Massachusetts Institute of Technology *Spring Term 2005*

8.02X Electricity and Magnetism

Quiz #3 Solutions

Monday, April 11 10:05-10:55am Room 26-100

The quiz has four questions. It is a closed book quiz. No calculators are allowed. A letter-size formula sheet can be used, but has to be signed and submitted together with the quiz.

LAST NAME

FIRST NAME

MIT ID#

RECITATION SECTION

			MARK YOURS
Rec01	MW11	B. Zeng	
Rec02	MW12	B. Zeng	
Rec03	MW1	D. Nagaj	
Rec04	TR11	G. Benedek	
Rec05	TR1	G. Benedek	
Rec06	TR2	G. Benedek	
Rec07	TR3	Daniel Nagaj	

Problem #1	
Problem #2	
Problem #3	
Problem #4	
TOTAL	

25

Problem 1 (~~20~~ points) Experiment EB

Suppose that experiment EB is performed with a gas that has an ionization potential of V_{ion} = 10V. For a gap of d=0.1mm you observe electric breakdown at a voltage difference across the spark gap of V_{gap} = 1000V.

(a) What is the mean free path of the electrons in the gas?

$$\frac{V_{ION}}{\lambda_{mfp}} = \frac{V_{gap}}{d} \quad b/c \quad q\,\lambda_{mfp} \cdot \frac{V_{gap}}{d} \geq q \cdot V_{ION}$$

$$\Rightarrow \frac{10V}{\lambda_{mfp}} = \frac{1000V}{10^{-4}m} \Rightarrow \lambda_{mfp} = 10^{-6}m = 1\mu m$$

Assume the experiment was repeated using the same gas and the same gap d = 0.1mm, but in an enclosure with only half the pressure and therefore only half the density of molecules? At which voltage would breakdown occur under these conditions? Explain your answer in a few sentences.

Same gas → same V_{ION}

Lower density → λ_{mfp} increases

$$\frac{V_{ION}}{\lambda_{mfp}} = \frac{V_{gap}}{d} \Rightarrow V_{gap} = 500V$$

(λ_{mfp} ↓ doubles)

The lower density allows electrons to travel farther before hitting a molecule. Therefore the field necessary for them to pick up enough energy to ionize molecules can be lower. For the same d, that means smaller V_{gap}.

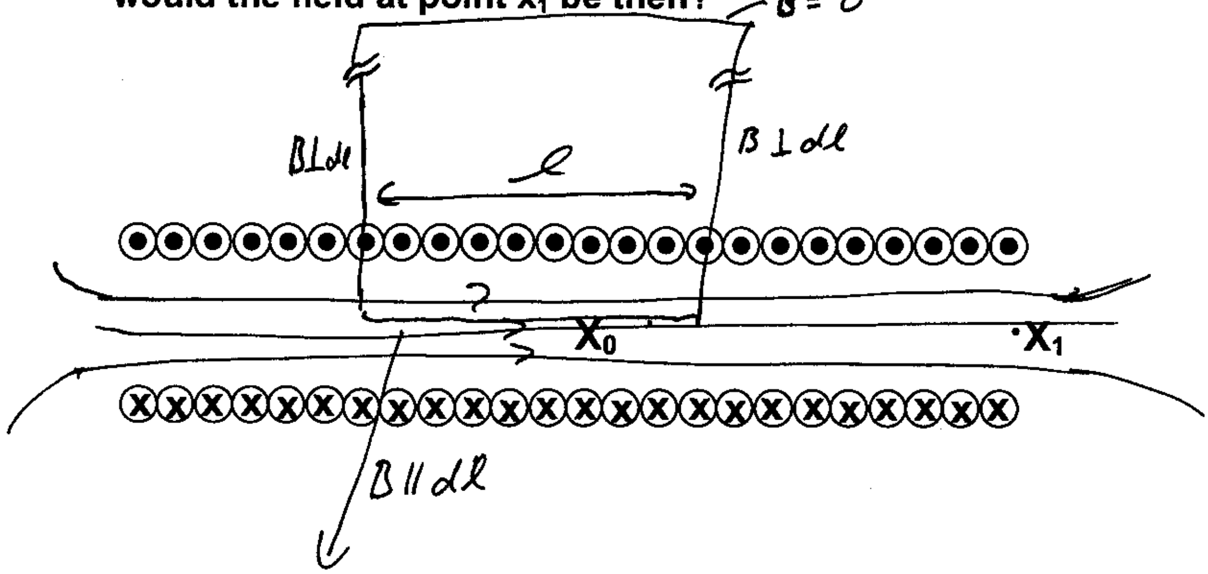
Problem 2 (25 points)

Shown below is the cross-section of a long solenoid with length L and number of windings N. The solenoid carries a current I.

5 **(a) Using fieldlines, sketch the magnetic field created by the solenoid.**

15 **(b) Using Ampere's Law and symmetry arguments, derive an expression for the magnitude of the magnetic field at the center (X_0) of the solenoid. Show work!**

5 **(c) Assume an identical solenoid was placed in close proximity to the first one, to the right of the first solenoid, carrying the same current I in the same direction. How big would the field at point x_1 be then?**



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} \Rightarrow B \cdot l = \mu N_l I \Rightarrow B = \mu_0 \frac{N}{L} \cdot I$$

c) $B(x_1) = B(x_0)$ b/c then it would look like one long solenoid (and at the edge the field of the two solenoids would add up)

Problem 3 (25 points)

Shown below is a square conducting loop. The loop is not movable. The sides of the loop have length 1m. The right half of the loop is inside a uniform external magnetic field, which points out of the paper plane. The resistance of the loop is 1 Ohm.

