Network Interference Management via Interference Alignment

Viveck R. Cadambe
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(for Wireless Communications and Distributed Storage)

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m Equations, n Unknowns

$$
\begin{aligned}
y_1 &= v_{11}x_1 + v_{12}x_2 + \ldots + v_{1n}x_n \\
y_2 &= v_{21}x_1 + v_{22}x_2 + \ldots + v_{2n}x_n \\
&\vdots \\
y_m &= v_{m1}x_1 + v_{m2}x_2 + \ldots + v_{mn}x_n
\end{aligned}
$$

$$\Leftrightarrow \mathbf{y} = \mathbf{V}_1 x_1 + \mathbf{V}_2 x_2 + \ldots + \mathbf{V}_n x_n$$

We want to derive $x_1, x_2, \ldots, x_n$ from $y_1, y_2, \ldots, y_m$

m Equations, n Unknowns

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✔ $m \geq n$, $\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_n$ linearly independent

$m < n$,

m Equations, n Unknowns

$$
\begin{aligned}
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$$\Leftrightarrow \mathbf{y} = \mathbf{V}_1 x_1 + \mathbf{V}_2 x_2 + \ldots + \mathbf{V}_n x_n$$

We want to derive $x_1, x_2, \ldots, x_n$ from $y_1, y_2, \ldots, y_m$

✔ $m \geq n$, $\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_n$ linearly independent

✘ $m < n$, In general, not resolvable

m Equations, n Unknowns - A slightly different problem

$$\mathbf{y} = \mathbf{V}_1 x_1 + \mathbf{V}_2 x_2 + \ldots + \mathbf{V}_n x_n$$

We want to derive x_1 from $y_1, y_2, \ldots, y_m$

✓ $m \geq n$, $\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_n$ linearly independent

If $m < n$, then ...??

m Equations, n Unknowns - A slightly different problem

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If $m < n$, then ...?? ⟵ Interference Alignment

Interference Alignment

Example: No. of Equations = 2, No. of Unknowns = 3

Interference Alignment

Example: No. of Equations = 2, No. of Unknowns = 3

$$4 = x_1 + x_2 + 2x_3 \quad \text{x2}$$

$$7 = x_1 + 2x_2 + 4x_3 \quad \text{x(-1)}$$

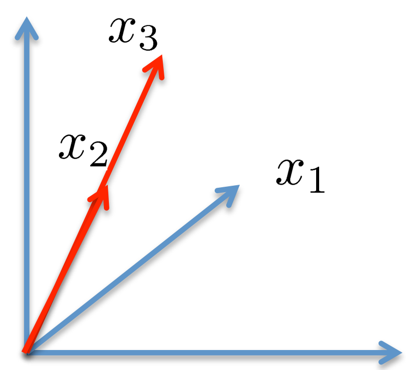
+

$$1 = x_1$$

Interference Alignment

Example: No. of Equations = 2, No. of Unknowns = 3

$$\begin{pmatrix} 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} x_1 + \begin{pmatrix} 1 \\ 2 \end{pmatrix} x_2 + \begin{pmatrix} 2 \\ 4 \end{pmatrix} x_3$$



x_2, x_3 *align*

Interference Alignment

$$\mathbf{y} \quad = \quad \mathbf{V}_1 x_1 + \mathbf{V}_2 x_2 + \ldots + \mathbf{V}_n x_n$$

If $m < n$, then, to uniquely resolve x_1

$$\mathbf{V}_1 \notin \text{span}(\{\mathbf{V}_2, \mathbf{V}_3, \ldots, \mathbf{V}_n\})$$
$$\Rightarrow \dim(\{\mathbf{V}_2, \mathbf{V}_3, \ldots, \mathbf{V}_n\}) \leq m - 1$$

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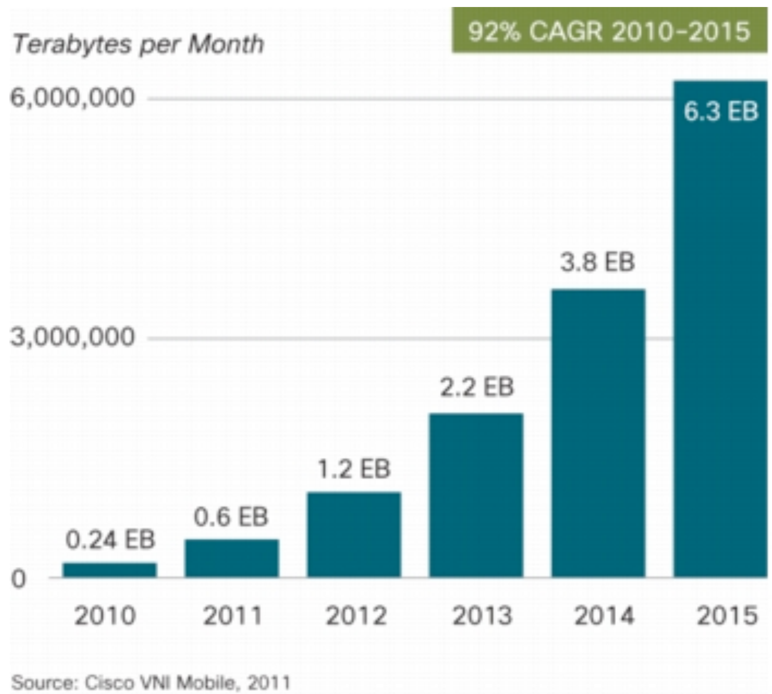
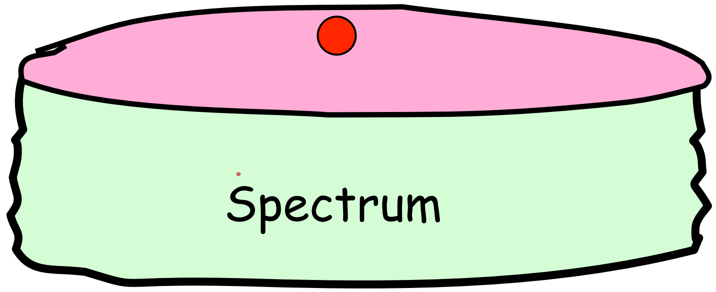
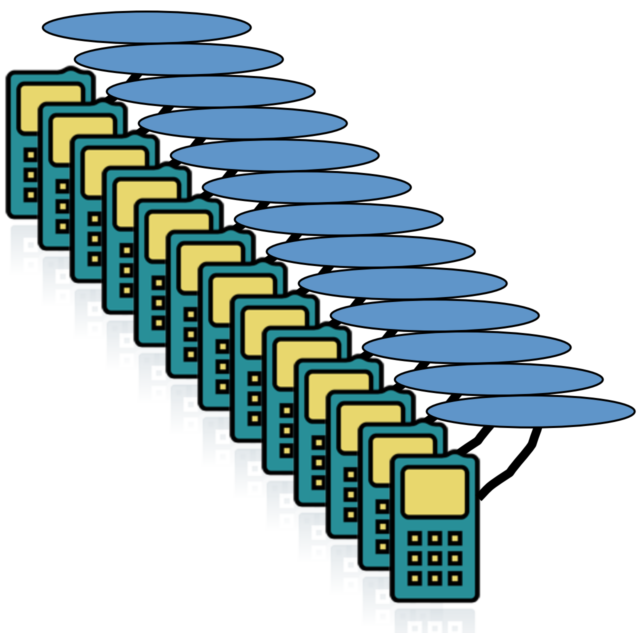
"Slim and sleek as it is, the iPhone is really the Hummer of cellphones." *The NY Times*

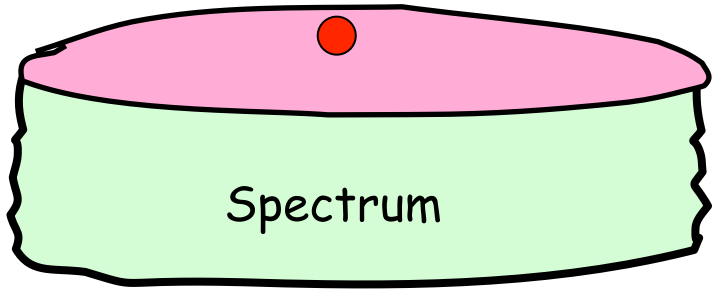


Worldwide mobile traffic projected to see a 26-fold increase by 2015.

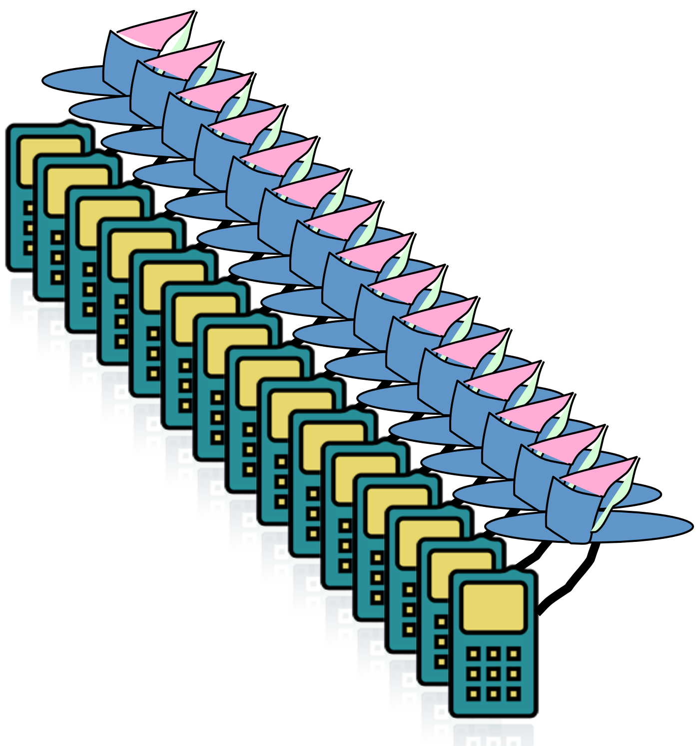
But the (usable) bandwidth remains the same!

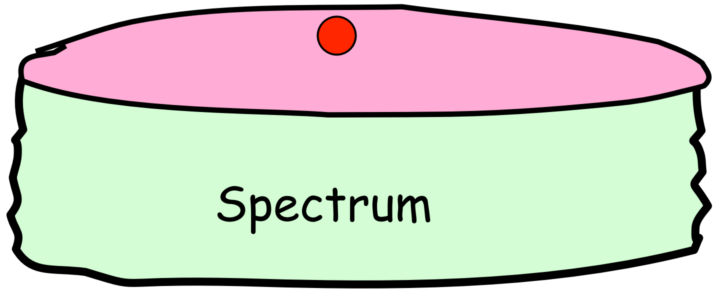

Spectrum



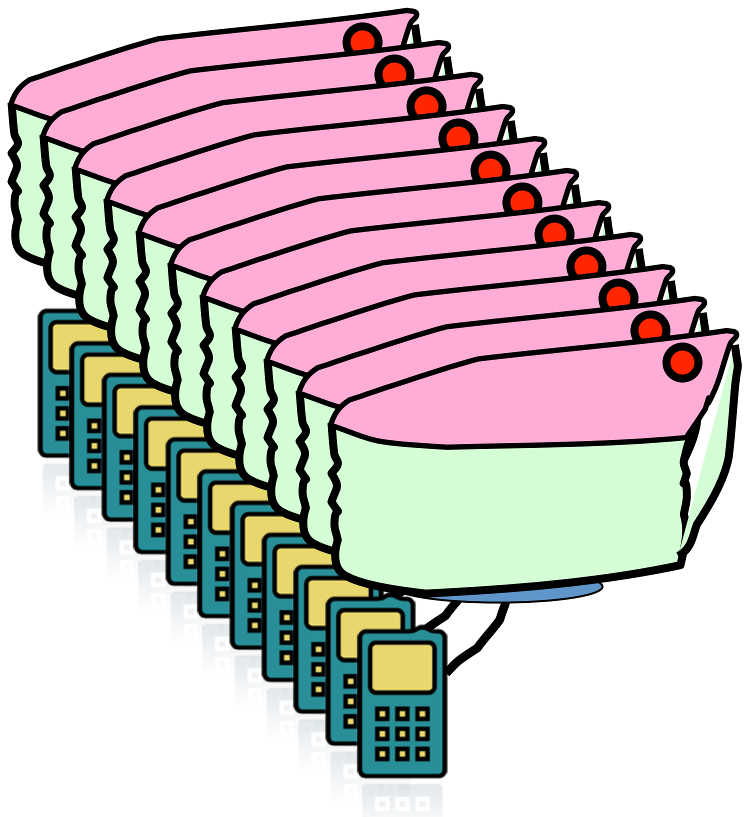


State of Art
Divide spectrum like a cake

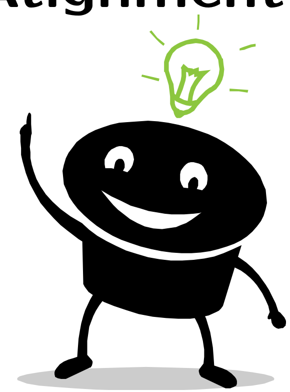


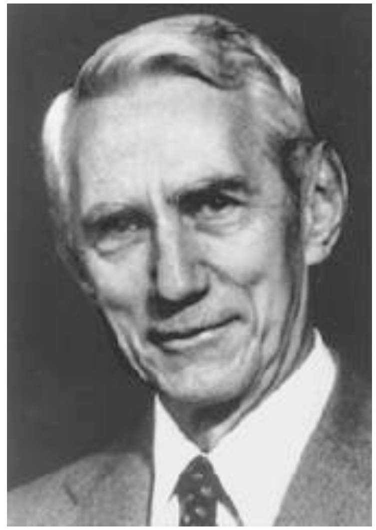


Everyone gets half the cake!!

