Estimate the number of states in a scattering process

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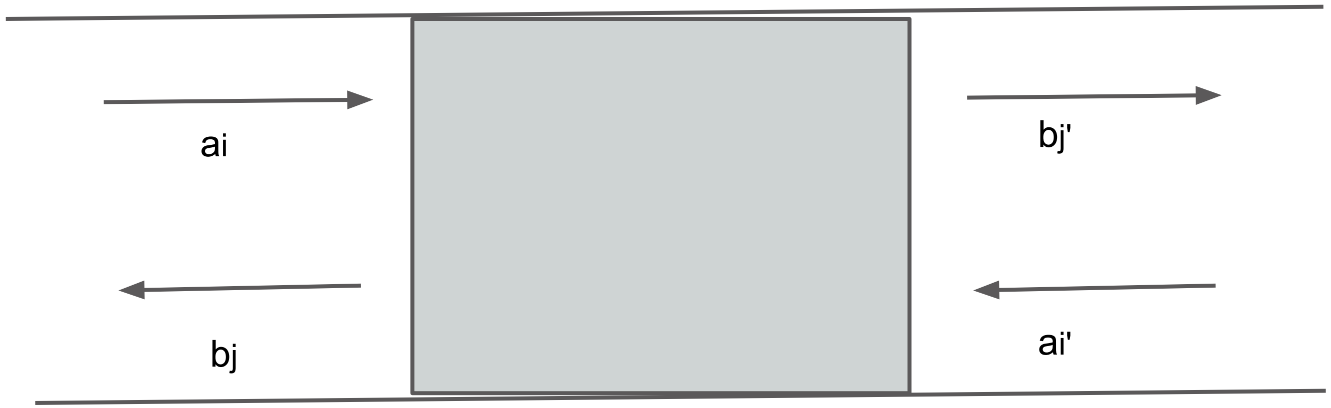
5/16/2012

Scattering Matrix

$\Psi_{in} = \Sigma[a_i * \psi_i]$

$\Psi_{out} = \Sigma[b_j * \psi_j]$

$\Psi_{out} = S * \Psi_{in}$



Random scattering, how could we get the number of states?

How many real eigenvalues are in a n by n matric?

Real eigenvalue indicates one existing state at equilibrium.

So now, we can convert the previous questions into:

How many real eigenvalues are there?

Math part

-$En=sqrt(Pi)*\Gamma((n+1)/2)/\Gamma(n/2)$

-Its asympototic series:

$En=sqrt(Pi*n/2)*(1-1/4n+1/32n\hat{}2...)$

-From here, we know that, when n is large, the expectation value of real eigenvalues decays at a similar rate as sqrt(n).

-Also, for even n: $En=sqrt(2)\ \Sigma((4k-1)!!/(4k)!!)$, $k=[0,n/2-1]$

for odd n: $En=1+sqrt(2)\Sigma((4k-3)!!/(4k-2)!!)$

$k=[1,(n-1)/2]$

Edelman A, Kostlan E, July 3,1993

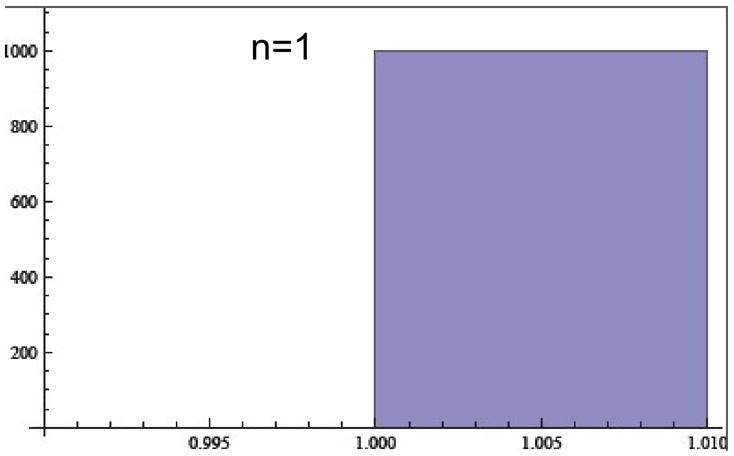
Distribution of the number of real Eigenvalues, k

