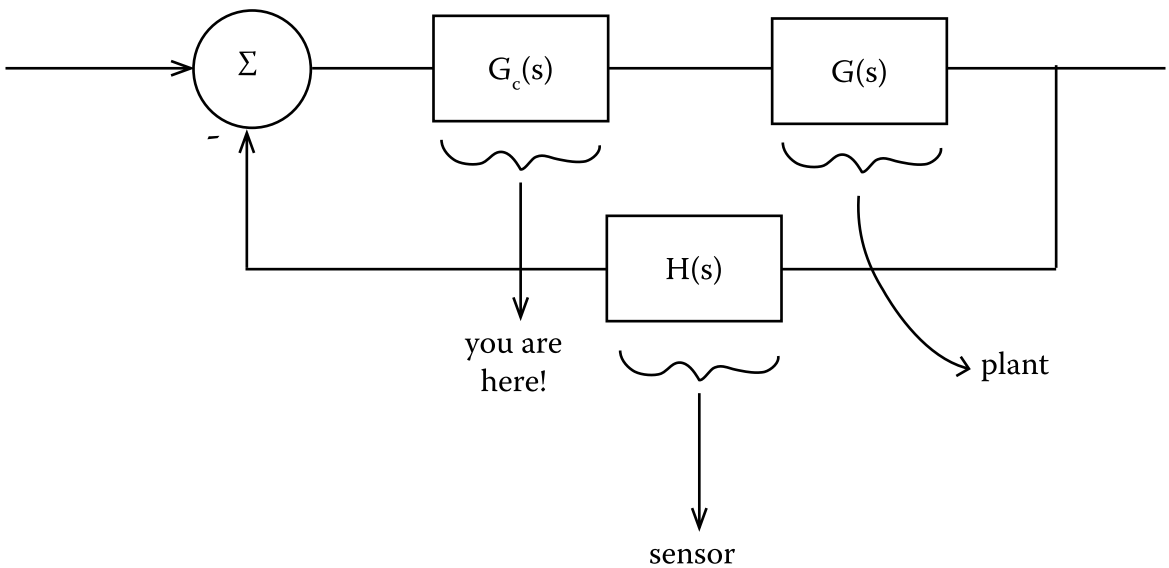
6.302 Feedback Systems

Recitation 16: Compensation
Prof. Joel L. Dawson

Bode Obstacle Course is one technique for doing "compensation," or designing a feedback system to make the closed-loop behavior what we want it to be. To review:



For this recitation, we're going to look at a few common choices for compensation strategies. We'll begin by looking at gain setting, the easiest, most straigtforward, technique.

$\boxed{\text{CLASS EXERCISE}}$

In the above diagram, let's assume that H(s) = 1, and that $G_c(s)$ is just k. That's right: you've decided to use gain setting as a compensation strategy. Let the plant be

$$G(s) = \frac{10^6}{(10^{-2}s + 1)\,(10^{-4}s + 1)}$$

1) For k=1, find ω_c, ϕ_m, and DC steady-state error.

2) Find k such that $\phi_m = 45^\circ$. Then, find ω_c and DC steady-state error.

How did we do? Well, we certainly got our phase margin. But look at what we lost: we're about a 100x slower now, and about 10,000x worse on the DC steady-state error spec. We've given up just about everything!

(Silver lining: noise rejection at high frequencies is improved.)

Look at this example, and try to understand the fundamental issue: with gain-setting, all performance specs are coupled to each other. You cannot, for example, set your DC gain and closed-loop bandwidth seperately.

This is a hint that frequency-dependent compensation might be a good thing...

