Decorrelating Decision-Feedback Multiuser Detector for Synchronous Code-Divison Multiple-Access Channel

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Abstract—**Several multiuser detectors for code-division multiple-access (CDMA) systems have been studied recently. We propose a new decorrelating decision-feedback detector (DF) for synchronous CDMA which utilizes decisions of the stronger users when forming decisions for the weaker ones. The complexity of DF is linear in the number of users, and it requires only one decision per user. Performance gains with respect to the linear decorrelating detector are more significant for relatively weak users, and the error probability of the weakest user approaches the single-user bound as interferers grow stronger. The error rate of DF is compared to those of the decorrelator and the two-stage detector.**

I. Introduction

We consider a code-division multiple-access (CDMA) system where the receiver observes the sum of synchronously transmitted signals from several users in additive noise. The synchronous assumption holds in several important practical systems [2], [3] and provides a simple model for studying detection algorithms which in their general form can be applied to the asynchronous case and to channels with fading and multipath. Since the conventional detector often fails to produce reliable decisions for the CDMA channel, several new multiuser detectors have been proposed [1]–[4]. The optimum receiver achieves low error probability at the expense of high computational complexity [1]. When the number of users is large, it is desirable to use a simple but reliable suboptimum detector. The linear decorrelating detector, e.g., [2] (the decorrelator) can significantly outperform the conventional receiver. If a synchronous CDMA system is modeled as a time-varying single-user channel with intersymbol interference (ISI), the decorrelator becomes analogous to the zero-forcing equalizer [6, p. 358], since it eliminates multiuser interference at the expense of increased noise power. Decision-feedback equalizers often have significantly lower error rates than linear detectors in single-user channels [6, p. 383], and have been shown superior to linear detectors in several multi-user systems [7], [8]. This observation motivates our study of decision-feedback detection for synchronous CDMA. Other recent approaches to multiuser detection include multistage detectors [3], [5]. They improve the performance of the decorrelator under certain operating conditions (e.g., when interfering users are stronger than the user under consideration), and their error rate approaches that of the optimum detector as energies of the interfering users grow. However, several stages of decisions result in higher complexity relative to the linear and decision-feedback detectors.

The decorrelating decision-feedback detector proposed in this paper is suitable for synchronous CDMA systems, since it utilizes the difference in users' energies. Its forward and feedback filters are chosen to eliminate all multiuser interference provided that the feedback data are correct. Decisions for users are made in the order for decreasing received energies. The receiver for each user linearly combines sampled outputs of the matrix matched filter with decisions of all stronger interfering users. Thus, the receiver for the strongest user does not involve feedback, and is equivalent to the decorrelator. On the other hand, the detector for the weakest user utilizes decisions of all other users and ideally (i.e., when feedback symbols are correct) achieves the performance of the single-user system.

In [4], a successive cancellation method was proposed for a coded CDMA system. Although the current paper was originally written without the knowledge of that technique, the ideas are quite similar. However, the emphasis of this work and its extension to the asynchronous case [9] is on the receiver filter optimization in addition to the cancellation of stronger users.

With the exception of the decorrelator [2], all of the multiuser detectors studied in [1]–[5] and the present approach require the knowledge of users' energies. Thus, although these detectors offer theoretical performance advantages over the conventional detector, they rely on the ability to obtain accurate estimates of users' energies in time-variant CDMA channels. These have to be estimated using the conventional tracking methods. In the coded case, provided that the users are ranked in the order of decreasing strength, more accurate estimates of the users' energies can be calculated using the re-encoded signal [4]. We briefly outline a technique for estimating energies in coded and uncoded systems for the simple synchronous CDMA model studied in the current paper.

In Section II, the discrete-time vector model for synchronous CDMA and the linear decorrelating detector are considered. In Section III, we derive a white-noise discrete-time model, the decorrelating decision-feedback detector and describe a method for estimating energies. Performance estimates, simulation results and a comparison with the decorrelator and the two-stage detector are presented in Section III.

