

Earley's algorithm & Taking a chance on parsing

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The Menu Bar

- Administrivia
- Three Loops to Rule them All, Three Loops to Bind them (review of CKY complexity)
- How to do better

Ordinary CKY algorithm: 3 loops to rule them all

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Initialize:
For  $j = 1, \dots, n, k = 1, \dots, K$ 
 $table(j, j) \leftarrow \{N_k | N_k \rightarrow w_j\}$ 

For  $j = 1$  to  $n$ 
  For  $i = j - 1$  downto 1
    For  $s \leftarrow i$  to  $j$ 
      For  $N_k \rightarrow N_l N_m$  s.t.  $N_{l,m} \in G$  &
         $N_l \in table(i, s)$  &
         $N_m \in table(s + 1, j)$ 
         $table(i, j) \leftarrow table(i, j) \cup N_k$ 
         $split(i, j, k) \leftarrow \{s, l, m\}$ 
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Probabilistic CKY

Modify CKY to find the greatest probability $\pi(i, j, k)$,
given nonterminals $k = 1, \dots, K$
Sentence is n words long, w_1, w_2, \dots, w_n
Spans word positions (i, j) , i.e., $(1, n, Start)$
Data structures: $table(i, j)$, $splits(i, j, k)$, $\pi(i, j, k)$

Overview of Earley's Algorithm

- Proceeds incrementally, left-to-right
 - Before it reads word 5, it has already built all hypotheses that are consistent with first 4 words
 - Reads word 5 & attaches it to immediately preceding hypotheses. Might yield new constituents that are then attached to hypotheses immediately preceding *them* ...
 - E.g., attaching D to A \rightarrow B C . D E gives A \rightarrow B C D . E
 - Attaching E to that gives A \rightarrow B C D E .
 - Now we have a complete A that we can attach to hypotheses immediately preceding the A, etc.

Earley's algorithm

- Use table as before; 1 column per word
- The table-entries are called *states* and are represented with *dotted-rules*.

S \rightarrow • VP A VP is predicted

NP \rightarrow Det • Noun An NP is in progress

VP \rightarrow V NP • A VP has been found

Dotted rules: run-time conversion to binary branching

- S \rightarrow • VP [0,0] • A VP is predicted at the start of the sentence
- NP \rightarrow Det • Noun [1,2] • An NP is in progress; the Det goes from 1 to 2
- VP \rightarrow V NP • [0,3] • A VP has been found starting at 0 and ending at 3

Our Example Grammar

ROOT \rightarrow S
 S \rightarrow NP VP NP \rightarrow Papa
 NP \rightarrow Det N N \rightarrow caviar
 NP \rightarrow NP PP N \rightarrow spoon
 VP \rightarrow VP PP V \rightarrow ate
 VP \rightarrow V NP P \rightarrow with
 PP \rightarrow P NP Det \rightarrow the
 Det \rightarrow a

0 Papa 1 ate 2 the 3 caviar 4 with 5 a 6 spoon 7

First Try: Recursive Descent

ROOT → S	VP → V NP	NP → Papa	V → ate
S → NP VP	VP → VP PP	N → caviar	P → with
NP → Det N	PP → P NP	N → spoon	Det → the
NP → NP PP			Det → a

- 0 ROOT → . S 0
 - 0 S → . NP VP 0
 - 0 NP → . Papa 0
 - 0 NP → Papa . 1
 - 0 S → NP . VP 1
 - 1 VP → . VP PP 1
 - 1 VP → . VP PP 1
 - 1 VP → . VP PP 1
 - 1 VP → . VP PP 1
- oops, stack overflowed
no fix after all
- must transform grammar to eliminate left-recursive rules

Use a Parse Table

- Earley's algorithm resembles recursive descent, but solves the left-recursion problem. No recursive function calls.
- Use a parse table as we did in CKY, so we can look up anything we've discovered so far. "Dynamic programming - AWP"
- Entries in column 5 look like (3, S → NP . VP)
 - (but we'll omit the → etc. to save space)
 - Built while processing word 5
 - Means that the input substring from 3 to 5 matches the initial NP portion of a S → NP VP rule
 - Dot shows how much we've matched as of column 5
 - Perfectly fine to have entries like (3, S → is it . true that S)