Dominant-Pole Compensation

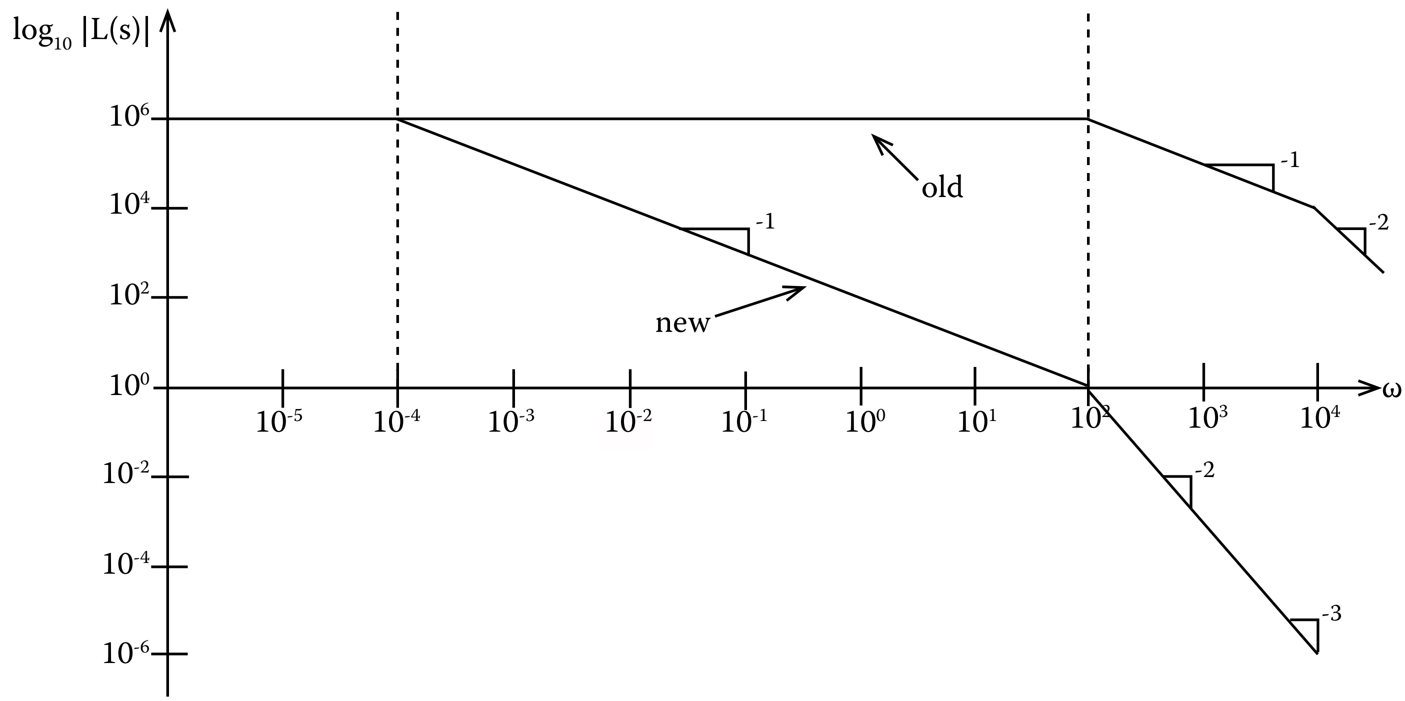
Conceptually, what we're going to do is "cross over before all those pesky poles make a difference." Consider adding a really slow pole at 10^{-4} rps. Now,

$$L(s) = \frac{1}{(10^4 s + 1)} \quad \frac{10^6}{(10^{-2}s + 1)(10^{-4}s + 1)}$$

DC steady-state error: $\lim_{s \to 0} \cancel{s} \left(\frac{1}{\cancel{s}}\right) \frac{1}{1 + \frac{10^6}{p(s)}} \approx 10^{-6}$

So we haven't given up anything with regards to steady-state error.

How's the frequency response looking?



Phase margin $\approx 45^\circ$, $\omega_c \approx 10^2$ rps. Okay, so we lost bandwidth. But, we did not lose steady-state error performance. More importantly, within a certain range we can use k and τ (time constant of our new pole) to set ω_c and steady-state error independently. Nice!

Dominant-pole compensation is often used when the plant is tough to stabilize by other means, or when the plant is hard to characterize.