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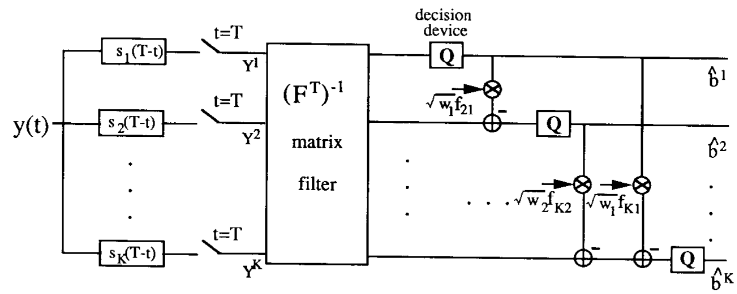

Fig. 1. Matched-filter receiver and the decorrelating decision-feedbackdetector for synchronous CDMA.

II. The System Model and the Decorrelator

Consider a synchronous CDMA system with K users and a set of preassigned normalized signature waveforms $s_k(t)$, $k = 1, \cdots, K$ where each waveform is restricted to a symbol interval of duration T, $\int_0^T s_i^2(t)\,dt = 1$, and signature waveforms are linearly independent. The input alphabet of each user is antipodal binary $\boldsymbol{X} = \{-1, +1\}$. The input bit sequence for each user is i.i.d., equiprobable, and independent of other users. It is sufficient to consider a single transmission. Suppose the input vector is given by $\boldsymbol{b} = (b_1, \cdots, b_K)^T$, $b_k \in \boldsymbol{X}$, and assume that input energies w_k are nondecreasing, i.e., $w_1 \geq w_2 \geq \cdots \geq w_K$. (We initially assume that the receiver knows the energies.) When a bank of filters matched to the set of signature waveforms is followed by samplers at time T, the following discrete-time output vector arises (Fig. 1) [2]

$$\boldsymbol{y} = \boldsymbol{R}\boldsymbol{W}\boldsymbol{b} + \boldsymbol{z}\,, \tag{1}$$

where $\boldsymbol{R}$ is a $K \times K$ positive definite matrix of signature waveform cross-correlations, and $\boldsymbol{W}$ is a diagonal matrix with $W_{i,i} = \sqrt{w_i}$, $i = 1, \cdots, K$. The term $\boldsymbol{z}$ is a Gaussian noise vector with the $K \times K$ autocorrelation matrix $\boldsymbol{R}(\boldsymbol{z}) = \sigma^2\boldsymbol{R}$ where $R(\boldsymbol{z})_{i,j} = E(z_i, z_j)$.

The objective of a multiuser detector is to recover the input data vector $\boldsymbol{b}$ given the output vector $\boldsymbol{y}$. In the linear decorrelating detector, the matrix filter $\boldsymbol{R}^{-1}$ followed by a set of decision devices (sign detectors) is applied to $\boldsymbol{y}$. The output of the matrix filter is

$$\hat{\boldsymbol{y}} = \boldsymbol{W}\boldsymbol{b} + \tilde{\boldsymbol{z}} \tag{2}$$

where $\tilde{\boldsymbol{z}}$ is a Gaussian noise vector with the autocorrelation matrix $\boldsymbol{R}(\tilde{\boldsymbol{z}}) = \sigma^2\boldsymbol{R}^{-1}$. Thus, the probability that the kth input is recovered incorrectly is

$$Pe_k(\text{dec}) = Q\left(\sqrt{w_k/\left[\sigma^2(R^{-1})_{k,k}\right]}\right) \tag{3}$$

where the Q-function is $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}\,dy$.

III. White Noise Discrete-Time Model and the Decorrelating Decision-Feedback Detector

In synchronous CDMA, a white noise model can be obtained by factoring the positive definite matrix of cross-correlations as $\boldsymbol{R} = \boldsymbol{F}^T\boldsymbol{F}$ where $\boldsymbol{F}$ is a lower triangular matrix (see Cholesky decomposition algorithm [11, p. 88]). If the filter with response $\left(\boldsymbol{F}^T\right)^{-1}$ is applied to the sampled output of the matched filter (1), the resulting output vector is

$$\check{\boldsymbol{y}} = \boldsymbol{F}\boldsymbol{W}\boldsymbol{b} + \boldsymbol{n} \tag{4}$$

where $\boldsymbol{n}$ is a white Gaussian noise vector with the autocorrelation matrix $\boldsymbol{R}(\boldsymbol{n}) = \sigma^2\boldsymbol{I}$. ($\boldsymbol{I}$ is the $(K \times K)$ identity matrix.) The discrete-time models (4) and (1) correspond to the outputs of the standard and whitened matched filters, respectively, in single-user channels with ISI [6, p. 353].

