Statistical Learning Theory and Applications 9.520/6.860 in Fall 2016

Class Times: Monday and Wednesday 1pm-2:30pm in 46-3310 Units: 3-0-9 H,G Web site: <u>http://www.mit.edu/~9.520/</u>

Email Contact : <u>9.520@mit.edu</u>

Instructors: Tomaso Poggio, Lorenzo Rosasco Guest lectures: Charlie Frogner, Carlo Ciliberto, Alessandro Verri TAs: Hongyi Zhang, Max Kleiman-Weiner, Brando Miranda, Georgios Evangelopoulos

Web: http://www.mit.edu/~9.520/ Office Hours: Friday 2-3 pm, 46-5156 (Poggio Lab lounge)

Further Info:9.520/6.860 is currently NOT using the Stellar system. Registration: Fill online registration form. Mailing list:Registered students will be added in the course mailing list (9520students)

Class

http://www.mit.edu/~9.520/

Class 2: Mathcamps

• Functional analysis (~45mins)

Linear Algebra

Basic notion and definitions: matrix and vectors norms, positive, symmetric, invertible matrices, linear systems, condition number.

Functional Analysis:

Linear and Euclidean spaces scalar product, orthogonality orthonormal bases, norms and semi-norms, Cauchy sequence and complete spaces Hilbert spaces, function spaces and linear functional, Riesz representation theorem, convex functions, functional calculus.

• Probability (~45mins)

Probability Theory:

Random Variables (and related concepts), Law of Large Numbers, Probabilistic Convergence, Concentration Inequalities.

9.520: Statistical Learning Theory and Applications

- Course focuses on regularization techniques for supervised learning.
- Support Vector Machines, manifold learning, sparsity, batch and online supervised learning, feature selection, structured prediction, multitask learning.
- Optimization theory critical for machine learning (first order methods, proximal/splitting techniques).
- In the final part focus on emerging deep learning theory

The goal of this class is to provide the theoretical knowledge and the basic intuitions needed to use and develop effective machine learning solutions to a variety of problems.

Class

http://www.mit.edu/~9.520/

Rules of the game:

- Problem sets: 4
- Final project: 2 weeks effort, you have to give us title + abstract before November 23
- Participation: check-in/sign in every class
- Grading: Psets (60%) + Final Project (30%) + Participation (10.0%)

Slides on the Web site (most classes on blackboard) Staff mailing list is 9.520@mit.edu Student list will be 9.520students@mit.edu Please fill form (independent of MIT/Harvard registration)!!

send email to us if you want to be added to mailing list

Class

http://www.mit.edu/~9.520/

Material: Most classes on blackboard.

Book draft:

Rosasco and T. Poggio, Machine Learning: a Regularization Approach, MIT-9.520 Lectures Notes, Manuscript, Dec. 2015 (chapters will be provided).

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Tentative dates

Problem Sets (due dates will be 11 days) Problem Set 1: 26 Sep. (due: 10/05) Problem Set 2: 12 Oct. (due: 10/24) Problem Set 3: 26 Oct. (due: 11/07) Problem Set 4: 14 Nov. (due: 11/23)

Final projects: Announcement/projects are open: Nov. 16 Deadline to suggest/pick suggestions (title/abstract): Nov. 23 Submission: Dec. xx

Final Project

The course project can be:

- Research project (suggested by you): Review, theory and/or application (~4 page report in NIPS format).
- Wikipedia articles (suggested list by us): Editing or creating new Wikipedia entries on a topic from the course syllabus.
- Coding (suggested by you or us): Implementation of one of the course algorithms and integration on the open-source library GURLS (Grand Unified Regularized Least Squares) https://github.com/LCSL/ GURLS
- Research project reports will be archived online (on a dedicated page on our web)
- Wikipedia entries links will be archived (on a dedicated <u>page</u> on our web), <u>https://docs.google.com/document/d/</u>
 <u>1RpLDfy1yMBNaSGqsdnl7w1GgzgN4lb-wPaLwRJJ44mA/edit</u>

Class <u>http://www.mit.edu/~9.520/</u>: big picture

- Classes 3-9 are the core: foundations + regularization
- Classes 10-22 are state-of-the-art topics for research in and applications of — ML
- Classes 23-25 are partly unpublished theory on multilayer networks (DCLNs)

Class

http://www.mit.edu/~9.520/

- Today is big picture day...
- Be ready for quite a bit of material
- If you need a complete renovation of your Fourier analysis or linear algebra background...you should not be in this class.

