Multiuser Detection

Summary for 6.975 EECS Graduate Seminar in Communications

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The *multiuser detection* problem applies when we are sending data on the uplink channel from a handset to a base station. The base station must demodulate and decode signals from $K-1$ interfering handsets. A related problem known as *interference cancellation* arises on the down-link channel from the base station to the handset in which the handset must separate the signal intended for it, from the signals that the base station intends for other handsets in the cell. Methods for directly mitigating interference from neighboring cells have generally not been addressed, probably because of the current practical difficulties of sharing large quantities of time sensitive information between different cells.

There is an enormous literature on multiuser detection, but a few resources may be helpful. The main resource is Verdu's book [Verdu 1998]. The Fall 11/8/2000 presentation by C. Emre Koksal, entitled "Linear Multiuser Receivers" for this seminar is quite helpful.

Matched Filter

In a Gaussian channel

$$p(y_i|x_i) = \exp\left(-\frac{1}{2\sigma^2}\int [y(t) - x_i(t)]^2 dt\right) \tag{1}$$

where x is the transmitted signal and y is the received signal, if we hypothesize two possible transmitted signals x_i and x_j, we can determine which is more likely to have actually been sent by comparing the likelihoods to see which is greater (maximum)

$$p(y_i|x_i) \quad <> \quad p(y|x_j), \tag{2}$$

$$-\frac{1}{2\sigma^2}\int[y(t)-x_i(t)]^2dt \quad <> \quad -\frac{1}{2\sigma^2}\int[y(t)-x_j(t)]^2dt, \tag{3}$$

$$\int[y(t)-x_i(t)]^2dt \quad <> \quad \int[y(t)-x_j(t)]^2dt, \tag{4}$$

$$\int y(t)x_i(t)dt-\frac{1}{2}\int x_i(t)^2dt \quad <> \quad \int y(t)x_j(t)dt-\frac{1}{2}\int x_j(t)^2dt. \tag{5}$$

This last relation states that to minimize the chance of a wrong decision as to which x was actually sent, we must choose the x with the minimum mean squared distance to y. We can use the *sufficient statistic*

$$\int y(t)x_i(t)dt, \tag{6}$$

to make an optimal decision if $x_1,\ldots,x_m$ are linearly independent.

In general, $g(y)$ is a sufficient statistic for θ if given the observations y, the conditional distribution $p(y|g(y))$ does not depend on θ. In other words, $g(y)$ contains all of the information in y necessary to infer θ.

We can apply the above argument to the single user version of the CDMA channel. The received signal is

$$y(t)=Axs(t)+\sigma n(t), t\in[0,T], \tag{7}$$

where the deterministic signature sequence (spreading code) s has unit energy, the noise n is white and Gaussian, and the transmitted symbol $b\in -1,1$. We can use a correlator with a model $h(t)$ of the signature waveform $s(t)$ to make an estimate

$$\hat{x}=sgn\left(\int h(t)y(t)dt-\frac{1}{2}\int x_i(t)^2dt\right) \tag{8}$$

of the correct transmitted symbol x. As long as the noise is uncorrelated, this will produce the most accurate result if h is a (nonzero) multiple of s. This linear detector is known as a matched filter and in the single user environment it is optimal in the sense that the SNR is maximized. Another way to say this is that the matched filter yields the maximum-likelihood (ML) detector for the single-user channel [**?**].

In a multiuser channel, we can write the received signal as

$$\mathbf{Y} = \sum_{i=1}^{K} \mathbf{X}_i \mathbf{s}_i + \mathbf{W} \tag{9}$$

and we can use a bank of matched filters to demodulate each independent user. In order to do this each matched filter must either be synchronized to the bit epochs of its corresponding transmitter or we must over-sample the received signal and use an asynchronous detector architecture. Synchronization in multiuser detectors (called acquisition and tracking) will not be discussed in this chapter. If the matched filters are perfectly synchronized and the signature sequences s_k for all the k users are all orthogonal to one another (linearly independent), then the matched filter is optimal and the results are the same as for the single user problem.

