MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Fall 2000

EXAM 3
FORMULA SHEET
Monday, November 20, 2000

Equations introduced in Chapter 1:

$$\vec{\mathbf{v}} = \frac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{d}t}\,; \qquad \vec{\mathbf{a}} = \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\mathbf{r}}}{\mathrm{d}t^2}\,; \qquad \vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}_0 + \int_0^{t_1} \vec{\mathbf{v}}\,\mathrm{d}t\,; \qquad \vec{\mathbf{v}}(t_1) = \vec{\mathbf{v}}_0 + \int_0^{t_1} \vec{\mathbf{a}}\,\mathrm{d}t\ .$$

For *constant* acceleration $\vec{\mathbf{a}}$, if $\vec{\mathbf{r}} = \vec{\mathbf{r}}_0$ and $\vec{\mathbf{v}} = \vec{\mathbf{v}}_0$ at time $t = 0$, then

$$\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t$$

$$\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2\ .$$

For one-dimensional motion with constant acceleration a:

$$v^2 = v_0^2 + 2a(x - x_0)\ .$$

For circular motion at constant speed v:

$$a = \frac{v^2}{r}\ ,$$

where r is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{\mathbf{r}}$ and velocity $\vec{\mathbf{v}}$, its position and velocity relative to an observer with position $\vec{\mathbf{r}}_0$ and velocity $\vec{\mathbf{v}}_0$ are given respectively by

$$\vec{\mathbf{r}}' = \vec{\mathbf{r}} - \vec{\mathbf{r}}_0\ , \qquad \vec{\mathbf{v}}' = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0\ .$$

Average velocity and acceleration are given by

$$\vec{\mathbf{v}}_{\text{average}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}\ , \qquad \vec{\mathbf{a}}_{\text{average}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}\ .$$

Equations introduced in Chapter 2:

$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$	(Newton's second law);
$\vec{\mathbf{F}} = -\dfrac{GMm}{r^2}\hat{\boldsymbol{r}}$	(the gravitational force between two particles);
$\vec{\mathbf{F}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq}{r^2}\hat{\boldsymbol{r}}$	(the electrostatic force between two particles);
$F_x = -kx$	(Hooke's law);
$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x$	(for a particle near a point of stable equilibrium; equation leads to simple harmonic motion);
$x = A\sin\omega t$	(a solution to the above equation; any solution can be written this way if we choose $t = 0$ when $x = 0$);
$\omega = 2\pi f$	(relation between angular frequency and frequency);
$T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$	(period of an oscillator).

Equations introduced in Chapter 4:

1 Dimension	**3 Dimensions**	**Description**
$W \equiv F\Delta x$	$W \equiv \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{\Delta r}}$	Work done by a constant force $\vec{\mathbf{F}}$
$W \equiv \int F(x)\,\mathrm{d}x$	$W \equiv \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$	Work done by a varying force $\vec{\mathbf{F}}$
$U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,\mathrm{d}x$	$U(\vec{\mathbf{r}}_p) \equiv U_0 - \int_{\vec{\mathbf{r}}_0}^{\vec{\mathbf{r}}_p} \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{dr}}$	Potential energy derived from force $\vec{\mathbf{F}}$
$F = -\dfrac{\mathrm{d}U}{\mathrm{d}x}$	$\vec{\mathbf{F}} = \left[-\dfrac{\partial U}{\partial x}, -\dfrac{\partial U}{\partial y}, -\dfrac{\partial U}{\partial z}\right]$	Force derived from potential energy

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta \qquad \text{(scalar (or dot) product of two vectors)}$$

$$= a_x b_x + a_y b_y + a_z b_z$$

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2 \qquad \text{(kinetic \& potential energy for projectile)}$$

$$\frac{1}{2}mv^2 + U(x) = \text{constant} \qquad \text{(energy conservation)}$$

$$W = K_f - K_i \qquad \text{(work-energy theorem)}$$

$$U = \frac{1}{2}kx^2 \qquad \text{(potential energy for spring force)}$$

$$U = mgh \qquad \text{(gravitational potential energy, near Earth)}$$

$$U = -\frac{GMm}{r} \qquad \text{(gravitational potential energy, spherical bodies)}$$

Equations introduced in Chapter 5:

$$\vec{\mathbf{F}}_{\mathrm{AB}} = -\vec{\mathbf{F}}_{\mathrm{BA}} \qquad \text{(Newton's third law);}$$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \qquad \text{(momentum);}$$

$$\frac{\mathrm{d}\vec{\mathbf{P}}_{\mathrm{tot}}}{\mathrm{d}t} = 0 \qquad \text{(conservation of momentum in absence of external force)}$$

$$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \qquad \text{(Newton's second law in terms of momentum);}$$

$$\vec{\mathbf{r}}_{\mathrm{cm}} \equiv \frac{1}{M_{\mathrm{tot}}} \sum_i m_i \vec{\mathbf{r}}_i \qquad \text{(position of center of mass);}$$

$$\vec{\mathbf{v}}_{\mathrm{cm}} \equiv \frac{\mathrm{d}\vec{\mathbf{r}}_{\mathrm{cm}}}{\mathrm{d}t} = \frac{1}{M_{\mathrm{tot}}} \sum_i m_i \vec{\mathbf{v}}_i \qquad \text{(velocity of center of mass);}$$

$$\vec{\mathbf{F}}_{\mathrm{tot}}^{\mathrm{ext}} = M_{\mathrm{tot}} \vec{\mathbf{a}}_{\mathrm{cm}} = \frac{\mathrm{d}\vec{\mathbf{P}}_{\mathrm{tot}}}{\mathrm{d}t} \qquad \text{(acceleration of a system of particles);}$$

$$\vec{\mathbf{P}}_{\mathrm{tot}} = \sum_i m_i \vec{\mathbf{v}}_i = M_{\mathrm{tot}} \vec{\mathbf{v}}_{\mathrm{cm}} \qquad \text{(momentum of a system of particles);}$$

$$K_{\mathrm{tot}} = \frac{1}{2} M_{\mathrm{tot}} v_{\mathrm{cm}}^2 + \sum_i \frac{1}{2} m_i \left(\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{\mathrm{cm}}\right)^2 \qquad \text{(K.E. of a system of particles);}$$

