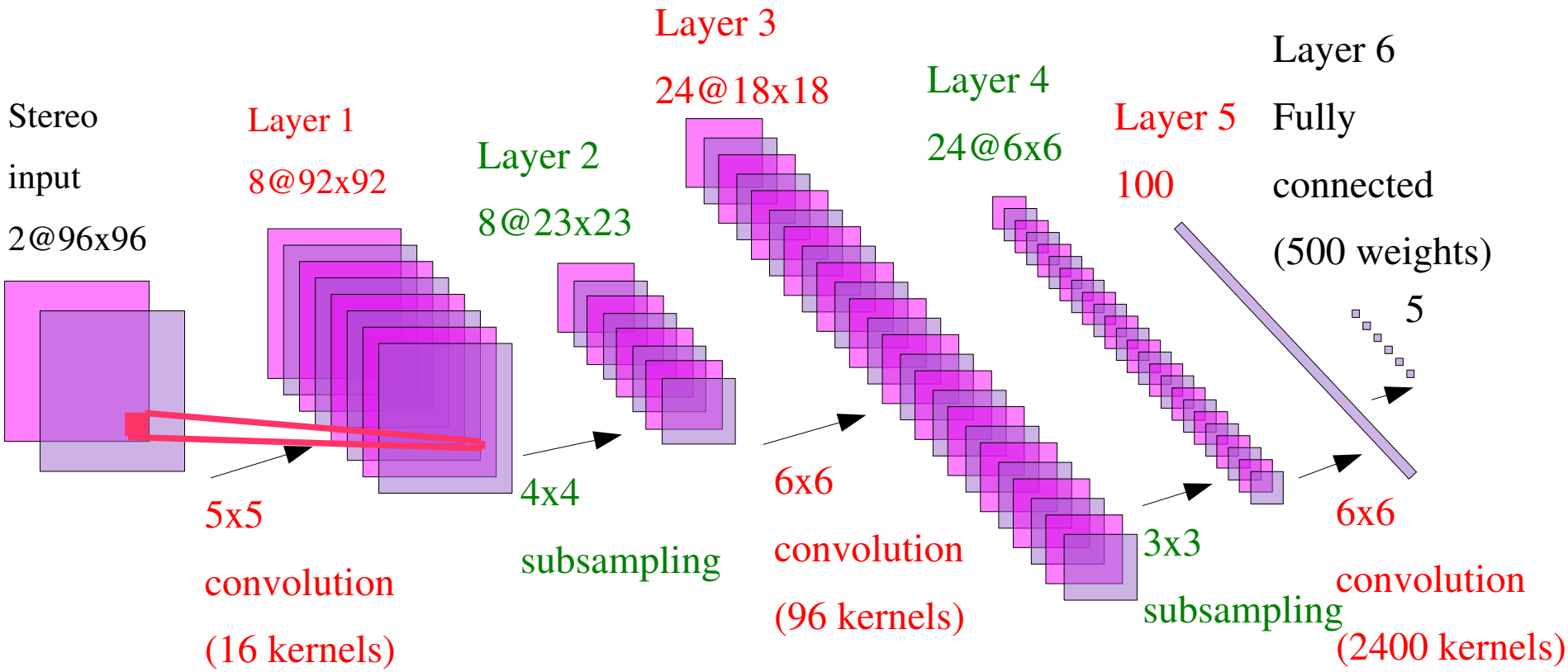
- **QUESTION: How do we find (or learn) the filters?**

Convolutional Net Architecture, Supervised Learning



- Convolutional layers (simple cells): all units in a feature plane share the same weights
- Pooling/subsampling layers (complex cells): for invariance to small distortions.
- **Supervised gradient-descent learning using back-propagation**
- **The entire network is trained end-to-end. All the layers are trained simultaneously.**

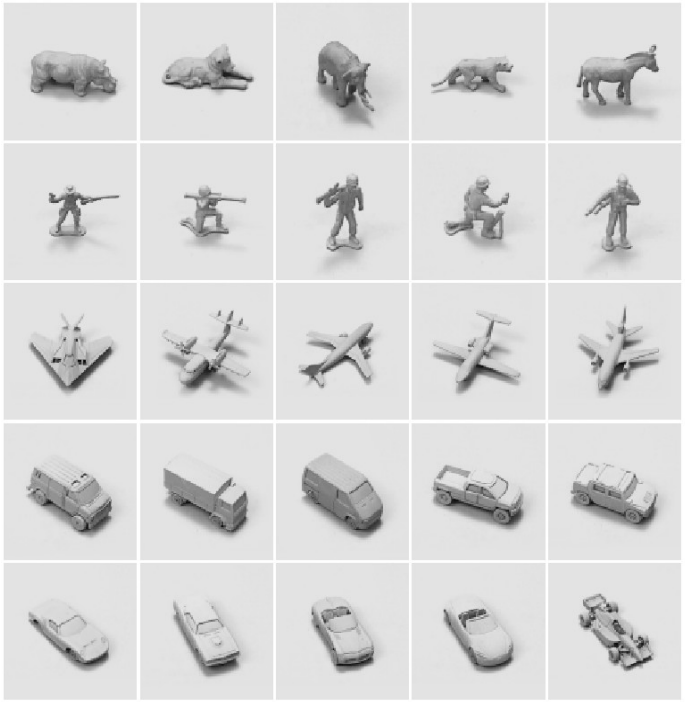
Convolutional Network for Object Recognition



- **96X69 input, 90,857 free parameters, 3,901,162 connections.**
- The architecture alternates convolutional layers (feature detectors) and subsampling layers (local feature pooling for invariance to small distortions).
- **The entire network is trained end-to-end** (all the layers are trained simultaneously).
- A gradient-based algorithm is used to minimize a supervised loss function.

Generic Object Detection and Recognition with Invariance to Pose and Illumination

- **50** toys belonging to 5 categories: **animal, human figure, airplane, truck, car**
- **10** instance per category: 5 instances used for training, 5 instances for testing
- **Raw dataset: 972** stereo pair of each object instance. **48,600** image pairs total.
- **For each instance:**
  - **18 azimuths**
    - 0 to 350 degrees every 20 degrees
  - **9 elevations**
    - 30 to 70 degrees from horizontal every 5 degrees
  - **6 illuminations**
    - on/off combinations of 4 lights
  - **2 cameras (stereo)**
    - 7.5 cm apart
    - 40 cm from the object

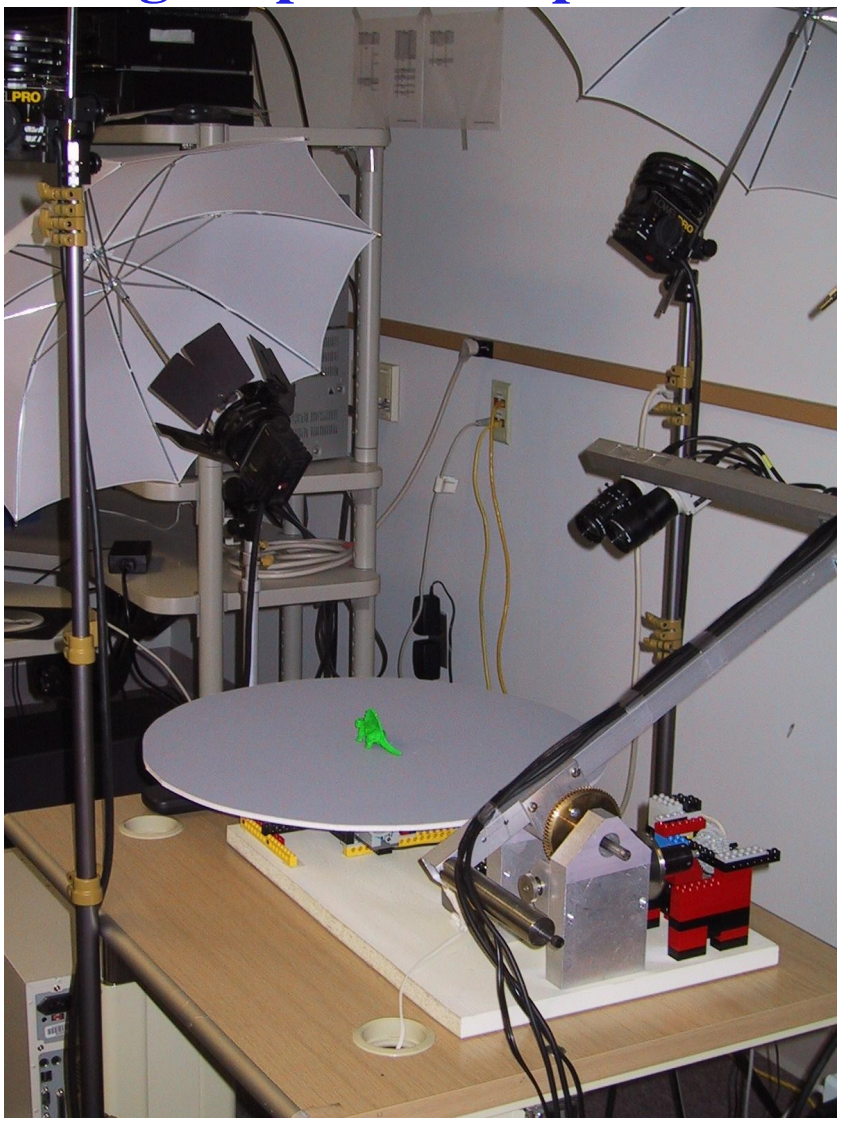


Training instances

Test instances

Data Collection, Sample Generation

Image capture setup

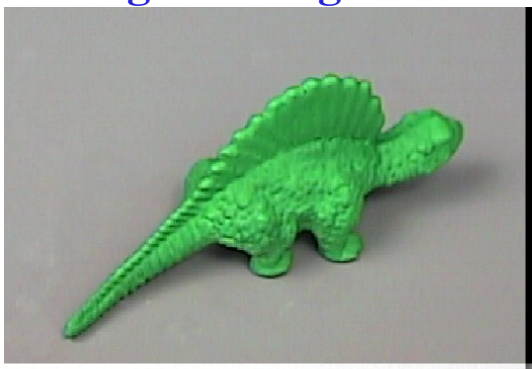


Objects are painted green so that:

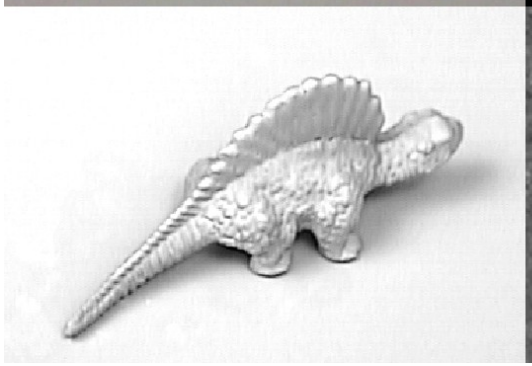
- all features other than shape are removed
- objects can be segmented, transformed,