Summary of today's overview

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The problem of intelligence: how it arises in the brain and how to replicate it in machines

The problem of (human) intelligence is one of the great problems in science, probably the greatest.

Research on intelligence:

- a great intellectual mission: understand the brain, reproduce it in machines
- will help develop intelligent machines

These advances will be critical to of our society's

- future prosperity
- education, health, security
- solve all other great problems in science



Brains Minds+ Machines

Science + Engineering of Intelligence

CBMM's <u>main</u> goal is to make progress in the science of intelligence which enables better engineering of intelligence.



Third Annual NSF Site Visit, June 8 – 9, 2016

Interdisciplinary



Centerness:

collaborations across different disciplines and labs



Recent Stats and Activities



Brains Minds+ Machines

Third CBMM Summer School, 2016

Recent Stats and Activities

Summer school at Woods Hole: Our flagship initiative, very good!

Brains, Minds & Machines Summer Course An intensive three-week course will give advanced students a "deep end" introduction to the problem of intelligence





Intelligence in games: the beginning





Third Annual NSF Site Visit, June 8 – 9, 2016





Third Annual NSF Site Visit, June 8 – 9, 2016

Recent progress in Al

Science

natureINSIGHT







The 2 best examples of the success of new ML

- · AlphaGo
- Mobileye







More PERSON IN THE NEWS

Kyle Bass

James Comey All al-Naimi

He describes Mr Lee as the "Roger Federer of Go", and for some the computer program's achievement is akin to a robot taking to the lawns of Wimbledon and beating the legendary tennis champion.

"I think it is pretty hage but, ultimately, it will be for

THE BIG READ







Real Engineering: Mobileye





Real Engineering: Mobileye





Third Annual NSF Site Visit, June 8 – 9, 2016

History





History: same hierarchical architectures in the cortex, in models of vision and in deep networks





Third Annual NSF Site Visit, June 8 – 9, 2016

The Science of Intelligence

The science of intelligence was at the roots of today's engineering success

We need to make another basic effort on it

- for the sake of basic science
- for the engineering of tomorrow



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Statistical Learning Theory: **supervised** learning (~1980-2010)



Given a set of I examples (data)

$$\{(x_1, y_1), (x_2, y_2), \dots, (x_{\ell}, y_{\ell})\}$$

Question: find function f such that

$$f(x) = \hat{y}$$

is a good predictor of y for a future input x (fitting the data is not enough!)

Statistical Learning Theory: prediction, not description



Generalization:

estimating value of function where there are no data (good generalization means predicting the function well; important is for empirical or validation error to be a good proxy of the prediction error)

Statistical Learning Theory: supervised learning



Statistical Learning Theory: part of mainstream math not just statistics (Valiant, Vapnik, Smale, Devore...)

BULLETIN (New Series) OF THE AMERICAN MATHEMATICAL SOCIETY Volume 39, Number 1, Pages 1–49 S 0273-0979(01)00923-5 Article electronically published on October 5, 2001

ON THE MATHEMATICAL FOUNDATIONS OF LEARNING



FELIPE CUCKER AND STEVE SMALE

The problem of learning is arguably at the very core of the problem of intelligence, both bi

T. Poggio and C.R. Shelton

INTRODUCTION

(1) A main theme of this report is the relationship of approximation to learning and the primary role of sampling (inductive inference). We try to emphasize relations of the theory of learning to the mainstream of mathematics. In particular, there are large roles for probability theory, for algorithms such as *least squares*, and for tools and ideas from linear algebra and linear analysis. An advantage of doing this is that communication is facilitated and the power of core mathematics is more easily brought to bear. Statistical Learning Theory: supervised learning

There is an unknown **probability distribution** on the product space $Z = X \times Y$, written $\mu(z) = \mu(x, y)$. We assume that X is a compact domain in Euclidean space and Y a bounded subset of \mathbb{R} . The **training set** $S = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_n, y_n)\} = \{z_1, ..., z_n\}$

consists of *n* samples drawn i.i.d. from μ .

 \mathcal{H} is the **hypothesis space**, a space of functions $f : X \to Y$.

A **learning algorithm** is a map $L : Z^n \to \mathcal{H}$ that looks at S and selects from \mathcal{H} a function $f_S : \mathbf{x} \to y$ such that $f_S(\mathbf{x}) \approx y$ in a predictive way.