Lag Compensation

We were able to get 45° of phase margin with dominant-pole. Let's see how well we can do with a Lag Compensator:

$$G_c(s) = \frac{\tau s + 1}{\alpha \tau s + 1}$$

Just to keep things fair, we'll continue to shoot for 45° of phase margin.

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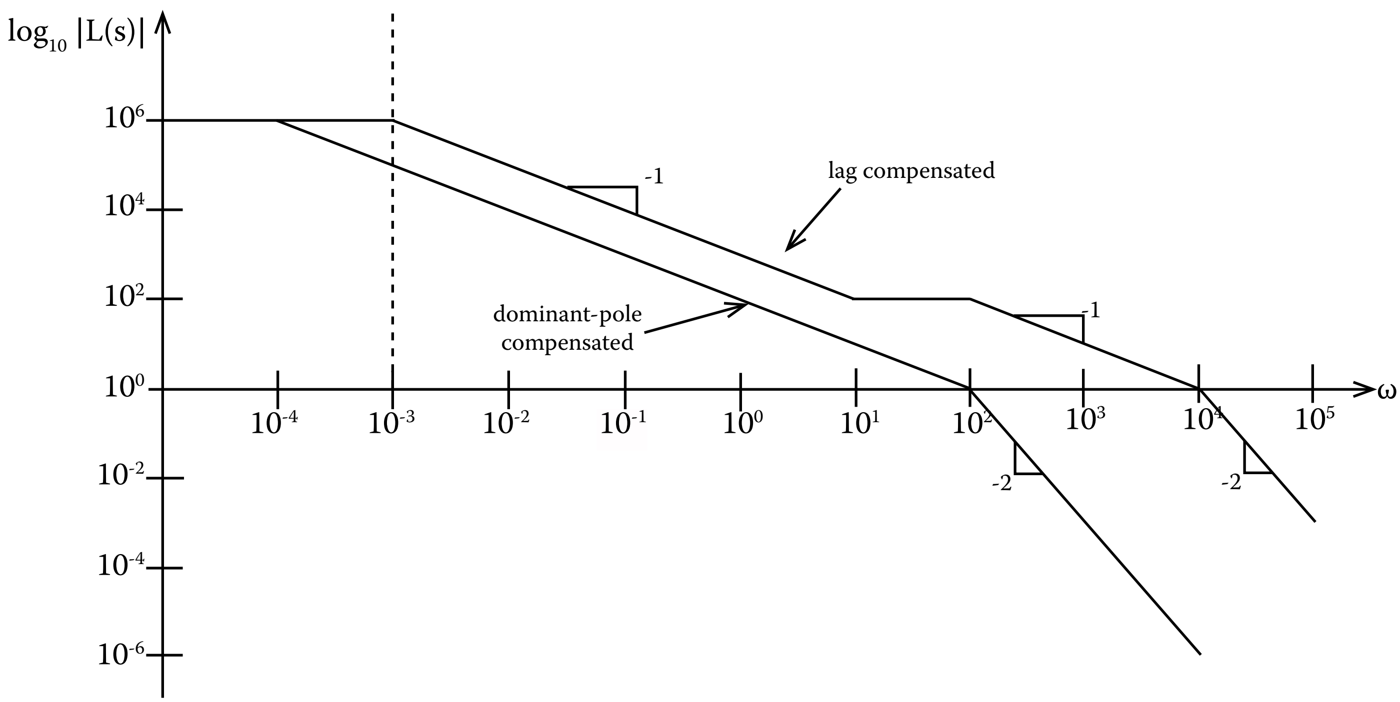
Because the lag compensator has the pole at a lower frequency than the zero, we're going to get a phase dip that we don't want to happen anywhere near crossover. I'm going to be conservative and place the zero of the lag a decade below the dynamics of the plant:

$$G_c(s) = \frac{(10^{-1}s + 1)}{(10^{3}s + 1)} \qquad \alpha = 10^4$$

$$L(s) = \left(\frac{(10^{-1}s + 1)}{(10^{3}s + 1)}\right)\left(\frac{10^6}{(10^{-2}s + 1)\,(10^{-4}s + 1)}\right)$$