  and composited onto various backgrounds

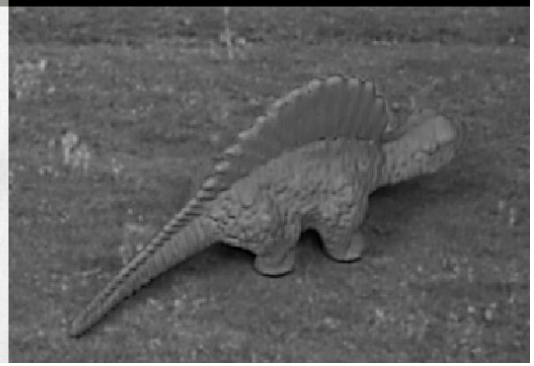
Original image



Object mask

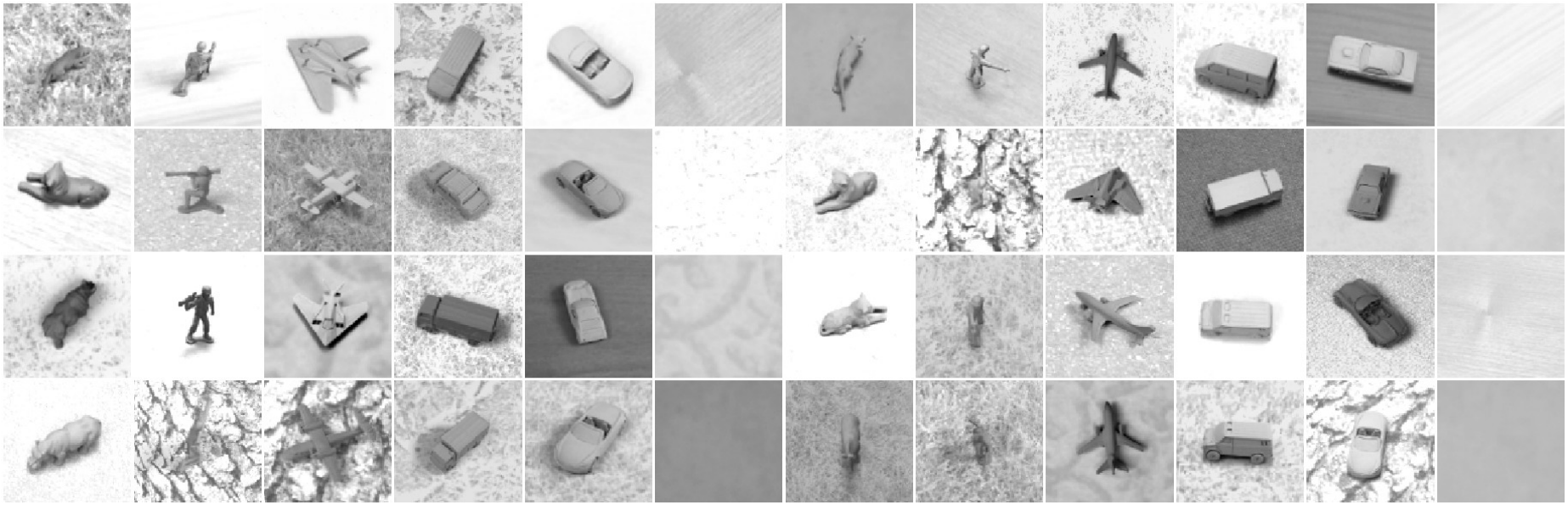


Shadow factor

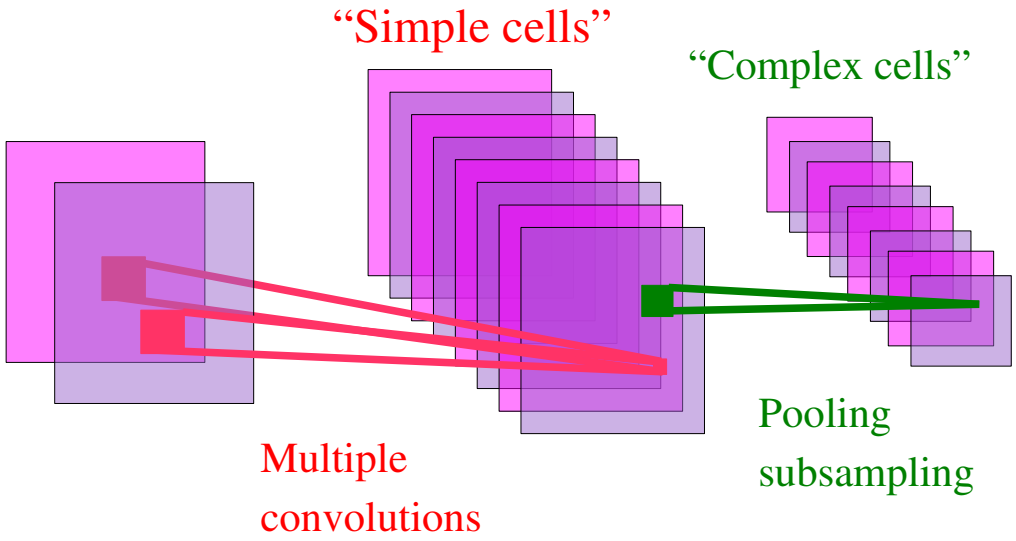


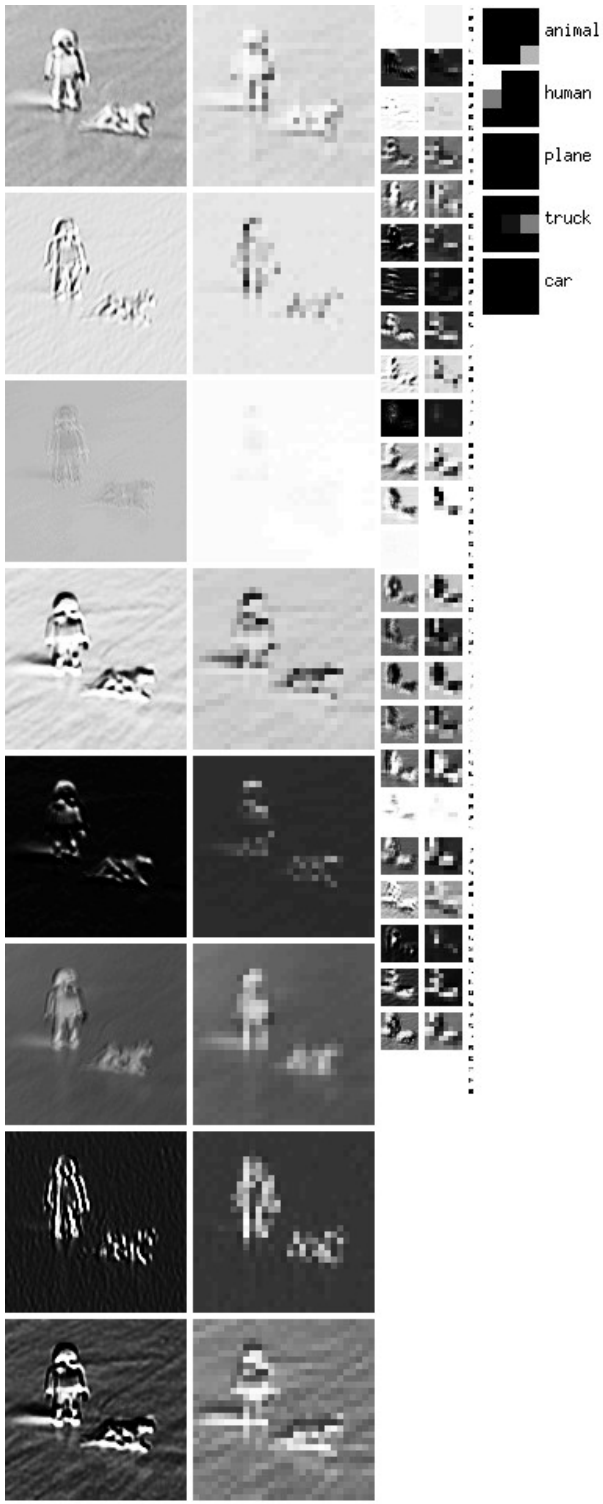
Composite image

Textured and Cluttered Datasets

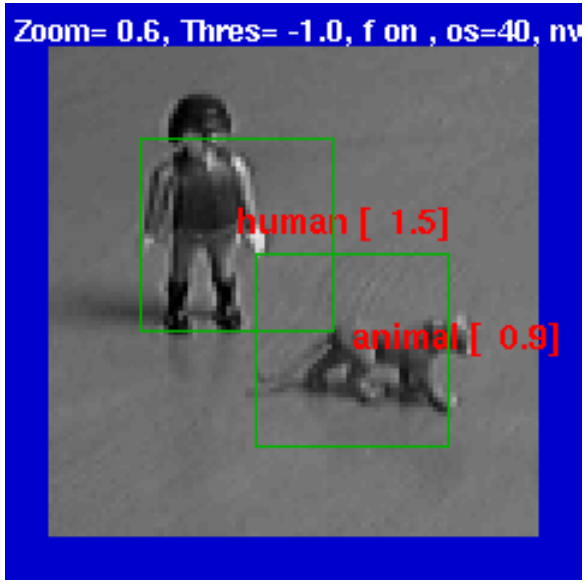


Alternated Convolutions and Subsampling



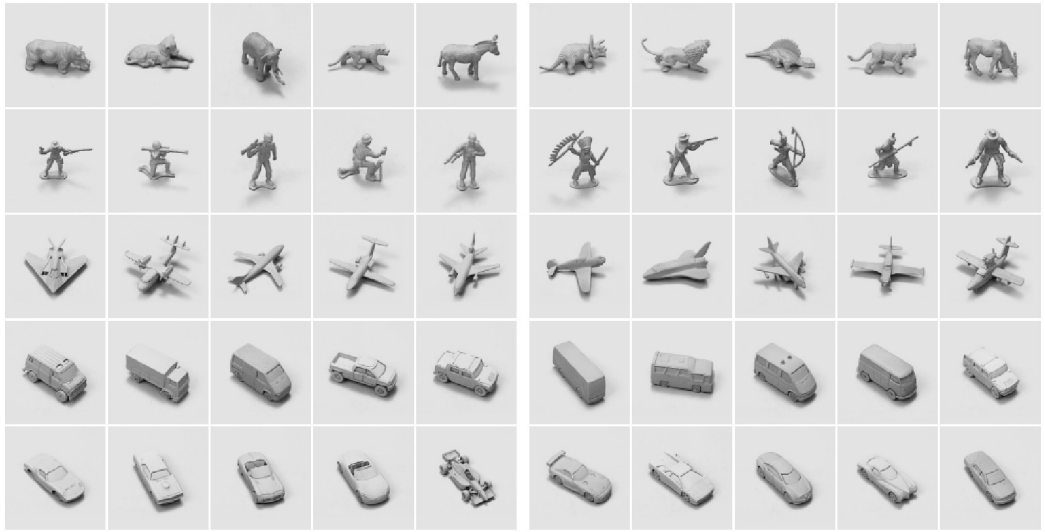


- **Local features are extracted everywhere.**
- **pooling/subsampling layer builds robustness to variations in feature locations.**



Normalized-Uniform dataset: Test Error Rates

- **Linear Classifier on raw stereo images:** **30.2% error.**
- **K-Nearest-Neighbors on raw stereo images:** **18.4% error.**
- **K-Nearest-Neighbors on PCA-95:** **16.6% error.**
- **Pairwise SVM on 96x96 stereo images:** **11.6% error**
- **Pairwise SVM on 95 Principal Components:** **13.3% error.**
- **Convolutional Net on 96x96 stereo images:** **5.8% error.**



Training instances Test instances

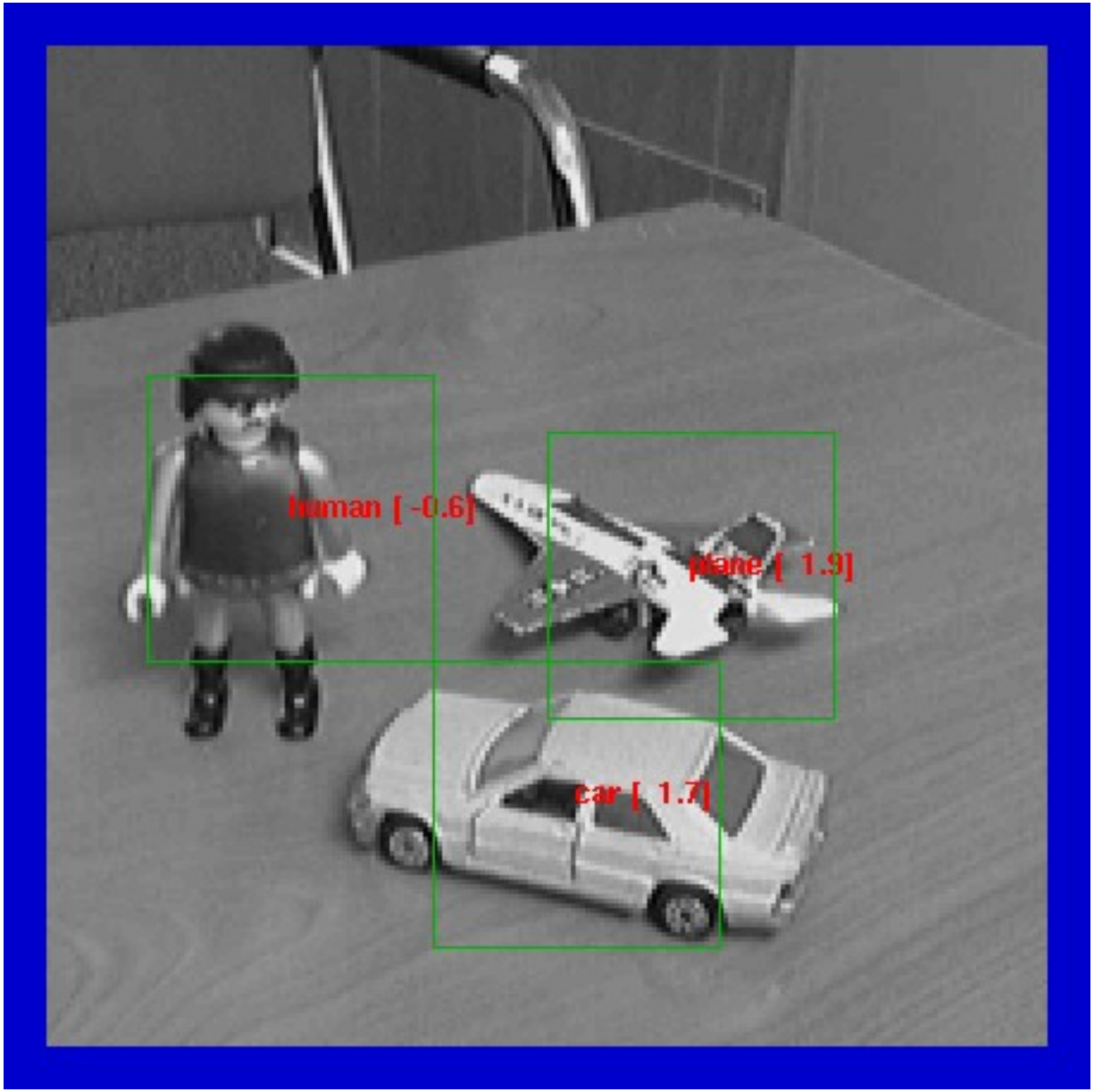
Jittered-Cluttered Dataset

- Jittered-Cluttered **Dataset:**
- **291,600** stereo pairs for training, **58,320** for testing
- Objects are jittered: position, scale, in-plane rotation, contrast, brightness, backgrounds, distractor objects,...
- Input dimension: 98x98x2 (approx 18,000)

Experiment 2: Jittered-Cluttered Dataset

- 291,600 training samples, 58,320 test samples
- SVM with Gaussian kernel — 43.3% error
- Convolutional Net with binocular input: — 7.8% error
- Convolutional Net + SVM on top: — 5.9% error
- Convolutional Net with monocular input: — 20.8% error
- Smaller mono net (DEMO): — 26.0% error
- Dataset available from http://www.cs.nyu.edu/~yann

Examples (Monocular Mode)

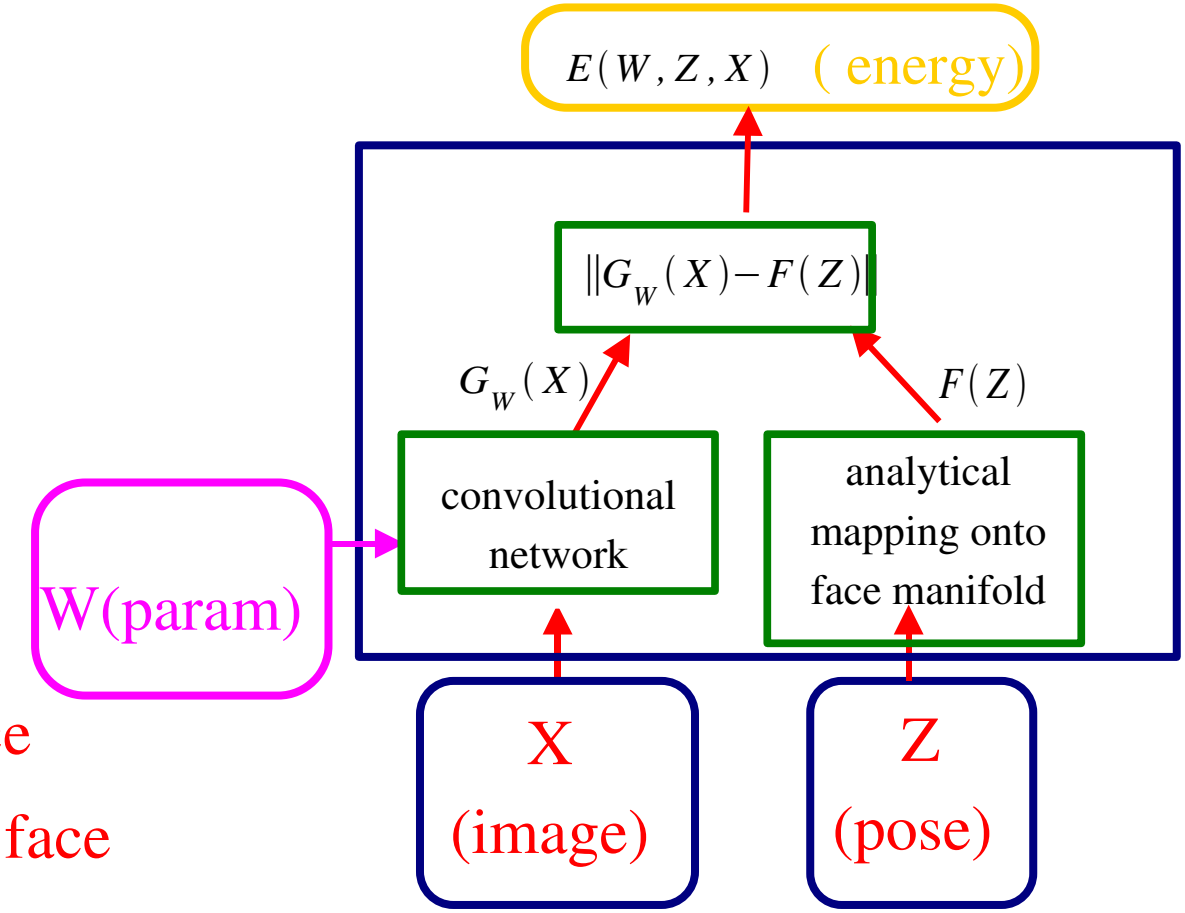


Face Detection and Pose Estimation with a Convolutional EBM

- **Training:** 52,850, 32x32 grey-level images of faces, 52,850 selected non-faces.
- Each training image was used 5 times with random variation in scale, in-plane rotation, brightness and contrast.
- **2nd phase:** half of the initial negative set was replaced by false positives of the initial version of the detector .

$$E^*(W,X) = \min_Z ||G_W(X) - F(Z)||$$

$$Z^* = \operatorname{argmin}_Z ||G_W(X) - F(Z)||$$

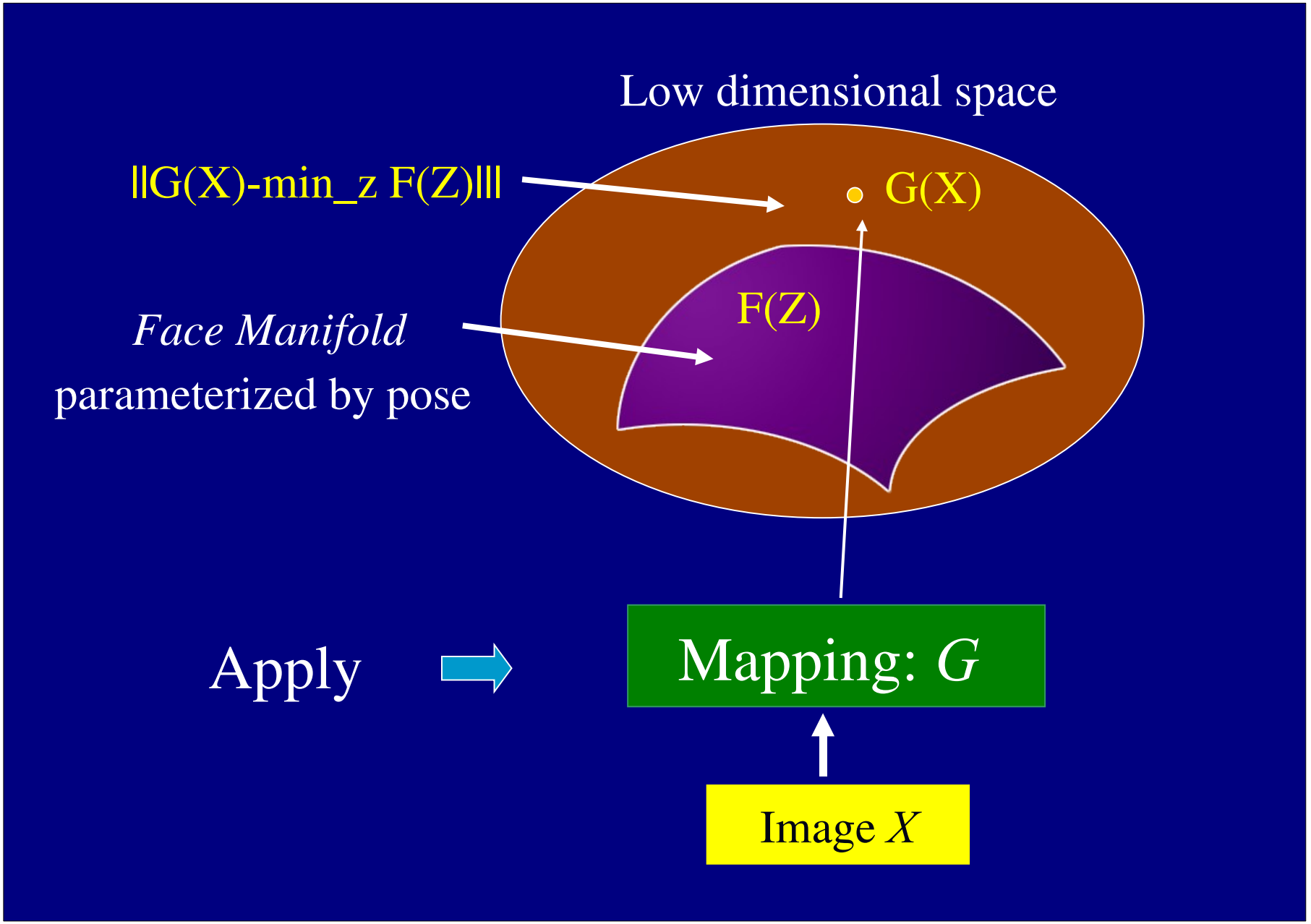


Small E*(W,X): face

Large E*(W,X): no face

[Osadchy, Miller, LeCun, NIPS 2004]