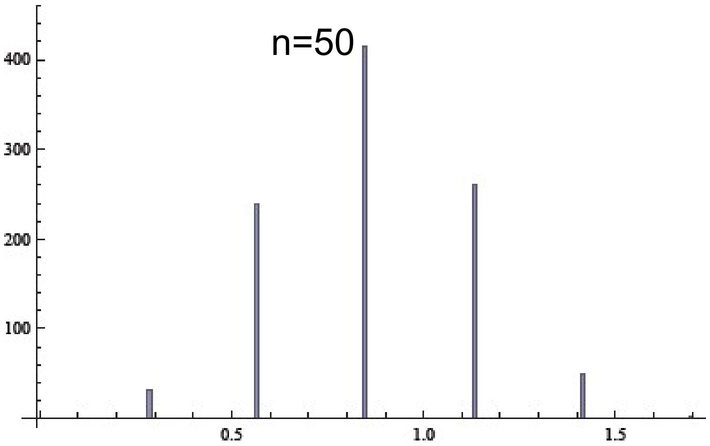
-Monte Carlo

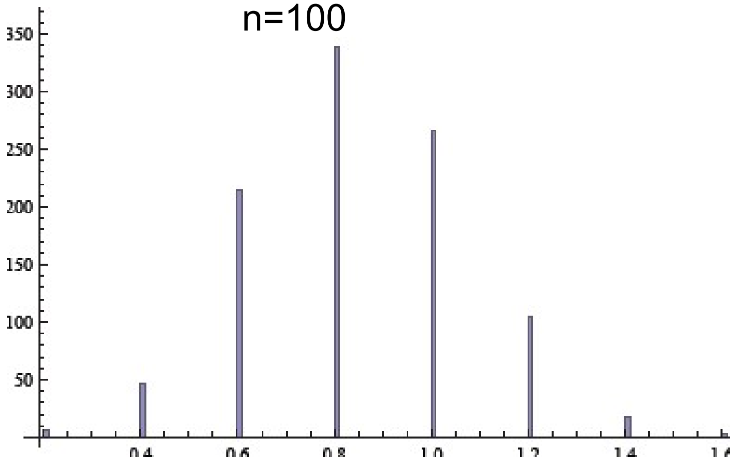
-1000 trials

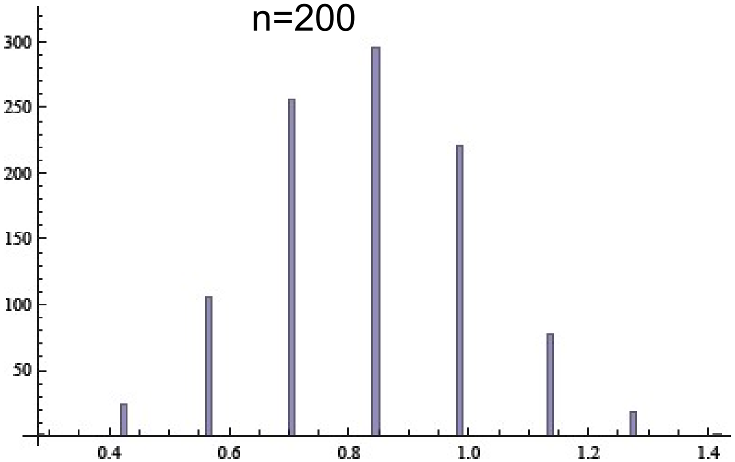
-k: array of the number of real eigenvalues for 1000 n by n matrices.

