6.962 Week 12 Summary of Discussion

Presenter: Michael Lopez
Date: Dec. 6, 2000
Topic: Multiuser Detection

1 Introduction

Last week's discussion focused on equalization of communication channels in the presence of both additive noise and channel dispersion. When two or more users transmit different signals, however, the interference that they cause on each other becomes structured noise which should be dealt with in a different manner.

Traditionally, it was believed that taking advantage of the structure of this interference in the equalizer would either not significantly increase performance or would be computationally prohibitive. Interference was either avoided, by transmitting different messages along orthogonal waveforms, or was treated as unstructured at the receiver. Recently, with the growth of wireless communication, commercial incentives to increase the number of users and bitrates, and the rise in available computation power, there has been widespread interest in improving upon this.

The main concept behind multiuser detection is very simple, and optimal detection is often just an extension of known single-user techniques. In digital communications, maximum likelihood equalizers take advantage of the fact that the transmitted symbol sequence must come from among a finite number of possiblities. When two or more signals are transmitted simultaneously, this just increases the number of possibilities that must be considered. Joint detection involves choosing from among this larger number of possiblities. An early and often-cited paper on multiuser detection [8] presented just this for the case where the users transmit asynchronously, *i.e.,* with different timing offsets. Considerable gains were shown over the "conventional" detector which treats the interference as unstructured noise. The complexity of joint detection, however, generally increases exponentially with the number of users, so finding reduced-complexity techniques becomes crucial.

Verdu's book [9] provides a good survey of results in multiuser detection and includes extensive references.

2 Synchronous Discrete-time Model

Most multiuser detection results are written in the context of direct sequence code division multiple access (DS-CDMA). We will therefore also use this vocabulary, but it is worth noting that the basic concepts apply to any system which uses multiple dimensions or "degrees of freedom," such as multiple antennas at the receiver.

Suppose K users each generate one bit, $b_k, k = 1, 2, \ldots, K$. (Most results generalize easily to other input alphabets.) Also assume that the users are perfectly synchronized and that there is no channel dispersion, so that no optimality is lost by considering only one symbol duration at a time.

Each user, instead of sending the bit b_k over the entire symbol duration, breaks up the symbol duration into N smaller segments, called chips, and sends the sequence $b_k \left[s_k[1]\ s_k[2]\ \ldots\ s_k[N]\right]^T = b_k \boldsymbol{s}_k$. The sequence $\boldsymbol{s}_k$ is called the signature sequence for user k. What is received during this symbol interval, then, is a superposition of all K users' sequences, each received with amplitude a_k, and white Gaussian noise,

$$y[n] = \sum_{k=1}^{K} a_k b_k s_k[n] + w_n, \qquad n = 1, 2, \ldots, N,$$

or in vector form,

$$\boldsymbol{y} = SA\boldsymbol{b} + \boldsymbol{w},$$

where the $\boldsymbol{s}_k$ is the kth column of S and A is diagonal with elements a_k. Some authors instead consider the matched filter outputs, $\hat{\boldsymbol{y}} = S^T \boldsymbol{y}$. Either method provides sufficient statistics for optimal detection.

If N, the "processing gain," is at least as large as the number of users, K, then all the $\boldsymbol{s}_k$'s could in principle be designed to be orthogonal, and optimal detection would degenerate to single-user detection. However, this would require much coordination, and places a cap on the total number of simultaneous users. In addition, continuous-time effects such as timing offsets and channel dispersion can sometimes be modeled as deviations from orthogonality. In fact, somewhat non-orthogonal codes are often chosen in practice because the cross correlations remain low under timing offsets. Many researchers even assume the entries of S to be $i.i.d$ random variables and have produced very interesting results [5, 10].

The signal amplitudes a_k for the different users play an important role in detection. These can result from different transmission strengths, different distances to from the receiver, or from (flat) fading. One popular term in the literature is the "*near-far* problem," when a strong interferer can drown out a weaker user's signal. Two of the linear detectors discussed in the next section attempt to compensate for differing signal strengths, while the successive interference cancellor in Section 4 tries to take advantage of the fact that some users are more reliable than others.

3 Linear Detectors

Had there been only one user and $N = 1$, then the optimal (maximum likelihood) detector would simply be a slicer: make a decision based on the sign of y. For $N > 1$, we just preceed the slicer with a matched filter to the signature $\boldsymbol{s}$.