Statistical Learning Theory

Given a function f, a loss function V, and a probability distribution μ over Z, the **expected or true error** of f is:

$$I[f] = \mathbb{E}_z V[f, z] = \int_Z V(f, z) d\mu(z)$$
(1)

which is the **expected loss** on a new example drawn at random from μ . The **empirical error** of *f* is:

$$I_S[f] = \frac{1}{n} \sum V(f, z_i) \tag{2}$$

A very natural requirement for *f*_S is distribution independent **generalization**

$$\forall \mu, \lim_{n \to \infty} |I_S[f_S] - I[f_S]| = 0 \text{ in probability}$$
(3)

In other words, the training error for the solution must converge to the expected error and thus be a "proxy" for it. Otherwise the solution would not be "predictive".

Statistical Learning Theory: generalization follows from control of complexity

The ERM problem does not have a *predictive* solution in general (just fitting the data does not work).

Choosing an appropriate hypothesis space H (for instance a compact set of continuous functions) can guarantee generalization. A necessary and sufficient condition for generalization is that H is uGC.

Related concept, measuring complexity of the hypothesis space, are:

VC dimension, V_gamma dimension, Rademacher numbers..

Statistical Learning Theory: the learning problem should be well-posed



- A problem is well-posed if its solution
- exists, unique and

J. S. Hadamard, 1865-1963

is stable, eg depends continuously on the data (here examples)

This is an example of foundational results in learning theory...
Statistical Learning Theory: foundational theorems

Conditions for <u>generalization</u> in learning theory have deep, almost philosophical, implications:

they can be regarded as equivalent conditions that guarantee a theory to be predictive (that is scientific)

- theory must be chosen from a small hypothesis set
- theory should not change much with new data...most of the time (stability)

Classical algorithm: Regularization in RKHS (eg. kernel machines)

$$\min_{f \in H} \left[\frac{1}{n} \sum_{i=1}^{n} V(f(x_i) - y_i) + \lambda \| \|f\|_{K}^{2} \right]$$

implies

$$f(\mathbf{x}) = \sum_{i=1}^{n} \alpha_{i} K(\mathbf{x}, \mathbf{x}_{i})$$

Equation includes splines, Radial Basis Functions and SVMs (depending on choice of K and V).

For a review, see Poggio and Smale, 2003; see also Schoelkopf and Smola, 2002; Bousquet, O., S. Boucheron and G. Lugosi; Cucker and Smale; Zhou and Smale...

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Remark (for later use):

Classical kernel machines correspond to shallow networks



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Learning





Theorems on foundations of learning Predictive algorithms



COMPUTATIONAL NEUROSCIENCE: models+experiments Sung & Poggio 1995, also Kanade& Baluja....

How visual cortex works







GetorC

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E.







Some other examples of past ML applications from my lab

Computer Vision

- Face detection
- Pedestrian detection
- Scene understanding
- Video categorization
- Video compression
- Pose estimation
- Graphics
- Speech recognition
- Speech synthesis
- Decoding the Neural Code
- **Bioinformatics**
- Text Classification
- Artificial Markets
- Stock option pricing

Decoding the neural code: Matrix-like read-out from the brain



The end station of the ventral stream in visual cortex is IT



Reading-out the neural code in AIT



77 objects,8 classes

Chou Hung, Gabriel Kreiman, James DiCarlo, Tomaso Poggio, Science, Nov 4, 2005

Recording at each recording site during passive viewing



- 77 visual objects
- 10 presentation repetitions per object
- presentation order randomized and counter-balanced

Example of one AIT cell



Decoding the neural code ... using a classifier



We can decode the brain's code and read-out from neuronal populations: reliable object categorization (>90% correct) using ~200 <u>arbitrary</u> AIT "neurons"

Video speed: 1 frame/sec

Actual presentation rate: 5 objects/sec



Hung, Kreiman, Poggio, DiCarlo. Science 2005

We can decode the brain's code and read-out from neuronal populations:

reliable object categorization using ~100 <u>arbitrary</u> AIT sites

- [100-300 ms] interval
- 50 ms bin size



Learning: image analysis



\Rightarrow Bear (0° view)



\Rightarrow Bear (45° view)

Learning: image synthesis

UNCONVENTIONAL GRAPHICS



$$\Theta$$
 = 0° view \Rightarrow



$$\Theta$$
 = 45° view \Rightarrow

Mary101



A- more in a moment

Tony Ezzat, Geiger, Poggio, SigGraph 2002

1. Learning

System learns from 4 mins of video face appearance (Morphable Model) and speech dynamics of the person

2. Run Time

For any speech input the system provides as output a synthetic video stream







B-Dido



C-Hikaru



D-Denglijun



E-Marylin



F-Katie Couric



G-Katie



H-Rehema



I-Rehemax

A Turing test: what is real and what is synthetic?



