Iterative Multiuser Detection

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Multiuser Detection

- *multiuser detection* uplink channel from a handset to a base station. Base station must demodulate/decode $K - 1$ interfering handset signals.

- *interference cancellation* down-link channel from the base station to the handset. Handset must separate the signal intended for it from others

useful supplementary text: Sergio, Verdu “Multiuser Detection”
Gaussian Channel

\[ p(y|x_i) = \exp \left( -\frac{1}{2\sigma^2} \int [y(t) - x_i(t)]^2 dt \right) \]  

(1)

\( x \) transmitted signal and \( y \) is the received signal.

Hypothesize two possible transmitted signals \( x_i \) and \( x_j \), which likelihood is greater (maximum)?

\[ p(y|x_i) \leftrightarrow p(y|x_j), \]  

(2)

\[ -\frac{1}{2\sigma^2} \int [y(t) - x_i(t)]^2 dt \leftrightarrow -\frac{1}{2\sigma^2} \int [y(t) - x_j(t)]^2 dt, \]  

(3)

\[ \int [y(t) - x_i(t)]^2 dt \leftrightarrow \int [y(t) - x_j(t)]^2 dt, \]  

(4)

\[ \int y(t)x_i(t)dt - \frac{1}{2} \int x_i(t)^2 dt \leftrightarrow \int y(t)x_j(t)dt - \frac{1}{2} \int x_j(t)^2 dt. \]  

(5)
Matched Filter

In the single user CDMA channel

\[ y(t) = Axs(t) + \sigma n(t), t \in [0, T], \quad (6) \]

signature sequence \( s \), transmitted symbol \( b \in -1, 1 \), \( h(t) = as(t) \)

\[ \hat{x} = sgn \left( \int h(t)y(t)dt - \frac{1}{2} \int x_i(t)^2 dt \right) \quad (7) \]

sufficient statistic:

\[ \int y(t)x_i(t)dt. \quad (8) \]
Multiuser channel, matrix form

\[ Y = \sum_{i=1}^{K} X_i s_i + \mathbf{W} \]  \hspace{1cm} (9)

IF

- bank of matched filters, 1 per user
- users perfectly synchronized, bit and chip
- signature sequences \( s_k \) linearly independent

Then = single user performance (optimum)
Nonorthogonal Signature Sequences

But maintaining strict orthogonality involves synchronization → Hard because of real-valued multi-path time delays.

Nonorthogonal is good anyway:

- # users is looser → graceful degradation of channel sharing
- Reliability depends on # simultaneous users not # potential users
- Trade reception quality for increased capacity
Nonorthogonal Signature Sequences

Matched filter is no longer optimal (*near-far problem*

But can make new receivers to exploit structure of the multi-access interference (MAI) to:

- Increased spectral efficiency
- Decreased output power
- Robustness against imbalances in the received powers
Decorrelator

- Linear like matched filter (MF)

- Uses information from all users unlike MF

Inverts the channel → Leaves received signal without interference
→ But increases the noise.
Decorrelator

Received signal:

\[ Y = S \mathbf{X} + \mathbf{W}, \]  

(10)

X data bits, S signature sequence matrix, W noise.
S for single-user, bit and chip synchronous channel:

\[
\begin{bmatrix}
  s_{1,1} \\
  s_{1,2} \\
  \vdots \\
  s_{1,N} \\
  s_{2,1} \\
  s_{2,2} \\
  \vdots \\
  s_{2,N} \\
  \vdots \\
  s_{Tu,1} \\
  s_{Tu,2} \\
  \vdots \\
  s_{Tu,N}
\end{bmatrix}
\]

(11)
S for multi-user, bit and chip synchronous channel:

\[
\begin{bmatrix}
  s_{1,1}^1 & \ldots & s_{1,1}^K \\
  s_{1,2}^1 & \ldots & s_{1,2}^K \\
  \vdots & & \vdots \\
  s_{1,N}^1 & \ldots & s_{1,N}^K \\
  s_{2,1}^1 & \ldots & s_{2,1}^K \\
  s_{2,2}^1 & \ldots & s_{2,2}^K \\
  \vdots & & \vdots \\
  s_{2,N}^1 & \ldots & s_{2,N}^K \\
  \vdots & & \vdots 
\end{bmatrix}
\] (12)
Decorrelator

Bank of matched filters: multiplying $S^T Y$:

$$
R = S^T S X + S^T W
$$

(13)

Decorrelator, also multiply by $(S^T S)^{-1}$:

$$
U = (S^T S)^{-1} R = X + (S^T S)^{-1} S^T W
$$

(14)

Decorrelator $= (S^T S)^{-1} S^T$. 
**Decorrelator**

No knowledge of the received power necessary

Solves near-far problem: Performance is independent of the power of interfering users
Optimum Multiuser Detector (Nonlinear)

