Upper Bounds on the Lifetime of Wireless Sensor Networks

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Abstract

In this paper, we ask a fundamental question concerning the limits of energy efficiency of wireless sensor networks - what is the upper bound on the lifetime of a sensor network that collects data from a specified region using a certain number of energy-constrained nodes? The answer to this question is valuable for two main reasons. First, it allows calibration of real world data-gathering protocols and an understanding of factors that prevent these protocols from approaching fundamental limits. Second, the dependence of lifetime on factors like the region of observation, the source behavior within that region, basestation location, number of nodes, radio path loss characteristics, efficiency of node electronics and the energy available on a node, is exposed. This allows architects of sensor networks to focus on factors that have the greatest potential impact on network lifetime. By employing a combination of theory and extensive simulations of constructed networks, we show that in all data gathering scenarios presented, there exist networks which achieve lifetimes equal to or >85% of the derived bounds. Hence, depending on the scenario, our bounds are either tight or near-tight.

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1 Introduction

Rapid commoditization and increasing integration of micro-sensors, digital signal processors and short-range radio electronics on a single node has lead to the idea of distributed, wireless networks that have the potential to collect data more cost effectively, autonomously and robustly compared to a few macro-sensors [1, 2, 3]. Applications of massively distributed sensor networks include seismic, acoustic, medical and intelligence data gathering and climate, equipment monitoring etc. Since these integrated sensor nodes have compact form factors and are wireless, they are highly energy constrained. Furthermore, replenishing energy via replacing batteries on up to tens of thousands of nodes (in possibly harsh terrain) is infeasible. Hence, it is well accepted that one of the key challenges in unlocking the potential of such data gathering sensor networks is conserving energy so as to maximize their post-deployment active sensing lifetime [1].

Any effort targeted at increasing network lifetime must necessarily be two-pronged. Firstly, the node and the physical layer itself must be made as energy efficient as possible [4, 5, 6]. Secondly, the *collaborative strategies* which govern how nodes co-operate to sense data must be energy efficient. Most work in this latter area has been directed towards energy-aware routing [7, 8, 9, 10].

In this paper, our key objective is neither proposing new energy-aware routing heuristics nor new protocols aimed at increasing network lifetime. Instead, it is to explore the fundamental limits of data gathering lifetimes [11]. Our motivation for doing so is several-fold. First, bounds on achievable lifetime of sensor networks allow one to calibrate the performance of collaborative strategies and protocols being proposed regularly. Unlike collaborative strategies, which are mostly heuristic due to the combinatorially explosive nature of the problem, the proposed bounds are crisp and widely applicable. Second, in order to prove that the proposed bounds are tight or near tight, we construct real networks and simulate data gathering and show that their lifetimes often come arbitrarily close to optimal. This exercise gives an insight into near-optimal data gathering strategies if the user has some level of deployment control. Third, in bounding lifetime, we expose its dependence on source behavior, region of observation, basestation location, number of nodes, available initial energy, path loss and radio energy parameters. This allows us to see what factors have the most impact on lifetime and consequently where engineering effort is best expended.

2 Background and Terminology

In this section, we describe the operation of wireless sensor networks in more detail, characterize source behavior, present node energy models and define the lifetime of a network and the lifetime bound problem.

2.1 Data Gathering Sensor Networks

The goal of a sensor network is to gather information from a specified region of observation, say $\mathcal{R}$, and relay this information to an energy unconstrained *basestation*, B (figure 1).

[Figure 1 about here.]

This information originates due to one or more *sources* located in $\mathcal{R}$. At any given instant, nodes in a sensor network can be classified as *live* or *dead* depending on whether they have any energy left or not. Live nodes collaborate to ensure that whenever a source resides in $\mathcal{R}$, it is sensed and the resultant data relayed to the basestation. In the collaborative model we assume, live nodes play one of three roles:

- Sensor: The node observes the source via an integrated sensor, digitizes this information, post-processes it and produces data which must be relayed back to the basestation.
- Relay: The node simply forwards the received data onward without any processing.
- Powered down: The node is live but does not participate in either sensing or relaying the data.

Note that nodes can change their roles dynamically with time (although their *locations* are fixed). Hence, a node might be sensing a source for a while, but when the source moves to a different location, this node might act as a relay or power itself down.

2.2 Characterizing Source Behavior

In addition to specifying the region of observation, it is necessary to specify how the source resides in $\mathcal{R}$. Source behavior is seldom known in a deterministic manner at the time the network is deployed. Rather, one must make do with stochastic knowledge of source behavior. In this paper, we use a simple but effective stochastic model - the spatial probability distribution function (p.d.f.) of a source, $l_{source}(x, y)$, with the usual properties,

$$\Pr(\text{Source} \in \mathcal{A}) = \iint_{\mathcal{A}} l_{source}(x, y)\, \mathrm{d}x\, \mathrm{d}y \tag{1}$$

$$\iint_{\mathcal{R}} l_{source}(x, y)\, \mathrm{d}x\, \mathrm{d}y = 1 \tag{2}$$

Note that the spatial p.d.f., $l_{source}(x, y)$, is conditioned on the event that the source is in fact in $\mathcal{R}$. Next, note that sources cannot be observed beyond certain distances. In this paper, we assume circularly observable sources with a radius of observation equal to d_S i.e. only nodes less than distance d_S away can observe the source. Finally, we assume that observing a source entails relaying data at a certain *source rate*, r measured in bits per second, that originiates at the sensing node.

2.3 Modelling Node Energy Behavior

While there is a plurality of wireless sensor node implementations [2, 12, 13], the overall architecture is identical. Every node has a sensor, analog preconditioning and data conversion circuitry, general purpose and digital signal processors, and a radio link. Since we are dealing with nodes that are either sensors or relays, the key energy parameters are the energy needed to sense a bit (E_{sense}), receive a bit (E_{rx}) and transmit a bit over a distance d (E_{tx}). Assuming a $1/d^n$ path loss model [14], these take the form,

$$E_{tx} = \alpha_{11} + \alpha_2 d^n \tag{3}$$

$$E_{rx} = \alpha_{12} \tag{4}$$

$$E_{sense} = \alpha_3 \tag{5}$$

where α_{11} is the energy/bit consumed by the transmitter electronics (including energy costs of finite startup times), α_2 accounts for energy dissipated in the transmit op-amp (including op-amp inefficiencies), α_{12} is the energy/bit consumed by the receiver electronics and α_3 is the energy cost of sensing a bit. Hence, the energy consumed per second (i.e. power) by a node acting as a relay that receives data and then transmits it d meters onward is,

$$\begin{aligned} P_{relay}(d) &= (\alpha_{11} + \alpha_2 d^n + \alpha_{12})r \\ &\equiv (\alpha_1 + \alpha_2 d^n)r \end{aligned} \qquad (6)$$

where r is the number of bits relayed per second. Typical values for radios are $\alpha_1 = 180$nJ/bit and $\alpha_2 = 10$pJ/bit/m^2 (n=2) or 0.001pJ/bit/m^4 (n=4) [15].

2.4 Defining Lifetime

There are several possible definitions of the lifetime of a sensor network, each suitable in a different context. Before we present those, note that for our purpose, a network is always in one of three states:

Active There is a source present in $\mathcal{R}$ and the network is obeying its contract (i.e. the source is being sensed and the data relayed back to the basestation).

Failure There is a source present in $\mathcal{R}$ but network is not obeying its contract.

Dormant No source present in the region.

In non-mission-critical applications, a reasonable definition of lifetime is the cumulative active time of the network (i.e. whenever the network is active its lifetime clock is ticking, otherwise not). In mission-critical applications, lifetime is defined as the cumulative active time of the network *until the first failure*. In this paper, we adopt this latter definition of lifetime. Note that active lifetime is different from *physical* lifetime. For instance a sensor network deployed to detect tank intrusion can "live on" *ad infinitum* (ignoring battery and physical degradation etc.) in the absence of activity. But it can only actively detect, say, 1000 hours of tank intrusion.