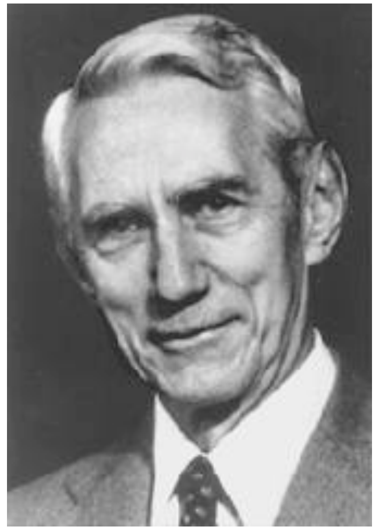


Interference Alignment





Claude E. Shannon



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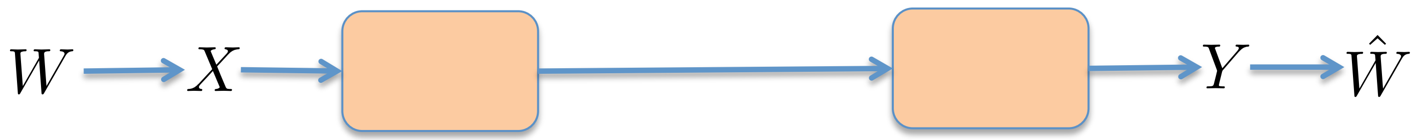
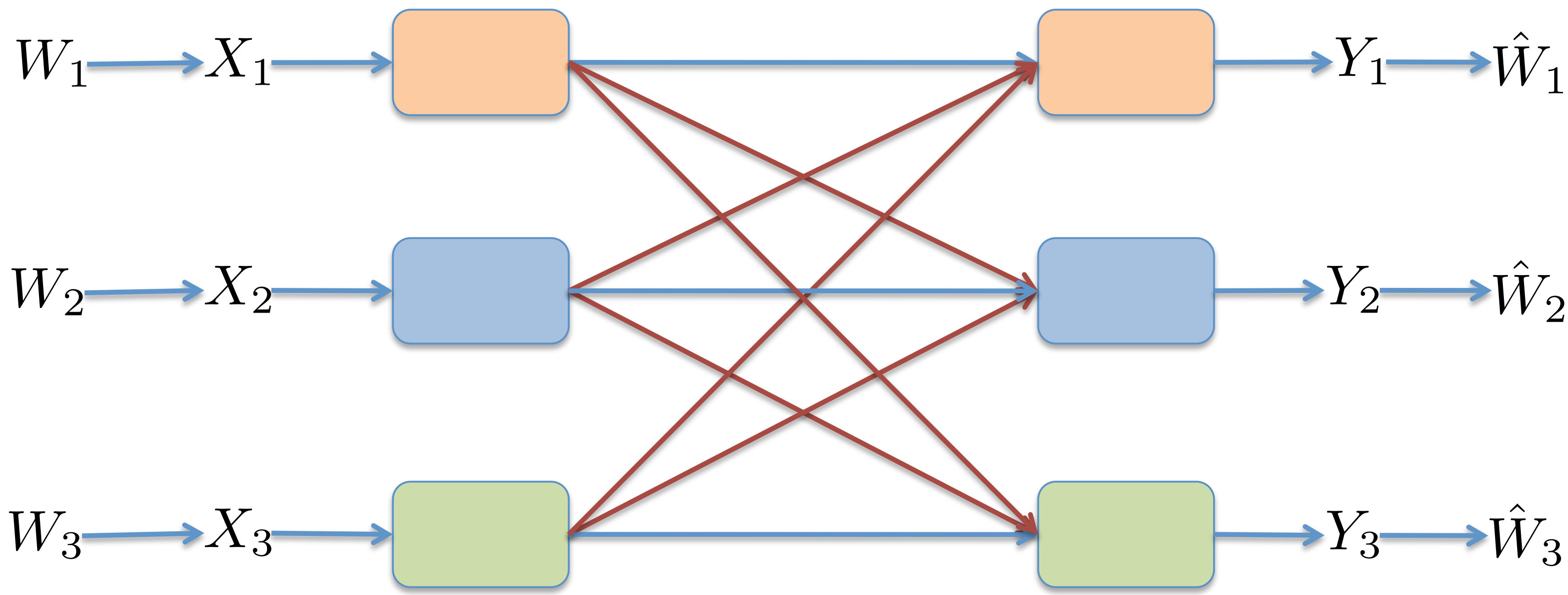


$$Y = X + Z, \quad Z \sim \mathcal{N}(0, \sigma^2)$$

$$\text{Max. Data Rate} = \text{Capacity} = B \log(1 + \text{SNR}) \text{ bits/sec}$$

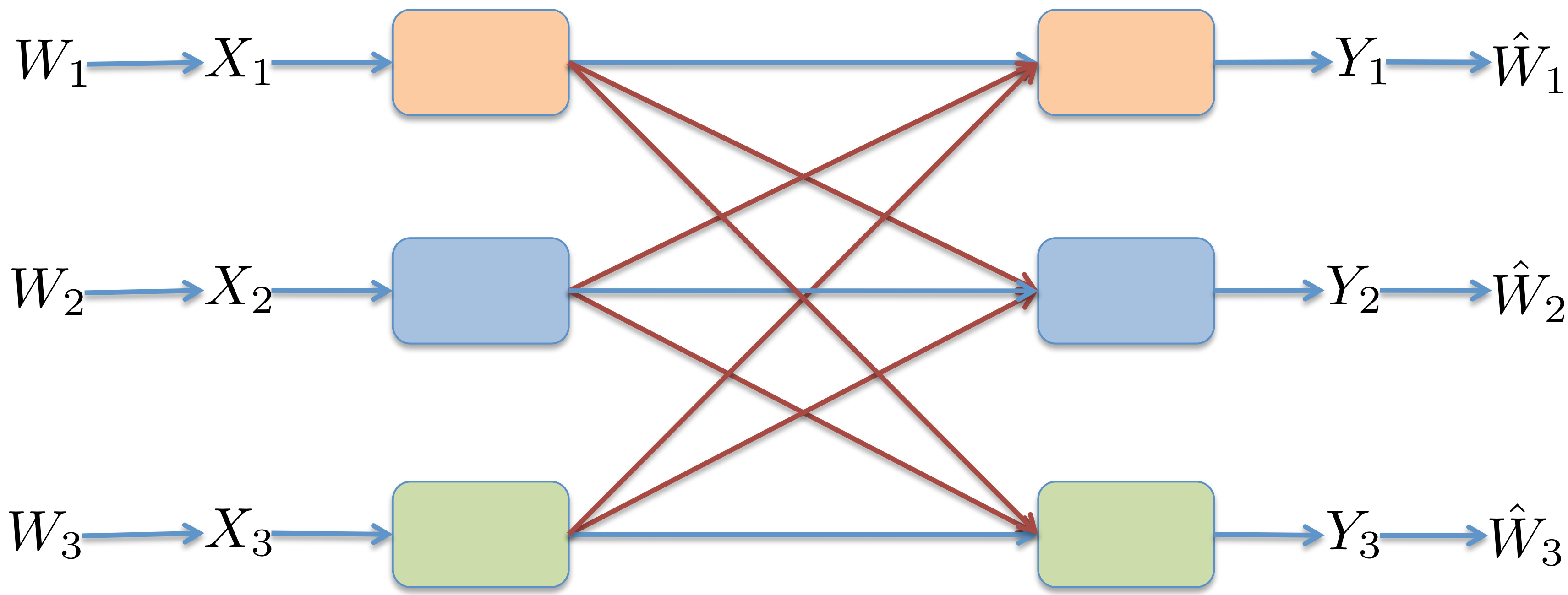
$$B = \text{Bandwidth}$$
$$\text{SNR} = \text{Ratio of Signal Power to Noise Power}$$

K users sharing the same bandwidth and space



$$Y_i = \sum_{j,i} H_{j,i} X_i + Z_i$$

The K-User Interference Network



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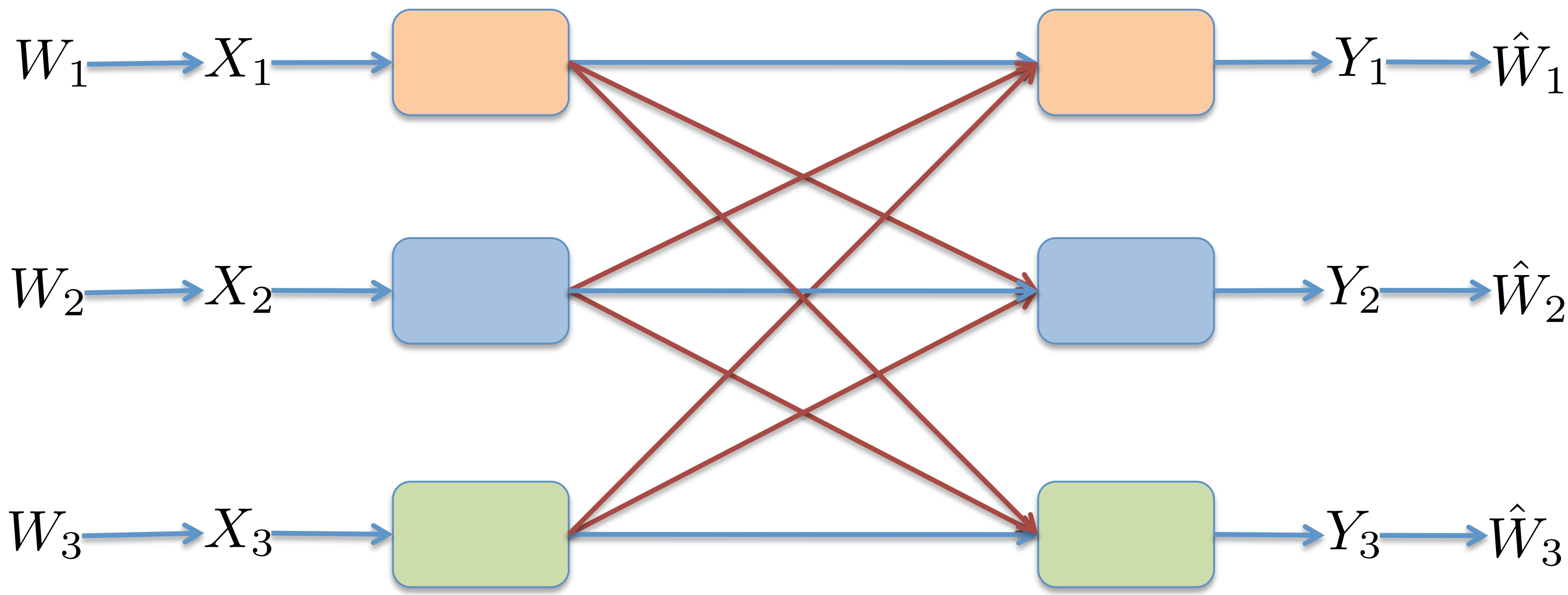


$$Y_i = \sum_{j,i} H_{j,i} X_i + Z_i$$

Capacity, 40 year old open problem, even for K=2!

Let us look at Capacity Approximations!

$$\mathrm{Rate} = B\log(1 + \mathrm{SNR})$$

$$\text{Rate} = B \log(1 + \text{SNR})$$

$$\text{Degrees of Freedom} = \text{Capacity-Prelog}$$

[Zheng-Tse 02]

Degrees of Freedom = Bandwidth Utilization Factor = Capacity-Prelog

[Zheng-Tse 02]

High SNR approximation of Network Capacity

$$\text{Degrees of Freedom} = \lim_{SNR \to \infty} \frac{\text{Network Capacity}}{\text{Single User Capacity}}$$

A network has d degrees of freedom iff

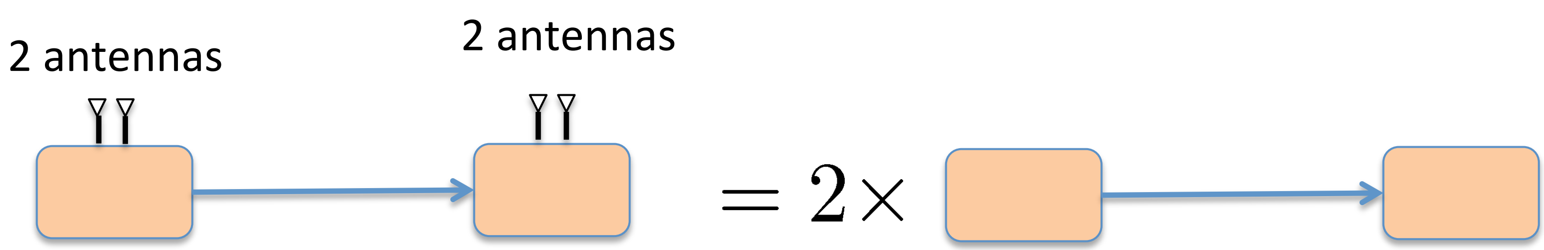
$$C(\text{SNR}) = dB \log(\text{SNR}) + o(\log(\text{SNR}))$$

$$\text{SNR} = \frac{\text{Total Network Signal Power}}{\text{Total Network Noise Power}}$$

B = Total Bandwidth available

De-emphasize noise, emphasize Interference

Intuitively, number of interference-free links (at high SNR).