Bode Plot



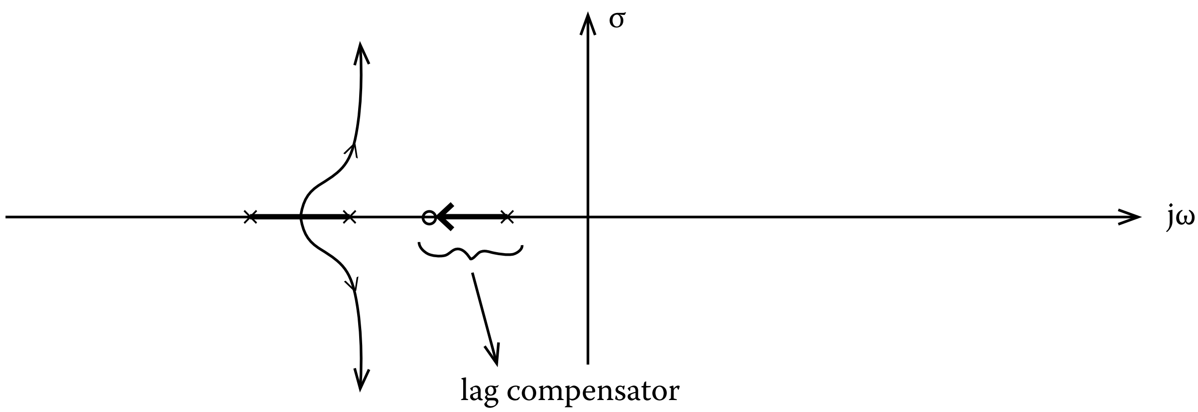
Phase margin ≈ 45°, $\omega_c \approx 10^4$ rps, <u>and</u> we kept our DC gain where it was. Notice, too, that our loop transmission is higher than the dominant pole case for all frequencies above 10^{-4} rps. → Better dynamic tracking.

Compared to the <u>uncompensated</u> case, we decreased L(s) at high frequencies, while leaving the low frequency parts as unbothered as we could.

Lag Compensation and Pole-zero Doublets

Lag compensation is often said to lead to “pole-zero doublets”... and to “long this in in the step response.” Let’s investigate...

In the example we’ve been using, the root locus looks as follows:



Notice that a lag compensator in the forward path results in a closed-loop pole and a closed-loop zero that lie very close to one another. Pole-zero pairs like this are called doublets.

Look at the step response of a doublet:

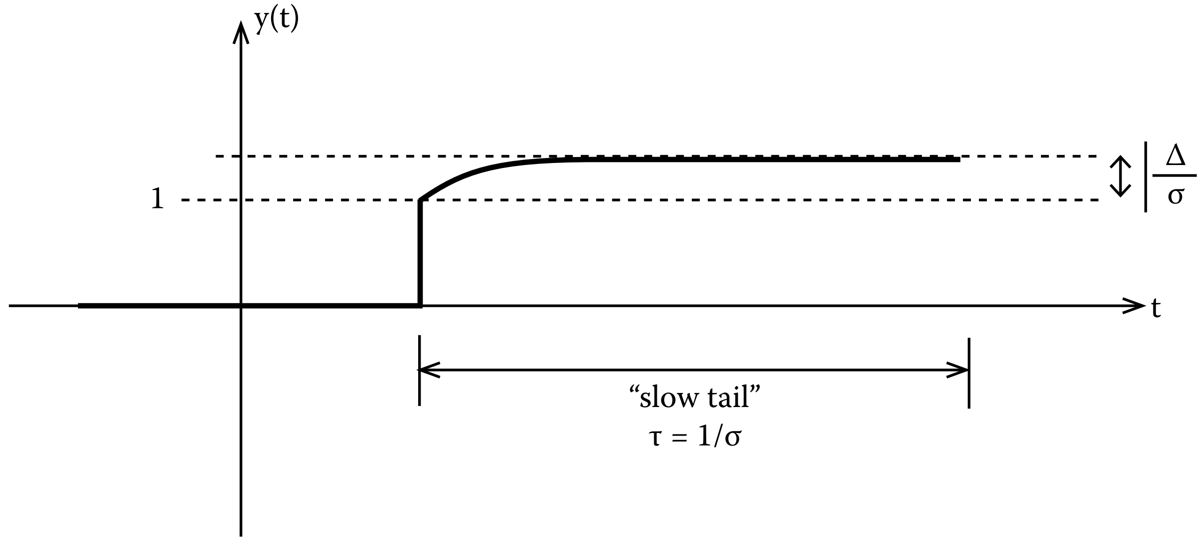
$$\frac{1}{s}\;\frac{(s+\sigma+\Delta)}{s+\sigma} = \frac{A}{s} + \frac{B}{s+\sigma}$$