2.5 Factors Affecting Lifetime

Factors affecting lifetime of energy limited systems can be methodically listed down by invoking the theory of power-aware systems developed in [16, 17]. In the context of sensor networks, lifetime depends on the following factors and their variations (with time and possibly space):

- **Inputs and/or input statistics**: This dimension includes factors like the topology of the region to be sensed, the topology of the network, number of sources and their characteristics etc. For instance, the lifetime of a network sensing five sources will generally be very different from the lifetime if it was sensing just one. Similarly, the lifetime of a sensor network detecting tank intrusion is dependent on the locomotive behavior of the tank (i.e. does it breach the perimeter uniformly or is it more likely to breach certain intervals more than others?). In this paper, we capture the topology of the region being sensed and the source location behavior (via the spatial p.d.f.). While we formulate our bounds for the case of a single source and its location p.d.f., the straightforward generalization to multiple sources with their respective p.d.f.s will be apparent to the reader.

- **Desired output quality**: As one would expect, higher quality sensing leads to shorter lifetimes. The two-step approach we take in this paper to quantitatively link desired quality and lifetime is as follows. In the first step, desired quality is used to derive the minimum rate needed and the minimum number of sensors that must observe the source[1]. The details of this step merit a paper on their own! It suffices to say that rate-distortion based information theoretic arguments provide a fundamental characterization of the quality-rate tradeoff [1]. Consider the tank intrusion application again. Once the user specifies a particular temporal and spatial resolution (i.e. locate the tank to within 2 meters every 10 seconds), information theoretic arguments provide reasonable lower bounds on the rate of the stream needed (i.e. the network must support a rate of 10 kbps for the above resolution). Linking the desired quality to the minimum number of sensors needed usually draws on stochastic signal processing formalisms in the multi-observer context. For instance, if we assume that environmental noise is spatially uncorrelated, using three sensors and simple delay-and-sum beamforming will increase the SNR (and loosely speaking quality) by three times [18]. In this paper, we allow explicit incorporation of the rate expected from the network. Also, the derivation of the bound generalizes easily to the case when more than one node is required to sense.

- **Tolerable latency**: In a well designed network, higher tolerable la-

[1]As one would expect, these two factors are not totally unrelated.

tency is generally exploited to yield higher lifetimes. The key mechanism enabling this latency-lifetime tradeoff is scaling the voltage of node electronics [19, 20]. Specifically, power consumption varies as the square of the voltage and delay varies in an inverse manner with voltage. Hence higher delays permit lower energy consumption. In this paper, we do not address tolerable latency explicitly. Rather, it is factored in implicitly via the node energy models.

- **The ambient environment**: The ambient temperature, noise characteristics, behavior of the wireless channel all affect the cost of computation and communication in a sensor network. These costs are captured by our node energy models.

- **The state of the network**: Networks with different initial states exhibit different lifetimes and our approach takes this into account.

We are now ready to state the problem of bounding lifetime in sensor networks.

The Lifetime Bound Problem: Given the region of observation ($\mathcal{R}$), the source radius of observability (d_S), the node energy parameters (α_1, α_{11}, α_{12}, α_2, α_3 and n), the number of nodes deployed (N), the initial energy in each node (E), what is the upper bound on the active lifetime (t) of *any* network established using these nodes which gathers data from a source residing in $\mathcal{R}$ with spatial location behavior $l_{source}(x, y)$.

We will first attack the simpler problem of transmitting a bit over distance D while minimizing the overall energy consumed (section 3) and then use the results to derive network lifetime bounds (section 4).

3 Characteristic Distance and Minimum Energy Relays

A recurring theme in bounding lifetimes of data gathering networks is the problem of establishing a data link with a certain rate r between a radio transmitter (at A) and a receiver (at B) separated by D meters. There are several ways of doing this. One can transmit directly from A to B or one can use several intervening nodes acting as *relays* so as to prevent any node from having to spend too much transmit energy (figure 2).

[Figure 2 about here.]

Definition 1 (Minimum Energy Relay). *We refer to the scheme that transports data between two nodes such that the sum of the rate of energy dissipation over all nodes is minimized as a* minimum energy relay.

If we introduce $K-1$ relays between A and B (figure 2), then the overall rate of dissipation is given by,

$$P_{link}(D) = \left(-\alpha_{12} + \sum_{i=1}^{K} P_{relay}(d_i) \right) r \tag{7}$$

Often we omit the rate term (r) and it is understood that the expression has been written for unit rate i.e. $r = 1$. The $-\alpha_{12}$ term accounts for the fact that the node at A need not spend any energy receiving. We disregard the receive energy needed at B mainly because in most applications of minimum energy relays, B corresponds to the energy unconstrained basestation. Even if B was not a basestation, its receive energy is independent of the location and number of intervening relays anyway and hence we can safely ignore it. We now present some simple properties of minimum energy relays.

Lemma 2. *Minimum energy relays have all nodes collinear and no directed link has a negative projection on vector* $\overrightarrow{AB}$.

Proof. Consider an arrangement in which either of these statements does not hold. Then, carry out the following two step transformation (figure 3):

- Step I: Move every node to its projection on AB.
- Step II: Create a link in the direction AB between every two adjacent nodes.

We claim that the overall power dissipation in the transformed network is no more than that in the original one. This is obvious since the projection of a vector cannot exceed the length of a vector. The result follows. □

[Figure 3 about here.]

Hence, given $K-1$ intervening nodes, the problem is merely one of placing these nodes along the line AB to minimize overall energy. The following lemma tells us how.

Lemma 3. *Given D and the number of intervening relays $(K-1)$, $P_{link}(D)$ is minimized when all the hop distances (i.e. $d_i s$) are made equal to $\frac{D}{K}$. This result holds for all radios with convex power versus distance curves i.e. whose energy per bit is a convex function of the distance over which the bit is transmitted.*

Proof. Recall Jensen's inequality for convex functions,

$$\forall \{\lambda_i\}, \lambda_i \in \mathbb{R}^+ \text{ such that } \sum_i \lambda_i = 1$$

$$f\left(\sum_i \lambda_i x_i\right) \le \sum_i \lambda_i f(x_i)$$

with equality if and only if all the x_is are equal. Since $P_{relay}(d) = \alpha_1 + \alpha_2 d^n$ is strictly convex, it follows that,

$$P_{relay}\left(\frac{d_1 + d_2 + \cdots + d_K}{K}\right) \le \frac{P_{relay}(d_1) + P_{relay}(d_1) + \cdots + P_{relay}(d_K)}{K}$$

$$KP_{relay}\left(\frac{D}{K}\right) \le P_{relay}(d_1) + P_{relay}(d_1) + \cdots + P_{relay}(d_K)$$

$$KP_{relay}\left(\frac{D}{K}\right) - \alpha_{12} \le P_{link}(D) \tag{8}$$

with equality if and only if all the d_is are equal (to D/K). The result follows. □

Corollary 4. *The minimum energy relay for a given distance D has either no intervening hops or some number of equidistant hops.*

It is instructive to point out that while making hops equidistant seems like an obvious thing to do, it works only due to the convexity of the radio's power-distance curve. This is illustrated in figure 4. The radio in 4(a) does not have a uniform path loss characteristic. Like all practical radios, its path loss index n increases with distance [15]. Since a "piecewise convex function" retains its convexity, the radio curve stays convex in this case. It follows that minimum energy relays using this radio must have equidistant hops. However, extremely coarse granularity of power control in a radio can lead to loss of covexity as illustrated in figure 4(b). For radios like this, equidistant spacing does *not* lead to the lowest energy solution.