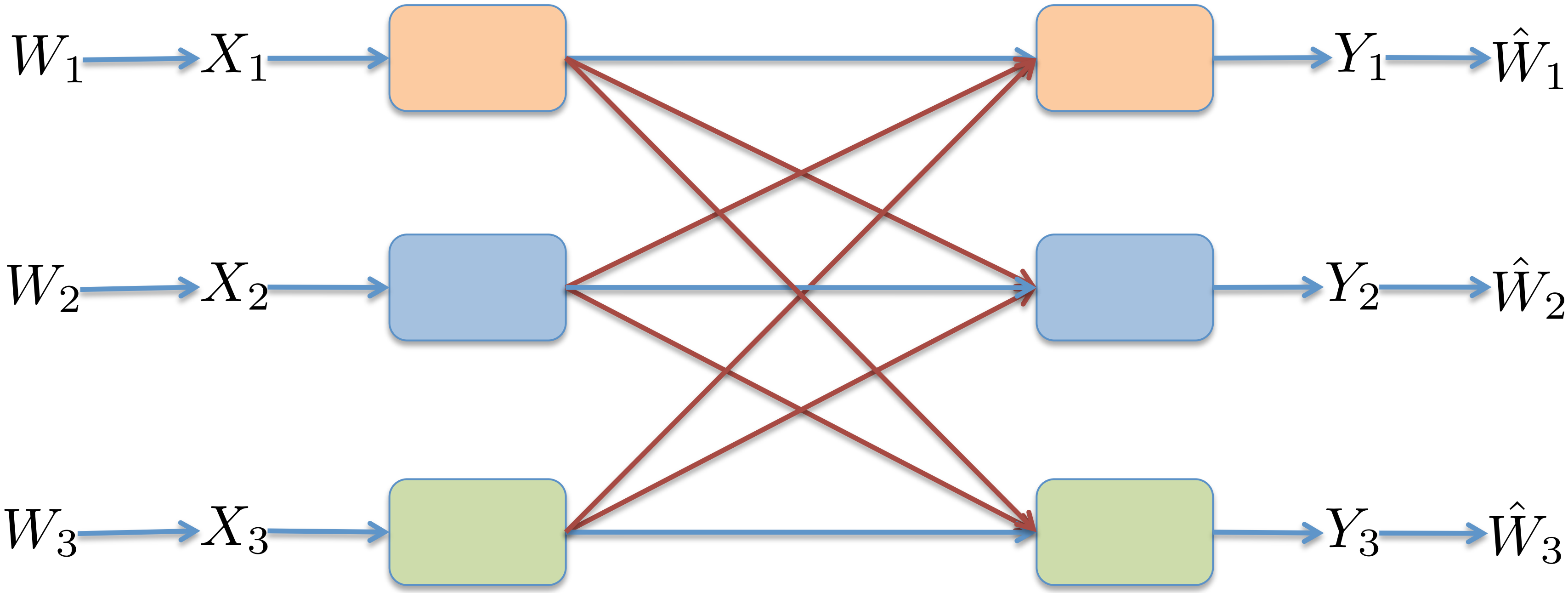


$$Y = HX + Z,$$

$$C(\text{SNR}) = 2 \cdot B \log(\text{SNR}) + o(\log(\text{SNR}))$$

[Telatar 95]

The K-User Interference Network

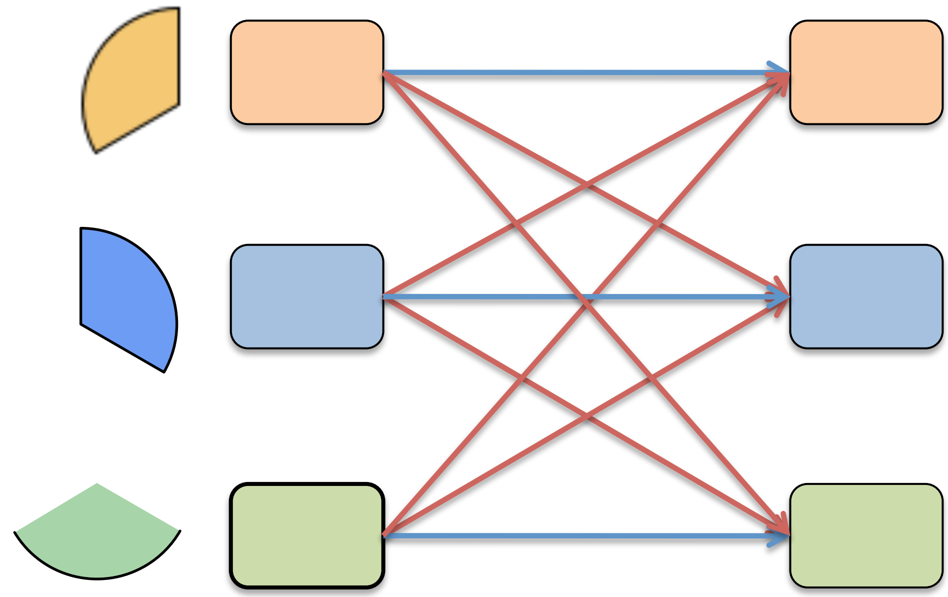


$$Y_i = \sum_{j,i} H_{j,i} X_i + Z_i$$

Question : What is the number of Degrees of Freedom?

The K-User Interference Network - Current Design

Spectrum



Orthogonalization (cake-cutting)

Each user gets $\frac{1}{K}$ of the cake

SUBOPTIMAL

Per user capacity $= \frac{B}{K}\log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$

$$\text{Sum-Rate} = 1 \cdot B\log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$$

~~Conjectured to be DoF optimal~~

[Cadambe-Jafar 08]

Interference Alignment

Each user gets half the cake

$$\text{Per-User Capacity} = \frac{1}{2}B\log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$$

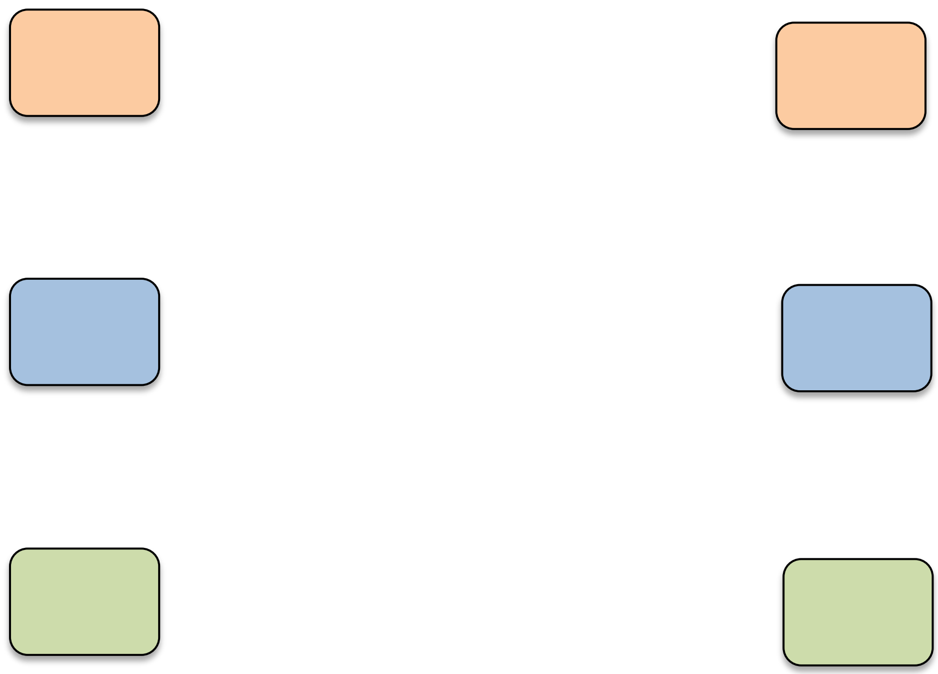
$$\text{Sum-Capacity} = \frac{K}{2}B\log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$$

Cake = Bandwidth-utilization factor of an interference-free user (high SNR).

Toy Example for Interference Alignment

[Cadambe-Jafar 08]

K speaker-listener pairs in a room, K conversations
How long can each conversation be active in one hour ?



Toy Example for Interference Alignment

[Cadambe-Jafar 08]

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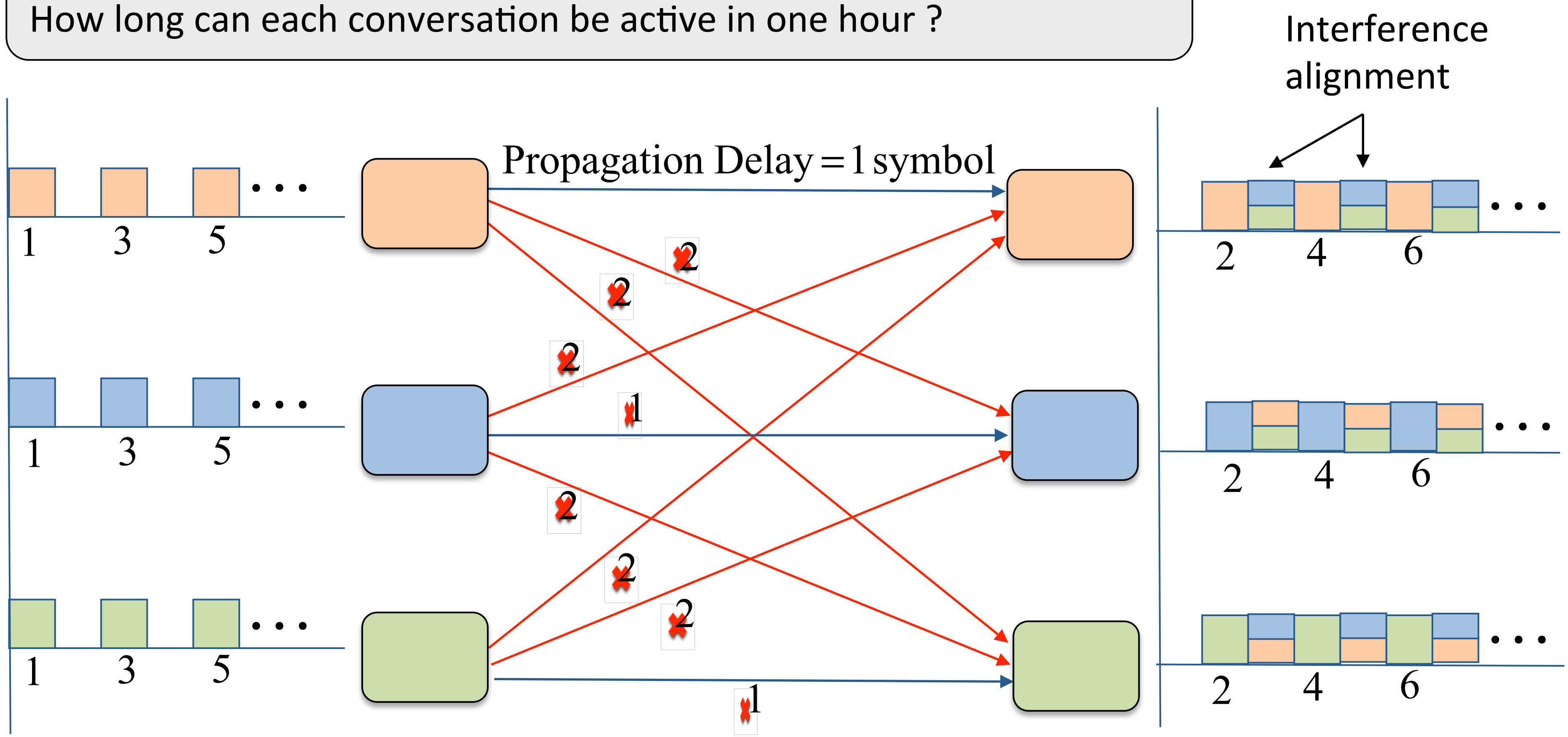


Each user gets half the cake because of interference alignment.

What is Interference Alignment?

