
8.02x – Problem Set 11 Solutions

Problem 1 (10 points)

How to tell whether sound is an EM wave? Let us just list some of the obvious differences. Sound waves do not propagate in vacuum, they need some medium (gas, solid, or liquid) to support them. The speeds of propagation have very different magnitudes for sound and EM waves.

However, to be sure to tell the two types of waves apart, we should measure their effects. Simply said, you can see light and hear sound, but not the reverse. An antenna (a metal rod) can “catch” EM waves, they would make the free charges move in it. This effect can be measured. However, you will not get any effect by trying to induce charge motion (or EM fields) by sound. On the other hand, you can hear a sound wave (your ears measure pressure changes), but not an EM wave.

Sound is a *longitudinal* mechanical wave, molecules of air bumping and pushing each other, making the signal spread towards the receiver. Longitudinal means the displacements of air molecules are in the direction of the signal. On the other hand, EM waves are *transverse*, meaning that the displacements (electric and magnetic fields) change in the direction perpendicular to the signal direction.

Y&F 29.38 (10 points)

a) Let us use the microscopical Ohm’s law. The electric field is proportional to resistivity and current density.

$$E = \rho j = \frac{\rho I}{A} = \frac{(2.0 \times 10^{-8} \Omega \text{m})(16 \text{A})}{2.1 \times 10^{-6} \text{m}^2} = 0.15 \frac{\text{V}}{\text{m}}. \quad (1)$$

b) The field is changing at rate

$$\frac{dE}{dt} = \frac{d}{dt} \left(\frac{\rho I}{A} \right) = \frac{\rho}{A} \frac{dI}{dt} = \frac{2.0 \times 10^{-8} \Omega \text{m}}{2.1 \times 10^{-6} \text{m}^2} (4000 \text{A/s}) = 38 \frac{\text{V}}{\text{ms}}. \quad (2)$$

c) The displacement current density in the wire is (taking $\epsilon \approx \epsilon_0$ for copper)

$$j_D = \epsilon_0 \frac{dE}{dt} = 3.4 \times 10^{-10} \frac{\text{A}}{\text{m}^2}. \quad (3)$$

d) The magnitude of the displacement current is

$$I_D = A j_D = (2.1 \times 10^{-6} \text{m}^2)(3.4 \times 10^{-10} \text{A/m}^2) = 7.14 \times 10^{-16} \text{A}, \quad (4)$$

which is many orders of magnitude smaller than the conduction current. Therefore the magnetic field is very well approximated only by the magnetic field from the conduction current,

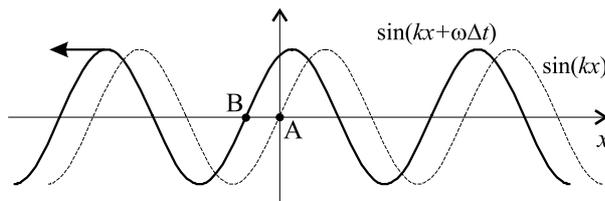
$$B_c = \frac{\mu_0 I_c}{2\pi r} = \frac{\mu_0}{2\pi} \frac{16\text{A}}{0.06\text{m}} = 5.33 \times 10^{-5}\text{T}. \quad (5)$$

Just for comparison, the magnitude of the magnetic field due to the displacement current is totally negligible:

$$B_D = \frac{\mu_0 I_D}{2\pi r} = \frac{\mu_0}{2\pi} \frac{7.14 \times 10^{-16}\text{A}}{0.06\text{m}} = 2.38 \times 10^{-21}\text{T}. \quad (6)$$

Y&F 32.6 (10 points)

a) Our EM wave has $\vec{B}(x, t) = \hat{y}B_0 \sin(kx + \omega t)$. Sketching $\sin(kx + \omega t)$ for time $t = 0$ and $t = \Delta t$ we realize it propagates in the $-\hat{x}$ direction. In detail, point A ($x = 0$) is where the amplitude is zero for time $t = 0$. At time $t = \Delta t$, point B where the amplitude is zero can be found from $k\Delta x + \omega\Delta t = 0$, giving us $\Delta x = -(\omega/k)\Delta t$. This means the wave has moved to the left.



b) We know the relationship between the wavevector and the wavelength, $k = 2\pi/\lambda$. The wavelength is the distance the wave travels in one time period, $\lambda = cT = c/f$. The frequency of the wave is then

$$f = \frac{kc}{2\pi} = \frac{(1.38 \times 10^4 \text{rad/m})(3 \times 10^8 \text{m/s})}{2\pi} = 6.59 \times 10^{11} \text{Hz}, \quad (7)$$

giving us also the angular frequency ω ,

$$\omega = 2\pi f = 4.14 \times 10^{11} \frac{\text{rad}}{\text{s}}. \quad (8)$$

c) The direction of the electric field is such that $\vec{E} \times \vec{B}$ points in the direction of propagation. If \vec{B} has direction \hat{y} , \vec{E} must then point in the \hat{z} direction for the wave to propagate in the $-\hat{x}$ direction. The magnitude of E is simply $|E| = c|B|$, so we get

$$\begin{aligned} \vec{B}(x, t) &= \hat{y}B_0 \sin(kx + \omega t), \\ \vec{E}(x, t) &= \hat{z}(cB_0) \sin(kx + \omega t) = \\ &= \hat{z}(2.48\text{V/m}) \sin \left[\left(1.38 \times 10^4 \frac{\text{rad}}{\text{m}} \right) x + \left(4.14 \times 10^{12} \frac{\text{rad}}{\text{s}} \right) t \right]. \end{aligned} \quad (9)$$