k/sqrt[n], 1000 trials

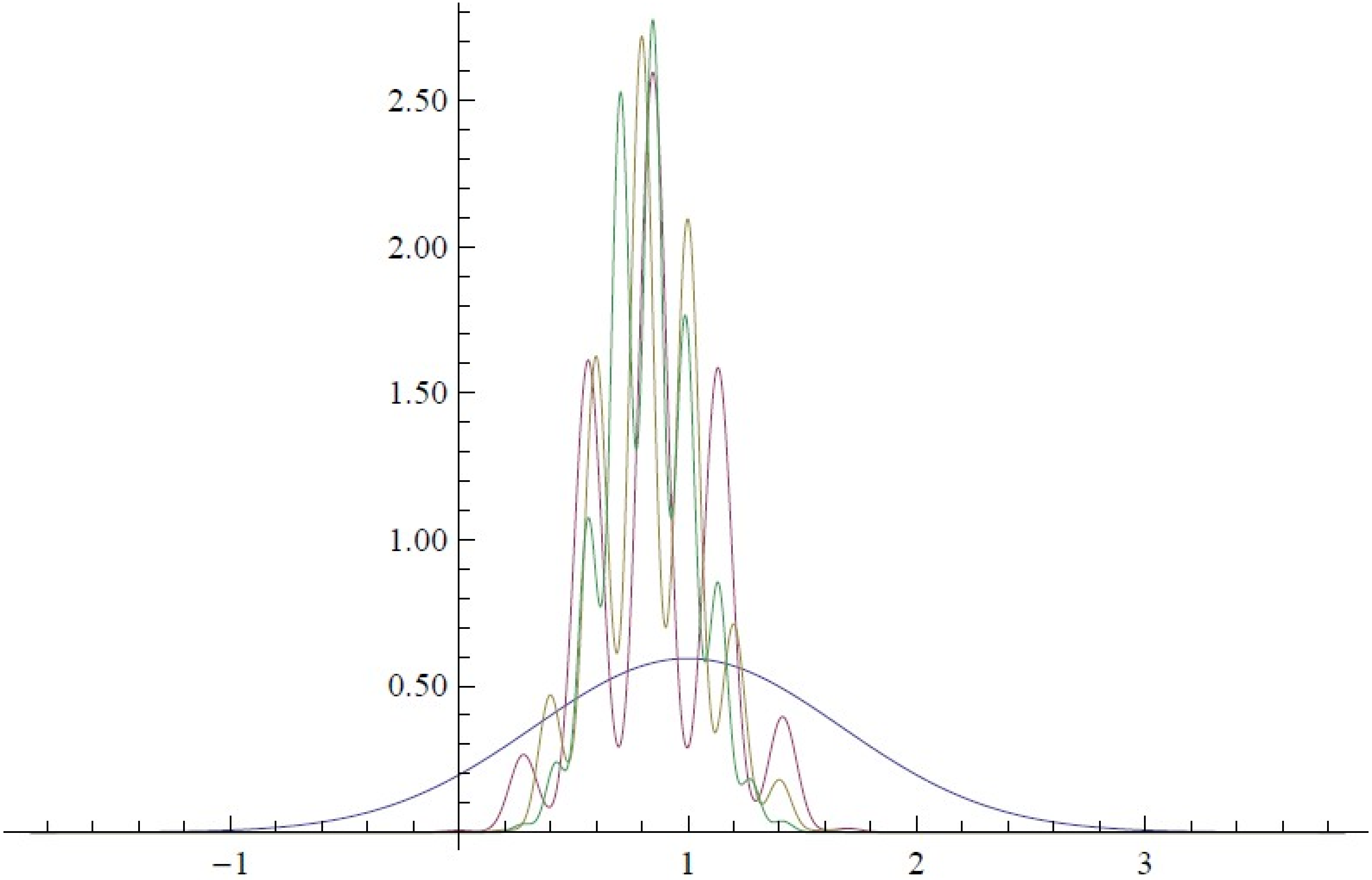






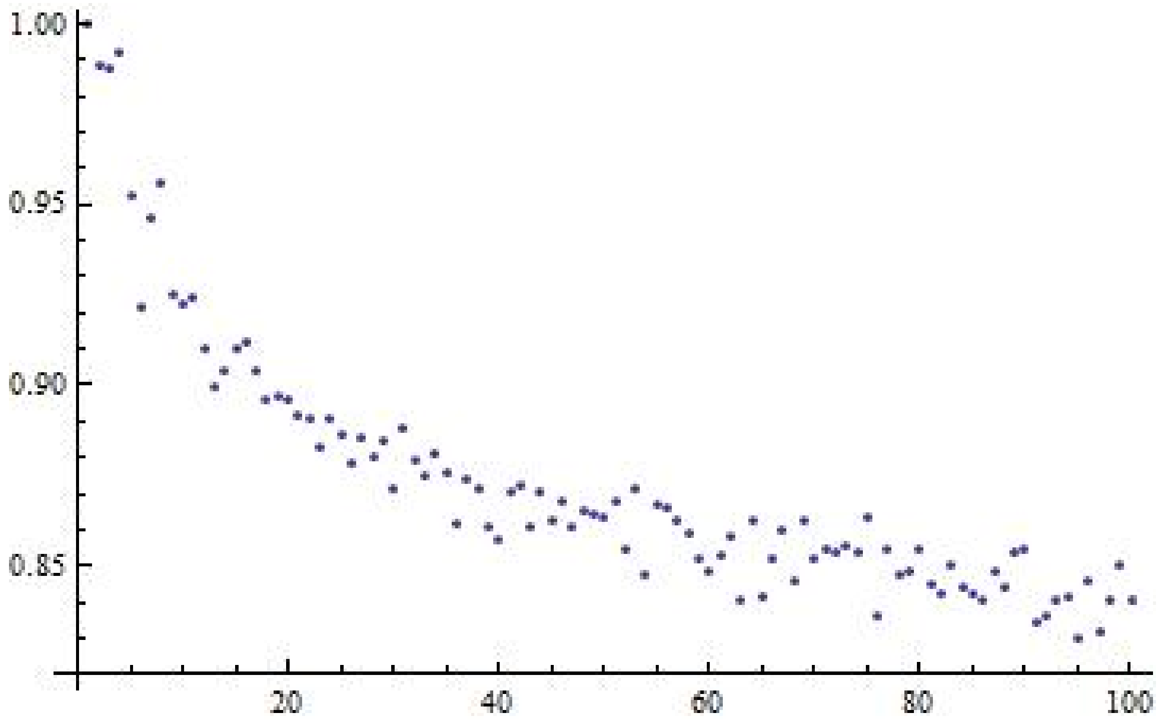