[Maddah-Ali-Motahari-Khandani 06, Jafar-Shamai 06]

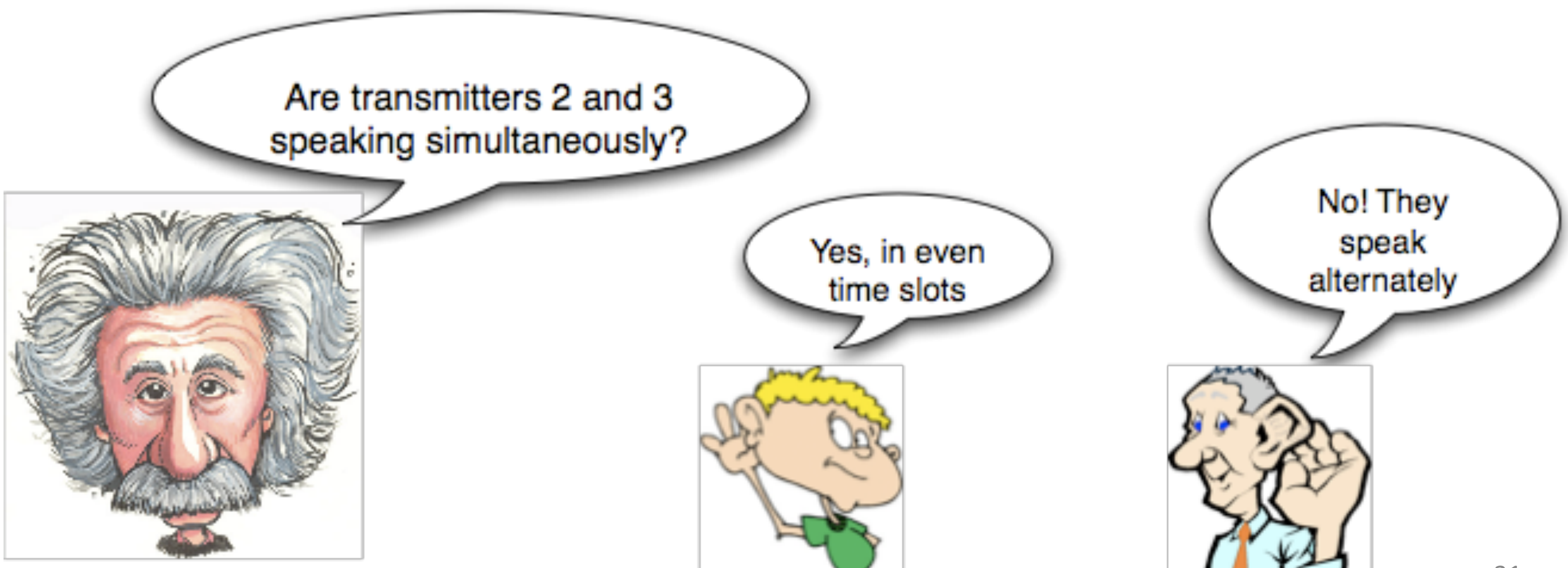
Signals are designed so that, at the receivers

Interfering signals overlap,

Desired signals are distinguishable

Works because

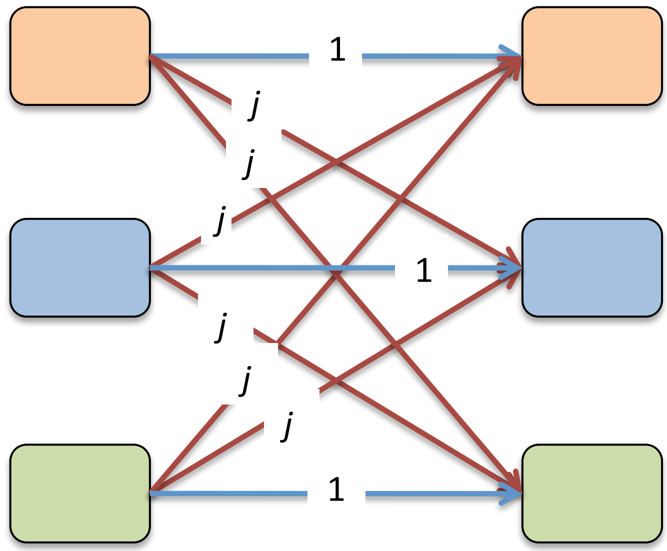
Each receiver sees a different picture



Receiver 1 Receiver 2

Toy Example- Interference Alignment in Phase

[Cadambe-Jafar 08]

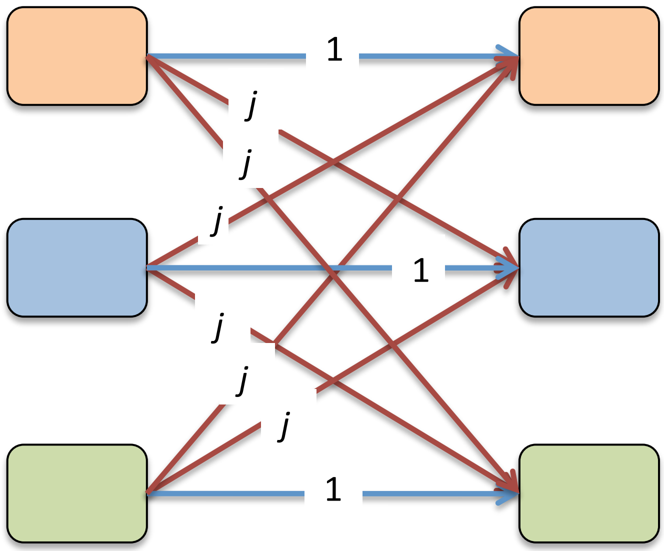


$j = \sqrt{-1}$

Single-User Capacity $= B \log(1 + \text{SNR})$, Cake$=B$

Toy Example- Interference Alignment in Phase

[Cadambe-Jafar 08]



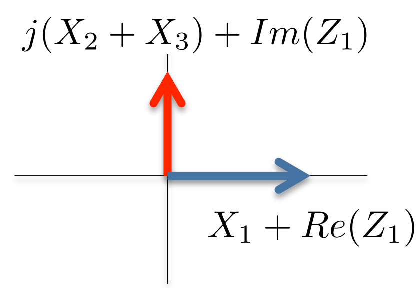
$j = \sqrt{-1}$

$$\text{Single-User Capacity} = B \log(1 + \text{SNR}), \text{ Cake} = B$$

Transmit Real Symbols.

Interference Aligns along imaginary dimension.

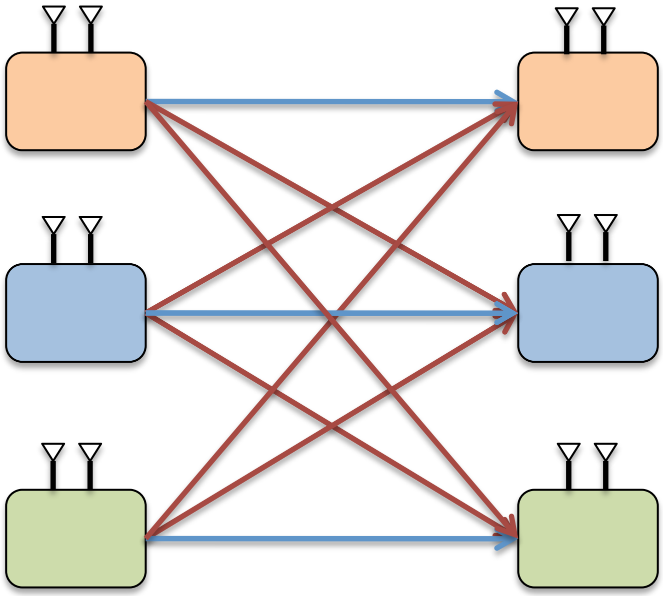
Each user gets half the cake!



$$\text{per-user-capacity} = \frac{1}{2} B \log(\text{SNR}) + o(\log(\text{SNR}))$$

3 Users, 2 antennas at each transmitter and receiver

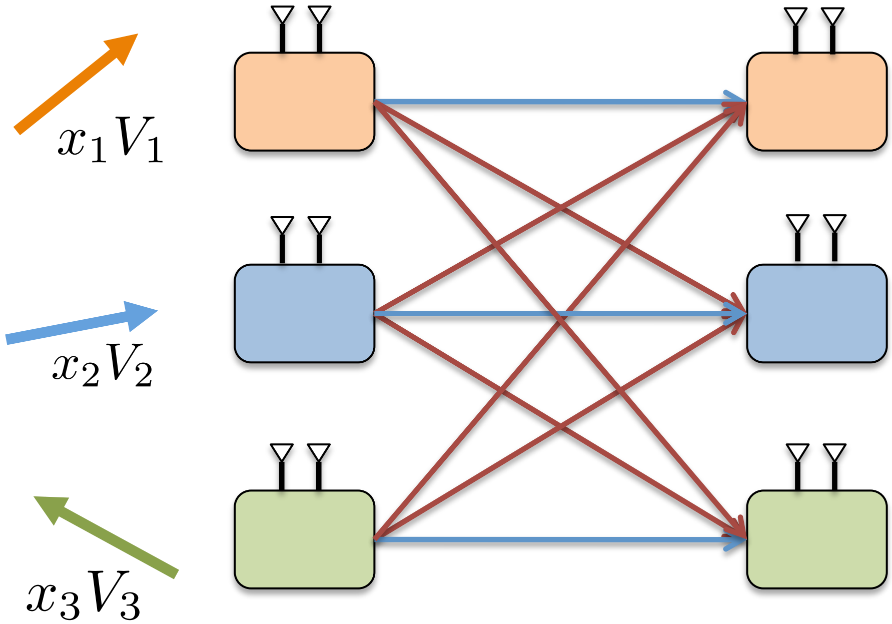
[Cadambe-Jafar 08]



Single-User Capacity $= 2B\log(\mathrm{SNR}) + o(\log(\mathrm{SNR}))$, Cake $= 2B$

3 Users, 2 antennas at each transmitter and receiver

[Cadambe-Jafar 08]



Single-User Capacity $= 2B\log(\text{SNR}) + o(\log(\text{SNR}))$, Cake $= 2B$

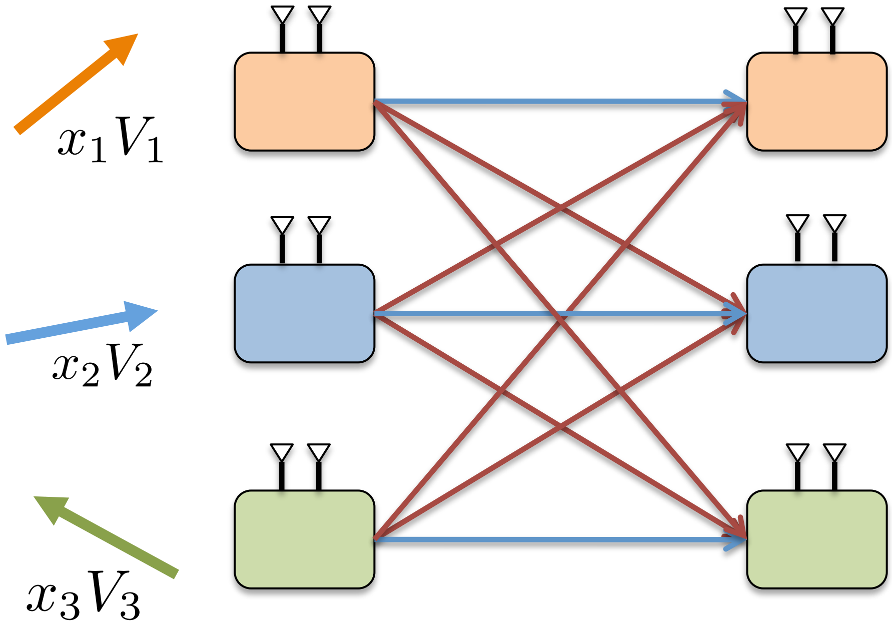
With Interference Alignment,

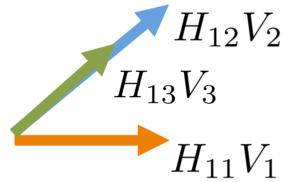
Each user gets half the cake!

Each user gets $1 \cdot B\log(SNR) + o(\log(\text{SNR}))$

3 Users, 2 antennas at each transmitter and receiver

[Cadambe-Jafar 08]





$$H_{12}V_2 = \lambda_1 H_{13}V_3$$

Single-User Capacity $= 2B\log(\text{SNR}) + o(\log(\text{SNR}))$, Cake $= 2B$

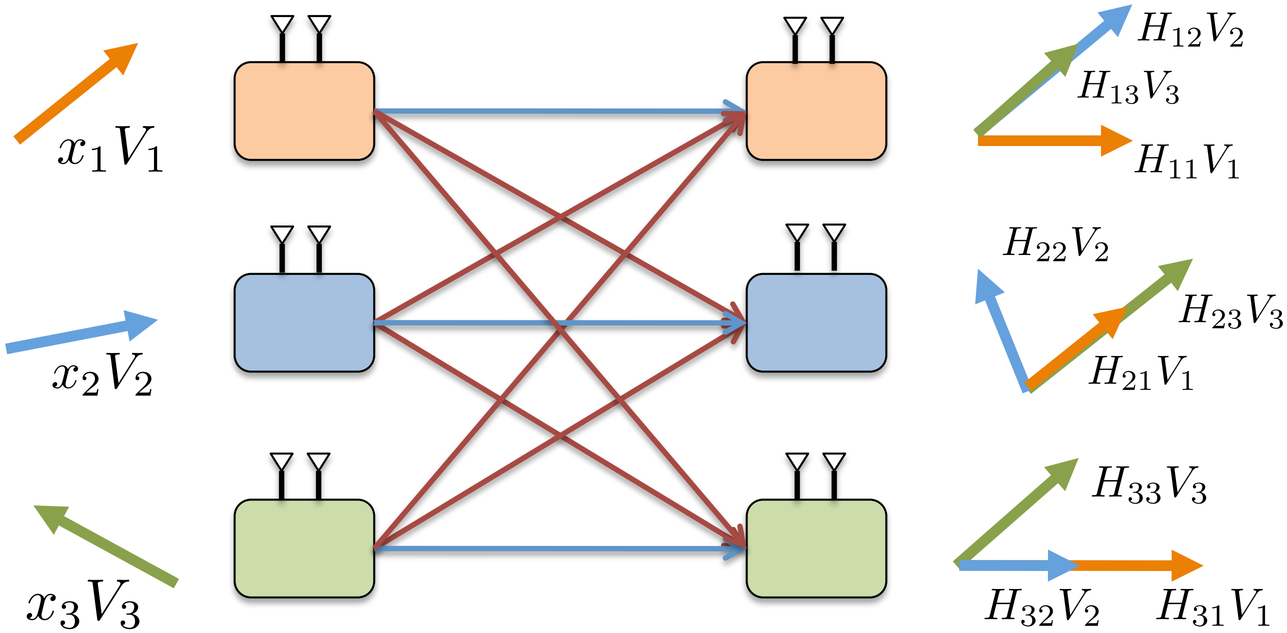
With Interference Alignment,

Each user gets half the cake!