An important question is, "how many mutually orthogonal signals with (approximate) duration T and (approximate) bandwidth B can be constructed? In fact, no signal is strictly time-limited or band-limited, so instead we ask that the amount of signal energy that lays outside the time interval $[0, T]$ or outside the band $[-B, B]$ does not exceed a bound ϵ. No explicit answer for the number of orthogonal signals in terms of B, T and ϵ is known, but unless BT is very small, the answer is essentially $2BT$. Therefore, a K-user orthogonal CDMA system employing antipodal modulation at the rate of R bits per second requires bandwidth approximately equal to

$$B = \frac{1}{2} RK. \tag{10}$$

Nonorthogonal Spreading Sequences

However, we need not always enforce the strict condition that the signals be orthogonal in CDMA. For example, the performance of the matched filter receiver will be degraded in the presence of nonorthogonal interferers, but we still may be able to correctly demodulate the signals from each user if the interference is bounded. This still requires careful selection of the signature waveforms so that their cross-correlations are fairly low (compared to the signature waveform energies $\|s_i\|^2 =$

$\langle s_i, s_i \rangle$) [Abend 1970]. More to the point, maintaining strict orthogonality involves maintaining strict synchronization among all the users of the system, which is a very difficult control problem in a real world channel due to real-valued multi-path time delays.

Even if global symbol and/or spreading sequence synchronization were possible, removing the restriction of orthogonal signature waveforms has several major benefits that make CDMA attractive for multiuser communications systems:

- The users can be asynchronous and yet "quasi-orthogonality" can be maintained by adequate design of the signature waveforms.
- The number of signature users is no longer constrained to twice the duration-bandwidth product of the signature waveforms.
- Channel sharing experiences graceful degradation; Reliability depends on the number of simultaneous users rather than on the (much larger) number of potential users. Therefore, unlike orthogonal multiaccess, it is possible to trade off reception quality for increased capacity.

With nonorthogonal CDMA, the simple matched filter is no longer optimal (even in the presence of white Gaussian noise). For example it suffers from the *near-far problem*: any interferer that is sufficiently powerful at the receiver causes arbitrarily high performance degradation. Furthermore, by designing receiver demodulation schemes that unlike the matched filter, take into account the structure of the multiaccess interference (MAI), it is possible to design a system with increased spectral efficiency, decreased output power, and robustness against imbalances in the received powers of various users.

The matched filter still plays an important role in multiuser detection, however, since the output of a bank of matched filters provides a minimal sufficient statistic for detection [Verdu 1986].

Decorrelator

The decorrelator, like the matched filter, is a linear multiuser detector, but unlike the matched filter, it uses information from all of the other users to remove interference. The decorrelator inverts the channel leaving the received signal without interference but by doing so also increases the noise. The advantage of the decorrelator is that no knowledge of the received power is necessary and its performance is independent of the power of interfering users so that it solves the near-far problem. Both synchronous and asynchronous decorrelators have been considered, but here we consider only the synchronous case as the generalization to the asynchronous case is relatively straightforward. Equation 9 can be written in matrix form as

$$\mathbf{Y} = S\mathbf{X} + \mathbf{W}, \tag{11}$$

where $\mathbf{X} = [X_1 \dots X_K]^T$ and $S = [\mathbf{s}_1 \dots \mathbf{s}_k]^T$ is the $N \times K$ matrix of signature sequences. For one user S would look like this,

$$\begin{bmatrix} s_{1,1} & & & \\ s_{1,2} & & & \\ \vdots & & & \\ s_{1,N} & & & \\ & s_{2,1} & & \\ & s_{2,2} & & \\ & \vdots & & \\ & s_{2,N} & & \\ & & \ddots & s_{T_u,1} \\ & & & s_{T_u,2} \\ & & & \vdots \\ & & & s_{T_u,N} \end{bmatrix} \tag{12}$$

and extended to multiple users it takes the form

$$
\begin{bmatrix}
s_{1,1}^1 & \cdots & s_{1,1}^K & & & & \\
s_{1,2}^1 & \cdots & s_{1,2}^K & & & & \\
\vdots & & \vdots & & & & \\
s_{1,N}^1 & \cdots & s_{1,N}^K & & & & \\
& & & s_{2,1}^1 & \cdots & s_{2,1}^K & \\
& & & s_{2,2}^1 & \cdots & s_{2,2}^K & \\
& & & \vdots & & \vdots & \\
& & & s_{2,N}^1 & \cdots & s_{2,N}^K & \\
& & & & & & \ddots
\end{bmatrix} \tag{13}
$$