Face Manifold



Probabilistic Approach: Density model of joint P(face,pose)

Probability that image X is a face with pose Z

$$P(X,Z) = \frac{\exp(-E(W,Z,X))}{\int_{X,Z \in \text{images,poses}} \exp(-E(W,Z,X))}$$

Given a training set of faces annotated with pose, find the W that maximizes the likelihood of the data under the model:

$$P(\text{faces} + \text{pose}) = \prod_{X,Z \in \text{faces+pose}} \frac{\exp(-E(W,Z,X))}{\int_{X,Z \in \text{images,poses}} \exp(-E(W,Z,X))}$$

Equivalently, minimize the negative log likelihood:

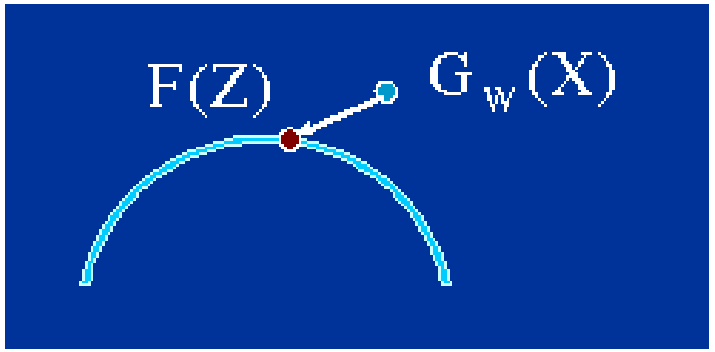
$$\mathcal{L}(W, \text{faces} + \text{pose}) = \sum_{X,Z \in \text{faces+pose}} E(W,Z,X) + \log \left[\int_{X,Z \in \text{images,poses}} \exp(-E(W,Z,X)) \right]$$

COMPLICATED

Energy-Based Contrastive Loss Function

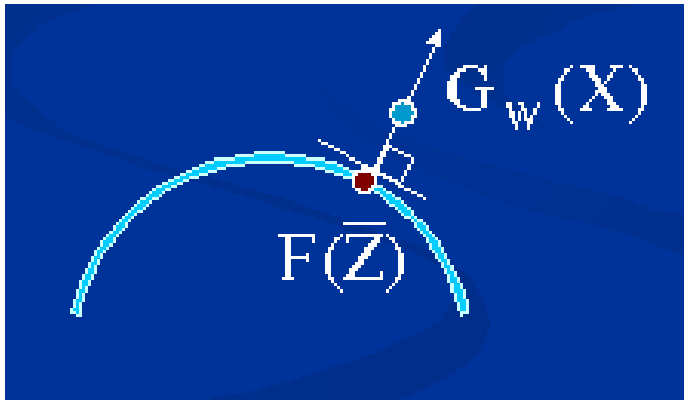
$$\mathcal{L}(W) = \frac{1}{|\mathrm{f}+\mathrm{p}|} \sum_{X,Z \in \mathrm{faces+pose}} \left[L^+\left(E(W,Z,X)\right)\right] + L^-\left(\min_{X,Z \in \mathrm{bckgnd,poses}} E(W,Z,X)\right)$$

$$L^+\left(E(W,Z,X)\right) = E(W,Z,X)^2 = ||G_W(X) - F(Z)||^2$$



Attract the network output Gw(X) to the location of the desired pose F(Z) on the manifold

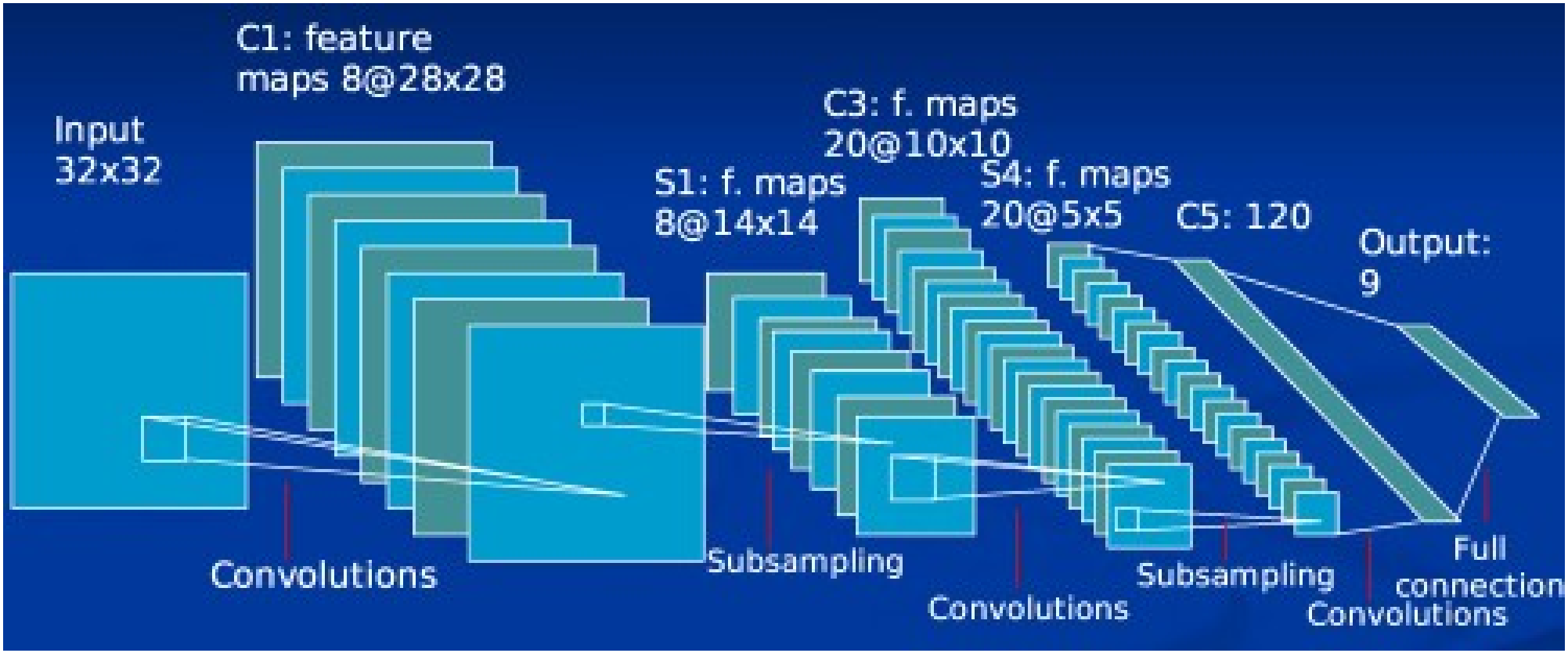
$$L^-\left(\min_{X,Z \in \mathrm{bckgnd,poses}} E(W,Z,X)\right) = K \exp\left(-\min_{X,Z \in \mathrm{bckgnd,poses}} ||G_W(X) - F(Z)||\right)$$



Repel the network output Gw(X) away from the face/pose manifold

Convolutional Net Architecture for Face Detection

[LeCun et al. 1988, 1989, 1998, 2005]



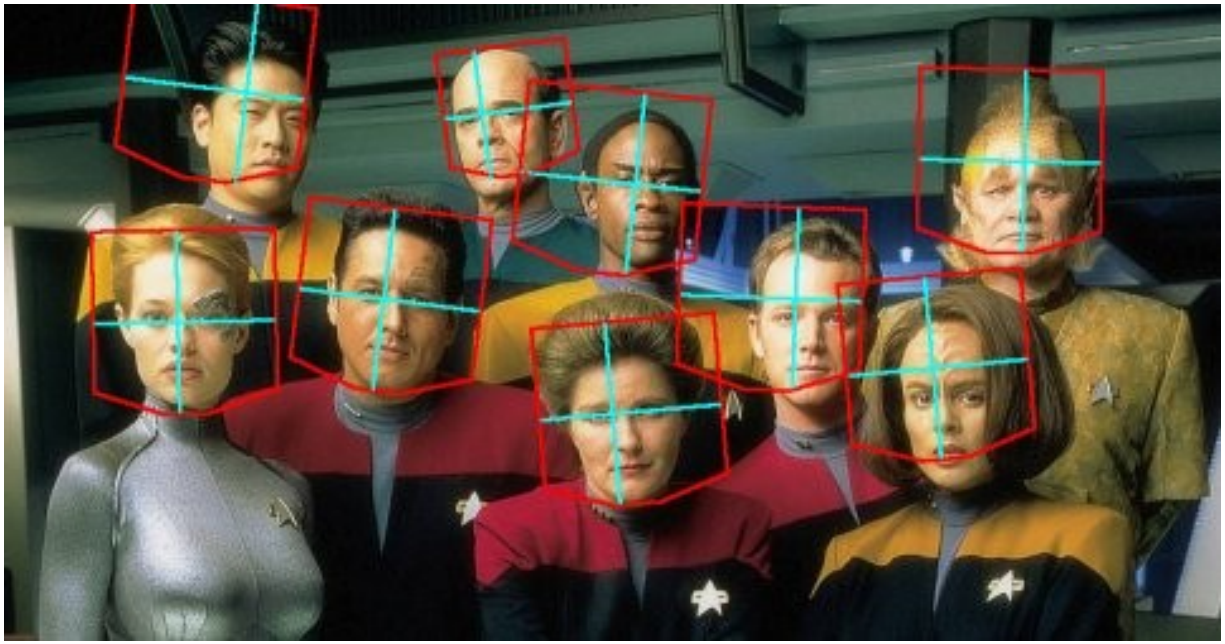
Hierarchy of local filters (convolution kernels),

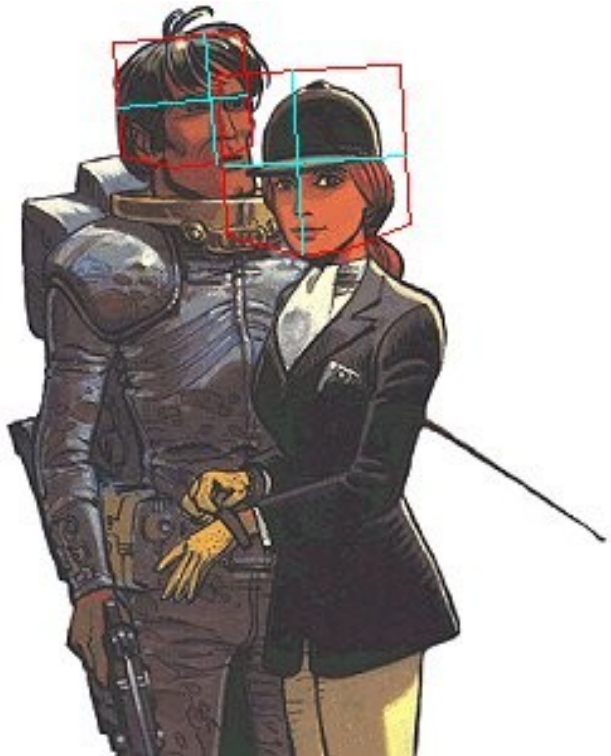
sigmoid pointwise non-linearities, and spatial subsampling

All the filter coefficients are learned with gradient descent (back-prop)

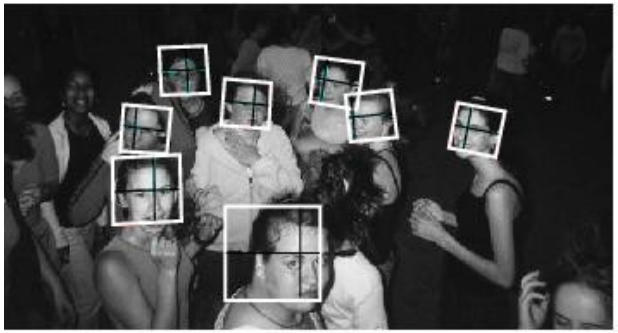
Face Detection: Results

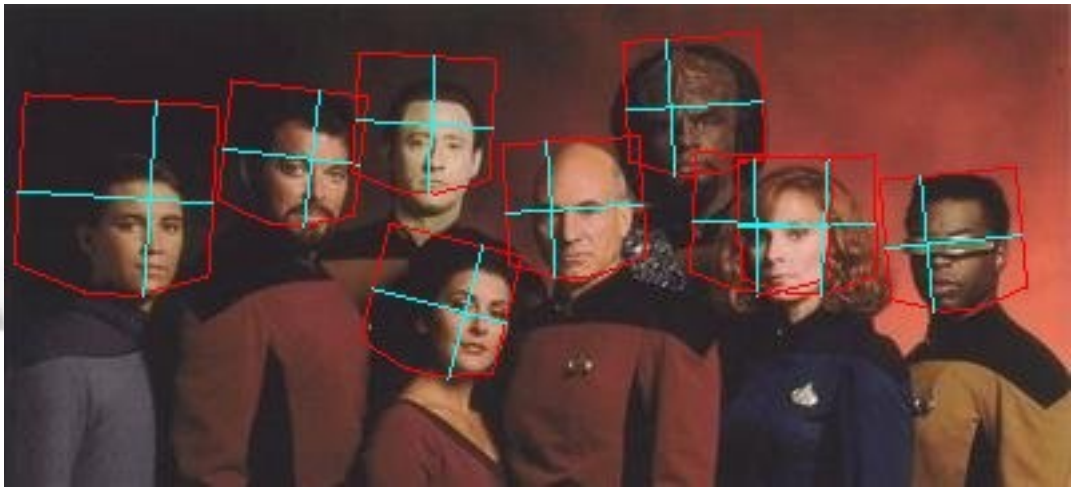
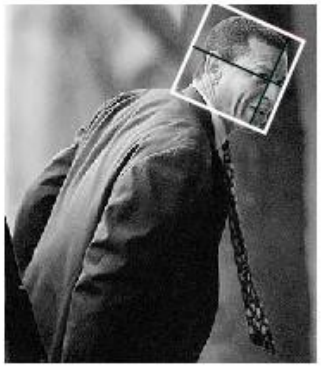
Data Set->	TILTED		PROFILE		MIT+CMU	
False positives per image->	4.42	26.9	0.47	3.36	0.5	1.28
Our Detector	90%	97%	67%	83%	83%	88%
Jones & Viola (tilted)	90%	95%	x		x	
Jones & Viola (profile)	x		70%	83%	x	



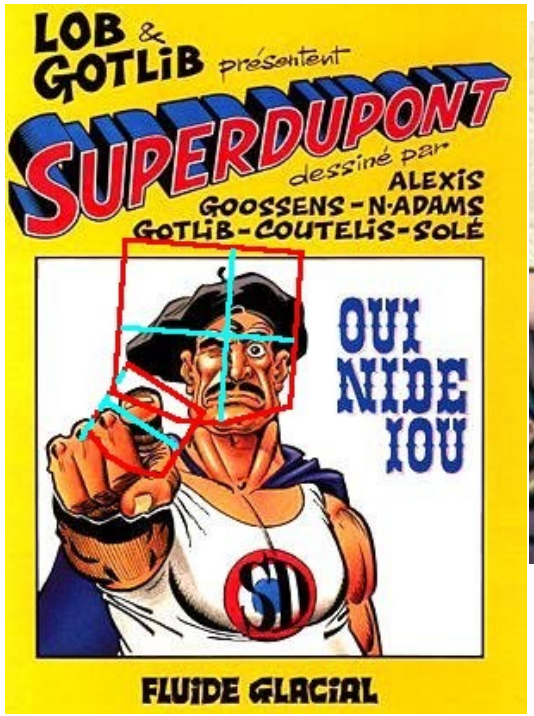


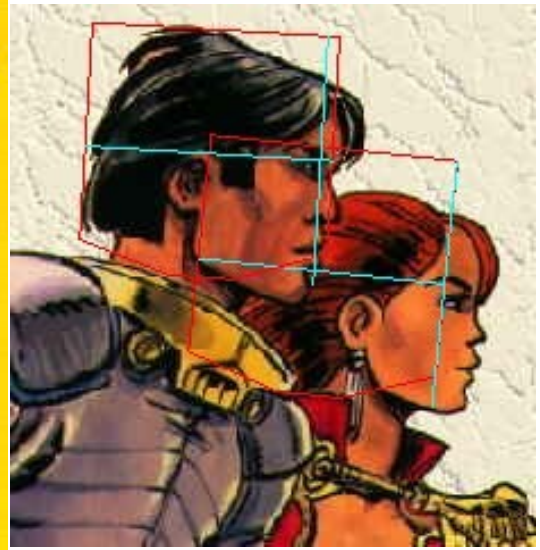
Face Detection and Pose Estimation: Results









