Universal Portfolios with Side Information

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Brian Chen

October 16, 2002

Review of

> T.M. Cover and E. Ordentlich, "Universal Portfolios with Side Information," *IEEE Trans. on Info. Thy.*, vol. 42, no. 2, Mar. 1996, pp. 348–363.

The general investment problem concerns the allocation of wealth among m assets (stocks) to generate high returns with low risk or uncertainty. Cover and Ordentlich consider this problem from an information theoretic perspective in the case where a side information sequence aids the investment decisions but no assumptions are made on the relative likelihoods (probabilities) of stock returns sequences or side information sequences nor on the relationship between the stock returns and side information. The authors present a method of constructing a sequential portfolio that is universal with respect to the class of so-called state-constant rebalanced (state-CRB) portfolios in the sense that for every stock returns and side information sequence, this universal portfolio asymptotically achieves the same growth rate of wealth as the best state-CRB portfolio for that particular stock returns and side information sequence. Thus, a universal portfolio without hindsight matches the performance of the best state-CRB portfolio chosen with hindsight.

A key insight in the development of these universal portfolios is that the average of a set of exponentials grows exponentially as fast as the largest exponential. Thus, one can construct portfolios that are universal with respect to a class of portfolios by constructing a portfolio that is an average over that class. In their paper, Cover and Ordentlich demonstrate how to apply this intuition — which applies straightforwardly in the finite portfolio class case — to the state-CRB case, which involves a continuum of portfolios. Only the results of their derivations are presented in the talk.

The authors also highlight connections between universal portfolios and universal data compression problems. In particular, portfolio allocations in investment correspond to source probabilities in compression, CRB portfolios correspond to independent and identically distributed (iid) sources, and growth rates correspond to compression rates. Thus, a portfolio that is universal with respect to the class of CRB portfolios corresponds to a source code that is universal for iid sources.

Because universal portfolios' asymptotic growth rates track the growth rate of the best portfolio within a class for every stock and side information sequence, one might be tempted to believe that one can devise very good investment strategies without much knowledge of the underlying stock returns model. However, such reasoning is misleading for a number of reasons. First, the class over which a portfolio is universal may not contain good portfolios. Secondly, and more importantly, convergence in asymptotic growth rate does not imply convergence in ending wealth, a fact that is not necessarily emphasized in the paper. In particular, while the growth rate of the universal portfolio may converge asymptotically to that of the best portfolio chosen in hindsight, the ratio of the *wealths* generated may decrease polynomially to zero. Thus, the conventional wisdom that universal algorithms are useful in the case of long sequence lengths n, where $\frac{\log n}{n}$ is small, does not seem to apply to the investment scenario. In fact, a bound on the ratio of the wealth generated by the universal portfolio to the wealth generated by the best state-CRB portfolio is actually maximum at $n = 1$ and decreases with n.

In contrast, in universal data compression problems asymptotic compression rate, the counterpart to asymptotic growth rate, is the quantity of interest since file sizes are linearly, rather than exponentially, related to these rates. Conversely, the counterpart to ending wealth, the exponential of compression rate, does not seem to be a significant quantity in data compression. Thus, while a mathematical relationship between investment and data compression does exist, one must use care in applying intuition developed in the data compression context to the investment context.