A bank of matched filters would mean filtering $\mathbf{Y}$ by multiplying it by S^T yielding,

$$\mathbf{R} = S^T S \mathbf{X} + S^T \mathbf{W} \tag{14}$$

If however, we also multiply by $(S^T S)^{-1}$, then we get

$$\mathbf{U} = (S^T S)^{-1} \mathbf{R} = \mathbf{X} + (S^T S)^{-1} S^T \mathbf{W} \tag{15}$$

We define the overall linear filter $N = (S^T S)^{-1} S^T$. This is called the decorrelator. The covariance of N, K_N is

$$K_N = (S^T S)^{-1} \sigma^2. \tag{16}$$

The Optimum Multiuser Detector (Nonlinear)

The optimum receiver for a DS/CDMA asynchronous Gaussian multiple access channel was first shown by Verdu [Verdu 1986] and is covered in a chapter in his book [Verdu 1998]. Although theoretically optimal, the optimum detector for this channel is unfortunately not computationally feasible in practical systems [Verdu 1989]. It is important, however, as an upper bound on performance and as a starting point for designing reduced complexity decoders. The optimum detector for the synchronous channel computational feasible, however, so we examine it first.

The matched filter assumed knowledge of the signature waveform and its timing (synchronization). The optimum detector requires this knowledge as well as knowledge of the amplitude for each of the users and the total noise level. The optimum MAP detector is defined as,

$$\hat{\mathbf{b}}_{MAP} = \arg \max_{b \in \{-1,1\}^{(P+1)K}} P(\mathbf{b}|\mathbf{y}) \tag{17}$$

$$\hat{\mathbf{b}}_{MAP} = \arg \max_{b \in \{-1,1\}^{(P+1)K}} P(\mathbf{b}|\mathbf{y})p(\mathbf{y}) \tag{18}$$

$$\hat{\mathbf{b}}_{ML} = \arg \max_{b \in \{-1,1\}^{(P+1)K}} p(\mathbf{y}|\mathbf{b})P(\mathbf{b}). \tag{19}$$

By applying Bayes' rule and assuming that the vector $\mathbf{b} \in \{-1,1\}^{(P+1)K)}$ was transmitted and that the probability of $\mathbf{b}$ is $P(\mathbf{b}) = 2^{-(P+1)K}$, ie. all the transmitted data vectors are independent and equally likely, we can write the optimum maximum likelihood detector,

$$\hat{\mathbf{b}}_{ML} = \arg \max_{b \in \{-1,1\}^{(P+1)K}} P(\mathbf{y}|\mathbf{b}) \tag{20}$$

where P is the number of symbols in the signature sequence, and K is again the number if users, and where $\hat{\mathbf{b}}$ is the data vector that maximizes the pdf of received vector y. This is known as maximum likelihood sequence estimation (MLSE).

Optimum Multiuser Detector for Two-User Synchronous Channel

If the received signal for the two-user synchronous channel is

$$y(t) = A_1 x_1 s_1(t) + A_2 b_2 s_2(t) + \sigma n(t), t \in [0, T], \tag{21}$$

then the individual minimum probability of error decision for user 1 is obtained by selecting the value of $b_1 \in -1, +1$ that maximizes the a posteriori probability

$$P[b_1|y(t), 0 \leq t \leq T] \tag{22}$$

and analogously for user 2. We could also ask for a joint minimum probability of error by requiring that the receiver select the pair (b_1, b_2) that maximizes the joint a posteriori probability

$$P[(b_1, b_2)|y(t), 0 \leq t \leq T]. \tag{23}$$

We can write the single user optimum detector 22 in terms of joint optimum detector 23

$$P[b_1|y(t), 0 \leq t \leq T] = P[(b_1, +1)|y(t), 0 \leq tlT] + P[(b_1, -1)|y(t), 0 \leq tlT]. \tag{24}$$