Face Detection with a Convolutional Net

Training The Layers of a Convolutional Net Unsupervised

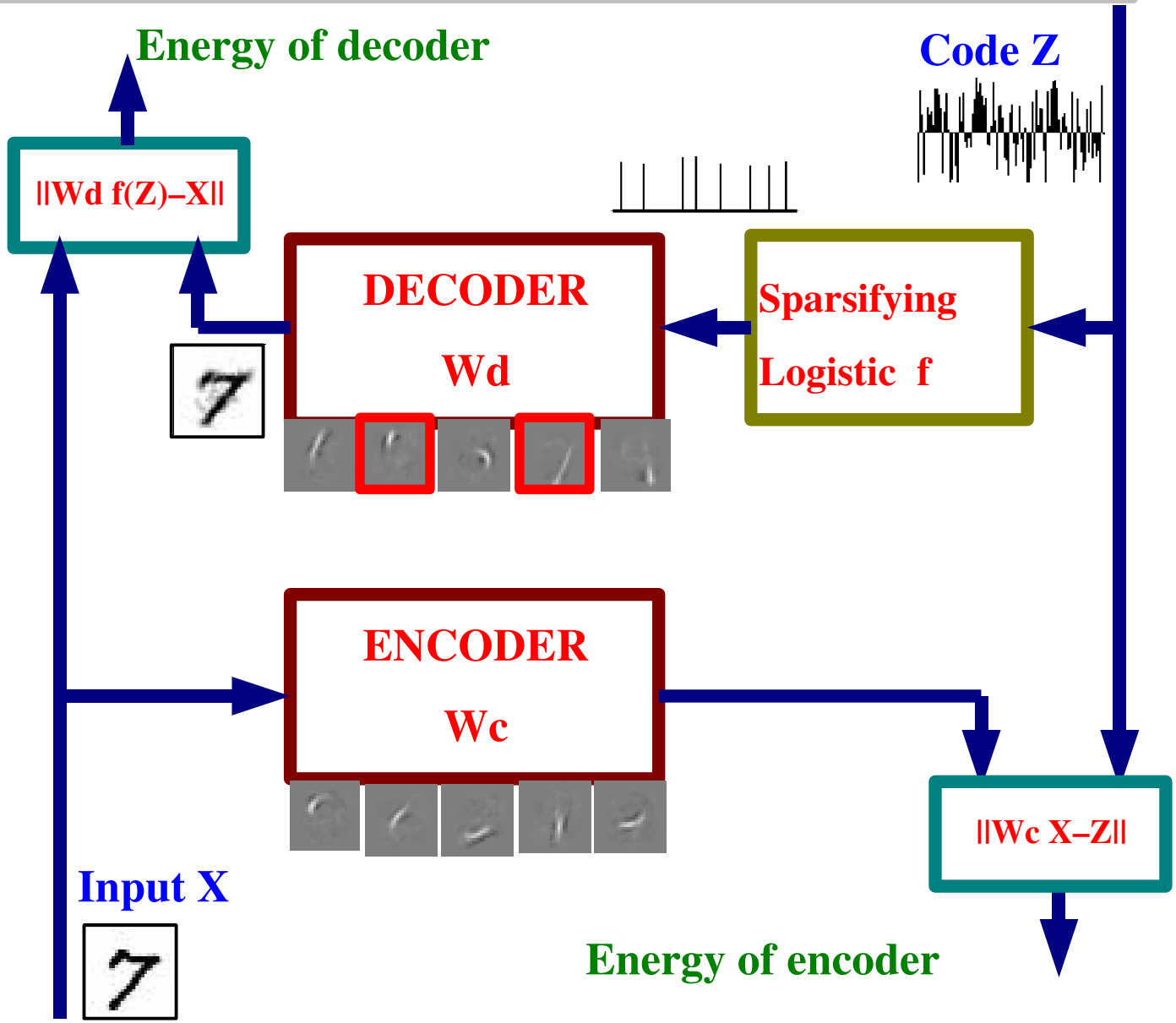
Supervised training of convolutional nets requires too

labeled many training samples

- **Extract windows from the images**
- **Train an unsupervised feature extractor on those windows**
- **Use the resulting features as the convolution kernels of a convolution network**
- **Repeat the process for the second layer**
- **Train the resulting network supervised.**

Encoder/Decoder Architecture for learning Sparse Feature Representations

- **Algorithm:**
  - 1. find the code Z that minimizes the reconstruction error AND is close to the encoder output
  - 2. Update the weights of the decoder to decrease the reconstruction error
  - 3. Update the weights of the encoder to decrease the prediction error



Sparsifying Logistic

- **Maps a code vector into a sparse code vector with components between 0 and 1 (most of which are near zero).**
  - Essentially: a sigmoid function with a large adaptive threshold.
  - Z_i input unit
  - $\bar{Z}_i$ corresponding output unit

$$\bar{z}_i(k) = \frac{\eta\, e^{\beta z_i(k)}}{\xi_i(k)}, \quad i \in [1..m], k \in [1..P], \eta \in (0,1), \beta > 0$$

$$\xi_i(k) = \eta\, e^{\beta z_i(k)} + (1-\eta)\xi_i(k-1)$$

- **Expanding the denominator:**

$$\bar{z}_i(k) = \frac{\eta\, e^{\beta z_i(k)}}{\eta\, e^{\beta z_i(k)} + \eta(1-\eta)\, e^{\beta z_i(k-1)} + \eta(1-\eta)^2\, e^{\beta z_i(k-2)} + \ldots}$$

- **equivalent to a sigmoid with a large threshold:**

$$\bar{z}_i(k) = \frac{1}{1 + e^{-\beta[z_i(k) - \frac{1}{\beta}\log(\frac{1-\eta}{\eta}\xi_i(k-1))]}}$$

Sparsifying Logistic

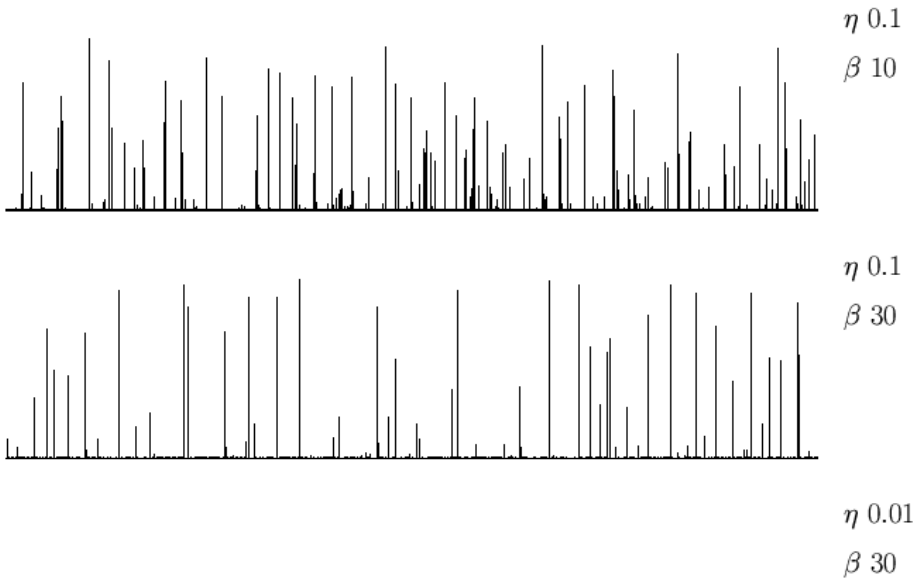
$$\bar{z}_i(k) = \frac{\eta\, e^{\beta z_i(k)}}{\xi_i(k)}, \quad i \in [1..m], k \in [1..P]$$

$$\xi_i(k) = \eta\, e^{\beta z_i(k)} + (1-\eta)\xi_i(k-1)$$

EXAMPLE

Input: random variable uniformly distributed in [-1,1]

Output: a Poisson process with firing rate determined by η and β .



- Increasing β the gain is increased and the output takes almost binary values.

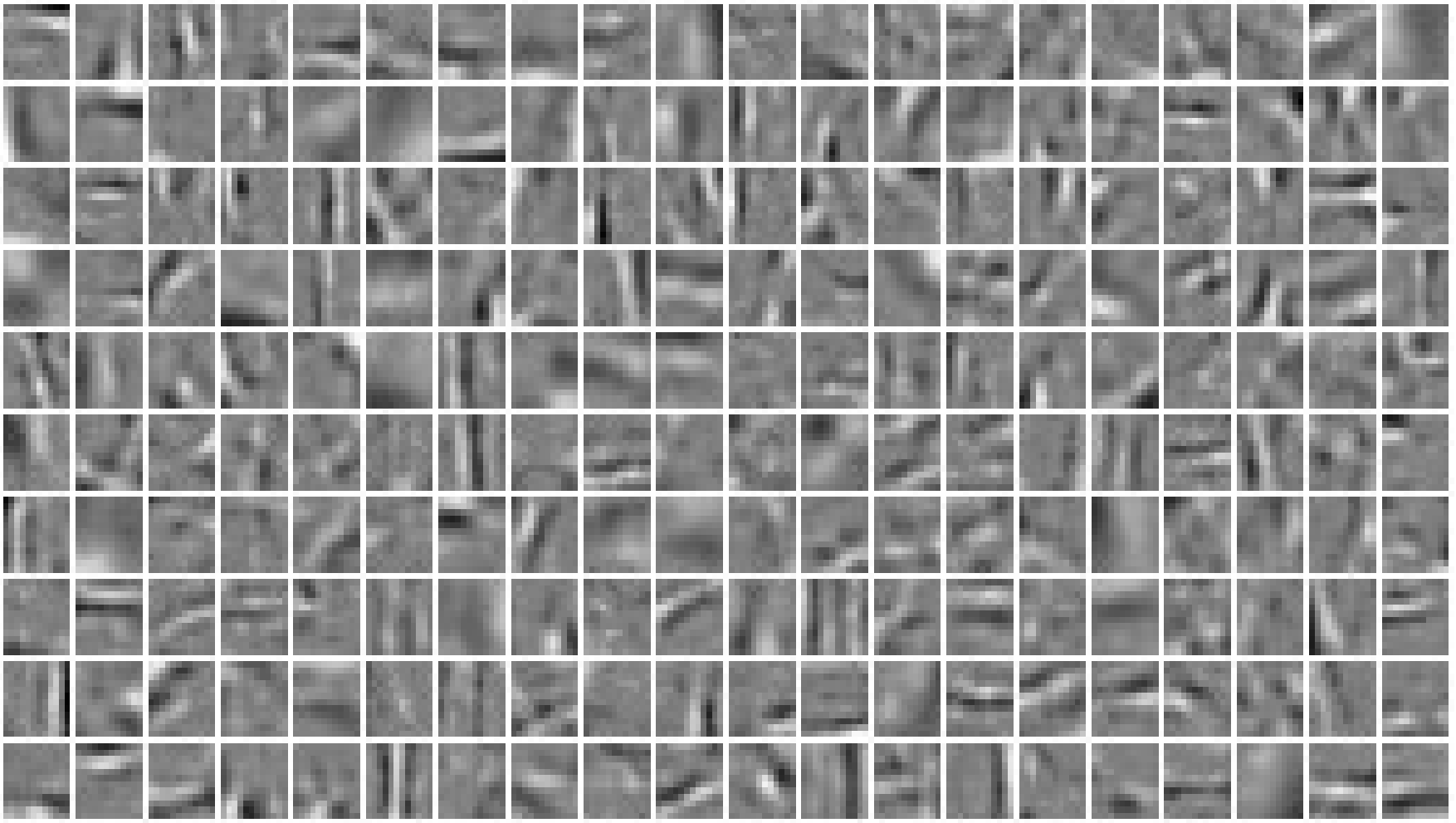
- Increasing η more importance is given to the current sample, a spike will be more likely to occur.

Natural image patches - Berkeley

Berkeley data set

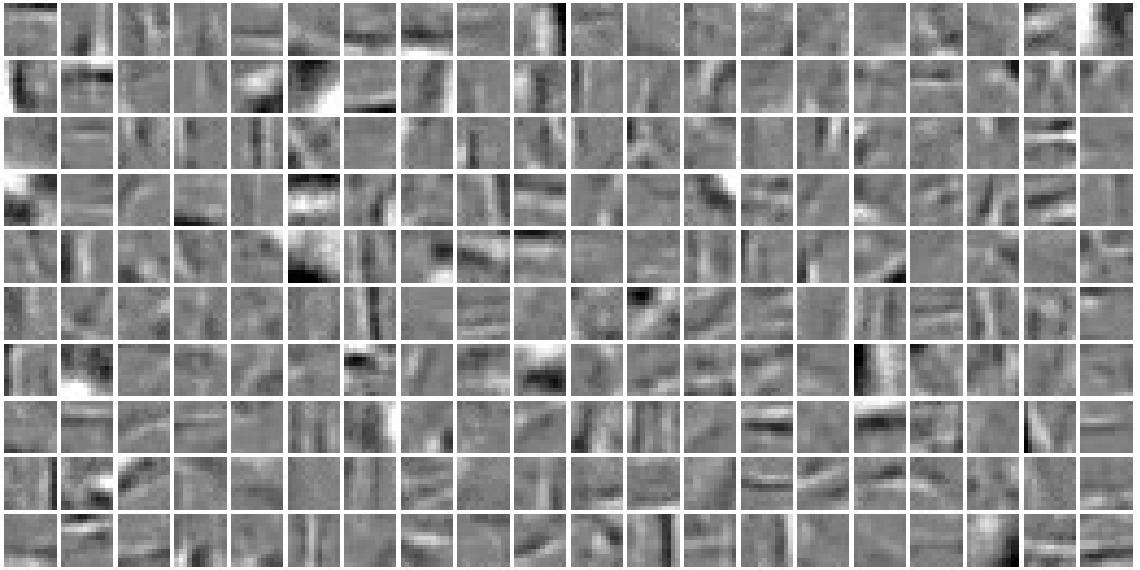
- 100,000 12x12 patches
- 200 units in the code
- η
- β 0.02
- 1
- learning rate 0.001
- L1 regularizer 0.001
- fast convergence: < 30min.

Natural image patches - Berkeley



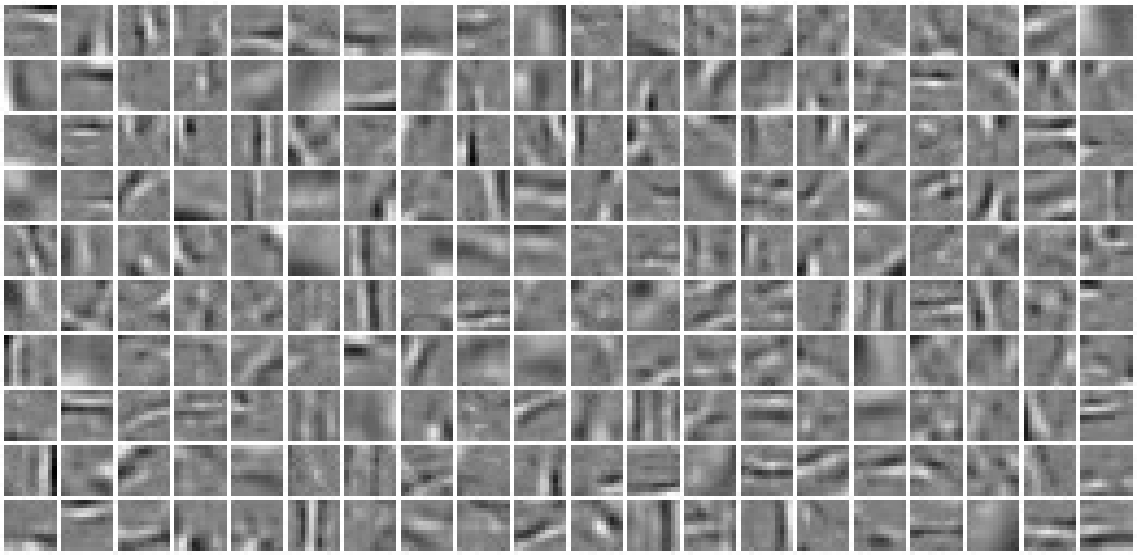
200 decoder filters (reshaped columns of matrix $\mathbf{W_d}$)