Each user gets $1 \cdot B\log(SNR) + o(B\log(\text{SNR}))$

3 Users, 2 antennas at each transmitter and receiver

[Cadambe-Jafar 08]



$$H_{12}V_2 = \lambda_1 H_{13} V_3$$

$$H_{21}V_1 = \lambda_2 H_{23} V_3$$

$$H_{31}V_1 = \lambda_3 H_{32} V_2$$

Single-User Capacity $= 2B\log(\text{SNR}) + o(\log(\text{SNR}))$, Cake $= 2B$

With Interference Alignment,

Each user gets half the cake!

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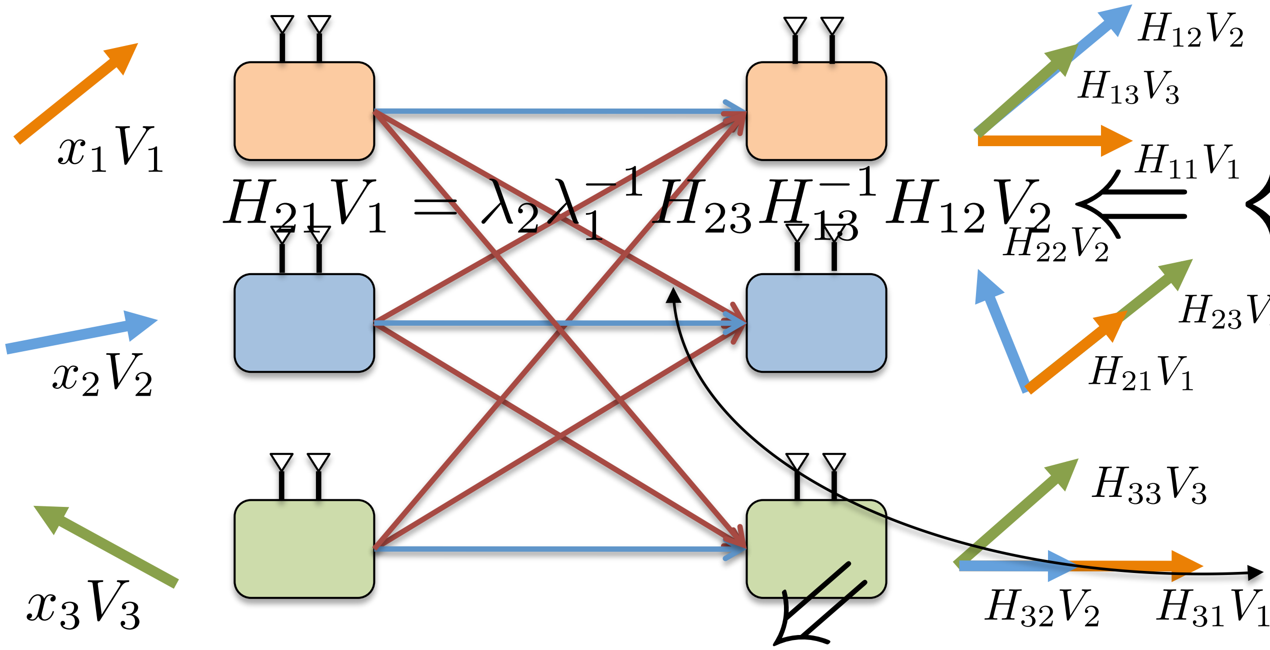
3 Users, 2 antennas at each transmitter and receiver

[Cadambe-Jafar 08]



$$H_{21}V_1 = \lambda_2\lambda_1^{-1}H_{23}H_{13}^{-1}H_{12}V_2 \Longleftarrow$$

$$\left\{ \begin{array}{l} H_{12}V_2 = \lambda_1 H_{13}V_3 \\ H_{21}V_1 = \lambda_2 H_{23}V_3 \end{array} \right.$$

$$H_{31}V_1 = \lambda_3 H_{32}V_2$$

$$V_1 = \lambda\, TV_1$$

$$T = H^{-1} H \, H^{-1} H \, H^{-1} H$$

Single-User Capacity $= 2B\log(\text{SNR}) + o(\log(\text{SNR}))$, Cake $= 2B$

V_1 eigen-vector of T

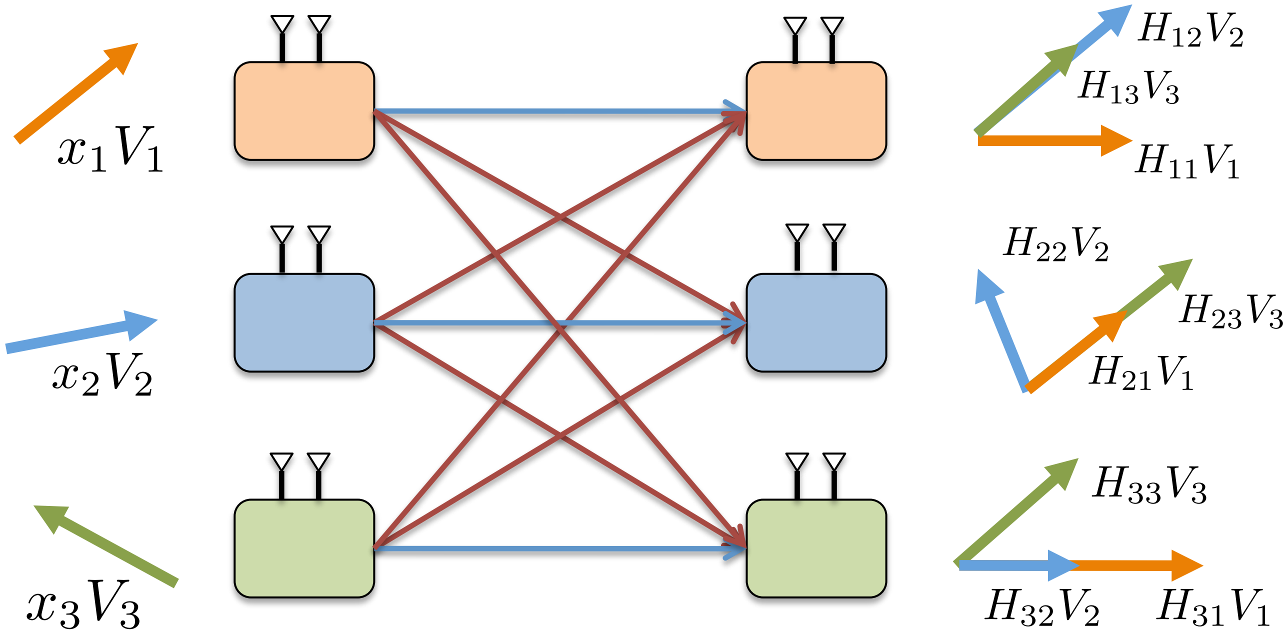
With Interference Alignment,

Each user gets **half** the cake!

Each user gets $1 \cdot B\log(SNR) + o(B\log(\text{SNR}))$

3 Users, 2 antennas at each transmitter and receiver

[Cadambe-Jafar 08]



$$H_{12}V_2 = \lambda_1 H_{13}V_3$$

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Single-User Capacity $= 2B\log(\text{SNR}) + o(\log(\text{SNR}))$, Cake $= 2B$

With Interference Alignment,

Each user gets half the cake!

Each user gets $1 \cdot B\log(SNR) + o(B\log(\text{SNR}))$

What about more than 3 users?

For every set of 3 users,

$$V_1 = \lambda \, T V_1$$

What about more than 3 users?

$$\text{span}(V) = \text{span}(T_1 V)$$
$$\text{span}(V) = \text{span}(T_2 V)$$
$$\vdots$$
$$\text{span}(V) = \text{span}(T_N V)$$

$$\text{rank}(V) = \frac{\text{rank}(T_i)}{2}$$

V invariant subspace of $T_1, T_2, .., T_N$

Infeasible!

How about finding approximately invariant subspaces?

Asymptotic Interference Alignment [Cadambe-Jafar 08]

$$\mathrm{span}(V) \approx \mathrm{span}(T_1 V)$$
$$\mathrm{span}(V) \approx \mathrm{span}(T_2 V)$$
$$\vdots$$
$$\mathrm{span}(V) \approx \mathrm{span}(T_N V)$$

$$\mathrm{rank}(V) = \frac{\mathrm{rank}(T_i)}{2}$$

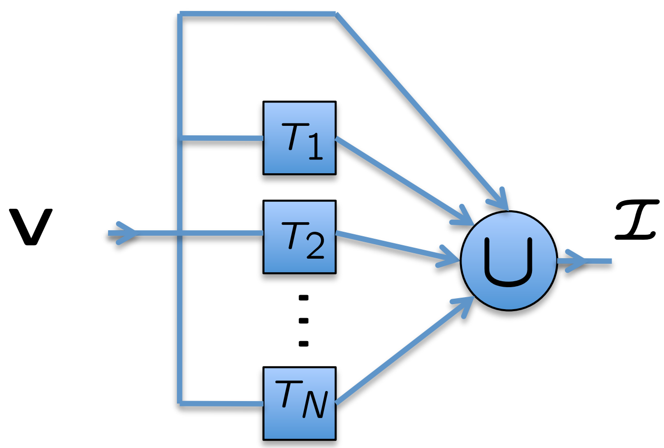
Asymptotic Solution $\rightarrow$ Diagonal T_is of arbitrarily large dimension

Diagonal T_is $\rightarrow$ Code over multiple frequencies (Vector Coding)
Frequency-Selective Channels

Goal: Simultaneously satisfy "N" Alignment Constraints:
$\text{span}(\mathbf{V}) \equiv \text{span}(T_1\mathbf{V}) \equiv \text{span}(T_2\mathbf{V}) \equiv \cdots \equiv \text{span}(T_N\mathbf{V})$

[Cadambe, Jafar, IT08]

Goal: Make $\mathbf{V} \equiv \mathcal{I}$



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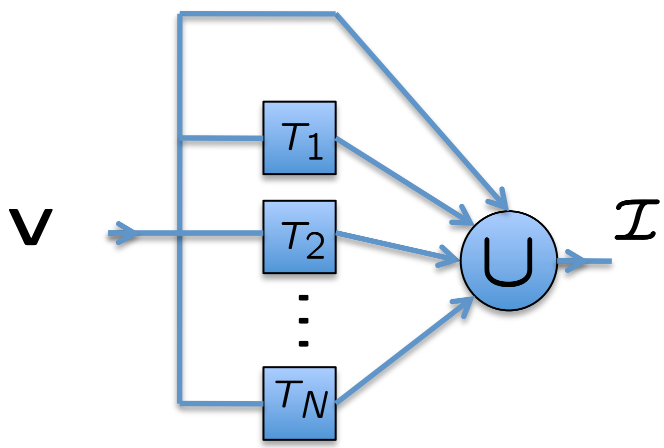

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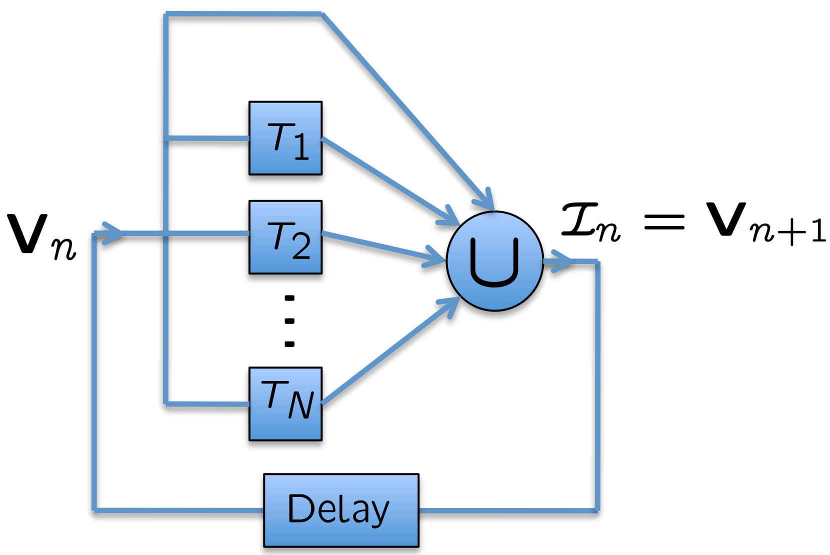