Since the transmitted data are equiprobable, the joint MAP estimate is the maximum likelihood estimate. In practice, the individual and joint optimum decision strategies will only give different decisions in very noisy situations where the probability of error is very large. For the received signal given in equation 21, the joint optimum decisions for two users are given by

$$\hat{b}_1 = sgn\left(A_1 y_1 + \frac{1}{2}|A_2 y_2 - A_1 A_2 \rho| - \frac{1}{2}|A_2 y_2 + A_1 A_2 \rho|\right), \tag{25}$$

and

$$\hat{b}_2 = sgn\left(A_2 y_2 + \frac{1}{2}|A_1 y_1 - A_1 A_2 \rho| - \frac{1}{2}|A_1 y_1 + A_1 A_2 \rho|\right), \tag{26}$$

where A_1, A_2 are the amplitudes of the received signals from users 1 and 2, and $\rho_{ij} = \mathbf{R}$ is the signature sequence crosscorrelation matrix given by

$$\rho_{ij} = \langle s_i, s_j \rangle = \int_0^T s_i(t) s_j(t) dt \tag{27}$$

which for two users reduces to

$$\rho = \int_0^T s_1(t)s_2(t)dt. \tag{28}$$

The optimum decisions for the users taken individually is very similar in form,

$$\hat{b}_1 = sgn\left(y_1 - \frac{\sigma^2}{2A_1}\log\frac{\cosh\left[\frac{A_2 y_2 + A_1 A_2 \rho}{\sigma^2}\right]}{\cosh\left[\frac{A_2 y_2 - A_1 A_2 \rho}{\sigma^2}\right]}\right), \tag{29}$$

but the absolute value function is replaced by cosh. For large signal to noise ratios ($A_1, A_2 \gg 0$), the individual optimum decision converges to the jointly optimum decision as the cosh function more and more approximates an absolute value function.

Optimum Multiuser Detector for the Asynchronous Channel

Now we turn to the optimum detector in the asynchronous channel. The detector in the asynchronous channel is computationally complex for a simple reason; making optimum decisions in the asynchronous channel requires observation of the entire frame of transmitted bits. Suppose we want to decode a particular bit, $b_1[0]$. The conventional matched filter or decorrelator detectors as well as the optimum detector in the synchronous channel can all decode a single bit at a time. This is not the case in the asynchronous channel. For example, in the two-user case, a given bit such as $b_1[0]$ will overlap user's bits bits $b_2[-1]$ and $b_2[0]$ to a greater or lesser degree. To be optimum, the detector should use information about the values of those interfering bits by extending the observation interval from $[0, T]$ to $[\tau - T, \tau + T]$. However, once we do this, the new observation interval contains bits $b_1[-1]$ and $b_1[1]$ so these must be included in the estimate by again extending the observation interval. Unless the entire bit frame is included, the decision on bit $b_1[0]$ or any other bit will be suboptimal.

We again examine the two user case to simplify the exposition. We will need the following definitions. There are $2M + 1$ transmitted bits from each of our 2 users,

$$b_{k+i2} = b_k[i], k = \{1, 2\}, i = \{-M, \ldots, M\}. \tag{30}$$

Our objective is to compute a $\mathbf{b}$ that maximizes

$$f(\{y(t), t \in [-MT, MT+2T]\}|\mathbf{b}) = \exp\left(-\frac{1}{2\sigma^2}\int_{-MT}^{MT+2T}[y(t) - S_t(\mathbf{b})]^2 dt\right) \quad (31)$$

where

$$S_t(\mathbf{b}) = \sum_{k=1}^{K}\sum_{i=-M}^{M} A_k b_k[i] s_k(t - iT - \tau_k). \quad (32)$$

Maximizing this only involves maximizing the term

$$\omega(\mathbf{b}) = 2\int S_t(\mathbf{b}) y(t) dt - \int S_t^2(\mathbf{b}) dt, \quad (33)$$

The observations enter into the (nonlinear) function to be optimized exclusively via the outputs of matched filters, so once again, y is a sufficient statistic for $\mathbf{b}$.

A detailed exposition is beyond the scope of this review, but suffice it to say that the Viterbi algorithm can be applied to this optimization problem. The intuition is that decoding each bit only depends on the bits immediately before and after it, so we can draw a trellis and a forward-backward type dynamic programming algorithm can be applied to calculate the maximum likelihood estimates for each bit.

With the maximum likelihood sequence estimation (MLSE) receiver operating in low noise conditions, the user with the lowest probability of detection error experiences the same performance as the user in a single user system. The complexity of the MLSE receiver is not exponential in the length of the spreading code, but it does scale exponentially $O(2^K)$ with the number of users K and is classified as NP-hard [Verdu 1989]. Due to the complexity of the MLSE receiver, a number of reduced complexity receivers have been developed, which we will examine next.