[Figure 4 about here.]

We have proved that the hops must be equidistant for convex radios. We now derive the relation between the optimal number of hops and D.

Lemma 5. *The optimal number of hops (K_{opt}) is always one of,*

$$K_{opt} = \left\lfloor \frac{D}{d_{char}} \right\rfloor \quad or \quad \left\lceil \frac{D}{d_{char}} \right\rceil \tag{9}$$

where the distance d_{char}, called the characteristic distance, *is* independent of D *and is given by,*

$$d_{char} = \sqrt[n]{\frac{\alpha_1}{\alpha_2(n-1)}} \tag{10}$$

Proof. From (8), we have, for equidistant hops,

$$P_{link}(D) = KP_{relay}\left(\frac{D}{K}\right) - \alpha_{12}$$

Let us optimize $P_{link}(D)$ w.r.t K treating K as a real instead of a natural i.e. assuming $K \in \mathbb{R}^+$. Then, by taking the partial derivative of $P_{link}(D)$ w.r.t K and setting it to zero, we get,

$$K_{opt(continuous)} = \frac{D}{d_{char}} \tag{11}$$

with d_{char} given by (10). Next, note that if $f(x)$ is strictly convex then, $xf(a/x)$ is convex too[2]. This implies that $KP_{relay}\left(\frac{D}{K}\right)$ is convex in K. Hence, once we calculate the minima for $K \in \mathbb{R}^+$ as given by (11), the minima for $K \in \mathbb{N}$ is given by (9). □

Corollary 6. *The power needed to relay a stream with unit rate over distance D can be bounded thus:*

$$P_{link}(D) \geq \alpha_1 \frac{n}{n-1} \frac{D}{d_{char}} - \alpha_{12} \tag{12}$$

with equality if and only if D is an integral multiple of d_{char}.

[2]Quick proof: The second derivative of $xf(a/x)$ w.r.t. x is $(a^2/x^3)f''(a/x)$. But f is convex, which implies $f''(a/x)$ is positive. Hence, $(a^2/x^3)f''(a/x)$ is strictly positive for positive a, x, proving the convexity of $xf(a/x)$.

Proof. From the lemma above, it follows that,

$$
\begin{aligned}
P_{link}(D) &\geq K_{opt(continuous)} P_{relay}\left(\frac{D}{K_{opt(continuous)}}\right) - \alpha_{12} \\
&= \frac{D}{d_{char}}\left(\alpha_1 + \alpha_2 d_{char}^n\right) - \alpha_{12} \\
&= \frac{D}{d_{char}}\left(\alpha_1 + \alpha_2 \frac{\alpha_1}{\alpha_2(n-1)}\right) - \alpha_{12} \\
&= \alpha_1 \frac{n}{n-1}\frac{D}{d_{char}} - \alpha_{12}
\end{aligned}
$$

with equality in the first step if and only if K_{opt} is the same as $K_{opt(continuous)}$ or equivalently from (9) when D/d_{char} is integral. □

The corollary above makes several important points:

- For any loss index n, the energy costs of transmitting a bit can always be made *linear* with distance.
- For any given distance D, there is a certain optimal number of intervening nodes acting as relays that must be used ($K_{opt} - 1$). Using more or less than this optimal number leads to energy inefficiencies.
- The most energy efficient relays result when D is an integral multiple of the characteristic distance (d_{char}).

4 Bounding Lifetime

We are now ready to derive upper bounds on the lifetime of sensor networks for a variety of source behavior.

4.1 Activity Fixed at a Point

The simplest sensor network is one that harvests data from a fixed source located at a distance d_B away from the basestation (figure 5).

[Figure 5 about here.]

It is easy to see that we have to establish a link of length equal to at least $d = d_B - d_S$ and sustain the source rate (say r) over this link. If we denote the energy dissipation in the *entire* network by $P_{network}$ then it follows from our discussion on minimum energy relays that,

$$\begin{aligned} P_{network} &\geq P_{link}(d) + P_{sensing} \\ &\geq \left(\alpha_1 \frac{n}{n-1} \frac{d}{d_{char}} - \alpha_{12}\right) r + \alpha_3 r \end{aligned} \tag{13}$$

Clearly, achieving a lifetime of say, t_{point}, demands that the total energy consumed be no greater than the total energy available at the start, i.e.,

$$t_{point} P_{network} \leq \sum_{i=1}^{N} e_i$$

which reduces to,

$$t_{point} \leq \frac{N.E}{\left(\alpha_1 \frac{n}{n-1} \frac{d}{d_{char}} - \alpha_{12} + \alpha_3\right) r} \tag{14}$$

for the case of a N node network with e_i (i.e. the energy of node i at deployment) set to E. While the bound in (14) is exact, we often use the following approximation noting that in most networks of practical significance, the cost of relaying data dominates,

$$t_{point} \leq t_{point_{max}} = \frac{N.E}{\alpha_1 \frac{n}{n-1} \frac{d}{d_{char}} r} \tag{15}$$

One can eliminate d_{char} in (15) to obtain,

$$t_{point_{max}} = \frac{N.E}{\frac{n}{n-1} \sqrt[n]{\alpha_1^{n-1} \alpha_2 (n-1)} (d_B - d_S) r} \tag{16}$$

which simplifies to,

$$t_{point_{max}} = \frac{N.E}{2\sqrt{\alpha_1 \alpha_2} (d_B - d_S) r} \tag{17}$$

for the simplest path loss model ($n = 2$). To put this in perspective, the above bound tells us that a 1000 node network has the potential to listen to human conversation a kilometer away for about 128 hours if every node started out with a mere 2 J of energy (using the typical energy parameters mentioned in section 2.3 and assuming that the speech is only compressed to 16 kbps).

Proposition 7. *The bound in (15) is tight when* $d = d_B - d_S$ *is an integral multiple of* d_{char} *and* N *is an integral multiple of* $\frac{d}{d_{char}}$ *i.e. with these conditions satisfied, there exist networks whose lifetime equals the upper bound.*

Proof. We present a proof by construction. Let $d_B - d_S = d = M d_{char}$, $M \in \mathbb{N}$, and $N = PM$, $P \in \mathbb{N}$. Then form P parallel "backbones" of $M = \frac{d_B - d_S}{d_{char}}$ nodes as shown in figure 6 and use exactly one backbone whenever the network is active.

[Figure 6 about here.]

Each node in any given backbone dissipates $(\alpha_1 + \alpha_2 d_{char}^n)r$ when active which evaluates to $\frac{n}{n-1}\alpha_1 r$. Hence the lifetime achieved by a backbone is $\frac{E}{\frac{n}{n-1}\alpha_1 r}$. The overall lifetime is simply $P = \frac{N}{M} = \frac{N}{\frac{d_B - d_S}{d_{char}}}$ times this lifetime and thus given by $\frac{N}{\frac{d_B - d_S}{d_{char}}} \frac{E}{\frac{n}{n-1}\alpha_1 r}$ which is the same as the bound derived in (17). □

To experimentally validate the derived bounds, a custom network simulator was used to simulate networks that gathered data from a point source using nodes with the energy behavior described earlier. Figure 7 charts the lifetimes achieved by networks with different values of N and d.

[Figure 7 about here.]

As predicted, some networks do achieve a lifetime equal to the bound. The networks used to obtain this data did *not* have random topologies. Rather, the networks and the collaborative strategies were designed with the express intention of defeating the upper bounds. Hence, the tightness apparent in figure 7 should not be interpreted as the lifetime expected of general networks but rather as the *best* possible lifetimes that *some* networks can achieve. Note that any network can achieve an arbitrarily poor lifetime and hence the issue of worst possible lifetime is vacuous.