Smoothout

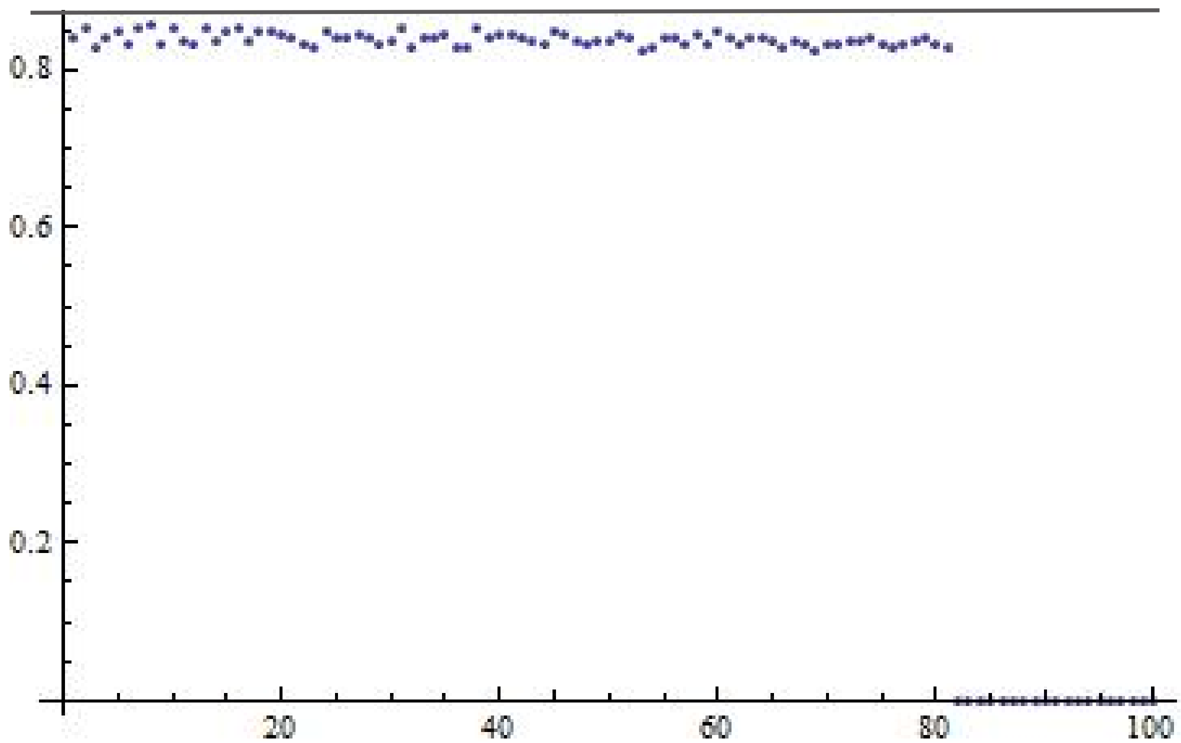


Mean,1000 trials

Mean[k/sqrt[n]]=Mean[k]/sqrt[n], n = [1,100]