In the more general model, there are 2^K possible $\boldsymbol{b}$ vectors that could have been sent. Given a received sequence $\boldsymbol{y}$, the optimal detector would construct all 2^K possible constellation points $S\boldsymbol{b}$ and then do a minimum distance search from $\boldsymbol{y}$.

One would like the decision device to be as simple as the single-user detector rather than involve an exponential search. One extension from the single-user receiver is to preceed each slicer with some linear combination of the elements of $\boldsymbol{y}$ (not necessary a matched filter). If we consider all K decisions together, this involves multiplying $\boldsymbol{y}$ by a $K \times N$ matrix H, as shown in Fig. 1. The filters most often discussed in the literature are:

- "Conventional": Matched filter, $H = S^T$. Ignores the interference in the design of the filter.

- Decorrelator [2]: If the columns of $\boldsymbol{s}_k$ are linearly independent, then one can completely remove multiaccess intereference. If $K = N$, for

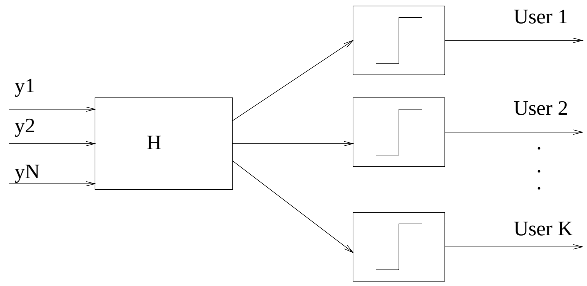

Figure 1: Detector with linear front end.

instance, then $H = S^{-1}$.

- MMSE-linear [3]: Choose the filter which will have minimum variance going into the slicer, taking into account both noise and interference.

The conventional linear receiver will have serious problems when number of users is large or in the *near-far* scenario. The decorrelator's performance does not depend on the powers of interferers and therefore is better in *near-far* scenarios, but it ignores the effect of noise enhancement and does not even exist when $K > N$. The MMSE-linear detector finds the optimal compromise, given only second-order statistics, between boosting signal energy and reducing interference. These three linear receivers are compared for large systems (where K and N both grow large, but according to a given ratio) in fading channels in [5] and [10].

4 Interference Cancellors

The linear detector for user k can be thought of as projecting $\boldsymbol{y}$ onto some direction and then slicing. Along with removing interference, the projection, however, can also remove useful signal energy. To avoid this, a "genie" would tell the receiver the values of all the other users' symbols, and then user k could simply subtract off their effects.

Of course, the actual symbol values are not available, but estimates are available from the linear detectors. One could therefore follow the linear detector with a second stage in which interference is cancelled, assuming that the other users' estimates from the first stage are correct. This is sometimes

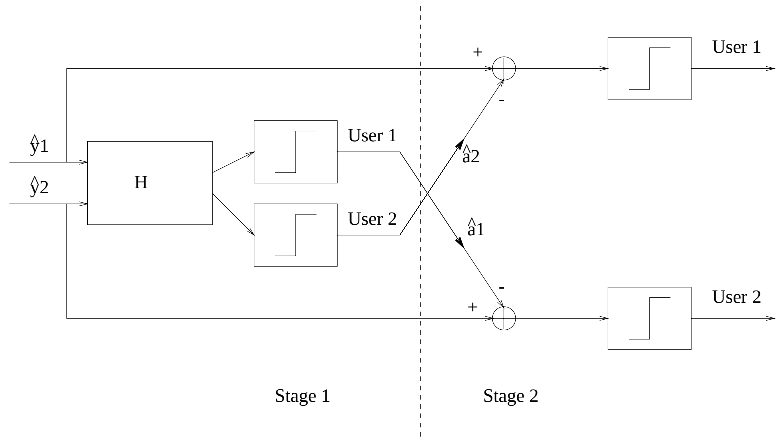

Figure 2: Parallel Interference Cancellor

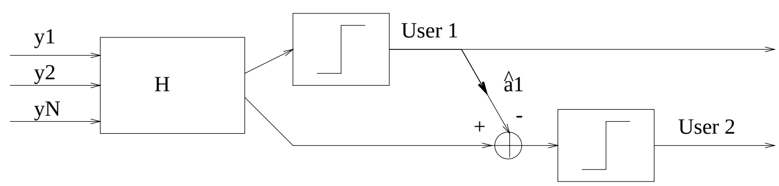

Figure 3: Successive Interference Cancellor

called a parallel interference cancellor or multistage detector [7], and is shown in Fig. 2.