Since the components of the noise vector $\boldsymbol{n}$ in (4) are uncorrelated, the optimum (maximum likelihood) detector for synchronous CDMA [3] has the Euclidean metric $\|\check{\boldsymbol{y}} - \hat{\boldsymbol{t}}\|^2 = \sum_{k=1}^K \left(\check{y}_k - \hat{t}_k\right)^2$ where $\hat{\boldsymbol{t}}$ is the signal associated with an input $\hat{\boldsymbol{b}}$, i.e., $\hat{\boldsymbol{t}} = \boldsymbol{F}\boldsymbol{W}\hat{\boldsymbol{b}}$. Both the metric and the expression for the probability of error of the optimal detector have a simpler derivation when the model (4) is used instead of (1).

The model (4) also gives rise to the decorrelating decision-feedback detector (DF). The kth component of $\check{\boldsymbol{y}}$ is given by $\check{y}_k = F_{k,k}\sqrt{w_k}b_k + \sum_{i=1}^{k-1} F_{k,i}\sqrt{w_i}b_i + n_k$. Since this expression does not contain a multiuser interference term for the strongest user ($k = 1$), a decision for this user is made first: $\hat{b}_1 = \text{sgn}(\check{y}_1)$. Multiuser interference for the second user is $F_{2,1}\sqrt{w_1}b_1$. Since a decision for the first user is available, we can use feedback in estimating the second symbol. Thus, the second decision is $\check{y}_2 - F_{2,1}\sqrt{w_1}\hat{b}_1$. Similarly, for the $k$the user, multiuser interference depends on stronger users ($i = 1, \cdots, k-1$). Decisions for these users have already been made, and they can be used to form a feedback term (Fig. 1), i.e.,

$$\begin{aligned}\hat{b}_k &= \text{sgn}\left(\check{y}_k - \sum_{i=1}^{k-1} F_{k,i}\sqrt{w_i}\hat{b}_i\right)\\ &= \text{sgn}\left(F_{k,k}\sqrt{w_k}b_k + \sum_{i=1}^{k-1} F_{k,i}\sqrt{w_i}\right.\\ &\qquad\left.\cdot\left(b_i - \hat{b}_i\right) + n_k\right).\end{aligned} \tag{5}$$

To summarize, the decorrelating decision-feedback detector is characterized by a feedback filter $\boldsymbol{B} = \left(\boldsymbol{F} - \boldsymbol{F}^d\right)\boldsymbol{W}$, where $\boldsymbol{F}^d$ is a diagonal matrix obtained from $\boldsymbol{F}$ by setting all off-diagonal elements to zero. The filter is fed by the vector of decisions $\hat{b}$. The vector input to the set of decision devices is $\tilde{y}-B\hat{b}=F^dWb+\left(F-F^d\right)W\left(b-\hat{b}\right)+n$. Since B is lower triangular with zeros along the diagonal, only previously made decisions (i.e., $\hat{b}_{k-1},\hat{b}_{k-2},\cdots,\hat{b}_1$) are required for forming the input to the kth quantizer. DF corresponds to the zero-forcing decision-feedback equalizer for ISI channels [6, p. 383] since it attempts to cancel all multiuser interference. The strictly lower triangular B corresponds to the purely causal feedback filter used in single-user systems.

Of course, it is possible to remove the requirement that received energies are nondecreasing. However, the multistage method studied in [3] and our analysis indicate that feedback is primarily beneficial when interfering users are stronger. This observation motivated our choice of the receiver structure.