L-real-synth

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How do the learning machines described by classical learning theory -such as kernel machines -- compare with brains?

□ One of the most obvious differences is the ability of people and animals to learn from very few examples ("poverty of stimulus" problem).

□ A comparison with real brains offers another, related, challenge to learning theory. Classical "learning algorithms" correspond to one-layer architectures. The cortex suggests a hierarchical architecture.

Thus...are hierarchical architectures with more layers important perhaps for the sample complexity issue?

50, No. 5, 537-544, 2003. The Mathematics of Learning: Dealing with Data Tomaso Poggio and Steve Smale

Computation in a neural net



 $f(\mathbf{x}) = f_L(\dots f_2(f_1(\mathbf{x})))$

mite	container ship	motor scooter	leopard
mite	container ship	motor scooter	leopard
black widow	lifeboat	go-kart	Jaguar
cockroach	amphibian	moped	cheetah
tick	fireboat	bumper car	snow leopard
starfish	drilling platform	golfcart	Egyptian cat
grille	mushroom	cherry	Madagascar cat
convertible	agaric	dalmatian	squirrel monkey
grille	mushroom	grape	spider monkey
pickup	jelly fungus	elderberry	titi
beach wagon	gill fungus	ffordshire bullterrier	indri
fire engine	dead-man's-fingers	currant	howler monkey

Krizhevsky et al. NIPS 2012

Computation in a neural net



Classical kernel machines are equivalent to shallow networks

Kernel machines...

$$f(\mathbf{x}) = \sum_{i}^{l} c_{i} K(\mathbf{x}, \mathbf{x}_{i}) + b$$

can be "written" as shallow networks: the value of K corresponds to the "activity" of the "unit" for the input and the correspond to "weights"



Deep and shallow networks: universality

Theorem Shallow, one-hidden layer networks with a nonlinear $\phi(x)$ which is not a polynomial are universal. Arbitrarily deep networks with a nonlinear $\phi(x)$ (including polynomials) are universal.



Theorem:

why and when are deep networks better than shallow network?



Theorem (informal statement)

Suppose that a function of *d* variables is compositional. Both shallow and deep network can approximate f equally well. The number of parameters of the shallow network depends exponentially on *d* as $O(\varepsilon^{-d})$ with the dimension whereas for the deep network depends linearly on *d* that is $O(d\varepsilon^{-2})$



The curse of dimensionality, the blessing of compositionality

For compositional functions deep networks — but not shallow ones — can avoid the curse of dimensionality, that is the exponential dependence on the dimension of the network complexity and of its sample complexity.

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CBMM: motivations

Key recent advances in the engineering of intelligence have their roots in basic science of the brain



Recognition in Visual Cortex



- It is in the family of "Hubel-Wiesel" models (Hubel & Wiesel, 1959: qual. Fukushima, 1980: quant; Oram & Perrett, 1993: qual; Wallis & Rolls, 1997; Riesenhuber & Poggio, 1999; Thorpe, 2002; Ullman et al., 2002; Mel, 1997; Wersing and Koerner, 2003; LeCun et al 1998: not-bio; Amit & Mascaro, 2003: not-bio; Hinton, LeCun, Bengio not-bio; Deco & Rolls 2006...)
- As a biological model of object recognition in the ventral stream – from V1 to PFC -- it is *perhaps* the most quantitatively faithful to known neuroscience data

Riesenhuber & Poggio 1999, 2000; Serre Kouh Cadieu Knoblich Kreiman & Poggio 2005; Serre Oliva Poggio 2007

Hierarchical feedforward models of the ventral stream



Using goal-driven deep learning models to understand sensory cortex

Daniel L K Yamins^{1,2} & James J DiCarlo^{1,2}



Using goal-driven deep learning models to understand sensory cortex

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Thus...are hierarchical architectures with more layers the answer to the sample complexity issue? Notices of the American Mathematical Society (AMS), Vol.

50, No. 5, 537-544, 2003. The Mathematics of Learning: Dealing with Data Tomaso Poggio and Steve Smale

Today's science, tomorrow's engineering: learn like children learn

<u>The first phase</u> (and successes) of ML: supervised learning, big data: $n \rightarrow \infty$



from programmers...

...to labelers...

... to computers that learn like children...

<u>The next phase</u> of ML: implicitly supervised learning, learning like children do, small data: $n \rightarrow 1$

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