4.2 Activity Distributed Along a Line

We now consider the case of harvesting information from a source that is located along a line (S_0S_1) of length d_N as shown in figure 8.

[Figure 8 about here.]

The minimal power for sensing the source at a unit rate is[3],

$$P_{network}(x) \geq P_{link}(d(x)) \geq \frac{n}{n-1}\alpha_1 \frac{d(x)}{d_{char}} \tag{18}$$

For the case when the source is located along S_0S_1 with equal probability $\frac{1}{d_N}$, the expected overall rate of dissipation for sensing is,

$$\begin{aligned} P_{network} &\geq \int_{x=d_B}^{x=d_B+d_N} P_{network}(x) l_{source}(x) \mathrm{d}x \\ &\geq \int_{x=d_B}^{x=d_B+d_N} \frac{n}{n-1}\alpha_1 \frac{d(x)}{d_{char}} \frac{1}{d_N} \mathrm{d}x \\ &\geq \frac{n}{n-1}\alpha_1 \frac{d_{linear}}{d_{char}} \end{aligned} \tag{19}$$

where,

$$d_{linear} = \frac{d_1 d_2 - d_3 d_4 + d_W^2 \ln(\frac{d_1+d_2}{d_3+d_4})}{2d_N} - d_S \tag{20}$$

Hence, the bound on the lifetime of a network gathering data from a source that resides on a line with equal probability is,

$$t_{linear\,max} = \frac{N.E}{\frac{n}{n-1}\alpha_1 \frac{d_{linear}}{d_{char}} r} \tag{21}$$

When S_0S_1 passes through the basestation B i.e. $d_W = 0$ (or S_0, S_1 and B are collinear) we have,

$$t_{collinear\,max} = \frac{N.E}{\frac{n}{n-1}\alpha_1 \frac{d_B + \frac{d_N}{2} - d_S}{d_{char}} r} \tag{22}$$

Figure 9 plots the lifetime achieved by *non*-collinear networks gathering data from a source that resides along a line. For each simulated network, the lifetimes have been normalized to the upper bound in (21). Clearly, the bound is near tight.

[3]Recall that we are ignoring the α_3 and $-\alpha_{12}$ term.

[Figure 9 about here.]

Proposition 8. *The bound for the collinear case (22) is tight.*

Proof. Consider the case when,

- d_N is an integral multiple of $2d_S$ i.e. $d_N = M(2d_S), M \in \mathbb{N}$.
- $2d_S$ is an integral multiple of d_{char} i.e. $2d_S = Ld_{char}, L \in \mathbb{N}$.
- $d_B + d_S$ is an integral multiple of d_{char} i.e. $d_B + d_S = Td_{char}, T \in \mathbb{N}$.

[Figure 10 about here.]

Then, we can arrange nodes as shown in figure 10. Note that the key idea is to establish M minimum energy relays. For this we need exactly $\sum_{m=1}^{M}(L(m-1)+T) = M(L(M-1)/2+T)$ nodes. If the number of nodes available is an integral multiple of this number, then we achieve the bound with equality (the proof is an exact analogue of that in proposition (7)). □

4.3 Activity Distributed Over a Rectangular Region

Consider harvesting information from a source that resides in a rectangle (fig. 11).

[Figure 11 about here.]

Assuming $l_{source}(x, y)$ to be uniform, we have,

$$\begin{aligned}
P_{network} &= \iint_R P_{network}(x,y) l_{source}(x,y)\,\mathrm{d}x\,\mathrm{d}y \\
&\geq \int_{x=d_B}^{x=d_B+d_N} \int_{y=-d_W}^{y=d_W} P_{link}(d(x,y)) \frac{1}{2d_W d_N}\,\mathrm{d}x\,\mathrm{d}y \\
&\geq \int_{x=d_B}^{x=d_B+d_N} \int_{y=-d_W}^{y=d_W} \frac{n}{n-1}\alpha_1 r \frac{\sqrt{x^2+y^2}-d_S}{2d_W d_N d_{char}}\,\mathrm{d}x\,\mathrm{d}y \\
&\geq \frac{n}{n-1}\alpha_1 r \frac{d_{rect}}{d_{char}}
\end{aligned} \tag{23}$$

where

$$d_{rect} = -d_S + \frac{1}{12 d_N d_W}\Big[4 d_W (d_1 d_2 - d_3 d_4) + \cdots$$

$$2 d_W^3 \ln(\frac{d_1 + d_2}{d_3 + d_4}) + d_3^3 \ln(\frac{d_4 - d_W}{d_4 + d_W}) + d_1^3 \ln(\frac{d_2 + d_W}{d_2 - d_W})\Big] \tag{24}$$

Hence, the bound on expected lifetime is,

$$t_{rectangle_{max}} = \frac{N.E}{\frac{n}{n-1}\alpha_1 r \frac{d_{rect}}{d_{char}}} \tag{25}$$

This implies for instance, that a 1000 node network where each node started out with 2 J has the potential to detect finite velocity tank intrusions in a kilometer by kilometer area (located a kilometer away from the basestation) for roughly 7 years! Figure 12 plots the lifetime of networks gathering data from sources that reside in rectangles. Once again, the lifetime of each network has been normalized to the bound in (25) and the tightness of the bound is apparent.

[Figure 12 about here.]

4.4 Activity Distributed Over a Sector

Consider a source that resides in a sector as shown in figure 13.

[Figure 13 about here.]

In a manner analogous to earlier derivations, we can show that the expected lifetime is bounded by,

$$\mathcal{L}_{sector} = \frac{N.E}{\frac{n}{n-1}\alpha_1 r \frac{d_{sector}}{d_{char}}} \tag{26}$$

where,

$$d_{sector} = \frac{2\theta d_R^3 - d_B d_R d_W - d_B^3 \ln\left(\frac{d_R + d_W}{d_B}\right)}{3(\theta(d_B^2 + d_W^2) - d_B d_W)} - d_S$$

For the special case of a basestation at the center of a semi-circle (i.e. $d_B = 0, \theta = \pi$) we get,

$$\mathcal{L}_{semi-circle} = \frac{N.E}{\frac{n}{n-1}\alpha_1 r \frac{\frac{2}{3} d_R - d_S}{d_{char}}} \tag{27}$$

Figure 14 compares the lifetime of some semi-circular data gathering networks compared against the upper bound.

[Figure 14 about here.]

5 Bounding Lifetime by Partitioning

The following theorem is useful in deriving lifetime bounds for source regions that can be partitioned into sub-regions for which the bounds are already known, or easier to compute.

Theorem 9. *The lifetime bound* per unit initial energy, $\tau(\mathcal{R})$, *of a network gathering data from a source region $\mathcal{R}$ that can be partitioned into Q disjoint regions $\mathcal{R}_j, j \in [1, Q]$ with their corresponding lifetime bounds (also normalized), $\tau(\mathcal{R}_j)$ is given by,*

$$\tau(\mathcal{R}) = \left(\sum_{j=1}^{Q} \frac{p_j}{\tau(\mathcal{R}_j)} \right)^{-1}$$

where p_j is the probability that a source resides in region $\mathcal{R}_j$.

Proof. First note that the lifetime bound (not normalized) is obtained by dividing the total energy available at the start by the expected rate of dissipation. Hence, we have,

$$t(\mathcal{R}_j) = \frac{E(\mathcal{R}_j)}{P(\mathcal{R}_j)} \quad \Rightarrow \quad \tau(\mathcal{R}_j) = \frac{1}{P(\mathcal{R}_j)}$$

where $E(\mathcal{R}_j)$ denotes the initial energy in region $\mathcal{R}_j$. Our task is simply to express the bound of the entire network in terms of the available parameters, which we do as follows,

$$t(\mathcal{R}) = \frac{E(\mathcal{R})}{P(\mathcal{R})} \Rightarrow \frac{t(\mathcal{R})}{E(\mathcal{R})} = \left(\sum_{j=1}^{Q} p_i P(\mathcal{R}_j) \right)^{-1} \Rightarrow \tau(\mathcal{R}) = \left(\sum_{j=1}^{Q} \frac{p_i}{\tau(\mathcal{R}_j)} \right)^{-1}$$

□

As an illustration, consider a source residing equiprobably along a line with the basestation's (i.e. B's) projection on S_0S_1 lying *within* the segment rather than outside it (fig. 15).