An important measure of performance for a decision-feedback detector is the signal-to-noise ratio at the input to the decision device under the assumption of correct previous decisions. From (5), the signal-to-noise ratio for the DF is

$$\mathrm{SNR}_k=F_{k,k}^2w_k/\sigma^2\,. \tag{6}$$

Given the same order of making decisions, this SNR is the largest achievable by any decision-feedback detector which attempts to cancel all multi-user interference. To show this, consider a $(K\times K)$ matrix filter M applied to the white-noise output (4) which result in a discrete-time model $MFWb+\acute{z}$, where the interference matrix MF is lower triangular. Note that this requires M to be lower triangular, and the autocorrelation of the noise is $R(\acute{z})=\sigma^2MM^T$. The lower triangular structure of the interference matrix allows to construct a decision-feedback detector with the kth signal-to-noise ratio $M_{k,k}^2F_{k,k}^2w_k/(\sigma^2\sum_{i=1}^kM_{k,i}^2)$. We observe that this signal-to-noise ratio is maximized for each k if $M=I$, and the optimality of DF results.

It can be shown that DF is equivalent to a noise-canceling detector obtained from the discrete-time model (2). Since the inverse of the autocorrelation matrix R is given by $R^{-1}=F^{-1}\left(F^T\right)^{-1}$, (2) can be rewritten as

$$\tilde{y}=Wb+F^{-1}n \tag{7}$$

where n is the white noise vector in (4). Since F^{-1} is lower triangular, i.e., $\tilde{y}_k=\sqrt{w_k}b_k+\sum_{i=1}^k\left(F^{-1}\right)_{k,i}n_i$, we can construct a detector which uses earlier decisions of the noise sequence $\hat{n}_1,\cdots,\hat{n}_{k-1}$ to reduce the variance of the noise in the kth component of (2) (or (7)). The kth decision of this detector is given by $\hat{b}_k=\mathrm{sgn}\left(\tilde{b}_k\right)$ where

$$\tilde{b}_k=\tilde{y}_k-\sum_{i=1}^{k-1}\left(F^{-1}\right)_{k,i}\hat{n}_i\,, \tag{8}$$

and the kth noise-decisions is $\hat{n}_k=F_{k,k}\left(\tilde{b}_k-\sqrt{w_k}\hat{b}_k\right)$. This noise-canceling detector is related to a partial feedback multistage detector studied in [5]. In the second stage of that detector, a subset of previous noise estimates is used to reduce the noise variance in (2). We observe that if this subset is restricted to stronger users (i.e., users $1,\cdots,k-1$ when making a decision for the kth user), the second stage of the detector in [5] and DF (or the noise-cancelling detector described above) have the same inputs to the decisions devices provided previous decisions are correct.

In time-varying CDMA channels (e.g., mobile radio), the energies of users need to be estimated and updated for proper cancellation of multiuser interference. Consider the noise-cancelling detector described in the previous paragraph. Suppose the receiver does not know the energies, and a single transmission occurs. Given the output of the decorrelating filter (7), it is reasonable to estimate $\sqrt{w_k}$ as $\sqrt{\hat{w}_k}=|\tilde{y}_k|$. This implies that the kth noise-decision is $\hat{n}_k=0$. Thus, the resulting detector does not cancel the noise, and is equivalent to the decorrelator. If the energies vary slowly, it is possible to improve upon this estimation method by averaging the absolute values of previous N outputs, i.e., the estimate at time n is $\sqrt{\hat{w}_k}(n)=\sum_{i=0}^{N-1}\alpha_i|\tilde{y}_k(n-i)|$ where α_i is a nonnegative nonincreasing sequence of length N, i.e., $\alpha_i\geq\alpha_{i+1}$, and $\sum_{i=0}^{N-1}\alpha_i=1$. If $N=\infty$, the estimator can be implemented as an IIR filter. (If the receiver knows the ranking of energies, but not their values, the estimate of energy of the kth user can be computed after cancellation, i.e., the components of the sequence $\tilde{b}_k(i)$ (8) can be averaged to obtain less noisy estimates.) Of course, the weights α_i have to be chosen appropriately. If the energy varies slowly, an accurate estimate is obtained by choosing a large N and a slowly decreasing sequence α_i. However, if the variations are fast, more emphasis should be placed on more recent components of the sequence $\tilde{y}_k$. When the energies jump unpredictably, we obtain the decorrelator as shown above. Note that the decorrelator is the maximum-likelihood detector when the energies are unknown, e.g., [2]. It also achieves the optimum near-far resistance [2]. If there are "jumps" and "quiet periods" in the energy values, the receiver can detect these and switch between the decorrelating and the feedback modes.