(a) At time t=0, the magnitude of the field is B=2T. What is the magnitude of the magnetic flux through the loop at this time? $\Phi = B \cdot A = 2T \times 0.5\,m^2 = 1\,Tm^2 = 1\,Wb$

2pts
−1 for wrong units

(b) Starting at time t=1 sec, the field is ramped from B=2T to B=0 over the course of 1 sec with a constant rate. What is the magnitude of the induced EMF at t=1.5 sec during the ramp. Show work! Because the change is linear, we can take $\frac{d\Phi}{dt} = \frac{\Delta\Phi}{\Delta t}$. The flux changes from $1\,Tm^2$ to $0\,Tm^2$ in 1s, so $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\Delta\Phi}{\Delta t} = -\frac{(-1\,Tm^2)}{1s} = 1V$

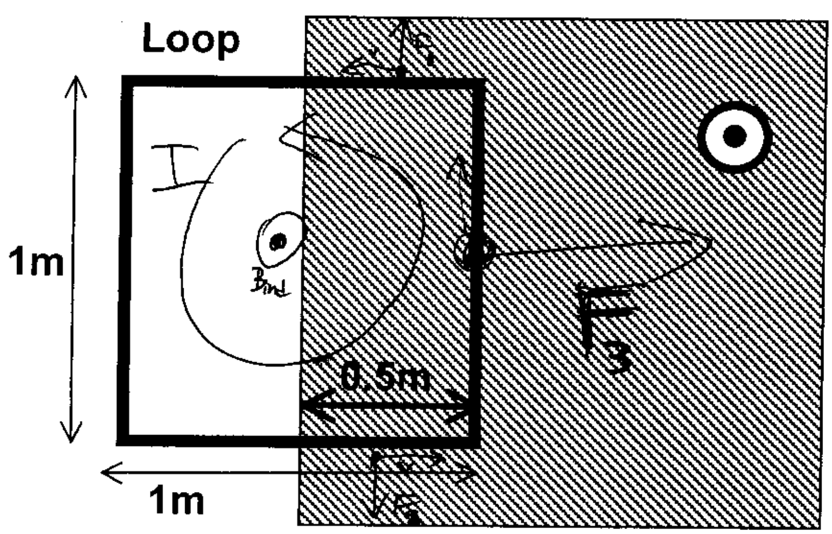
11 pts

(c) What is the direction and magnitude of the induced current at t=1.5 sec? The flux Φ decreases, so the induced field wants to increase it. Therefore B_{ind} is ⊙ and the current is counterclockwise. $I = \frac{\mathcal{E}}{R} = \frac{1V}{1\Omega} = 1A$

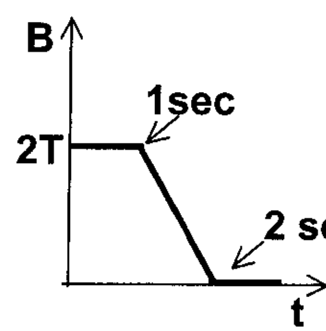
7pts

(d) What is the direction and magnitude of the net magnetic force on the loop at t=1.5 sec?
The forces on the top & bottom cancel out, the only force left is $F_3 = B.I.L = (1T).(1A).(1m) = 1N$

5pts



Magnetic Field, pointing out of paper plane



Problem 4 (25 points)

Shown below is the cross-section of a parallel plate capacitor carrying a charge +Q (top) and -Q (bottom). The potential difference between the plates is ΔV, the plates are separated by a distance *d*.

An electron with charge e = -1.6 10^{-19}C and velocity v is entering the capacitor from the left.

(a) On the figure, show the direction of the electric field in the capacitor.

(b) What direction should an external magnetic field have, such that the electron is not deflected inside the capacitor? Since $e<0$, the electric force $\vec{F}_E = e\vec{E} = |eE|\,\hat{y}$

therefore the magnetic force must be in $-\hat{y}$ direction:

$\vec{F}_B = e\vec{v}\times\vec{B} = -|e||v|\,\hat{x}\times\hat{B} \propto -\hat{y} \Rightarrow \hat{x}\times\vec{B} \propto \hat{y}$

$\Rightarrow \vec{B} = |B|(-\hat{z})$ as shown in figure

(c) What should the magnitude of the field be in terms of the quantities given, such that the electron is not deflected inside the capacitor? Show work!

Since $\vec{v}$, $\vec{F}_E$, $\vec{F}_B$ are perpendicular to each other:

$\vec{F}_{total} = e[\vec{E} + \vec{v}\times\vec{B}] = 0 \Rightarrow \vec{E} = -\vec{v}\times\vec{B}$

$\Rightarrow |E| = |vB| \Rightarrow |B| = \left|\frac{E}{v}\right|$ Note $|E| = \frac{\Delta V}{d}$

$\Rightarrow \boxed{B = \frac{\Delta V}{v d}}$