6.962 Week 10 Summary of Discussion

The Capacity of Wireless Networks

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1 Introduction

In the last three weeks, we have talked about how well we could communicate over a wireless channel given the problems of multi-path fading and interference from other simultaneous users. This week we step back a bit and look at a wireless network of randomly distributed nodes with no centralized control, and how much data throughput that network will be able to support.

1.1 Motivation

With all of the hype about Bluetooth capable devices being developed and internet-capable cellular phones and futuristic "smart homes" where computers, phones, PDAs, and appliances are all wirelessly connected, wireless networks are (or could be) everywhere.

1.2 System Model

The network model in [1] is a collection of identical nodes located in a region of one square meter. This can either be a disk of unit area, or more conveniently the surface of a sphere to prevent edge effects. Each node is capable of transmitting at W bits per second over the shared wireless channel. The transmissions are omnidirectional (although this is not always required), and received signal power decays with distance r as $\frac{1}{r^\alpha}$ where $\alpha > 2$. The distance is measured as the great circle distance when the nodes are located on the surface of a sphere. There is no centralized control to allocate resources or route data flow. Nodes may help each other communicate by relaying each other's data packets, or by buffering data packets before relaying them.

There are two types of networks considered, *Arbitrary Networks*, in which the node locations, sources, destinations, and traffic patterns are all arbitrary, and *Random Networks*, in which they are all randomly chosen from a uniform distribution.

2 Arbitrary Networks

First, we need to describe what is required for a transmission to be received successfully by the destination node. Two models are used. Let X_i be the ith node of the network. It also designates the location of that node.

Protocol Model: If X_i transmits over a subchannel or the entire wireless channel to X_j, then the transmission succeeds if

$$|X_k - X_j| \geq (1 + \Delta)|X_i - X_j|$$

for all other X_k which are simultaneously transmitting over the same frequency bands. The quantity $\Delta > 0$ models a guard zone specified by the network protocol to prevent neighboring nodes from transmitting on the same subchannel at the same time.

Physical Model: Given the same situation as in the Protocol Model, the transmission succeeds if

$$\frac{\frac{P_i}{|X_i - X_j|^\alpha}}{N + \sum_{k \in \mathcal{T}, k \neq i} \frac{P_k}{|X_k - X_j|^\alpha}} \geq \beta$$

where $\mathcal{T}$ is the set of all nodes which are transmitting at the same time as X_i, and P_k is the transmit power level used by node X_k. The quantity β represents the minimum signal-to-interference ratio (SIR) necessary for successful reception, N is the ambient noise, and α represents the exponent with which the power decays with transmitted distance.

The *transport capacity* of a network is the sum of products of bits that are transported across the network multiplied by the distance each bit is carried. For example, if the network is able to support every second 3 communications of 1, 2, and 3 bits which are transported a distance of 2 meters each, then the transport capacity is 12 bit-meters per second.

Under the Protocol Model, the transport capacity of an Arbitrary Network is $\mathcal{O}(W\sqrt{n})$ bit-meters per second, if the node locations, traffic patterns, and transmission ranges are optimally chosen. This translates to a throughput capacity per node of $\mathcal{O}(\frac{W}{\sqrt{n}})$.

Under the Physical Model, transport capacity of $\mathcal{O}(W\sqrt{n})$ is feasible, but $\mathcal{O}(Wn^{\frac{\alpha-1}{\alpha}})$ is not. Feasible here means that the network on average is able to support the given data rate.

Thus, in both models, the throughput capacity per node as the number of nodes increases falls off as $\frac{1}{\sqrt{n}}$, which says that the effective data rate per node decreases to zero as the number of nodes increase because each node spends proportionally more time routing traffic from other nodes.

3 Random Networks

In a Random Network, the locations of the nodes are independent and identically distributed on the surface of a sphere or on a disk, both one square meter in area. The destination for each node X_i is independently chosen as the node X_j which is closest to a randomly chosen point Y_j. The Y_j are uniform and identically distributed. All nodes are assumed to transmit at the same nominal power, giving each node a range r. As before, there are two models for when a transmission is successful.

Protocol Model: If X_i transmits over a subchannel to X_j, then the transmission succeeds if

$$|X_i - X_j| \leq r$$

and for every other node X_k that is simultaneously transmitting over the same subchannel,

$$|X_k - X_j| \geq (1 + \Delta)r$$

As before, $\Delta > 0$ models a guard zone specified by the network protocol to prevent neighboring nodes from transmitting on the same subchannel at the same time.