- NOT computationally feasible
- upper bound on performance
- starting point for reduced complexity decoders
Optimum Multiuser Detector (Nonlinear)

• required to know:
  – signature waveform
  – timing (synchronization)
  – amplitude of each user
  – noise level
Two User Synchronous Optimum Multiuser Detector (is feasible)

Individual minimum probability of error for user 1: MAP value of $b_1 \in \{-1, +1\}$

$$P[b_1|y(t), 0 \leq t \leq T]$$  \hspace{1cm} (15)

Joint (Both Users) Minimum Probability of Error: Select the pair $(b_1, b_2)$ that jointly maximizes APP:

$$P[(b_1, b_2)|y(t), 0 \leq t \leq T].$$ \hspace{1cm} (16)

If transmitted data are equiprobable $\rightarrow$ joint MAP = ML
Individual optimum similar to joint optimum

Received signal:

\[ y(t) = A_1 x_1 s_1(t) + A_2 b_2 s_2(t) + \sigma n(t), \quad t \in [0, T], \quad (17) \]

Joint optimum decisions for two users are given by

\[ \hat{b}_1 = \text{sgn} \left( A_1 y_1 + \frac{1}{2} |A_2 y_2 - A_1 A_2 \rho| - \frac{1}{2} |A_2 y_2 + A_1 A_2 \rho| \right), \quad (18) \]

\( A_1, A_2 \) are amplitudes of users, \( \rho_{ij} = \mathbf{R} \) is signature sequence crosscorrelation matrix

\[ \rho = \int_0^T s_1(t) s_2(t) dt. \quad (19) \]
Joint optimum similar to Individual Optimum

\[ \hat{b}_1 = \text{sgn} \left( y_1 - \frac{\sigma^2}{2A_1} \log \frac{\cosh \left[ \frac{A_2y_2 + A_1A_2\rho}{\sigma^2} \right]}{\cosh \left[ \frac{A_2y_2 - A_1A_2\rho}{\sigma^2} \right]} \right), \]  

(20)

absolute value function is replaced by \( \cosh \).

for large SNR, individual optimum decision converges to joint optimum, \( \cosh \to |\cdot| \).
Iterative Multiuser Detection

Suboptimal (but lower complexity) non-linear detectors:

- Multistage receivers

- Decision feedback equalizers (DFE)

- Xie et al. trellis-based suboptimal MLSE (much better than MF)

- Iterative decoders (Turbo/Factor graph inspired)
With randomly generated spreading codes:

- (and many users) → synchronous or asynchronous average performance is the same
- It is theoretically possible to achieve single-user performance
Prior work combining convolutional decoding and CDMA decoding

Giallorenzi et al.: Optimal MLSE with convolutional error correction coding (ECC)

They jointly estimate CDMA and ECC

Complexity exponential in K (# users) and # of states in ECC.

Find a way to factorize
The channel output

\[ e_t = A_t d_t + n_t, \]  

(21)
where

\[ d_t = (d_t^{(1)}, \ldots, d_t^{(K)})^T \in \{+1, -1\}^K \]  \hspace{1cm} (22)

is the data vector.

\[ \mathbf{A}_t = (s_t^1, \ldots, s_t^K) \in \{-1/\sqrt{N}, \ldots, +1/\sqrt{N}\}^{N \times K} \]  \hspace{1cm} (23)

is the bank of spreading codes, one spreading code for each user.
Matched filter (MF) output

\[ y_t = A_t^T A_t d_t + A_t^T n_t \]
\[ = H_t d_t + z_t \]  \hspace{1cm} (24)

where \( H_t = A_t^T A_t \) is the crosscorrelation matrix of the spreading sequences, \( z_t \) and \( n_t \) are the correlated and uncorrelated noise vectors, respectively.
Decomposition of Iterative Multiuser Receiver with Channel Coding
The Algorithm:

1. Matched filter channel output $\rightarrow$ conditional channel probabilities $p(y_t | d_t)$, (multivariate Gaussian conditional probabilities).

2. The metric generator then calculates the marginal probabilities $p(y_t | d^{(k)}_t)$ for the $k$th decoder.

3. The single user soft-in/soft-out FEC decoders then generate the a posteriori coded bit probabilities $Pr\{d^{(k)}_t = d|y^{(k)}\}$ for user $k$ for coded block size 0 to $L - 1$. 
4. The a posteriori coded bit probabilities are then used as a priori information for the metric generator on the next iteration.

5. Output from the single user’s decoder can be taken as bit estimates after a suitable number of iterations.
**Algorithm Complexity**

Joint detection of DS/CDMA channel and FEC code → $\mathcal{O}(2^K + K\nu)$

When partition the receiver: separate FEC decoder and DS/CDMA channel decoder → $\mathcal{O}(2^K + 2^\nu)$