Use a Parse Table

- Entries in column 5 look like (3, S → NP . VP)
- What does it mean if we have this entry?
 - *Unknown right context:* *Doesn't* mean we'll necessarily be able to find a VP starting at column 5 to complete the S.
 - *Known left context:* *Does* mean that some dotted rule back in column 3 is looking for an S that starts at 3.
 - So if we actually do find a VP starting at column 5, allowing us to complete the S, then we'll be able to attach the S to something.
 - And when that something is complete, it too will have a customer to its left ...
 - In short, a top-down (i.e., goal-directed) parser: it chooses to start building a constituent not because of the input but because that's what the left context needs. In **the spoon**, won't build **spoon** as a verb because there's no way to use a verb there.
 - So any hypothesis in column 5 *could* get used in the correct parse, if words 1-5 are continued in just the right way by words 6-n.

Operation of the Algorithm

- Process all hypotheses one at a time in order. (Current hypothesis is shown in blue.)
- This may add **new hypotheses** to the end of the to-do list, or try to add **old hypotheses** again.
- Process a hypothesis according to what follows the dot
 - If a word, **scan** input and see if it matches
 - If a nonterminal, **predict** ways to match it
 - (we'll predict blindly, but could reduce # of predictions by *looking ahead* k symbols in the input and only making predictions that are compatible with this limited *right context*)
 - If nothing, then we have a complete constituent, so **attach** it to all its customers

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa

predict the kind of NP we are looking for
(actually we'll look for 3 kinds: any of the 3 will do)

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a

predict the kind of Det we are looking for *(2 kinds)*

0
0 ROOT . S
0 S . NP VP
0 NP . Det N
0 NP . NP PP
0 NP . Papa
0 Det . the
0 Det . a

predict the kind of NP we're looking for
but we were already looking for these so don't add duplicate goals! Note that this happened when we were processing a left-recursive rule.

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP		
0 NP . Det N		
0 NP . NP PP		
0 NP . Papa		
0 Det . the		
0 Det . a		

scan: the desired word is in the input!

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP		
0 NP . Det N		
0 NP . NP PP		
0 NP . Papa		
0 Det . the	scan: failure	
0 Det . a		

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP		
0 NP . Det N		
0 NP . NP PP		
0 NP . Papa		
0 Det . the		
0 Det . a	scan: failure	

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP		
0 NP . Papa		
0 Det . the		
0 Det . a		

attach the newly created NP (which starts at 0) to its customers (incomplete constituents that *end* at 0 and have NP after the dot)

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the		
0 Det . a		

predict

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	predict
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a		

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	predict
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	predict
0 Det . the	1 PP . P NP	
0 Det . a	1 V . ate	

0	Papa	1
0 ROOT . S	0 NP Papa .	
0 S . NP VP	0 S NP . VP	
0 NP . Det N	0 NP NP . PP	
0 NP . NP PP	1 VP . V NP	
0 NP . Papa	1 VP . VP PP	
0 Det . the	1 PP . P NP	predict
0 Det . a	1 V . ate	
	1 P . with	

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP			
0 NP . Det N	0 NP NP . PP			
0 NP . NP PP	1 VP . V NP			
0 NP . Papa	1 VP . VP PP			
0 Det . the	1 PP . P NP			
0 Det . a	1 V . ate	scan: success!		
	1 P . with			

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP			
0 NP . Det N	0 NP NP . PP			
0 NP . NP PP	1 VP . V NP			
0 NP . Papa	1 VP . VP PP			
0 Det . the	1 PP . P NP			
0 Det . a	1 V . ate			
	1 P . with	scan: failure		