Rather than wait until all users have been detected using the linear detector, we could start cancelling interference as soon as some users' estimates are available. This results in the successive interference cancellor [1], shown in Fig. 3 for two users. For user k, the filter H is set up to mitigate the interference of later users, while interference from earlier users is simply canceled by assuming that previous decisions were correct. It is important that the stronger users are equalized first, because all the other users will rely on these decisions. Because the weaker users can take fuller advantage of the reliable decisions of the strong users, this type of receiver appears to perform better than the parallel interference cancellor when the users have unequal power [4].

5 Design of Signature Sequences

The previous two sections have focused on reducing the complexity of the optimal joint detector, which is in general NP-hard and is not practical except in small systems. This is not true for all sequences, however. The matched filter detector is optimal for orthogonal sequences, for example. Researchers have also determined that joint detection of M-sequences is equivalent to a min-cut network problem which can be solved in polynomial time [6].

6 Comparison with Single-User Equalization

As stated in the introduction, multiuser detection deals with many of the same issues as single-user equalization. It is not surprising, therefore, that multiuser receivers often take familiar forms, with multiple-access interference replacing the role of intersymbol interference (ISI). The decorrelator and MMSE-linear multiuser detectors can be compared to the zero-forcing and MMSE-linear equalizers. (The condition of linear independence of signature sequences replaces that of spectral nulls in determining the existence of the decorrelator, with relation to the zero-forcing linear equalizer.) Interference cancellors can be connected with the decision-feedback equalizer (DFE), in that they suboptimally assume that certain previous decisions are correct.

When the symbol timing of the various users is not synchronized, or when frequency-selective channels are encountered, the optimal joint detector can be run with a version of the Viterbi algorithm [8], similarly to single-user maximimum likelihood equalization over ISI channels. Rather than estimating the jointly optimal sequence, the user-by-user, symbol-by-symbol estimates can be found by substituting the maximum *a posteriori* (MAP or APP) algorithm for the Viterbi algorithm.

The correspondence is not exact, however (at least for typical equalization scenarios). The fact that all users can interfere with one another would correspond to a model where the ISI is as long as the number of symbols, in both directions, and the symbol sequence must be terminated with that many zeros. The asynchronous case corresponds to a cyclostationary channel model rather than a time-invariant channel.

As with equalization, knowledge of parameters such as user powers and signature sequences are an issue. Blind or adaptive forms of some of these multiuser detectors have been developed to learn parameters and track changes over time.

References

[1] Alexandra Duel-Hallen. Decorrelating decision-feedback multiuser detector for synchronous code-division multi-access channel. *IEEE Trans. Comm.*, 41(2):285–290, Feb. 1993.

[2] Ruxandra Lupas and Sergio Verdu. Linear multiuser detectors for synchronous code-division multiple-access channels. *IEEE Trans. Info. Theory*, 35(1):123–136, January 1989.

[3] Upamanyu Madhow and Michael L. Honig. MMSE interference suppression for direct-sequence spread-spectrum CDMA. *IEEE Trans. Communications*, 42(12):3178–3188, December 1994.

[4] Pulin Patel and Jack Holtzman. Performance comparison of a DS/CDMA system using a successive interference cancellation (IC) scheme and a parallel IC scheme under fading. In *Proc. ICC '94*, pages 510–514, 1994.

[5] David N.C Tse and Stephen V. Hanly. Linear multiuser receivers: effective interference, effective bandwidth and user capacity. *IEEE Trans. Info. Theory*, 44(2):641–657, March 1998.

[6] Sennur Ulukus and Roy D. Yates. Optimum multiuser detection is tractable for synchronous CDMA systems using m-sequences. *IEEE Comm. Letters*, 2(4):89–91, Apr. 1998.

[7] Mahesh K. Varanasi and Behnaam Aazhang. Near-optimum detection in synchronous code-division multiple-access systems. *IEEE Trans. Comm.*, 39(5):725–736, May 1991.

[8] Sergio Verdu. Minimum probability of error for asynchronous gaussian multiple-access channels. *IEEE Trans. Info. Theory*, 32(1):85–96, January 1986.

[9] Sergio Verdu. *Multiuser detection*. Cambridge University Press, 1998.

[10] Sergio Verdu and Shlomo Shamai. Spectral efficiency of CDMA with random spreading. *IEEE Trans. Info. Theory*, 45(2):622–640, March 1999.