[Figure 15 about here.]

While the lifetime expression derived in section 4.2 (for sources residing on a line) can't be used directly, it can be used via partitioning $\mathcal{R}$ into three regions as shown in figure 15 thus,

$$\frac{d_N}{\tau(\mathcal{R})} = \frac{d_B}{\tau(\mathcal{R}_1)} + \frac{d_B}{\tau(\mathcal{R}_2)} + \frac{d_N - 2d_B}{\tau(\mathcal{R}_3)} \tag{28}$$

Note that $\tau(\mathcal{R}_1)$ ($=\tau(\mathcal{R}_2)$) and $\tau(\mathcal{R}_3)$ can both be obtained using (21).

6 Conclusions

The key challenge in networks of energy constrained wireless integrated sensor nodes is maximizing network lifetime. In this paper, we derived upper bounds on the lifetime of data gathering sensor networks for a variety of scenarios assuming node energy models based on $1/d^n$ path loss behavior. Using both analytical arguments and extensive network simulations, the bounds were shown to be tight for some scenarios and near-tight (better than 85%) for the rest. Lastly, we presented a technique that allows bounding lifetime by partitioning the problem into sub-problems for which the bounds are already known or easier to derive. We hope that the work presented here will enable a rigorous understanding of the fundamental limits of the energy efficiency of wireless sensor networks.

Acknowledgments

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References

[1] G. J. Pottie, "Hierarchical information processing in distributed sensor networks," in *International Symposium on Information Theory*, p. 163, 1998. 1, 1

[2] G. Pottie and W. Kaiser, "Wireless integrated network sensors," *Communications of the ACM*, vol. 43, pp. 51–58, May 2000. 1, 2.3

[3] J. M. Kahn, R. H. Katz, and K. S. J. Pister, "Next century challenges: Mobile networking for smart dust," in *Proceedings of the Fifth Annual ACM/IEEE International Conference on Mobile Computing and Networking (MobiCom '99)*, Aug. 1999. 1

[4] K. Bult *et al.*, "Low power systems for wireless microsensors," in *Proceedings of the International Symposium on Low Power Electronics and Design (ISLPED'96)*, pp. 17–21, Aug. 1996. 1

[5] A. Chandrakasan *et al.*, "Design considerations for distributed microsensor systems," in *Custom Integrated Circuits Conference (CICC)*, pp. 279–286, May 1999. 1

[6] A.-S. Porret, T. Melly, C. C. Enz, and E. A. Vittoz, "A low-power low-voltage transceiver architecture suitable for wireless distributed sensors network," in *Proceedings of the International Symposium on Circuits and Systems (ISCAS 2000)*, vol. 1, (Geneva), pp. 56–59, 2000. 1

[7] S. Singh, M. Woo, and C. Raghavendra, "Power-aware routing in mobile ad hoc networks," in *Proceedings of the Fourth Annual ACM/IEEE International Conference on Mobile Computing and Networking (MobiCom'98)*, pp. 181–190, Oct. 1998. 1

[8] J.-H. Chang and L. Tassiulas, "Energy conserving routing in wireless adhoc networks," in *Proceedings of IEEE INFOCOM 2000*, vol. 1, pp. 22–31, 2000. 1

[9] W. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-Efficient Routing Protocols for Wireless Microsensor Networks," in *Proc. 33rd Hawaii International Conference on System Sciences (HICSS '00)*, Jan. 2000. 1

[10] V. Rodoplu and T. H. Meng, "Minimum energy mobile wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 17, pp. 1333–1344, Aug. 1999. 1

[11] M. Bhardwaj, T. Garnett, and A. Chandrakasan, "Upper bounds on the lifetime of sensor networks," in *Proceedings of the IEEE International Conference on Communication (ICC'01)*, (Helsinki, Finland), June 2001. 1

[12] J. M. Rabaey, M. J. Ammer, J. L. da Silva, Jr., D. Patel, and S. Roundy, "Picoradio supports ad hoc ultra-low power wireless networking," *Computer*, vol. 33, pp. 42–48, July 2000. 2.3

[13] R. Min *et al.*, "An architecture for a power-aware distributed microsensor network," in *Proceedings of the IEEE Workshop on Signal Processing Systems:Design and Implementation (SiPS'00)*, 2000. 2.3

[14] T. Rappaport, *Wireless Communications: Principles & Practice*. New Jersey: Prentice-Hall, Inc., 1996. 2.3

[15] W. Heinzelman, *Application-Specific Protocol Architectures for Wireless Networks*. PhD thesis, Massachusetts Institute of Technology, 2000. 2.3, 3

[16] M. Bhardwaj, R. Min, and A. Chandrakasan, "Power-aware systems," in *Proceedings of the 34th Asilomar Conference on Signals, Systems, and Computers*, Nov. 2000. 2.5

[17] M. Bhardwaj, R. Min, and A. Chandrakasan, "Quantifying and Enhancing Power-Awareness in VLSI Systems," *IEEE Transactions on Very Large Scale Integration (VLSI) Systems (To appear)*, 2001. 2.5

[18] K. Yao, C. Reed, R. Hudson, and F. Lorenzelli, "Beamforming performance of a randomly distributed sensor array system," in *Workshop on Signal Processing Systems (SIPS 97 - Design and Implementation)*, pp. 438–447, 1997. 1

[19] R. Min, T. Furrer, and A. Chandrakasan, "Dynamic voltage scaling techniques for distributed microsensor networks," in *Proceedings of the IEEE Computer Society Workshop on VLSI*, pp. 43–46, 2000. 1

[20] A. Wang and A. Chandrakasan, "Energy efficient system partitioning for distributed wireless sensor networks," in *Proceedings of the 2001 International Conference on Acoustics, Speech and Signal Processing (ICASSP'01), (To appear)*, 2001. 1

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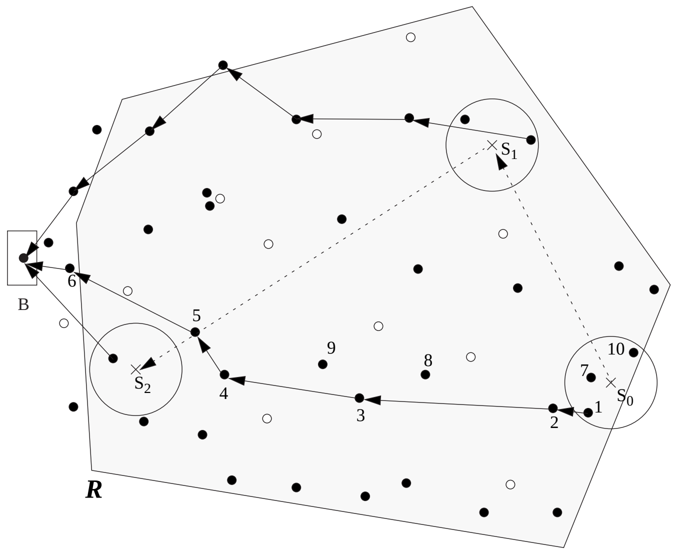


Figure 1: A sensor network gathering data from a circularly observable source (denoted by a $\times$) residing in the shaded region $\mathcal{R}$. Live nodes are denoted by $\bullet$ and dead ones by $\circ$. The basestation is marked B. When the source is at S_0, node 1 acts as the sensor and nodes $2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$ form the relay path. This is not the only possible role assignment that allows the source to be sensed. For instance, node 7 could act as the sensor and nodes $1 \rightarrow 2 \rightarrow 8 \rightarrow 9 \rightarrow 5$ could form the relay path. While nodes 1, 7 and 10 can all sense simultaneously, we assume in this example that only one sensor needs to observe the source. Finally, note how the sensor and relay paths must change as the source moves from S_0 to S_1 to S_2.