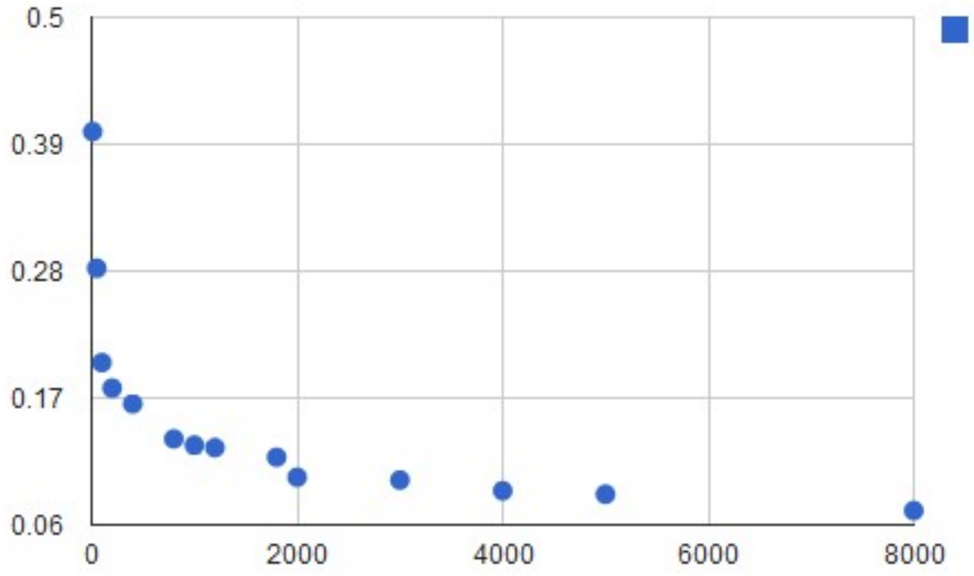
n=[100,200], 1000 trials



About 80 points are plot.