Iterative Multiuser Detection

There have been a variety of suboptimal (but lower complexity) non-linear detectors. Multistage receivers are receivers in which decisions made by the first stage are used for interference cancellation in the second stage. Decision feedback equalizers (DFE) using this principle have been known for quite some time. Xie et al. also proposed a sequential decoding scheme based on the original trellis decoder of Verdu which decoded the MLSE solution in a suboptimal way but with substantially better performance than a matched filter.

Following on success of turbo codes and the understanding of their decoding on probability graphs, there has been much recent interest in designing iterative multiuser receivers following the same principles. Here we discuss a receiver that is derived from the maximum a posteriori (MAP) criterion for the joint received signal, but uses only single user decoders. "Iterating" the system is utilized to greatly improve performance. I carefully follow the exposition in a paper by Reed *et al.* found in the bibliography of this document.

Each user uses a single user turbo code for forward error correction in addition to a randomly generated signature sequence (spreading code). With randomly generated spreading codes, performance is on average the same for synchronous or asynchronous system, given a large number of users. Furthermore, using random codes (which are not at all guaranteed to be orthogonal) it is theoretically possible to achieve single-user performance.

Giallorenzi *et al.* [Giallorenzi 1996] formulated an optimal multiuser sequence estimator for an asynchronous DS-CDMA system where each user employs convolutinonal error control coding. They found that the complexity per bit of information using the MLSE solution grows exponentially with the number of users in the system and the number of states in each user's encoder. Rather than jointly estimate the spreading sequence and the error control code, various proposals to factor the graph into subsystems to reduce the overall complexity of the decoding/demodulation task while maintaining performance.

We describe the trellis-based system proposed by Reed *et al.* which decomposes the receiver into a separate equalizer (multiuser detector) and decoder. The multiuser decoder is derived from a maximum a posteriori (MAP) criterion, that maximizes the probability of a correct symbol decision. This MAP-based multiuser receiver is concatenated with a single user soft-in/soft-out trellis decoder for the single-user turbo code employed by the transmitters.

The System and the Channel

For this discussion, the uplink communication system transmits convolutional coded, discrete time, perfect square pulses (no pulse shaping or inter-symbol interference (ISI), and has no synchronization errors or multipath. The channel model adds Gaussian noise of variance σ^2 with samples taken synchronously at the chip rate.

K users transmit L coded bits $d_t^{(k)} \in \{+1, -1\}$, where $k \in \{1, \ldots, K\}$ is the user number and $t \in \{0, \ldots L-1\}$ indexes the bits. The spread sequence employed by user k on bit t consists of N chips denoted by

$$s_t^{(k)} \in \{-1/\sqrt{N}, \ldots, +1/\sqrt{N}\}^N. \tag{34}$$

The chips of the spreading code are generated randomly and independently for each user k and for every bit t which is statistically the same as using a pseudorandom sequence that is much longer than the spreading length N. As can be seen in figure 0-1, the channel sums the encoded signals from all of the users and adds Gaussian noise.

The channel output e_t at time t can be written as

$$e_t = \mathbf{A}_t d_t + n_t \mathbb{C}^N \tag{35}$$

where

$$\mathbf{A}_t = (s_t^1, \ldots, s_t^K) \in \{-1/\sqrt{N}, \ldots, +1/\sqrt{N}\}^{N \times K} \tag{36}$$