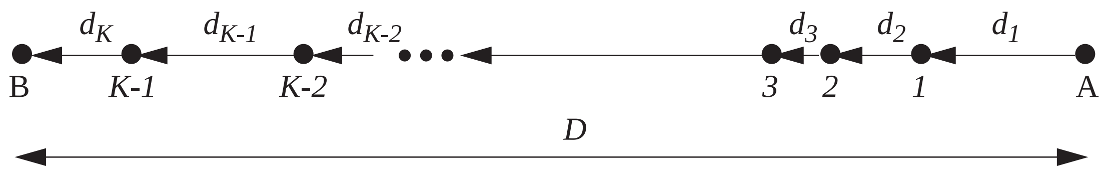


Figure 2: Introducing $K-1$ relay nodes between A and B to reduce energy needed to transmit a bit.

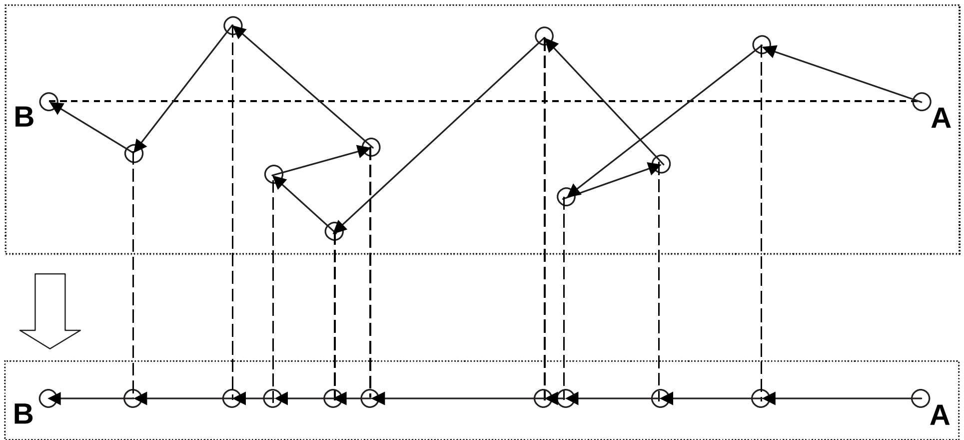
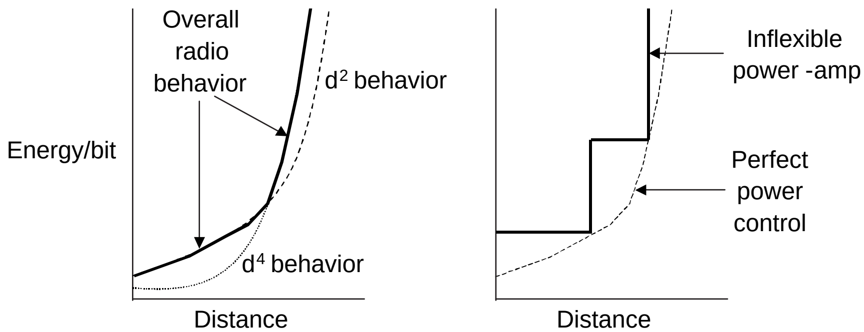


Figure 3: Every non-collinear network with links that have negative projections along AB can be transformed into a more efficient collinear network with links only in the AB direction.



(a) A Convex Radio Curve

(b) A Non-Convex Radio Curve

Figure 4: Two deviations from ideal radios are shown here. The radio on the left retains a convex energy-distance curve while the radio on the right does not. Hence, while equidistant hops are optimal for the radio on the left, they are rarely optimal for the radio on the right.

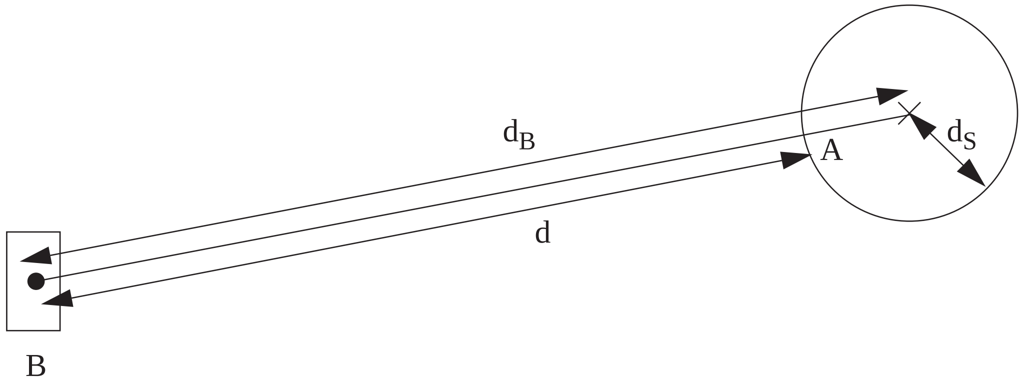


Figure 5: Gathering data from a circularly observable, fixed point source d_B away from the basestation (B). The source location is marked by $\times$.

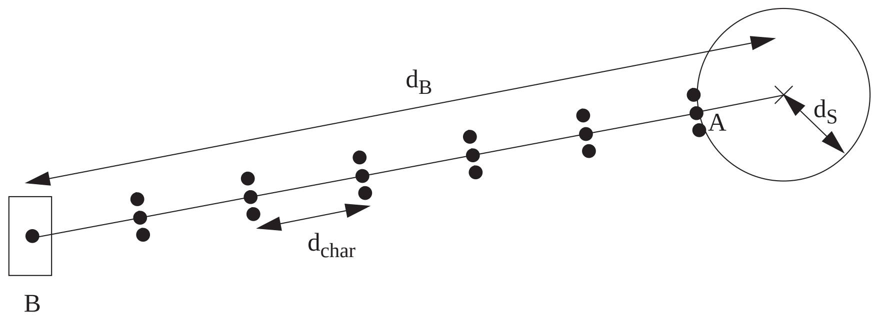


Figure 6: An example network that achieves the upper bound on lifetime. Here, $d_B - d_S = 6d_{char}$ i.e. $M = 6$ and $N = 18 = 3(6)$ i.e. $P = 3$. Thus there are 3 parallel minimum-energy backbones of 6 nodes each.