Natural image patches - Berkeley



Encoder *direct* filters

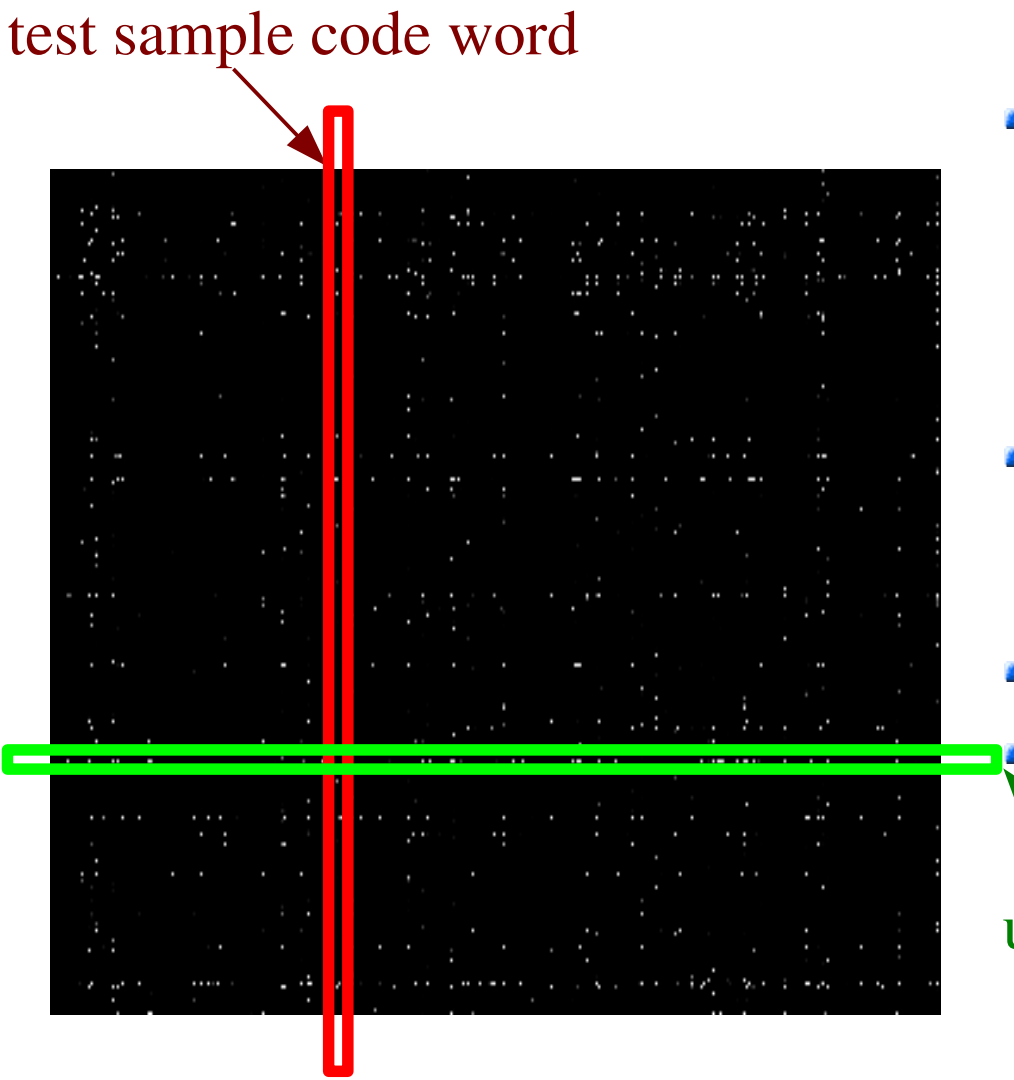
(rows of $\mathbf{W_c}$)



Decoder *reverse* filters

(cols. of $\mathbf{W_d}$)

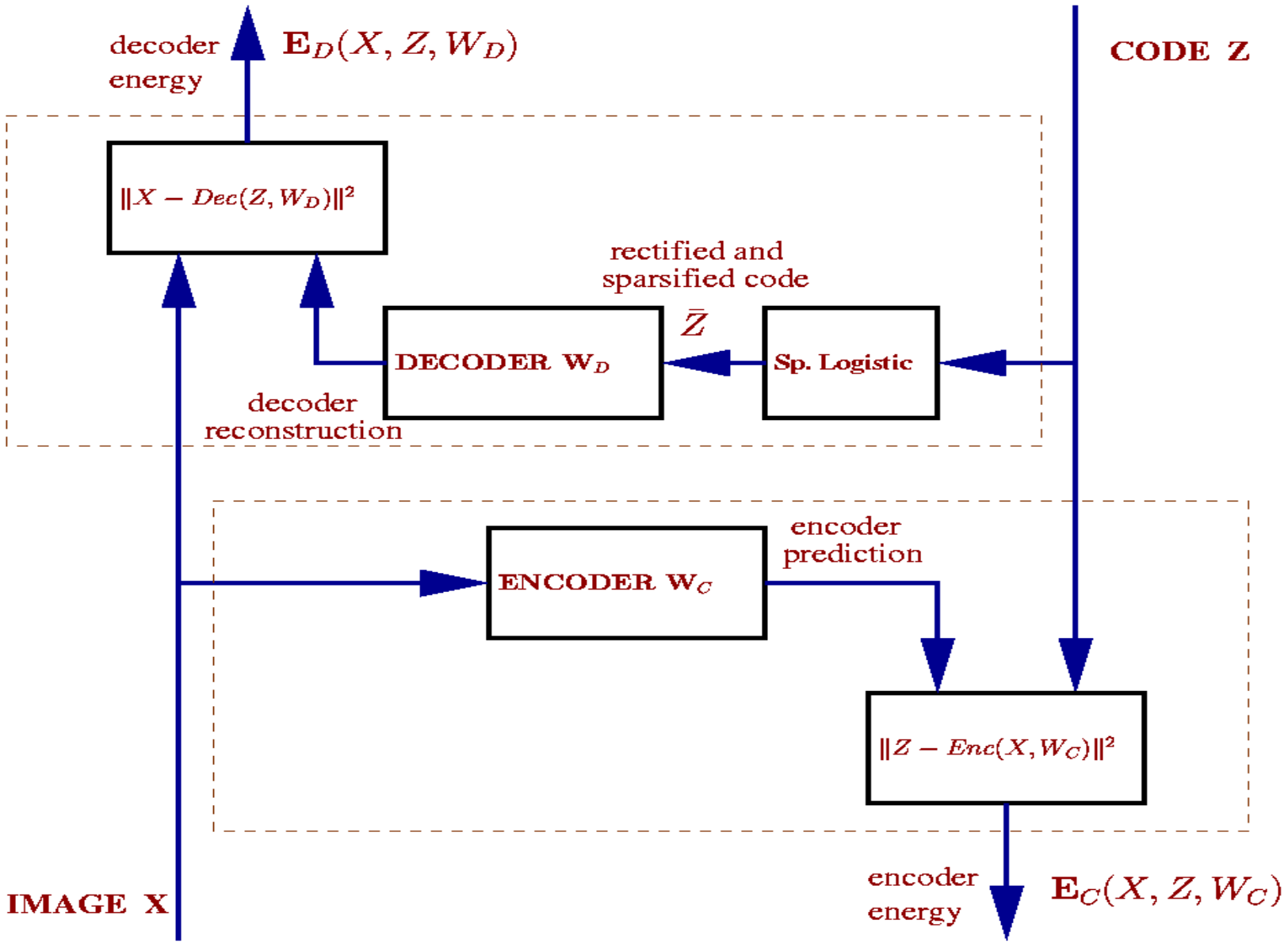
Natural image patches - Berkeley



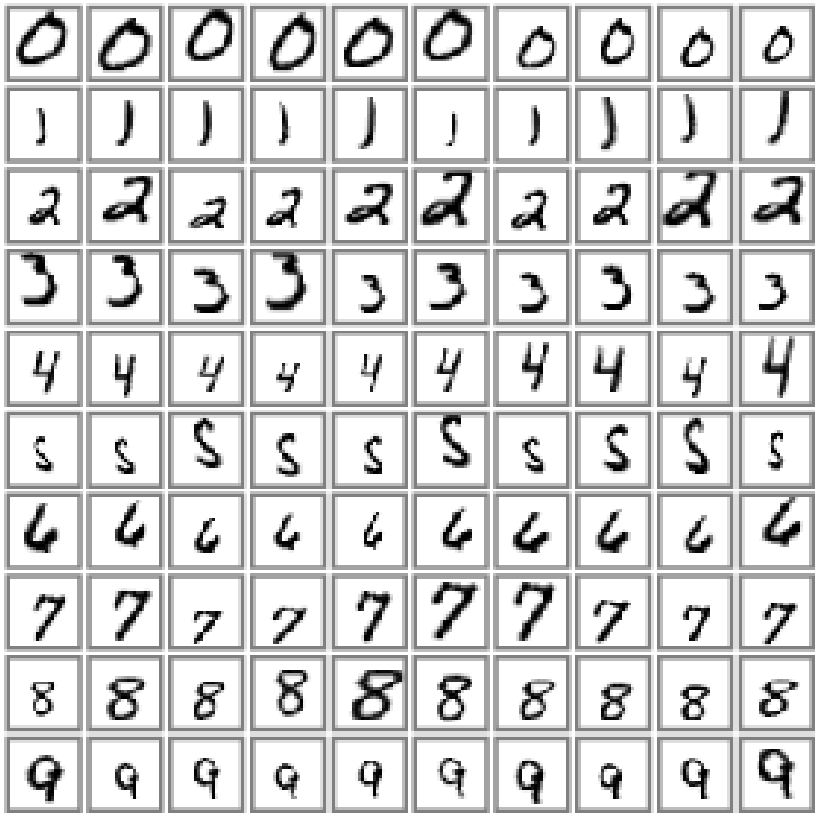
code words from 200 randomly selected test patches

- codes are:
  - sparse
  - almost binary
  - quite decorrelated
- in testing codes are produced by propagating the input patch through encoder and Sparsifying Logistic
- β
  - controls sparsity
  - controls the "bit content" in each code unit

What about an autoencoder?

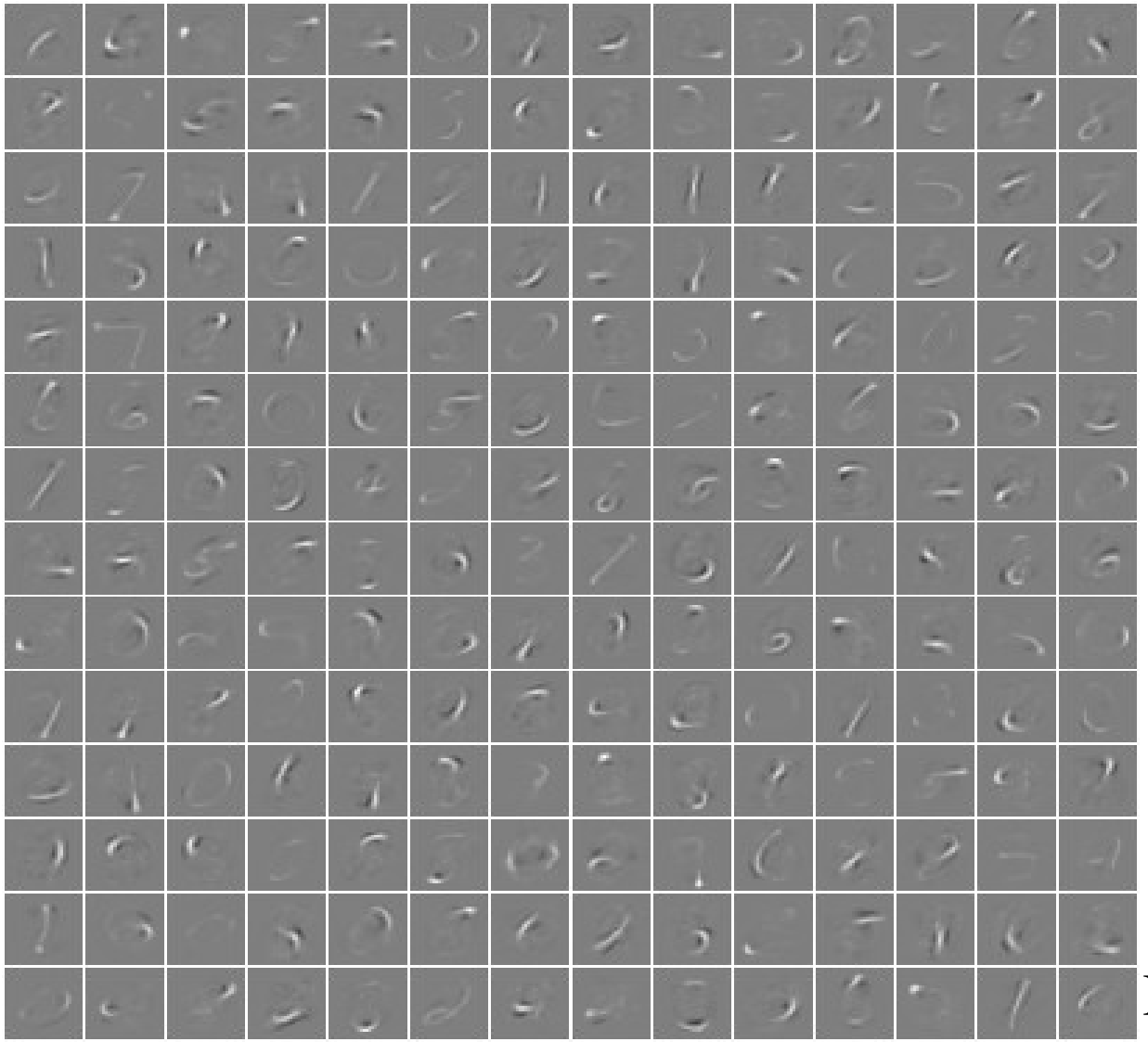


MNIST Dataset



- Handwritten Digit Dataset MNIST: 60,000 training samples, 10,000 test samples

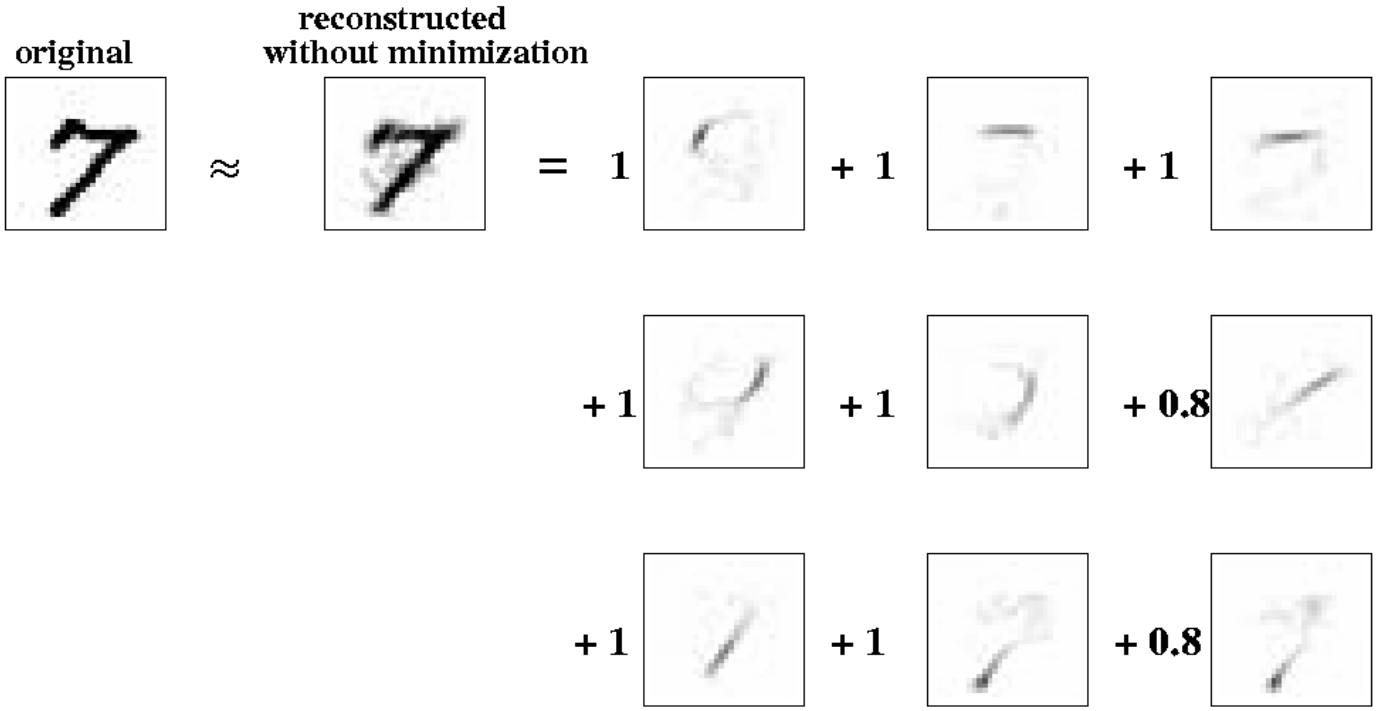
Handwritten digits - MNIST

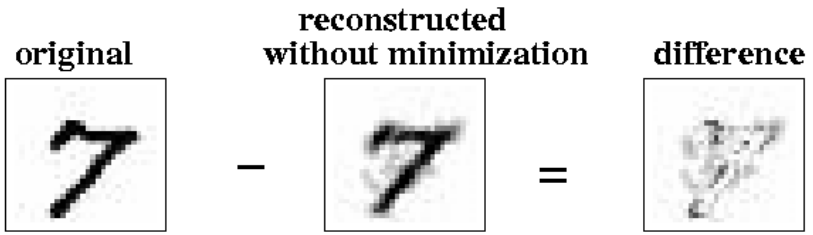


- 60,000 28x28 images
- 196 units in the code
- η 0.01
- β 1
- learning rate 0.001
- L1, L2 regularizer 0.005

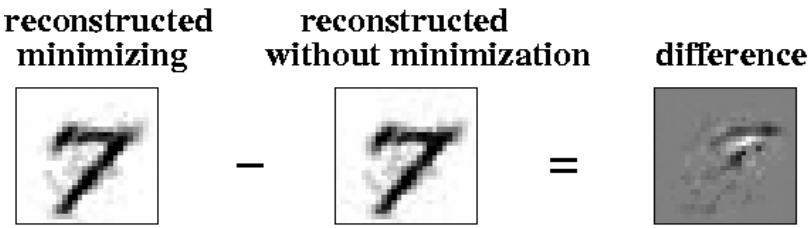
Encoder *direct* filters

Handwritten digits - MNIST





forward propagation through encoder and decoder



after training there is no need to minimize in code space

Best Results on MNIST (from raw images: no preprocessing)