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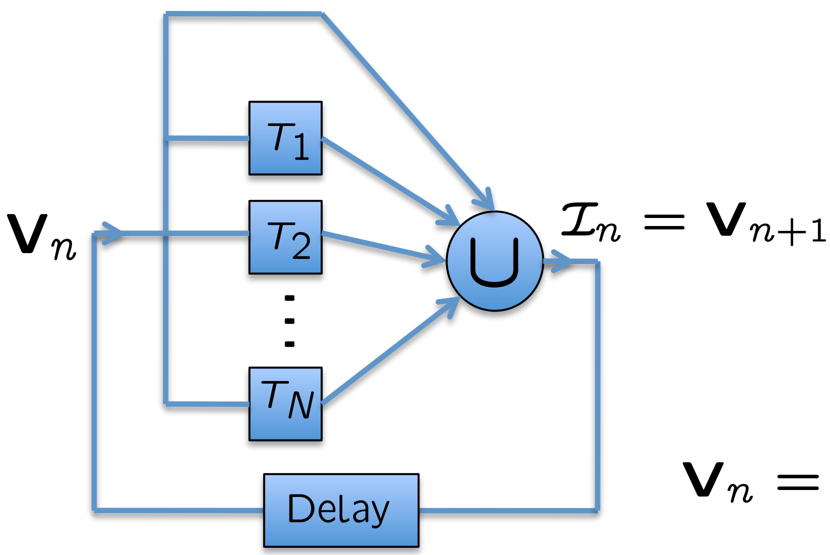


Initialize: $\mathbf{V}_0 = \mathbf{1}$

$$\mathbf{V}_1 = \{\mathbf{1}, T_1\mathbf{1}, \cdots, T_N\mathbf{1}\}$$

$$\mathbf{V}_2 = \{\mathbf{1}, \cdots, T_i\mathbf{1}, \cdots, T_iT_j\mathbf{1}, \cdots, T_i^2\mathbf{1}\}$$

$$\mathbf{V}_n = \{T_1^{\alpha_1}T_2^{\alpha_2}\cdots T_N^{\alpha_N}\mathbf{1}, \alpha_1 + \cdots + \alpha_N \leq n\}$$

$$\mathbf{V}_{n+1} = \{T_1^{\alpha_1}T_2^{\alpha_2}\cdots T_N^{\alpha_N}\mathbf{1}, \alpha_1 + \cdots + \alpha_N \leq n+1\}$$

$$|\mathbf{V}_n| = \binom{n+N}{n}$$

$$|\mathcal{I}| = \binom{n+N+1}{n+1}$$

$$\frac{|\mathbf{V}|}{|\mathcal{I}|} = \frac{n+1}{n+N+1} \to 1 \text{ as } n \to \infty$$

Key : T_is commute

Asymptotic Interference Alignment [Cadambe-Jafar 08]

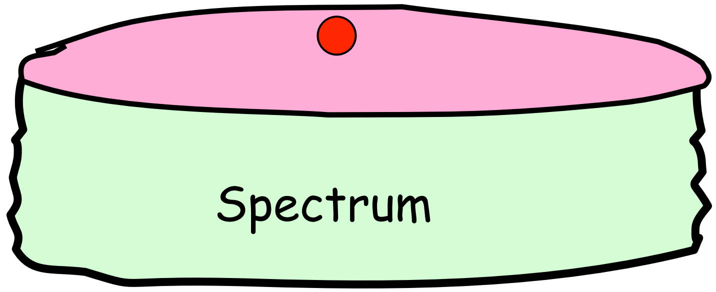
$$\begin{aligned}\text{span}(V) &\approx \text{span}(T_1 V)\\ \text{span}(V) &\approx \text{span}(T_2 V)\\ &\vdots\\ \text{span}(V) &\approx \text{span}(T_N V)\end{aligned}$$

$$\text{rank}(V) = \frac{\text{rank}(T_i)}{2}$$

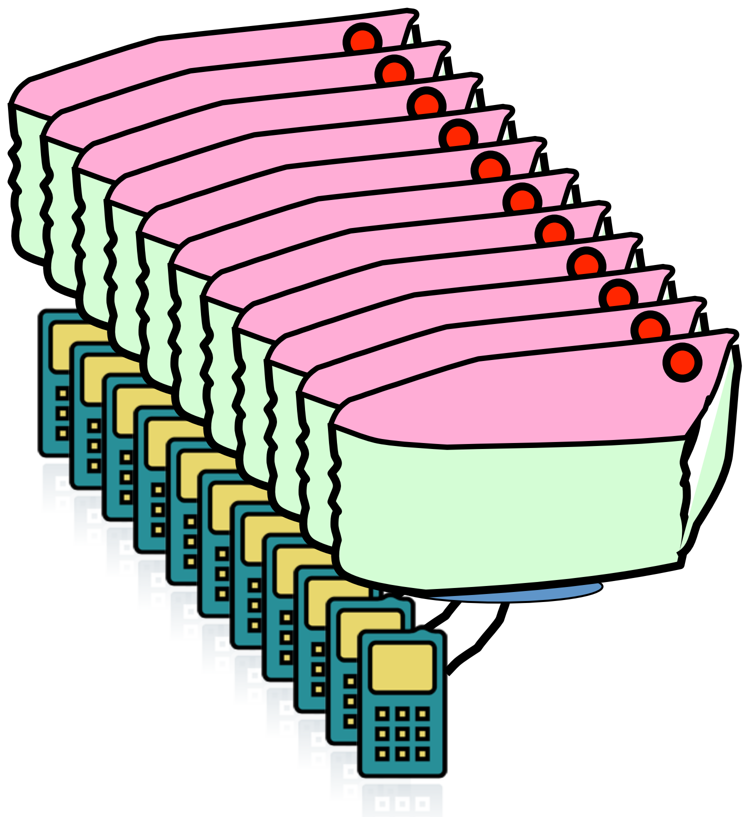
Asymptotic Solution $\rightarrow$ Diagonal T_is of arbitrarily large dimension

Diagonal T_is $\rightarrow$ Code over multiple frequencies (Vector Coding)
Frequency-Selective Channels

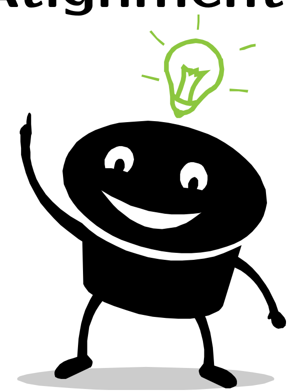
Can be done over constant (non-frequency-selective) channels by exploiting rational dimensions [Motahari et. al. 09]



Everyone gets half the cake!!



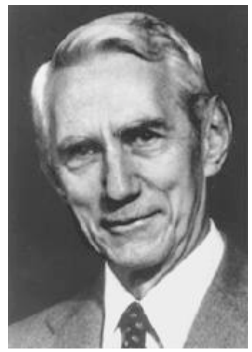
Interference Alignment



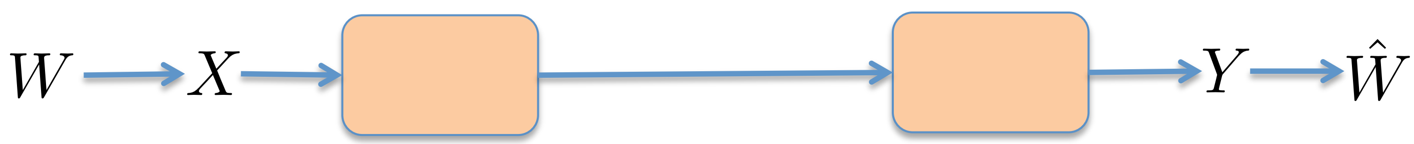
Asymptotic Alignment scheme widely applicable

X channels [Cadambe-Jafar 09], MIMO Int. Channels [Gou-Jafar 10], Compound Broadcast and Interference Networks [Wang et. al 10], Distributed Storage Problems [Cadambe-Jafar-Maleki 10, Suh-Ramchandran 10], Network Coding, [Ramkrishnan et. al. 10]

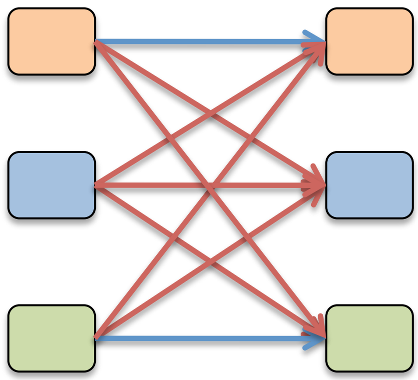
Design principles of current wireless systems inspired by


Claude E. Shannon

information theory of single user systems



Theoretical foundations of today's networks

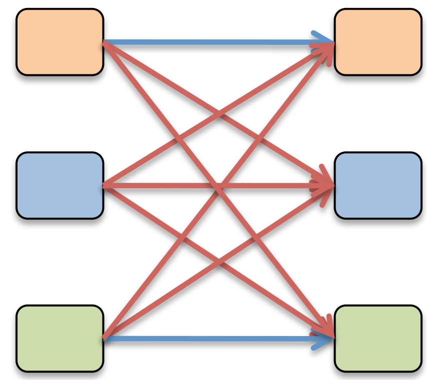


Random Codes

Circularly Symmetric Signaling

Multicarrier Systems,
Whole = sum of parts

New Theoretical Insights for Networks because of alignment



Structured Codes*

Asymmetric Signaling*

Multicarrier Systems,
Whole > sum of parts

*[Cadambe-Jafar-Shamai 08, Nazer-Gastpar 08, Bresler-Parekh-Tse 07, Cadambe-Jafar 09, Cadambe-Jafar-Wang 09, Cadambe-Jafar-08, Shankar et. al 08, Cadambe-Jafar 10]

Single User Channels versus Multi-User Channels

Main Challenge in Single-User Channels - Robustness to Noise

Insight : Use Random codes. [Shannon 1948]

(Eg. Turbo Codes)

Main Challenge in Multi-User Channels - Robustness to Interference

Insight : Interference Alignment, Structured codes!

(Eg. Lattice Codes)

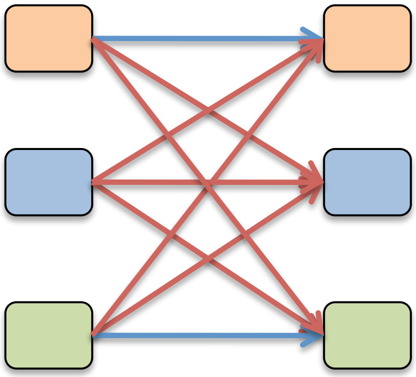
Random codes do not align interference

[Cadambe-Jafar-Shamai Trans. IT 08, Nazer-Gastpar 08, Bresler-Parekh-Tse Trans. IT 10]

except under special circumstances.

[Cadambe-Jafar Allerton-10]

Theoretical foundations of today's networks



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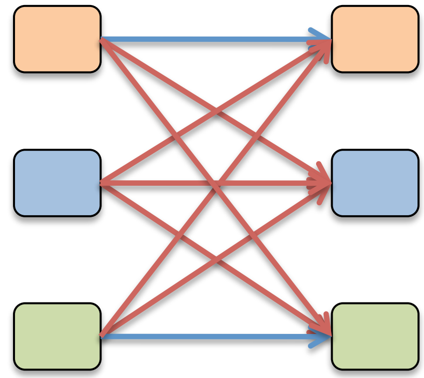

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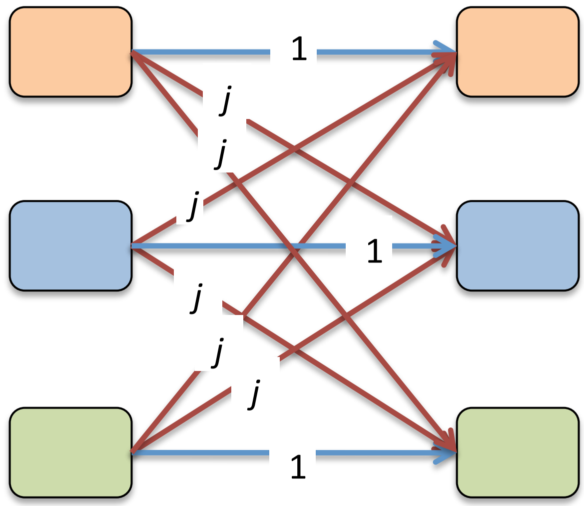
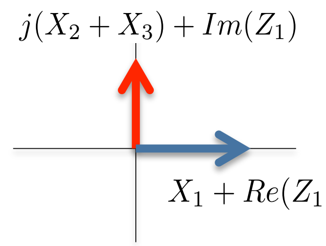
Interference Alignment in Phase



$j = \sqrt{-1}$

Transmit Real Symbols.

Interference Aligns along imaginary dimension.