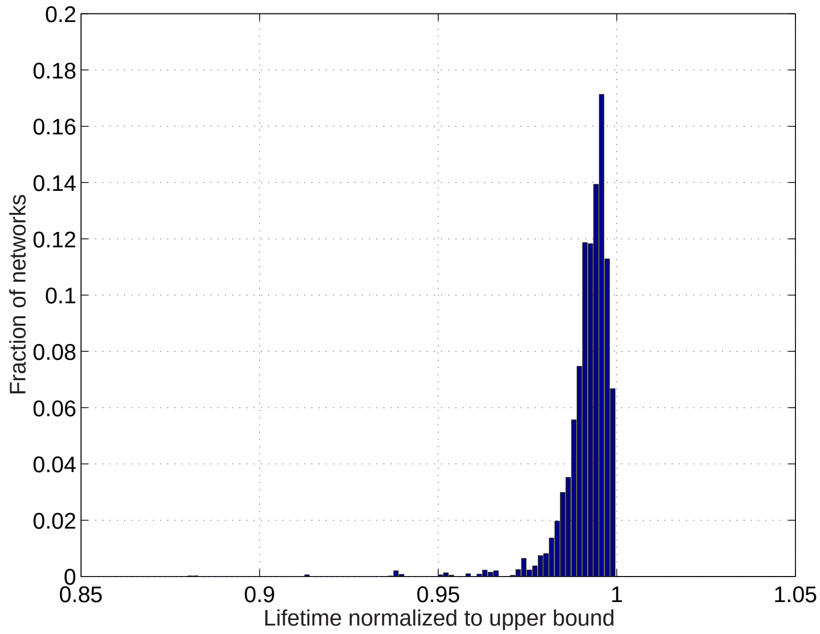


Figure 7: Observed lifetimes of >50,000 actually constructed networks that gather data from a fixed point source. For each network, the lifetime was determined via simulation and then normalized to the upper bound in (15). These networks had $400 \geq N \geq 100$ and $20d_{char} \geq d_B - d_S \geq 0.1d_{char}$.

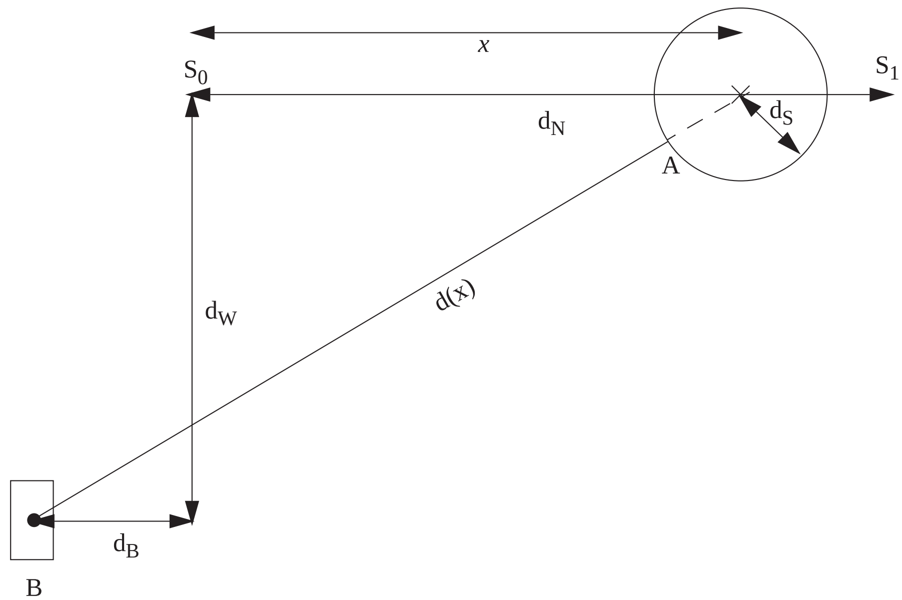


Figure 8: Gathering data from a source that resides on a line (S_0S_1). We use $d_1 = d_B + d_N$, $d_2 = BS_1$, $d_3 = d_B$ and $d_4 = BS_0$ in (20) to make (21) easier to read.

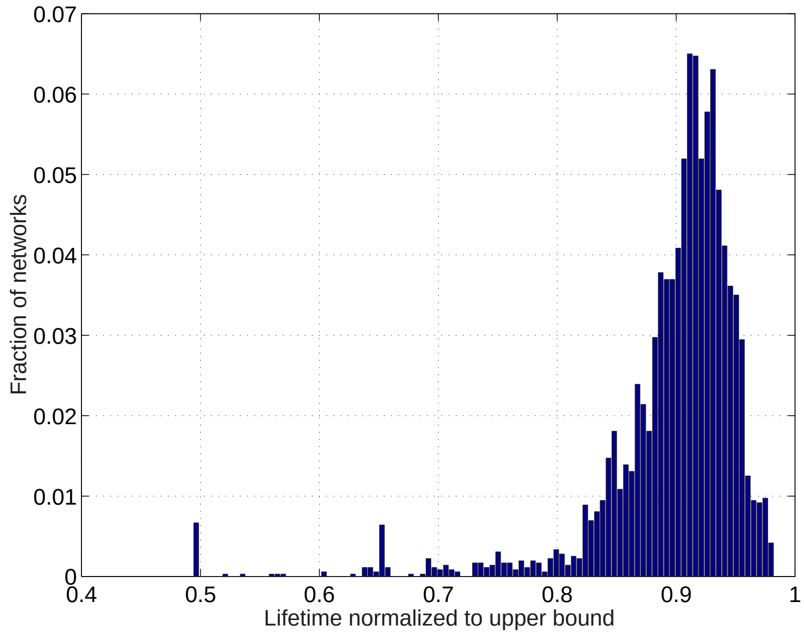


Figure 9: Observed lifetimes of ≈3500 networks that gather data from a source residing on a line ($400 \geq N \geq 100$, $d_B \leq 10d_{char}$, $d_W \leq 7.5d_{char}$, $11d_{char} \geq d_N \geq d_{char}$, $1.5d_{char} \geq d_S \geq 0.25d_{char}$).

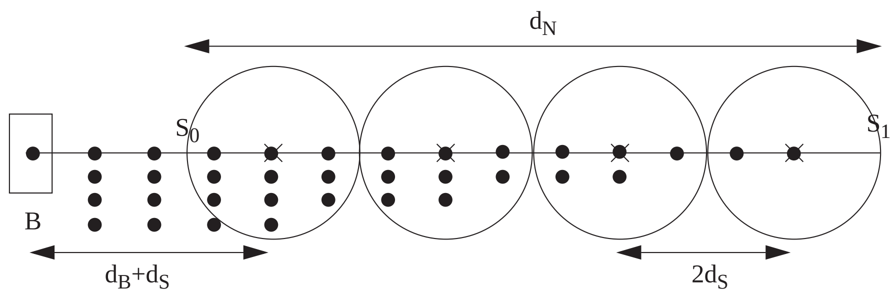


Figure 10: An example network that achieves the upper bound on lifetime. Here, $d_N = 4(2d_S)$ i.e. $M = 4$, $2d_S = 3d_{char}$ i.e. $L = 3$, $d_B + d_S = 4d_{char}$ i.e. $T = 4$. We establish $M = 4$ minimum energy relays consisting of 4,7,10 and 13 nodes respectively.