CLASSIFIER	DEFORMATION	ERROR	Reference
Knowledge-free methods			
2-layer NN, 800 HU, CE		1.60	Simard et al., ICDAR 2003
3-layer NN, 500+300 HU, CE, reg		1.53	Hinton, in press, 2005
SVM, Gaussian Kernel		1.40	Cortes 92 + Many others
Unsupervised Stacked RBM + backprop		0.95	Hinton, Neur Comp 2006
Convolutional nets			
Convolutional net LeNet-5,		0.80	Ranzato et al. NIPS 2006
Convolutional net LeNet-6,		0.70	Ranzato et al. NIPS 2006
Conv. net LeNet-6- + unsup learning		0.60	Ranzato et al. NIPS 2006
Training set augmented with Affine Distortions			
2-layer NN, 800 HU, CE	Affine	1.10	Simard et al., ICDAR 2003
Virtual SVM deg-9 poly	Affine	0.80	Scholkopf
Convolutional net, CE	Affine	0.60	Simard et al., ICDAR 2003
Training et augmented with Elastic Distortions			
2-layer NN, 800 HU, CE	Elastic	0.70	Simard et al., ICDAR 2003
Convolutional net, CE	Elastic	0.40	Simard et al., ICDAR 2003
Conv. net LeNet-6- + unsup learning	Elastic	0.39	Ranzato et al. NIPS 2006

Training Convolutional Filters

CLASSIFICATION EXPERIMENTS

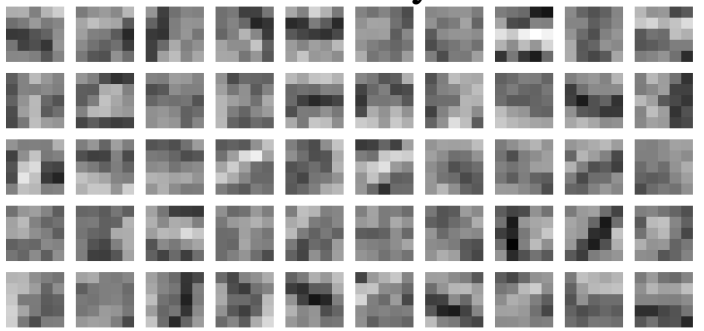
IDEA: improving supervised learning by pre-training with the unsupervised method (*)

sparse representations & *lenet6* (1->50->50->200->10)

- The baseline: *lenet6* initialized randomly

Test error rate: 0.70%. Training error rate: 0.01%.

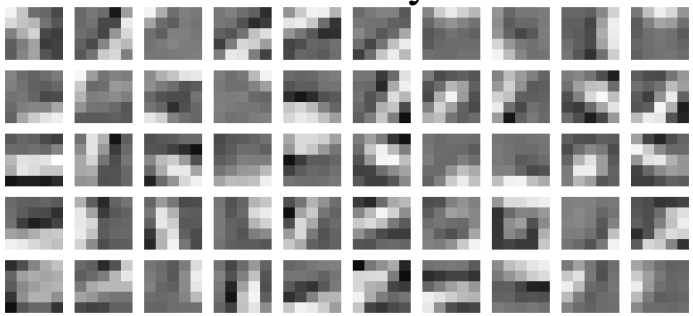
filters in first conv. layer



- *Experiment 1*
  - Train on 5x5 patches to find 50 features
  - Use the scaled filters in the encoder to initialize the kernels in the first convolutional layer

Test error rate: 0.60%. Training error rate: 0.00%.

filters in first conv. layer

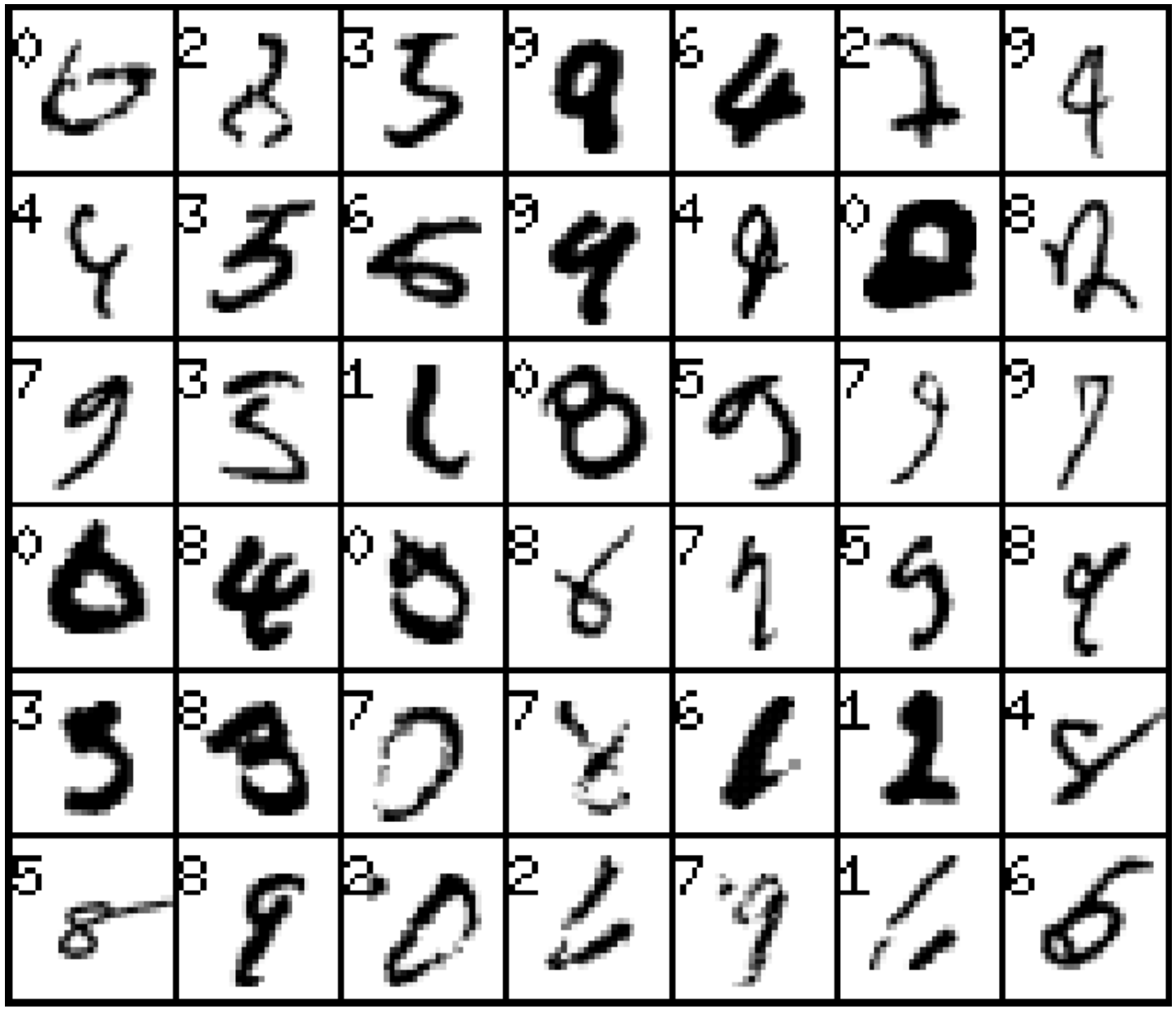


- *Experiment 2*
  - Same as experiment 1, but training set augmented by elastically distorted digits (random initialization gives test error rate equal to 0.49%).

Test error rate: 0.39%. Training error rate: 0.23%.

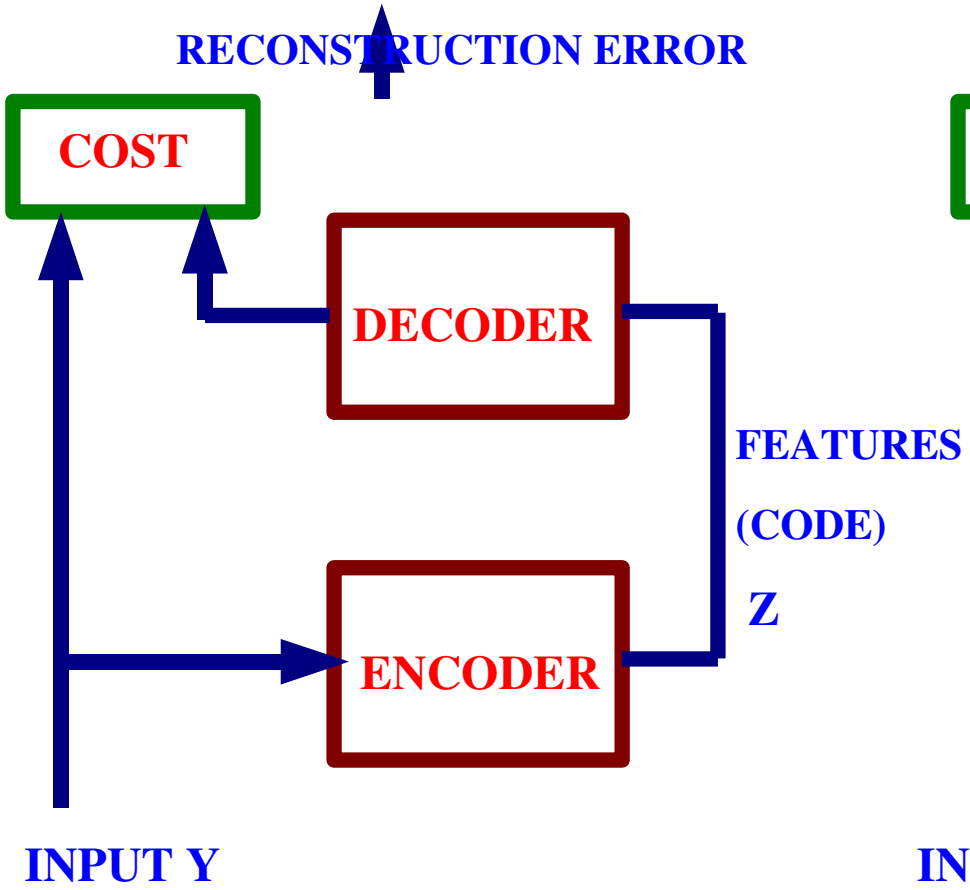
(*)[Hinton, Osindero, Teh "*A fast learning algorithm for deep belief nets*" Neural Computaton 2006]

MNIST Errors (0.42% error)