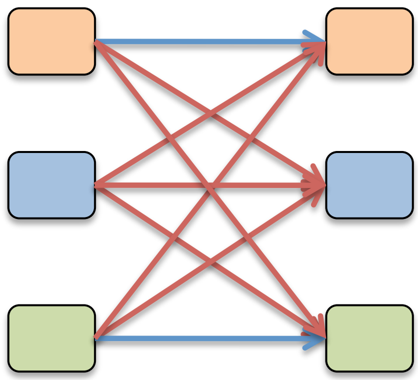
Unlike single user channels,

Asymmetric Complex Signals are useful in aligning interference

Characterized the gains of asymmetric complex signaling for networks with *random* channels

[Cadambe-Jafar-Wang Trans. IT 10]

Theoretical foundations of today's networks



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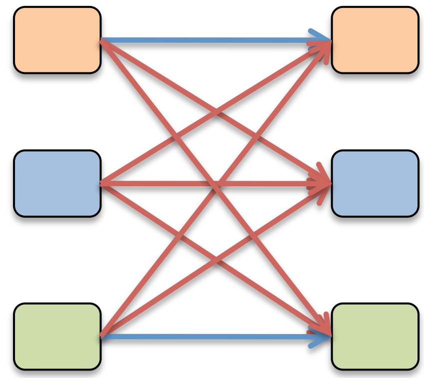

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Asymmetric Signaling*

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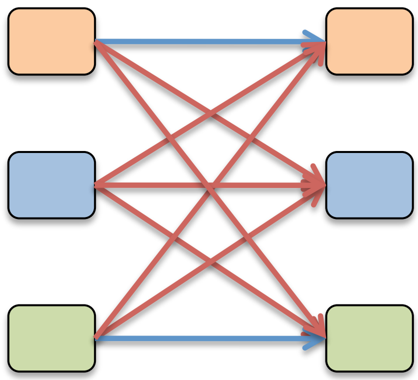

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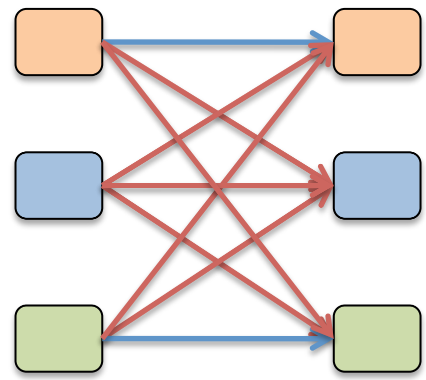

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The road ahead - making Interference Alignment Practical

Interference Alignment

faces challenges of

Limited SNR, Channel State Information,
Synchronization, Signaling Dimensions

The road ahead - making Interference Alignment Practical

Distributed*

Retrospective*

Ergodic*

Interference Alignment

Delay-based*

Subspace*

Blind*

faces challenges of

Limited SNR, Channel State Information,
Synchronization, Signaling Dimensions

*[Cadambe-Jafar 07, 08, Suh-Tse 08,Grokop-Tse-Yates 08, Nazer et. al. 09, Jafar 09, Wang-Guo-Jafar 10, Maddah-Ali-Tse 10, Maleki-Jafar-Shamai 10]

The road ahead - making Interference Alignment Practical

Distributed*

Retrospective*

Ergodic*

Interference Alignment

Delay-based*

Subspace*

Blind*

Feedback

Relays

Co-operation

Transmit/receive co-operation

Wired Backhauls

face challenges of

Limited SNR, Channel State Information, Synchronization, Signaling Dimensions

*[Cadambe-Jafar 07, 08, Suh-Tse 08,Grokop-Tse-Yates 08, Nazer et. al. 09, Jafar 09, Wang-Guo-Jafar 10, Maddah-Ali-Tse 10, Maleki-Jafar-Shamai 10]

Network Interference Management via Interference Alignment

(for Wireless Communications and Distributed Storage)

Viveck R. Cadambe

University of California, Irvine

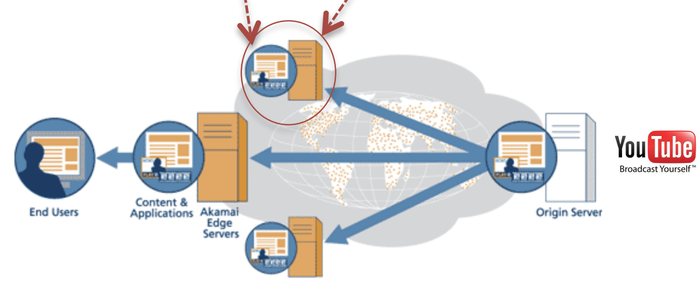
Servers (RAID-type systems)

Servers (RAID-type systems)

Data Centers

Servers (RAID-type systems)

Data Centers

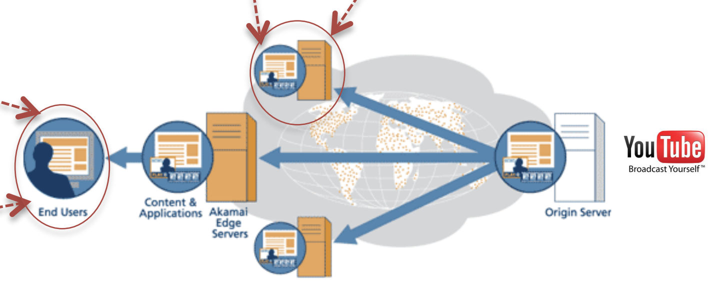


Content Delivery Networks

Servers (RAID-type systems)

Data Centers

Peer-to-Peer Storage



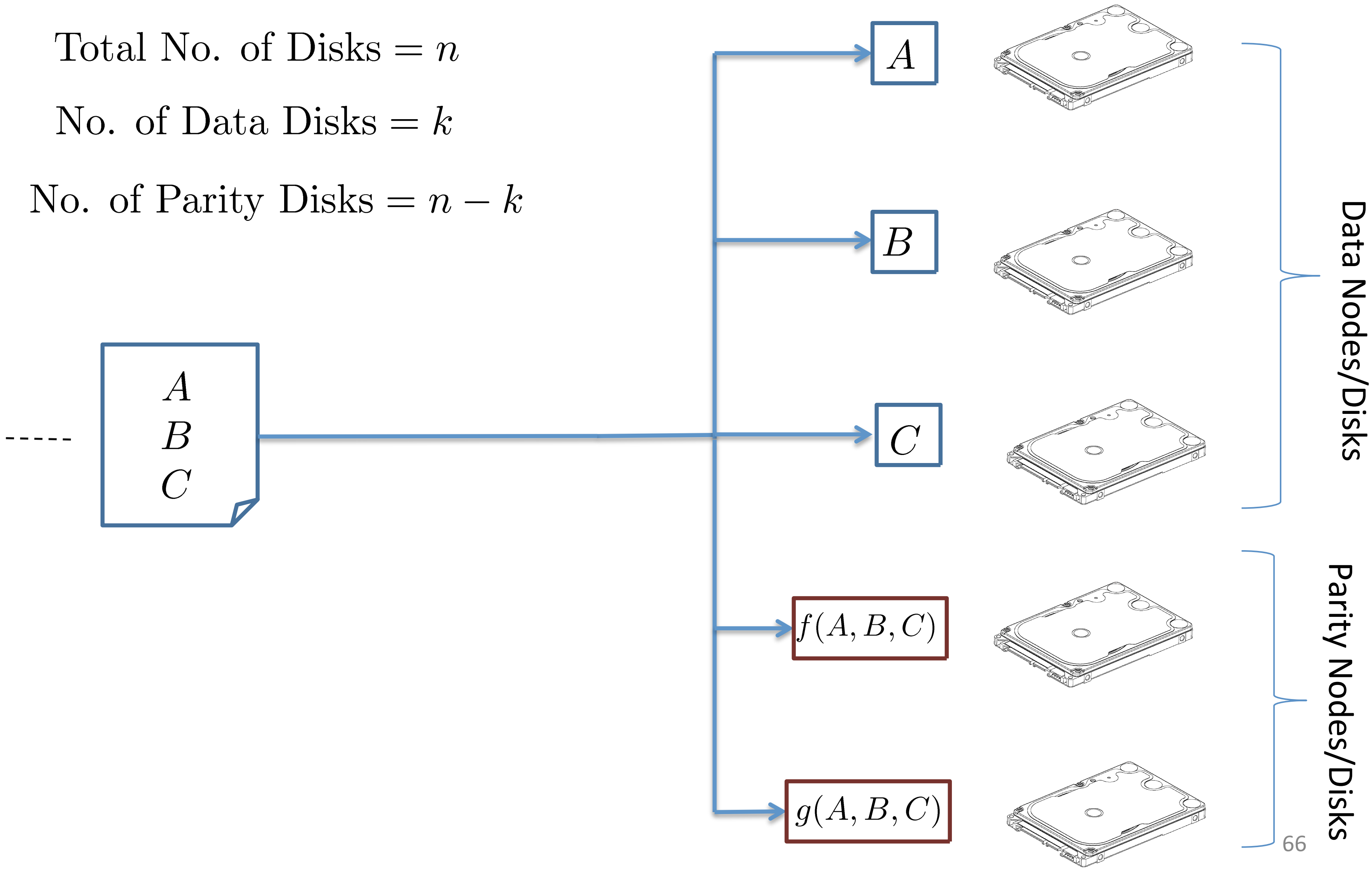
Content Delivery Networks

Distributed Storage System

Total No. of Disks $= n$

No. of Data Disks $= k$

No. of Parity Disks $= n - k$



Trade-offs in Distributed Storage Systems

Reliability v/s  **Cost of Storage**

 "My data is my life…"

Replication widely used today.

Use Erasure Coding

Replication

$n = 4$

$k = 2$

$A, B \rightarrow 1$ unit

A

B

A

B

MDS* Codes

A

B

$A + B$

$A + 2B$

1 Disk Failure	Download 1 unit	Download 2 unit
2 Disk Failure	Data Loss	Download 2 unit

*MDS = Maximum Distance Separable

Replication

$n = 4$

$k = 2$

$A, B \rightarrow 1$ unit

A

B

A

B

MDS* Codes

A

B

$A + B$

$A + 2B$

	Replication	MDS* Codes
1 Disk Failure	Download 1 unit	Download 2 unit
2 Disk Failure	Data Loss	Download 2 unit

*MDS = Maximum Distance Separable

(n, k) MDS Code - Pros and Cons

1 Disk Fails	Download k units
$n-k$ Disks Fail	Download k units

Highest redundancy (from worst-case failure perspective)

Slow Recovery of single disk failure

- Single node failure, most common failure scenario
- Recovery important before other nodes fail, so speed of recovery matters.
- Avoid Network Congestion!

(n, k) MDS Code - Pros and Cons

1 Disk Fails	Download k units

Reduced? If so, how small? [Dimakis et. al 08, Wu-Dimakis 09]

Special case of multi-source (non-multicast) network coding

(n, k) MDS Code - Pros and Cons

1 Disk Fails	Download k units

Reduced? If so, how small? [Dimakis et. al 08, Wu-Dimakis 09]

Special case of multi-source (non-multicast) network coding

Yes*, download only $\frac{n-1}{n-k} < k$ units

[Cadambe-Jafar-Maleki 10, Suh-Ramchandran 10]

Matches cut-set bound of [Dimakis et. al. 08]

$(n = 9, k = 6)$ Speed of repair $\rightarrow$ doubly faster

*[Suh-Ramchandran 09, Shah et. al. 08] previously solved it for $n \geq k/2$

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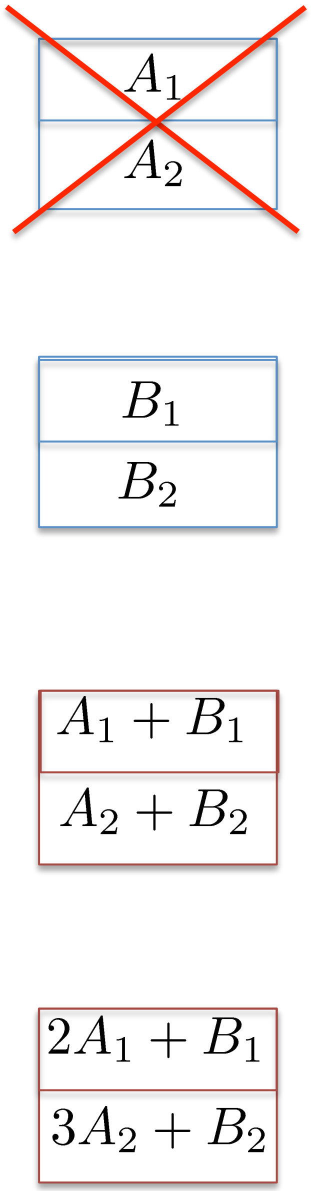
$(n = 9, k = 6)$ Speed of repair $\rightarrow$ doubly faster

