MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 | Fall 1998

FINAL EXAMINATION FORMULA SHEET

Posted Wednesday, December 9, 1998

Equations introduced in Chapter 1:

$$\vec{\mathbf{v}} = \frac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{d}t}\,; \qquad \vec{\mathbf{a}} = \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\mathbf{r}}}{\mathrm{d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,\mathrm{d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,\mathrm{d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\text{average}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\text{average}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Equations introduced in Chapter 2:

$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$	(Newton's second law);
$\vec{\mathbf{F}} = -\dfrac{GMm}{r^2}\hat{\mathbf{r}}$	(the gravitational force between two particles);
$\vec{\mathbf{F}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq}{r^2}\hat{\mathbf{r}}$	(the electrostatic force between two particles);
$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x$	(for a particle near a point of stable equilibrium; equation leads to simple harmonic motion);
$x = A\sin\omega t$	(a solution to the above equation; any solution can be written this way if we choose $t = 0$ when $x = 0$);
$\omega = 2\pi f$	(relation between angular frequency and frequency);
$T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$	(period of an oscillator).

Equations introduced in Chapter 4:

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2$	(kinetic & potential energy for projectile)
$W = \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$	(work done by constant force $\vec{\mathbf{F}}$)
$W = \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$	(work done by varying force $\vec{\mathbf{F}}$)
$W = K_f - K_i$	(work-energy theorem)
$\frac{1}{2}mv^2 + U(x) = \text{constant}$	(energy conservation)
$F = -\dfrac{\mathrm{d}U}{\mathrm{d}x}$	(force derived from potential energy)
$U = \frac{1}{2}kx^2$	(potential energy for spring force)
$U = mgh$	(gravitational potential energy, near Earth)
$U = -\dfrac{GMm}{r}$	(gravitational potential energy, spherical bodies)
$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \lvert\vec{\mathbf{a}}\rvert\lvert\vec{\mathbf{b}}\rvert\cos\theta$ $= a_x b_x + a_y b_y + a_z b_z$	(scalar (or dot) product of two vectors)

Equations introduced in Chapter 5:

$\vec{\mathbf{F}}_{\mathrm{AB}} = -\vec{\mathbf{F}}_{\mathrm{BA}}$ (Newton's third law);

$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$ (momentum);

$\vec{\mathbf{F}} = \dfrac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$ (Newton's second law in terms of momentum);

$\vec{\mathbf{r}}_{\mathrm{cm}} \equiv \dfrac{1}{M_{\mathrm{tot}}} \sum_i m_i \vec{\mathbf{r}}_i$ (position of center of mass);

$\vec{\mathbf{F}}^{\mathrm{ext}}_{\mathrm{tot}} = M_{\mathrm{tot}} \vec{\mathbf{a}}_{\mathrm{cm}}$ (acceleration of a system of masses);

$\vec{\mathbf{P}}_{\mathrm{tot}} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\mathrm{tot}} \vec{\mathbf{v}}_{\mathrm{cm}}$ (momentum of a system of masses);

$K_{\mathrm{tot}} = \frac{1}{2} M_{\mathrm{tot}} v_{\mathrm{cm}}^2 + \sum_i \frac{1}{2} m_i \left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\mathrm{cm}}\right)^2$ (K.E. of a system of masses);

$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\,\mathrm{d}t = \int_{t_1}^{t_2} \dfrac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\,\mathrm{d}t = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1$ (impulse-momentum theorem).

Equations introduced in Chapter 6:

$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right|$ (kinetic friction);

$\left|\vec{\mathbf{F}}_s\right| \leq \mu_s \left|\vec{\mathbf{N}}\right|$ (static friction);

$\vec{\mathbf{F}}_{\mathrm{fict}} = -m\vec{\mathbf{a}}(t)$ (Fictitious force in linearly accelerating frame).