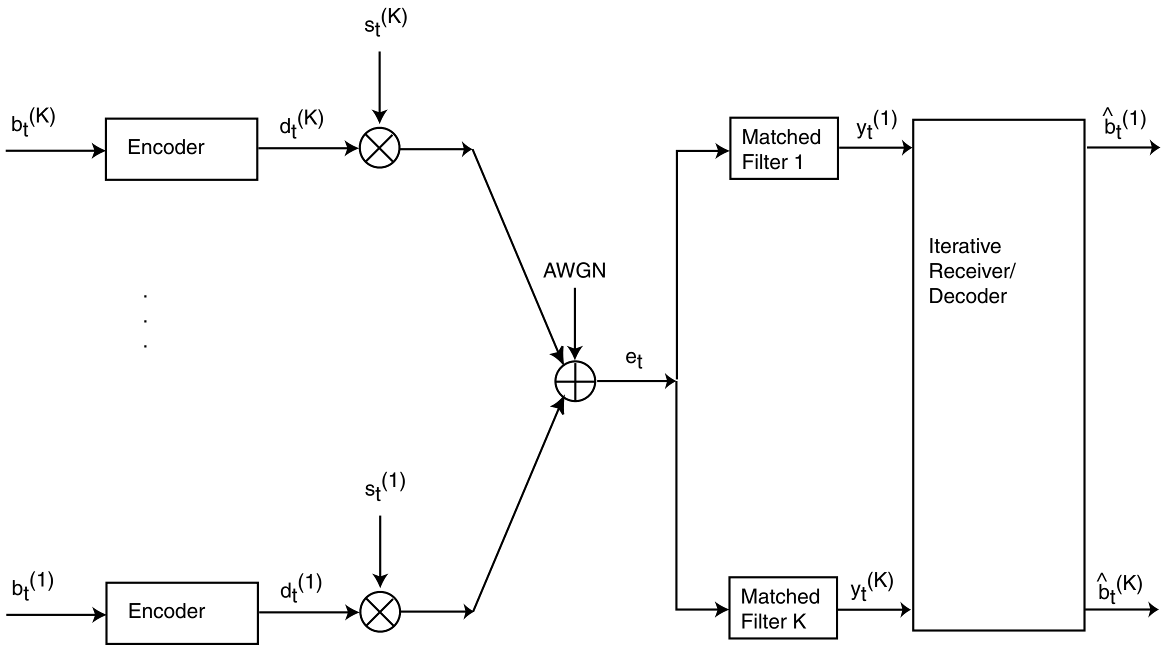

Figure 0-1: Convolutional coded synchronous multiuser channel

is the bank of spreading codes, one spreading code for each user. The matched filter (MF) output y_t at time t is then

$$\begin{aligned} y_t &= \mathbf{A}_t^T \mathbf{A}_t d_t + \mathbf{A}_t^T n_t \in \mathbb{C}^K \\ &= \mathbf{H}_t d_t + z_t \in \mathbb{C}^K \end{aligned} \tag{37}$$

where

$$d_t = (d_t^{(1)}, \ldots, d_t^{(K)})^T \in \{+1, -1\}^K \tag{38}$$

is the data vector, $\mathbf{H}_t = \mathbf{A}_t^T \mathbf{A}_t \in \mathbb{R}^{K \times K}$ is the crosscorrelation matrix of the spreading sequences, and z_t and n_t are the correlated and uncorrelated noise vectors, respectively.

Decomposing the Receiver

The receiver takes the matched filter channel output and generates the conditional channel probabilities $p(y_t|d_t)$ (multivariate Gaussian conditional probabilities). The metric generator then calculates the marginal probabilities $p(y_t|d_t^{(k)})$ for the kth decoder. The single user soft-in/soft-out FEC decoders then generate the a posteriori coded bit probabilities $Pr\{d_t^{(k)} = d|y^{(k)}\}$ for user k for coded block size 0 to $L-1$. The a posteriori coded bit probabilities are then used as a priori information for the metric generator on the next iteration. This information flow is shown in figure 0-2. The output from the single user's decoder can be taken as the system's bit estimates after a suitable number of iterations. Probabilities related to the coded bits $d_t^{(k)}$ are iterated around the receiver, while the information bits $b_j^{(k)}$ are only generated when a decision by the receiver is finally required.