The above discussion applies to relatively high signal-to-noise ratio systems. When the channel is very noisy, coding has to be employed. In [4], a cancellation method was proposed for a coded system. Similarly, DF can be applied to encoded signals. In this case, received sequences of stronger users (extend (4) or (7) to sequences) need to be decoded first, and then re-encoded and fed back to the receivers of weaker users. To estimate the energy of the kth user, the re-encoded kth sequence can be correlated with the sequence $\tilde{b}_k(i)$ (8) (see [4]). In this case, ranking of energies has to be performed *a priori* based on previous estimates and other tracking methods.

IV. Analysis and Examples

First, assume that the energies of users are estimated correctly. The signal-to-noise ratio (6) gives rise to the probability of error of DF for the kth user under the assumption of correct previous decisions

$$\tilde{P}e_k(\mathrm{DF})=Q(F_{k,k}\sqrt{w_k}/\sigma)\,. \tag{9}$$

It is easy to show that $F_{k,k}^2\geq1/\left(R^{-1}\right)_{k,k}$. Note that for the strongest user ($k=1$), $F_{1,1}^2=1/\left(R^{-1}\right)_{1,1}$ and the estimate (9) gives the probability of error since the receiver for this user does not utilize feedback. Therefore, the error rates of the DF (9) and the decorrelator (3) are the same for the strongest user. For $k > 0$, the last inequality is tight provided that multiuser interference affects the kth user. Thus, an improvement over the performance of the decorrelator is suggested by the comparison of (9) and (3). (When DF and the other methods [1]–[3] are compared to the decorrelator, it should be taken into account that the decorrelator has the following advantage: it does not require the knowledge of energies.) Finally, for the weakest user, $F_{K,K}^2 = 1$, and the ideal performance of DF (9) agrees with the error probability of the single-user system given by

$$Pe_k(\text{SU}) = Q(\sqrt{w_k}/\sigma). \tag{10}$$

To find the exact error rate of DF for the kth user, one has to average the conditional error probability given a particular error pattern for users $1, \cdots, k-1$ over all such error patterns:

$$P_k = \frac{1}{2} E_{\Delta b_1, \cdots, \Delta b_{k-1}} \cdot Q\left(\frac{F_{k,k}\sqrt{w_k} + \sum_{i=1}^{k-1} F_{k,i}\sqrt{w_i}\Delta b_i}{\sigma}\right) \tag{11}$$

where the error pattern for the ith user is $\Delta b_i = (b_i - \hat{b}_i)$. For example, consider a two-user system with $R_{1,2} = r$. Then $F_{1,1} = \sqrt{1 - r^2}$ and $F_{2,1} = r$. The error rate for the stronger user ($k = 1$) is given by the error rate of the decorrelator: $P_1 = Q(\sqrt{w_1(1 - r^2)}/\sigma)$, and the input to the decision device of the second user is $\sqrt{w_2}b_2 + \sqrt{w_1}r(b_1 - \hat{b}_1) + n_2$. By averaging over all possible values taken on by $b_1 - \hat{b}_1$, we derive the error rate for the weaker user

$$Pe_2(\text{DF}) = (1 - P_1)Q\left(\frac{\sqrt{w_2}}{\sigma}\right) + \frac{P_1}{2}\left[Q\left(\frac{\sqrt{w_2} - 2r\sqrt{w_1}}{\sigma}\right) + Q\left(\frac{\sqrt{w_2} + 2r\sqrt{w_1}}{\sigma}\right)\right]. \tag{12}$$

This expression also gives the error rate of the second stage detector in [3]. We use (12) and numerical results obtained in [3] to analyze performance of DF in this two-user system. Suppose the energy of the second user w_2 is fixed. As w_1 grows (P_1 becomes smaller), the first Q-function dominates (12), and the error rate of the weaker user approaches the single-user bound (10). Thus, DF is attractive in near–far situations. The expression (12) also gives insight into the behavior of DF with the order of users interchanged, i.e., when $w_1 < w_2$. This analysis is of interest if the users are not ranked properly. In this case, the second and third terms in (12) (error propagation) are more significant, and feedback is not as useful. In fact, in a certain region of values taken by w_1, the error probability of DF is higher than that of the decorrelator (in the Figs. 2, 3 of [3], they differ at most by a factor of 20). Finally, as w_1 decreases, the DF detector again performs better than the decorrelator and finally approaches the error rate of the single-user system. However, the gain is not significant, and for small w_1, the conventional detector is a better choice than either DF or the decorrelator.