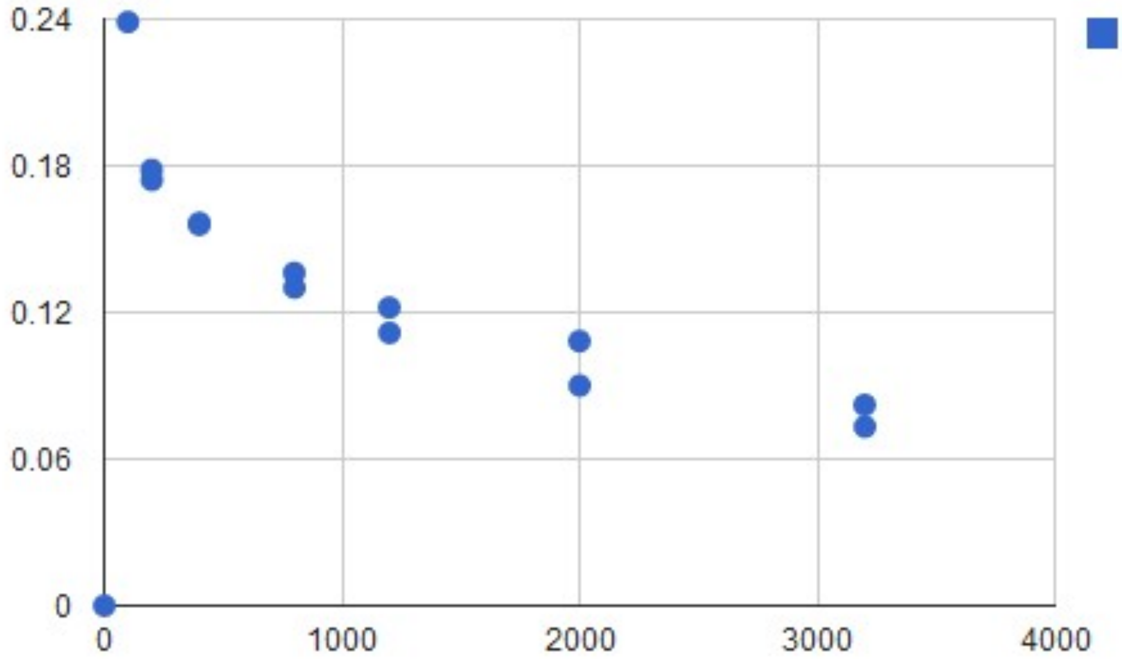
Standard Deviation

STD[k/sqrt[n]] approaches to 0 as n goes to infinity.



100 trials with n by n matrix

500 trials



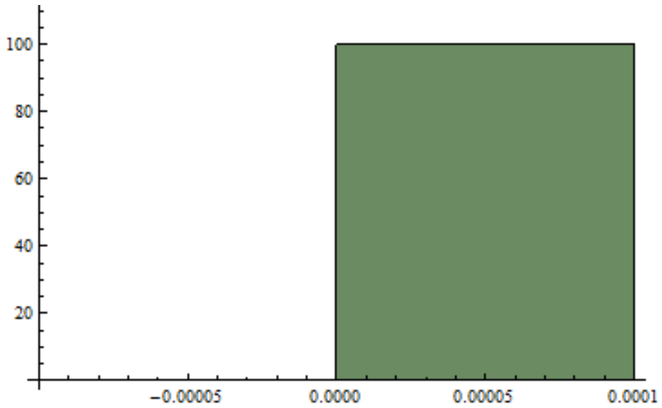
2 lines:
upper one for even number;
lower one for odd number

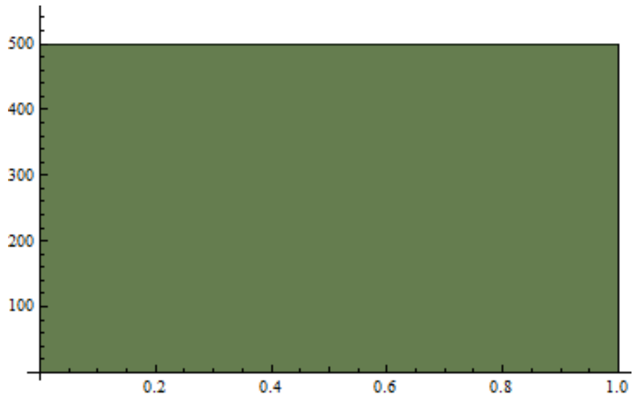
Unitary Matrix

Is there any characteristics for random unitary Matrix?

U is designed as:
U= MatrixExp[i*(R+Transpose(R))],
where R is a random matrix.

Eigenvalues





Full zeros.