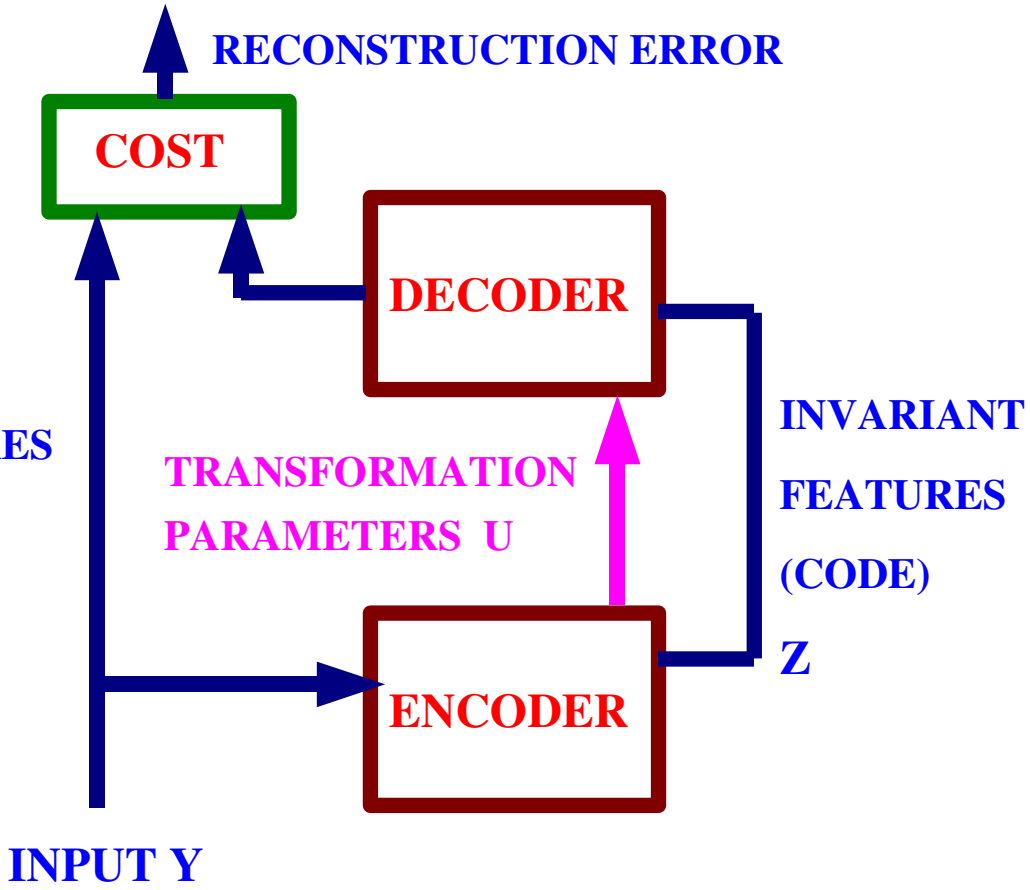


Learning Invariant Feature Hierarchies

Learning Shift Invariant Features



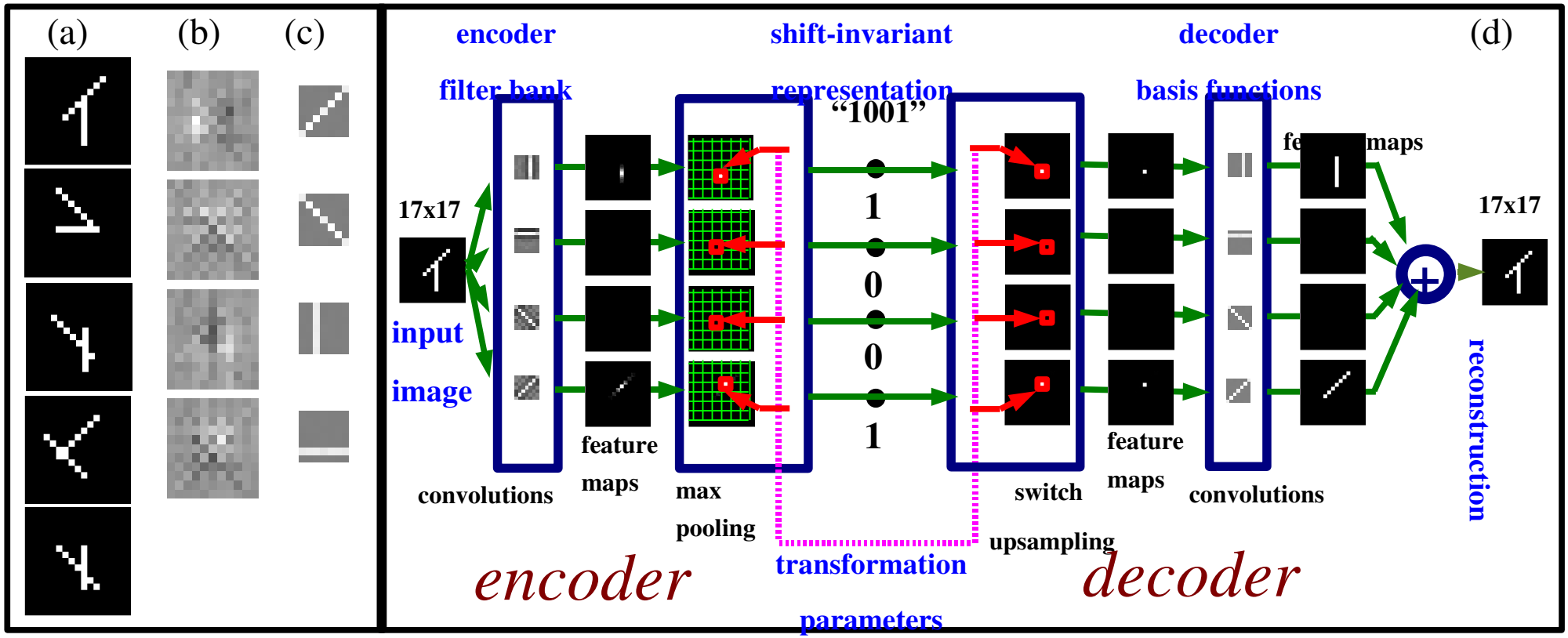
Standard Feature Extractor



Invariant Feature Extractor

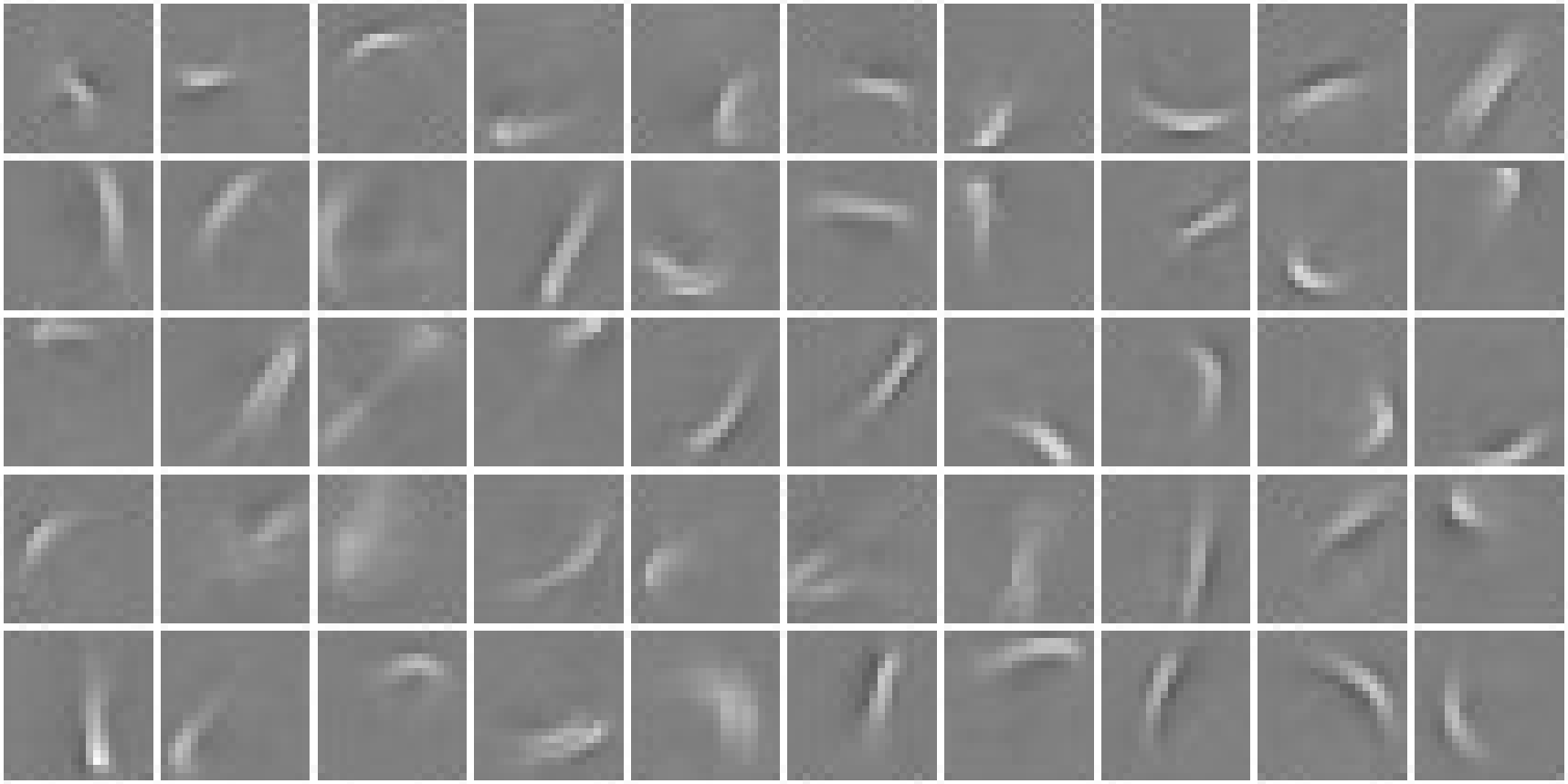
Learning Invariant Feature Hierarchies

Learning Shift Invariant Features



Shift Invariant Global Features on MNIST

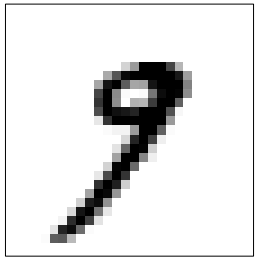
- **Learning 50 Shift Invariant Global Features on MNIST:**
  - 50 filters of size 20x20 movable in a 28x28 frame (81 positions)
  - movable strokes!

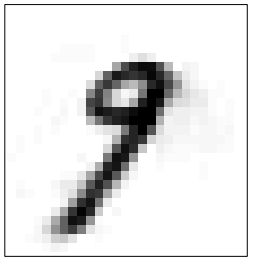


Example of Reconstruction

- Any character can be reconstructed as a linear combination of a small number of basis functions.

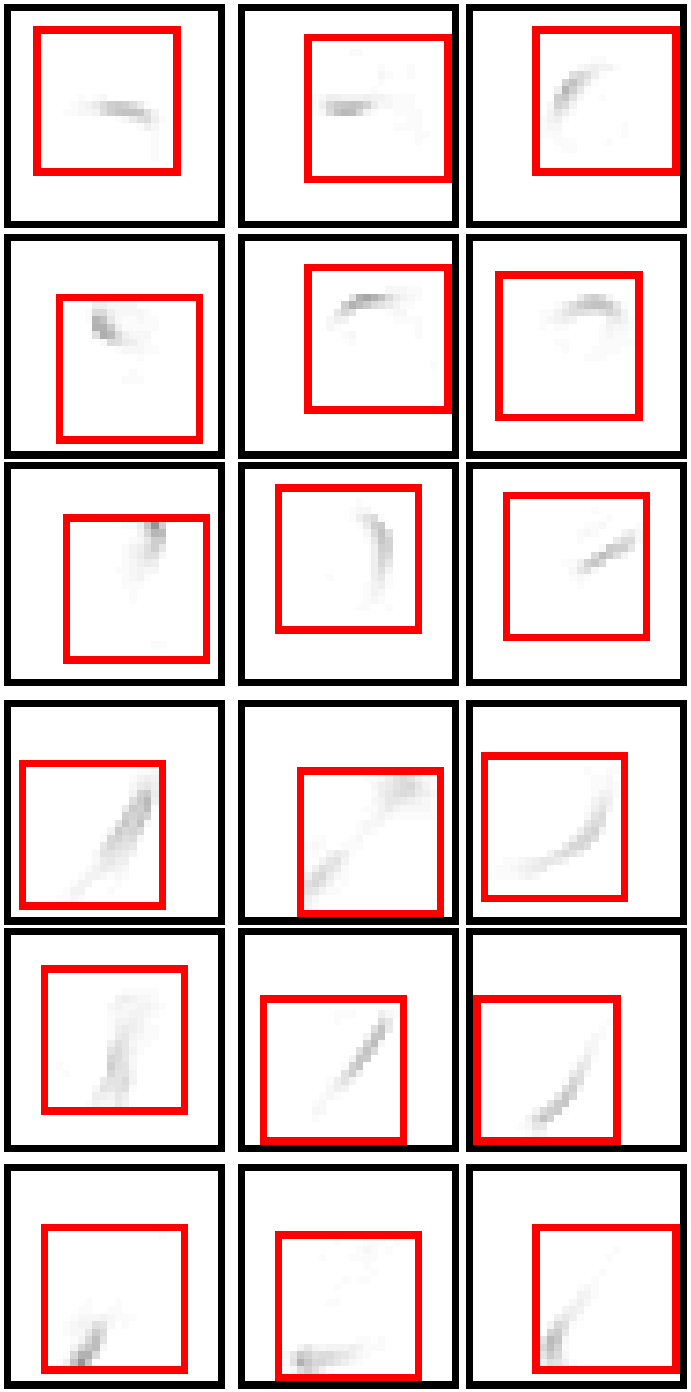
ORIGINAL DIGIT ≅ RECONS-TRUCTION =





ACTIVATED DECODER BASIS FUNCTIONS

(in feed-back layer)



red squares: decoder bases

Learning Invariant Filters in a Convolutional Net

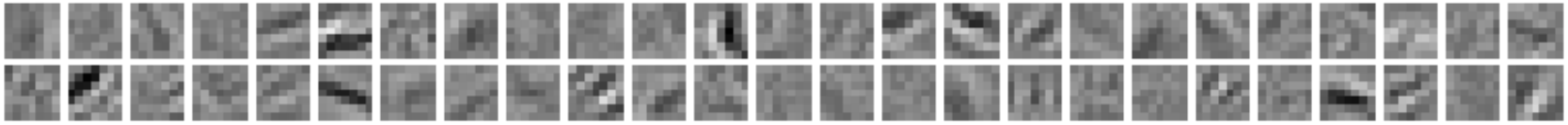


Figure 1: 50 7x7 filters in the first convolutional layer that were learned by the network trained supervised from *random* initial conditions with 600K digits.

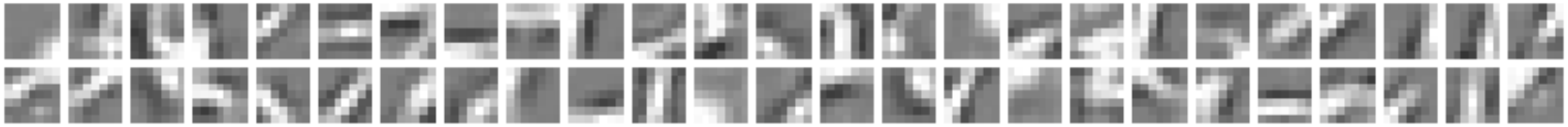


Figure 2: 50 7x7 filters that were learned by the unsupervised method (on 60K digits), and that are used to initialize the first convoltional layer of the network.

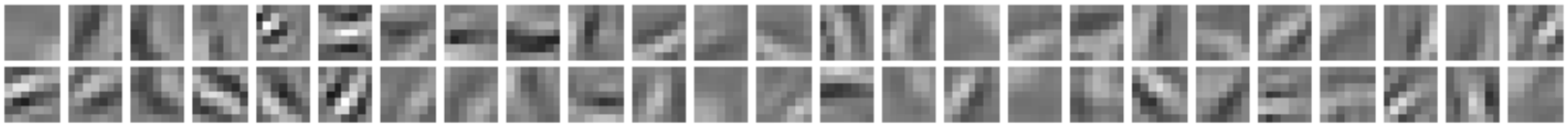
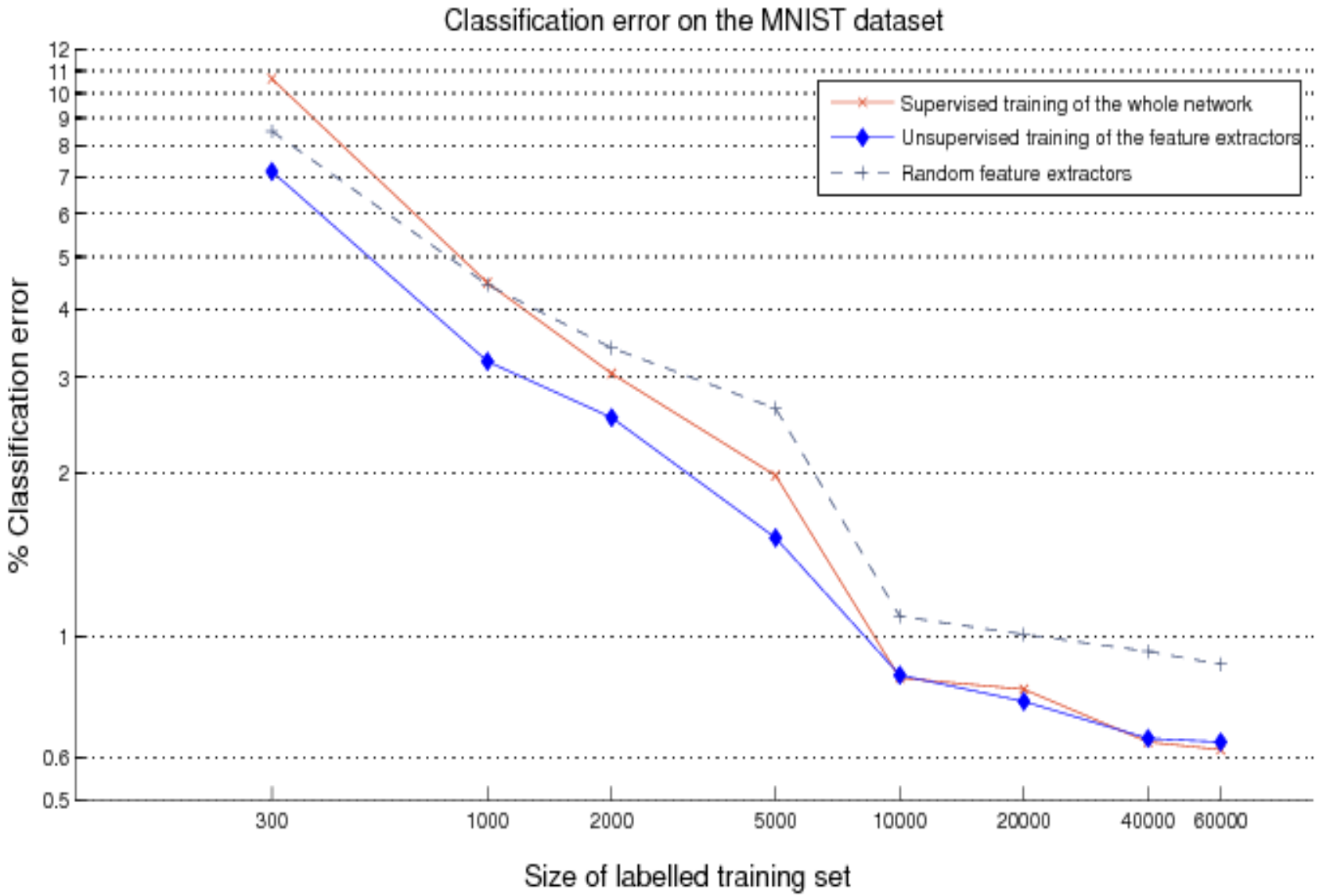


Figure 3: 50 7x7 filters in the first convolutional layer that were learned by the network trained supervised from the initial conditions given by the *unsupervised method* (see fig.2) with 600K digits.

Influence of Number of Training Samples

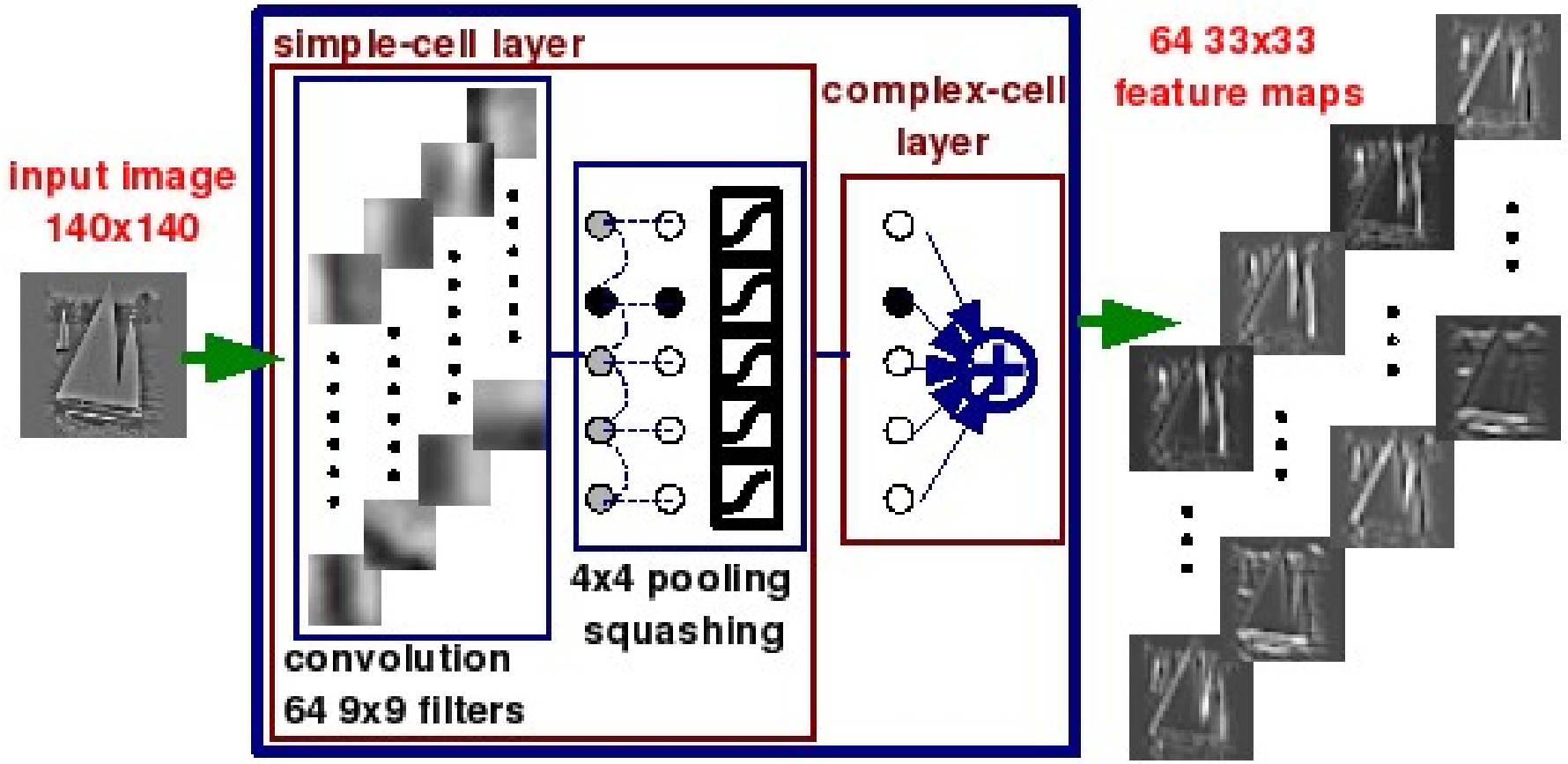


Generic Object Recognition: 101 categories + background

- **Caltech-101 dataset: 101 categories**
  - accordion airplanes anchor ant barrel bass beaver binocular bonsai brain brontosaurus buddha butterfly camera cannon car_side ceiling_fan cellphone chair chandelier cougar_body cougar_face crab crayfish crocodile crocodile_head cup dalmatian dollar_bill dolphin dragonfly electric_guitar elephant emu euphonium ewer Faces Faces_easy ferry flamingo flamingo_head garfield gerenuk gramophone grand_piano hawksbill headphone hedgehog helicopter ibis inline_skate joshua_tree kangaroo ketch lamp laptop Leopards llama lobster lotus mandolin mayfly menorah metronome minaret Motorbikes nautilus octopus okapi pagoda panda pigeon pizza platypus pyramid revolver rhino rooster saxophone schooner scissors scorpion sea_horse snoopy soccer_ball stapler starfish stegosaurus stop_sign strawberry sunflower tick trilobite umbrella watch water_lilly wheelchair wild_cat windsor_chair wrench yin_yang
- **Only 30 training examples per category!**
- **A convolutional net trained with backprop (supervised) gets 20% correct recognition.**
- **Training the filters with the sparse invariant unsupervised method**