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP	1 VP V . NP		
0 NP . Det N	0 NP NP . PP			
0 NP . NP PP	1 VP . V NP			
0 NP . Papa	1 VP . VP PP			
0 Det . the	1 PP . P NP			
0 Det . a	1 V . ate			
	1 P . with			

attach

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP	1 VP V . NP		
0 NP . Det N	0 NP NP . PP	2 NP . Det N		
0 NP . NP PP	1 VP . V NP	2 NP . NP PP		
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP			
0 Det . a	1 V . ate			
	1 P . with			

predict

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP	1 VP V . NP		
0 NP . Det N	0 NP NP . PP	2 NP . Det N		
0 NP . NP PP	1 VP . V NP	2 NP . NP PP		
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

predict (these next few steps should look familiar)

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP	1 VP V . NP		
0 NP . Det N	0 NP NP . PP	2 NP . Det N		
0 NP . NP PP	1 VP . V NP	2 NP . NP PP		
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

predict

0	Papa	1	ate	2
0 ROOT . S	0 NP Papa .	1 V ate .		
0 S . NP VP	0 S NP . VP	1 VP V . NP		
0 NP . Det N	0 NP NP . PP	2 NP . Det N		
0 NP . NP PP	1 VP . V NP	2 NP . NP PP		
0 NP . Papa	1 VP . VP PP	2 NP . Papa		
0 Det . the	1 PP . P NP	2 Det . the		
0 Det . a	1 V . ate	2 Det . a		
	1 P . with			

scan (this time we fail since Papa is not the next word)

0	Papa	1	ate	2	the	3
0 ROOT . S	0 NP Papa .	1 V ate .			2 Det the .	
0 S . NP VP	0 S NP . VP	1 VP V . NP				
0 NP . Det N	0 NP NP . PP	2 NP . Det N				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa				
0 Det . the	1 PP . P NP	2 Det . the				
0 Det . a	1 V . ate	2 Det . a				
	1 P . with					

scan: success!

0	Papa	1	ate	2	the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .			
0 S . NP VP	0 S NP . VP	1 VP V . NP				
0 NP . Det N	0 NP NP . PP	2 NP . Det N				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa				
0 Det . the	1 PP . P NP	2 Det . the				
0 Det . a	1 V . ate	2 Det . a				
	1 P . with					

0	Papa	1	ate	2	the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .			
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N			
0 NP . Det N	0 NP NP . PP	2 NP . Det N				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa				
0 Det . the	1 PP . P NP	2 Det . the				
0 Det . a	1 V . ate	2 Det . a				
	1 P . with					

0	Papa	1	ate	2	the	3
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .			
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N			
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar			
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon			
0 NP . Papa	1 VP . VP PP	2 NP . Papa				
0 Det . the	1 PP . P NP	2 Det . the				
0 Det . a	1 V . ate	2 Det . a				
	1 P . with					

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N					
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar					
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon					
0 NP . Papa	1 VP . VP PP	2 NP . Papa						
0 Det . the	1 PP . P NP	2 Det . the						
0 Det . a	1 V . ate	2 Det . a						
	1 P . with							

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N					
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar					
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon					
0 NP . Papa	1 VP . VP PP	2 NP . Papa						
0 Det . the	1 PP . P NP	2 Det . the						
0 Det . a	1 V . ate	2 Det . a						
	1 P . with							

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar					
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon					
0 NP . Papa	1 VP . VP PP	2 NP . Papa						
0 Det . the	1 PP . P NP	2 Det . the						
0 Det . a	1 V . ate	2 Det . a						
	1 P . with							

attach

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa						
0 Det . the	1 PP . P NP	2 Det . the						
0 Det . a	1 V . ate	2 Det . a						
	1 P . with							

**attach
(again!)**

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa					0 S NP VP .	
0 Det . the	1 PP . P NP	2 Det . the					1 VP VP . PP	
0 Det . a	1 V . ate	2 Det . a						
	1 P . with							

**attach
(again!)**

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP				
	1 P . with							

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP				
	1 P . with			0 ROOT S .				

attach
(again!)

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP				
	1 P . with			0 ROOT S .				

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP				
	1 P . with			0 ROOT S .				