Orthogonal Matrix

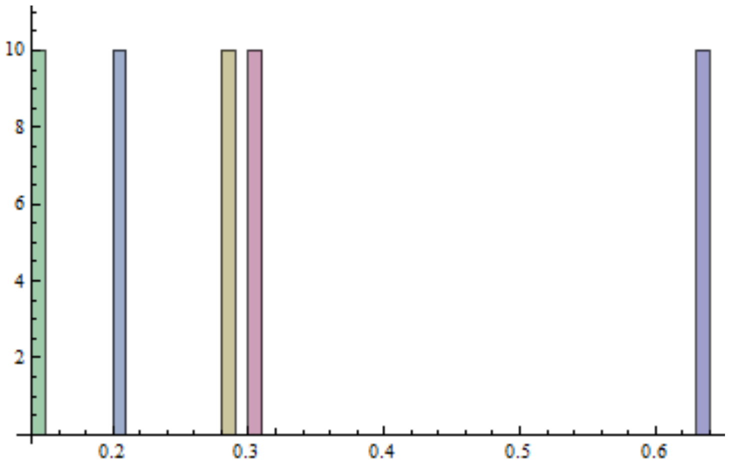
For a unitary matrix with all real elements, orthogonal Matrix is designed as:

O=MatQ= QRDecompositon[M].

Number of real eigenvalues is 2, which are 1 and -1. [n is even.]

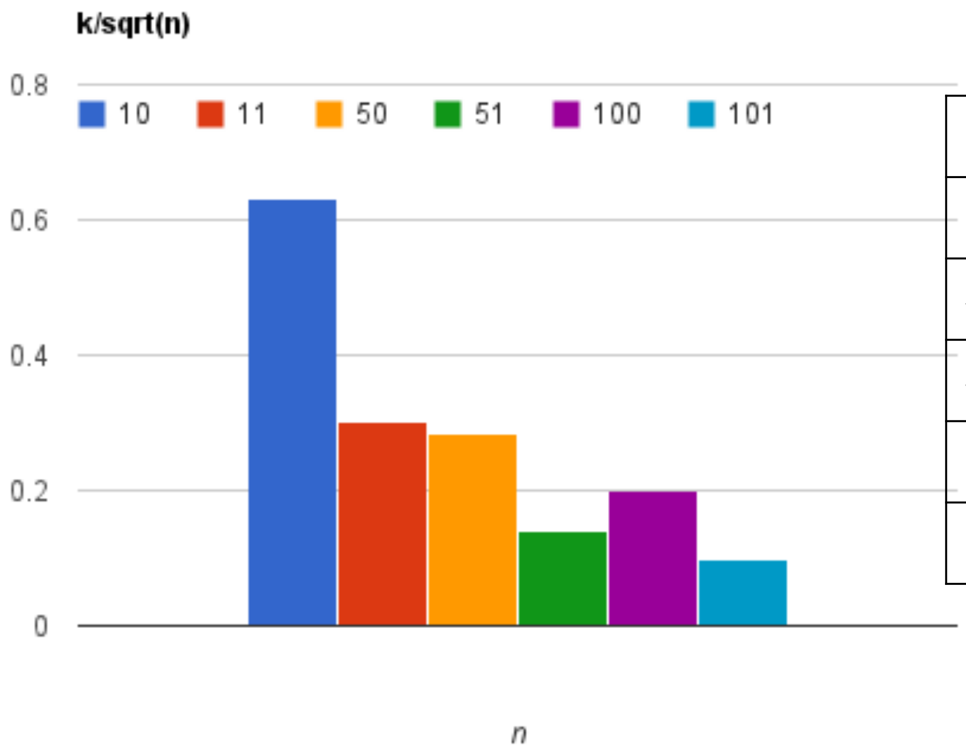
Number of real eigenvalues is 1, which is either 1 or -1. [n is odd.]

10 trials



10	.632
11	.3015
50	.2828
51	.14
100	.2
101	.0995

1000 trials



10	.632
11	.3015
50	.2828
51	.14
100	.2
101	.0995

And STD are all zeros.

