18.338 Eigenvalues of Random Matrices

Problem Set 2

Due Date: Wed Feb. 25, 2015

Reading and Notes

Read chapter *9* and chapter *10* of the class notes. Please again give your feedback especially high level style and where things did not make sense, in addition to spelling or technical errors

Homework

Do at least four out of the following problems (Computational/Mathematical problems are denoted as C/M. Exercise with numbers and pages are from the class notes.

1. (C) Exercise 5.1 (p82)
2. (C) Exercise 5.3 (p82)
3. (C) Exercise 5.4 (P83)
4. (M) Exercise 9.1 (p153)
5. (M) Exercise 9.2 (p153)
6. (M) Exercise 10.4 (p189)
7. (M) Exercise 10.5 (p189)

Cauchy-Binet: Based on Corollary 12.1. (p211), prove the following.

8. (M) Let $\mathrm{d}_i = \begin{cases} 1+z & \text{if } i=5 \\ 1 & \text{if } i \neq 5 \end{cases}$ Also let $A^TA = I_p$. Prove

$$\begin{aligned} \det(A^T \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1+z & \\ & & & \ddots \\ & & & & 1 \end{pmatrix} A) &= 1 + z \sum_{\text{Some } i=5} A \begin{pmatrix} i_1 & \dots & i_p \\ i & \dots & p \end{pmatrix}^2 \\ &= 1 + z\|A(S,:)\|^2 \end{aligned}$$

9. (M) Let $\mathrm{d}_i = Z_i$ symbolic.

$$\det(A^TDA) = \sum A \begin{pmatrix} i_1 & \dots & i_p \\ 1 & \dots & p \end{pmatrix}^2 Z_{i_1} \dots Z_{i_p}$$

Displays squares of elements of $\mathrm{Pl}(A)$ with symbolic labels.

10. (M) Let $\mathrm{d}_i = 1 + Z_i$. Prove

$$\begin{aligned} \det(A^TDA) = & \sum_{i_1 < \cdots < i_p} A \begin{pmatrix} i_1 & \dots & i_p \\ 1 & \dots & p \end{pmatrix}^2 + \sum_{\substack{k=1,\cdots,p \\ i_k=i, i_l=j}} Z_i A \begin{pmatrix} i_1 & \dots & i_p \\ 1 & \dots & p \end{pmatrix}^2 \\ & + \sum_{\substack{i_1<\cdots<i_p \\ 1\leqslant k<l\leqslant p \\ i_k=j, i_l=j, i_1<\cdots<i_p}} Z_i Z_j A \begin{pmatrix} i_1 & \dots & i_p \\ 1 & \dots & p \end{pmatrix}^2 + \cdots \end{aligned}$$