For a large number of users, exact computation of error probability becomes complex (see, for example, the analysis for the multistage detector in [3]). The problem is similar to computing the exact probability of a decision-feedback equalizer for a single-user system. Although this probability can be bounded analytically, computer simulations are most commonly used in determining the error rate and the effects of error propagation [6]. We show results of simulations for a four-user system in Example 2. It is possible to study general performance trends by considering dominating terms in the expression for the error rate (11). Suppose the energy w_k is fixed. Then the error rate of DF for the kth user does not depend on energies of users $k+1, \cdots, K$, and approaches the ideal error probability (9) as energies of users $1, \cdots, k-1$ grow. In particular, the error rate of the weakest user tends to the error probability of the single-user system (10). It is interesting to note that if the energies of users $1, \cdots, i$ grow, but $w_{i+1}, \cdots, w_k$ remain fixed, the limiting error rates of DF for users $i+1, \cdots, k$ are found by studying the $(k-i)$-user white-noise model with a lower triangular interference matrix (a submatrix of $\boldsymbol{F}$) whose m, nth component ($m \geq n$, $m = 1, \cdots, k-i$) is given by $F_{i+m,i+n}$. We can construct a decision-feedback detector for this model and find its error rate for the mth user. This error probability is approached by the error rate of DF for the $(i+m)$th user in the original K-user system as energies of users $1, \cdots, i$ grow. Example 2 illustrates the situation for a four-user system.

Finally, we investigate the effect of errors in estimating energies on the performance of DF. (Note that incorrect ranking was discussed following (12).) Suppose the ith energy estimate is $\sqrt{\hat{w}_i}$, and define $\Delta w_i = \sqrt{w_i} - \sqrt{\hat{w}_i}$. The input to the kth decision device is then $F_{k,k}\sqrt{w_k}b_k + \sum_{i=1}^{k-1} F_{k,i}(\sqrt{w_i}(b_i - \hat{b}_i) + \Delta w_i \hat{b}_i) + n_k$. Assuming correct previous decisions, the remaining interference is $\sum_{i=1}^{k-1} F_{k,i}\Delta w_i \hat{b}_i$. If Δw_i are large relative to w_k, DF suffers from multiuser interference similarly to the conventional detector. However, using estimation techniques described in the previous section, we can keep $\sqrt{\hat{w}_i}$ close to the output value $\tilde{y}_i$ (7) in the case of rapidly varying energies, and obtain more accurate estimates when energies vary slowly. Then, as discussed in Section III (for the equivalent noise-canceling detector), the performance of DF is at least as good as that of the decorrelator.

Example 1: We consider the bandwidth-efficient two-user synchronous CDMA system studied in [3] with signal cross-correlation $R_{1,2} = r = 0.7$. The error probability of DF for the second user is given by (12). Fig. 2 depicts error probabilities for the second user versus the difference between input signal-to-noise ratios where $\text{SNR}(2) = w_2^2/\sigma^2 = 11$ dB. We observe that DF has lower error rate than the decorrelator (3). As the first user grows stronger, the performance of DF approaches that of the single-user system (10).

As was pointed out earlier, the error probabilities of the two-stage detector and DF are the same for the weaker user in a two-user system. In addition to the data in Fig. 2, Fig. 2 of [3] also shows error rates of the conventional, decorrelator, optimum, optimum linear [2], and three-stage detectors for this

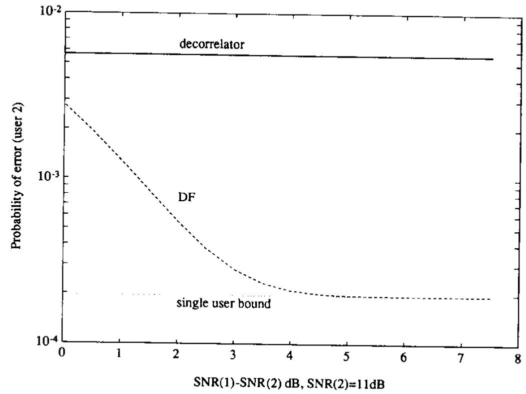

Fig. 2. Error probabilities for two-user channel, $r = 0.7$. (Example 1).

example. We observe that performance of DF is close to that of the optimal detector, and other methods have higher error rates.