RMT for scattering matrix

In the circular ensemble,

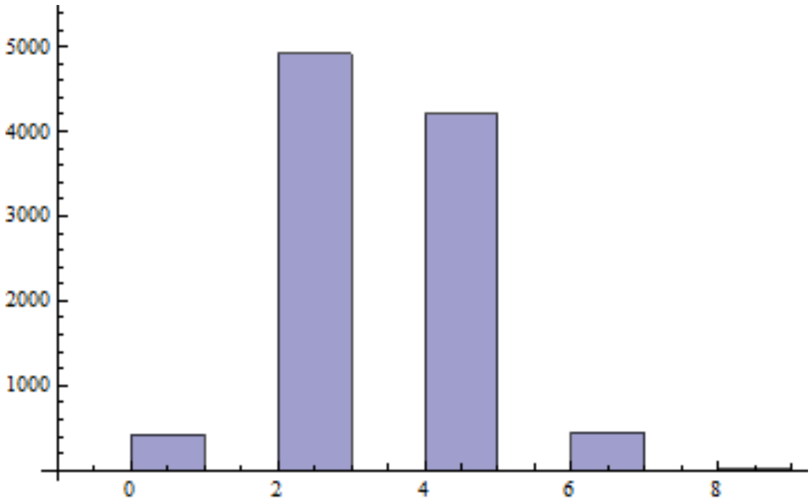
for $\beta=1$, S is COE
for $\beta=2$, S is CUE
for $\beta=4$, we do not care that much.

For other cases, where S is completely a random matrix, we could apply the previous results that as n becomes large enough, En is about 0.8 with a standard deviation of 0. Hence, we can estimate the possible number states available before and after the scattering.

Extra slide 1

Extended table:

n=10

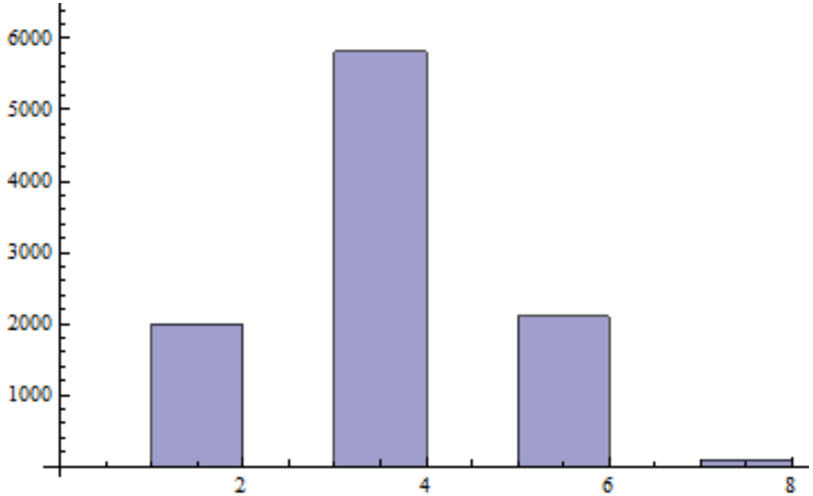


k	$P_{10}(k)$	
10	1/(4193304*sqrt(2))	1.68*10^-7
8	(236539-320sqrt(2))/536870912sqrt(2)	3.1*10^-4
6	/	0.0444***
4	/	0.421***
2	(1216831949-594932556sqrt(2))/536870912*sqrt(2)	0.49
0	-1146637039+834100651sqrt(2)/526870912sqrt(2)	0.043

Extra slide 2

Extended table:

n=11



k	$P_{11}(k)$	
11	1/(134217728*sqrt(2))	5.27*10^-9
9	(-320+333123sqrt(2)) /8589934592sqrt(2)	3.87*10^-5
7	/	8.9*10^-3***
5	/	0.2102***
3	/	0.5818***
1	-12606311702+106298452511sqrt (2)/8589934592sqrt(2)	0.1997

Reference

Edelman A, Kostlan E,*How many Eigenvalues of a Random Matrix are Real*, July 3,1993

C. W. J. Beenakker, *Random-matrix theory of thermal conduction in superconducting quantum dotsRandom-matrix theory of thermal conduction in superconducting quantum dots.* Apr 2010

Michael V. Moskalets,*Scattering matrix approach to non-stationary quantum transport*