0	Papa	1	ate	2	the	3	caviar	4
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP				
	1 P . with			0 ROOT S .				
				4 P . with				

0	Papa	1	ate	2	the	3	caviar	4	with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .					
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .						
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .						
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP						
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .						
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP						
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP						
	1 P . with			0 ROOT S .						
				4 P . with						

0	Papa	1	ate	2	the	3	caviar	4	with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .					
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP					
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .						
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP						
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .						
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP						
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP						
	1 P . with			0 ROOT S .						
				4 P . with						

0	Papa	1	ate	2	the	3	caviar	4	with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .					
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP					
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N					
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP					
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa					
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP						
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP						
	1 P . with			0 ROOT S .						
				4 P . with						

0	Papa	1	ate	2	the	3	caviar	4	with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .					
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP					
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N					
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP					
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa					
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the					
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a					
	1 P . with			0 ROOT S .						
				4 P . with						

0	Papa	1	ate	2	the	3	caviar	4	with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .					
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP					
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N					
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP					
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa					
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the					
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a					
	1 P . with			0 ROOT S .						
				4 P . with						

0	Papa	1	ate	2	the	3	caviar	4	with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .					
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP					
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N					
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP					
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa					
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the					
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a					
	1 P . with			0 ROOT S .						
				4 P . with						

0	Papa	1	ate	2	the	3	caviar	4	with	5
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar .	4 P with .					
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP					
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N					
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP					
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .	5 NP . Papa					
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP	5 Det . the					
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP	5 Det . a					
	1 P . with			0 ROOT S .						
				4 P . with						

	ate	2	the	3	caviar	4	with	5	a	6
.	1 V ate .		2 Det the .		3 N caviar .		4 P with .		5 Det a .	
P	1 VP V . NP		2 NP Det . N		2 NP Det N .		4 PP P . NP			
PP	2 NP . Det N		3 N . caviar		1 VP V NP .		5 NP . Det N			
P	2 NP . NP PP		3 N . spoon		2 NP NP . PP		5 NP . NP PP			
P	2 NP . Papa				0 S NP VP .		5 NP . Papa			
P	2 Det . the				1 VP VP . PP		5 Det . the			
	2 Det . a				4 PP . P NP		5 Det . a			
					0 ROOT S .					
					4 P . with					

	ate	2	the	3	caviar	4	with	5	a	6
.	1 V ate .		2 Det the .		3 N caviar .		4 P with .		5 Det a .	
P	1 VP V . NP		2 NP Det . N		2 NP Det N .		4 PP P . NP		5 NP Det . N	
PP	2 NP . Det N		3 N . caviar		1 VP V NP .		5 NP . Det N			
P	2 NP . NP PP		3 N . spoon		2 NP NP . PP		5 NP . NP PP			
P	2 NP . Papa				0 S NP VP .		5 NP . Papa			
P	2 Det . the				1 VP VP . PP		5 Det . the			
	2 Det . a				4 PP . P NP		5 Det . a			
					0 ROOT S .					
					4 P . with					

	ate	2	the	3	caviar	4	with	5	a	6
.	1 V ate .		2 Det the .		3 N caviar .		4 P with .		5 Det a .	
P	1 VP V . NP		2 NP Det . N		2 NP Det N .		4 PP P . NP		5 NP Det . N	
PP	2 NP . Det N		3 N . caviar		1 VP V NP .		5 NP . Det N		6 N . caviar	
P	2 NP . NP PP		3 N . spoon		2 NP NP . PP		5 NP . NP PP		6 N . spoon	
P	2 NP . Papa				0 S NP VP .		5 NP . Papa			
P	2 Det . the				1 VP VP . PP		5 Det . the			
	2 Det . a				4 PP . P NP		5 Det . a			
					0 ROOT S .					
					4 P . with					

	ate	2	the	3	caviar	4	with	5	a	6
.	1 V ate .		2 Det the .		3 N caviar .		4 P with .		5 Det a .	
P	1 VP V . NP		2 NP Det . N		2 NP Det N .		4 PP P . NP		5 NP Det . N	
PP	2 NP . Det N		3 N . caviar		1 VP V NP .		5 NP . Det N		6 N . caviar	
P	2 NP . NP PP		3 N . spoon		2 NP NP . PP		5 NP . NP PP		6 N . spoon	
P	2 NP . Papa				0 S NP VP .		5 NP . Papa			
P	2 Det . the				1 VP VP . PP		5 Det . the			
	2 Det . a				4 PP . P NP		5 Det . a			
					0 ROOT S .					
					4 P . with					