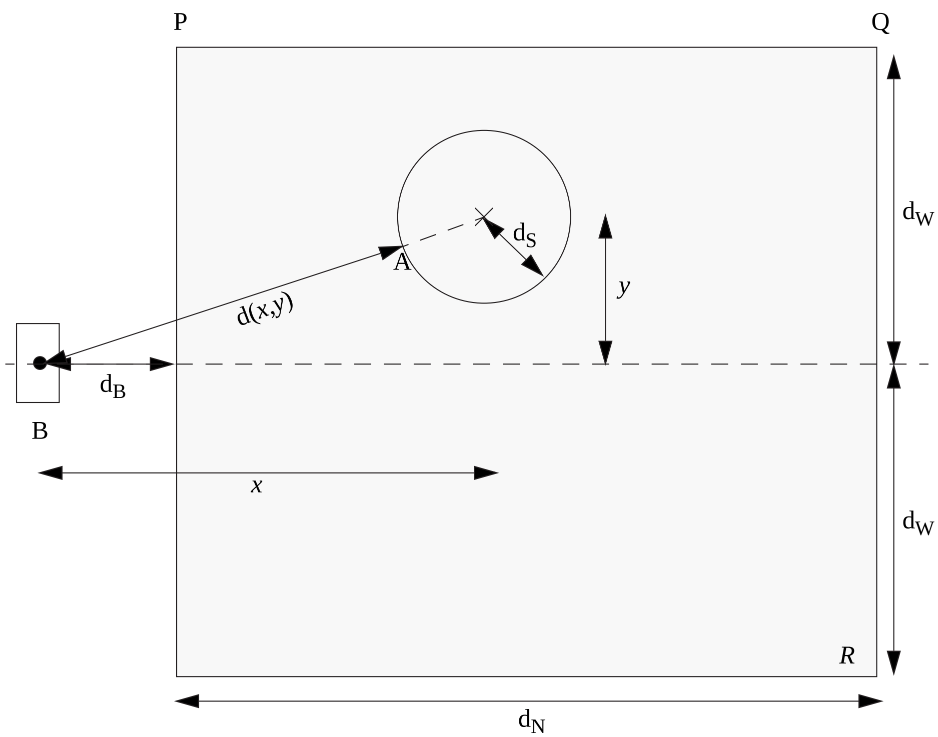


Figure 11: Gathering data from a source that resides in a $2d_w$ by d_N rectangle that is d_B away from the basestation (B). We use $d_1 = d_B + d_N$, $d_2 = BQ$, $d_3 = d_B$ and $d_4 = BP$ in (24) to make (23) easier to read.

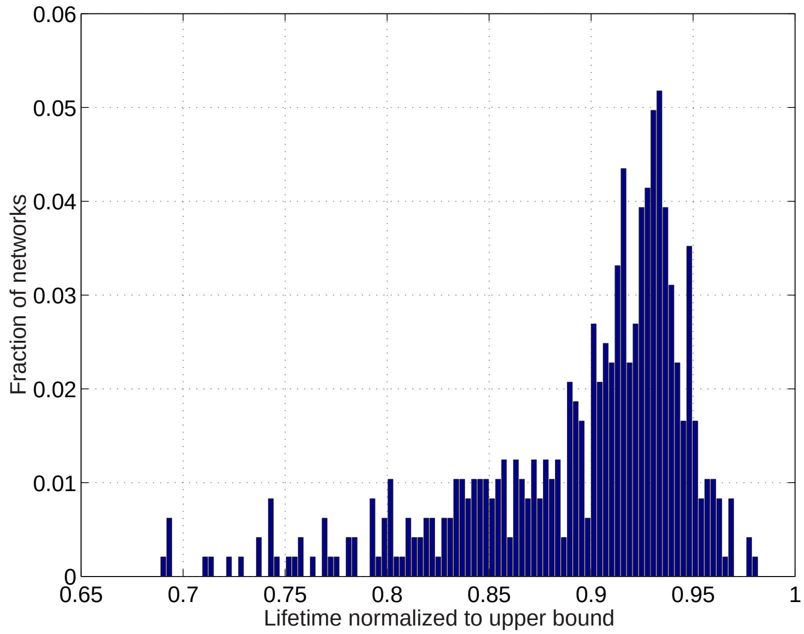


Figure 12: Observed lifetimes of ≈500 networks that gather data from sources residing in rectangles ($1000 \geq N \geq 100$, $d_B \leq 10d_{char}$, $6.75d_{char} \geq 2d_W \geq 0.5d_{char}$, $13.5d_{char} \geq d_N \geq d_{char}$, $1.5d_{char} \geq d_S \geq 0.25d_{char}$).

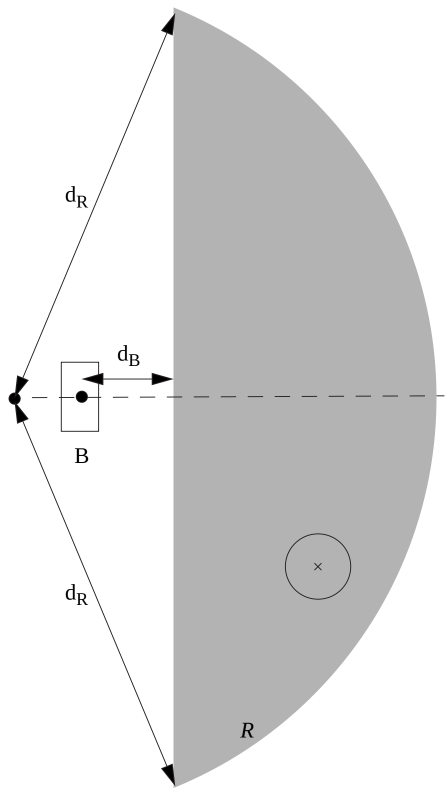


Figure 13: Gathering data from a source that resides in a sector (subtending angle 2θ at the centre) d_B away from the basestation (B).

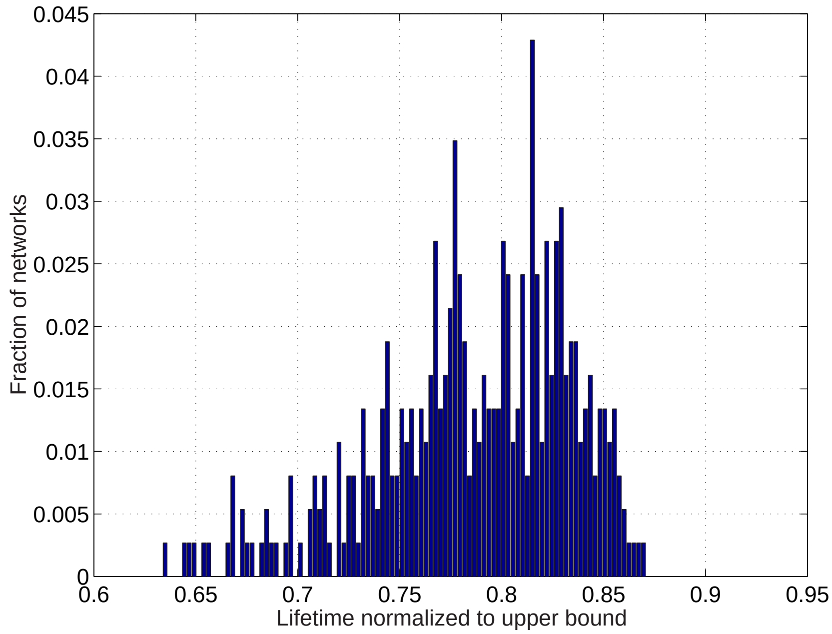


Figure 14: Observed lifetimes of ≈350 networks that gather data from sources residing in semi-circles ($1000 \geq N \geq 100$, $d_B = 0$, $10d_{char} \geq d_R \geq 1.25d_{char}$, $2d_{char} \geq d_S \geq 0.2d_{char}$).

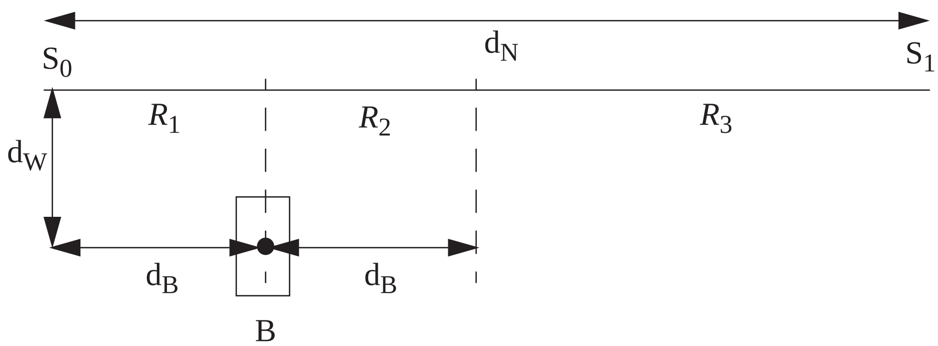


Figure 15: Bounding lifetime when source moves along a line, but with basestation "between" S_0S_1.