Physical Model: Given the same situation as in the Protocol Model, all of the transmitting nodes choose a power level P for all of their transmissions, and a transmission is successful if

$$\frac{\frac{P}{|X_i - X_j|^\alpha}}{N + \sum_{k \in \mathcal{T}, k \neq i} \frac{P}{|X_k - X_j|^\alpha}} \geq \beta$$

where $\mathcal{T}$ is the set of all nodes which are transmitting at the same time as X_i. The quantity β represents the minimum signal-to-interference ratio (SIR) necessary for successful reception, N is the ambient noise, and α represents the exponent with which the power decays with transmitted distance.

The throughput capacity (bits per second per node) of a Random Network with the Protocol Model is $\mathcal{O}\left(\frac{W}{\sqrt{n \log n}}\right)$ in both the case of the spherical surface and the planar disk. For the Physical Model, $\mathcal{O}\left(\frac{W}{\sqrt{n \log n}}\right)$ is feasible, and $\mathcal{O}\left(\frac{W}{\sqrt{n}}\right)$ is not.

4 Implications and Tradeoffs Involved

Since the throughput for both Arbitrary and Random Networks decreases with n, these networks would not scale well with large numbers of nodes. The authors suggest that networks in which nodes only need to communicate with their neighbors would not have as great restrictions on throughput because there would not be as much relaying of traffic. A network in which the nodes are clustered into small groups (or "cells"), with one node in each cell designated to provide inter-cell access. This would also greatly reduce the power transmitted by most of the nodes.

Dividing the bandwidth of the wireless channel into subchannels doesn't affect the results either. It is possible to increase the transport capacity of the network by adding relay-only nodes, but this is very inefficient. If there are n nodes in the network, adding kn relay-only nodes provide less than a factor $\sqrt{k+1}$ increase in throughput.

Another possibility would be to increase the power of each transmission so that the total number of relay hops that a packet would have to use to get to the destination node. Increasing the transmit power increases the amount of interference for other nodes, which turns out to be much worse than forcing packets to take more hops. In an extreme case, increasing the transmit power so that any two nodes will be able to communicate directly, then only one pair may communicate at any given moment, so the throughput goes as $\mathcal{O}\left(\frac{1}{n}\right)$.

Ideally then, we would want to have the transmission power as small as possible. The Physical Model bounds for both Random and Arbitrary Networks show that if α is larger, then performance can actually increase because interference between nodes is reduced. We could also reduce the transmission power to be as small as possible, but if it is too small, then the network starts to lose connectivity.

5 Networks with Mobile Nodes

In all of the analyses above, all the nodes were fixed. It was stated in the Implications section that lack of perfect node location due to mobility would result in a decrease in capacity. However, it is shown in [2] that we can actually increase the capacity of a network by exploiting the mobility of the nodes, raising the throughput to $\mathcal{O}(1)$. However, the price for this is that the delay and memory requirements increase greatly with large numbers of nodes.

The increase in throughput comes from the idea of multiuser diversity, in which the mobility of the nodes means that sometimes the channel that each node "sees" is better than other times. It would be best to transmit between nodes when they are close together, saving both power and number of hops needed. But simply waiting until the source is close to the destination node is worse than relaying. When a node wants to transmit a packet, it transmits it to its nearest neighbor. All packets are relayed at most one time, so a relay node only passes on a stored packet to the destination. In the steady state, all nodes can act as relays for every other node, so a node will have its nearest neighbor as a destination for one of its stored packets with high probability. The price for this is the large amount of storage needed (a node needs to hold at least one packet for every other node in the system), and the potentially long delay in packet transmission.

6 Wireless Network Standards

Bluetooth and 802.11 are two IEEE standards for wireless network protocols. I will give a few details on how these systems (mostly Bluetooth) are implemented should time and energy permit.

References

[1] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory*, vol. 46, March 2000.

[2] M. Grossglauser and D. Tse, "Mobility Increases the Capacity of Ad-hoc Wireless Networks," *preprint.*

[3] A. Iwata, C. Chiang, G. Pei, M. Gerla, and T. Chen, "Scalable Routing Strategies for Ad Hoc Wireless Networks," *IEEE Journal on Selected Areas in Communications*, vol. 17, August 1999.

[4] B. Crow, I. Widjaja, J. Kim, and P. Sakai, "IEEE 802.11 Wireless Local Area Networks," *IEEE Communications Magazine*, September 1997.

[5] J. Haartsen, "The Bluetooth Radio System," *IEEE Personal Communications*, Febuary 2000.