	ate	2	the	3	caviar	4	with	5	a	6	spoon	7
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .						
VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N							
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar							
PP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon							
PP	2 NP . Papa		0 S NP VP .	5 NP . Papa								
PP	2 Det . the		1 VP VP . PP	5 Det . the								
	2 Det . a		4 PP . P NP	5 Det . a								
			0 ROOT S .									
			4 P . with									

	ate	2	the	3	caviar	4	with	5	a	6	spoon	7
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .						
VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	5 NP Det N .						
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar							
PP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon							
PP	2 NP . Papa		0 S NP VP .	5 NP . Papa								
PP	2 Det . the		1 VP VP . PP	5 Det . the								
	2 Det . a		4 PP . P NP	5 Det . a								
			0 ROOT S .									
			4 P . with									

	ate	2	the	3	caviar	4	with	5	a	6	spoon	7
.	1 V ate .	2 Det the .	3 N caviar .	4 P with .	5 Det a .	6 N spoon .						
VP	1 VP V . NP	2 NP Det . N	2 NP Det N .	4 PP P . NP	5 NP Det . N	5 NP Det N .						
PP	2 NP . Det N	3 N . caviar	1 VP V NP .	5 NP . Det N	6 N . caviar	4 PP P NP .						
PP	2 NP . NP PP	3 N . spoon	2 NP NP . PP	5 NP . NP PP	6 N . spoon	5 NP NP . PP						
PP	2 NP . Papa		0 S NP VP .	5 NP . Papa								
PP	2 Det . the		1 VP VP . PP	5 Det . the								
	2 Det . a		4 PP . P NP	5 Det . a								
			0 ROOT S .									
			4 P . with									

	0	Papa	1	ate	2	the	3	caviar	4	with	a	spoon	7
.	0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar	6 N spoon .						
VP	0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .						
PP	0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .						
PP	0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP						
PP	0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .						
PP	0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .						
	0 Det . a	1 V . ate	2 Det . a		4 PP . P NP								
		1 P . with			0 ROOT S .								
					4 P . with								

0	Papa	1	ate	2	the	3	caviar	4	with a spoon	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar	6 N spoon .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP				
	1 P . with			0 ROOT S .		1 VP V NP .				
				4 P . with		2 NP NP . PP				
						0 S NP VP .				
						1 VP VP . PP				
						7 P . with				
						0 ROOT S .				

0	Papa	1	ate	2	the	3	caviar	4	with a spoon	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar	6 N spoon .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP				
	1 P . with			0 ROOT S .		1 VP V NP .				
				4 P . with		2 NP NP . PP				
						0 S NP VP .				
						1 VP VP . PP				
						7 P . with				
						0 ROOT S .				

0	Papa	1	ate	2	the	3	caviar	4	with a spoon	7
0 ROOT . S	0 NP Papa .	1 V ate .	2 Det the .	3 N caviar	6 N spoon .				
0 S . NP VP	0 S NP . VP	1 VP V . NP	2 NP Det . N	2 NP Det N .		5 NP Det N .				
0 NP . Det N	0 NP NP . PP	2 NP . Det N	3 N . caviar	1 VP V NP .		4 PP P NP .				
0 NP . NP PP	1 VP . V NP	2 NP . NP PP	3 N . spoon	2 NP NP . PP		5 NP NP . PP				
0 NP . Papa	1 VP . VP PP	2 NP . Papa		0 S NP VP .		2 NP NP PP .				
0 Det . the	1 PP . P NP	2 Det . the		1 VP VP . PP		1 VP VP PP .				
0 Det . a	1 V . ate	2 Det . a		4 PP . P NP		7 PP . P NP				
	1 P . with			0 ROOT S .		1 VP V NP .				
				4 P . with		2 NP NP . PP				
						0 S NP VP .				
						1 VP VP . PP				
						7 P . with				
						0 ROOT S .				