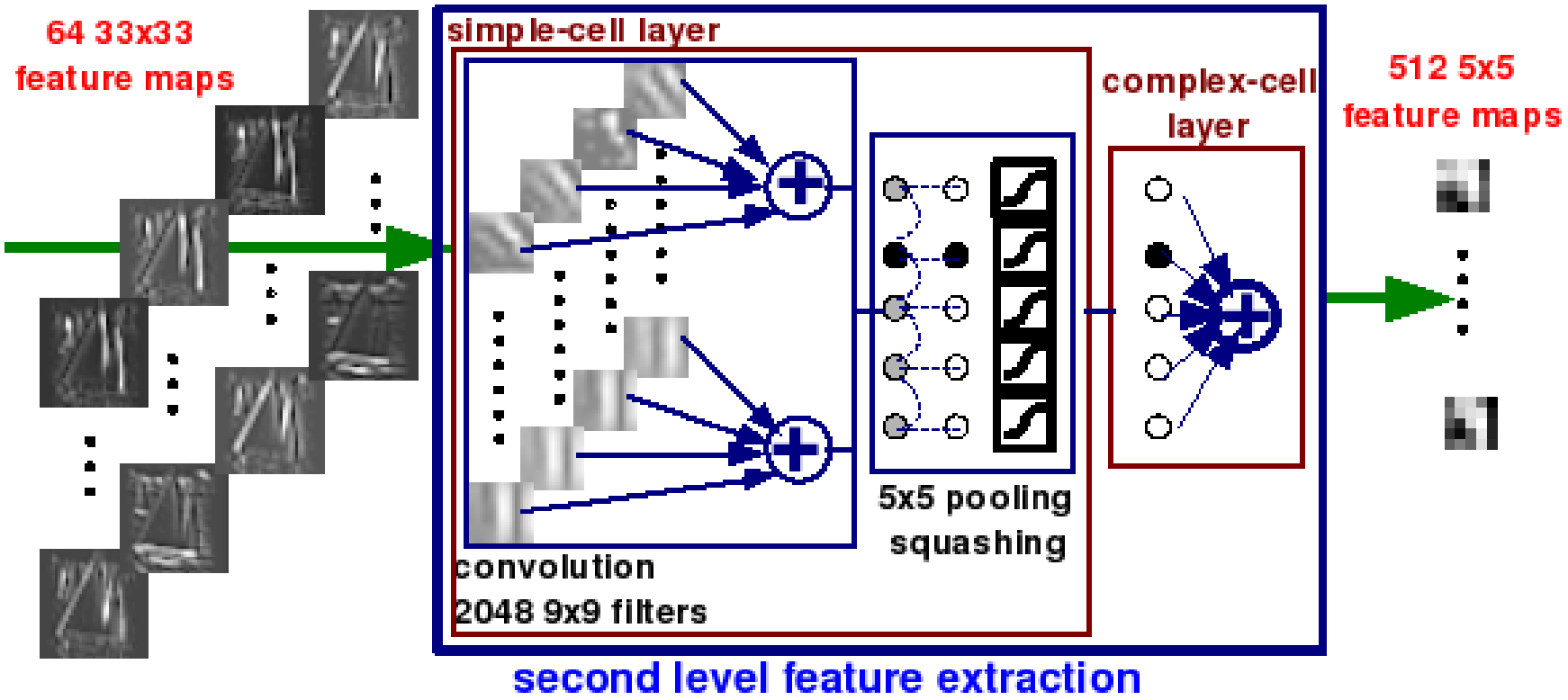
Training the 1st stage filters

- 12x12 input windows (complex cell receptive fields)
- 9x9 filters (simple cell receptive fields)
- 4x4 pooling



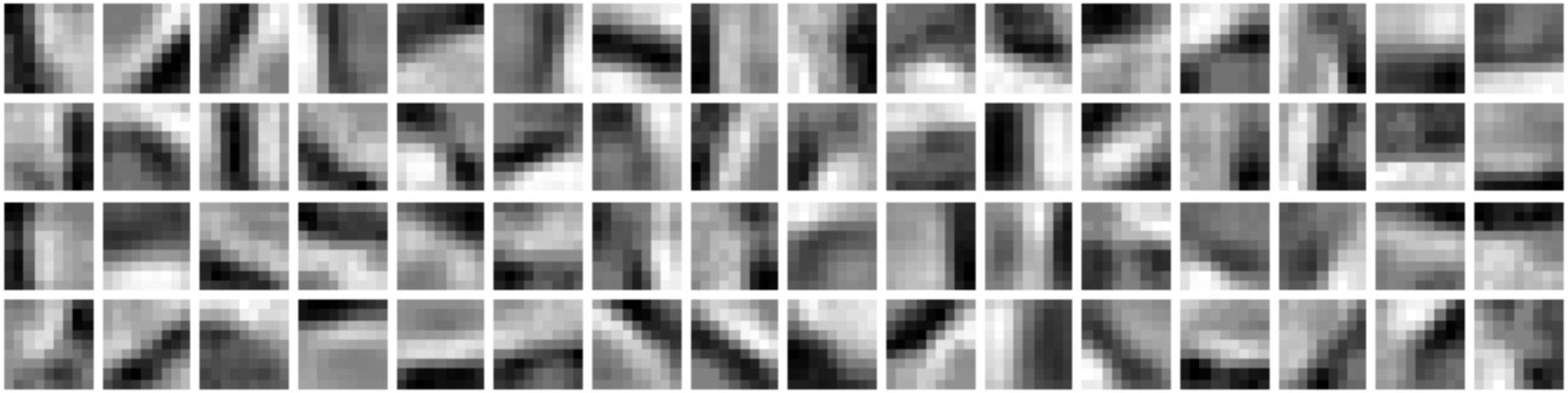
Training the 2nd stage filters

- 13x13 input windows (complex cell receptive fields on 1st features)
- 9x9 filters (simple cell receptive fields)
- Each output feature map combines 4 input feature maps
- 5x5 pooling

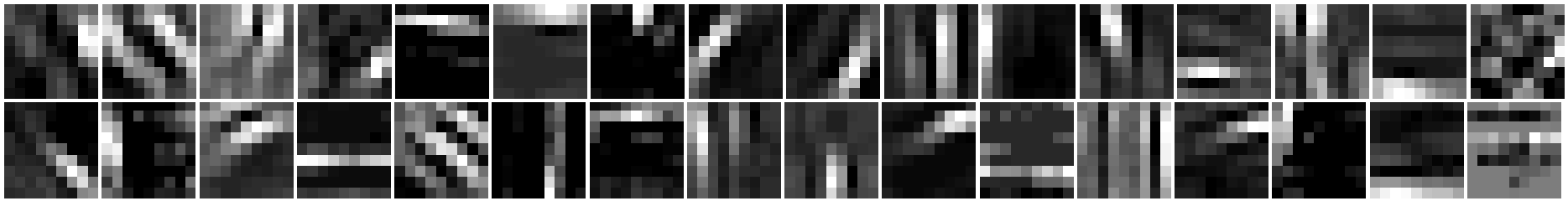


Generic Object Recognition: 101 categories + background

- 9x9 filters at the first level

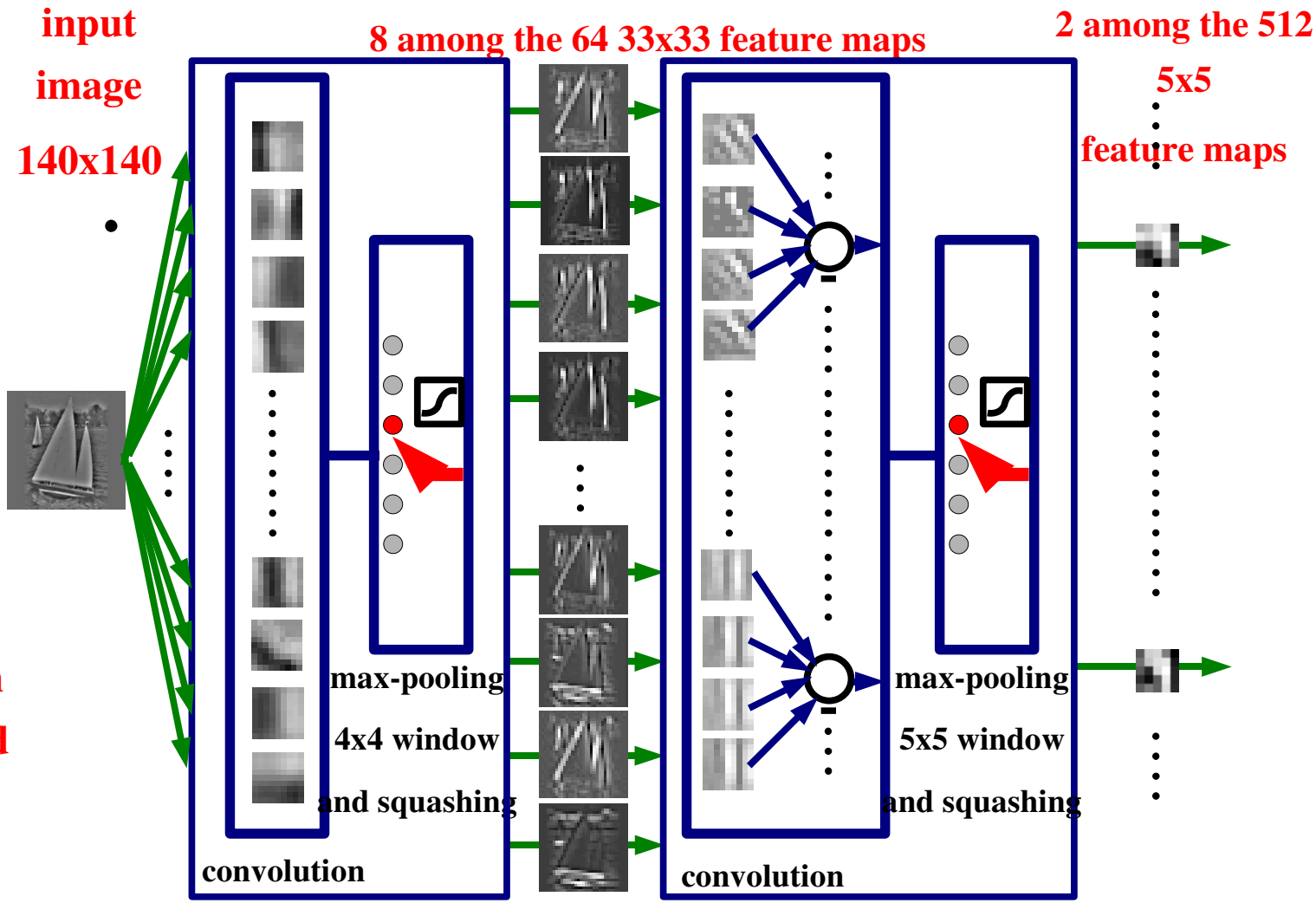


- 9x9 filters at the second level

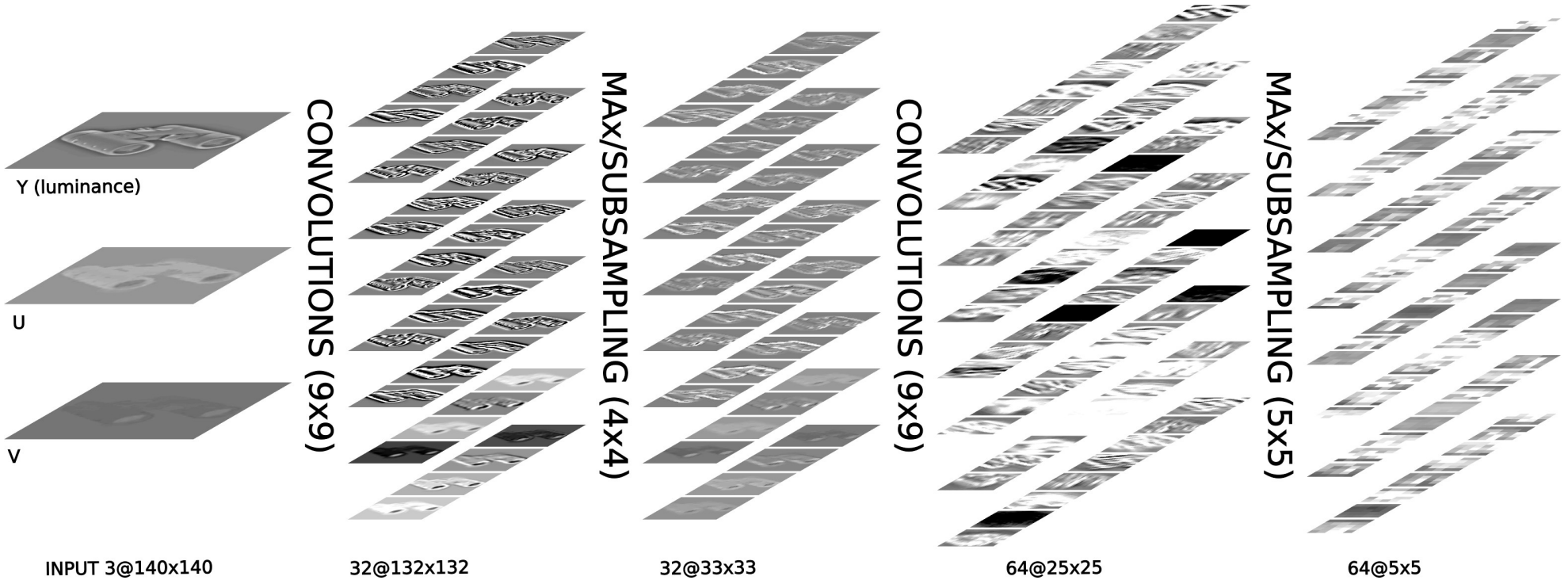


Shift-Invariant Feature Hierarchies on Caltech-101

- 2 layers of filters trained unsupervised
- supervised classifier on top.
- 54% correct on Caltech-101 with 30 examples per class
- 20% correct with purely supervised backprop



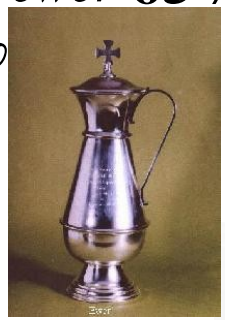
Another Architecture for Caltech-256



Recognition Rate on Caltech 101


dollar 100%

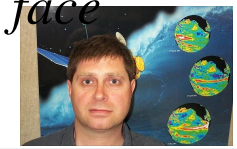

w. chair 92%


ewer 65%


bonsai 34%


beaver 13%

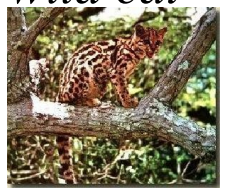

skate 100%


face 84%


anchor 41%


lotus 22%


cougar body 12%


wild cat 1%

BEST

WORST


okapi 100%


minaret 91%


cellphone 83%


joshua t. 47%


sea horse 37%


ant 16%

metronome 100%

background 3%

Practical Conclusion

- **The Multi-stage Hubel-Wiesel Architecture can be trained to recognize almost any set of objects.**
  - Supervised gradient descent learning requires too many examples
  - Unsupervised learning of each layer reduces the number of necessary training samples
- **Invariant feature learning preserves the nature of each feature, but throws away the instantiation parameters (position).**
- **Invariant feature hierarchies can be trained unsupervised**
  - on large training sets: the recognition rate is almost as good as supervised gradient descent learning
  - on small training sets: the recognition rate is much better.