Equations introduced in Chapter 8:

TRANSLATION (one dimension)		ROTATION (about fixed axis)	
Name	Symbol	Name	Symbol
Position	x	Orientation	θ
Velocity	$v = \dfrac{\mathrm{d}x}{\mathrm{d}t}$	Angular velocity	$\omega = \dfrac{\mathrm{d}\theta}{\mathrm{d}t}$
Acceleration	$a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$	Angular acceleration	$\alpha = \dfrac{\mathrm{d}\omega}{\mathrm{d}t}$
Mass	$M = \sum_i m_i$	Moment of inertia	$I = \sum_i m_i R_i^2$
Force	F	Torque	$\tau = F_\perp R$ $= \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$
Force equation	$\sum_i \vec{\mathbf{F}}^{\mathrm{ext}} = M\vec{\mathbf{a}}_{\mathrm{cm}}$	Torque equation	$\sum_i \tau^{\mathrm{ext}} = I\alpha$
Momentum	$p = Mv$	Angular momentum	$L = I\omega$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Work done	$\vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$	Work done	$\tau\,\Delta\theta$

Other equations introduced in this unit:

$v_r = 0$; $v_\perp = R\omega$ (velocity of point on rotating body);

$a_r = -\dfrac{v^2}{R} = -R\omega^2$; $a_\perp = R\alpha$ (acceleration of point on rotating body);

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \tau^{\text{ext}} &= I_{\text{cm}}\alpha = \frac{\mathrm{d}L}{\mathrm{d}t} \end{aligned}\right\}$$ (combined translational and rotational motion);

$v = \pm R|\omega|$ (rolling without slipping);

$I_{||} = I_{\text{cm}} + Md^2$ (parallel-axis theorem);

$I_z = I_x + I_y$ (perpendicular-axis theorem).

TABLE OF STANDARD MOMENTS OF INERTIA:

Slender uniform rod of length ℓ, axis through center and perpendicular to axis of rod	$\frac{1}{12}m\ell^2$
Rectangular plate with dimensions $a \times b$, axis along one of the b edges	$\frac{1}{3}ma^2$
Thin-walled hollow cylinder of radius R, axis along axis of cylinder	mR^2
Uniform solid cylinder of radius R, axis along axis of cylinder	$\frac{1}{2}mR^2$
Thin-walled hollow sphere of radius R, axis through center	$\frac{2}{3}mR^2$
Solid uniform sphere of radius R, axis through center	$\frac{2}{5}mR^2$

Equations introduced in Chapter 9:

$$c_x = a_y b_z - a_z b_y \; ; \quad c_y = a_z b_x - a_x b_z \; ; \quad c_z = a_x b_y - a_y b_x \; .$$
(vector cross product, component form);

$$|\vec{\mathbf{c}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$$
(magnitude of vector cross product);

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$$
(velocity of atom on rotating body with a fixed point);

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i$$
(angular momentum, as vector product);

$$\vec{\boldsymbol{\tau}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i$$
(vector torque, as vector product);

$$\vec{\boldsymbol{\tau}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$$
(torque equation);

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \vec{\boldsymbol{\tau}}^{\text{ext}} &= \frac{\mathrm{d}\vec{\mathbf{L}}_{\text{cm}}}{\mathrm{d}t} \end{aligned}\right\}$$
(combined translational and rotational motion);

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{p}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times m_i \vec{\mathbf{v}}_{\text{rel},i}$$
(angular momentum decomposition);

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{F}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times \vec{\mathbf{F}}_i$$
(torque decomposition).

Equations introduced in Chapter 11:

$PV = \frac{2}{3} N \left\langle \frac{1}{2} m v^2 \right\rangle$	(pressure of an ideal gas);
$\left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3}{2} kT$	(definition of kinetic temperature);
$PV = NkT = \mathcal{N}RT$	(ideal-gas law);
$\Delta U = Q - W$	(first law of thermodynamics);
$\Delta W = P\,\Delta V$	(work done by expanding gas).

* Useful constants:

$$R = 8.3 \text{ J/mol.K}$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

$$\text{Avogadro's Number} = 6.02 \times 10^{23} \text{ molecules/mole.}$$

Equations introduced in Chapter 12:

$P_2 - P_1 = -\rho g (y_2 - y_1)$	(Pascal's Law: pressure in a liquid as a function of height, for a stationary liquid);
$A_2 v_2 = A_1 v_1$	(equation of continuity for steady flow);
$P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant}$	(Bernoulli's equation);
$\gamma = \frac{F}{\ell} = \frac{U}{A}$	(surface tension).