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\, \mathrm{d}t = \int_{t_1}^{t_2} \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}\, \mathrm{d}t = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \qquad \text{(impulse-momentum theorem).}$$

Equations introduced in Chapter 6:

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \le \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction);}$$

$$\vec{\mathbf{F}}_{\text{fict}} = -m\vec{\mathbf{a}}(t) \qquad \text{(fictitious force in linearly accelerating frame).}$$

Equations introduced in Chapter 8:

Most of the equations in this chapter are most easily remembered in the context of the analogous equations for linear motion in one dimension:

TRANSLATION (one dimension)		ROTATION (about fixed axis)	
Name	Symbol	Name	Symbol
Position	x	Orientation	θ
Velocity	$v = \frac{\mathrm{d}x}{\mathrm{d}t}$	Angular velocity	$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$
Acceleration	$a = \frac{\mathrm{d}v}{\mathrm{d}t}$	Angular acceleration	$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$
Mass	$M = \sum_i m_i$	Moment of inertia	$I = \sum_i m_i R_i^2$
Force	F	Torque	$\boldsymbol{\tau} = F_\perp R$ $= \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$
Force equation	$\sum_i \vec{\mathbf{F}}^{\text{ext}} = M\vec{\mathbf{a}}_{\text{cm}}$	Torque equation	$\sum_i \boldsymbol{\tau}^{\text{ext}} = I\alpha$
Momentum	$p = Mv$	Angular momentum	$L = I\omega$
Kinetic energy	$\frac{1}{2}Mv^2$	Kinetic energy	$\frac{1}{2}I\omega^2$
Work done	$\vec{\mathbf{F}} \cdot \overrightarrow{\boldsymbol{\Delta r}}$	Work done	$\boldsymbol{\tau}\,\Delta\theta$

Other equations introduced in this chapter:

$v_r = 0\ ; \quad v_\perp = R\omega$	(velocity of point on rotating body);
$a_r = -\dfrac{v^2}{R} = -R\omega^2\ ; \quad a_\perp = R\alpha$	(acceleration of point on rotating body);
$v = \pm R\lvert\omega\rvert$	(rolling without slipping);
$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \boldsymbol{\tau}^{\text{ext}} &= I_{\text{cm}}\alpha = \frac{\mathrm{d}L}{\mathrm{d}t} \end{aligned}\right\}$	(combined translational and rotational motion);
$K_{\text{tot}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$	(kinetic energy for combined translational and rotational motion);
$I_\parallel = I_{\text{cm}} + Md^2$	(parallel-axis theorem);
$I_z = I_x + I_y$	(perpendicular-axis theorem).

TABLE OF STANDARD MOMENTS OF INERTIA:

Slender uniform rod of length ℓ, axis through center and perpendicular to axis of rod	$\frac{1}{12}m\ell^2$
Rectangular plate with dimensions $a \times b$, axis along one of the b edges	$\frac{1}{3}ma^2$
Thin-walled hollow cylinder of radius R, axis along axis of cylinder	mR^2
Uniform solid cylinder of radius R, axis along axis of cylinder	$\frac{1}{2}mR^2$
Thin-walled hollow sphere of radius R, axis through center	$\frac{2}{3}mR^2$
Solid uniform sphere of radius R, axis through center	$\frac{2}{5}mR^2$

Equations introduced in Chapter 9:

$$c_x = a_y b_z - a_z b_y \ ; \quad c_y = a_z b_x - a_x b_z \ ; \quad c_z = a_x b_y - a_y b_x \ .$$
(vector cross product, component form);

$$|\vec{\mathbf{c}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$$
(magnitude of vector cross product);

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$$
(velocity of atom in rotating body with a fixed point);

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_P + \vec{\boldsymbol{\omega}} \times \left(\vec{\mathbf{r}} - \vec{\mathbf{r}}_P\right)$$
(velocity of atom in rotating body, general case);

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i$$
(angular momentum, as vector product);

$$\vec{\boldsymbol{\tau}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i$$
(vector torque, as vector product);

$$\vec{\boldsymbol{\tau}} = \frac{\mathrm{d}\vec{\mathbf{L}}}{\mathrm{d}t}$$
(torque equation);

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \\ \sum \vec{\boldsymbol{\tau}}^{\text{ext}} &= \frac{\mathrm{d}\vec{\mathbf{L}}_{\text{cm}}}{\mathrm{d}t} \end{aligned}\right\}$$
(combined translational and rotational motion);

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{p}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times m_i \vec{\mathbf{v}}_{\text{rel},i}$$
(angular momentum decomposition);

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{F}}_{\text{tot}} + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times \vec{\mathbf{F}}_i$$
(torque decomposition).