$$A = \frac{\sigma+\Delta}{\sigma}$$

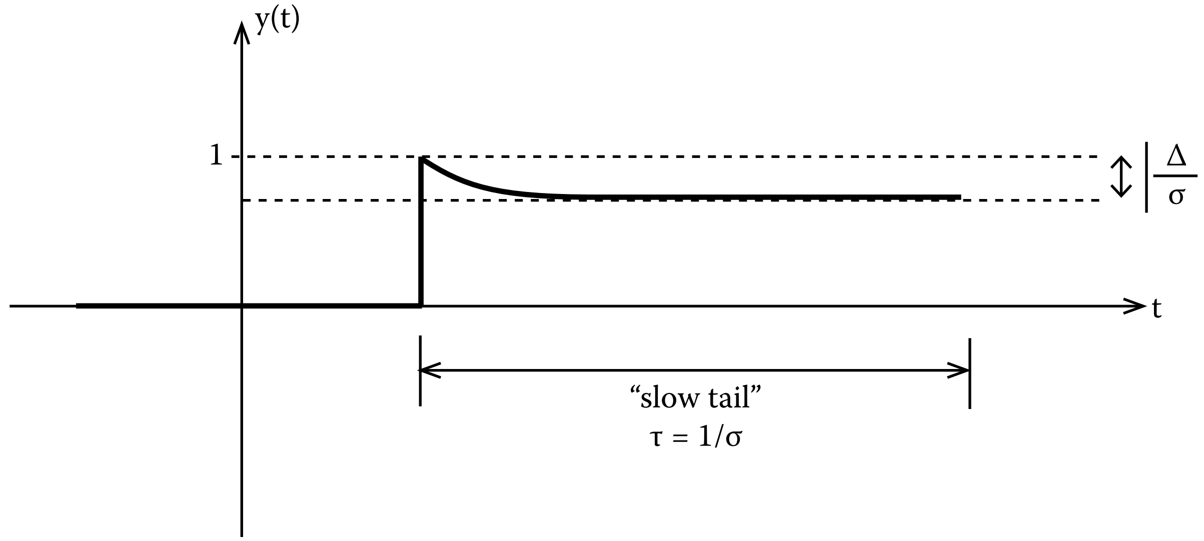
$$B = \frac{1}{-\sigma}\left(\frac{-\sigma+\sigma+\Delta}{1}\right) = -\frac{\Delta}{\sigma}$$

$$y(t) = \frac{\sigma+\Delta}{\sigma}\,\mu(t) - \frac{\Delta}{\sigma}\,e^{-\sigma t}\,\mu(t)$$

For $\Delta > 0$, the low-frequency gain is higher than the high-frequency gain. We wind up with a step response that looks like



For $\Delta < 0$, the high-frequency gain is higher than the low-frequency gain. For a step response, we get:



If settling time is important to us, we may not be able to use a compensator. At the very least, we must put the zero at as high a frequency as possible.