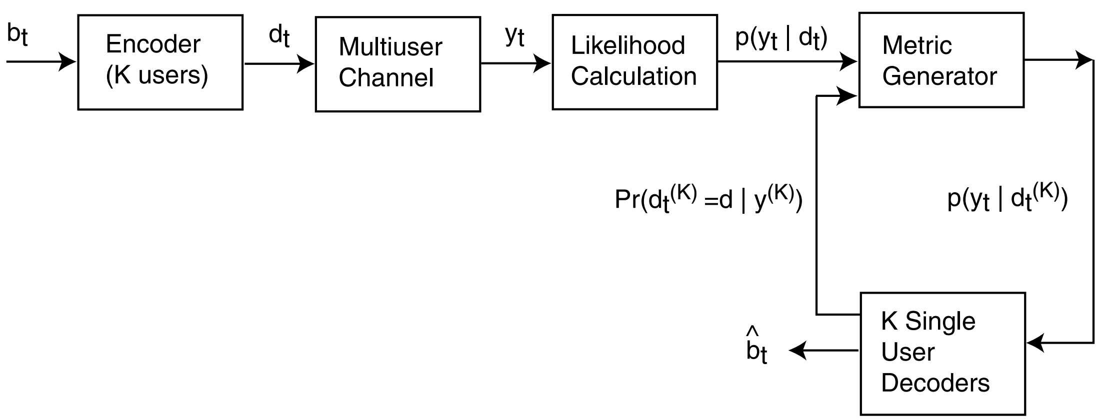

Figure 0-2: A decomposition of an iterative convolutional coded multiuser receiver

From the matched filter, we know that the conditional probability of y_t is

$$p(y_t|d_t) = \frac{1}{(2\pi)^{\frac{K}{2}}|H_t\sigma^2|^{\frac{1}{2}}} \exp\{-\frac{1}{2\sigma^2}\left(y_t^T H_t^{-1} y_t - 2y_t^T d_t + d_t^T H_t d_t\right)\} \tag{39}$$

The MAP decision rule for the metric generator sets

$$
\begin{aligned}
\hat{d}_t &= \arg\max_{d_t} Pr\{d_t|y_t\} && (40)\\
&= \arg\max_{d_t} p(y_t|d_t) Pr\{d_t\}. && (41)
\end{aligned}
$$

Since there are K users using binary symbol alphabets, there are 2^K hypotheses of the coded bits sent at time t. If each user encodes their coded bit sequence with a code with memory ν then there will be 2^ν possible states. When the DS/CDMA channel and the FEC code are jointly detected, the total complexity will be $\mathcal{O}(2^{K+K\nu})$. When we partition the receiver into a separate decoder for the FEC decoder and the DS/CDMA channel decoder, the complexity is reduced to $2^K + 2^\nu$.

APP Inputs to the FEC Decoder

Say there are n coded bits $d_t^{(k)}, \ldots, d_{t+n-1}^{(k)}$ for every information bit b_j, denoted as $d_j^{(k)}$. We want to calculate the a posteriori probability

$$
Pr\{d_{t'}^{(k)} = d|y^{(k)}\} = \sum_{m'} \sum_{d_j^{(k)}} Pr\{S_{j-1} = m'; d_j^{(k)}|y^{(k)}\} \tag{42}
$$

where S_j is the state at time j and m' ranges over all code states. Vector $d_j^{(k)}$ is the hypothesized channel bit vector for a particular user k for a particular FEC code trellis transition at time j, where $t \leq t' \leq t+n-1$. One way that t his can be calculated is by using the MAP algorithm. The FEC decoder takes as input the state transition probability.

APP Inputs to the Metric Generator

The a posteriori probabilities $p(d_t = d|y_t)$ are the outputs of the FEC decoder. We assign them as the a-priori input probabilities to the metric generator.

$$
Pr\{d_t^{(k)} = Pr\{d_t^{(k)}|y^{(k)}\}. \tag{43}
$$

This is okay since there are no correlations between the single user convolutional codes and the spreading codes. On the first iteration, these probabilities are set to $\frac{1}{2}$ for all k.

Computational Complexity Reduction

The computational complexity of this iterative receiver includes exponential complexity in the number of users just like the optimum decoder. The complexity is dominated by a 2^K term in the likelihood generation and the metric generation.

First we try to reduce the complexity of the likelihood generator. Since the first term in the exponential of equation 39 is independent of the variable d we can simplify it to

$$p(y_t|d_t) =_h \exp\{-\frac{1}{2\sigma^2}\left(2y_t^T d_t - d_t^T H_t d_t\right)\}. \tag{44}$$

Even so there are 2^K hypotheses that need to be tested by the likelihood generator and these likelihoods are then passed to the metric generator. This is the source of the complexity in these modules. Further methods, however, can be used to reduce the complexity, for example, by only computing an update of the least significant bit to the likelihoods in each iteration. A full discussion of such methods are beyond the scope of this review which only aims to bring together ideas from turbo decoding, factor graphs and message passing with the problem of multiuser detection.

When comparing this iterated multiuser detector against the performance of a decorrelator detector concatenated with the same turbo FEC that was used in the iterative decoder, we see that the iterative decoder performs at least 3dB better even with the complexity reduction techniques.

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