Example 2: In [3], multistage detectors are evaluated for a multiuser system based on a set of signature waveforms derived from Gold sequences of length seven. The objective of this study is to gain insight into the performance of multistage detectors in asynchronous bandwidth-efficient CDMA with many users. It is of interest to conduct a similar study here. We consider the same set of signature waveforms, and assume that four users are active. The corresponding matrix of cross-correlations is given in Fig. 3. The probability of error of the decorrelator (3), the single-user bound (10) and the simulated performance of DF are shown for the weakest user in Fig. 3. The input signal-to-noise ratio is $\mathrm{SNR}(4) = 11$ dB for the weakest user, and varies from 11 to 17 dB for other users. We observe that DF has significantly lower error rate than the decorrelator, and approaches the single-user bound as interfering users grow stronger. We also find that the performance of DF is close to that of the two-stage detector (see [3, Fig. 5(c)]), although in this case the two detectors are not the same. The second-stage detector uses outputs of the decorrelator to cancel interference terms, whereas DF feeds back its decisions for stronger users, which, in general, are better than those of the decorrelator (see below). Since both DF and two-stage detectors of [3], [5] are difficult to analyze, it would be desirable to compare these detectors for various examples using simulations for the asynchronous system, this is accomplished in [9], [10]. We also note that DF is simpler to implement than two-stage detectors, since it does not require a first stage.

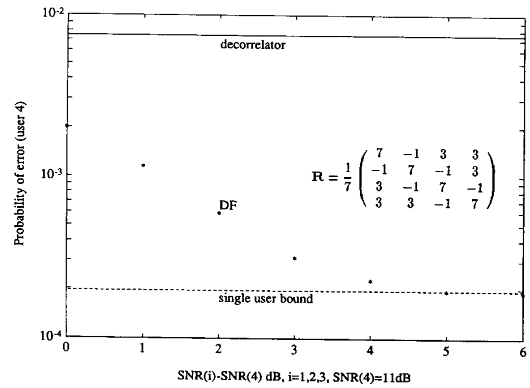

Fig. 3. Error probabilities for the first four-user channel of Example 2.

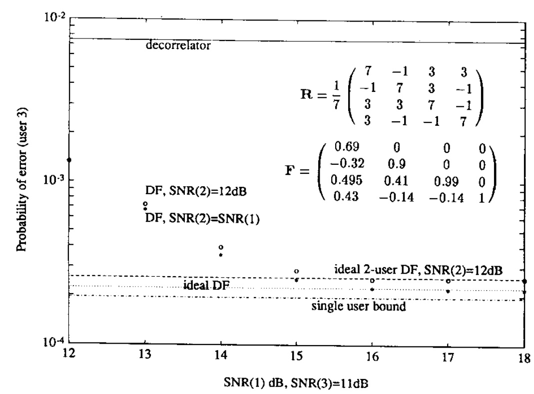

Fig. 4. Error probabilities for the second four-user channel of Example 2.

Next, we evaluate the error rate of DF for user 3 with $\mathrm{SNR}(3) = 11$ dB in the same four-user system, but with a different order of users. The matrix of cross-correlation and the corresponding matrix $\boldsymbol{F}$ are shown in Fig. 4 along with simulation results (individual points) and analytical performance estimates (lines). In the first simulation study, we allowed the energies of the first two users grow. We observe that the error rate of DF converges to the ideal performance (9) which is close to the single-user bound (10), since $F_{3,3} \approx 1$. In the second study, we fixed $\mathrm{SNR}(2) = 12$ dB, while the energy of the first user was allowed to grow. We find that the error rate of user 3 approaches the error rate of the decision-feedback detector for the weaker user in a white-noise two-user system with the interference matrix $\begin{pmatrix} F_{2,2} & 0 \\ F_{3,2} & F_{3,3} \end{pmatrix}$. The exact expression for this "ideal two-user DF" error probability is derived similarly to (12). We also observe that the decorrelator performs worse than DF for all operating points we have considered.

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