Use Interference Alignment, Vector Coding

*[Suh-Ramchandran 09, Shah et. al. 08] previously solved it for $n \geq k/2$

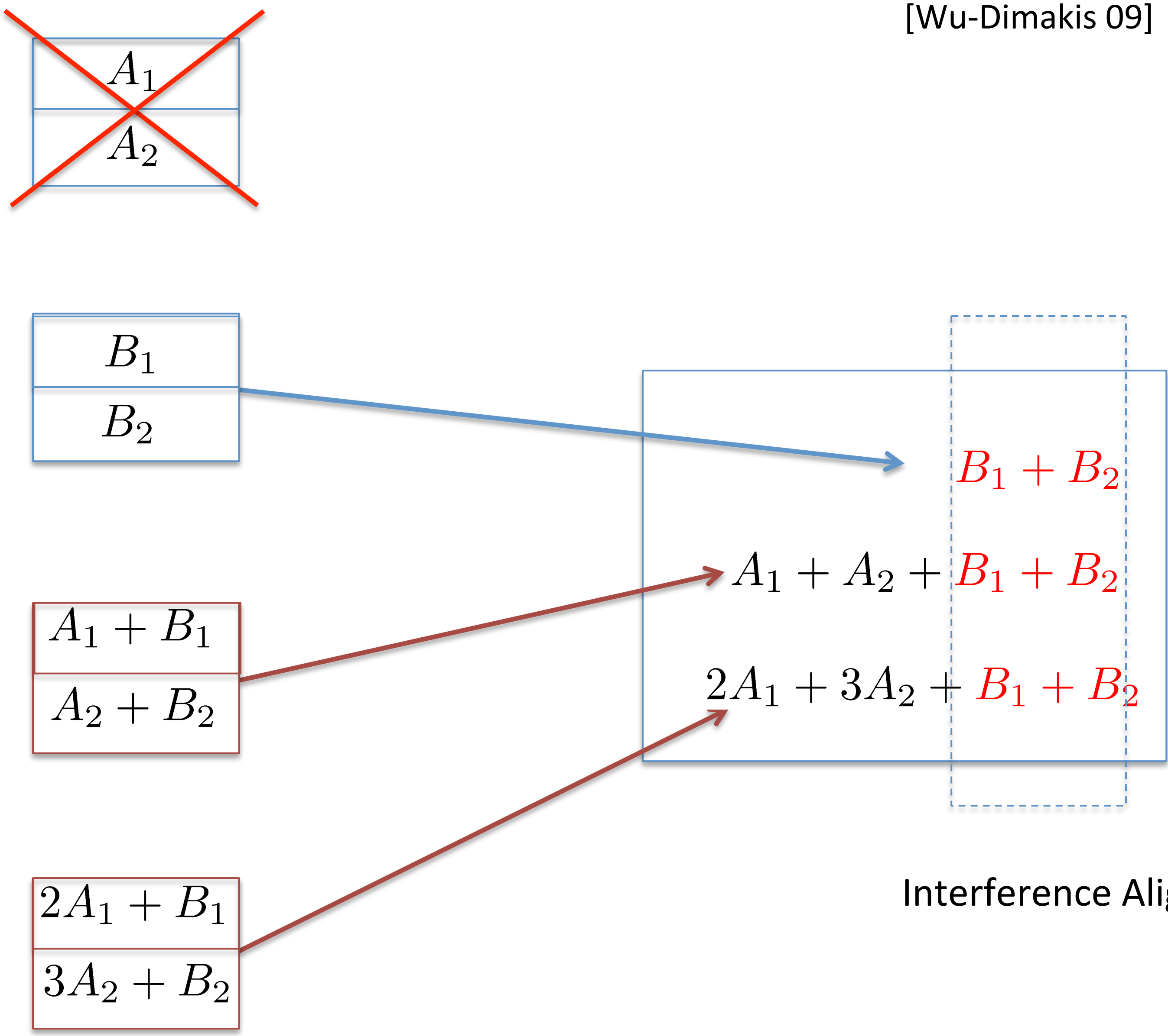
$$n = 4, k = 2$$ [Wu-Dimakis 09]



Trivial Repair : 4 linear combinations

$n = 4, k = 2,$ Repair with 3 Linear Combinations

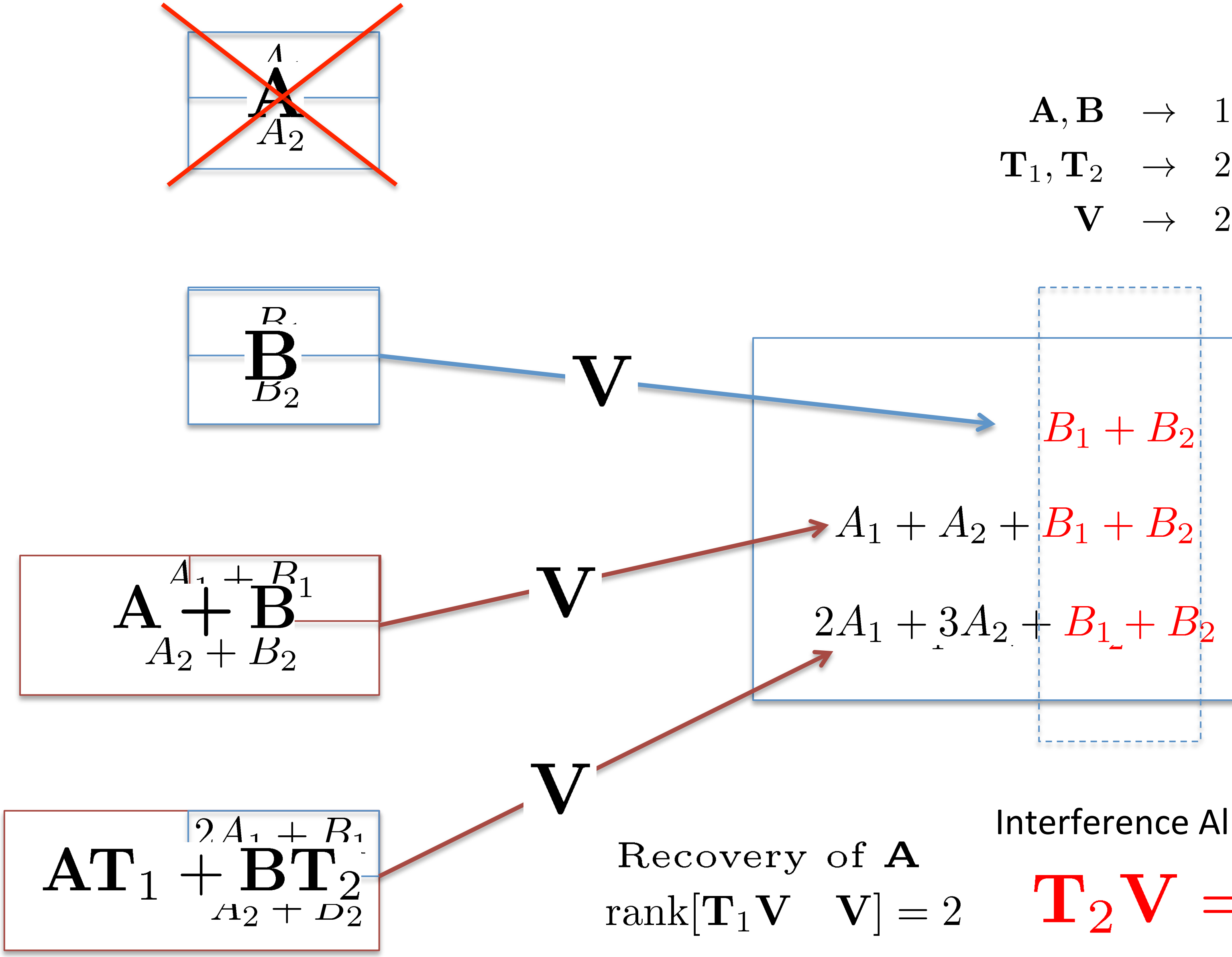
[Wu-Dimakis 09]



Interference Alignment!

$n = 4, k = 2$, Repair with 3 Linear Combinations

$$\mathbf{A}, \mathbf{B} \rightarrow 1 \times 2$$
$$\mathbf{T}_1, \mathbf{T}_2 \rightarrow 2 \times 2$$
$$\mathbf{V} \rightarrow 2 \times 1$$



Recovery of $\mathbf{A}$

$$\text{rank}[\mathbf{T}_1\mathbf{V} \quad \mathbf{V}] = 2$$

Interference Alignment!

$$\mathbf{T}_2\mathbf{V} = \lambda_1 \mathbf{V}$$

Repair for n=4, k=2

$$\mathbf{T}_1, \mathbf{T}_2 \quad \rightarrow \quad 2 \times 2$$
$$\mathbf{V}, \mathbf{W} \quad \rightarrow \quad 2 \times 1$$

Repair of Node 1

$$\mathbf{T}_2\mathbf{V} = \lambda_1 \mathbf{V}$$
$$\text{rank}[\mathbf{T}_1\mathbf{V} \quad \mathbf{V}] = 2$$

Repair of Node 2

$$\mathbf{T}_1\mathbf{W} = \lambda_2 \mathbf{W}$$
$$\text{rank}[\mathbf{T}_2\mathbf{W} \quad \mathbf{W}] = 2$$

$\mathbf{V}$ eigen-vector of $\mathbf{T}_2$
$\mathbf{W}$ eigen-vector of $\mathbf{T}_1$

Repair Vectors $\mathbf{V}_1, \mathbf{V}_2, \rightarrow$ Beamforming Vectors in Wireless Comm.
Coding matrices $\mathbf{T}_1, \mathbf{T}_2, \rightarrow$ Channel Matrices in Wireless Comm.
Artificial Channels applicable!

[Wu-Dimakis 08, Shah et.al. 08, Suh-Ramchandran 09]

Code=Channel Matrices, Repair vectors = Beamforming vectors

$$\text{span}(T_i \mathbf{V}) = \text{span}(\mathbf{V}), i = 1, 2, \ldots, N$$

Code (Channel Matrices) can be designed!

1. Asymptotic Alignment Codes

[Cadambe-Jafar-Maleki, 2010]

- Diagonal Channel (Code) Matrices

[Suh-Ramchandran, 2010]

- Approximate Alignment

2. Exact Finite Alignment-Based Codes

[Cadambe-Huang-Li ISIT 2011, Tamo-Wang-Bruck ISIT 2011]

- Non-trivial extensions of "Toy Alignment" Examples
- Tensor Product based Subspace Alignment

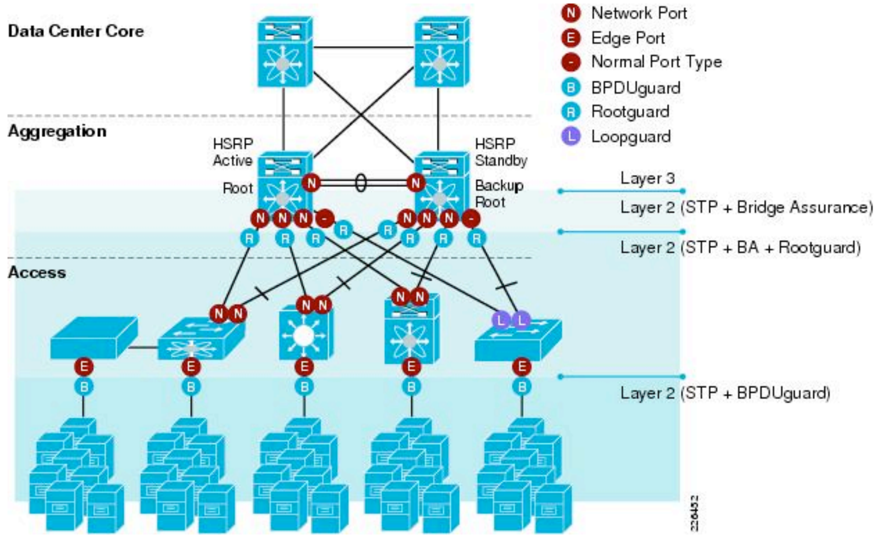
[Cadambe-Huang-Li-Jafar Arxiv 2011]

Future Work

Modeling Hierarchical Architecture of Data Centers for Repair

- Racks
- Machines
- Disks

all connected via switches/routers.

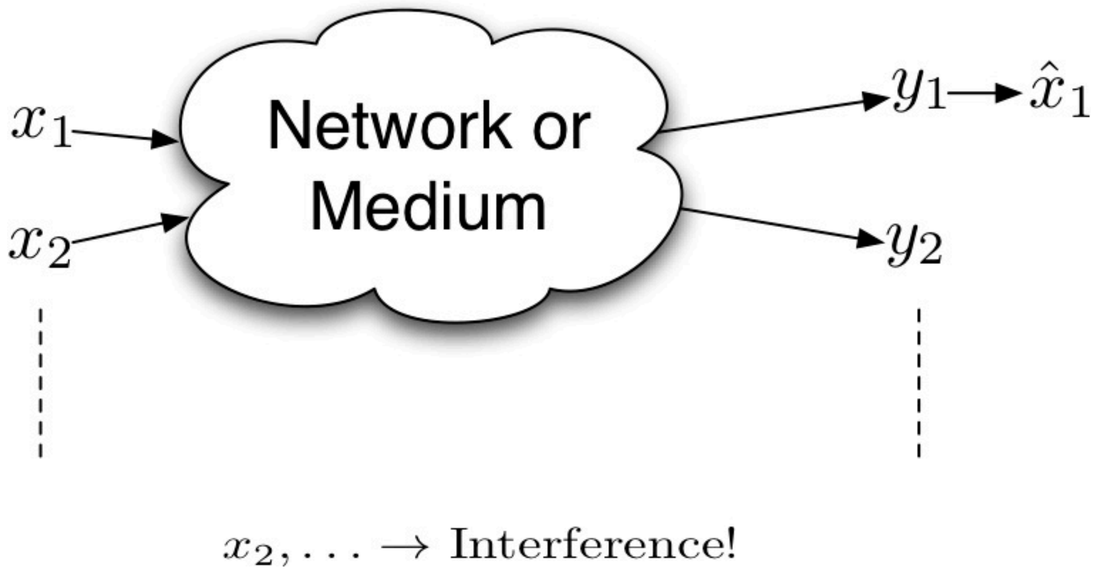


Multi-Source Network Coding

Routing $\leftrightarrow$ Orthogonalization

Structured Network Coding $\leftrightarrow$ Interference Alignment

Future Work



Broader view of alignment as an ingredient of Distributed Systems.

Thank you!

3 users 2 antennas